# Sub^n-leading modes for YM and gravity

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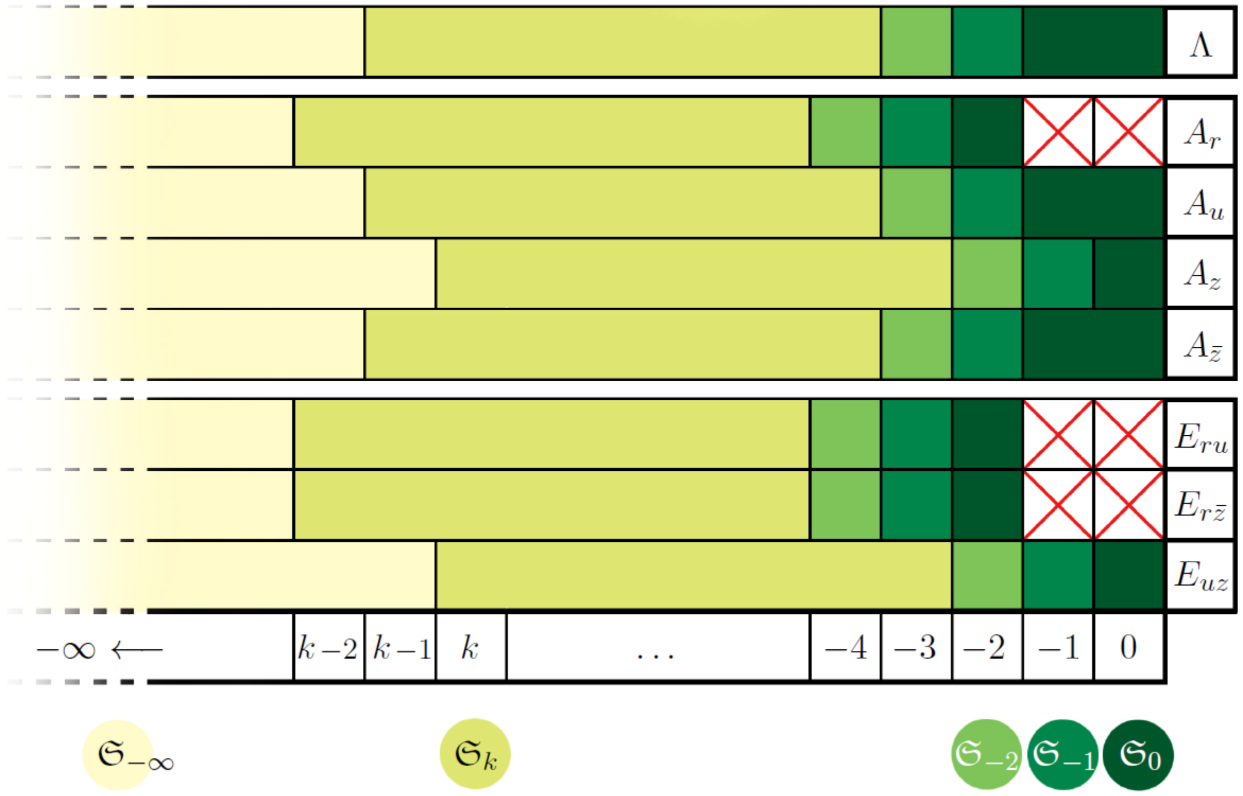
and work in progress

# Quick ad: Homotopy kinematic algebras at null infinity with F. Diaz-Jarramillo and G. Pizzolo

- Different perspective on projecting information to the boundary
- Kinematic algebras on slices near null infinity
- 2503.21035

SDYM



# Structure

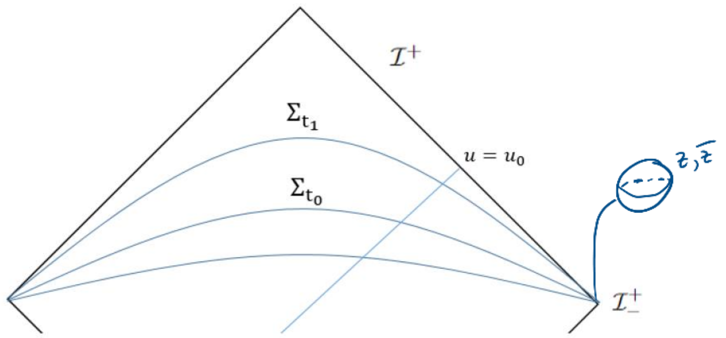
1. Sub^n-leading soft therems from symmetries
2. Stueckelberg trick
3. Boundary Lagrangian -> charges two ways
4. Charge algebra
5. Quasi-Universal terms (if time)

# Sub^n-leading soft theorems from symmetries

# Soft theorem (leading)

Bondi $(u, r, z, \bar{z})$

$$\lim_{\omega\to 0} A_{n+1} = \frac{1}{\omega} S^{(0)} A_n + \ldots$$




# Asymptotic symm (leading)

$$A_z = A_z^{(0)}(u, z, \bar{z}) + \frac{A^{(1)}(u, z, \bar{z})}{r} + \ldots$$

$$\Lambda = \Lambda(z, \bar{z})$$

$$\delta A_z = D_z \Lambda$$ preserves fall-off

## Soft theorem (leading)

$$\lim_{\omega\to 0} A_{n+1} = \frac{1}{\omega} S^{(0)} A_n + \ldots$$

## Ward identity

$$Q_\Lambda^{(0)} = \int_{S^2} \mathrm{tr}(\Lambda \mathcal{F}_{ru}^{(-2)}) \sqrt{g_{S^2}} dz d\bar{z}.$$

$\downarrow$

$r^2 \gamma_{z\bar{z}}$

## Asymptotic symm (leading)

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$$\delta A_z = D_z \Lambda$$

preserves fall-off

## Soft theorem (leading)

$$\lim_{\omega \to 0} A_{n+1} = \frac{1}{\omega} S^{(0)} A_n + \ldots$$

Ward identity

$$Q_f^+ = \frac{1}{4\pi G} \int_{\mathcal{I}_-^+} d^2 z \gamma_{z\bar{z}} f m_B,$$

## Asymptotic symm (leading)

$$g_{\mu\nu} = \ \ldots \ + r C_{zz} dz^2 + \ldots$$

$$\zeta = f\partial_u - \frac{1}{r}\left(D^z f \partial_z + D^{\bar{z}} f \partial_{\bar{z}}\right) + D^z D_z f \partial_r + \ldots, \qquad f = f(z, \bar{z})$$

$$\mathcal{L}_f C_{zz} = f \partial_u C_{zz} - 2 D_z^2 f.$$

preserves fall-off

# Subleading effects

$$\lim_{\omega \to 0} A_{n+1} = \underbrace{\left(\frac{1}{\omega} S^{(0)} + S^{(1)} + \ldots\right) A_n}_{\text{universal}} \quad + \quad \underbrace{\ldots}_{\text{non-universal}}$$

↑ from symmetries

# Subleading effects

$$\lim_{\omega\to 0} A_{n+1} = \underbrace{(\tfrac{1}{\omega} S^{(0)} + S^{(1)} + \ldots) A_n}_{\text{universal}} \quad + \quad \underbrace{\ldots}_{}$$

universal + non-universal

↑

project out

e.g. Hamada,Shiu,Li,Lin,Zhang, also in cosmology…

# Subleading effects

$$\lim_{\omega\to 0} A_{n+1} = \underbrace{\left(\frac{1}{\omega}S^{(0)} + S^{(1)} + \ldots\right)A_n}_{\text{universal}} \quad + \quad \underbrace{\ldots}_{\text{non-universal}}$$

↑

sometimes from symmetries

# Subleading QED

Low,Lysov,Pasterski,Strominger,Campiglia,Laddha, Peraza...

Soft theorem

$$\lim_{\omega\to 0} A_{n+1} = \underbrace{(\tfrac{1}{\omega}S^{(0)} + S^{(1)} + ...)A_n}_{\text{universal}} + \underbrace{\cdots}_{\text{non-universal}}$$

$$\frac{1}{\omega} S^0 = e\sum_k Q_k \frac{p_k\cdot\varepsilon^-}{p_k\cdot q} \sim \mathcal{O}(\omega^{-1})$$

$$S^{(1)} = -ie\sum_k Q_k \frac{q_\mu \varepsilon^-_\nu J^{\mu\nu}_k}{p_k\cdot q} \sim \mathcal{O}(\omega^0)$$

Ward identity

Asymptotic symmetry

$$\lambda(u,r,\hat{x}) = r\overset{(1)}{\lambda}(u,\hat{x}) + \overset{(0)}{\lambda}(u,\hat{x}) + O(r^{-\epsilon})$$

$$\overset{(1)}{\lambda}(u,\hat{x}) = \mu(\hat{x}), \qquad \overset{(0)}{\lambda}(u,\hat{x}) = u(1+\Delta/2)\mu(\hat{x}).$$

violates fall-off

# Generalise the problem

- Gauge field finite fall-off

$$\mathcal{A}_\mu = \sum_{n,k} A_\mu^{(-n;k)}(\vec{\mathbf{y}}) \frac{\log^k \mathfrak{r}}{\mathfrak{r}^n} \ ,$$

with $n, k$ such that $\lim_{r\to\infty} \mathcal{A}_\mu$ at most $\mathcal{O}(1)$.

- Divergent gauge parameter

$$\Lambda_+(x) = \sum_{n,k} \mathfrak{r}^n \log^k \mathfrak{r} \ \Lambda^{(n;k)}(\vec{\mathbf{y}}) \ ,$$

$\delta A_\mu$ violates fall-off of $A_\mu$

- May also be field dependent $\breve{\Lambda}_+ = \breve{\Lambda}_+(\mathcal{A}_\mu(x), \Lambda_+(x))$

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→ similar for gravity

# Stueckelberg trick

# Problem

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} $\delta A_\mu$ violates fall-off of $A_\mu$

→ symmetry - breaking

# Stueckelberg trick

- Promote gauge parameter of symmetry we want to restore to a field

$$\Lambda_+(x) \quad \rightarrow \quad \Psi_+(x),$$

with

$$\Psi(x)_+ = \sum_{n,k} \mathfrak{r}^n \log^k \mathfrak{r} \ \Psi^{(n;k)}(\vec{\mathbf{y}}).$$

- the phase space is now

$$\Gamma^{\text{ext}}_\infty := \Gamma^0 \times \{\Psi_+(x)\}$$

- $\Psi$ is Goldstone-type field

# Dressed fields

- Dressed gauge field

$$\tilde{\mathcal{A}}_\mu = e^{i\Psi_+}\mathcal{A}_\mu e^{-i\Psi_+} + ie^{i\Psi_+}\partial_\mu e^{-i\Psi_+}$$

→ comes from bulk

- Consistency condition:

$$\tilde{\mathcal{A}}'_\mu = e^{i\Lambda}\tilde{\mathcal{A}}_\mu e^{-i\Lambda} + ie^{i\Lambda}\partial_\mu e^{-i\Lambda} \quad \Rightarrow \quad \delta_\Lambda \tilde{\mathcal{A}}_\mu = \tilde{D}_\mu \Lambda$$

where

$$\Lambda = \Lambda^{(0)} + \Lambda_+$$

$\delta A_\mu$ unchanged

$\Lambda_+$ only transforms $\Psi$

# Transformations (group level)

- Rewrite dressing:

$$\tilde{A}_\mu = e^{i\Psi_+} A_\mu e^{-i\Psi_+} + i e^{i\Psi_+} \partial_\mu e^{-i\Psi_+}$$
$$\equiv s^{-1} A_\mu s + s^{-1} \partial_\mu s$$

Then

consistency condition $\Rightarrow$

$$s' = \eta^{-1} s \gamma$$

where

$$\eta = e^{-i\Lambda_0} \quad \text{and} \quad \gamma = e^{-i\Lambda}$$

# Transformations

- The Stueckelberg (Goldstone) field transforms as

$$(\mathcal{O}_X)^{-1} = \left(\frac{1-e^{-ad_X}}{ad_X}\right)^{-1}$$

$$\delta_\Lambda \Psi_+ = \mathcal{O}^{-1}_{-i\Psi_+}(\Lambda - e^{i\Psi_+}\Lambda^{(0)}e^{-i\Psi_+})$$

- Perturbatively

$$\delta^{[m]}_\Lambda \Psi_+ = \frac{B^+_m}{m!}(ad_{-i\Psi_+})^m\left[\Lambda + (-1+2\delta_{m,1})\Lambda^{(0)}\right]$$

- at 0th order in the field, it transforms via a shift

$$\delta^{[0]}_\Lambda \Psi_+ = \Lambda - \Lambda^{(0)} = \Lambda_+$$

i.e. Goldstone modes for the symmetry breaking in the bulk.

# Full Stueckelberging

- Recall we had

$$\begin{aligned}\tilde{\mathcal{A}}_\mu &= e^{i\Psi_+}\mathcal{A}_\mu e^{-i\Psi_+} + ie^{i\Psi_+}\partial_\mu e^{-i\Psi_+}\\ \delta\mathcal{A}_\mu &= D_\mu\Lambda^{(0)}\\ \delta\Psi_+ &= \Lambda_+ + \dots\end{aligned}$$

  We could instead do

$$\begin{aligned}\tilde{\mathcal{A}}_\mu &= e^{i\Psi}\mathcal{A}_\mu e^{-i\Psi} + ie^{i\Psi}\partial_\mu e^{-i\Psi}\\ \delta\mathcal{A}_\mu &= 0\\ \delta\Psi &= \Lambda + \dots\end{aligned}$$

  with

$$\Psi = \Psi^{(0)} + \Psi_+, \qquad \Lambda = \Lambda^{(0)} + \Lambda_+$$

- Will be useful for computing charges!

# Gravity

- Recall Stueckelberg starting point is symmetry transformation.
- In GR
$$\delta_\xi g_{\mu\nu} = \mathcal{L}_\xi g_{\mu\nu}$$
- Linear in $\xi$, will only work up to subleading order
- GCT:
$$g'_{\mu\nu}(x') = \frac{\partial x^\rho}{\partial x^{\mu'}} \frac{\partial x^\sigma}{\partial x^{\nu'}} g_{\rho\sigma}(x)$$

  is not fit for the purpose.

# Gravity

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- GCT:
$$g'_{\mu\nu}(x') = \frac{\partial x^\rho}{\partial x^{\mu'}} \frac{\partial x^\sigma}{\partial x^{\nu'}} g_{\rho\sigma}(x)$$
is not fit for the purpose.
- Instead we work with
$$g'_{\mu\nu}(x) = e^{\mathcal{L}_\xi} g_{\mu\nu}(x)$$
corresponding to
$$x^{\mu'} = e^{-\xi^\rho \partial_\rho} x^\mu$$

# Gravity

- Define
$$\tilde{g}_{\mu\nu} = e^{\mathcal{L}_V} g_{\mu\nu}$$
with $V$ a vector Stueckelberg field. We want to set
$$\delta g_{\mu\nu} = 0$$
and absorb all leading and subleading transformations in the transformation of the Stueckelberg vector.
- Consistency condition
$$\tilde{g}'_{\mu\nu} = e^{\mathcal{L}_\xi} \tilde{g}_{\mu\nu} \quad \Rightarrow \quad V' = \log(e^\xi e^V), \qquad V \equiv V^\mu \partial_\mu$$
- At linear order in $\xi$
$$\delta V = (\mathcal{O}_X)^{-1} e^{-\mathrm{ad}_V} \xi, \qquad (\mathcal{O}_X)^{-1} = \left(\frac{1-e^{-ad_X}}{ad_X}\right)^{-1}$$
so
$$\delta V = \xi + \ldots$$
Goldstone type

# 3. Boundary Lagrangian and charges

# Constructing charges

- The *bulk* action,

$$\tilde{S}[\tilde{A}_\mu] = \int_{\mathcal{M}} \mathrm{tr}\left(\tilde{\mathcal{F}}_{\mu\nu}\tilde{\mathcal{F}}^{\mu\nu}\right) dvol,$$

where $\tilde{\mathcal{F}}^{\mu\nu}$ is constructed from $\tilde{A}_\mu$, and takes the form

$$\tilde{\mathcal{F}}_{\mu\nu} \equiv \partial_\mu \tilde{A}_\nu - \partial_\nu \tilde{A}_\mu - i[\tilde{A}_\mu, \tilde{A}_\nu] = e^{i\Psi}\mathcal{F}_{\mu\nu}e^{-i\Psi}$$

- same as without $\Psi$ !
- Think of $\tilde{A}_\nu$ as a single field.

# Charges from covariant phase space formalism

- Alternative to Noether's procedure (useful for GR, field dependent symmetries, dealing with fluxes, renormalising quantities etc)
- $$\delta_1 L = \delta_1 \tilde{\mathcal{A}} \cdot \mathsf{EOM} + d\theta[\delta_1]$$

  then

  $$\Omega[\delta_1, \delta_2] = \int_{\partial \mathcal{M}} \delta_1 \theta[\delta_2] - (1 \leftrightarrow 2)$$

  Finally, take

  $$\delta_1 = \delta_\Lambda \quad \Rightarrow \quad \Omega = \delta_2 \tilde{Q}_\Lambda$$

# Constructing charges

- Covariant phase space formalism gives:

$$\tilde{Q}_\Lambda = \int_\Sigma \partial_\nu \, \mathrm{tr}(\Lambda \tilde{\mathcal{F}}^{\mu\nu}) dS_\mu$$
$$= \int_{S^2} \mathrm{tr} \left[ \Lambda \tilde{\mathcal{F}}_{ru} \sqrt{g_{S^2}} \right]^{(0)} dz d\bar{z}.$$

renormalised!

# Fall-offs in u (tree-level)

$$A_z^{(0)}(u, z, \bar{z}) = A_z^{(0,0)}(z, \bar{z}) + o(u^{-\infty}),$$

translates to

| | | | | |
|---|---|---|---|---|
| $o(u^{-\infty})$ | $F_{uz}^{(0)}$ | | | |
| $O(1)$ | $F_{uz}^{(-1)}$ | $\mathbf{F}_{\mathbf{ru}}^{(-2)}$ | $F_{z\bar{z}}^{(0)}$ | |
| $O(u)$ | $F_{uz}^{(-2)}$ | $\mathbf{F}_{\mathbf{ru}}^{(-3)}$ | $F_{z\bar{z}}^{(-1)}$ | $F_{rz}^{(-2)}$ |
| $O(u^2)$ | $F_{uz}^{(-3)}$ | $\mathbf{F}_{\mathbf{ru}}^{(-4)}$ | $F_{z\bar{z}}^{(-2)}$ | $F_{rz}^{(-3)}$ |
| $\cdots$ | | | | |
| $O(u^n)$ | $F_{uz}^{(-n-1)}$ | $\mathbf{F}_{\mathbf{ru}}^{(-\mathbf{n}-2)}$ | $F_{z\bar{z}}^{(-n)}$ | $F_{rz}^{(-n-1)}$ |

fall-off in r

# Lagrangian for Stueckelberg fields

- Recall

$$S = -\frac{1}{2}\int_{\mathcal{M}} *F \wedge F \quad \xrightarrow{Stueck} \quad \tilde{S} = -\frac{1}{2}\int_{\mathcal{M}} *\tilde{F} \wedge \tilde{F} = S$$

- Not very satisfying
- We are missing boundary terms !

# Boundary Lagrangian

- Now try

$$S = -\frac{1}{2}\int_M *F \wedge F + \int_{\partial M} j \wedge A$$

where

$$\delta S \quad \Rightarrow \quad j = *F|_{\partial M}$$

- Then

$$S \xrightarrow{Stueck} \tilde{S} = -\frac{1}{2}\int_M *\tilde{F} \wedge \tilde{F} + \int_{\partial M} \left[\tilde{j} \wedge \tilde{A}\right]^{(0)}$$

$$= -\frac{1}{2}\int_M *F \wedge F + \int_{\partial M} \left[j \wedge (A + dss^{-1})\right]^{(0)}$$

where

$$\tilde{j} = s^{-1} j s$$

# Constructing charges

$$\tilde{S} = -\frac{1}{2}\int_M *F \wedge F + \int_{\partial M} \left[j \wedge (A + dss^{-1})\right]^{(0)}$$

with $s = e^{-i\Psi_+}$

- Charges same as before

$$\tilde{Q}_\Lambda = \int_{S^2} \mathrm{tr}\left[\Lambda \tilde{\mathcal{F}}_{ru}\sqrt{g_{S^2}}\right]^{(0)} dz d\bar{z}$$

- Arise exclusively from boundary (holographic flavour)
- Also useful for entanglement entropy calculations

Leading order see Donnelly, Freidel, Blommaert, Mertens, Verschelde, Geiller, Jai-akson

# Charge Algebra

# Charge algebra

- Charge

$$\tilde{Q}_\Lambda = \int_{S^2} \mathrm{tr}\left[\Lambda \tilde{\mathcal{F}}_{ru}\sqrt{g_{S^2}}\right]^{(0)} dz d\bar{z}$$

- We can now define the charge density

$$\begin{aligned}\tilde{q}_\Lambda &= \mathrm{tr}\left(\sqrt{g_{S^2}}\Lambda \tilde{\mathcal{F}}_{ru}\right)^{(0)} \\ &= \mathrm{tr}\left(\sqrt{g_{S^2}}\Lambda e^{i\Psi}\mathcal{F}_{ru}e^{-i\Psi}\right)^{(0)}\end{aligned}$$

- Nice recursion relations to all orders.
- Generate (overleading)symmetries. Compute charge algebra!

# Charge algebra at sub^n-leading order

Order in $\Psi$

$$\{ \overset{n-k}{q}_{\Lambda_1^{(k)}}, \overset{n-j}{q}_{\Lambda_2^{(j)}} \} = \begin{cases} \overset{n-k-j}{q}_{-i[\Lambda^{(k)},\Lambda^{(j)}]} & \text{if } j+k \leq n, \\ 0 & \text{otherwise.} \end{cases}$$

$\Lambda = \sum r^k \Lambda^{(k)}$

# Infinite algebras in the SD sector

| | SD | Full |
|---|---|---|
| YM | $\left[S^{p,a}_m(\bar z), S^{q,b}_n(\bar z)\right] = -i\, f^{ab}{}_c\, S^{p+q-1,c}_{m+n}(\bar z)$ <br> S-alg | $\{ {}^{n-k}q_{\Lambda_1^{(k)}}, {}^{n-j}q_{\Lambda_2^{(j)}} \} = \begin{cases} {}^{n-k-j}q_{-i[\Lambda^{(k)},\Lambda^{(j)}]} & \text{if } j+k \leq n, \\ 0 & \text{otherwise.} \end{cases}$ |
| Grav | $[w^p_m, w^q_n] = \big(m(q-1) - n(p-1)\big)\, w^{p+q-2}_{m+n}$ <br> $W_{1+\infty}$ | ? |

# Current / future work

- **Finish gravity**
- **Loop level**
- **Relation to homotopy algebra language**
- **Faddeev-Kulish states?**

Thank you!

# 5. effective theories (if time)

# Quasi-universal terms

- Soft theorems

$$\lim_{\omega\to 0} A_{n+1} = \underbrace{(\tfrac{1}{\omega}S^{(0)} + S^{(1)} + \ldots)A_n}_{\text{universal}} \quad + \quad \underbrace{\ldots}_{\text{non-universal}}$$

- universal terms come from symmetries
- in some simple cases non-universal terms vanish
- Focus on subleading via

$$\lim_{\omega\to 0}(1+\omega\partial_\omega)A_{n+1} = S^{(1)}A_n \quad + \quad \ldots$$

- take a more interesting scenario

$$\lim_{\omega\to 0}(1+\omega\partial_\omega)A_{n+1} = S^{(1)}A_n \quad + \text{quasi-universal}$$

# Quasi-universal terms

- Soft theorems

$$\lim_{\omega\to 0} A_{n+1} = \underbrace{(\tfrac{1}{\omega}S^{(0)} + S^{(1)} + \ldots)A_n}_{\text{universal}} \quad + \quad \underbrace{\ldots}_{\text{non-universal}}$$

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# Quasi-universal terms

- QED with higher derivative interactions, e.g. $\phi F^2$ [Elvang,Jones,Naculich]

$$\lim_{\omega\to 0}(1+\omega\partial_\omega)A_{n+1} = S^{(1)}A_n \quad + \quad \tilde{S}^{(1)}[A_n]$$

where $S^{(1)}$ is the usual

$$S^{(1)} = -ie\sum_{k=1}^{n} Q_k \frac{p_s^\mu \varepsilon_s^\nu}{p_k \cdot p_s}\mathcal{J}_{k\mu\nu}$$

and $\tilde{S}^{(1)}$ is an *operator*

$$\tilde{S}_+^{(1)}A_n = \sum_{k=1}^{n}\frac{[sk]}{\langle sk\rangle}\mathcal{F}_k A_n, \quad \tilde{S}_-^{(1)} = (\tilde{S}_+^{(1)})^\dagger$$

where $\mathcal{F}_k$ is a particle changing operator.

# Quasi-universal terms from symmetries

$$\lim_{\omega\to 0}(1+\omega\partial_\omega)A_{n+1} = S^{(1)}A_n \quad + \quad \tilde{S}^{(1)}[A_n]$$

- [Laddha,Mitra] showed that both $S^{(1)}$ and $\tilde{S}^1$ follow as a Ward identity for an gauge symmetry which violates the fall-off of the fields

$$\Lambda(u,r,z,\bar{z}) = r\Lambda(z,\bar{z}) + \frac{u}{2}(D^2+2)\Lambda(z,\bar{z})$$

and, schematically, the charge

$$Q(\varphi,\delta\varphi) = \underbrace{Q_{old}(\varphi,\delta\varphi)}_{\text{from QED,universal}} + \underbrace{Q_{new}(\varphi,\delta\varphi)}_{\text{from e.g. } \phi F^2}$$

- $Q_{new}(\varphi,\delta\varphi)$ is subleading relative to $Q_{old}(\varphi,\delta\varphi)$ for a standard $\delta_\Lambda$,
- it becomes of the same order when we allow $\delta_{r\Lambda}$ and gives the quasi-universal term

# Quasi-universal terms from symmetries

$$\lim_{\omega\to 0}(1+\omega\partial_\omega)A_{n+1} = S^{(1)}A_n \quad + \quad \tilde{S}^{(1)}[A_n]$$

- Overleading part of gauge parameter acts on the Stueckelberg field

$$\Lambda(u,r,z,\bar{z}) = r\Lambda(z,\bar{z}) + \frac{u}{2}(D^2+2)\Lambda(z,zb)$$

- the symplectic potential is dressed with the Stueckelberg fields

$$\theta(\tilde{\varphi},\delta\tilde{\varphi}) = \theta_{old}(\tilde{\varphi},\delta\tilde{\varphi}) + \theta_{new}(\tilde{\varphi},\delta\tilde{\varphi})$$

- Universal and (mildly) non-universal terms arising from symmetries acting canonically on an extended phase space
- Different gauge choices can simplify things !