Constraints on Long-Range Forces in De Sitter Space

Callum R. T. Jones

to appear *with* Daniel Baumann, Kurt Hinterbichler, Austin Joyce, Hayden Lee, Jiajie Mei and Nathan Meurrens

Universiteit van Amsterdam

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Weinberg Soft Constraints in Minkowski Space

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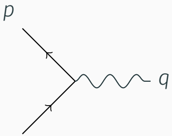

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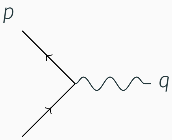

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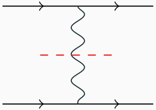

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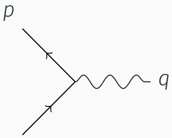

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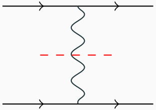

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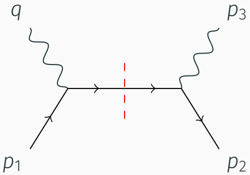

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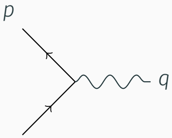

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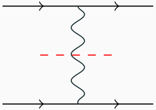

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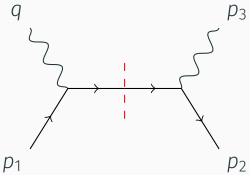

$$q_\mu M^\mu(q; p_1, p_2, p_3) \quad \overset{q \to 0}{\sim} \quad {\color{green} g}^2\,(p_1 \cdot \epsilon_q)^2\,(p_2 \cdot \epsilon_3)^3 + (1 \leftrightarrow 2) \quad \overset{!}{=} \quad 0.$$

Only solution is ${\color{green} g} = 0$; *no-go for spin-3 long-range forces* [Weinberg 1964].

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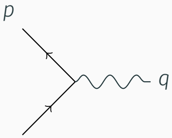

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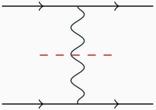

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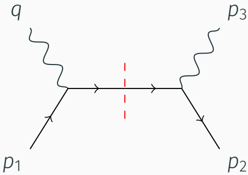

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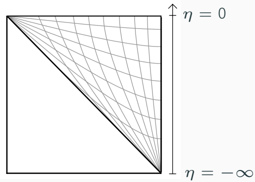
This talk: derive and analyze equivalent constraints in *de Sitter space*.

De Sitter Holography



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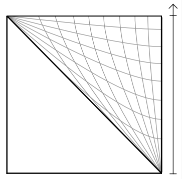

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Want a dS "amplitude": **Bunch-Davies wavefunction**

$$\Psi_{\rm BD}\left[\varphi\right] \quad \equiv \quad \int D\phi \; e^{iS[\phi]} \quad \cong \quad e^{iS_{\text{on-shell}}[\varphi]}$$

$$\phi(\eta, \mathrm{k}) \overset{\eta\to-\infty}{\sim} e^{+i\eta|\mathrm{k}|}, \qquad \phi(\eta, \mathrm{x}) \overset{\eta\to 0^+}{\sim} \eta^{d-\Delta}\varphi(\mathrm{x}), \qquad m^2 L_{\rm dS}^2 = \Delta(d-\Delta).$$

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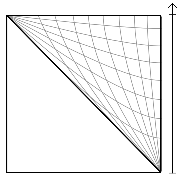

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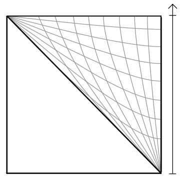

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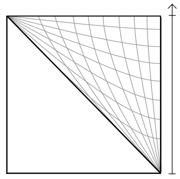

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Import standard CFT technology to "on-shell boostrap" bulk interactions

$$\langle J_1^a J_2^b J_3^c\rangle = f^{abc}\frac{c_1 V_1V_2V_3 + c_2\left(V_1H_{23} + V_2H_{31} + V_3H_{12}\right)}{P_{12}^{3/2}P_{23}^{3/2}P_{31}^{3/2}},$$

$c_1 = c_2 \Leftrightarrow$ Yang-Mills

$c_1 = 5c_2 \Leftrightarrow F^3$.

UIRs of De Sitter Isometries

Free field theories in dS_d $\Leftrightarrow$ **unitary irreducible representations** of $SO(d+1,1)$.

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Classified long-ago [Harish-Chandra; Bargmann; Gel'fand, Naimark 1947]

Symmetric traceless spin-s tensors, $m^2 L_{\mathrm{ds}}^2 = \Delta(d-\Delta) + (s-2)(d+s-2)$

- **Principal Series** (heavy spinning fields) $\Delta = \frac{d}{2} + i\nu, \quad \nu \in \mathbb{R}.$
- **Complementary Series** (light spinning fields) $1 < \Delta < d-1$
- **Exceptional Series:** (symmetric tensor gauge fields)

$$\Delta = d - 1 + t, \qquad \underbrace{t = 0, 1, \ldots, s-1.}_{\text{depth}}$$

For $t < s-1$, these fields are called **partially massless** and have no analogue in either Minkowski or AdS.

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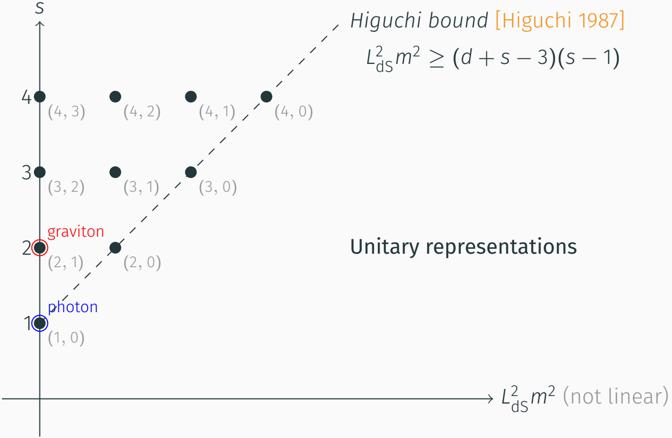
Main problem: are there consistent interacting theories for partially massless fields?

Partially Massless Gauge Fields



Higuchi bound [Higuchi 1987]

$$L_{\mathrm{dS}}^2 m^2 \geq (d+s-3)(s-1)$$

Unitary representations

Partially Massless Gauge Fields

Example: Partially massless spin-2

$$\left(\Box - \frac{2}{L_{\text{dS}}^2} - m^2\right)\Phi^{AB} = 0, \qquad \nabla_A \Phi^{AB} = 0, \qquad g_{AB}\Phi^{AB} = 0.$$

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Holographic dictionary for PM fields: [Dolan, Nappi, Witten 2001]

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$$\underbrace{X^{\mu}_{(1,0)} = J^\mu,}_{\text{photon}} \qquad \underbrace{X^{\mu\nu}_{(2,1)} = T^{\mu\nu},}_{\text{graviton}} \qquad \underbrace{X^{\mu\nu}_{(2,0)}, \quad \partial_\mu\partial_\nu X^{\mu\nu}_{(2,0)} = 0}_{\text{PM spin-2}}.$$

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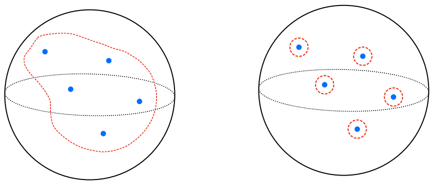

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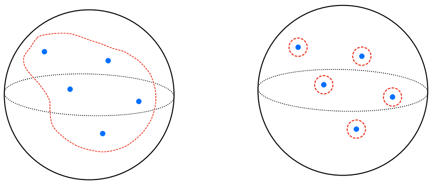

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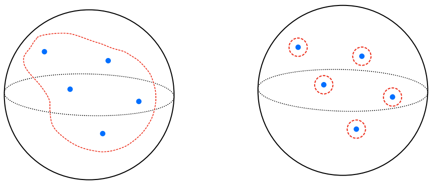

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Gives a (quadratic) sum rule on cubic couplings

$$\sum_i\left[c_{1i}\partial^{\#_{1i}}\langle\mathcal{O}_i\mathcal{O}_2\mathcal{O}_3\rangle + c_{2i}\partial^{\#_{2i}}\langle\mathcal{O}_1\mathcal{O}_i\mathcal{O}_3\rangle + c_{3i}\partial^{\#_{3i}}\langle\mathcal{O}_1\mathcal{O}_2\mathcal{O}_i\rangle\right] = 0.$$

Familiar Examples

Example 1: spin-1 current $X_{(1,0)}$ with scalar matter $\mathcal{O}_i \quad i = 1, 2, 3$

$$[Q_{(1,0)}, \mathcal{O}_i(x)] = q_i \mathcal{O}_i(x), \qquad \langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\rangle = \frac{\lambda_{123}}{x_{12}^{\Delta_1+\Delta_2-\Delta_3} x_{23}^{\Delta_2+\Delta_3-\Delta_1} x_{31}^{\Delta_3+\Delta_1-\Delta_2}}.$$

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If $\lambda_{123} \neq 0$ then must have $q_1 + q_2 + q_3 = 0$: *electric charge conservation.*

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Example 2: spin-2 depth-1 current $X_{(2,1)}$ with scalar matter $\mathcal{O}_i \quad i = 1, 2, 3$

$$[Q^{\mu}_{(2,1)}, \mathcal{O}_i(x)] = \kappa_i \partial^{\mu} \mathcal{O}_i(x).$$

Familiar Examples

Example 1: spin-1 current $X_{(1,0)}$ with scalar matter $\mathcal{O}_i \quad i = 1, 2, 3$

$$[Q_{(1,0)}, \mathcal{O}_i(x)] = q_i \mathcal{O}_i(x), \qquad \langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\rangle = \frac{\lambda_{123}}{x_{12}^{\Delta_1+\Delta_2-\Delta_3} x_{23}^{\Delta_2+\Delta_3-\Delta_1} x_{31}^{\Delta_3+\Delta_1-\Delta_2}}.$$

$$\langle [Q_{(1,0)}, \mathcal{O}_1\mathcal{O}_2\mathcal{O}_3]\rangle = 0 \quad \Rightarrow \quad (q_1 + q_2 + q_3)\,\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\rangle \overset{!}{=} 0.$$

If $\lambda_{123} \neq 0$ then must have $q_1 + q_2 + q_3 = 0$: *electric charge conservation.*

Example 2: spin-2 depth-1 current $X_{(2,1)}$ with scalar matter $\mathcal{O}_i \quad i = 1, 2, 3$

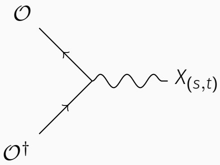
$$[Q^{\mu}_{(2,1)}, \mathcal{O}_i(x)] = \kappa_i \partial^{\mu} \mathcal{O}_i(x).$$

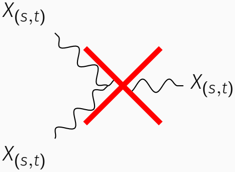
$$\langle [Q^{\mu}_{(2,1)}, \mathcal{O}_1\mathcal{O}_2\mathcal{O}_3]\rangle = 0 \quad \Rightarrow \quad \left(\kappa_1\partial_1^{\mu} + \kappa_2\partial_2^{\mu} + \kappa_3\partial_3^{\mu}\right)\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\rangle \overset{!}{=} 0.$$

If $\lambda_{123} \neq 0$ then must have $\kappa_1 = \kappa_2 = \kappa_3$: *Einstein equivalence principle.*

Un-Familiar Examples

As a warm-up, **partially massless electrodynamics** $\{X_{(s,t)}, \mathcal{O}\}$ [Deser, Waldron 2006]





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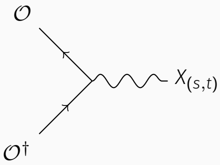

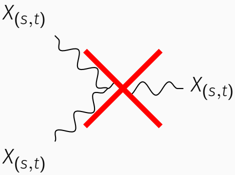

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$$[\underset{2}{Q_{(3,0)}}, \underset{\Delta}{\mathcal{O}(x)}] = ic \underset{2}{\Box} \underset{\Delta}{\mathcal{O}(x)}.$$

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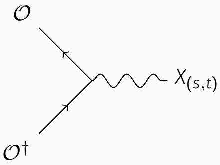

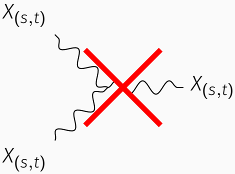

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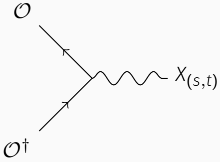

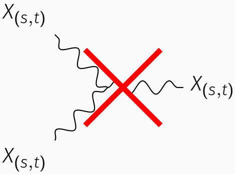

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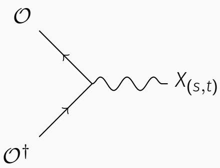

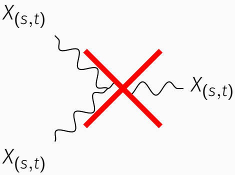

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Example 2: spin-2 depth-0, $\quad [D, Q_{(2,0)}] = Q_{(2,0)}.$

$$[\underset{1}{Q_{(2,0)}}, \underset{\Delta}{\mathcal{O}(x)}] = i c\, \underset{1}{\Box}^{1/2} \underset{\Delta}{\mathcal{O}(x)} \qquad ???$$

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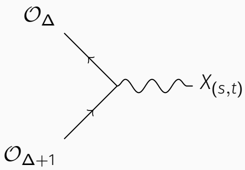

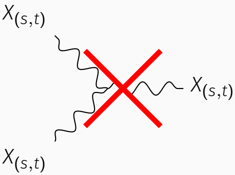

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$$[\underset{1}{Q_{(2,0)}}, \underset{\Delta}{\mathcal{O}_\Delta}(x)] = c\, \underset{\Delta+1}{\mathcal{O}_{\Delta+1}}(x),$$

$$c\langle X_{(2,0)}(z_1, x_1)\mathcal{O}_{\Delta+1}(x_2)\mathcal{O}_\Delta(x_3)\rangle + c\langle X_{(2,0)}(z_1, x_1)\mathcal{O}_\Delta(x_2)\mathcal{O}_{\Delta+1}(x_3)\rangle = 0.$$

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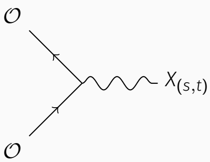
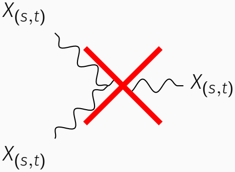




$$[Q_{(s,t)}, X_{(s,t)}] = 0.$$

Straightforward to generalize to arbitrary spin-s and depth-t, ex.

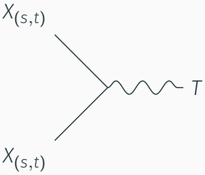
$$c\left(\Box_2^{\frac{1}{2}(s-t-1)}(z\cdot\partial_2)^t + \Box_3^{\frac{1}{2}(s-t-1)}(z\cdot\partial_3)^t\right)\langle X_{(s,t)}\mathcal{O}\mathcal{O}\rangle = 0, \qquad s \text{ even}, \quad t \text{ odd}.$$

No-go for partially massless electrodynamics.

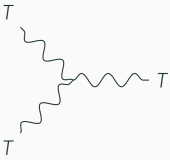
Generalizes previous no-go theorem for $s - t = 3$ [Sleight, Taronna 2021].

Coupling to Einstein Gravity

Gravity couples to everything universally



$\sim \quad \sqrt{G_N}$



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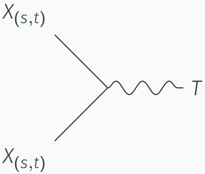

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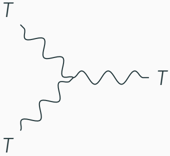

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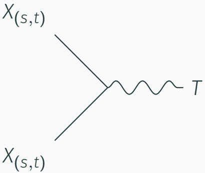

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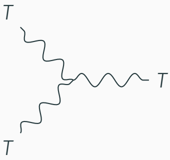

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Current algebra *reciprocity*

$$\langle [Q_{(s,t)}, T\, X_{(s,t)}] \rangle = \langle [Q_{(s,t)}, T]\, X_{(s,t)} \rangle + \langle T\, [Q_{(s,t)}, X_{(s,t)}] \rangle = 0, \quad \Longrightarrow \quad [Q_{(s,t)}, X_{(s,t)}(x)] \supset T(x),$$

Conformal + PM symmetry must combine into a larger **higher-spin algebra**.

Coupling to Einstein Gravity

Strategy: analyze two sum-rules that *cannot trivially vanish*

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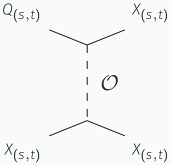
$$\langle [Q_{(s,t)}, X_{(s,t)} X_{(s,t)} X_{(s,t)}] \rangle = 0 \qquad \text{“Møller scattering”}$$

$\sum_{\mathcal{O}}$



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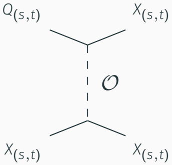
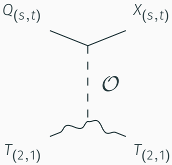


$$+ \quad (1 \leftrightarrow 2) \quad + \quad (1 \leftrightarrow 3) \quad = \quad 0.$$

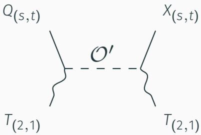
$$\langle [Q_{(s,t)}, X_{(s,t)}T_{(2,1)}T_{(2,1)}]\rangle = 0$$

“Compton scattering”

$$\sum_{\mathcal{O}}$$



$$+ \quad \sum_{\mathcal{O}'}$$



$$+ \quad (2 \leftrightarrow 3) \quad = \quad 0.$$

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   and then try to solve them simultaneously.

5. If there is *no solution*, then a model with the assumed spectrum is **rigorously excluded**. Can then try to enlarge the initial spectrum and start again at step 1.

Results: Graviton + PM Spin-2

Assume that the bulk contains a graviton and a PM spin-2 depth-0:

$$[Q_{(2,0)}, X_{(2,0)}] = c_1 \left[\Box - \frac{1}{d-1}(z\cdot\partial)(\partial\cdot D_z)\right] X_{(2,-1)} + c_2 T_{(2,1)} + c_3 (\partial\cdot D_z) X_{(3,0)},$$

$$[Q_{(2,0)}, T_{(2,1)}] = c_4 \left[\Box - \frac{2}{d}(z\cdot\partial)(\partial\cdot D_z)\right] X_{(2,0)} + c_5 (\partial\cdot D_z) X_{(3,1)}.$$

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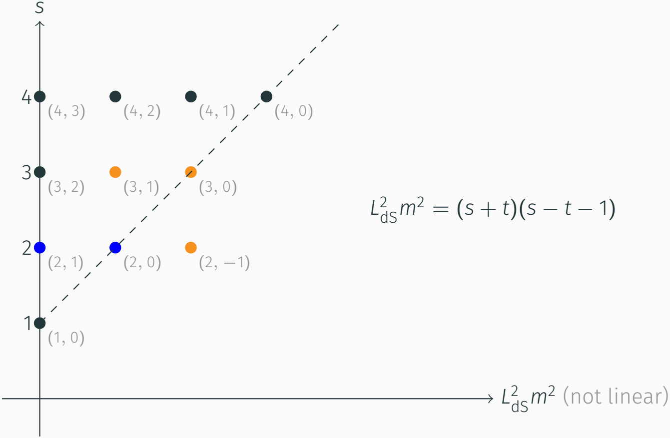
- In $d \geq 4$ the simplest **consistent theory** requires

$$\{T_{(2,1)}, X_{(2,0)}\} \quad \oplus \quad \{X_{(2,-1)}\}.$$

Results: Graviton + PM Spin-2



$$L_{\text{dS}}^2 m^2 = (s+t)(s-t-1)$$

Results: Graviton + PM Spin-3

Assume that the bulk contains a graviton and a PM spin-3 depth-0:

$$\left[Q_{(3,0)}, X_{(3,0)}\right] \sim c_1 \partial^2 X_{(1,0)} + c_2 \partial T_{(2,1)} + c_3 X_{(3,2)} + c_4 \partial X_{(4,1)} + c_5 \partial^2 X_{(5,0)} + c_6 \partial^3 X_{(2,-1)} + c_7 \partial^4 X_{(3,-2)} + c_8 \partial^3 X_{(4,-1)},$$

$$\left[Q_{(3,0)}, T_{(2,1)}\right] \sim c_9 \partial^4 X_{(2,-1)} + c_{10} \partial^3 X_{(3,0)} + c_{11} \partial^2 X_{(4,1)}.$$

We analyzed the constraints in $d = 3, 4, 5, 6, 7$ (dS_{d+1}):

Results: Graviton + PM Spin-3

Assume that the bulk contains a graviton and a PM spin-3 depth-0:

$$\begin{aligned}[Q_{(3,0)}, X_{(3,0)}] \sim\ & c_1 \partial^2 X_{(1,0)} + c_2 \partial T_{(2,1)} + c_3 X_{(3,2)} + c_4 \partial X_{(4,1)} + c_5 \partial^2 X_{(5,0)} \\ & + c_6 \partial^3 X_{(2,-1)} + c_7 \partial^4 X_{(3,-2)} + c_8 \partial^3 X_{(4,-1)},\end{aligned}$$

$$[Q_{(3,0)}, T_{(2,1)}] \sim c_9 \partial^4 X_{(2,-1)} + c_{10} \partial^3 X_{(3,0)} + c_{11} \partial^2 X_{(4,1)}.$$

We analyzed the constraints in $d = 3, 4, 5, 6, 7$ (dS_{d+1}):

- **No go** for minimal theory $\{X_{(3,0)}, T_{(2,1)}\}$ in all d.

Results: Graviton + PM Spin-3

Assume that the bulk contains a graviton and a PM spin-3 depth-0:

$$\begin{aligned}\left[Q_{(3,0)}, X_{(3,0)}\right] \sim\ & c_1 \partial^2 X_{(1,0)} + c_2 \partial T_{(2,1)} + c_3 X_{(3,2)} + c_4 \partial X_{(4,1)} + c_5 \partial^2 X_{(5,0)} \\ & + c_6 \partial^3 X_{(2,-1)} + c_7 \partial^4 X_{(3,-2)} + c_8 \partial^3 X_{(4,-1)},\end{aligned}$$

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Results: Graviton + PM Spin-3

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How do we know these remain consistent solutions when we impose more constraints?

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Top-down approach: try to directly construct a CFT with partially conserved currents.

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Million won question: are the bulk duals of the non-unitary $k>1$ models the only consistent PM theories in de Sitter?

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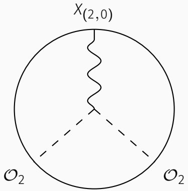

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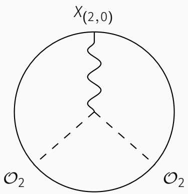

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Bulk gauge invariance **broken by boundary conditions**

$$\partial_\mu\partial_\nu X^{\mu\nu}_{(2,0)} \propto \underbrace{\mathcal{O}_1\mathcal{O}_2}_{\text{double-trace}}$$

PM gauge symmetry Higgsed at one-loop [Porrati 2001].

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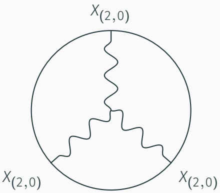

$$\underset{2}{\partial_\mu \partial_\nu} \underset{2}{X^{\mu\nu}_{(2,0)}} \quad = \quad g : \underset{2}{X_{(2,0)\mu\nu}} \underset{2}{X^{\mu\nu}_{(2,0)}} :$$

Standard quantization boundary conditions break PM gauge symmetry.

PM spin-2 gains mass correction at one-loop, $\Delta_X = 2 + \mathcal{O}\left(\frac{1}{N}\right)$.

Final Thoughts

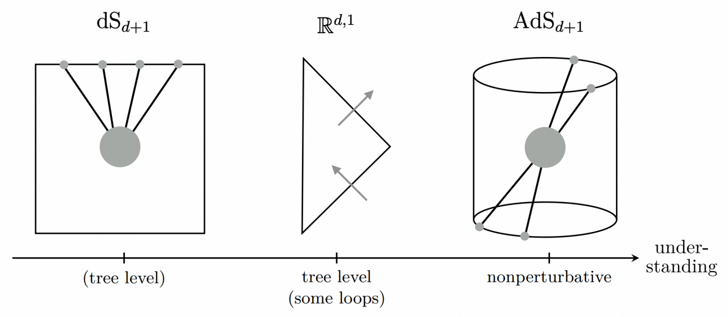

Image credit: "Snowmass White Paper: The Cosmological Boostrap" [2203.08121]

Basic problem: classify consistent gauge theories in maximally symmetric spacetimes.

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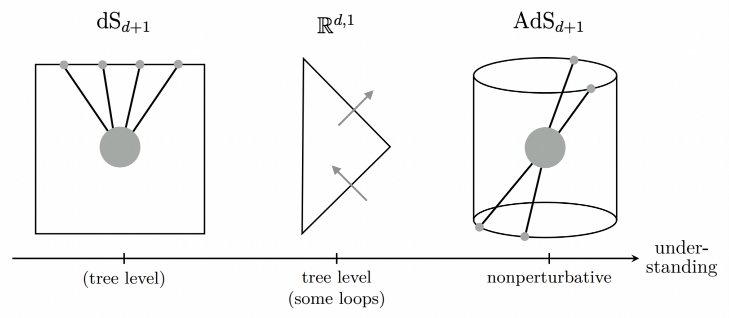

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