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JHEP 01 (2025) 111 (2410.22988 [hep-th])
Phys.Rev.D 111 (2025) 12, L121702 (2501.07372 [hep-th])

Compactly packaging PM dynamics:
a fresh look at the eikonal

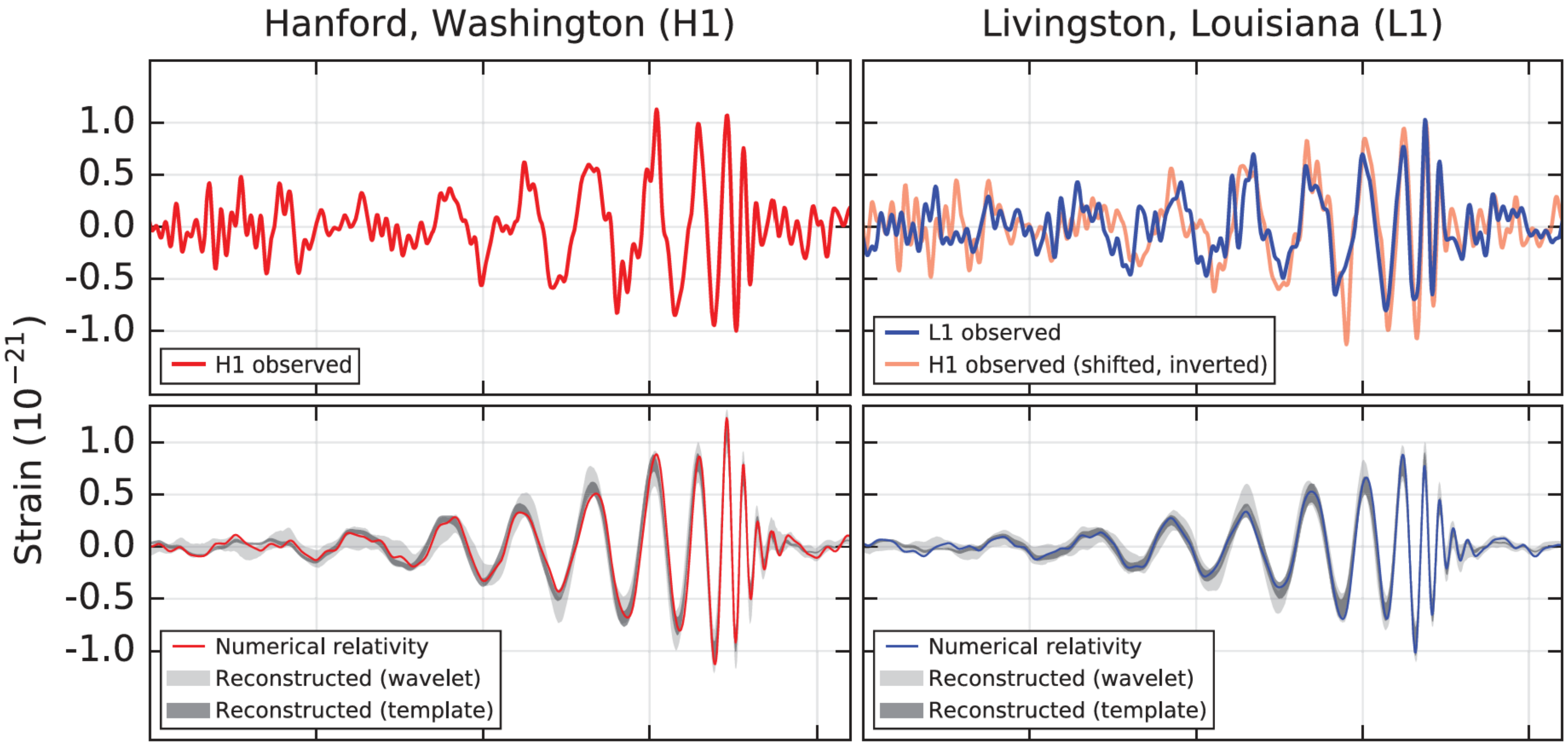
Jung-Wook Kim [MPI for Gravitational Physics (AEI), Potsdam]

Works in collaboration with : Joon-Hwi Kim, Sangmin Lee [2405.17056]

Joon-Hwi Kim, Sungsoo Kim, Sangmin Lee [2410.22988]

PM dynamics: Why did we get here?

- 10 years since GW150914



LIGO Scientific & Virgo [1602.03837]

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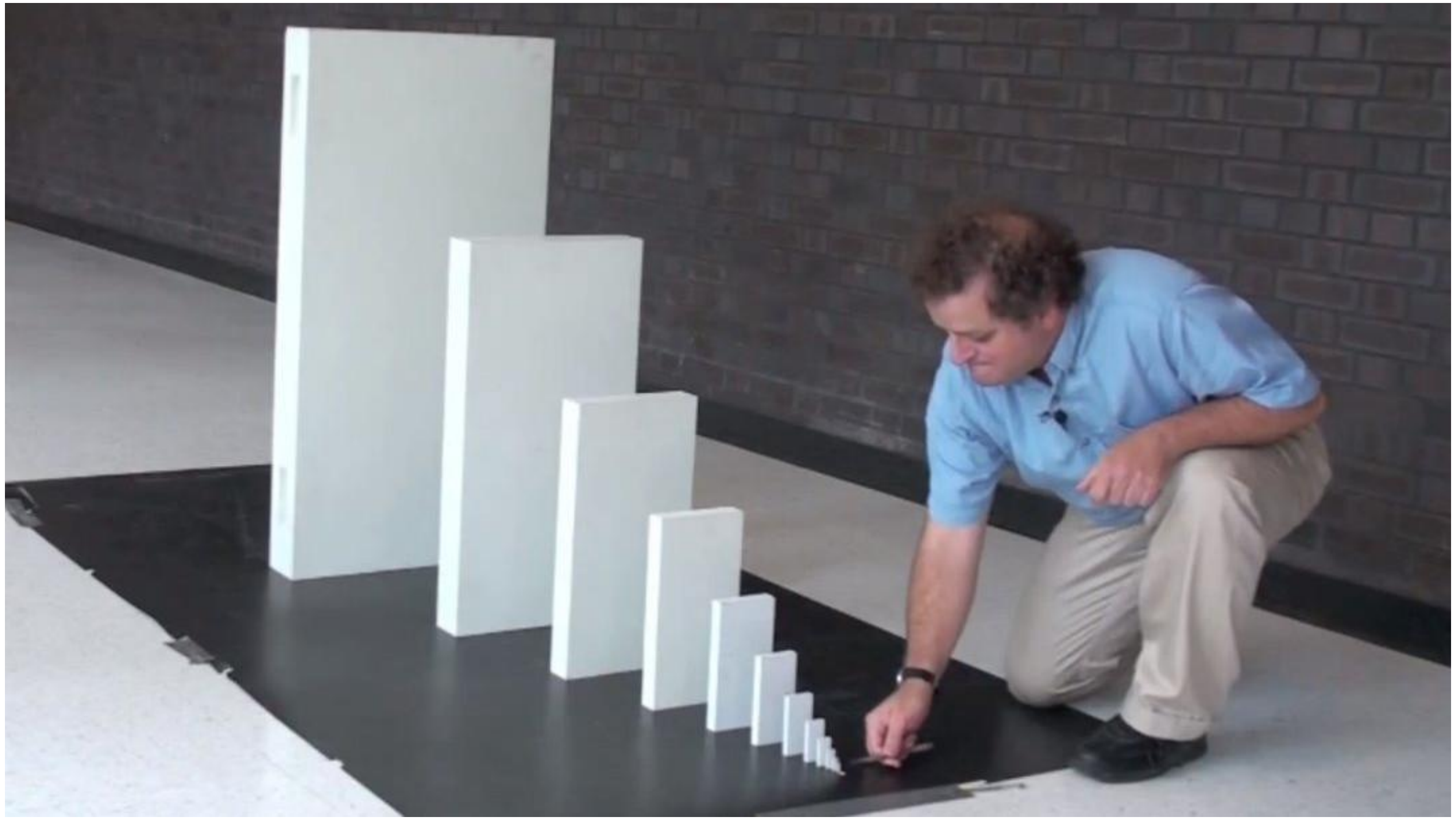

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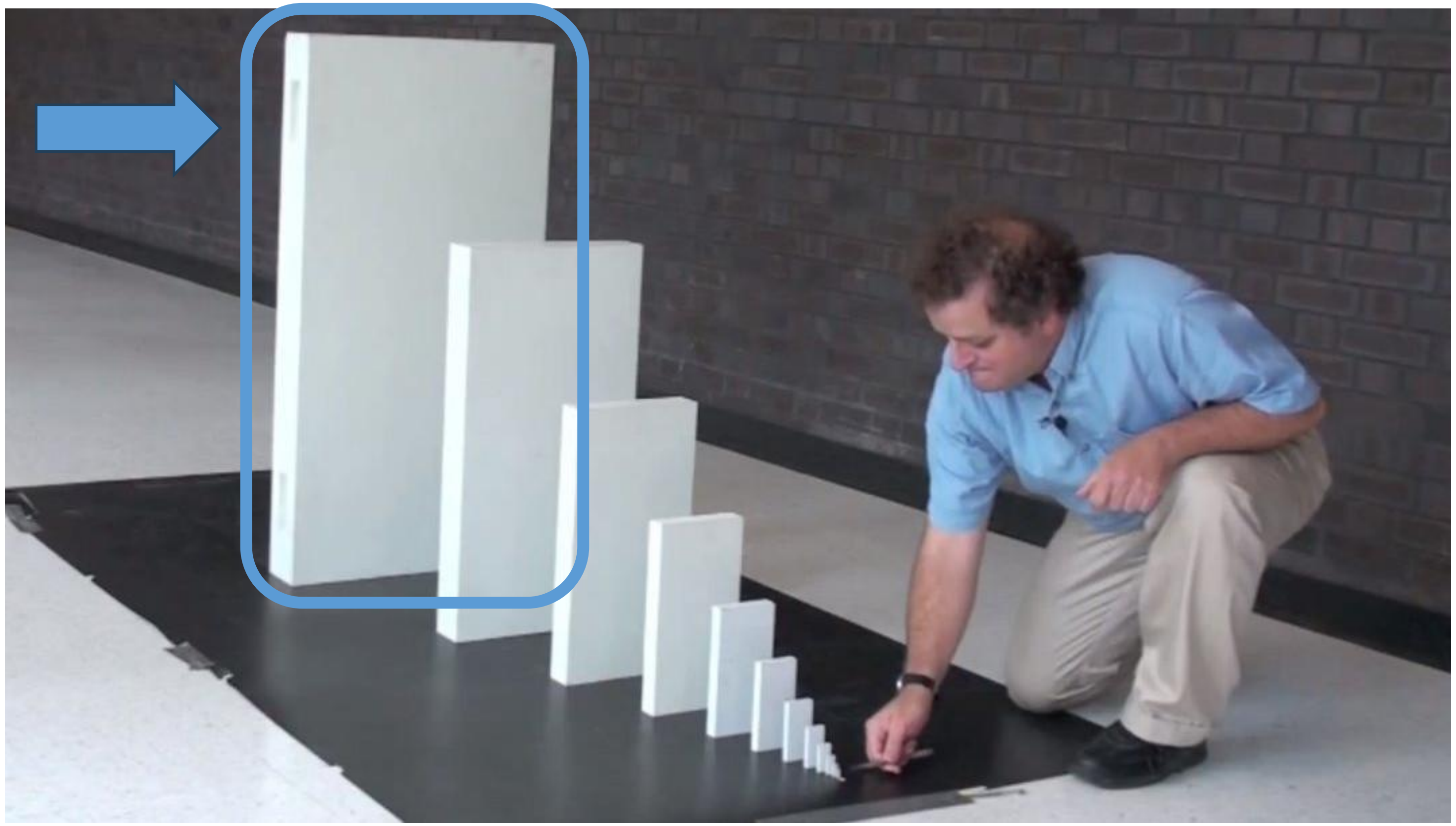

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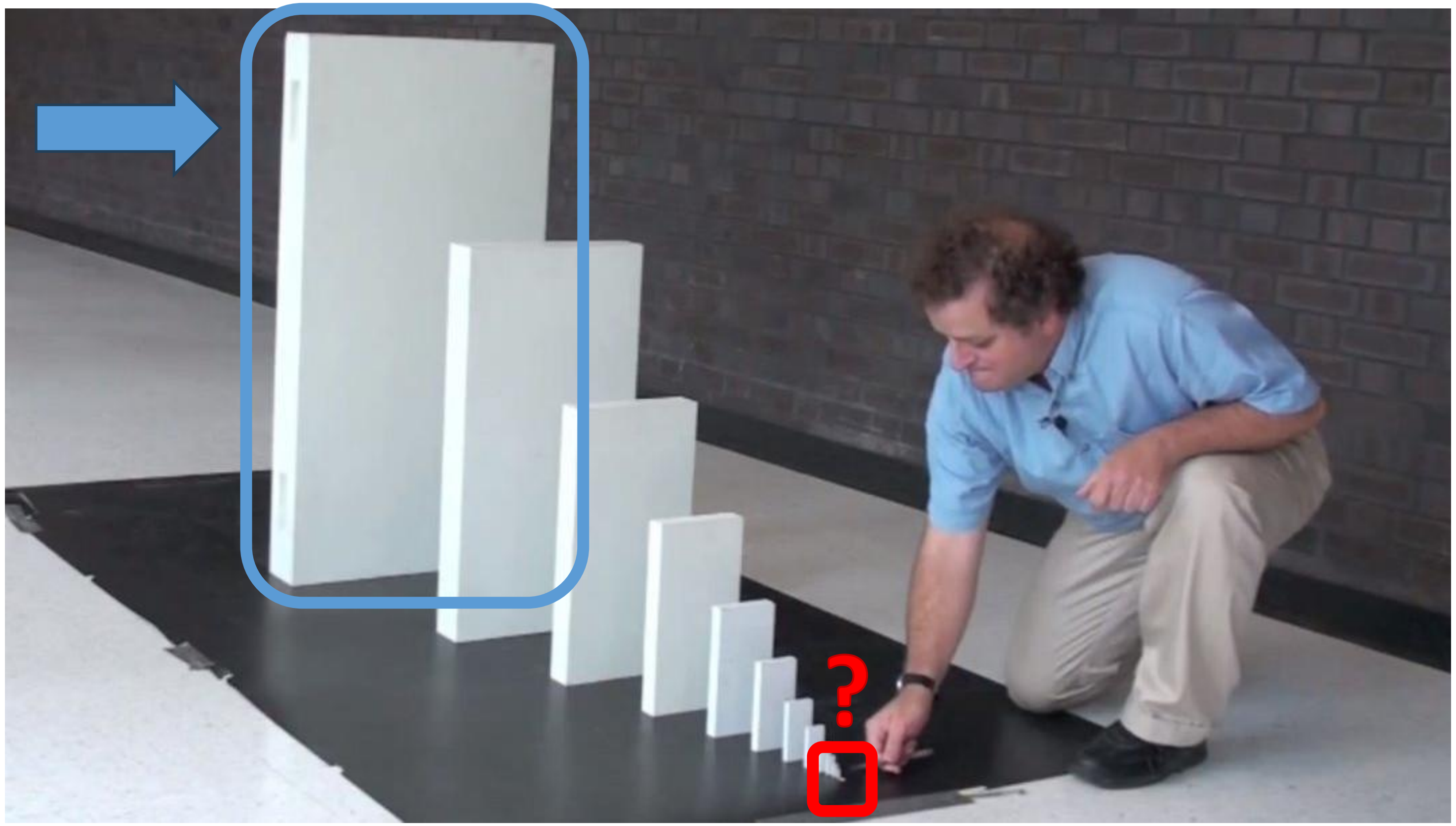

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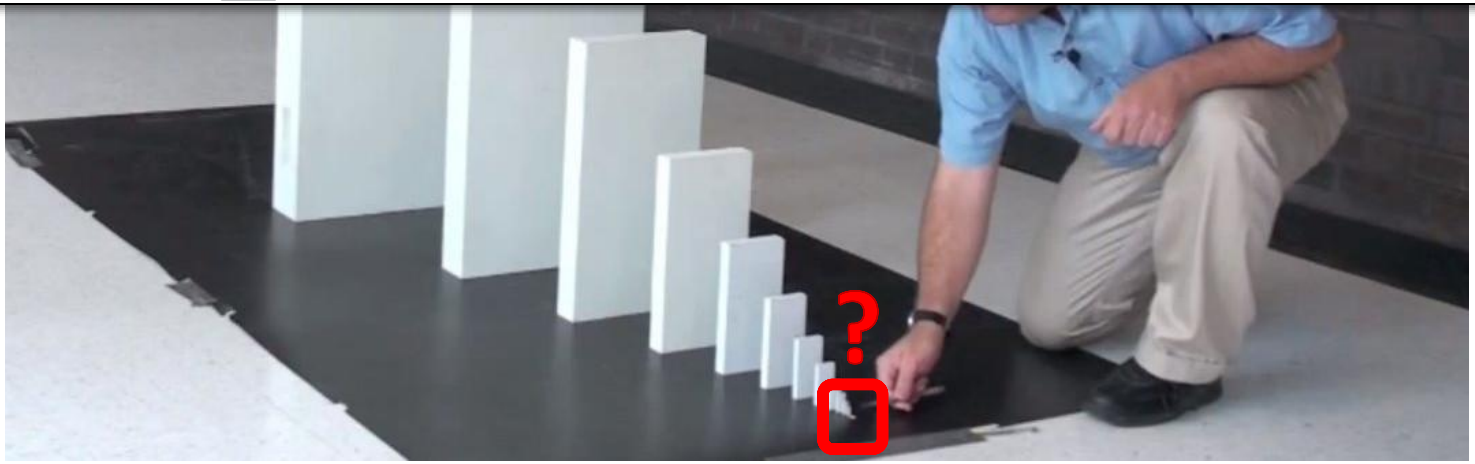
High-energy gravitational scattering and the general relativistic two-body problem

Thibault Damour*
Institut des Hautes Etudes Scientifiques, 35 route de Chartres, 91440 Bures-sur-Yvette, France

(Received 29 October 2017; published 26 February 2018)



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We derive several consequences of our second post-Minkowskian Hamiltonian: (i) the need to use special phase-space gauges to get a tame high-energy limit; and (ii) predictions about a (rest-mass independent) linear Regge trajectory behavior of high-angular-momenta, high-energy circular orbits. Ways of testing these predictions by dedicated numerical simulations are indicated. We finally indicate a way to connect our classical results to the quantum gravitational scattering amplitude of two particles, and we urge amplitude experts to use their novel techniques to compute the two-loop scattering amplitude of scalar masses, from which one could deduce the third post-Minkowskian effective one-body Hamiltonian.

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Scattering Amplitudes and the Conservative Hamiltonian for Binary Systems at Third Post-Minkowskian Order

Zvi Bern,[1] Clifford Cheung,[2] Radu Roiban,[3] Chia-Hsien Shen,[1] Mikhail P. Solon,[2] and Mao Zeng[4]
[1]*Mani L. Bhaumik Institute for Theoretical Physics, University of California at Los Angeles, Los Angeles, California 90095, USA*
[2]*Walter Burke Institute for Theoretical Physics, California Institute of Technology, Pasadena, California 91125*
[3]*Institute for Gravitation and the Cosmos, Pennsylvania State University, University Park, Pennsylvania 16802, USA*
[4]*Institute for Theoretical Physics, ETH Zürich, 8093 Zürich, Switzerland*

(Received 28 January 2019; published 24 May 2019)

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- 10 years since GW150914: huge literature on PM dynamics
- Keywords from Damour's request
  - Effective-one-body (EOB): weak-field → analytic strong-field [Buonanno, Damour]
    - Resum by leveraging background-probe limit
    - Motivation: GW loudest in strong-field regime
  - Post-Minkowskian (PM): (gauge-inv.) scattering dynamics
    - Singularities *physical* → leverage for resummation (e.g. $\mathcal{L}$-resum [Damour, Rettegno])
- What else can we leverage from PM? How should we do it?
  - Spin resummation / Through the eikonal (= scattering generator)

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Why spin resummation?

- Waveform models perform worse for high spins/mass ratios
  - Comparison with numerical relativity (NR) simulations

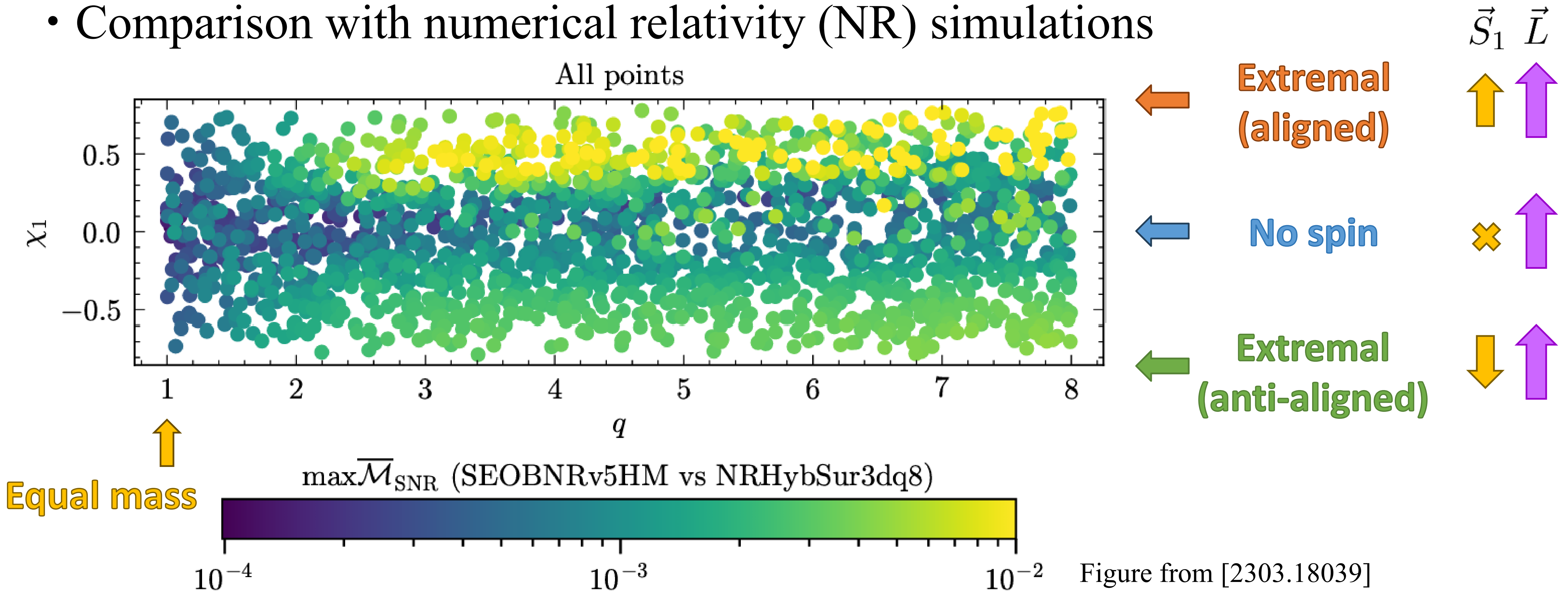
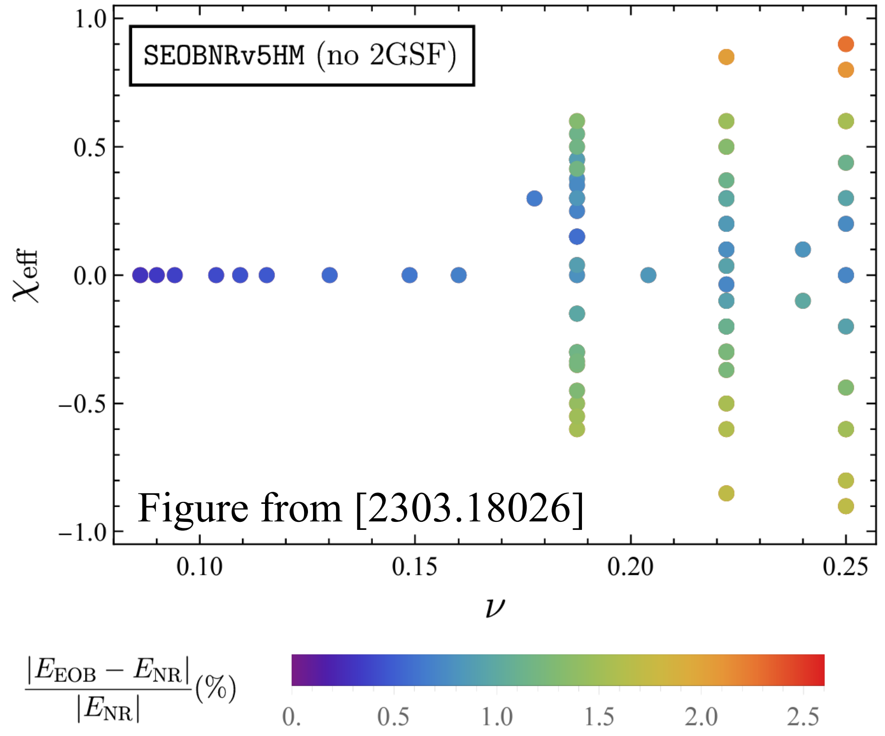


Figure from [2303.18039]

Why spin resummation?

- Waveform models perform worse for high spins/mass ratios
  - Comparison with numerical relativity (NR) simulations
- Limiting factor for (next-gen) GW physics
  - "we ascertain that current waveforms can accurately recover the distribution of masses in the LVK astrophysical population, **but not spins**"

[Dhani, Völkel, Buonanno, Estelles, Gair, Pfeiffer, Pompili, Toubiana]

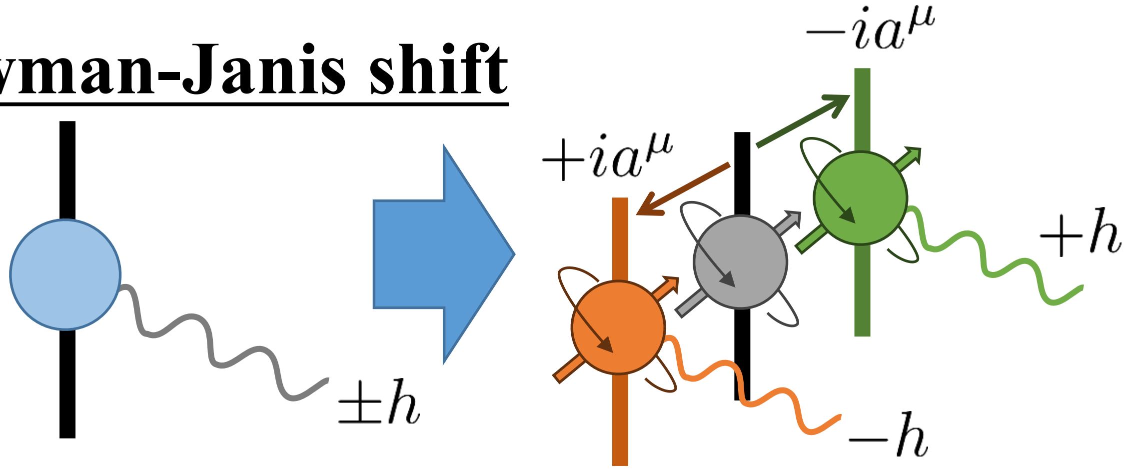


Parameter	Notation	Astrophysically relevant range
Total mass in the detector frame	M	$10^5 - 10^7 M_\odot$
Mass ratio (> 1)	q	$1 - 10$
Dimensionless spin	$\max\lvert\chi_i\rvert$	$0 - 0.998$
Eccentricity entering LISA band	$e_{\rm init}$	$0 - 0.99$
Eccentricity at last stable orbit	$e_{\rm merge}$	< 0.1
Signal to noise ratio	SNR	$10 - 10^4$

Table from [2311.01300]

Why spin resummation?

- Why expect spin to be resummable?
  - Black hole spin moments given by **Newman-Janis shift**
  - Kerr couples minimally to gravitons
    [Guevara, Ochirov, Vines; Chung, Huang, JWK, Lee]
    - Minimal coupling ≈ on-shell NJ shift
      [Arkani-Hamed, Huang, O'Connell]



- Expectation: *singularities* of binary dynamics governed by NJ shift
  - Resummations leverage singularities ⇒ *singularities* > *pert. exactness*
  - Several known models implement NJ shift dynamically
    - Worldsheet [Guevara, Maybee, Ochirov, O'connell, Vines], Twistor worldline [JH Kim, JWK, Lee], Bootstrap [Bjerrum-Bohr, Chen, Skowronek], Higher-spin gauge [Cangemi, Chiodaroli, Johansson, Ochirov, Pichini, Skvortsov]

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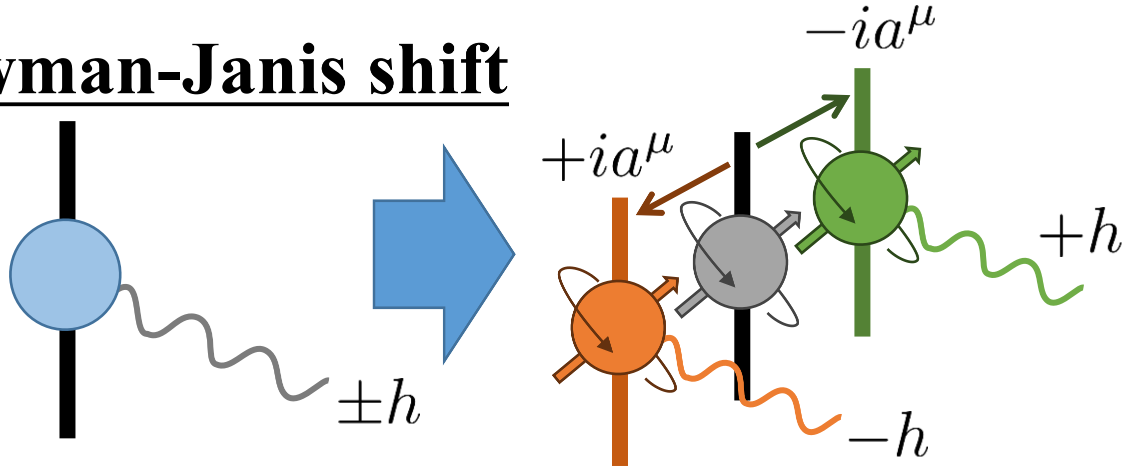

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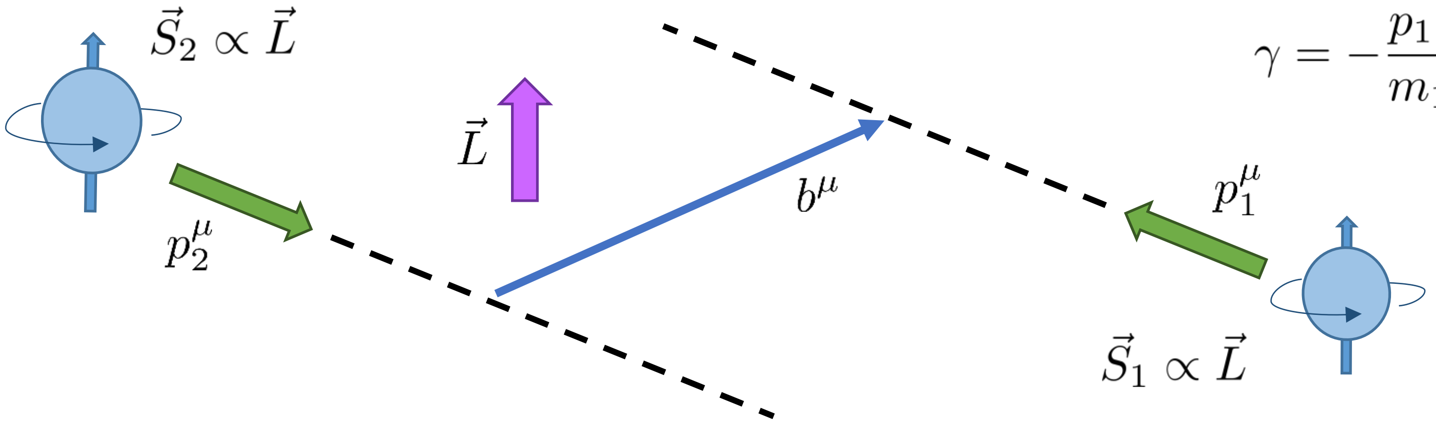
Spin resummation @ NLO

[JH Kim, JWK, Lee]

- Spin-resummed 2PL twistor worldline eikonal
  - Eikonal is a double infinite sum of 2F1, but simplifies in special kinematics
  - Aligned-spin configuration: *square roots*

[Aoude, Haddad, Helset; Damgaard, Hoogeveen, Luna, Vines; Chen, JWK, Wang; Bohnenblust, Cangemi, Johansson, Pichini]

$$\chi_{(2,\text{aligned})} = \frac{(q_1 q_2)^2 \left(b^2 + \frac{(\zeta-2)\gamma}{(\gamma^2-1)} \epsilon[b, v_1, v_2, a] + \frac{\gamma^2(1-\zeta)+\zeta}{\gamma^2-1} a^2 \right)}{32\pi m_1 \sqrt{\gamma^2-1}\,(b^2-a^2)^{3/2}} + (1 \leftrightarrow 2)$$



$$\gamma = -\frac{p_1 \cdot p_2}{m_1 m_2}\,,\ a_1^\mu = \frac{S_1^\mu}{m_1} = \zeta a^\mu\,,\ a^\mu = a_1^\mu + a_2^\mu$$

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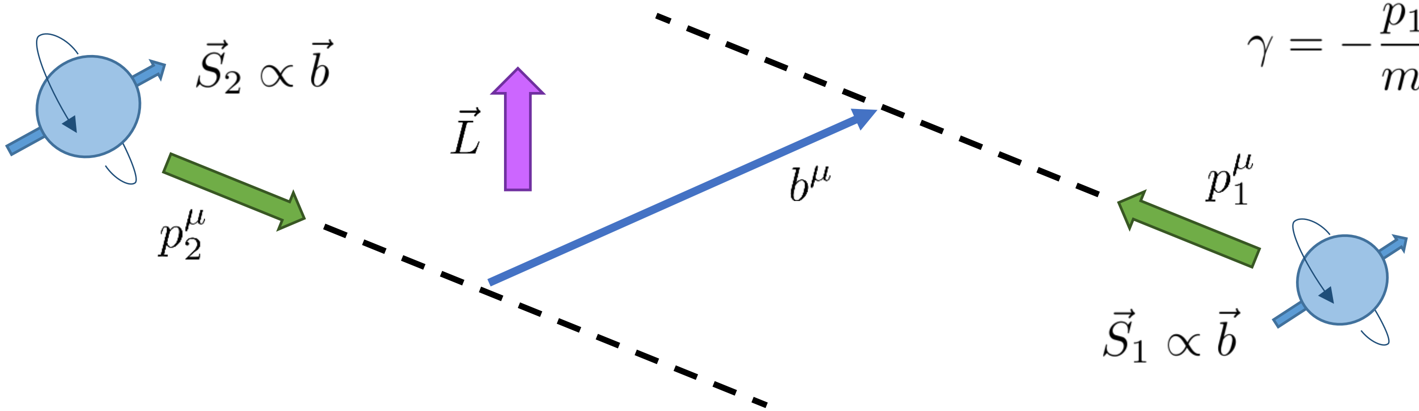
  - Axial scattering configuration: *elliptic integrals*

$$\chi_{(2,\text{axial})} = \frac{(q_1 q_2)^2 \sqrt{b^2}}{16\pi^2 m_1 (\gamma^2-1)^{3/2}} \left[\frac{\gamma^2(\zeta-1)-\zeta}{b^2} K(-\frac{a^2}{b^2}) - \frac{\gamma^2(\zeta-2)-(\zeta-1)}{b^2+a^2} E(-\frac{a^2}{b^2}) \right] + (1 \leftrightarrow 2)$$



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What is the eikonal?

- Eikonal = **scattering generator** (See also Riccardo's talk) [JH Kim, JWK, Lee; Gonzo, Shi]

$$\mathcal{O}_{\text{out}} = e^{\{\chi, \bullet\}}[\mathcal{O}_{\text{in}}] = \mathcal{O}_{\text{in}} + \{\chi, \mathcal{O}_{\text{in}}\} + \frac{1}{2!}\{\chi, \{\chi, \mathcal{O}_{\text{in}}\}\} + \frac{1}{3!}\{\chi, \{\chi, \{\chi, \mathcal{O}_{\text{in}}\}\}\} + \cdots$$

  - (KMOC) + ($S = e^{\frac{i}{\hbar}N}$) + ($\hbar \to 0$) [Kosower, Maybee, O'Connell; Lehmann, Symanzik, Zimmermann; Damgaard, Plante, Vanhove; Hansen]

$$\mathcal{O}_{\text{out}} = S^\dagger \mathcal{O}_{\text{in}} S = e^{-\frac{i}{\hbar}N} \mathcal{O}_{\text{in}} e^{+\frac{i}{\hbar}N} = \sum_{n=0}^{\infty} \frac{1}{n!} \left(-\frac{i}{\hbar}\right)^n \underbrace{[N, [N, [N, \cdots, [N,}_{n \text{ times}} \mathcal{O}_{\text{in}}]] \cdots]]$$

- *Magnus series*: power series expansion of N-matrix [JH Kim, JWK, S Kim, Lee]
  - Commutator expansion ⇒ causal propagators (tree graphs)
    - Especially relevant for (classical) WQFT [Mogull, Plefka, Steinhoff; Jakobsen, Sauer]
  - Real-valued & IR-finite 3PM eikonal from WQFT [JH Kim, JWK, S Kim, Lee]

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The role of nested brackets

- Impact parameter space is "noncommutative" (due to $(b \cdot p_i) = 0$)

$$\{b^\mu, b^\nu\} \doteq \frac{b^\mu w_1^\nu - b^\nu w_1^\mu}{m_1} - \frac{b^\mu w_2^\nu - b^\nu w_2^\mu}{m_2}$$

$$\gamma = -(v_1 \cdot v_2)$$
$$p_i^\mu = m_i v_i^\mu$$
$$w_i \cdot v_j = -\delta_{ij}$$

- Poisson brackets = *causality cuts* [JH Kim, JWK, S Kim, Lee]
  - Special case of Peierls brackets (≈ covariantised Poisson brackets)
  - Join vertices by retarded-minus-advanced Green's ftn.

$$-i\{V_1, V_2\} = \sum_\phi \left[(\Delta V_1^+) \xrightarrow{\phi} (\Delta V_2^+) - (\Delta V_1^+) \xleftarrow{\phi} (\Delta V_2^+) \right]$$

  - Poisson brackets can be computed *diagrammatically*
    - ($\mathcal{X}$ from Magnus series) + ($e^{\{\mathcal{X}, \bullet\}}[p_1^\mu]$ as c. cuts) = (impulse diagrams = rooted trees)

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Adopt manifestly orthogonal definition of IP!

$$b^\mu = \Delta x^\mu + (\Delta x \cdot v_1) w_1^\mu + (\Delta x \cdot v_2) w_2^\mu$$

$$\Delta x^\mu := x_1^\mu - x_2^\mu \,, \; w_1^\mu := \frac{\gamma v_2^\mu - v_1^\mu}{\gamma^2 - 1} \,, \; w_2^\mu := \frac{\gamma v_1^\mu - v_2^\mu}{\gamma^2 - 1}$$

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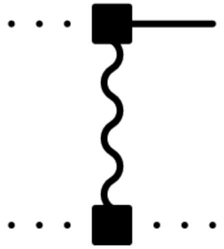
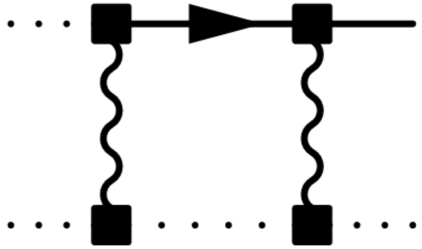
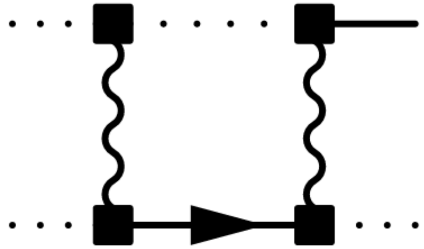
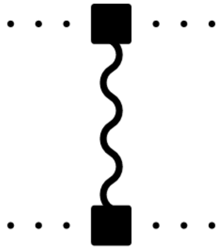
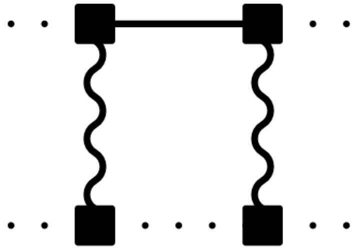
Nested brackets in twistor WL

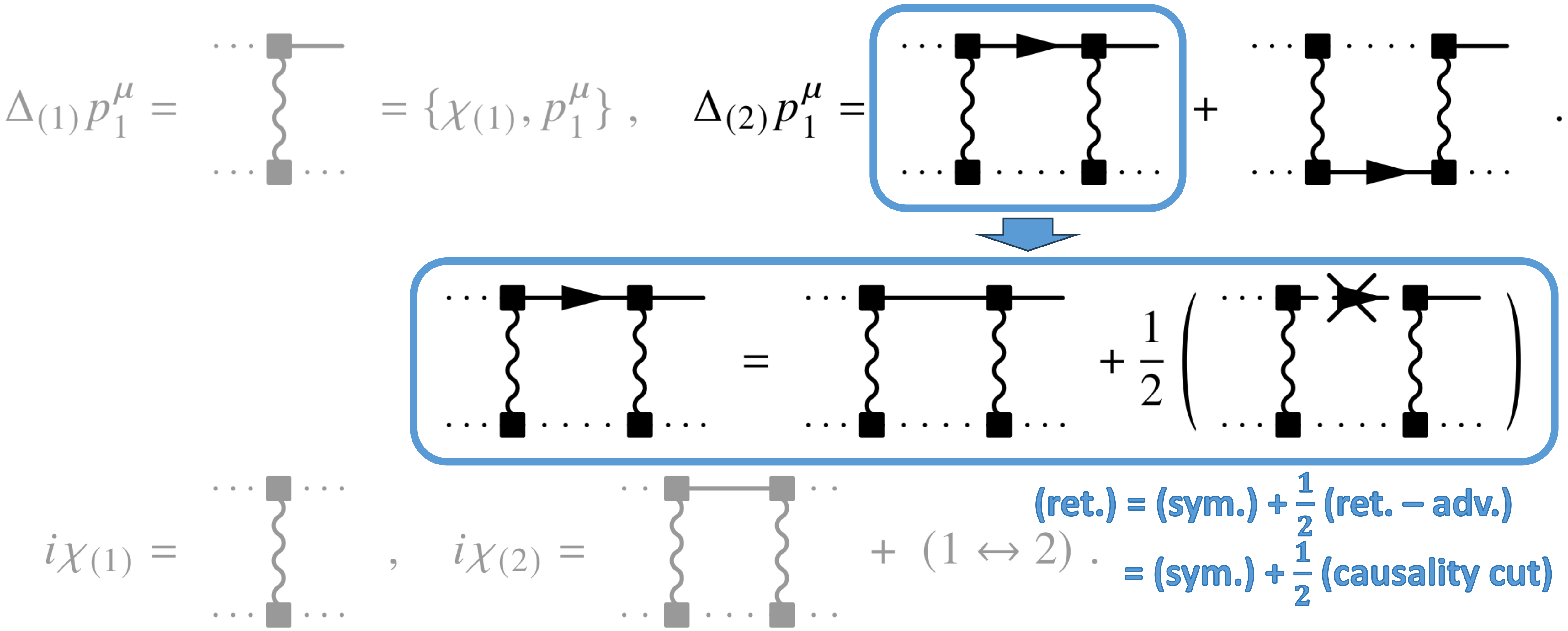
[JH Kim, JWK, Lee; JH Kim, JWK, S Kim, Lee]

$\Delta_{(1)} p_1^\mu =$  $= \{\chi_{(1)}, p_1^\mu\}$, $\Delta_{(2)} p_1^\mu =$  $+$  .

$i\chi_{(1)} =$  , $i\chi_{(2)} =$  $+ \ (1 \leftrightarrow 2)$.

Nested brackets in twistor WL

[JH Kim, JWK, Lee; JH Kim, JWK, S Kim, Lee]



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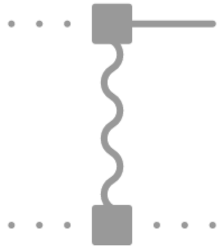
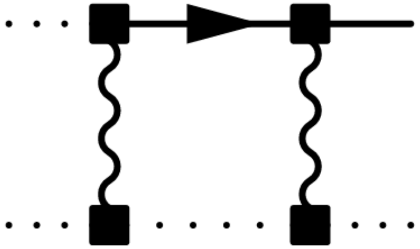
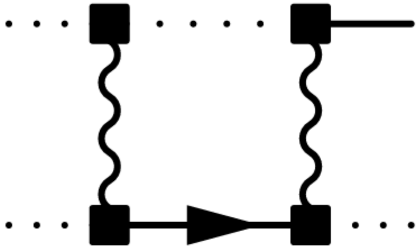
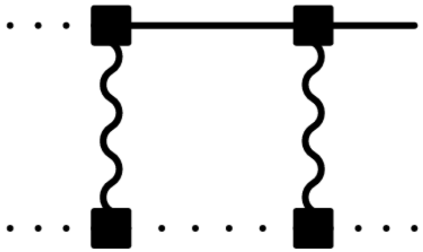
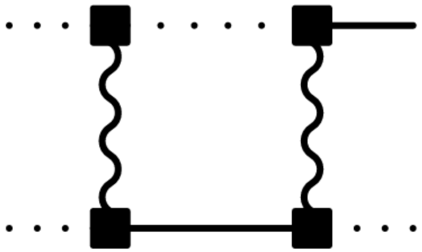
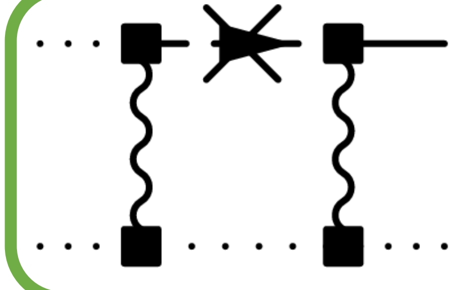
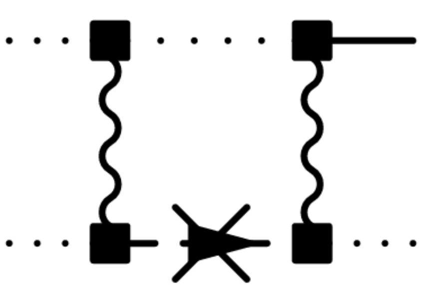
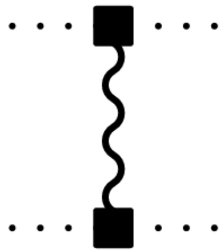
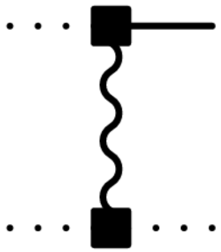
$\Delta_{(1)} p_1^\mu =$  $= \{\chi_{(1)}, p_1^\mu\}$, $\Delta_{(2)} p_1^\mu =$  $+$  .

(sym.) = (eikonal)

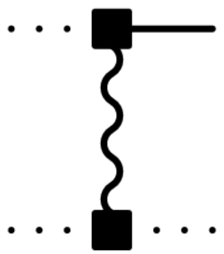
 $+$  $= \{\chi_{(2)}, p_1^\mu\}$

(c. cut) = (nested P.B.)

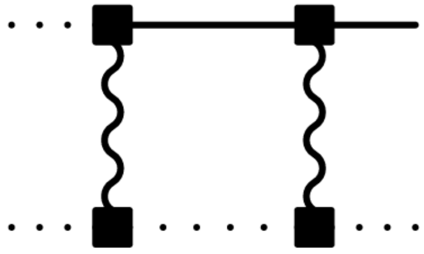
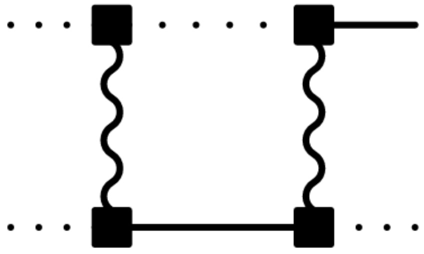
 $+$  $= -i \Big\{$  ,  $\Big\} = \{\chi_{(1)}, \{\chi_{(1)}, p_1^\mu\}\}$

Nested brackets in twistor WL

[JH Kim, JWK, Lee; JH Kim, JWK, S Kim, Lee]

$\Delta_{(1)} p_1^\mu =$  $= \{\chi_{(1)}, p_1^\mu\}$

(1PL eikonal)

 $+$  $= \{\chi_{(2)}, p_1^\mu\}$

(2PL eikonal)

$$\mathcal{O}_{\text{out}} = e^{\{\chi, \bullet\}}[\mathcal{O}_{\text{in}}] = \mathcal{O}_{\text{in}} + \{\chi, \mathcal{O}_{\text{in}}\} + \frac{1}{2!}\{\chi, \{\chi, \mathcal{O}_{\text{in}}\}\} + \cdots$$

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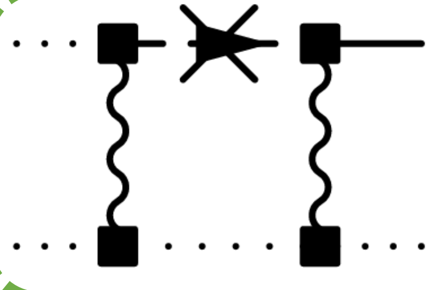
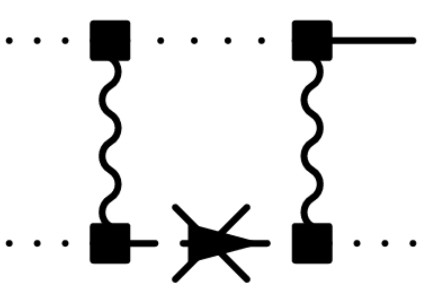
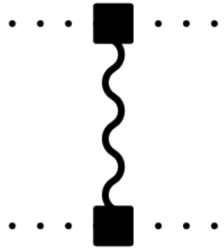
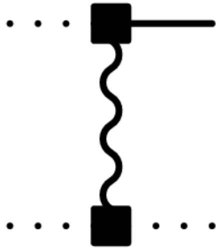
 $+$  $= -i \Big\{$  ,  $\Big\} = \{\chi_{(1)}, \{\chi_{(1)}, p_1^\mu\}\}$

Remarks on nested brackets

- The reorganisation extends to spinning observables
  - Impulse & spin kick @ $\mathcal{O}(G^3 S^4)$ (See also Mao's talk) [Akpinar, Cordero, Kraus, Smirnov, Zeng]
  - Dirac brackets = Poisson brackets (for variables manifestly satisfying constraints)
- Nested brackets implement rotations [JWK]

$$\{\chi, \bullet\} = \sum_{i=1,2} \{\chi, p_i^\mu\} \frac{\partial}{\partial p_i^\mu} + \sum_{i=1,2} \{\chi, S_i^{\mu\nu}\} \frac{\partial}{\partial S_i^{\mu\nu}} + \{\chi, b^\mu\} \frac{\partial}{\partial b^\mu}$$

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  - Nested brackets are (causality) cuts
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Scattering generator approach to 3PM impulse

- The 3PM eikonal [Damgaard, Plante, Vanhove; Brandhuber, Chen, Travaglini, Wen; JH Kim, JWK, S Kim, Lee]

$$\chi_{(1)} = -Gm_1m_2\frac{2\gamma^2-1}{\sqrt{\gamma^2-1}}\log|b^2|\,,$$

$$\chi_{(2)} = +\frac{3\pi}{4}G^2m_1m_2(m_1+m_2)\frac{5\gamma^2-1}{\sqrt{\gamma^2-1}}\frac{1}{|b^2|^{1/2}}\,,$$

$$\chi_{(3)} = \frac{G^3m_1m_2}{3\sqrt{\gamma^2-1}|b^2|}\left[(m_1^2+m_2^2)X_{(3,0)}+m_1m_2(X_{(3,1c)}+X_{(3,1r)})\right]\,,$$

$$X_{(3,0)} = \frac{\left(64\gamma^6-120\gamma^4+60\gamma^2-5\right)}{\left(\gamma^2-1\right)^2}\,,$$

$$X_{(3,1c)} = \frac{2\gamma(36\gamma^6-114\gamma^4+132\gamma^2-55)}{(\gamma^2-1)^2}-\frac{12(4\gamma^4-12\gamma^2-3)}{\sqrt{\gamma^2-1}}\operatorname{arccosh}(\gamma)\,,$$

$$X_{(3,1r)} = (2\gamma^2-1)^2\left[-2\frac{(5\gamma^2-8)}{(\gamma^2-1)^{3/2}}+\frac{6\gamma(2\gamma^2-3)}{(\gamma^2-1)^2}\operatorname{arccosh}(\gamma)\right]\,.$$

Scattering generator approach to 3PM impulse

- Reference available [Herrmann, Parra-Martinez, Ruf, Zeng; Jakobsen, Mogull, Plefka, Sauer; Kälin, Liu, Neef, Porto]
  - Misses (radiation loss) = (momentum lost by radiated graviton)

$$\Delta_{(3)}p_1^\mu = \Delta_{(3,c)}p_1^\mu + \Delta_{(3,rr)}p_1^\mu + \Delta_{(3,rl)}p_1^\mu$$

$$e^{\{\chi,\bullet\}}p_1^\mu\Big|_{\mathcal{O}(G^3)} = \Delta_{(3,c)}p_1^\mu + \Delta_{(3,rr)}p_1^\mu$$

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[2207.00569]

```
(Humboldt3PM // ListToPlus) - impulse3PMsimp (* exactly radiaton loss term *)
```

$$\frac{\gamma\,(33-112\gamma^2+165\gamma^4-70\gamma^6)\,G^3 m_1^2 m_2^2\,\pi\,\mathrm{ArcCosh}[\gamma]\,(u_\perp)_1{}^i}{16\,(-1+\gamma^2)^2\,(b_i\, b^i)^{3/2}} + \frac{(-5+\gamma\,(76+5\gamma\,(-30+\gamma\,(12+7\gamma))))\,G^3 m_1^2 m_2^2\,\pi\,\mathrm{Log}\left[\frac{1+\gamma}{2}\right]\,(u_\perp)_1{}^i}{8\sqrt{-1+\gamma^2}\,(b_i\, b^i)^{3/2}} +$$

$$\frac{(-1151+\gamma\,(3336+\gamma\,(-3148+3\gamma\,(304+\gamma\,(-113+2\,(92-35\gamma)\,\gamma)))))\,G^3 m_1^2 m_2^2\,\pi\,(u_\perp)_1{}^i}{48\,\left((-1+\gamma^2)\,(b_i\, b^i)\right)^{3/2}}$$

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- Which contributions did we miss?
  - (Graviton phase space contributions) = (diagrams from graviton c. cuts)
  - Computed from *radiation eikonal* as $\Delta_{(3,rl)} p_1^\mu = \frac{1}{2}\{\chi_{(1.5)}^{\rm rad}, \{\chi_{(1.5)}^{\rm rad}, p_1^\mu\}_\circ\}_\circ$ [JWK]

The radiation eikonal

[JWK]

- Missing contributions from graviton phase space / causality cuts
  - No vertex rules where graviton legs can be attached ⇒ add new vertices!
  - Formal implementation: turn on background $H_{\mu\nu}$
  - Practical implementation: add diagrams with external graviton legs

- Expanded set of eikonal-derived PM observables
  - Impulse radiation loss @ 3PM
  - Radiative observables: scattering waveform & radiated energy
  - New perspective on "KMOC cuts" in scattering WFs [Caron-Huot, Giroux, Hannesdottir, Mizera; Georgoudis, Heissenberg, Russo; Bini, Damour, De Angelis, Geralico, Herderschee, Roiban, Teng; Alessio, Gonzo, Shi]
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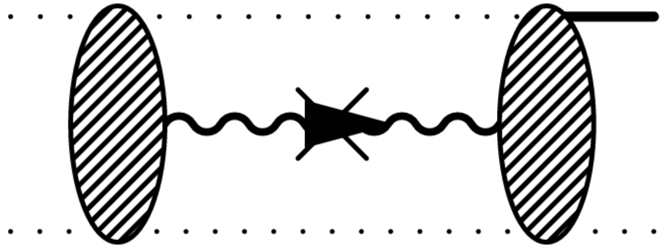
Impulse radiation loss @ 3PM

[JWK]

- Computed as $\Delta_{(3,rl)} p_1^\mu = \frac{1}{2}\{\chi^{\text{rad}}_{(1.5)}, \{\chi^{\text{rad}}_{(1.5)}, p_1^\mu\}_\circ\}_\circ$, *exact* match!

$$\Delta_{(3,rl)} P_1^\mu = \frac{1}{2}\int \frac{d^Dk}{(2\pi)^D}\frac{d^Dq}{(2\pi)^D}\left(\frac{\partial \chi^{H^1}_{(1.5)}}{\partial H_{\alpha\beta}(k)}\right) \times \{H_{\alpha\beta}(k), H_{\lambda\sigma}(q)\} \times \left(\frac{\partial}{\partial H_{\lambda\sigma}(q)}\left[-\frac{\partial \chi^{H^1}_{(1.5)}}{\partial X_{1\mu}}\right]\right)$$

$$= \frac{1}{2}\Bigg[$$  $$\Bigg]$$

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NLO scattering waveform

[JWK]

- Nested brackets (= cut contributions) compute rotated LO WF

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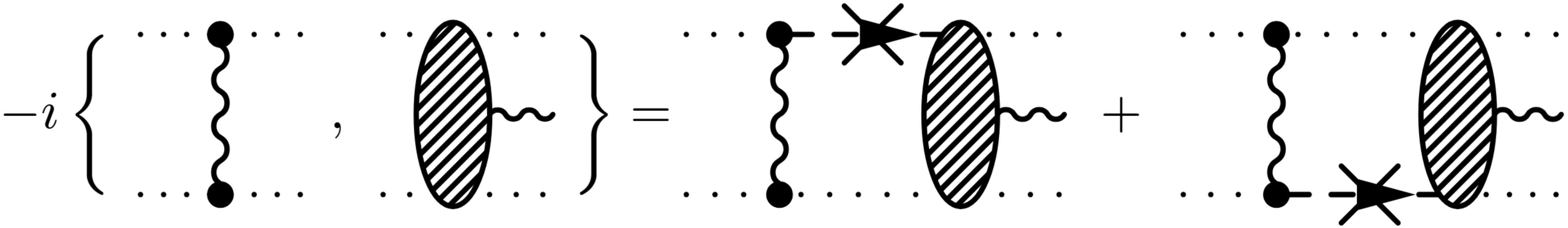

Take-home messages

- *Spin resummation* will become important for next-gen GW detectors
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  - Impact parameter space is "noncommutative": $\{b^\mu, b^\nu\} \neq 0$
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Some future directions

- B2B continuation for *nonseparable* HJ systems [Cho, Kälin, Porto; Gonzo, Shi]
  - Eikonal-based WF models for bound binaries? [JWK, Patil, Scheopner, Steinhoff (WIP)]
- Local-in-time / hereditary separation? [Dlapa, Kälin, Liu, Porto; Buonanno, Mogull, Patil, Pompili]
  - (hereditary contributions) $\stackrel{?}{=}$ (radiation eikonal contributions)
- Nonlinear radiation eikonal (e.g. $\chi_{(2)}^{H^2}$: quadratic in $H_{\mu\nu}$)
  - Contributes to NNLO WF / 5PM impulse [Driesse, Jakobsen, Mogull, Plefka, Sauer, Usovitsch; Bern, Herrmann, Roiban, Ruf, Smirnov, Smirnov, Zeng]
  - Is 6pt relevant for classical physics? [Cristofoli, Gonzo, Moynihan, O'Connell, Ross, Sergola, White]
- Spin resummation @ 3PL / 3PM (1SF corrections: relevant for EOB)
  - Obstruction: Integrals (Tensor Integral Generating Functions / Twisted FI) [Feng; Brunello, Crisanti, Giroux, Mastrolia, Smith; Brunello, De Angelis; Chen, JWK, Wang; JH Kim, JWK, Lim (WIP)]
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