All orders Structures in the Gravitational Scattering Ampltudes in the Regge Limit

Amplitudes 25

IZR in Collaboration with Michael Saavedra

Carnegie Mellon University

- 2412.04428 [hep-th]

We are interested in the form of the gravitational scattering amplitude in the Ultra-Planckian Limit:

Regimes:

$S \sim m^2 \gg t$	Inspirals
$S \gg m^2 \gg t$	High Energy: Rel. inspiral
$S \gg t \gg m^2$	Regge Limit

1) What is the structure of the peturbrative series in G (non-analytic dependence on t) for the amplitude as well as the classical observables?

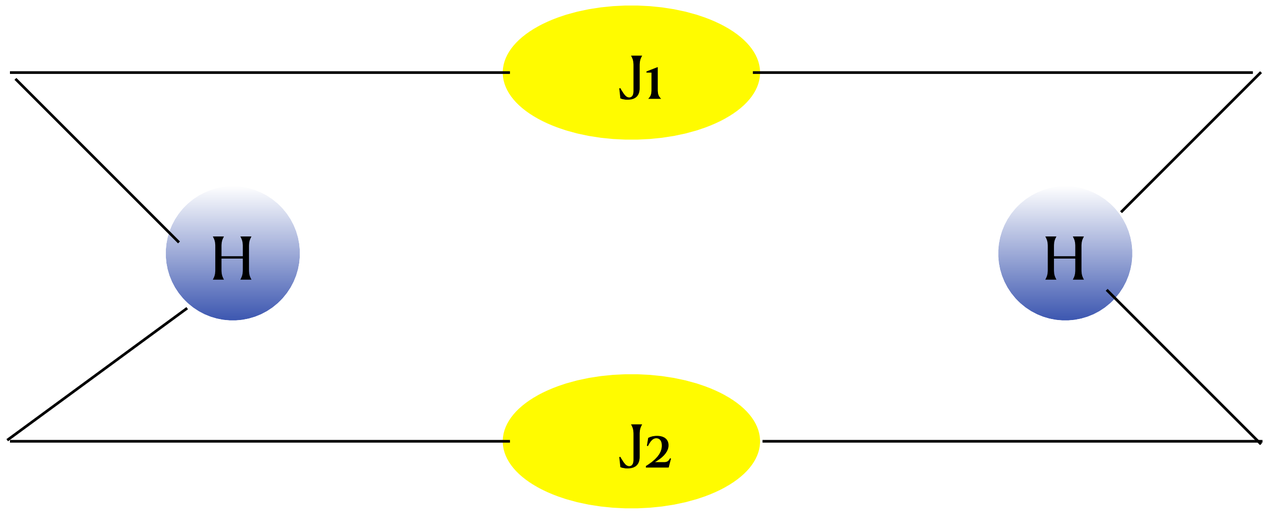
2) Can we sysematically perform resummations of non-analytic terms?

What can we learn from YM Theory?

Generic Hard Scattering

$$s \sim t \sim u \gg \Lambda_{QCD} \qquad \sigma \equiv \sigma(\alpha_s(s), \frac{1}{\epsilon_{IR}}, \log(s/\mu))$$

Drell-Yan



$$\sigma \sim \int J_1 \otimes J_2 \otimes H$$

Jest composed of collinear modes: $k^\mu_{J_1} \sim (Q, \Lambda^2/Q, \Lambda)$ $\quad k^\mu_{J_2} \sim (\Lambda^2/Q, Q, \Lambda)$

$k^\mu_H \sim (Q, Q, Q)$ (Off-shell)

EFT Approach:

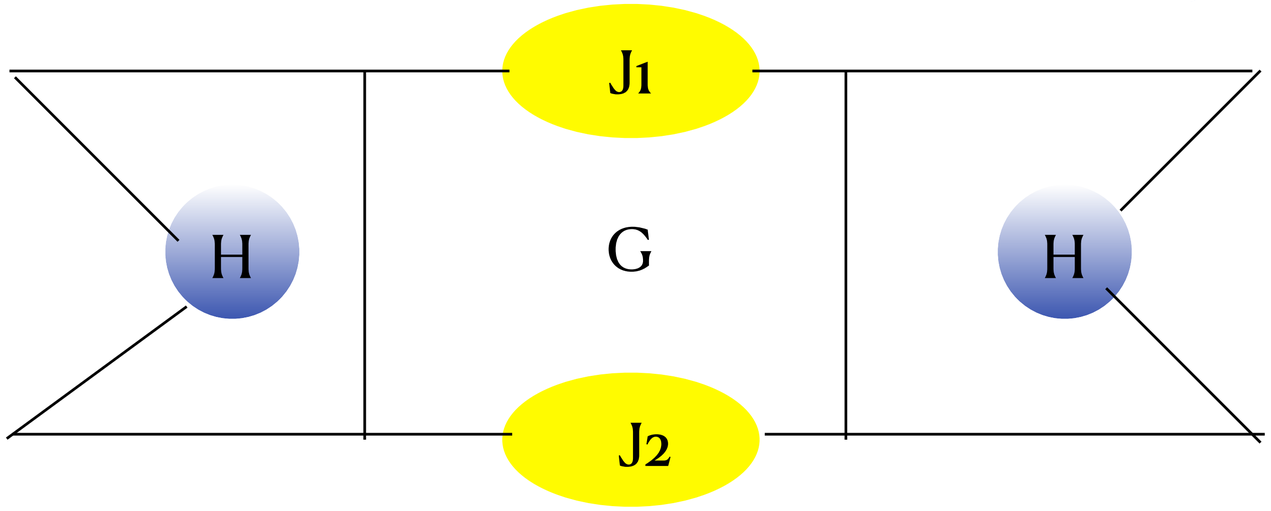
Emergent gauge symmetry G $\quad SU(N)_n \otimes SU(N)_{\bar{n}} \otimes SU(N)_s$

$$L = L_n + L_{\bar{n}} + L_s$$

Bauer et al: [hep-ph/0005275]

- [hep-ph/0011336] [hep-ph/0202088]

What can we learn from YM Theory?



$$\sigma \sim \int J_1 \otimes J_2 \otimes H$$

EFT needs Augmentation (GSCET)

IZR/Stewart

- [1601.04695]

Challenge is to prove that G does not contribute to all orders

$$k_G^\mu \sim (\Lambda^2, \Lambda^2, \Lambda)$$

(Off-shell)

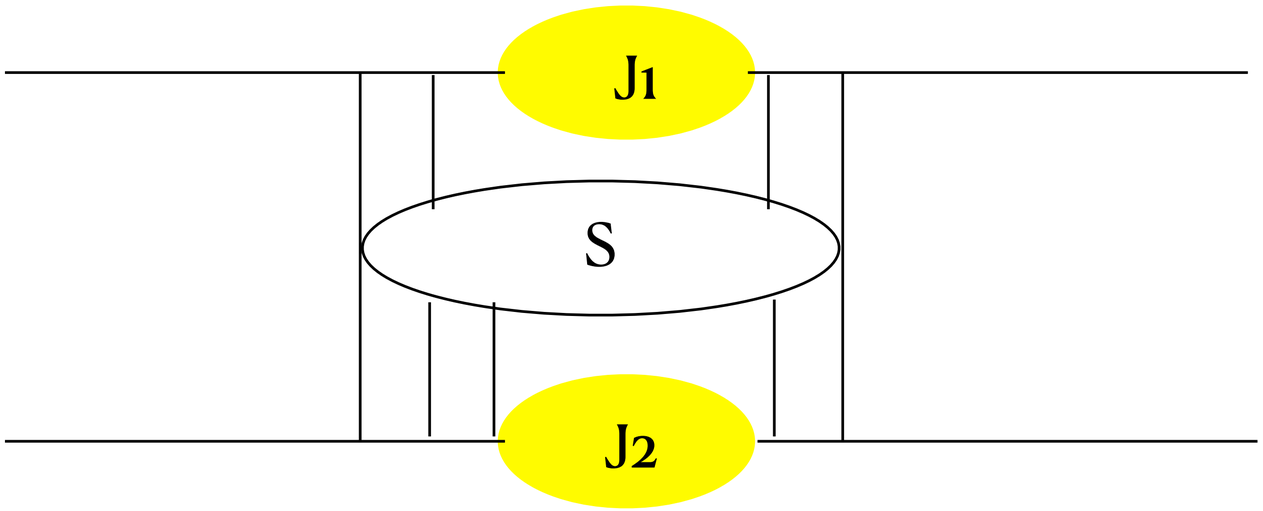
Challenge is to prove that G does not contribute to all orders

CSS (1985), EFT proof IZR/Stewart (un-published)

What can we learn from YM Theory?

Regge Limit: s>>t

Now the Glauber becomes the object of interest



$$M \equiv M(\frac{1}{\epsilon_{IR}}, \log(s/t))$$

To maintain calculational control we must have method to resum the Regge logs.

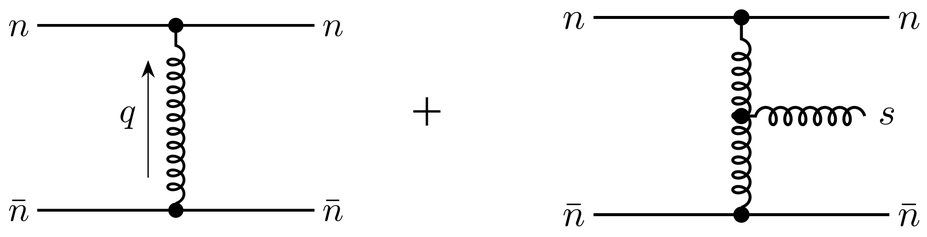
However, these logs are not RG logs, they are rapidity logs and necessitate a distinct treatment.

What can we learn from YM Theory?

GSCET:

$$\mathcal{O}_n^\phi \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_{\bar{n}}^\phi$$

One Gber exchange operator



Operating building blocks individual (G) gauge invariant.

(e.g.) $\mathcal{O}_n^{qB} = \overline{\chi}_n T^B \frac{\bar{\not{n}}}{2} \chi_n$ $\chi_n = W_n^\dagger \xi_n$

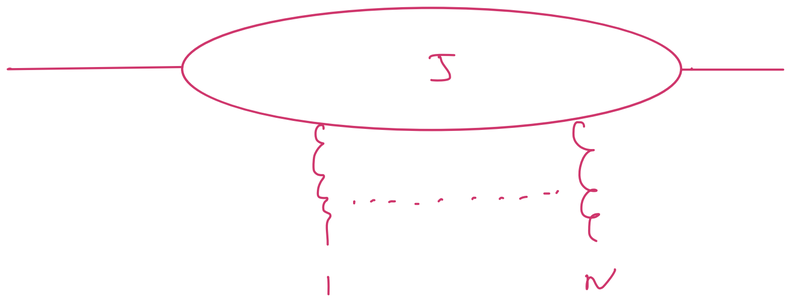
As a consequence each Feynam rule will be individually gauge invariant reproducing multiple full theort contributions.

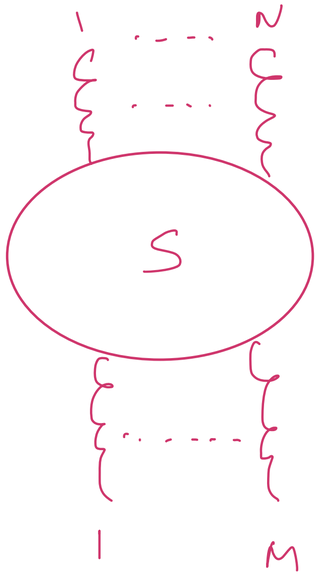
e.g. ``Lipatov vertex" for soft emission corresponding to multuple soft emissions will be gauge invariant.

What can we learn from YM Theory?

Despite breaking of factorization we can still write the amplitude in convolutive form

$$M^{\kappa\kappa'} = i \sum_{MN} \int \int_{\perp(N,M)} J_{\kappa N}(\{l_{\perp i}\}, \epsilon, \eta) S_{(N,M)}(\{l_{\perp i}\}; \{l'_{\perp i}\}, \epsilon, \eta) \bar{J}_{\kappa' M}(\{l'_{\perp i}\}, \epsilon, \eta)$$





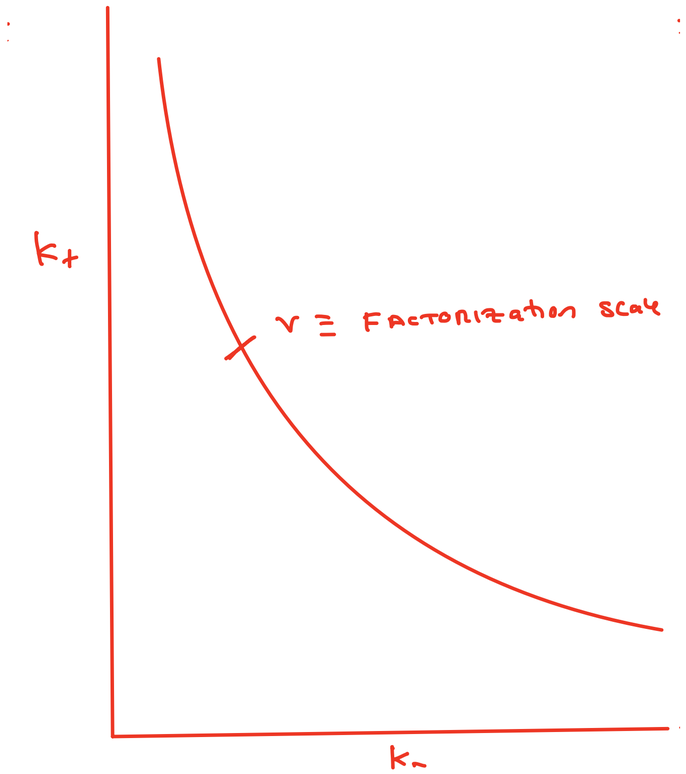
Decompose color channels into irreps

Gao et al) [2401.00931]

What can we learn from YM Theory?

Since we have artifically separated soft from collinear modes this introduces divergences what are not regulated by dim. Reg.

$$I = \int [d^d k] \frac{1}{(n \cdot k - \Delta_1)} \mid 2k_z/\nu \mid^{-\eta}$$



Need to introduce a rapidity factorization scale.

Generates polls in $\frac{1}{\eta}$

$$\frac{1}{\eta} + (\log(t/\nu), \log(p_+/\nu), \log(p_-/\nu))$$

Soft Col Acol

What can we learn from YM Theory?

Full Amplitude must be independent of rapidity regulator (Rapidity Renormalization Group (RRG)

Neil et al [1104.0881]

CSS (NPB 1985)

Korchemsky+Radyushkin (PLB 1987)

Balitsky [hep-ph/9509348]

$$\frac{d}{d\nu}(J_A \otimes S_{A,B} \otimes J_B) = 0$$

$$\nu \frac{\partial}{\partial \nu} J_{\kappa(i)} = \sum_{j=1}^{\infty} J_{\kappa(j)} \otimes \gamma^J_{(j,i)},$$

$$\nu \frac{\partial}{\partial \nu} S_{(i,j)} = -\sum_{k=1}^{\infty} \gamma^S_{(i,k)} \otimes S_{(k,j)} - \sum_{k=1}^{\infty} S_{(i,k)} \otimes \gamma^S_{(k,j)},$$

$$\nu \frac{\partial}{\partial \nu} \bar{J}_{\kappa'(i)} = \sum_{j=1}^{\infty} \gamma^{\bar{J}}_{(i,j)} \otimes \bar{J}_{\kappa'(j)}.$$

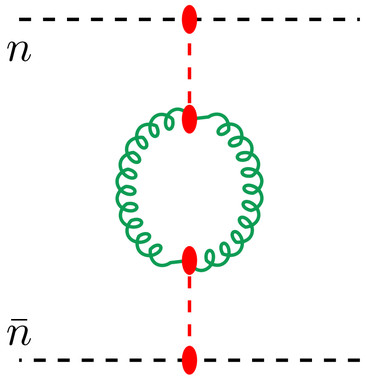
We can sum the logs by

running the soft/Jet functions to $\nu = (\sqrt{t}, \sqrt{s})$

What can we learn from YM Theory?

Running of S(1,1) leads to the Gluon Regge Trajectory

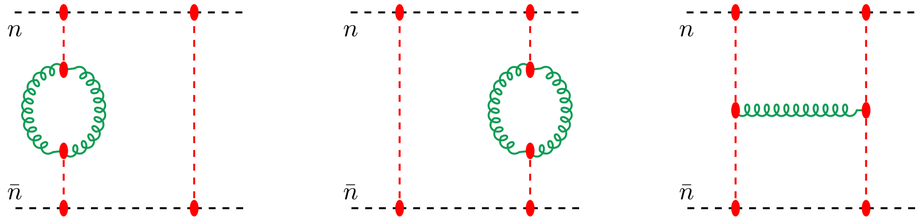
$$M \sim \left(\frac{s}{-t}\right)^{\alpha_R(t)}$$



$$\alpha_R(t) = \frac{\alpha_s C_A}{2\pi} \log(-t/m^2)$$

Running of S(2,2) leads to the BFKL equation

$$\nu \frac{\partial}{\partial \nu} S_{(2,2)}(k_{1\perp}, k_{2\perp}) = \frac{1}{2} \int \frac{[d^{d'} p_\perp]}{p_\perp^2 (p_\perp - q_\perp)^2} \left(\gamma_{(2,2)}(k_{1\perp}, p_\perp) S_{(2,2)}(p_\perp, k_{2\perp}) + S_{(2,2)}(k_{1\perp}, p_\perp) \gamma_{(2,2)}(p_\perp, k_{2\perp}) \right).$$



$$\gamma^{A_1 A_2; B_1 B_2}_{(2,2),\mathrm{YM}} = 4\alpha_s f^{A_1 B_1 C} f^{A_2 B_2 C} K_{\mathrm{YM}}(k_{1\perp}, k_{2\perp}) + 2\delta^{A_1 B_1} \delta^{A_2 B_2} \delta^{d-2}(k_{1\perp} - k_{2\perp}) k_{2\perp}^2 (k_{2\perp} - q_\perp)^2 (\alpha_R(k_{2\perp}) + \alpha_R(k_{2\perp} - q_\perp))$$

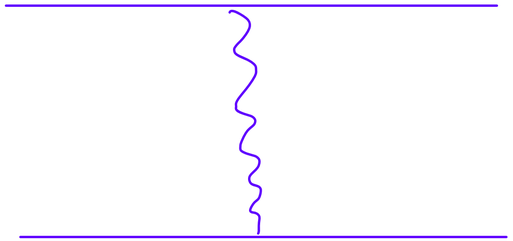
$$K_{\mathrm{YM}}(k_{1\perp}, k_{2\perp}) = q_\perp^2 + \frac{k_{2\perp}^2 (q_\perp - k_{1\perp})^2 + k_{1\perp}^2 (k_{2\perp} - q_\perp)^2}{(k_{2\perp} - k_{1\perp})^2}$$

The Gravitational Case

In Regge Gravity we have three expansion parameters:

$$\alpha_Q \equiv \frac{t}{M_{pl}^2} < 1 \qquad \alpha_C \equiv \frac{st}{M_{pl}^4} < 1 \qquad \lambda = \sqrt{t/s} \qquad \alpha_Q \ll \alpha_C$$

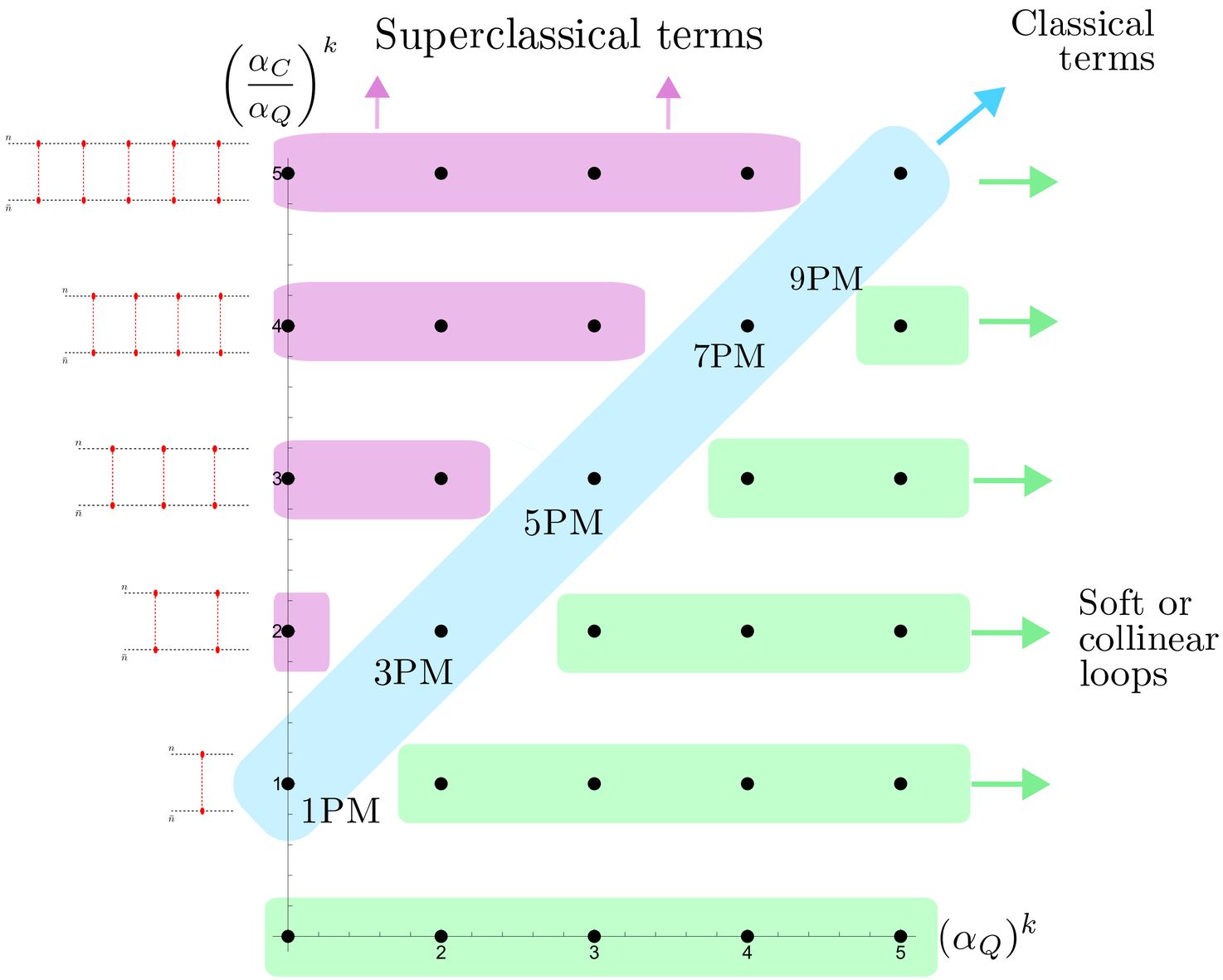
No BH production



$\sim \frac{\alpha_c}{\alpha_q}$!!

Which poses some challenges for power counting. However, given that the Regge limit is semi-classical we know that the Glauber ladders sum to the eikonal exponential in b space.

$$M(b,s) \sim \left((1 + [\sum_{i=0} \alpha_Q^i C_i(bs)]) e^{i\delta_{\rm Cl}^{(0)} \sum_{j=0} (\alpha_C^j D_j(bs))} - 1 \right)$$



Each soft/collinear loop will lead to one high power of log

$$M(nPM) \sim \log^{n-2}(s/t) + \log^{n-3}(s/t) + \ldots$$

GGSCET

Izr/Saavedra • 2412.04428 [hep-th]

Hard scattering GSCET:

Beneke and Kirilin • 1207.4926 [hep-ph]

Okui and Yunesi 1710.07685

Relevant Degrees of Freedom: $g^{s}_{\mu\nu}, g^{c}_{\mu\nu}, g^{\bar{n}}_{\mu\nu}, \phi_c . \phi_{\bar{n}}$

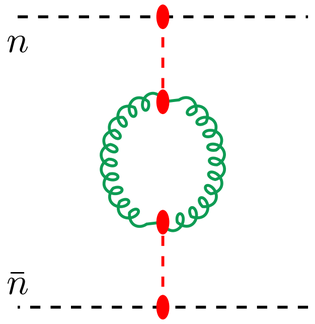
Symmetries: $Diff_s \times Diff_n \times Diff_{\bar{n}}$

The basic algorithm is the same the complication arises in that we need the full non-linear gravitational Wilson lines

Now we can build a collinear diff invariant operator which mediates one Glauber exchange

$$O_g = \bar{\chi}_n \chi_n \frac{1}{q^2} \bar{\chi}_{\bar{n}} \chi_{\bar{n}}$$

The (rapidity) anomalous dimension of this operator is the Regge trajectory (S(1,1))



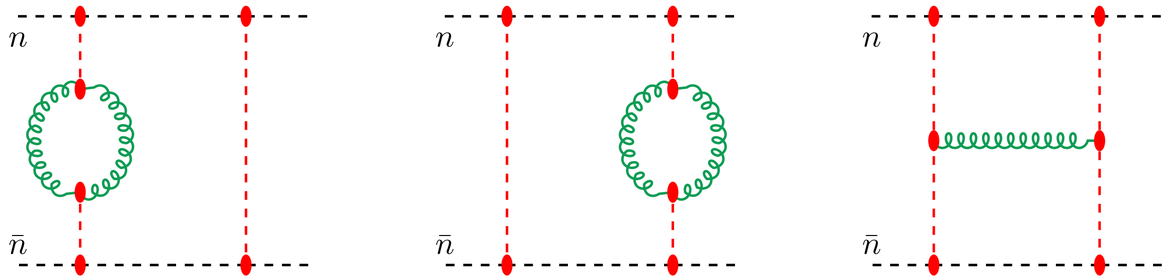
$$= -i \frac{\kappa^4 s^2 w^2}{8\pi\eta} (3-2\epsilon) q_\perp^2 \int \frac{[d^{d'} k_\perp]}{k_\perp^2 (k_\perp - q_\perp)^2}$$

$$= i \frac{\kappa^4 s^2 w^2}{32\pi^2 \eta} (3-2\epsilon) \frac{\Gamma(-\epsilon)^2 \Gamma(1+\epsilon)}{\Gamma(-2\epsilon)} \left(\frac{-t}{\bar{\mu}^2}\right)^{-\epsilon}$$

$$\omega_G(t) = \frac{\kappa^2 t}{16\pi^2} \left(-\frac{3}{\epsilon} + 3 \log \frac{-t}{\mu^2} + 2 + O(\epsilon) \right)$$

Agrees with Lipatov seems disagrees with Schnitzer et al.

BFKL Equation (renormalizing two Glauber Exchange)



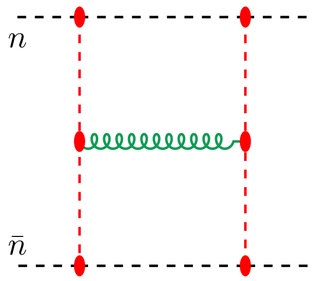
$$\gamma_{(2,2)}(k_\perp, p_\perp) = \frac{\kappa^2}{4\pi} K_{\mathrm{GR}}(k_\perp, p_\perp) + 2\delta^{d-2}(k_\perp - p_\perp) p_\perp^2 (p_\perp - q_\perp)^2 (p_\perp^2 \omega_G(p_\perp) + (p_\perp - q_\perp)^2 \omega_G(p_\perp - q_\perp)).$$

Agrees with Lipatov Result. $K_{GR} = K_{QCD}^2$

Allows us to extract higher PM classical logs without having to do higher loops integrals

Hunting Classical Logs

To calculate the leading classical log at (2N+1) PM we need to calculate the N loop correction to the (N+1,N+1) Glauber.



$$\delta_{\rm Cl}^{(2,\log)} = i\log\left(\frac{\nu^2}{-t}\right)\frac{G^3 s^2}{b^2}\frac{\left(\bar{\mu}^2 b^2\right)^{3\epsilon}}{\pi^{1-\epsilon}2^{4\epsilon}}\frac{\Gamma(1-3\epsilon)\Gamma(-\epsilon)^3}{3\Gamma(2\epsilon)}\left(\frac{3\Gamma(-\epsilon)\Gamma(1+\epsilon)^2}{\Gamma(-2\epsilon)^2} - 2\frac{(3-2\epsilon)\Gamma(1+2\epsilon)}{\Gamma(-3\epsilon)}\right),$$
$$= i\log(s)\frac{4G^3 s^2}{b^2}\left(-\frac{1}{\epsilon}+2\right)\frac{\left(\bar{\mu}^2 b^2\right)^{3\epsilon}}{\pi^{1-\epsilon}2^{4\epsilon}} + O(\epsilon). \qquad (7.6)$$

Agrees with DiVecchia et al [2008.12743],

also agrees with up to $O(\epsilon^3)$

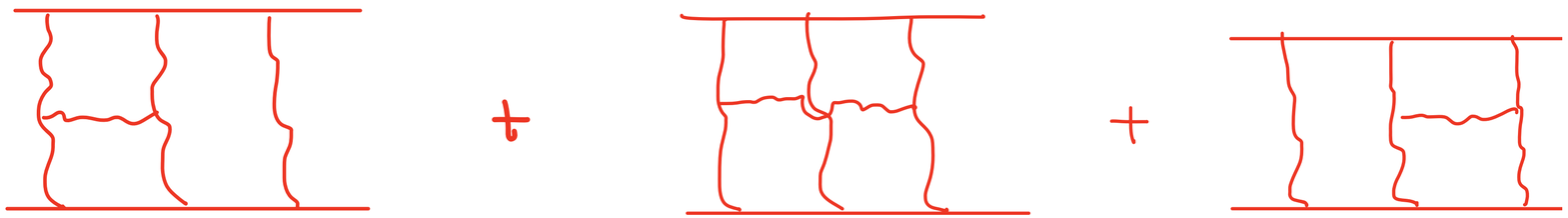
N=8 result Henn 1903.10075, DiVecchia et al [1911.11716

Now we can get the 5PM log by iterating RRG equation for S(3,3).

$$\gamma_{(N+1,N+1)} = -i^{N+1}(N+1)!\Bigg[\sum_{i<j}\frac{\kappa^2}{8\pi}K(k_{i\perp},k_{j\perp};\ell_{i\perp},\ell_{j\perp})\prod_{m\neq i,j}\ell_{m\perp}^2\delta^{d-2}(\ell_{m\perp}-k_{m\perp})$$

$$+\sum_j \omega_G(\ell_j)\prod_{m\neq j}\ell_{m\perp}^2\delta^{d-2}(\ell_{m\perp}-k_{m\perp})\Bigg].$$

Quantum



$$\nu\frac{\partial}{\partial\nu}S_{(N+1,N+1)}(\{k_{i\perp}\},\{\ell_{\perp_i}\}) = -\int_{\perp(N+1)}\Big(\gamma^{(1)}_{(N+1,N+1)}(\{k_{i\perp}\};\{\ell'_{j\perp}\})S_{(N+1,N+1)}(\{\ell'_{i\perp}\};\{\ell_{j\perp}\}) \tag{6.20}$$

$$+ S_{(N+1,N+1)}(\{k_{i\perp}\};\{\ell'_{j\perp}\})\gamma_{(N+1,N+1)}(\{\ell'_{i\perp}\};\{\ell_{j\perp}\})\Big)..$$

Solve iteratively:

$$S(l_i,k_i) = 16\log^2\nu\int\frac{[d^2l'_1][d^2l'_2]}{{l'_1}^2{l'_2}^2(q-l'_1-l'_2)}\gamma_{(3,3)}(l'_i;k_1,k_2,q-k_1-k_2)\gamma_{(3,3)}(l_i;l'_i)$$

$$iM(b) = \; i\pi^{\epsilon-2}2^{-8\epsilon}(G^5s^3)b^{10\epsilon-4}\exp(9\gamma\epsilon)\left(\frac{3}{8\pi^4\epsilon^2}-\frac{3}{\pi^4\epsilon}-\frac{3\zeta(3)}{4\pi^4}+\frac{5}{32\pi^2}+\frac{15}{2\pi^4}\right)\log^2(s)$$

Alessio,Del Duca,Gonzo,Rosi,IZR,Saavedra

What does this have to with the Relativistic Black Hole Inspiral case(s>>m^2>>t) ?

We will show that

1) The MSF diagrams do not contribute to the radiated momentum,

at all order in the PM expansion. No leading logs in radiated momentum up to

m^2/s corrections.

2) The leading log at each order in PM does not contribute to the impulse.

3) Leading log at each PM order will be identical to the massive case,

up to corrections of order m^2/s.

All of these claims follow from two emergent properties of the GGSCET

a) Reparameterization invariance (non-linear realization of Lorentz)

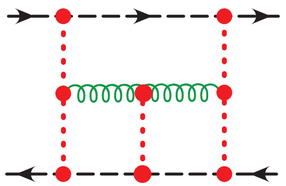
b) Rapidity Renormalization Invariance.

c) Factorization

1) Leading log at each PM order is maximally SF

- We can fix all the logs either by considering the Jets or the Soft, so we can focus on the Soft function at leading Log (collinears will untroduce powers of G.

- RPI dictates that $S_{(I,J)} \sim \delta_{IJ}$



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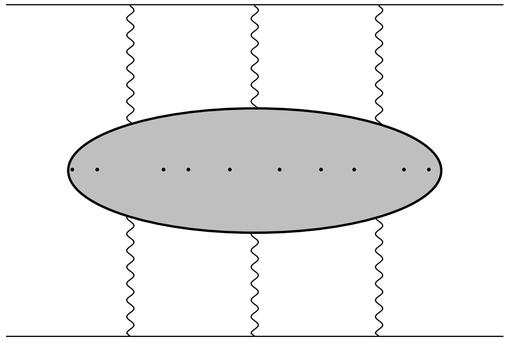
All non zero softs of the form



MSF terms

Which proves the assertion and since the softs are independent of the masses, the leading log in the massless case is identical to the massive case.

The vanishing of the leading log contribution to the radiated momentum follows since by covariance

$$\langle \mathcal{P}^\mu \rangle = A q_\perp^\mu + B p_1^\mu + C p_2^\mu$$

But the soft function is independent of p_1^μ, p_2^μ

So the leading log contribution (as well as the full set of gauge invariant diagrams at maximal SF) does not contribute to the radiated momentum.

Agrees at 3PM level with:

Di Vecchia et al. [2008.12743].

Herrmann et al. 2104.03957v1

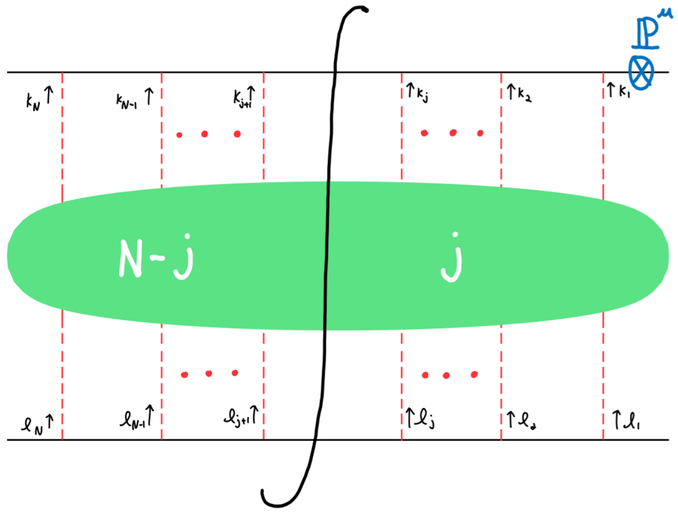
Leading Log (maximally SF) piece does not contribute to the impulse to all Order:

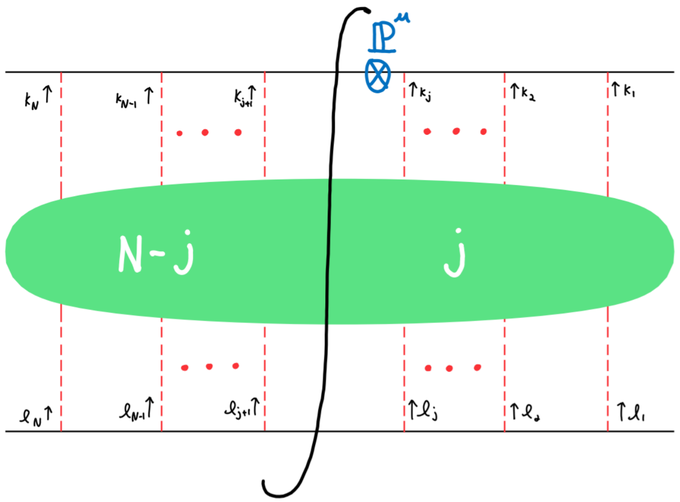
$$\langle \Delta p_n^\mu \rangle = \langle \psi_{\text{in}} | \, i[\mathbb{P}_n^\mu, T] \, | \psi_{\text{in}} \rangle + \langle \psi_{\text{in}} | \, T^\dagger [\mathbb{P}_n^\mu, T] \, | \psi_{\text{in}} \rangle$$

KMOC 1811.10950

$$\begin{aligned} \langle p_3, p_4 | \, i[\mathbb{P}_n^\mu, T] \, | p_1, p_2 \rangle &= i(p_3^\mu - p_2^\mu) \, \langle p_3, p_4 | \, T \, | p_1, p_2 \rangle \\ &= i q^\mu \mathcal{M}_{2 \to 2}, \end{aligned}$$

For second term:





Sum over all cuts gives q times uncat

How? In the EFT soft line and uncut lines are numerically same because the soft light-cone momentum do not appear in the collinear lines after multipole expanding. Thus soft are oblivous to the cuts. The integrals are then invariant under exchange of any Glauber momenta, thus if there are N Glaubers

Allows us to make the replacement $k_\perp^i \to -\frac{q_\perp}{N}$

Sumning over the N cuts then gives the sum of the second term in KMOC such that

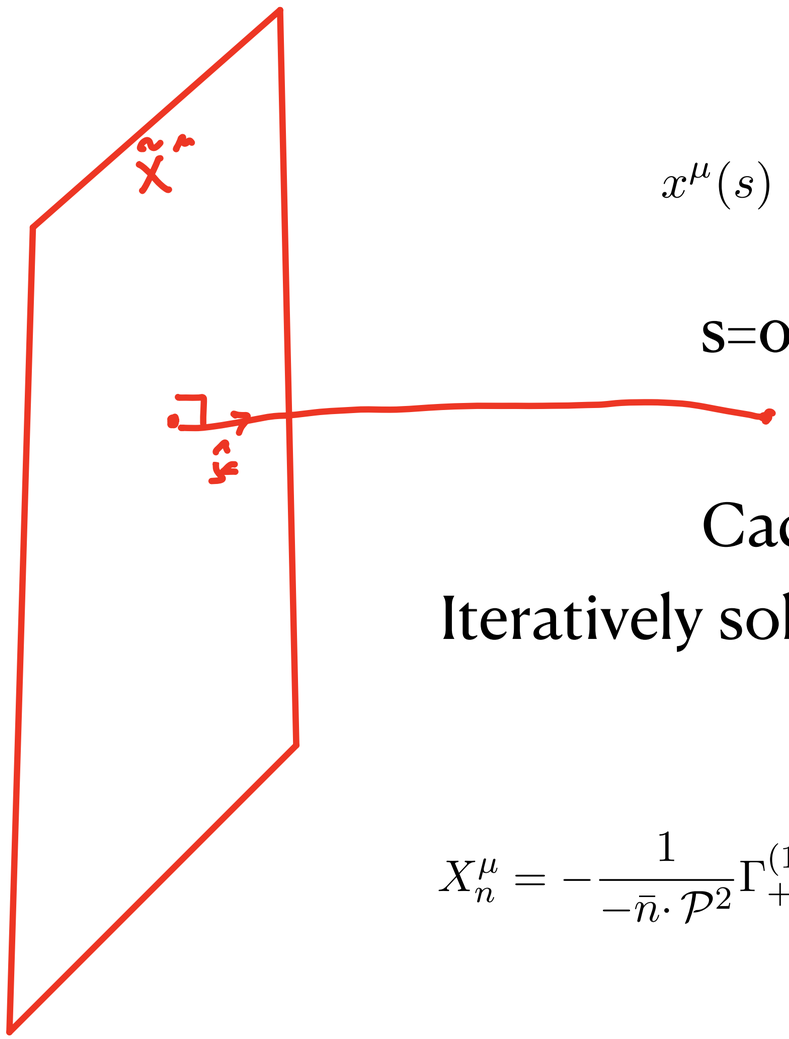
$$iq_\perp^\mu \mathcal{M}_{\text{amp}} + \mathcal{M}_{\text{cut}}^\mu = 0$$

We conclude that the leading logs diagrams do not contribute to the impulse at any order in PM expansion.

The massive theory is currently understudy

Teather the point to null infinity gives it a geometric meaning: i.e. choose the argument of the field such that it is invariant under diffs:

Giddings and Donnely (2015)



$$x^\mu(s) = \tilde{x}^\mu + v^\mu(0) + s\hat{z}^\mu$$

s=0

$$\phi(x^\mu - v^\mu(0))$$

Is the a diff invariant

Caclulate $v^\mu(0)$

Iteratively solve the geodesic equation $\frac{d}{ds}v^\mu(s) = -\Gamma^\mu(h)_{\rho\nu}v^\rho(s)v^\nu(s)$

$$X_n^\mu = -\frac{1}{-\bar{n}\cdot\mathcal{P}^2}\Gamma_{++}^{(1)\mu} - \frac{1}{-\bar{n}\cdot\mathcal{P}^2}\Gamma_{++}^{(2)\mu} - \frac{1}{-\bar{n}\cdot\mathcal{P}^2}\left(2\Gamma_{\nu+}^{(1)\mu}\frac{1}{-i\bar{n}\cdot\mathcal{P}}\Gamma_{++}^{(1)\nu} + \left(\mathcal{D}_\nu\Gamma_{++}^{\mu(1)}\right)\frac{1}{-\bar{n}\cdot\mathcal{P}^2}\Gamma_{++}^{(1)\nu}\right)$$