Amplitudes 2025, Seoul

Scattering in Self-Dual Backgrounds

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Black Hole Initiative, Harvard

Work with T. Adamo, L. Mason, G. Bogna

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- **Twistors** provide a path beyond.

- Many techniques developed for self-dual backgrounds.
  - Twistor actions and MHV rules for gluons [Adamo, Mason, AS ’20] [Adamo, Bogna, Mason, AS ’23, ’24] [Bogna, Mason ’23]
  - Twistor sigma models for gravitons [Adamo, Mason, AS ’21, ’22]
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- Shout-out to numerous other modern approaches!
  - Direct computations in non-sd plane wave spacetimes [Adamo, Casali, Mason, Nekovar…]
  - Cosmological polytopes [Arkani-Hamed, Benincasa, Postnikov…]
  - Bootstrap and single-valuedness techniques for AdS [Alday, Chester, Hansen…]

Twistor sigma models

- Arise as proto-celestial CFTs. [Adamo, Mason, AS '21]

$$S = \int_{\mathbb{P}^1} \frac{D\lambda}{\langle \lambda 1 \rangle^2 \langle \lambda 2 \rangle^2} \left(\mu^{\dot{0}} \bar{\partial} \mu^{\dot{1}} + h \right)$$

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- Applied to SD radiative spacetimes: $h = D\bar\lambda\,\partial_u^{-1}\bar\sigma$ [Adamo, Mason, AS '22]

Asymptotic shear

Today: *Toy model of a black hole*

- Taub-NUT space (generalization of Schwarzschild)

$$ds^2 = f(r)(dt - 2Na)^2 - \frac{dr^2}{f(r)} - (r^2 + N^2)(d\theta^2 + \sin^2\theta d\phi^2)$$

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- Real metric after Wick rotation $t \mapsto -it$, $r \mapsto r + M$

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Flat space wavefunction

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Dressing

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- $(--)$ 2-graviton tree-amplitude

$$\mathcal{M}(1^-2^-) = \int \mathrm{vol}\,(\gamma_{1\alpha\beta\gamma\dot{\gamma}}{\gamma_2}^{\alpha\gamma\beta\dot{\gamma}} - {\gamma_{1\gamma\alpha}}^{\alpha}{}_{\dot{\gamma}}{\gamma_2}^{\gamma}{}_{\beta}{}^{\beta\dot{\gamma}})$$

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$$= 4\pi^2 M\,(4M\omega_2)!\,(2\omega_1)^{4M\omega_2-2}\,\frac{(z_1-z_2)^{4M\omega_2+4}}{|\vec{k_1}+\vec{k_2}|^{8M\omega_2+2}}\,\delta_{\omega_1+\omega_2,0}$$

- Computed **non-perturbatively** in M.

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$$S = \int_{\mathbb{P}^1} \frac{D\lambda}{\langle\lambda 1\rangle^2 \langle\lambda 2\rangle^2} \left(\mu^{\dot{0}} \bar{\partial} \mu^{\dot{1}} + \frac{(\mu^{\dot{0}} \mu^{\dot{1}})^2}{4M} \frac{D\overline{\lambda}}{\|\lambda\|^4} \right)$$

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$$\mathcal{M}(1^- 2^- 3^+ \cdots n^+) = \int_{\mathbb{R}^4} d^4x \sqrt{|g|} \left\langle \underbrace{\widetilde{V}_1 \widetilde{V}_2}\, \underbrace{V_3 \cdots V_n} \right\rangle_{\text{conn. tree}}$$

− helicity vertex ops

+ helicity vertex ops

$$\mathcal{M}_{\mathrm{MHV}} = \delta(\textstyle\sum_j \omega_j)\, \frac{\langle 12\rangle^6}{\langle 1i\rangle^2\langle 2i\rangle^2} \int_{\mathbb{R}^3} d^3\vec{x}\; e^{i\sum_j \vec{k}_j\cdot\vec{x}}\left(1+\frac{2M}{r}\right)$$

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“Incidence relations”

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↓ Background deformed Hodges' matrix

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Related work & future directions

- “Single copy”: MHV gluon amplitude on ***Self-dual dyon***

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- Non-perturbative contributions to graviton scattering?

[See Bittleston's talk for gauge theory analog]