Dirac brackets for classical black hole scattering: from amplitudes to observables

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The two-body problem in GR: PM, PN, GSF and NR

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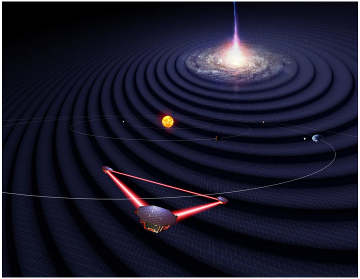
The two-body problem in GR: PM, PN, GSF and NR

- The recent discovery of gravitational waves calls for new analytical techniques to study the two-body problem.
- To model accurately the entire parameter space of the two-body dynamics, various communities need to work together: Gravitational self-force (GSF), Post-Minkowskian (PM), Post-Newtonian (PN) and numerical relativity (NR)



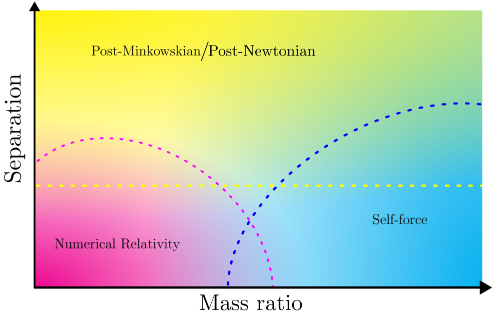
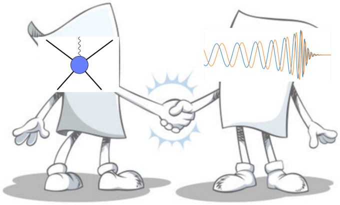


Image credit: adapted from 2304.09200 (Bern et al.)

Why QFT? Why Post-Minkowskian expansion?

- Why perturbative QFT? (adapted to scattering orbits. . . bound orbits?)

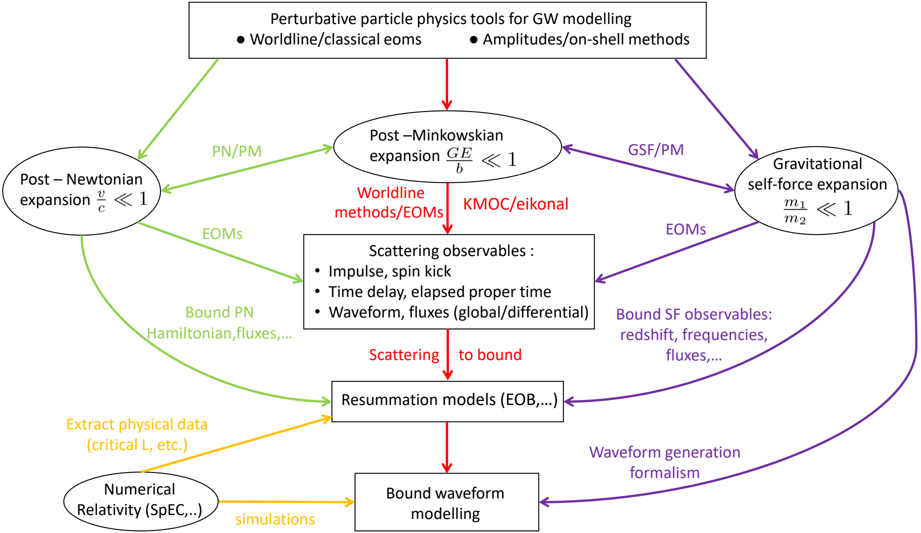
QFT tools (amplitudes/worldline) provide gauge-invariant, universal objects which encode in a compact and analytic way the perturbative dynamics for point particles.
New perspective on GR!



Advantages:
1) analytic compact expressions
2) many physical insights (clean setup)
3) scalable and flexible formalism
(spin, tidal effects, beyond GR)
4) great synergy with PN and GSF

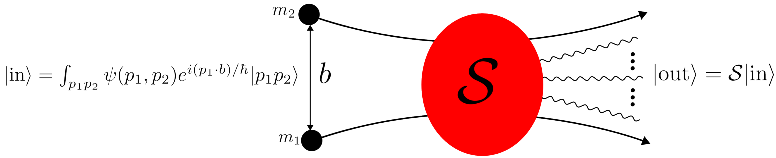
Disadvantages:
1) need scatter-to-bound map
2) need resummation scheme

Particle physics for GWs modelling: workflow



The Post-Minkowskian two-body scattering problem (I)

- Two-body scattering in GR: consider as initial state two massive particles separated by an impact parameter b^μ [Kosower,Maybee,O'Connell=KMOC]



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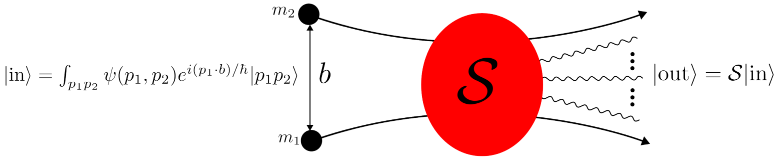
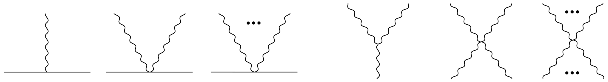


- The dynamics of the evolution is determined by the action

$$S = -\frac{1}{16\pi G_N}\int \mathrm{d}^4x\sqrt{-g}R + S_{\mathrm{matter}} + S_{\mathrm{GF}}\,,$$

where we perform the perturbative expansion

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}\,, \quad \kappa = \sqrt{32\pi G_N} \to \text{Post-Minkowskian expansion in } G_N\,.$$



The Post-Minkowskian two-body scattering problem (II)

- What is an efficient approach to describe the classical dynamics? Inspired by the eikonal/WKB, define the $\hat{N}$ operator [Damgaard,Hansen,Plante,Vanhove]

$$\mathcal{S} = \exp\left(i\hat{N}/\hbar\right) .$$

Matrix elements of $\hat{N}$ are directly related to $\mathcal{S} = 1 + i\hat{T}$.

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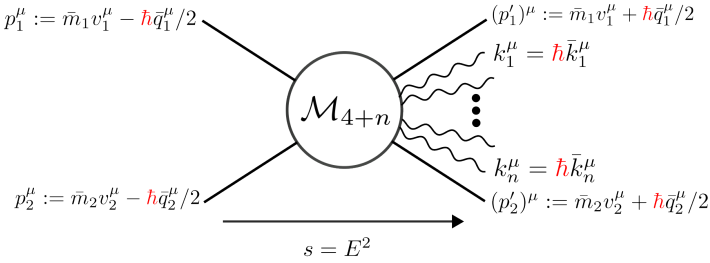
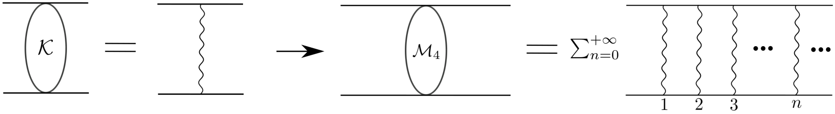
- Classical kinematics? Use heavy-particle effective theory for the amplitude $\mathcal{M}_{4+n}(p_1, p_2; p_1', p_2', k_1, \ldots, k_n)$ in the $\hbar \to 0$ limit [Brandhuber,Chen,Travaglini,Wen;Daamgard,Haddad,Helset;...]



N operator and classical S-matrix exponentiation (I)

- Matrix elements of $\hat{N}$ directly resum s-channel superclassical iterations! Focusing first on the potential/conservative region where $q = q_1 = -q_2$

$$\langle p_1' p_2' | \hat{N} | p_1 p_2 \rangle = \mathcal{K}\,(p_1, p_2; q)\,\hat{\delta}^4(q_1 + q_2)\,,$$



- How to make it manifest? Define the 2MPI kernel in impact parameter space

$$\widetilde{\mathcal{K}}^{\rm cl}\,(b) = \int \hat{\rm d}^4 q\,\hat{\delta}\,(2\bar{p}_1 \cdot q)\,\hat{\delta}\,(2\bar{p}_2 \cdot q)\,{\rm e}^{i(q\cdot b)/\hbar}\mathcal{K}^{\rm cl}\,(q)\ ,$$

so that the S-matrix exponentiates to a phase ($\rightarrow$ radial action!) [Bern,Parra-Martinez,Roiban,Ruf,Shen,Solon,Zeng;Adamo,RG;Bjerrum-Bohr, Plante,Vanhove;Damgaard,Hansen,Plante,Vanhove;Kol,O'Connell,Telem;. . .]

$$\hat{S}^{\rm cl}\Big|_{\rm cons} = \exp\left\{\frac{i}{\hbar}\widetilde{\mathcal{K}}^{\rm cl}\,(b)\right\}.$$

N operator and classical S-matrix exponentiation (II)

- How to incorporate dissipation? Define the 2MPI radiative kernels

$$\langle p_1' p_2' k_1' | \hat{N} | p_1 p_2 \rangle = \mathcal{K}_{5,\mathcal{R}}(q_1, q_2, k_1')\, \hat{\delta}^4(q_1 + q_2 - k_1'),$$
$$\langle p_1' p_2' k_1' k_2' | \hat{N} | p_1 p_2 \rangle = \mathcal{K}^A_{6,\mathcal{R}}(q_1, q_2, k_1', k_2')\, \hat{\delta}^4(q_1 + q_2 - k_1' - k_2'),$$
$$\langle p_1' p_2' k_2' | \hat{N} | p_1 p_2 k_1 \rangle = \mathcal{K}^B_{6,\mathcal{R}}(q_1, q_2, k_1, k_2')\, \hat{\delta}^4(q_1 + q_2 + k_1 - k_2'), \ldots$$

which include both connected and disconnected contributions.

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which include both connected and disconnected contributions.

- Novel coherent state expansion of the S-matrix in b-space [Alessio,RG,Shi]

$$\hat{S}^{\mathrm{cl}} = \exp\left\{ \frac{i}{\hbar} \left[\widetilde{\mathcal{K}}^{\mathrm{cl}}(b) + \frac{1}{\sqrt{\hbar}} \hat{\alpha}^{\mathrm{cl}}_{5,\mathcal{R}} + \frac{1}{\hbar} \hat{\alpha}^{\mathrm{cl}}_{6,\mathcal{R}} + \ldots \right] \right\},$$

$$i\, \hat{\alpha}^{\mathrm{cl}}_{5,\mathcal{R}} = \int_{k_1'} \left[i\, \widetilde{\mathcal{K}}^{\mathrm{cl}}_{5,\mathcal{R}}(b; k_1')\, a^\dagger(k_1') - \mathrm{h.c.} \right],$$

$$\hat{\alpha}^{\mathrm{cl}}_{6,\mathcal{R}} = \frac{1}{2!} \int_{k_1' k_2'} \left[\widetilde{\mathcal{K}}^{A\,\mathrm{cl}}_{6,\mathcal{R}}(b; k_1', k_2')\, a^\dagger(k_1') a^\dagger(k_2') + \widetilde{\mathcal{K}}^{B\,\mathrm{cl}}_{6,\mathcal{R}}(b; k_1, k_2')\, a^\dagger(k_2') a(k_1) + \mathrm{h.c.} \right]$$

Similar to the eikonal exponentiation [Amati,Ciafaloni,Veneziano;Cristofoli, RG,Moynihan,O'Connell,Ross,Sergola,White;Di Vecchia,Heissenberg,Russo, Veneziano;Fernandes,Lin] but in a different basis in b-space! Still conjectural ..

Dirac brackets for in-in scattering observables (I)

- We consider the variation of a classical observable $\hat{O}$ [KMOC],

$$\Delta \hat{O} = (\hat{S}^{\text{cl}})^{\dagger} \hat{O} \hat{S}^{\text{cl}} - \hat{O} \overset{BCH}{=} \sum_{n=1}^{+\infty} \frac{(-i)^n}{\hbar^n} \frac{1}{n!} \left[\left(\widetilde{\mathcal{K}}^{\text{cl}} + \sum_{j=1}^{+\infty} \frac{\hat{\alpha}_{4+j,\mathcal{R}}}{\hbar^{j/2}} \right)^{\odot n}, \hat{O} \right],$$

$$[(\hat{U})^{\odot n}, \hat{O}] := \underbrace{[\hat{U}, [\hat{U}, \dots, [\hat{U}}_{\text{n times}}, \hat{O}]]] \,,$$

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- In general, we have radiative ($\hat{O}_{\rm R}$) and two-body ($\hat{O}_{2-\rm body}$) observables

$$\hat{O}_{2-\rm body} = f_{\mathcal{O}}(v_1, v_2, s_1, s_2, b_1, b_2), \qquad \hat{O}_{\rm R} = g_{\mathcal{O}}(a, a^\dagger).$$

Is there a map commutators $\to$ classical brackets in our phase space?

$$[f(\cdot), f'(\cdot)] \to i\hbar\{f(\cdot), f'(\cdot)\}_{\rm DB}, \qquad [a(k), a^\dagger(k')] = 2E_k\,\hat{\delta}^3(k-k').$$

Yes, go back to worldline approach to spinning point particles!
[RG,Shi;Alessio,RG,Shi;Kim,Kim,Lee;Kim] (see also Jung-Wook's talk!)

Dirac brackets for in-in scattering observables (II)

- Consider the trajectories $x_1(\tau), x_2(\tau)$ with initial spin tensor $S_1^{\mu\nu}$ and $S_2^{\mu\nu}$

$$x_1^\mu(\tau) = v_1^\mu \tau + b_1^\mu, \qquad x_2^\mu(\tau) = v_2^\mu \tau + b_2^\mu,$$

The Poisson brackets for interacting spinning particles are $(i = 1, 2)$

$$\{b_i^\mu, v_i^\nu\}_{\rm PB} = \frac{\eta^{\mu\nu}}{m_i}, \quad \{S_i^{\mu\nu}, S_i^{\alpha\beta}\}_{\rm PB} = S_i^{\mu\beta}\eta^{\nu\alpha} - S_i^{\mu\alpha}\eta^{\nu\beta} - (\mu \leftrightarrow \nu),$$

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- Impose covariant SSC condition, on-shellness and transversality constraints

$$v_{i,\mu} S_i^{\mu\nu} = 0, \quad b \cdot v_i = 0, \quad v_i^2 = 1, \quad i = 1, 2$$

to obtain the covariant Dirac brackets in the kinematic phase space [RG,Shi]

$$\{b_i^\mu, v_j^\nu\}_{\rm DB}, \qquad \{b_i^\mu, s_j^\nu\}_{\rm DB}, \qquad \{b_i^\mu, b_j^\nu\}_{\rm DB}, \qquad \{s_i^\mu, s_j^\nu\}_{\rm DB}.$$

Generalize Hanson-Regge brackets from an on-shell scattering perspective!

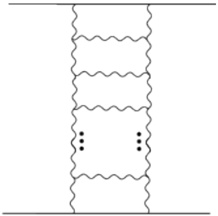
Impulse, spin kick and angular momentum

- Novel universal expression ($\lambda^\mu = \{v_i^\mu, s_i^\mu, b_i^\mu\}$) for the impulse Δv_i^μ, spin kick Δs_i^μ and impact parameter shift Δb_i^μ [Alessio,RG,Shi]

$$\langle \Delta\lambda^\mu \rangle = \underbrace{\sum_{n=1}^{+\infty} \frac{1}{n!} \{(\widetilde{\mathcal{K}}^{\rm cl})^{\odot n}, \lambda^\mu\}_{\rm DB}}_{\text{Conservative-like term}} - \frac{i}{2!} \underbrace{\int_k \left(\widetilde{\mathcal{K}}^{*\rm cl}_{5,\mathcal{R}}(k)\{\widetilde{\mathcal{K}}^{\rm cl}_{5,\mathcal{R}}(k), \lambda^\mu\}_{\rm DB} - \text{h.c}\right)}_{\text{Radiation reaction-like term}} + \ldots$$

where there are additional iterative terms involving $\widetilde{\mathcal{K}}^{\rm cl}$,$\widetilde{\mathcal{K}}^{\rm cl}_{5,\mathcal{R}}$,$\widetilde{\mathcal{K}}^{\rm cl}_{6,\mathcal{R}}$,. . . .
Graviton loops are quantum-suppressed [Britto,RG,Jehu;Cristofoli et al.]

$$\hbar \int_{k_1,k_2} \widetilde{\mathcal{K}}^{A\rm cl}_{6,\mathcal{R}}\{\widetilde{\mathcal{K}}^{A*\rm cl}_{6,\mathcal{R}}, \lambda^\mu\}_{\rm DB} \propto \mathcal{O}(\hbar)\,.$$

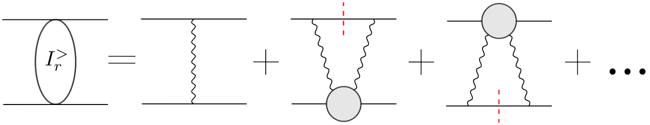


Need to clarify the role of higher-point kernels $\widetilde{\mathcal{K}}^{\rm cl}_{4+N,\mathcal{R}}$ with $N \geq 2$!

Observables for conservative spinning binaries

- New state-of-art results for observables of the conservative spinning dynamics

$$\widetilde{\mathcal{K}}^{\rm cl}\Big|_{\rm cons} = I_r^{>} = 2\int_{r_{\rm min}}^{+\infty} \mathrm{d}r\, p_{\rm r,COM} \to \langle \Delta\lambda^\mu \rangle = \sum_{n=1}^{+\infty} \frac{1}{n!}\{(I_r^{>,\epsilon})^{\odot n}, \lambda^\mu\}_{\rm DB}\,,$$



1. Test-body Δv_1^μ, Δs_1^μ at $\mathcal{O}(G^6 s_1 s_2^4)$ (constructing $I_r^{>,\epsilon}$ via integrability)[RG,Shi]
2. Δv_i^μ and Δs_i^μ up to $\mathcal{O}(G^2 s_1^{j_1} s_2^{j_2})$ with $j_1 + j_2 \leq 11$ [Alessio,RG,Shi], using the recent 2PM amplitude results [Bohnenblust,Cangemi,Johansson,Pichini]
3. Δv_i^μ, Δs_i^μ at $\mathcal{O}(G^3 s_1^0 s_2^4)$ [Akpinar,Febres-Cordero,Kraus,Smirnov,Zeng]

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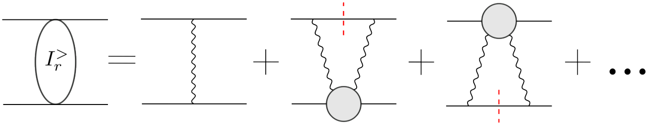

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- Various related observable calculations at 2PM [Chen,Chung,Huang,Jung-Wook;Bern,Kosmopoulos,Luna,Roiban,Teng;Liu,Porto,Yang;Aoude,Haddad, Helset;Bautista;Chen,Kim,Wang;...] and including dissipation at 3PM/4PM [Jakobsen,Mogull;Akpinar,Febres-Cordero,Kraus,Ruf,Smirnov,Zeng;Jakobsen, Mogull,Plefka,Sauer]. Can we find any structure? (see Andres & Mao talks!)

On-shell conservation laws and classical integrability

- In the phase space $\{b_i, v_i, s_i\}$ we consider a quantity $\mathcal{Q}$ (e.g.$y = v_1 \cdot v_2$). When is it conserved? In the conservative case, [Akpinar,Brown,RG,Zeng]

$$\{I_r^{>,\epsilon}, \mathcal{Q}\}_{\rm DB} = 0 \to \Delta \mathcal{Q} = 0\,.$$

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- New on-shell approach to classical integrability: find n charges in involution in a reduced $2n$ dimensional phase space $\mathcal{H}_{\mathrm{2B}}^{\mathrm{red}} = \left\{\vec{b}_\perp, \vec{u}_{\mathrm{rel}}, \vec{s}_{1\perp}, \vec{s}_{2\perp}\right\}$

$$\{\mathcal{Q}_i, \mathcal{Q}_j\}_{\mathrm{DB}} = 0\,, i, j = 1, \ldots n$$

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- Using $\mathcal{Q}_i = \{I_r^{>,\epsilon}, L_z, Q, Q_Y\}$[Compere,Druart,Vines;RG,Shi], evidence of integrability beyond the current state-of-art for a spinning probe –up to quartic order in spin– on a Kerr background, and for generic spinless-spinning scattering up to 2PM order! (See Mao's talk)

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- What happens when dissipation is included for the brackets?

Angular momentum, BMS and radiative Dirac brackets

- The angular momentum is similarly computed [Jakobsen,Mogull]

$$\langle \Delta J_i^{\mu\nu} \rangle = 2m_i(b_i^{[\mu} + \langle \Delta b_i^{[\mu} \rangle)(v_i^{\nu]} + \langle \Delta v_i^{\nu]} \rangle) + m_i \epsilon^{\mu\nu}{}_{\rho\sigma}(v_i^{\rho} + \langle \Delta v_i^{\rho} \rangle)(s_i^{\sigma} + \langle \Delta s_i^{\sigma} \rangle) \quad i = 1, 2\,.$$

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- The change in the angular momentum requires –besides $\widetilde{\mathcal{K}}^{\rm cl}$– also the static (disconnected) piece of $\mathcal{K}^{\rm cl}_{5,\mathcal{R}}$ [Di Vecchia,Heissenberg,Russo,Veneziano]

$$\mathcal{K}^{\rm cl}_{5,\mathcal{R}}(k) = \lim_{\omega^* \to 0} \left[\Theta\left(\omega^* - \omega\right) \mathcal{K}^{\rm stat}_{5,\mathcal{R}}(k) + \Theta\left(\omega - \omega^*\right) \mathcal{K}^{\rm dyn}_{5,\mathcal{R}}(k) \right] ,$$

$$\mathcal{K}^{\rm stat}_{5,\mathcal{R}}(k) = i \frac{\kappa}{2} \sum_{j=1,2,1',2'} \frac{\varepsilon_{\mu\nu}(k)\, p_j^\mu p_j^\nu}{\eta_j p_j \cdot k - i0} \,, \quad p_j'^\mu = p_j^\mu + (-1)^j Q^\mu \,.$$

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$$\mathcal{K}^{\mathrm{stat}}_{5,\mathcal{R}}(k) = i\frac{\kappa}{2} \sum_{j=1,2,1',2'} \frac{\varepsilon_{\mu\nu}(k)\, p_j^\mu p_j^\nu}{\eta_j p_j \cdot k - i0}\,, \quad p_j'^\mu = p_j^\mu + (-1)^j Q^\mu\,.$$

- We find that the static piece depends only on Δb_i^μ [Alessio,RG,Shi]

$$\langle \Delta J_i^{\mu\nu} \rangle^{\mathrm{stat}} \Big|_{\mathcal{O}(G^2)} = -2p_i^{[\mu} \langle \Delta b_i^{\nu]} \rangle^{\mathrm{stat}} \Big|_{\mathcal{O}(G^2)}\,.$$

Interesting perspective on the BMS frame dependence! [Veneziano,Vilkovisky; Riva,Vernizzi,Wong;Elkhidir,O'Connell,Roiban;Heissenberg,Russo;Veneziano;. . .]

Spinning waveform and fluxes

- We can also study the waveform [Cristofoli,RG,Kosower,O'Connell]

$$\langle \kappa h_{\mu\nu}(x)\rangle = \frac{\kappa}{4\pi|\vec{x}|}\sum_{\sigma}\int \frac{\mathrm{d}\omega}{2\pi}\left(e^{i\omega u}\varepsilon^{\sigma}_{\mu\nu}(\hat{n})\langle a_{\sigma}(\omega\hat{n})\rangle + \text{h.c.}\right)$$

$$\kappa\langle a_{\sigma}(\omega\hat{n})\rangle = i\kappa\widetilde{\mathcal{K}}^{\rm cl}_{5,\mathcal{R}}(\omega\hat{n}) + i\,\kappa\underbrace{\sum_{n=2}^{+\infty}\frac{1}{n!}\left\{(\widetilde{\mathcal{K}}^{\rm cl})^{\odot(n-1)},\widetilde{\mathcal{K}}^{\rm cl}_{5,\mathcal{R}}(\omega\hat{n})\right\}_{\rm DB}}_{\text{rotation } b_J=b\to b_{\rm eik}}$$

$$+\frac{\kappa}{2}\int_k \widetilde{\mathcal{K}}^{*\rm cl}_{5,\mathcal{R}}(k)\,\widetilde{\mathcal{K}}^{A*\rm cl}_{6,\mathcal{R}}(\omega\hat{n},k) - \frac{\kappa}{4}\int_k(\widetilde{\mathcal{K}}^{B\rm cl}_{6,\mathcal{R}}(\omega\hat{n},k)+\text{h.c.})\widetilde{\mathcal{K}}^{\rm cl}_{5,\mathcal{R}}(k) + \ldots$$

Simpler (causal) description [Kim;Alessio,RG,Shi] of the KMOC cut contribution found at one-loop order! [Caron-Huot,Giroux,Hannesdottir, Mizera;Georgoudis,Heissenberg,Russo;Bini,Damour,De Angelis,Geralico, Herderschee,Roiban,Teng] Lots to be understood for $\widetilde{\mathcal{K}}^{\rm cl}_{6,\mathcal{R}}$ and beyond ...

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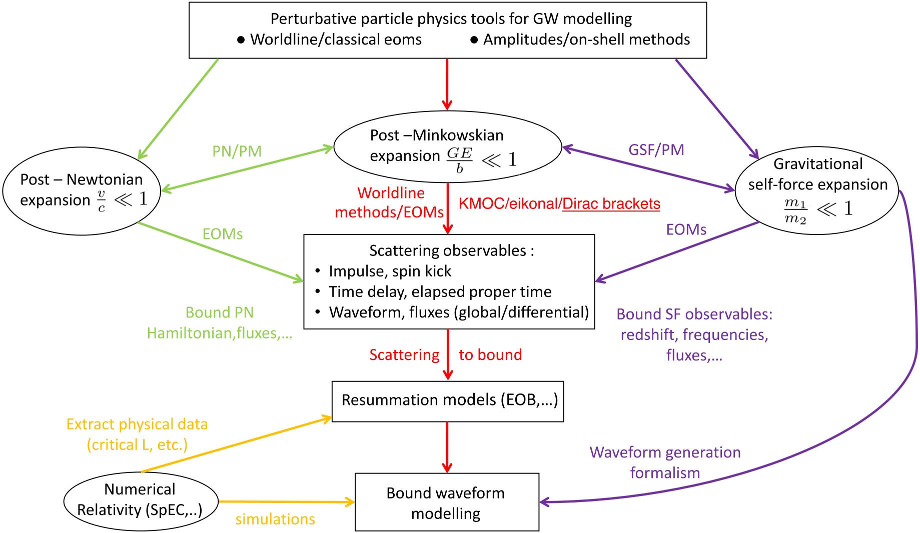
- Similar equations for spinning fluxes! We still need to translate all scattering observables to the corresponding bound ones and perform a resummation ...

Particle physics for GWs modelling: workflow

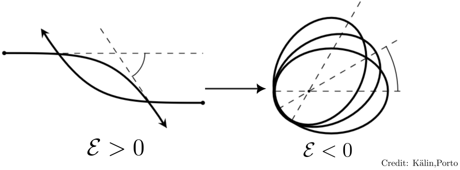


Open problems: scatter-to-bound and high-energy (I)

- Classical scattering amplitudes describe hyperbolic encounters. If we define

$$\mathcal{E} := \frac{E - m_1 - m_2}{\mu}\,, \qquad p_\infty^2 = -\tilde{p}_\infty^2 = \frac{E^2 - (m_1 + m_2)^2}{2 m_1 m_2}\,,$$

we have $\mathcal{E}, p_\infty^2 > 0$ for scattering orbits and $\mathcal{E}, p_\infty^2 < 0$ for bound orbits.



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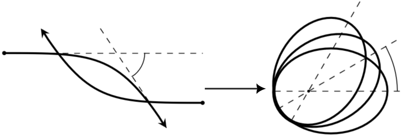

- Powerful analytic method to extract bound state physics from amplitudes: gauge invariant map between scattering and bound observables:

$$\boxed{\mathcal{O}^{>}(\mathcal{E} > 0, J, c_X, a_1, a_2, m_1, m_2) \to \mathcal{O}^{<}(\mathcal{E} < 0, J, c_X, a_1, a_2, m_1, m_2)}.$$

First derived in PM for aligned-spin binaries [Kälin,Porto] (hints in 1985 [Damour, DeRuelle]!), extended to fluxes [Cho,Kälin,Porto;Saketh,Vines, Steinhoff,Buonanno], waveforms [Adamo,RG,Ilderton]; proved recently at geodesic order [RG,Shi;RG,Lewis,Pound]; hints for misaligned spin [RG,Shi]

Open problems: scatter-to-bound and high-energy (II)

- For aligned-spin binaries we find a conjectural scatter-to-bound dictionary [Kälin,Porto;Saketh,Vines,Steinhoff,Buonanno;Cho,Kälin,Porto;Adamo,RG; Heissenberg;Adamo,RG,Ilderton;Damour,Deruelle;RG,Shi;RG,Lewis,Pound]

Bound observable	Scattering observable
$\Delta\Phi(\tilde{p}_\infty; L, a, c_X)$	$\chi(-i\tilde{p}_\infty; L, a, c_X) + \chi(+i\tilde{p}_\infty; L, a, c_X)$
$\frac{2\pi}{\Omega_r}(\tilde{p}_\infty; L, a, c_X)$	$\Delta t^\epsilon(-i\tilde{p}_\infty; L, a, c_X) + \Delta t^\epsilon(+i\tilde{p}_\infty; L, a, c_X)$
$\frac{2\pi}{\Omega_r}\langle z\rangle(\tilde{p}_\infty; L, a, c_X)$	$\Delta\tau^\epsilon(-i\tilde{p}_\infty; L, a, c_X) + \Delta\tau^\epsilon(+i\tilde{p}_\infty; L, a, c_X)$
$\Delta E^<_{\mathrm{rad}}(\tilde{p}_\infty; L, a, c_X)$	$\Delta E^>_{\mathrm{rad}}(-i\tilde{p}_\infty; L, a, c_X) + \Delta E^>_{\mathrm{rad}}(+i\tilde{p}_\infty; L, a, c_X)$
$\Delta J^<_{\mathrm{rad}}(\tilde{p}_\infty; L, a, c_X)$	$\Delta J^>_{\mathrm{rad}}(-i\tilde{p}_\infty; L, a, c_X) + \Delta J^>_{\mathrm{rad}}(+i\tilde{p}_\infty; L, a, c_X)$
$h^{<\mathrm{dyn}}(u; \tilde{p}_\infty, L, a, c_X)$	$h^{>\mathrm{dyn}}(u; +i\tilde{p}_\infty, L, a, c_X)$

which is valid at least up to 3PM/0SF/3PN order for integrated observables and tree-level/1PN for waveforms ($L = p_\infty b$, $a_i = \sqrt{s_i \cdot s_i}$ c_X tidal effects).

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- Breaks in the presence of (hereditary) tail effects appearing at higher orders! [Cho,Kälin,Porto; Dlapa,Liu,Kälin,Porto]! Important problem to solve, some preliminary progress for UV tail effects [Ivanov,Li,Parra-Martinez,Zhou]...

Open problems: scatter-to-bound and high-energy (III)

- Incorporating PM analytic data in an hybrid analytic/numerical model ($\mathcal{L}$-resummed/w_{EOB} [Damour,Rettegno], SEOBPM [Buonanno,Mogull,Patil, Pompili],...) shows problems at high-energies [Swain,Pratten,Schmidt;Hermann,Parra-Martinez,Ruf,Zeng;Dlapa,Kälin,Liu,Porto]

$$\Delta E|_{3\mathrm{PM}} \overset{y\to+\infty}{\sim} G^3 y^3 \,, \qquad \Delta E|_{4\mathrm{PM}} \overset{y\to+\infty}{\sim} G^4 y^3 \log(y) \,,$$

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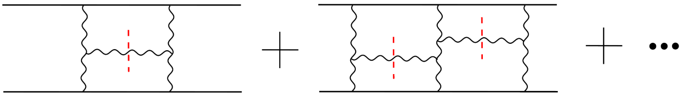

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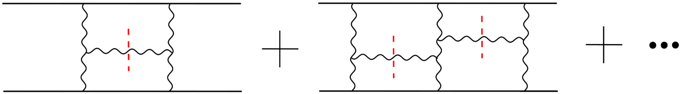

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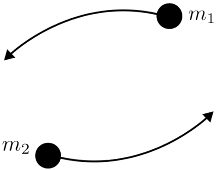
- The emitted energy is free of leading logs, consequence of SCET approach!

$$\Delta E|_{(2n+1)\rm PM} \overset{y\to+\infty}{\sim} 0 * G^{2n+1} y^{n+2} \log^n(y) + \text{subleading logs} \,.$$

  Important to understand the high-energy structure! (see Ira's talk)

Summary and future directions

- Promising application of novel particle physics tools for the binary dynamics in the Post-Minkowskian regime (field relatively new! ~ 8 years)



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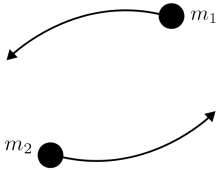

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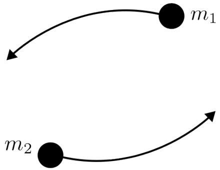

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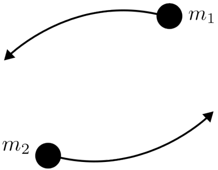

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- Resummation of perturbative methods is needed for direct application to LISA waveform modelling (EOB, GSF, ...)$\rightarrow$Exciting direction for the future!