Positivity in Renormalization

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Based on ArXiv: 2505.02910 with
You-Peng Liao and Chia-Hsien Shen

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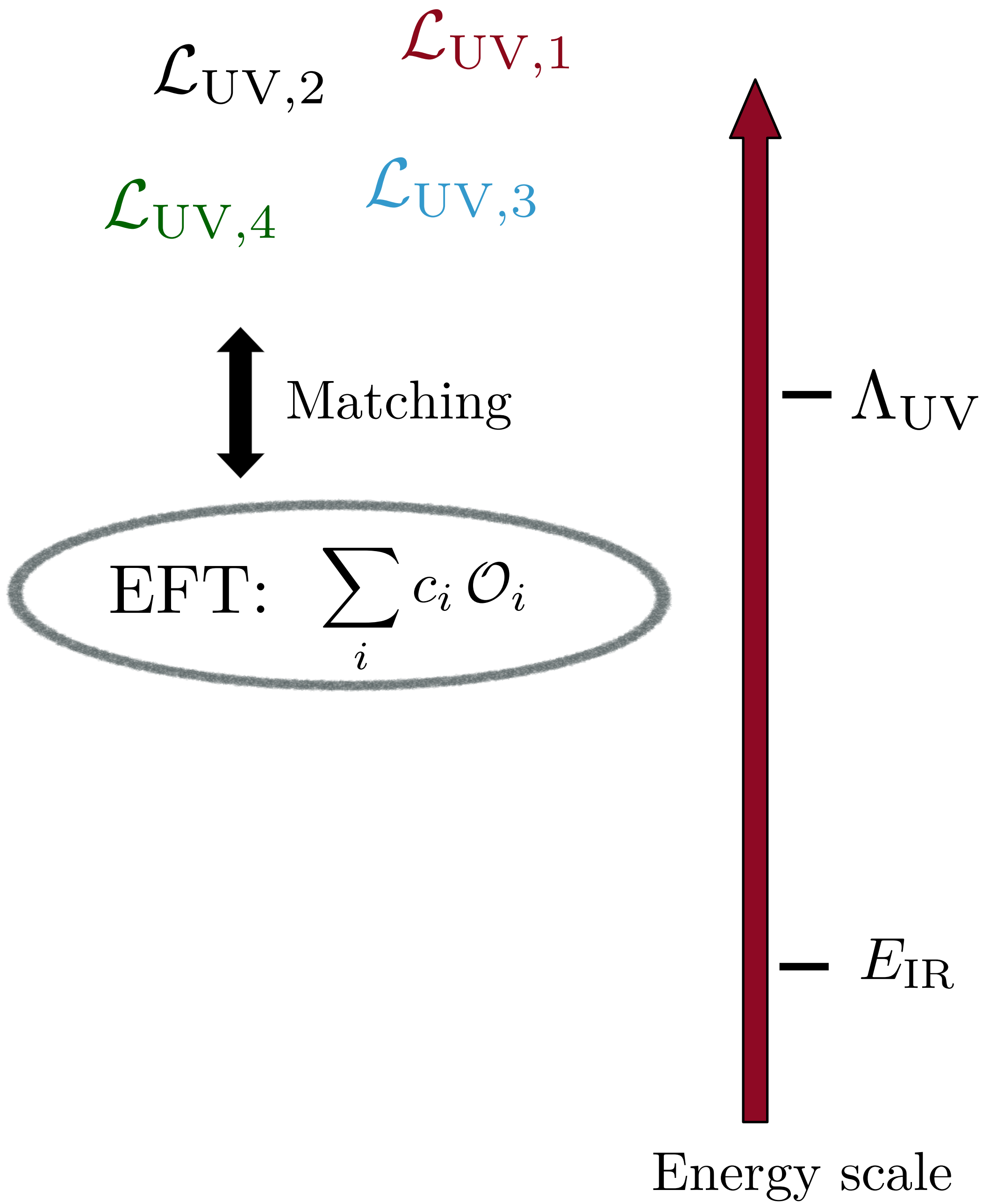
Effective field theory

Model independent description of high energy physics

Examples

- EFT of quantum gravity
- Chiral perturbation theory
- Standard model EFT (SMEFT)



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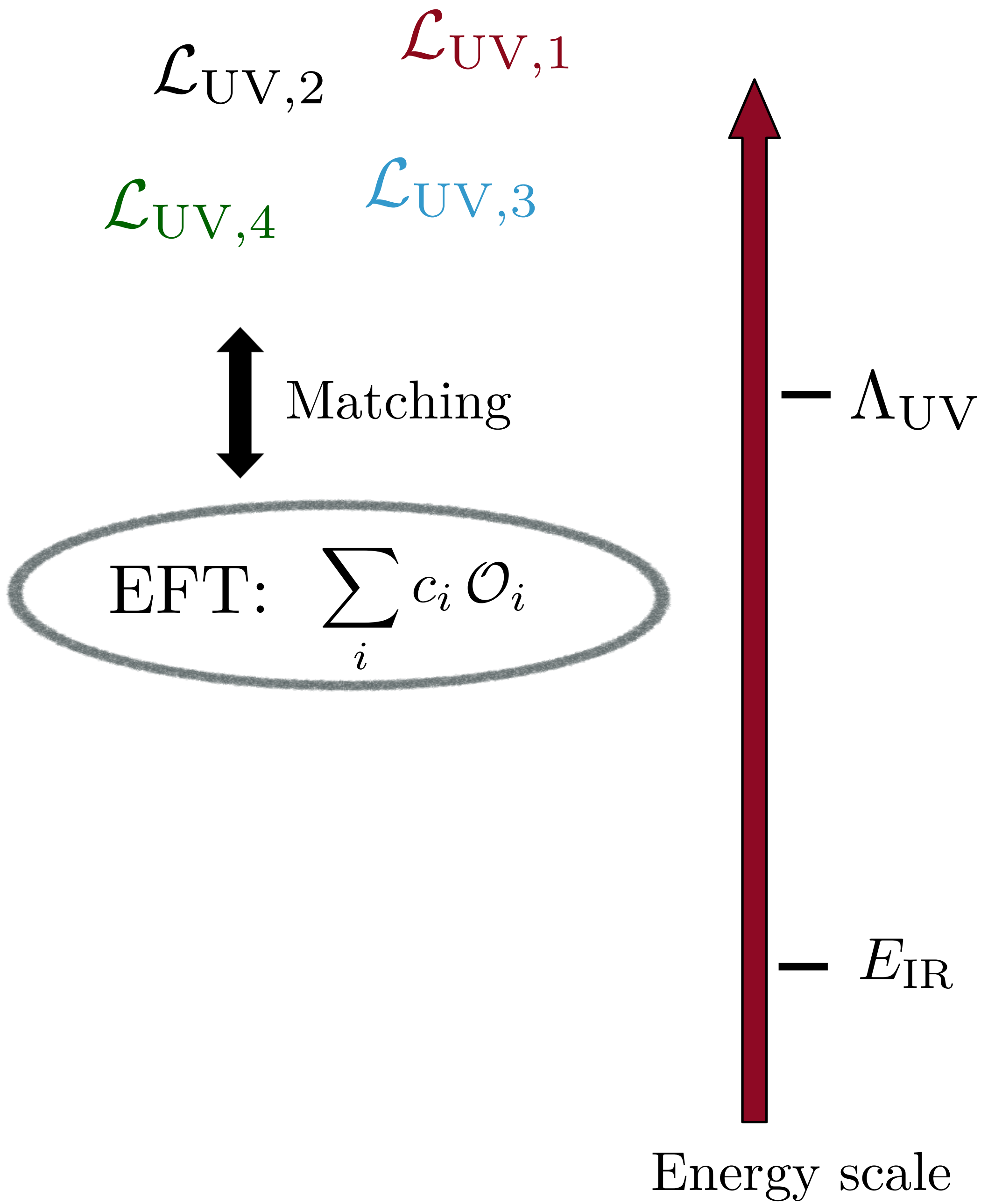
Large parameter space

SMEFT: 12 parameters at $O(\Lambda^{-1})$
2499 at $O(\Lambda^{-2})$
14750 at $O(\Lambda^{-4})$



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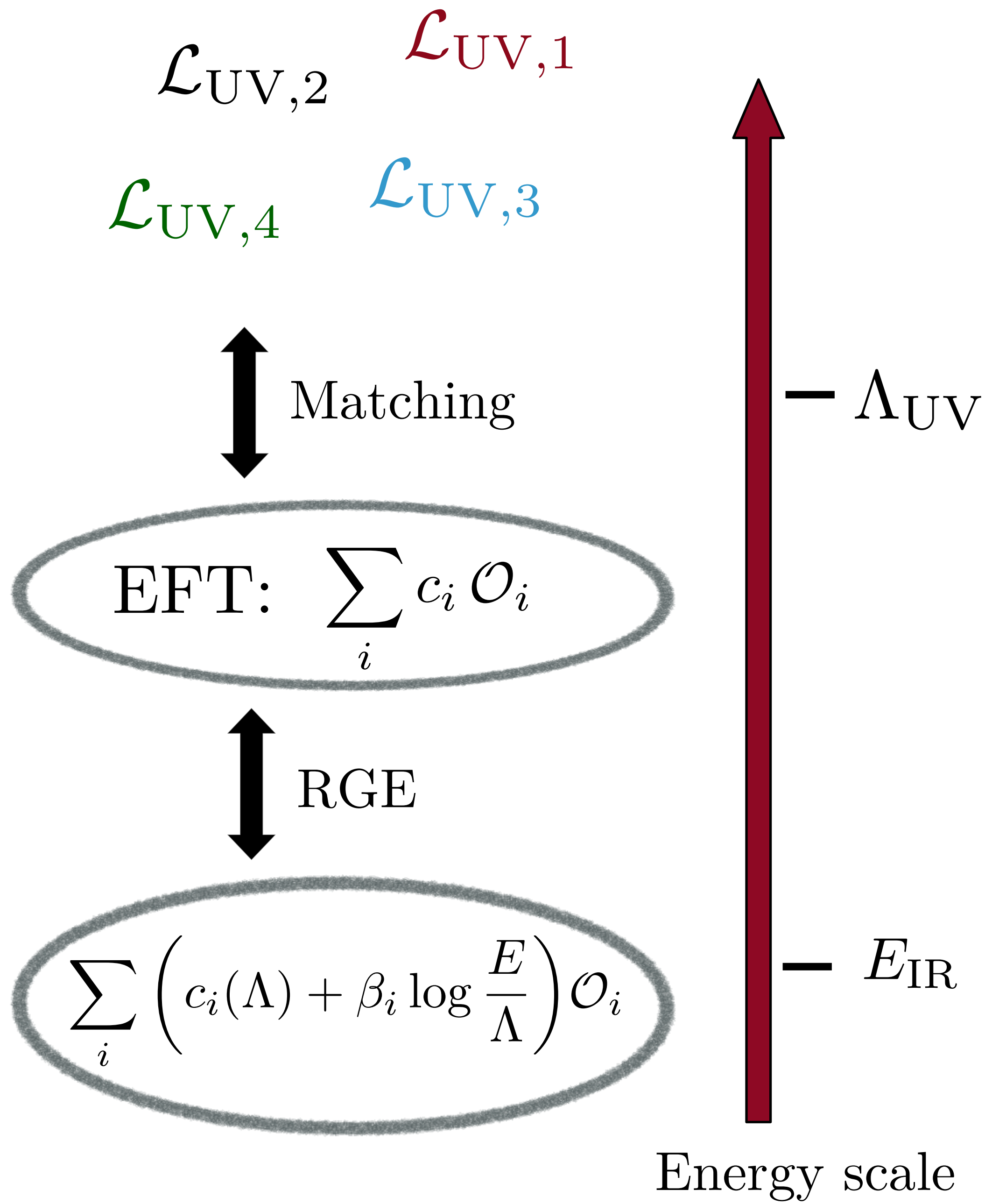

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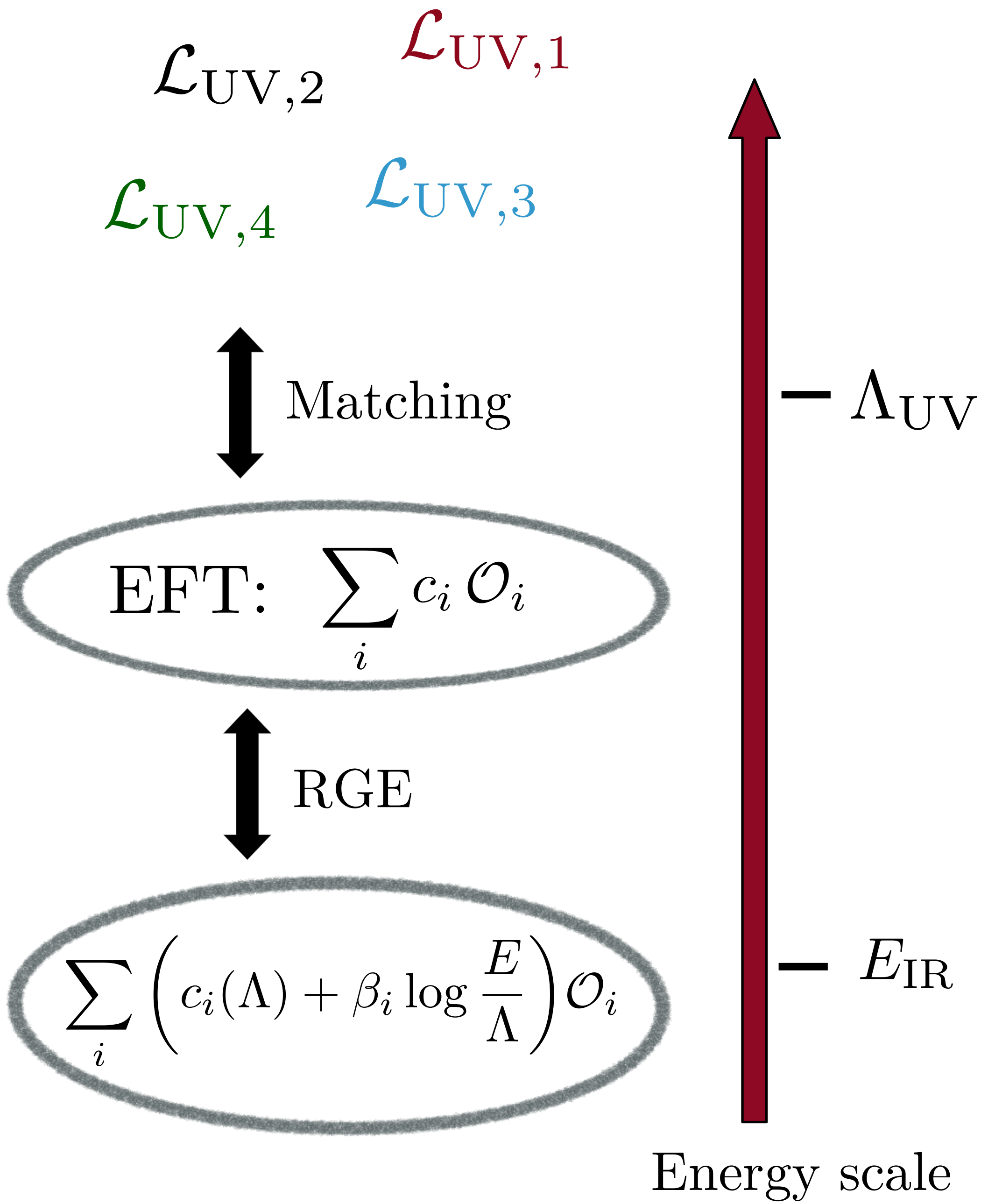
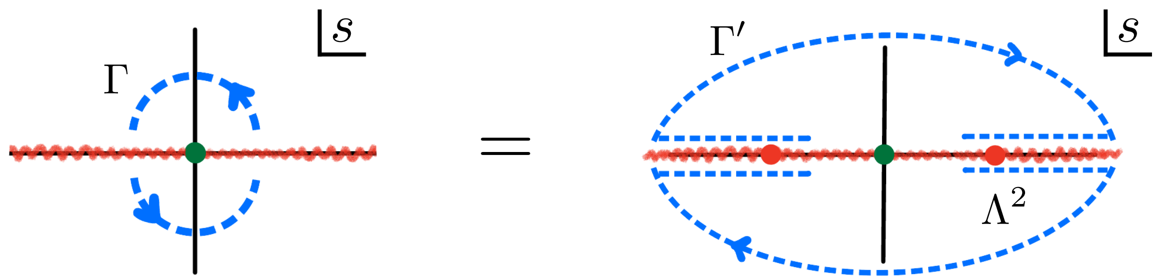
→ **Improve the understanding of universal structures**



Positivity bounds

- **EFTs have to be UV completed**
- Unitarity, locality, analyticity & Lorentz invariance of the UV completion constrain the parameter space of EFTs in the form of **positivity bounds**



$$\int_{\Gamma} ds\, \frac{\mathcal{A}_{\mathrm{EFT}}(s, t=0)}{s^3} = \int_{\Gamma'} ds\, \frac{\mathcal{A}_{\mathrm{UV}}(s, 0)}{s^3}$$

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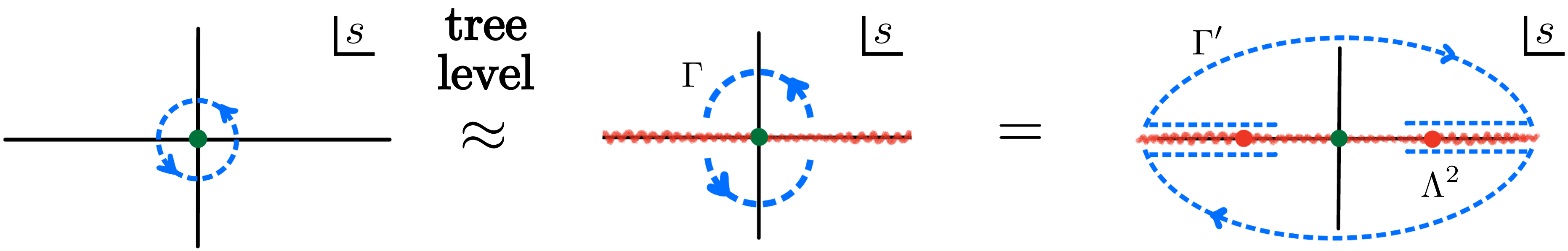

$$\oint ds\, \frac{\mathcal{A}_{\rm EFT}^{\rm tree}(s,0)}{s^3} \approx \int_{\Gamma} ds\, \frac{\mathcal{A}_{\rm EFT}(s,t=0)}{s^3} = \int_{\Gamma'} ds\, \frac{\mathcal{A}_{\rm UV}(s,0)}{s^3}$$

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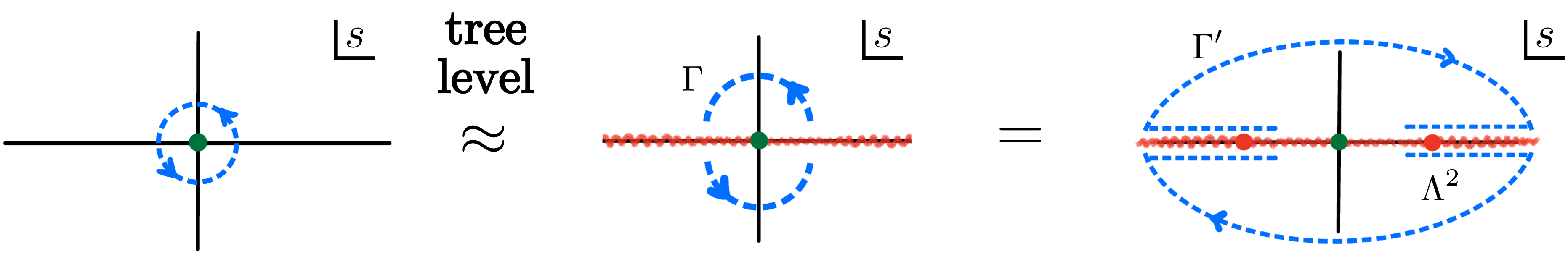

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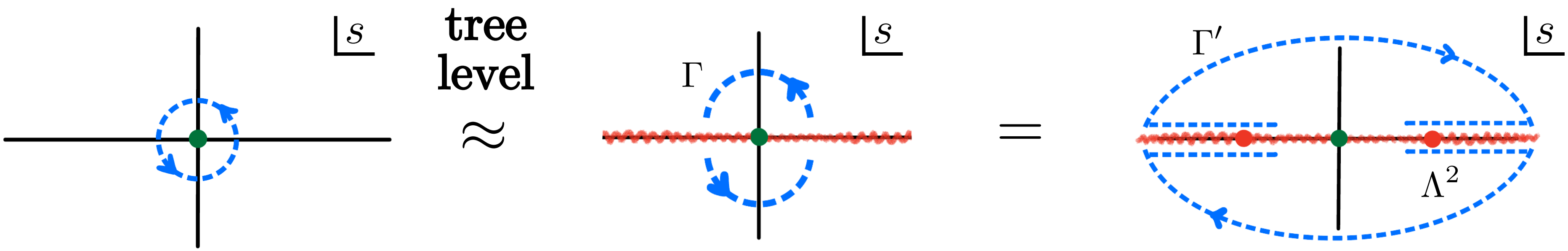

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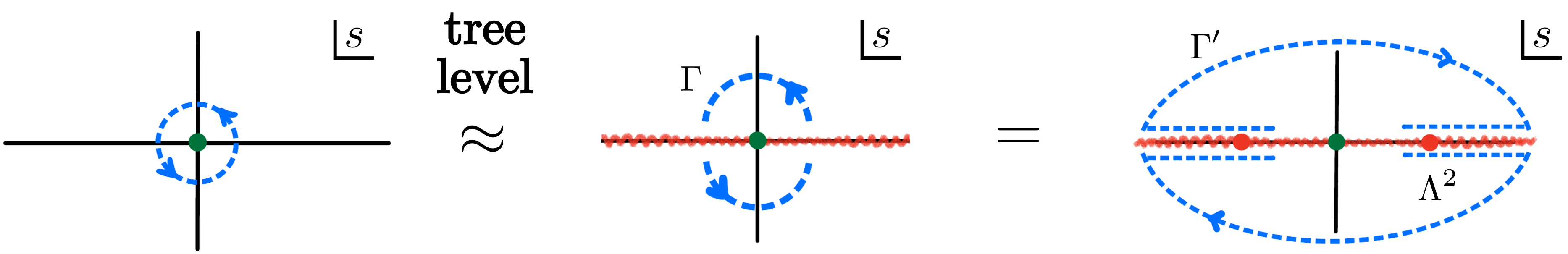

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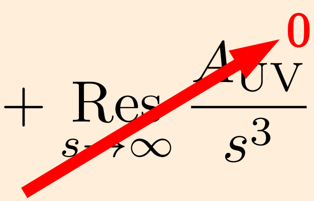

For example in the SMEFT:

$$c^{(2)}_{H^4D^4} > 0$$
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Positivity bounds in the SMEFT: Low, Rattazzi, Vichi (2009); Falkowski, Rychkov, Urbano (2012); Bellazzini, Martucci, Torre (2014); Englert, Giudice, Greljo, McCullough (2019); Bellazzini, Riva (2018); Zhang, Zhou (2018, 2020); Bi, Zhang, Zhou (2019); Remmen, Rodd (2019, 2020, 2020, 2022, 2024); Fuks, Liu, Zhang, Zhou (2020); Yamashita, Zhang, Zhou (2020); Gu, Wang, Zhang (2020); Bonnefoy, Gendy, Grojean (2020); Gu, Wang (2020); Trott (2020); Chala, Santiago (2021); Zhang (2021); Azatov, Ghosh, Singh (2021); Li, Zhou (2022); Li, Mimasu, Yamashita, Yang, Zhang, Zhou (2022); Li (2022), Ghosh, Sharma, Ullah (2022); Chen, Mimasu, Wu, Zhang, Zhou (2023); Gu, Shu (2023); Davighi, Melville, Mimasu, You (2023); Chala (2023); Chala, Li (2023); Altmannshofer, Gori, Lehmann, Zuo (2023); Hong, Wang, Zhou (2024); Ye, He, Gu (2024); ...

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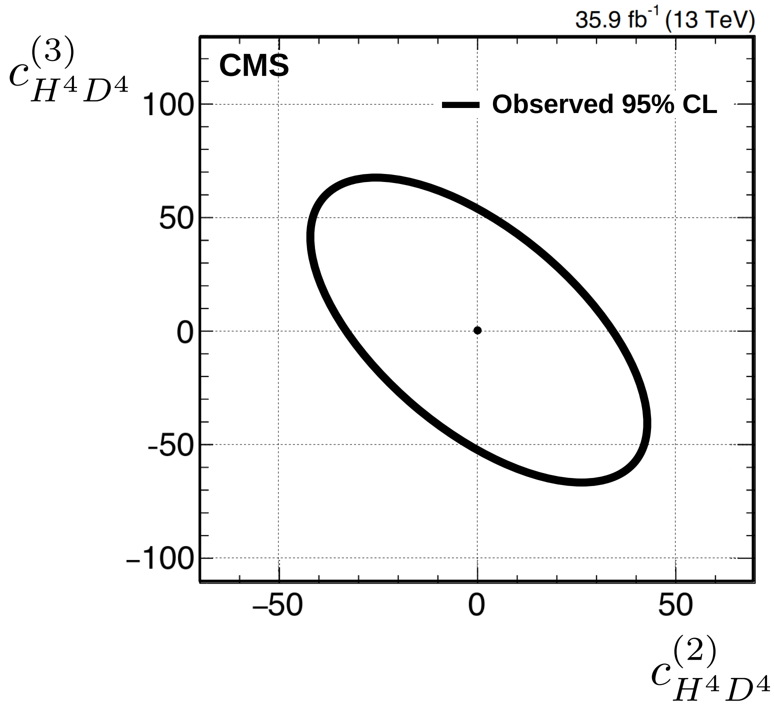
The CMS Collaboration (2019) [*adapted*]



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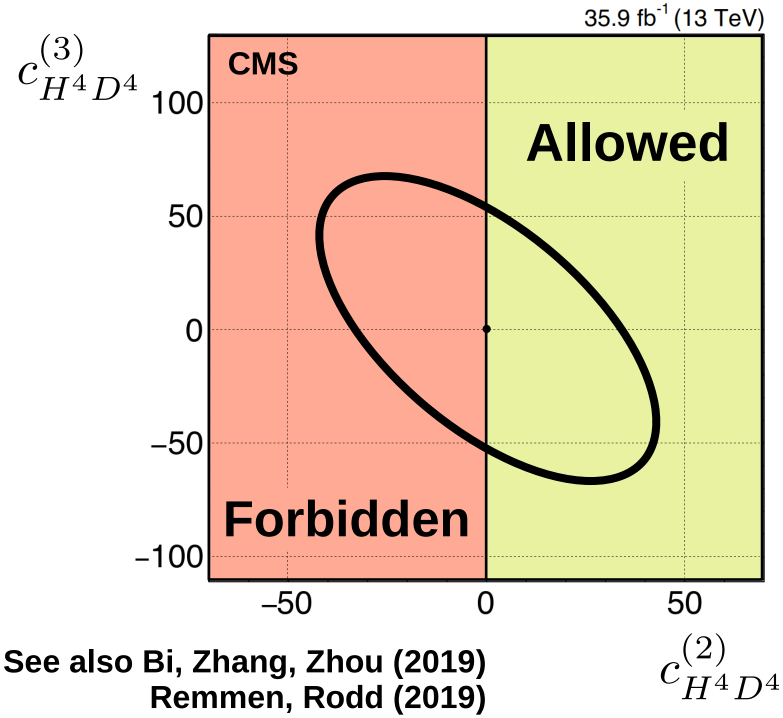

See also Bi, Zhang, Zhou (2019)
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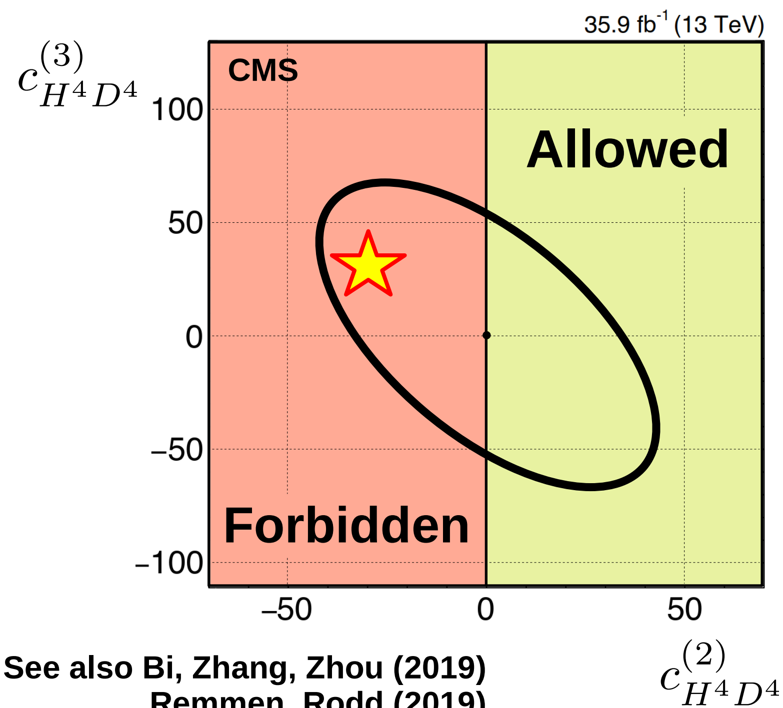

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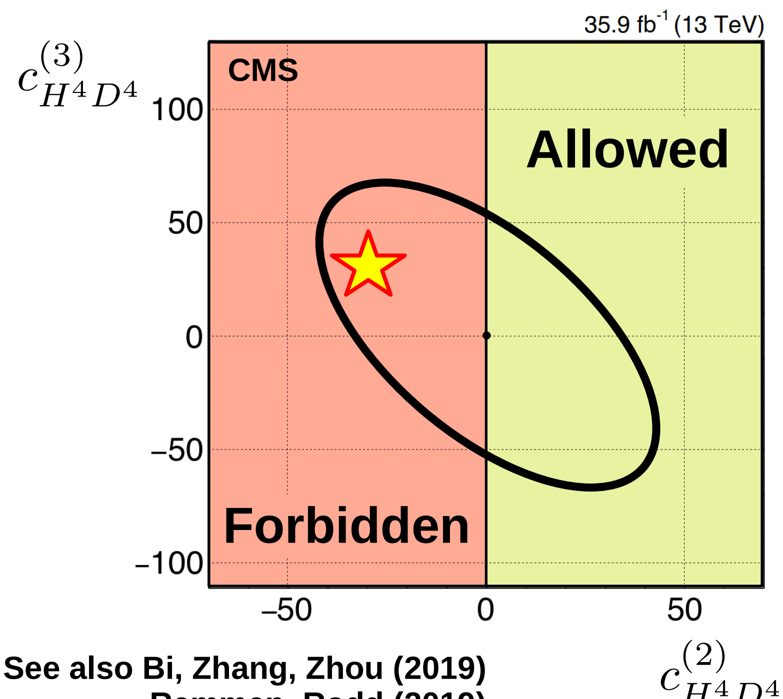

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What if new physics is discovered in the forbidden region?

How sharp should the boundary be?

At which energy scale?

Positivity bounds at loop level

- **Tree level approximation of EFT**: bounds on **individual** EFT couplings
- **At loop level in the EFT**: dispersion relations involve **all** EFT couplings

Arkani-Hamed, Huang, Huang (2021)
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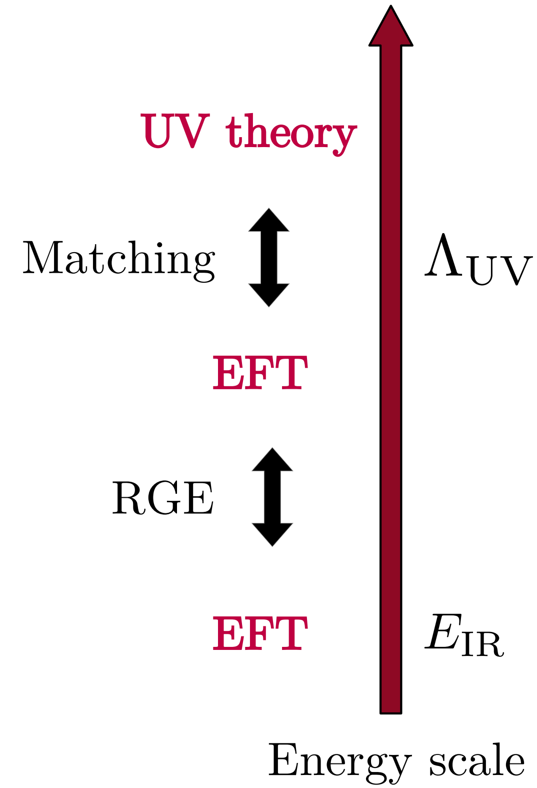

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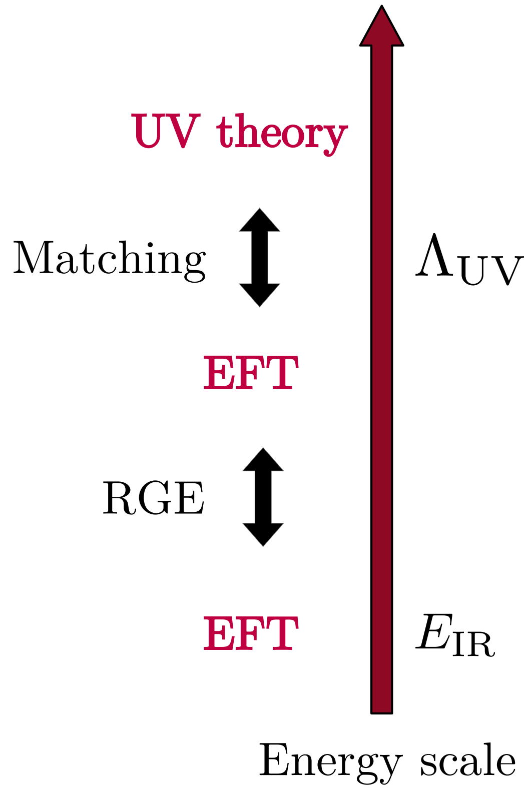
- **In the SMEFT**

Chala and Santiago (2022)
Chala (2023)
Chala and Li (2023)

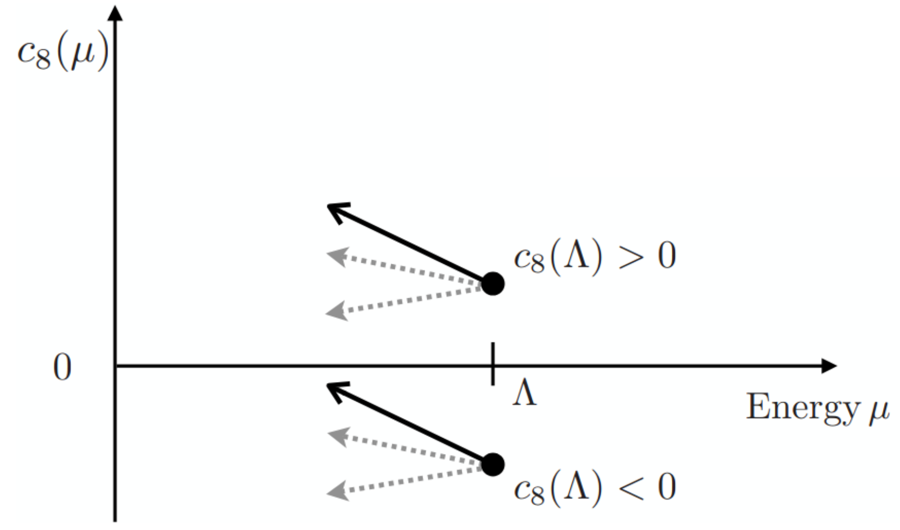
  - **Matching** may result in **negative couplings**
  - **Renormalization** may result in **negative couplings**
  - In other cases, positivity bounds impose **non-trivial constraints** on the RGE



Goal of this talk

Understand the impact of renormalization on positivity bounds

- Pure EFT argument
- Unitarity, analyticity and Lorentz invariance **of the EFT**
- For particles with any spin



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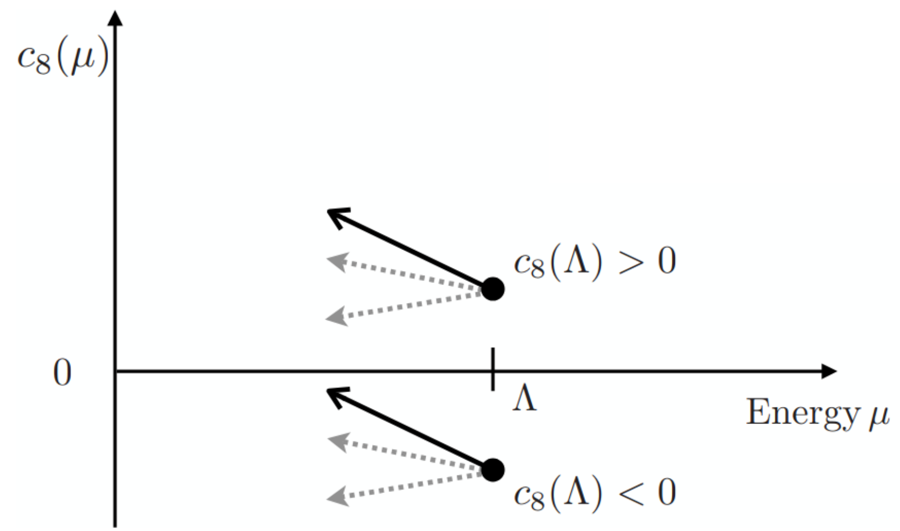
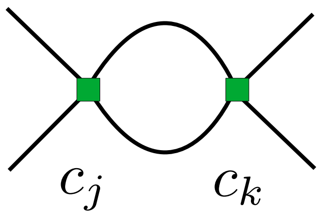


$$\Rightarrow \quad -\pi\,\beta_i\Big|_{\text{Forward}} \propto \int_{\text{LIPS}} |\mathcal{A}|^2 > 0$$

Positivity in Renormalization

Setup



- Bubble diagrams with two contact interactions
  - Massless particles
  - No three-point vertices
- ⇒ No infrared divergences
- ⇒ Single momentum flowing through the loop

Renormalization from amplitudes

- Amplitudes do not depend on the renormalization scale

$$\frac{d\,\mathcal{A}^{\text{tree}}}{d\log(\mu)} = \frac{i}{\pi} \sum_{S=s,t,u} \text{Disc}_S\, \mathcal{A}^{\text{1-loop}}$$

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Unitarity

$$= -\frac{1}{\pi}\sum_{S=s,t,u}\sum_{X}\int_{\text{LIPS}(X)}\langle\varphi_{\text{out},S}|\hat{A}^{\text{tree}}|X\rangle\langle X|(\hat{A}^{\text{tree}})^{\dagger}|\varphi_{\text{in},S}\rangle$$

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- Special case of a more general formula

$$\left(\gamma_{\mathcal{O}}^{(1)} - \gamma_{\text{IR}}^{(1)}\right)\langle p_1,\ldots,p_n|\mathcal{O}|0\rangle^{(0)} = -\frac{1}{\pi}\langle p_1,\ldots,p_n|\mathcal{M}\otimes\mathcal{O}|0\rangle^{(0)}$$

Caron-Huot and Wilhelm (2016)

with extensions to higher loops

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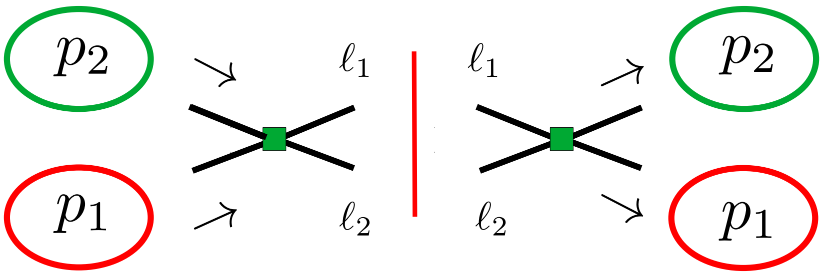
s-channel: $|\varphi_{\text{in}}\rangle = |\varphi_{\text{out}}\rangle$



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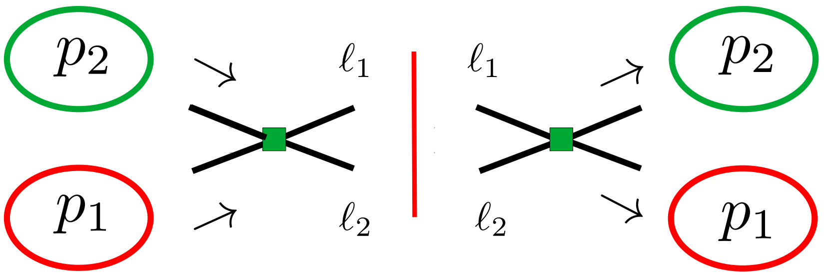

take $s > 0$: $\quad \frac{i}{\pi}\text{Disc}_s\,\mathcal{A}^{\text{1-loop}} = -\frac{1}{\pi}\sum_X\int_{\text{LIPS}(X)}|\mathcal{A}^{\text{tree}}_{\varphi\to X}|^2$

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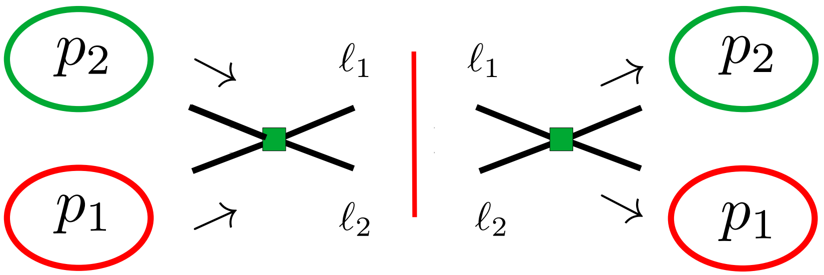

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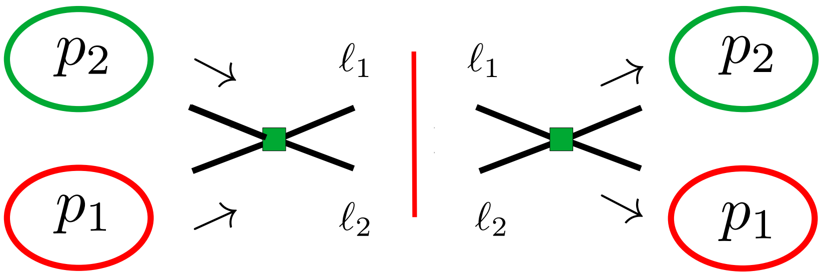

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$$= a_s\, s^n\,,\;\; \text{with } a_s < 0$$

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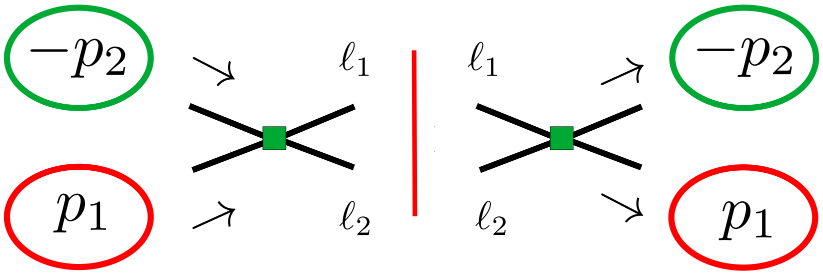

$$\text{take } u > 0: \quad \frac{i}{\pi}\text{Disc}_u\,\mathcal{A}^{\text{1-loop}} = -\frac{1}{\pi}\sum_X \int_{\text{LIPS}(X)} |\mathcal{A}^{\text{tree}}_{\varphi\to X}|^2 \; {\color{red}< 0}$$

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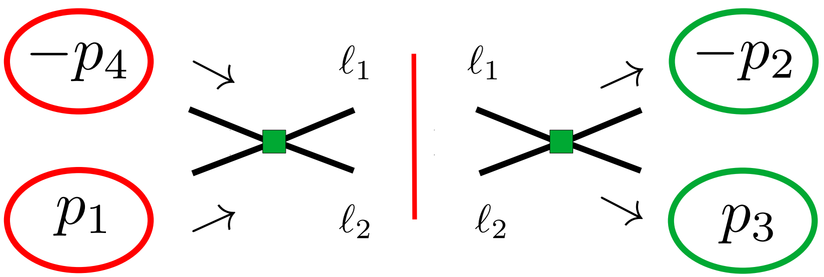
t-channel: $|\varphi_{\text{in}}\rangle \neq |\varphi_{\text{out}}\rangle$



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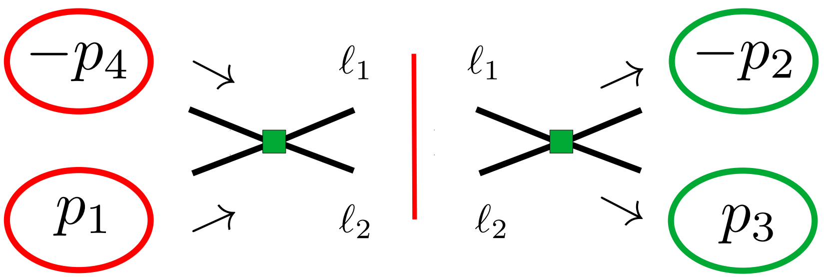

$$\frac{i}{\pi}\text{Disc}_t\,\mathcal{A}^{\text{1-loop}} \propto \sum_X \int_{\text{LIPS}(X)} \left(|\ell_1\rangle[\ell_1|\right)^{2|h_1|+n_1}\left(|\ell_2\rangle[\ell_2|\right)^{2|h_2|+n_2}$$

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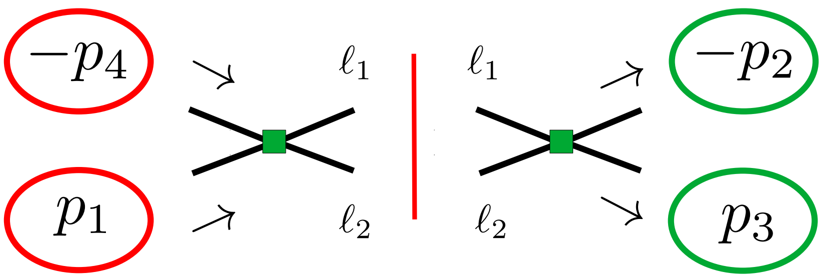

$$\frac{i}{\pi}\text{Disc}_t\,\mathcal{A}^{\text{1-loop}} \propto \sum_X \int_{\text{LIPS}(X)} (|\ell_1\rangle[\ell_1|)^{2|h_1|+n_1}(|\ell_2\rangle[\ell_2|)^{2|h_2|+n_2}$$

Lorentz invariance

$$\propto (p_1 - p_4)^{2|h_1|+2|h_2|+n_1+n_2} \quad \rightarrow 0$$

in the forward limit

- Only non-zero in the forward limit for ϕ^4-interactions

Result

- At mass dimension $2\,n+4$

$$\frac{d\,\mathcal{A}^{\text{tree}}_{2n+4}}{d\log(\mu)}\Bigg|_{\text{Forward}} = a_s\,s^n + a_u\,u^n\,,\ \text{with}\ a_s < 0\,,\ \text{and}\ a_u < 0$$

$$= (a_s + (-1)^n a_u)\,s^n$$

- Sign definite when n is even: mass dimension 8, 12, 16, ...

Result

- At mass dimension $2\,n + 4$

$$\left.\frac{d\,\mathcal{A}^{\text{tree}}_{2n+4}}{d\log(\mu)}\right|_{\text{Forward}} = a_s\, s^n + a_u\, u^n\,, \text{ with } a_s < 0\,, \text{ and } a_u < 0$$

$$= (a_s + (-1)^n a_u)\, s^n$$

- Sign definite when n is even: mass dimension 8, 12, 16, ...

- Cross terms are not sign definite $-\frac{1}{\pi}\sum_X \int_{\text{LIPS}(X)} |\mathcal{A}^{\text{tree}}_{\varphi\to X}|^2 < 0$

$$\mathcal{A}^{\text{tree}}_{\varphi\to X} = c_4\,\mathcal{A}^{(4)} + \frac{c_6}{\Lambda^2}\mathcal{A}^{(6)} + \frac{c_8}{\Lambda^4}\mathcal{A}^{(8)} + \ldots \implies \begin{cases} O(c_6^2) < 0 \\ O(c_4\, c_8) \text{ either sign} \end{cases}$$

Example

Mixing into H^4D^4 operators

$$16\pi^2 \frac{d\, c^{(1)}_{H^4D^4}}{d\log\mu} = \frac{1}{3}\left(-16\, c^2_{H\Box} + 32\, c_{HD}c_{H\Box} - 11\, c^2_{HD}\right) - \mathcal{G}_1^2 - 2\,\mathcal{G}_2^2 + \mathcal{F}_1^2 - \mathcal{F}_2^2 - \mathcal{F}_3^2,$$

$$16\pi^2 \frac{d\, c^{(2)}_{H^4D^4}}{d\log\mu} = -\frac{1}{3}\left(16\, c^2_{H\Box} + 16\, c_{HD}c_{H\Box} + 5\, c^2_{HD}\right) - \mathcal{G}_1^2 - \mathcal{F}_1^2 - \mathcal{F}_2^2,$$

$$16\pi^2 \frac{d\, c^{(3)}_{H^4D^4}}{d\log\mu} = \frac{1}{3}\left(-40\, c^2_{H\Box} - 16\, c_{HD}c_{H\Box} + 7\, c^2_{HD}\right) + 2\,\mathcal{G}_1^2 + \mathcal{G}_2^2 - \mathcal{G}_3^2 + 2\,\mathcal{F}_2^2 + \mathcal{F}_3^2.$$

Chala, Guedes, Ramos, Santiago (2021)
Helset, Jenkins, Manohar (2022)
Liao, JRN, Shen (2025)

$$\mathcal{G}_1^2 \equiv 6\, g_2^2 \left(c^2_{W^3} + c^2_{\widetilde{W}W^2}\right)$$

$$\mathcal{G}_2^2 \equiv 8 \left(c^2_{H^2WB} + c^2_{H^2\widetilde{W}B}\right)$$

$$\mathcal{G}_3^2 \equiv 16 \left(c^2_{H^2B^2} + c^2_{H^2\widetilde{B}B} + 3\left(c^2_{H^2W^2} + c^2_{H^2\widetilde{W}W}\right) + 8\left(c^2_{H^2G^2} + c^2_{H^2\widetilde{G}G}\right)\right)$$

$$\mathcal{F}_1^2 \equiv \frac{8}{3}\left[3\,(c_{Hd})^\dagger c_{Hd} + (c_{He})^\dagger c_{He} + 2\,(c^{(1)}_{HL})^\dagger c^{(1)}_{HL} + 6\,(c^{(1)}_{Hq})^\dagger c^{(1)}_{Hq} + 3\,(c_{Hu})^\dagger c_{Hu}\right]$$

$$\mathcal{F}_2^2 \equiv \frac{16}{3}\left[(c^{(3)}_{HL})^\dagger c^{(3)}_{HL} + 3\,(c^{(3)}_{Hq})^\dagger c^{(3)}_{Hq}\right]$$

$$\mathcal{F}_3^2 \equiv 8\,(c_{Hud})^\dagger c_{Hud}$$

Operators in the “Warsaw basis”
Grzadkowski, Iskrzynski, Misiak, Rosiek (2010)

Example

Mixing into $H^4 D^4$ operators

RGE of operators that can be probed in the forward limit:

$$16\pi^2 \, \frac{d\, c^{(2)}_{H^4 D^4}}{d \log \mu} =$$

$$16\pi^2 \, \frac{d\left(c^{(1)}_{H^4 D^4} + c^{(2)}_{H^4 D^4}\right)}{d \log \mu} =$$

$$16\pi^2 \, \frac{d\left(c^{(1)}_{H^4 D^4} + c^{(2)}_{H^4 D^4} + c^{(3)}_{H^4 D^4}\right)}{d \log \mu} =$$

Example

Mixing into $H^4 D^4$ operators

RGE of operators that can be probed in the forward limit:

$$16\pi^2 \frac{d\, c^{(2)}_{H^4D^4}}{d\log\mu} = -\frac{4}{3}(2\, c_{H\Box} + c_{HD})^2 - \frac{1}{3} c^2_{HD} \; - \mathcal{G}_1^2 - \mathcal{F}_1^2 - \mathcal{F}_2^2$$

$$16\pi^2 \frac{d\left(c^{(1)}_{H^4D^4} + c^{(2)}_{H^4D^4}\right)}{d\log\mu} = -\frac{4}{3}(c_{H\Box} - 2\, c_{HD})^2 - \frac{28}{3} c^2_{H\Box} \; - 2\,\mathcal{G}_1^2 - 2\,\mathcal{G}_2^2 - 2\,\mathcal{F}_2^2 - \mathcal{F}_3^2$$

$$16\pi^2 \frac{d\left(c^{(1)}_{H^4D^4} + c^{(2)}_{H^4D^4} + c^{(3)}_{H^4D^4}\right)}{d\log\mu} = -24\, c^2_{H\Box} - 3\, c^2_{HD} \; - \mathcal{G}_2^2 - \mathcal{G}_3^2$$

Example

Mixing into H^4D^4 operators

RGE of operators that can be probed in the forward limit:

$$16\pi^2\,\frac{d\,c^{(2)}_{H^4D^4}}{d\log\mu} = -\,\frac{4}{3}(2\,c_{H\Box}+c_{HD})^2-\frac{1}{3}c^2_{HD}\;-\mathcal{G}_1^2-\mathcal{F}_1^2-\mathcal{F}_2^2$$

$$16\pi^2\,\frac{d\left(c^{(1)}_{H^4D^4}+c^{(2)}_{H^4D^4}\right)}{d\log\mu} = -\,\frac{4}{3}(c_{H\Box}-2\,c_{HD})^2-\frac{28}{3}c^2_{H\Box}\;-2\,\mathcal{G}_1^2-2\,\mathcal{G}_2^2-2\,\mathcal{F}_2^2-\mathcal{F}_3^2$$

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In this basis and for these contributions

$$c_{H^4D^4}(\mu_{\mathrm{IR}}) \geq c_{H^4D^4}(\mu_{\mathrm{UV}})$$

Liao, JRN, Shen (2025)
See also Chala, Santiago (2022)

- **Full RG has more contributions:**

< 0

$$\frac{d\, c_i(\mu)}{d \log \mu} = \beta_i = \gamma_{ij}\, c_j\, \lambda_{\rm SM} + \gamma_{ijk}\, c_j\, c_k + \ldots$$

When do double insertions dominate?

- **Full RG has more contributions:**

$$\frac{d\, c_i(\mu)}{d \log \mu} = \beta_i = \gamma_{ij}\, c_j\, \lambda_{\text{SM}} + \underbrace{\gamma_{ijk}}_{<0}\, c_j\, c_k + \ldots$$

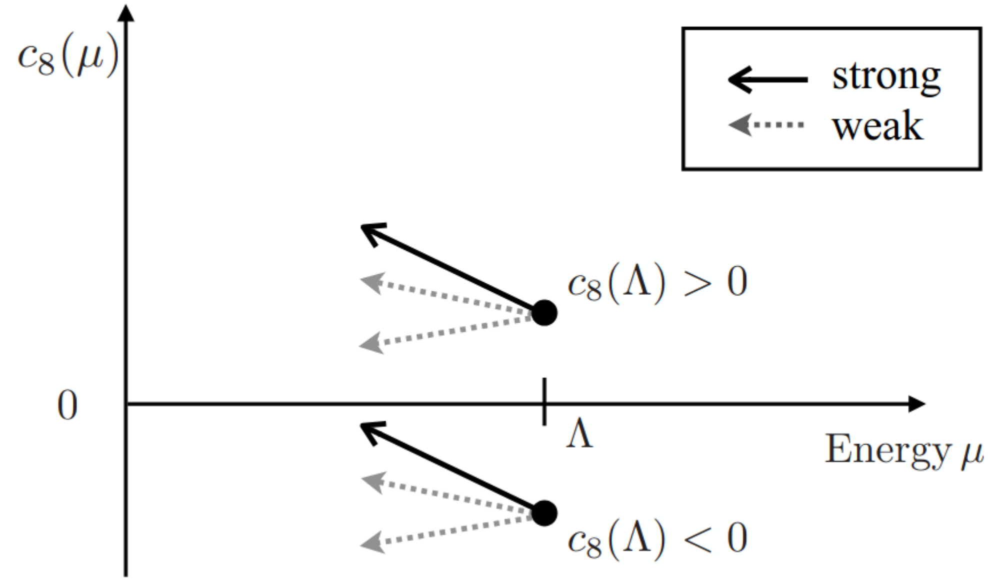
When do double insertions dominate?

- **"Weakly coupled" EFT**

$$c \ll 16\pi^2 \implies c^2 \approx c\, \lambda_{\text{SM}}$$

- **"Strongly coupled" EFT**

$$c \approx 16\pi^2 \implies c^2 > c\, \lambda_{\text{SM}}$$



e.g. chiral perturbation theory: $c_{p^4} \sim (c_{p^2})^2/(4\pi)^2$

Applications and extensions

RG of dimension-six operators

$$\mathcal{O}_5 = \epsilon^{ik}\epsilon^{jl} H_k H_l \bar{\ell}_i^c \ell_j$$

$$(\mathcal{O}_5)^2 \to H^4 D^2$$

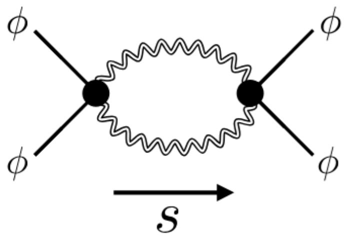
$$(\mathcal{O}_5)^2 \to H^2 \bar{\ell}\ell D$$

$$(\mathcal{O}_5)^2 \to \bar{\ell}^2 \ell^2$$

$$\boxed{\beta_i < 0}$$

EFT of quantum gravity

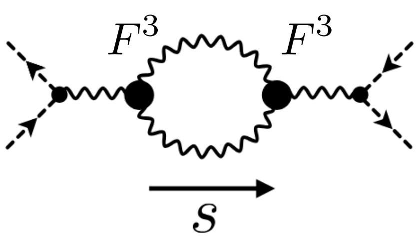
$$(R^2\phi^2)^2 \to \phi^4 \partial^8$$



$+ (s \to t) + (s \to u)$

Three-point interactions

$$(F^3)^2 \to H^4 D^4$$

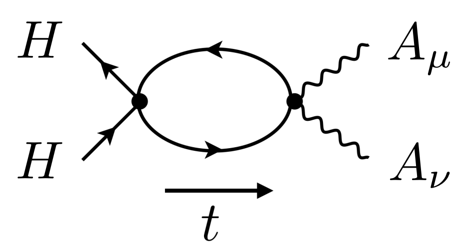


$+$ t-channel

Non-renormalization results

when only the t-channel exists, e.g.

$$H^4 D^2 \times H^2 F^2 \to H^2 F^2 D^2$$



Conclusion

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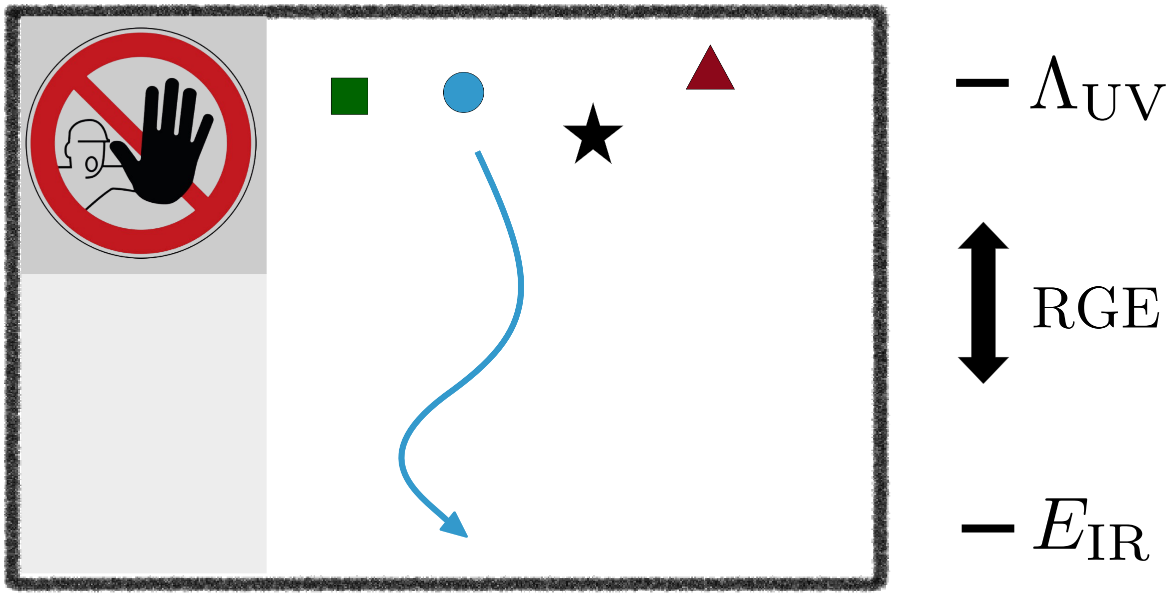
- Negative EFT parameters are in agreement with dispersion relations
- However, RG contributions from double insertions at dimension 8, 12, 16, ... are such that **EFT parameters grow towards the IR**
- This follows from EFT consistency (without assumptions on the UV completion)
- **Outlook:** three-point interactions, non-forward relations



“Parameter space of SMEFT”

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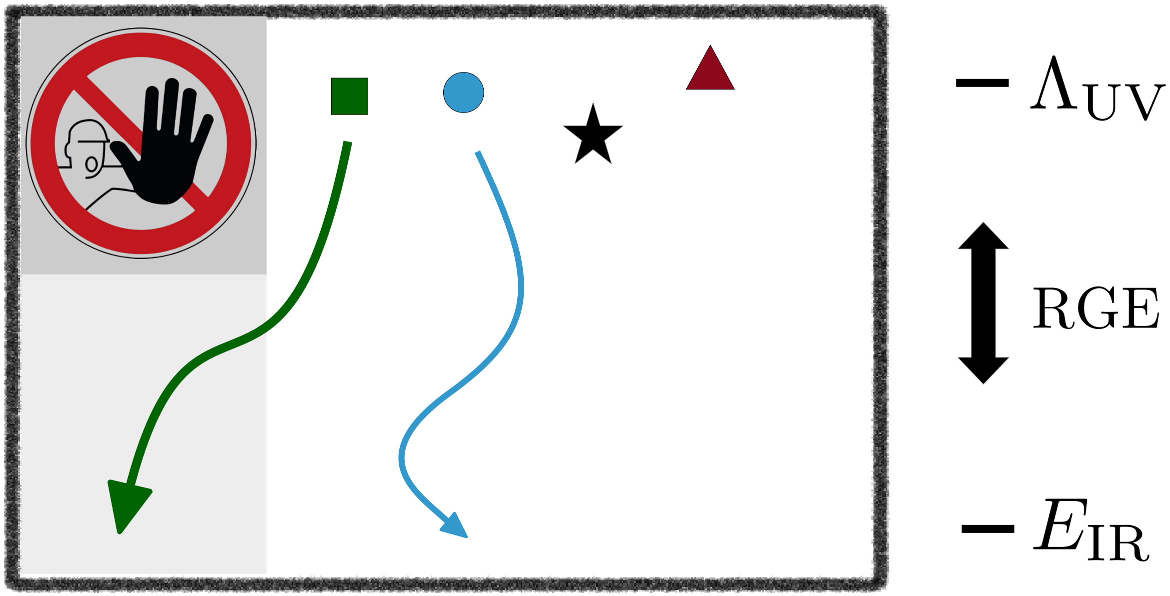

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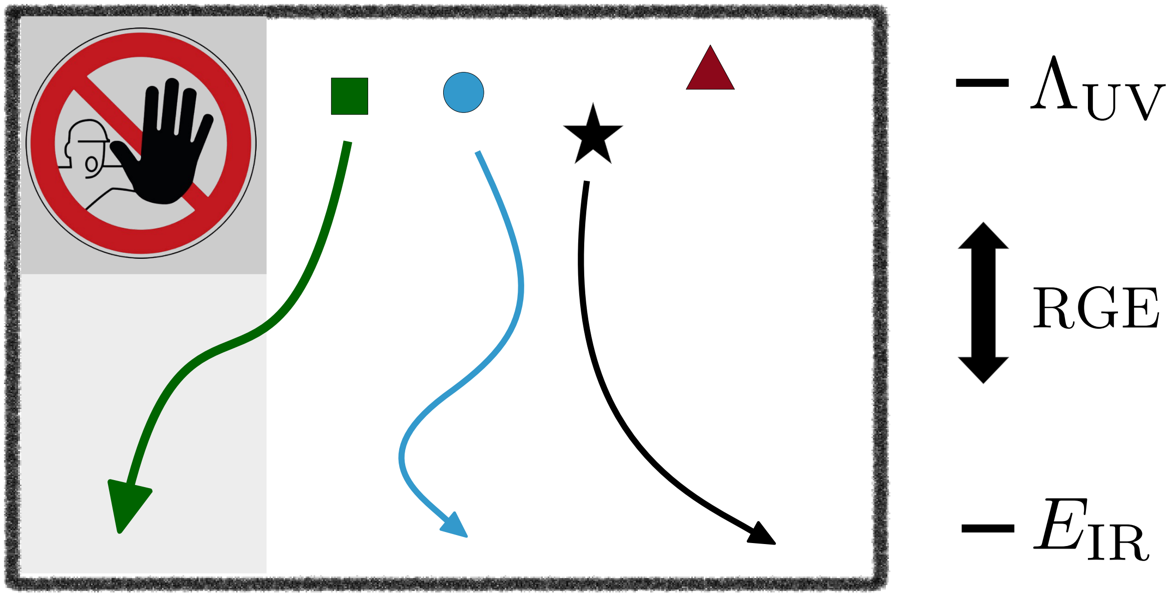

"Parameter space of SMEFT"

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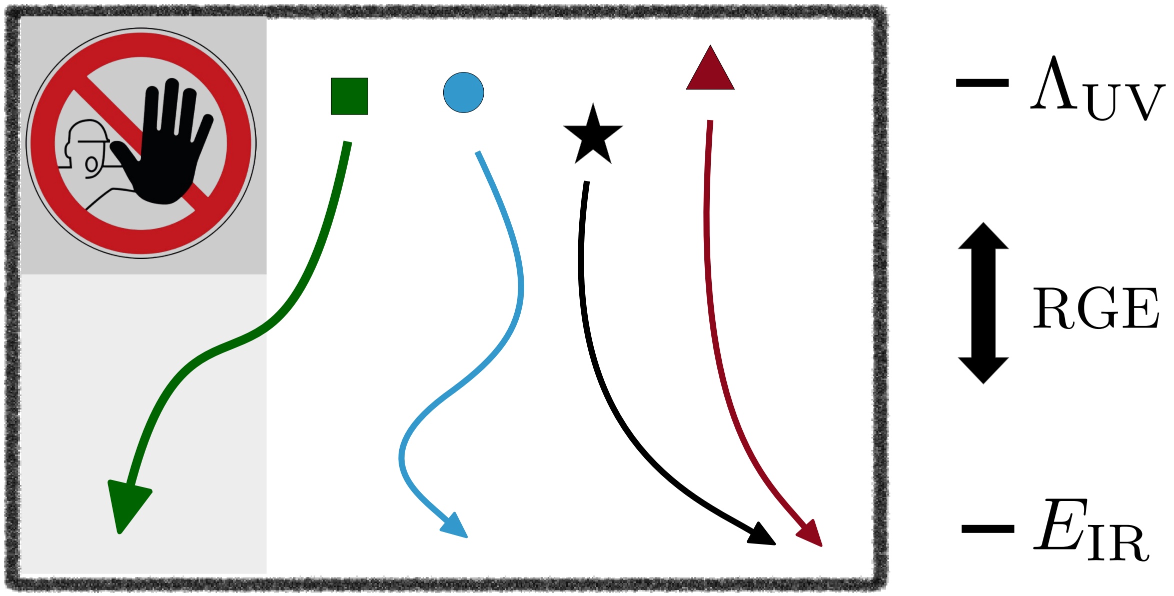

"Parameter space of SMEFT"

Thank you !

Backup slides

Why don't we just compute?

- Anomalous dimensions are basis dependent
- Helpful to identify universal structures of generic EFTs

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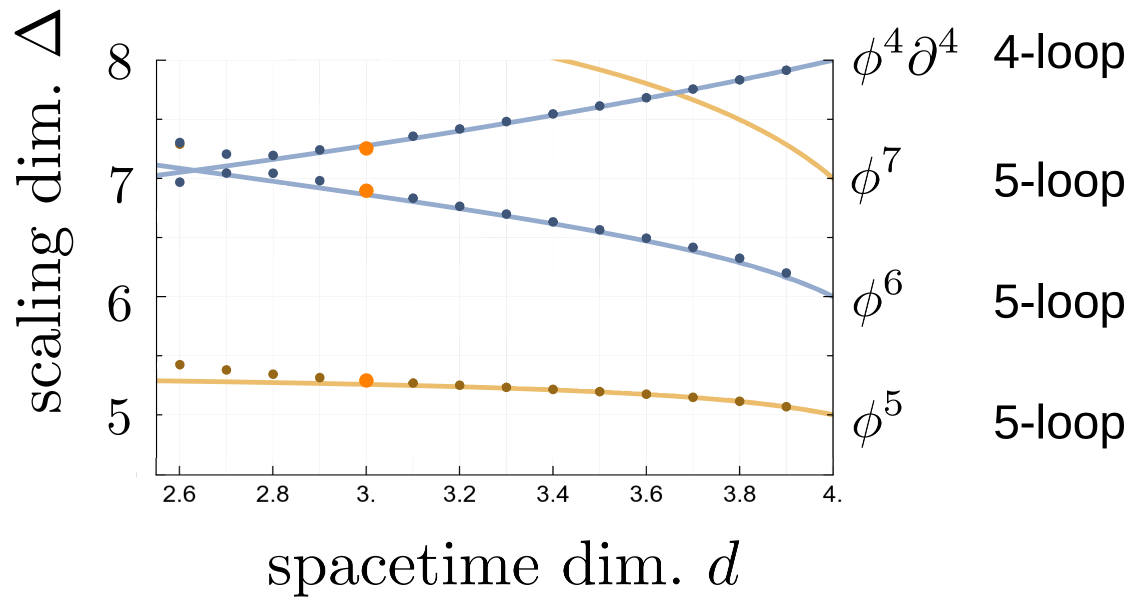
- Helpful to identify universal structures of generic EFTs

Work in progress

- Multi-loop renormalization of the most general scalar EFT
  - 5-loop for any dim-6 operator
  - 2-loop for any dim-8 operator
- Future extension to a general scalar-gluon EFT at multi-loop

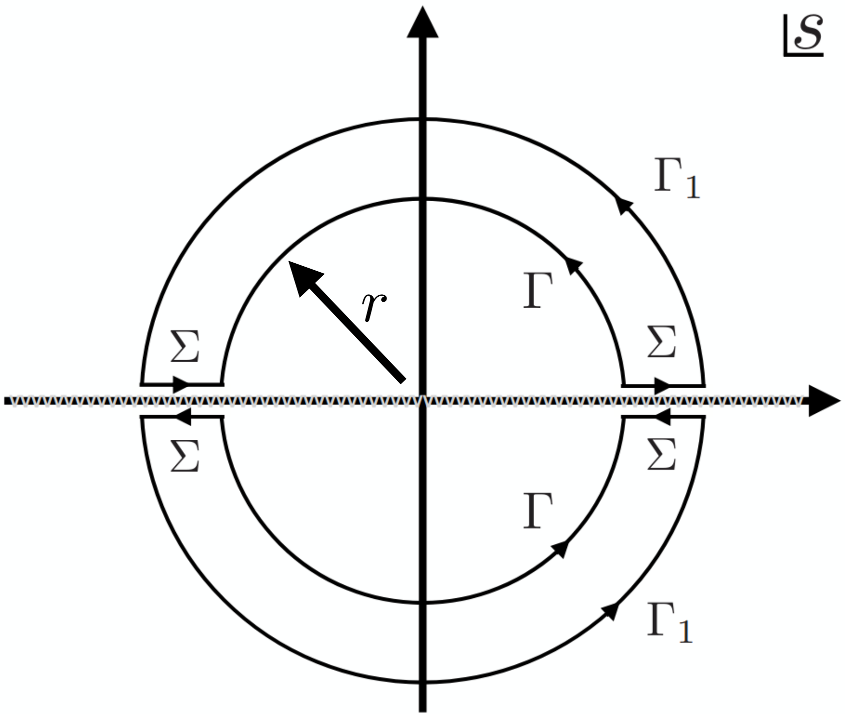
Henriksson, Herzog, Kousvos, JRN (2025 - expected)



Results for Ising CFT

Results from the numerical bootstrap
Simmons-Duffin (2016)
Henriksson, Kousvos, Reehorst (2022)

RG from dispersion relations



$$\text{arc}(r) = \frac{1}{2\pi i}\int_{\Gamma} \frac{ds}{s^3} A(s,t)$$

$$\text{arc}(r) = \text{arc}(r_1) + \frac{1}{2\pi i}\int_{\Sigma} \frac{ds}{s^3}\, \text{Disc} A(s,t)$$

$$\geq 0$$

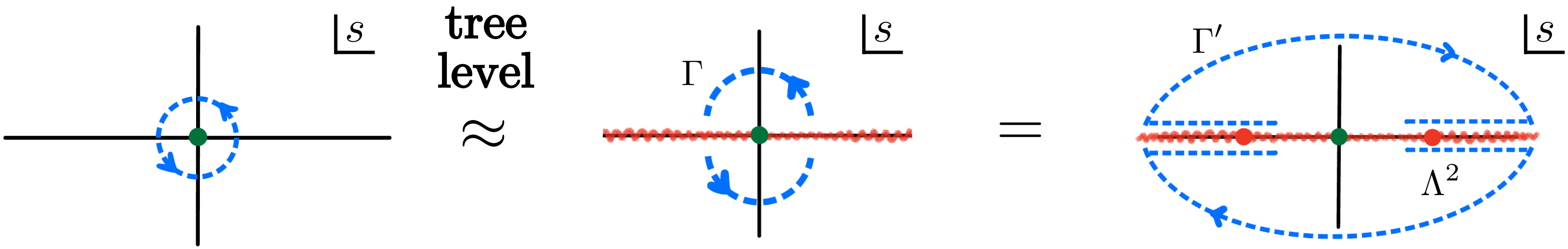
$$\Rightarrow \quad \frac{d}{dr}\text{arc}(r) \leq 0$$

Example in ϕ^4-theory: $\frac{d}{dr}\text{arc}(r) = -\frac{\beta_{\tilde{\lambda}}}{3\,r^3} + \frac{2}{\Lambda^4}\frac{\beta_8}{r} + \mathcal{O}\left(\frac{1}{\Lambda^6}\right) \leq 0$

See Liao, JRN, Shen (2025)

Positivity bounds at loop level

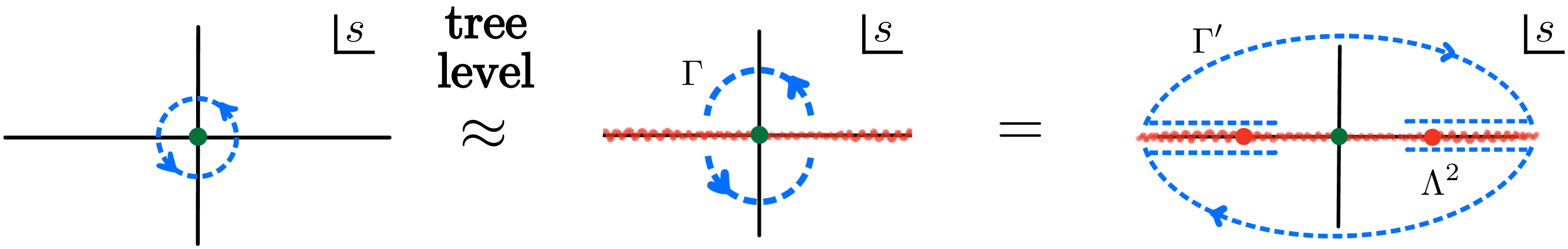
Not valid at loop level in EFT



$$c_4(s)$$

Not valid at loop level in EFT

Positivity bounds at loop level



- Example: Goldstone boson scattering in the forward limit

$$\mathcal{A}_{\text{EFT}}^{\text{1-loop}}(s, t \to 0) = c_2 \frac{s^2}{\Lambda^4} + \underbrace{\left(c_4(\mu) + \frac{1}{2}\beta_4 \left(\log \frac{-s}{\mu^2} + \log \frac{-u}{\mu^2}\right)\right)}_{c_4(s)} \frac{s^4}{\Lambda^8} + \ldots$$

$$c_2 + O(s) > 0$$

$$c_4(s) + O(s) > 0$$

$$c_6(s) > \frac{\beta_4}{2\, s^2} + O(1/s) \quad \Longrightarrow \quad c_6(s) < 0 \quad \text{is allowed}$$

Bellazzini, Miró, Rattazzi, Riembau, Riva (2021)
Arkani-Hamed, Huang, Huang (2021)

Dimension eight operators

Why study dimension eight operators in the SMEFT?

- Some observables may be dominated by dimension eight effects

  e.g. Degrande (2013); Liu, Pomarol, Rattazzi, Riva (2016)
  Alioli, Boughezal, Mereghetti, Petriello (2020)

- Helicity selection rules **Azatov, Contino, Machado, Riva (2016)**

- Logarithmically enhanced effects at dimension eight

  Grojean, Guedes, JRN, Salla (2024)

- Identify the UV completion from measured EFT parameters

  Zhang, Zhou (2020); Zhang (2021)

- Positivity bounds

  Pham, Truong (1985)
  Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi (2006)

Definition of operators

$$\mathcal{G}_1^2 \equiv 6\, g_2^2 \left(c_{W^3}^2 + c_{\widetilde{W}W^2}^2 \right),$$

$$\mathcal{G}_2^2 \equiv 8 \left(c_{H^2WB}^2 + c_{H^2\widetilde{W}B}^2 \right),$$

$$\mathcal{G}_3^2 \equiv 16 \left(c_{H^2B^2}^2 + c_{H^2\widetilde{B}B}^2 + 3 \left(c_{H^2W^2}^2 + c_{H^2\widetilde{W}W}^2 \right) + 8 \left(c_{H^2G^2}^2 + c_{H^2\widetilde{G}G}^2 \right) \right)$$

$$\mathcal{F}_1^2 \equiv \frac{8}{3} \left[3\, \mathrm{tr}\big((c_{Hd})^\dagger c_{Hd}\big) + \mathrm{tr}\big((c_{He})^\dagger c_{He}\big) + 2\, \mathrm{tr}\Big((c_{HL}^{(1)})^\dagger c_{HL}^{(1)}\Big) + 6\, \mathrm{tr}\Big((c_{Hq}^{(1)})^\dagger c_{Hq}^{(1)}\Big) + 3\, \mathrm{tr}\big((c_{Hu})^\dagger c_{Hu}\big) \right]$$

$$\mathcal{F}_2^2 \equiv \frac{16}{3} \left[\mathrm{tr}\Big((c_{HL}^{(3)})^\dagger c_{HL}^{(3)}\Big) + 3\, \mathrm{tr}\Big((c_{Hq}^{(3)})^\dagger c_{Hq}^{(3)}\Big) \right],$$

$$\mathcal{F}_3^2 \equiv 8\, \mathrm{tr}\big((c_{Hud})^\dagger c_{Hud}\big),$$

H^2F^2

$\mathcal{O}_{H^2B^2}$	$(H^\dagger H) B_{\mu\nu} B^{\mu\nu}$
$\mathcal{O}_{H^2\widetilde{B}B}$	$(H^\dagger H) \widetilde{B}_{\mu\nu} B^{\mu\nu}$
$\mathcal{O}_{H^2W^2}$	$(H^\dagger H) W^I_{\mu\nu} W^{I\mu\nu}$
$\mathcal{O}_{H^2\widetilde{W}W}$	$(H^\dagger H) \widetilde{B}^I_{\mu\nu} W^{I\mu\nu}$
$\mathcal{O}_{H^2G^2}$	$(H^\dagger H) G^A_{\mu\nu} G^{A\mu\nu}$
$\mathcal{O}_{H^2\widetilde{G}G}$	$(H^\dagger H) \widetilde{G}^A_{\mu\nu} G^{A\mu\nu}$
$\mathcal{O}_{H^2WB}$	$(H^\dagger \tau^I H) W^I_{\mu\nu} B^{\mu\nu}$
$\mathcal{O}_{H^2\widetilde{W}B}$	$(H^\dagger \tau^I H) \widetilde{W}^I_{\mu\nu} B^{\mu\nu}$

$\overline{\psi}\psi H^2 D$

$\mathcal{O}_{HL}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\overline{L}_\alpha \gamma^\mu L_\beta)$
$\mathcal{O}_{HL}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}^I_\mu H)(\overline{L}_\alpha \tau^I \gamma^\mu L_\beta)$
$\mathcal{O}_{Hq}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\overline{q}_\alpha \gamma^\mu q_\beta)$
$\mathcal{O}_{Hq}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}^I_\mu H)(\overline{q}_\alpha \tau^I \gamma^\mu q_\beta)$
$\mathcal{O}_{He}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\overline{e}_\alpha \gamma^\mu e_\beta)$
$\mathcal{O}_{Hu}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\overline{u}_\alpha \gamma^\mu u_\beta)$
$\mathcal{O}_{Hd}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\overline{d}_\alpha \gamma^\mu d_\beta)$
$\mathcal{O}_{Hud}$	$i(\widetilde{H}^\dagger D_\mu H(\overline{u}_\alpha \gamma^\mu d_\beta)$

H^4D^2

$\mathcal{O}_{HD}$	$(H^\dagger D_\mu H)(D^\mu H^\dagger H)$
$\mathcal{O}_{H\Box}$	$(H^\dagger H) D^2 (H^\dagger H)$

H^4D^4

$\mathcal{O}^{(1)}_{H^4D^4}$	$(D_\mu H^\dagger D_\nu H)(D^\nu H^\dagger D^\mu H)$
$\mathcal{O}^{(2)}_{H^4D^4}$	$(D_\mu H^\dagger D_\nu H)(D^\mu H^\dagger D^\nu H)$
$\mathcal{O}^{(3)}_{H^4D^4}$	$(D_\mu H^\dagger D^\mu H)(D_\nu H^\dagger D^\nu H)$

F^3

$\mathcal{O}_{W^3}$	$\epsilon^{IJK} W^{I\nu}_\mu W^{J\rho}_\nu W^{K\mu}_\rho$
$\mathcal{O}_{\widetilde{W}W^2}$	$\epsilon^{IJK} \widetilde{W}^{I\nu}_\mu W^{J\rho}_\nu W^{K\mu}_\rho$

$(\overline{\psi}\psi)\,(\overline{\psi}\psi)$

$\mathcal{O}_{LL}$	$(\overline{L}_\alpha \gamma_\mu L_\beta)\,(\overline{L}_\gamma \gamma^\mu L_\delta)$