Meetings for Two-Component Dark Matter, 02/19/2024

Warm Surprises from Cold Duets

Jeong Han Kim

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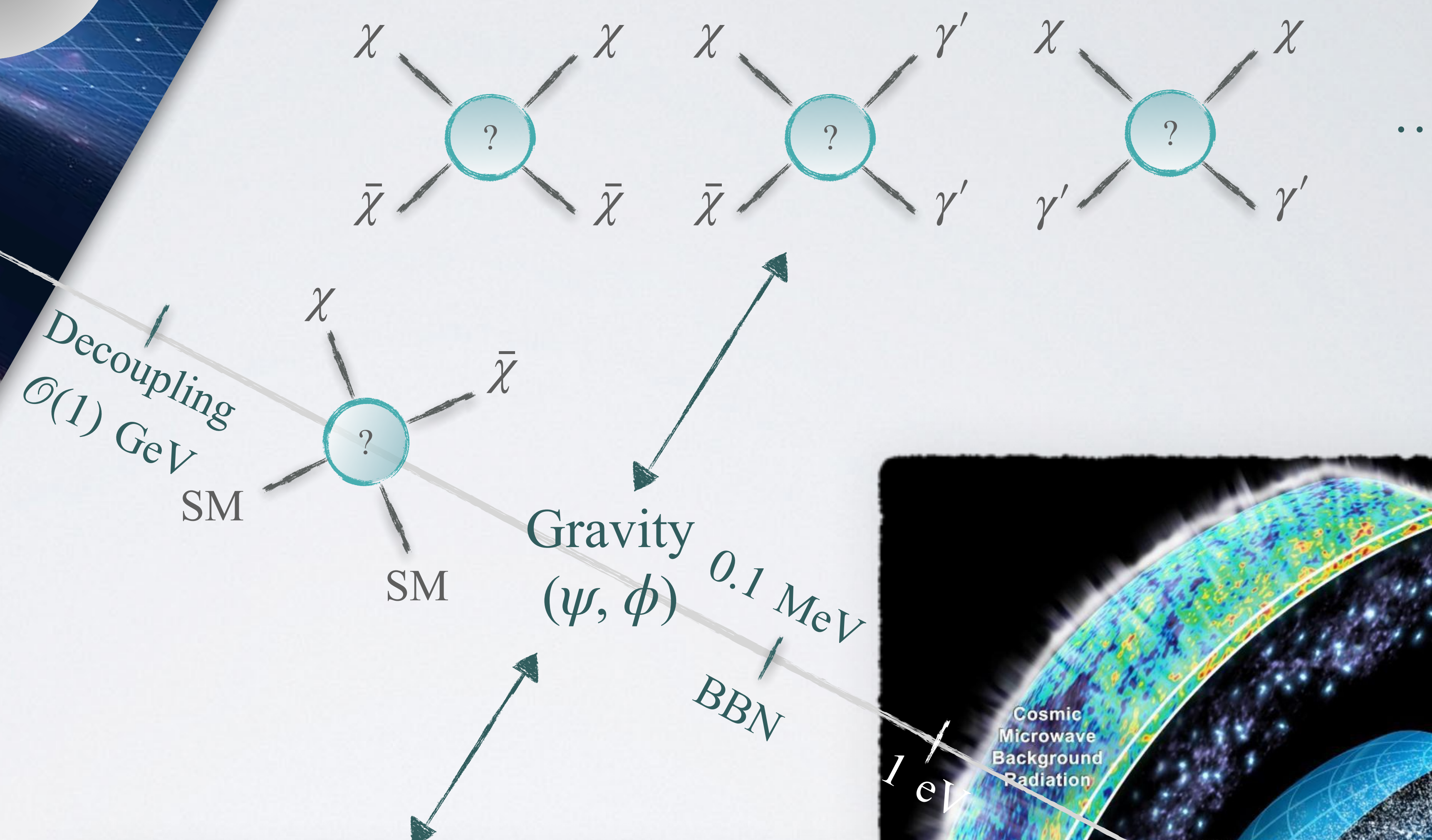
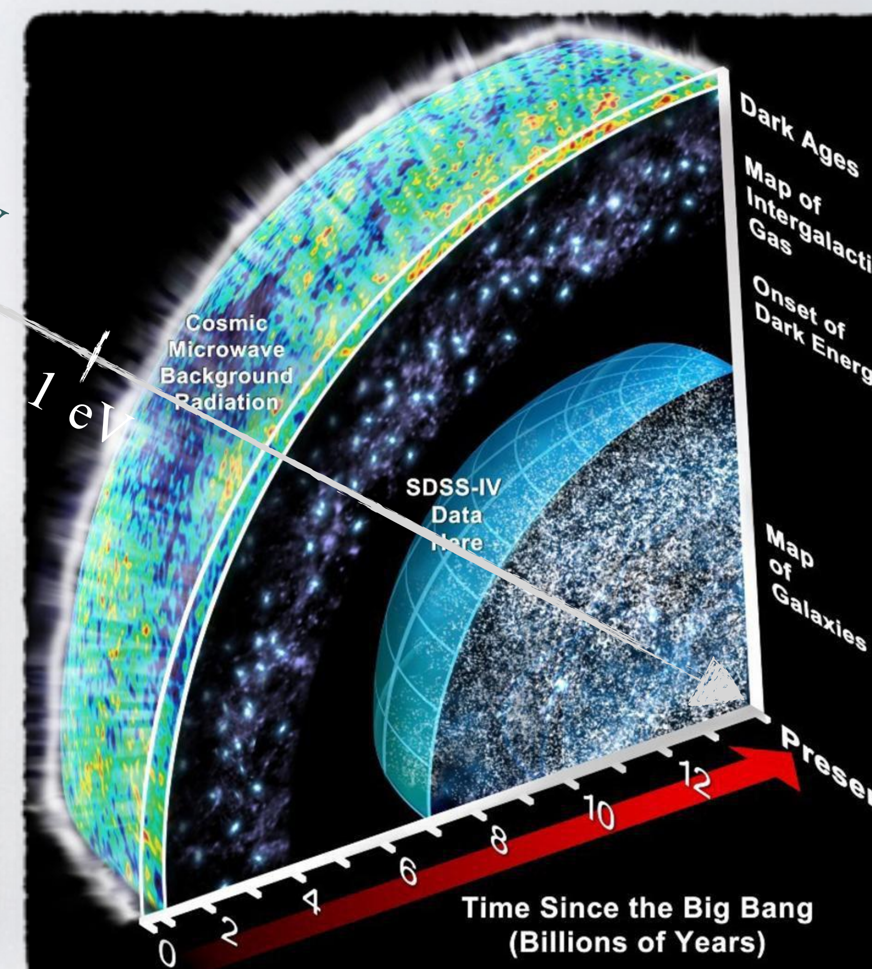
Based on :

Sehwan Lim, **Jeong Han Kim**, Kyoungchul Kong, Jong Chul Park - [arXiv:2312.07660]

Ayuki, Kamada, Hee Jung Kim, Jong Chul Park, Seodong Shin - [JCAP 10 (2022) 052]

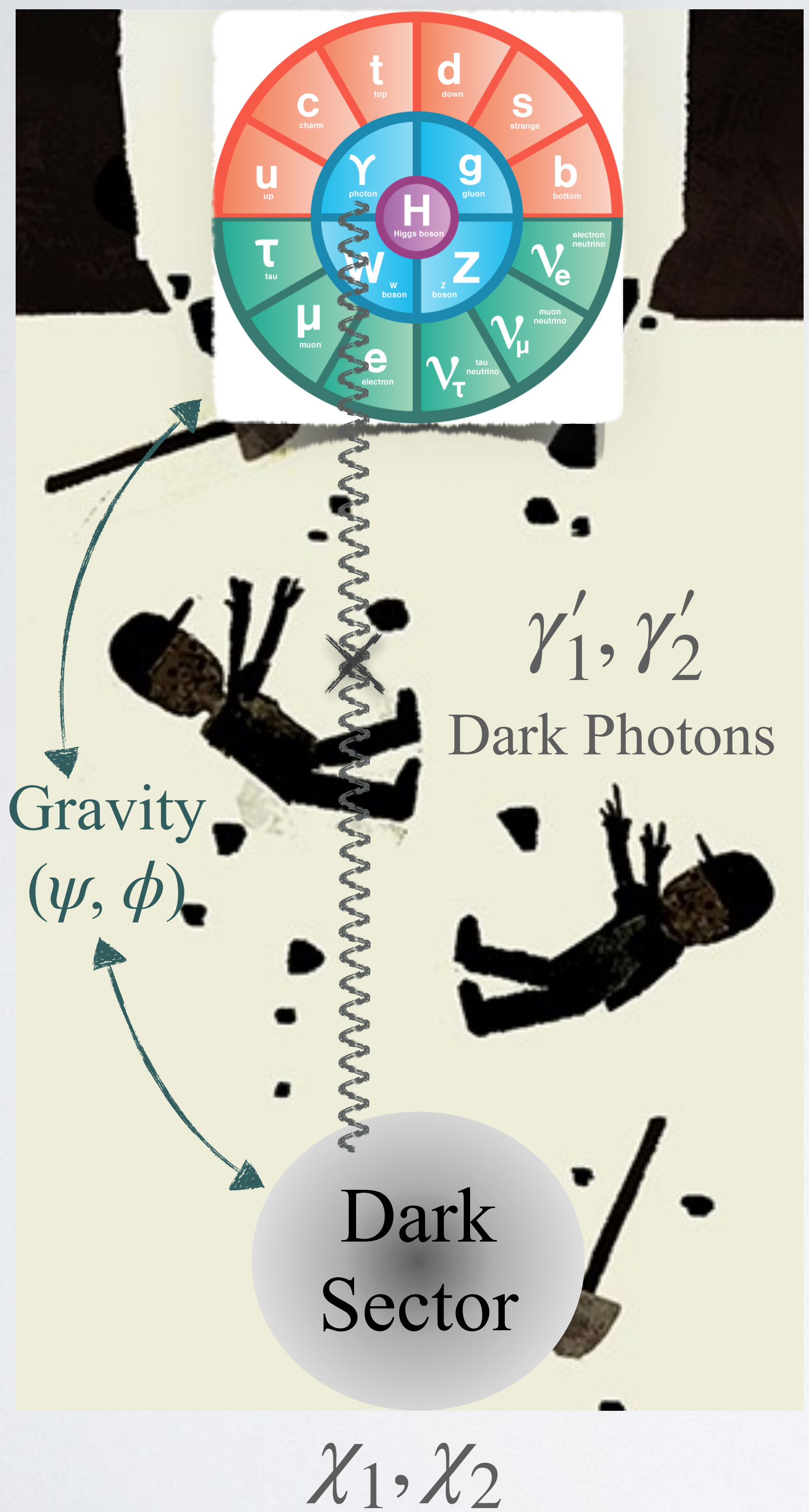
Motivation





- What is a hidden dynamics of a dark sector?
- What are useful cosmological data to illuminate them?
- Use the gravitational interaction as a main source to probe the dark sector.

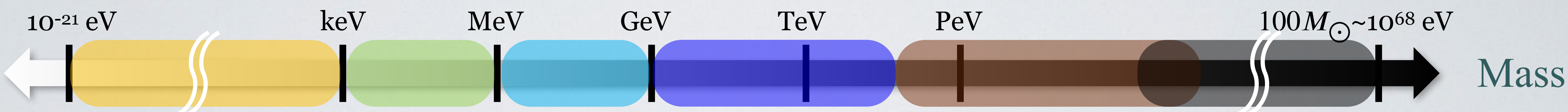
Motivation



- We take a simplified approach to explore the two-component dark matter.
- Exploring even a simplified model requires thorough analysis to identify its unique signatures in various cosmological data.
- This rigorous scrutiny paves the way for distinguishing one scenario from others.
- This is our strategy to unravel the complexities of the dark sector.

- Two-Component Scenarios (simplified model)

Simple Extension of ΛCDM



Ultralight
(QCD) axion,
hidden photon,
scalar field,
fuzzy

Superlight
sterile ν,
axino,
warm DM

Light
SIMP,
ELDER

WIMP

Superheavy

$m \sim \mathcal{O}(\text{keV})$

Warm Dark Matter (WDM)

WDM

+

Cold Dark Matter (CDM)

CDM

Mixed Cold and Warm Dark Matter (CWDM) (~27 %)

Boyarsky et al. [0812.0010]

Anderhalden et al. [1212.2967]

Maccio et al. [1202.2858]

...

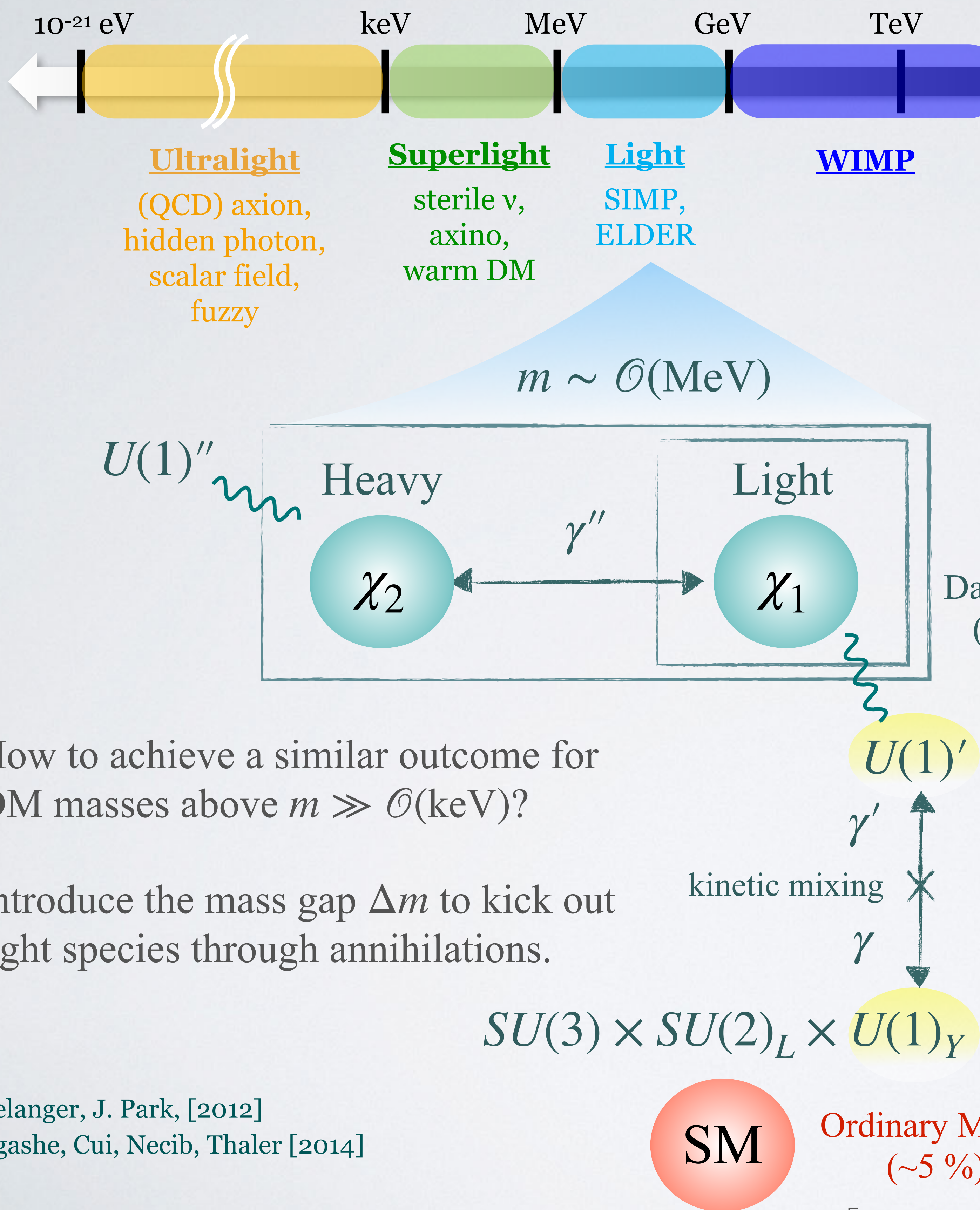
Dark Energy (~68 %)

Ordinary Matter (~5 %)

SM

- The WDM is able to free-stream and dampens density perturbations at small scales.
- To have a significant impact on astrophysical data, $m \sim \mathcal{O}(\text{keV})$.

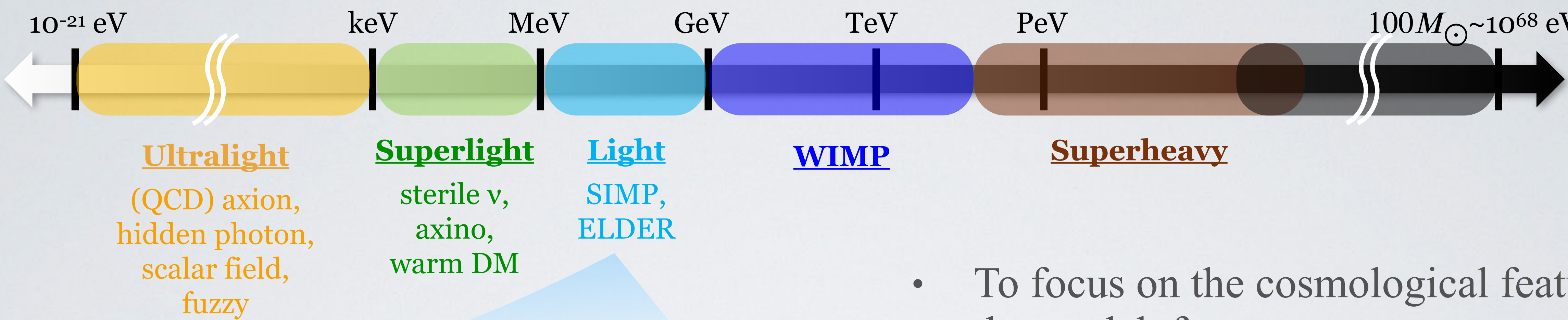
Simplified Two-Component DM

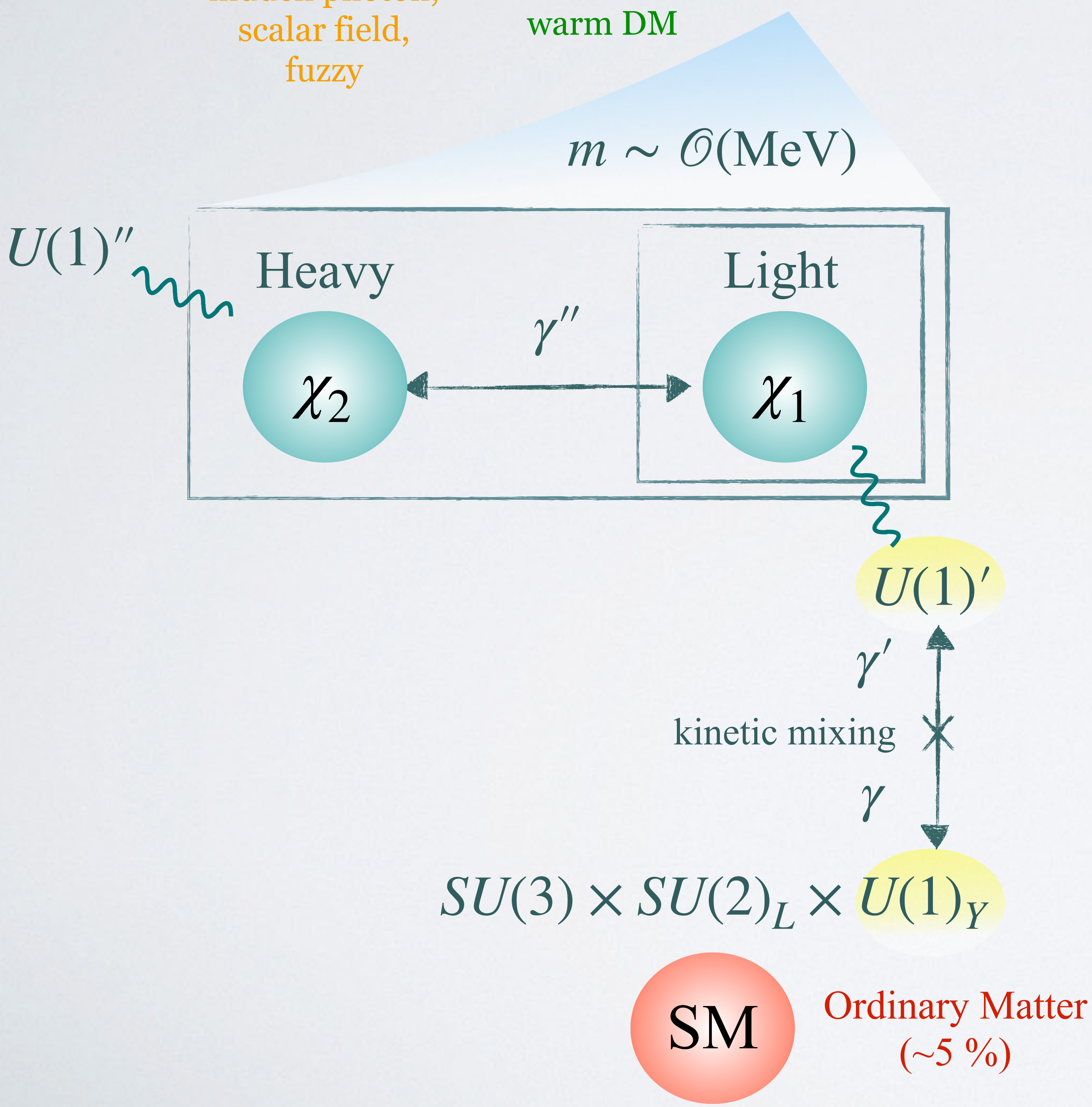


- How to achieve a similar outcome for DM masses above $m \gg \mathcal{O}(\mathrm{keV})$?
- Introduce the mass gap Δm to kick out light species through annihilations.

Belanger, J. Park, [2012]
Agashe, Cui, Necib, Thaler [2014]
...

Simplified Two-Component DM

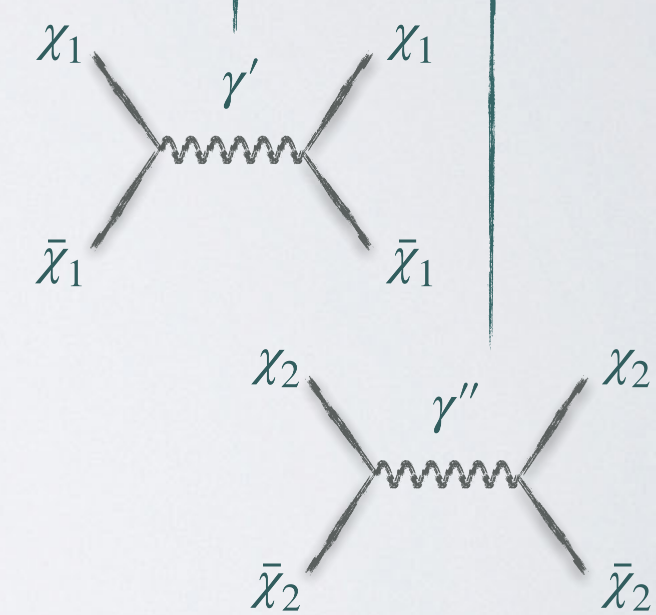




- To focus on the cosmological features of the model, free parameters are

m_1, m_2, Ω_{DM}, r_1, σ_{self1}, σ_{self2}

~ 0.27 (DM relic)

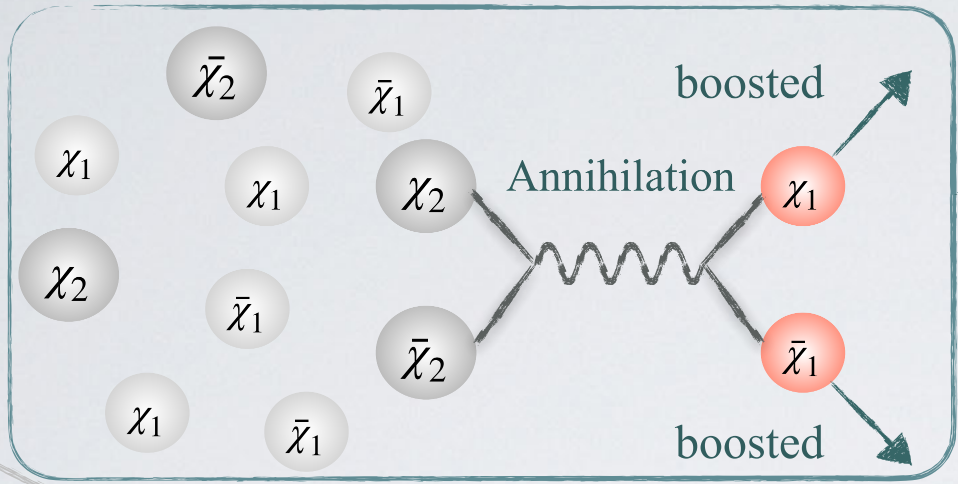


- The fraction of the final χ_1 relic is :

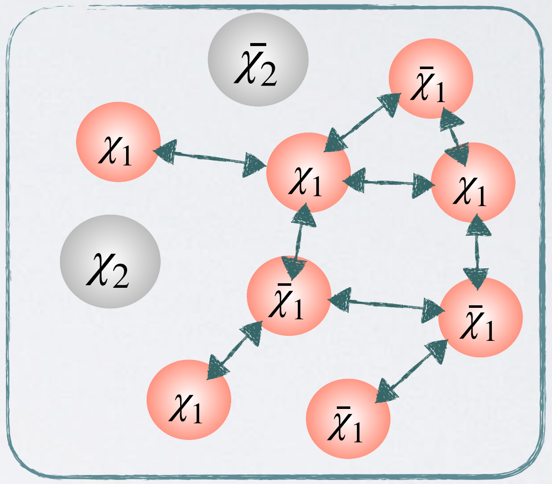
$$r_1 = \frac{\Omega_{\chi_1}}{(\Omega_{\chi_1} + \Omega_{\chi_2})}$$

"Self-Heating Effects"

1. The heavy χ_2 annihilates to light χ_1 which becomes boosted.



2. Sharing energies through self-interaction $\sigma_{11\to11}/m_{\chi_1}$ which increases the χ_1 temperature.



(with $\Gamma_{11\to11} > H$)

...

χ_2 Decoupling

χ_1 Decoupling

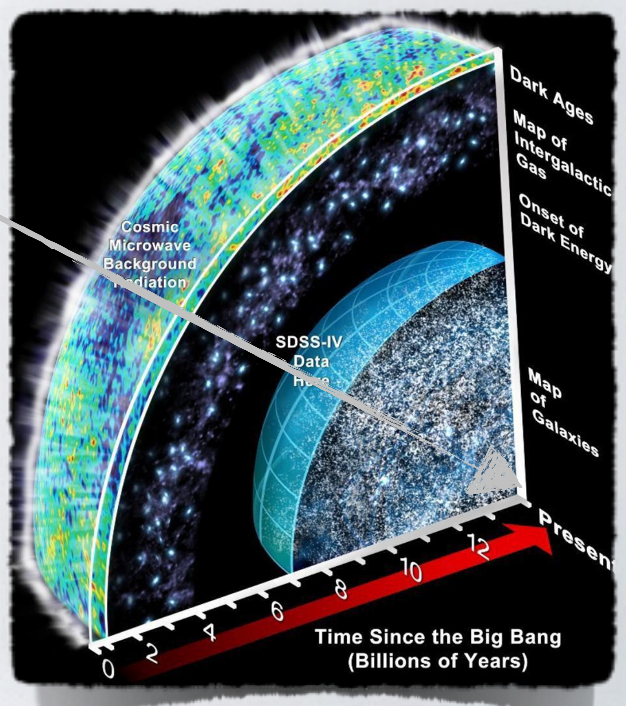
...

$$\gamma_{\rm heat} = \frac{2n_{\chi_2}^2 \langle \sigma v \rangle_{22\to11}}{3n_{\chi_1} T}(m_{\chi_2} - m_{\chi_1})$$

$$\dot{T}_{\chi_1} + 2HT_{\chi_1} \simeq \gamma_{\rm heat} T - 2\gamma_{\chi_1,SM}\left(T_{\chi_1} - T\right)$$

Cooling due the Hubble expansion

Kinetic scattering of χ_1 with a thermal bath

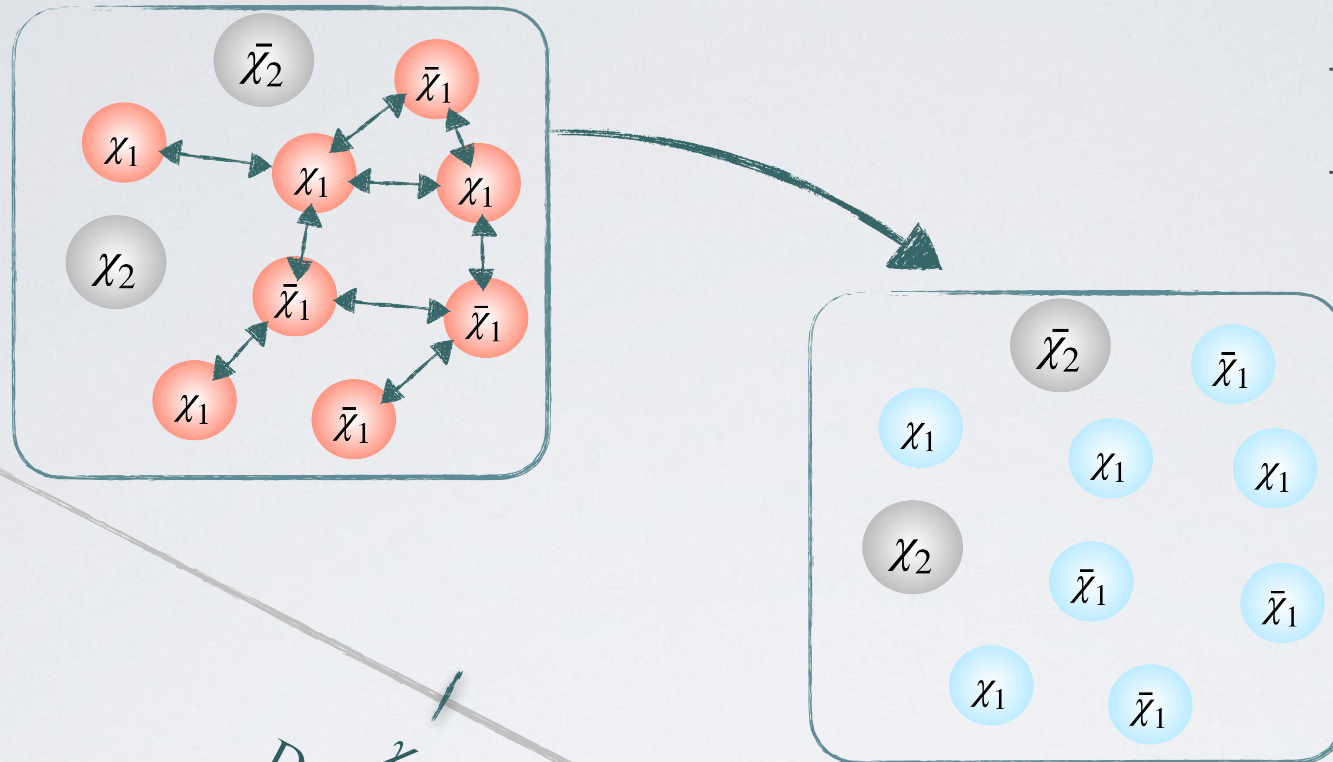


A. Kamada, H. Kim, J. Park, S. Shin [2021]

Sehwan Lim, **J. H. Kim**, K.C. Kong, J. Park [2023]

“Self-Heating Effects”

3. When the self-interaction rate drops below the Hubble scale, it starts to cool down.



...

χ_2 Decoupling

χ_1 Decoupling

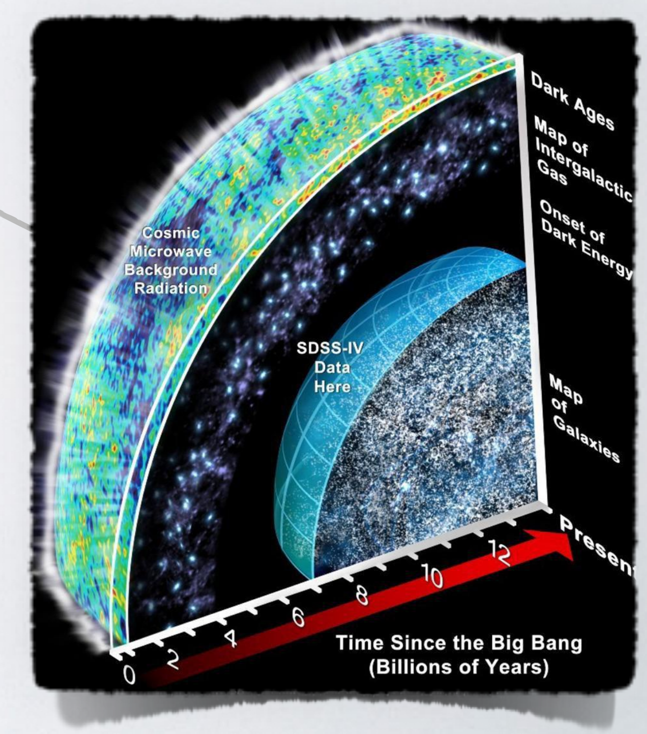
$T_{\rm dec,self}$

Decoupling temperature of the self-interaction

$$T_{\rm dec,self} \sim \left(\frac{0.3}{r_1}\right)^{2/3}\left(\frac{m_{\chi_1}}{100\ {\rm MeV}}\right)^{1/3}\left(\frac{1\,{\rm cm^2/g}}{\sigma_{11\to11}/m_{\chi_1}}\right)^{2/3}$$

$$\dot{T}_{\chi_1} + 2HT_{\chi_1} \simeq 0$$

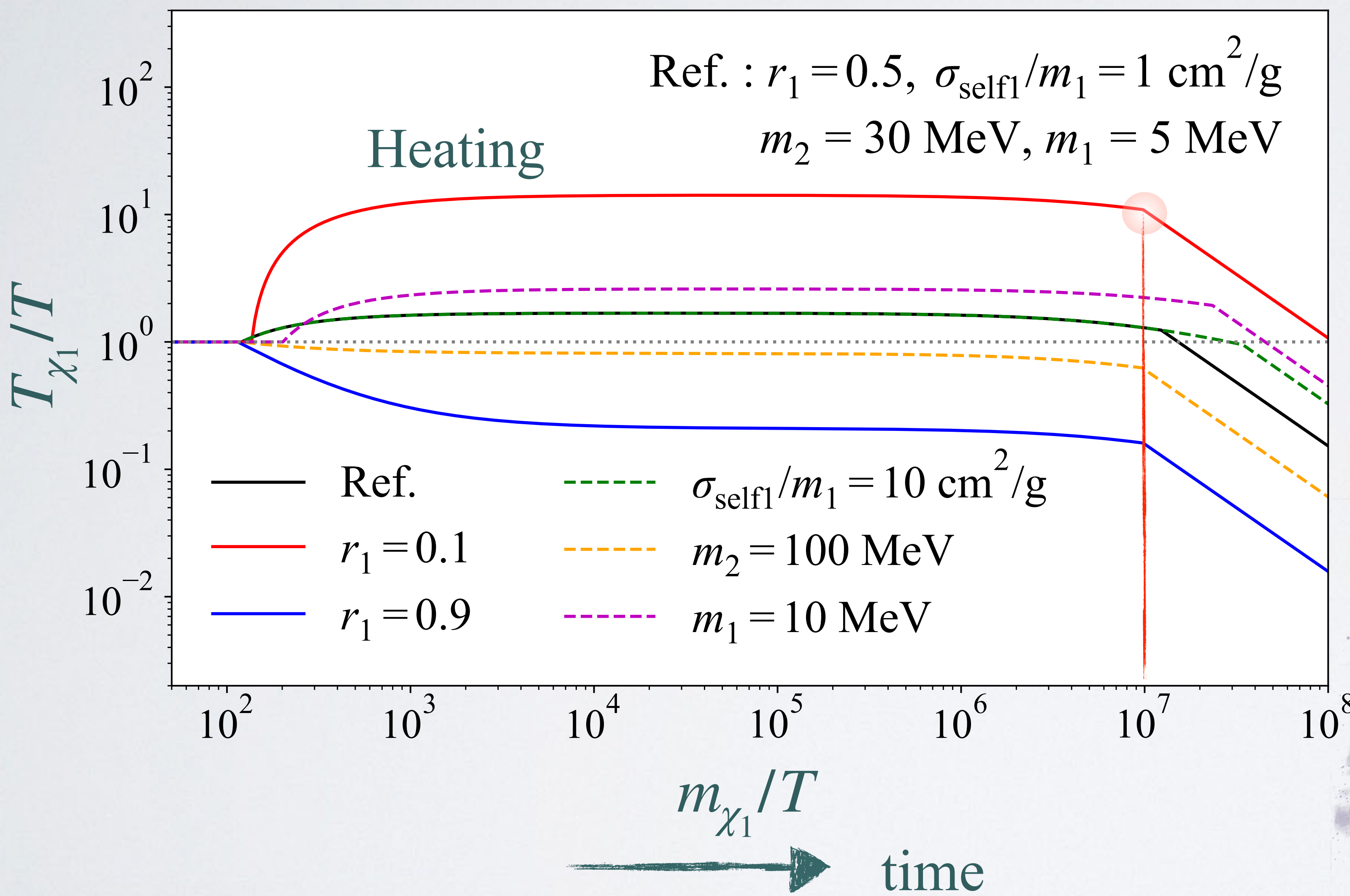
(when $\Gamma_{11\to11} < H$)

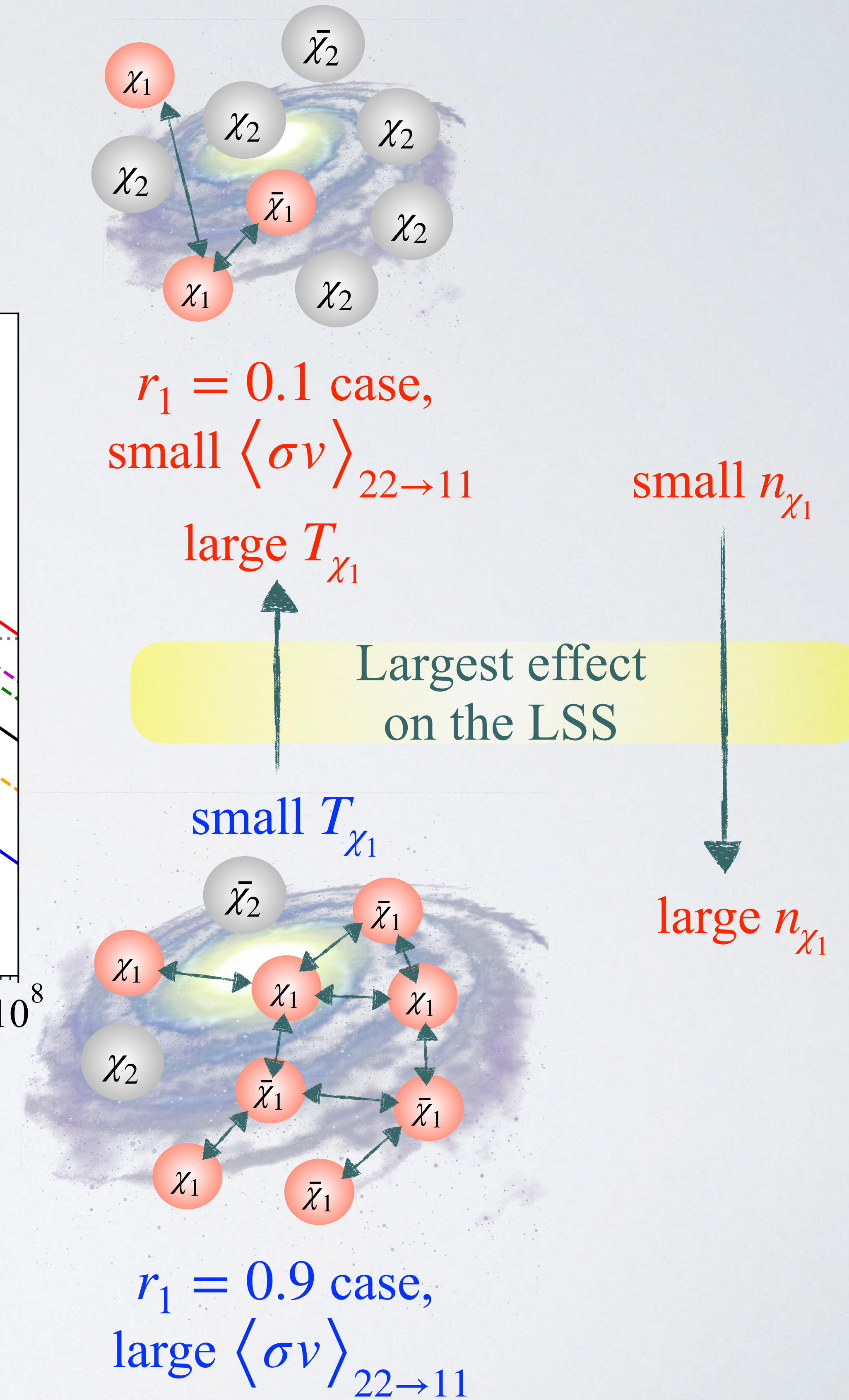


Temperature Evolution of Light χ_1

Ratio of relics :

$r_1 = \Omega_{\chi_1}/(\Omega_{\chi_1} + \Omega_{\chi_2})$



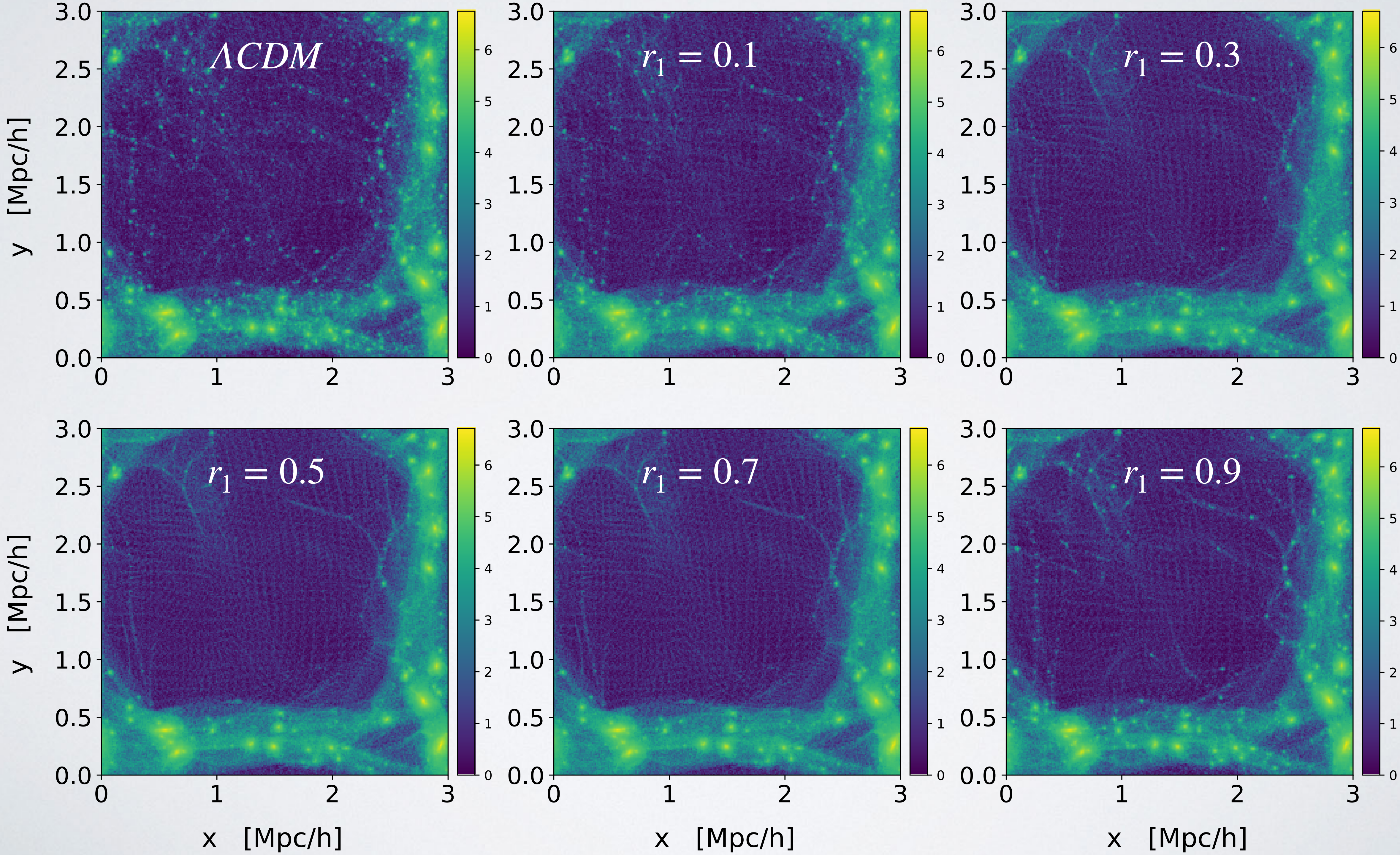


How Does the Structure Formation Change?

Sehwan Lim, **J. H. Kim**, K.C. Kong, J. Park [2023]

- There seem to be fewer subhalos in the two-component Universe.

(For fixed $\sigma_{11\to11}/m_{\chi_1} = 1\text{cm}^2/\text{g}$, $m_{\chi_2} = 30$ MeV, $m_{\chi_1} = 5$ MeV)



Perturbed Boltzmann Equations

Sehwan Lim, **J. H. Kim**, K.C. Kong, J. Park [2023]

- Use the FRW metric with the following convention

$$ds^2 = -(1+2\Psi)dt^2 + (1-2\Phi)a(t)^2\delta_{ij}dx^i dx^j$$

- Density contrasts δ_{χ_i} dictate amount of matter perturbations.

$$\rho_{\chi_i} = \bar{\rho}_{\chi_i}\left(1+\delta_{\chi_i}\right) \quad \text{(with } i = 1,\ 2\text{)}$$

- Perturbed velocities $\vec{v}_{\chi_i}$ of dark matters.

$$\theta_{\chi_i} = \nabla \cdot \vec{v}_{\chi_i}$$

- Perturbation equations for χ_2 . See also the lecture by Lam Hui

(number density)

$$n_{\chi_i,\mathrm{eq}} \simeq g_{\chi_i}\, e^{-m_{\chi_i}/T}\left(\frac{m_{\chi_i} T}{2\pi}\right)^{3/2}$$

(energy density)

$$\rho_{\chi_i,\mathrm{eq}} \simeq m_{\chi_i}\, n_{\chi_i,\mathrm{eq}}$$

(perturbation for $\rho_{\chi_i,\mathrm{eq}}$)

$$\delta_{\chi_i,\mathrm{eq}} = \frac{n_{\chi_i,\mathrm{eq}}}{\bar{n}_{\chi_i,\mathrm{eq}}} - 1$$

$$\frac{d\delta_{\chi_2}}{dt} + \frac{\theta_{\chi_2}}{a} - 3\frac{d\Phi}{dt} = \frac{\langle\sigma v\rangle_{22\to11}}{m_{\chi_2}\bar{\rho}_{\chi_2}}\left(-\Psi\left(\bar{\rho}^2_{\chi_2} - \frac{\bar{\rho}^2_{\chi_2,\mathrm{eq}}}{\bar{\rho}^2_{\chi_1,\mathrm{eq}}}\bar{\rho}^2_{\chi_1}\right) - \bar{\rho}^2_{\chi_2}\delta_{\chi_2} + \frac{\bar{\rho}^2_{\chi_2,\mathrm{eq}}}{\bar{\rho}^2_{\chi_1,\mathrm{eq}}}\bar{\rho}^2_{\chi_1}\left(2\delta_{\chi_2,\mathrm{eq}} - \delta_{\chi_2} - 2\delta_{\chi_1,\mathrm{eq}} + 2\delta_{\chi_1}\right)\right)$$

$$\frac{d\theta_{\chi_2}}{dt} + H\theta_{\chi_2} + \frac{\nabla^2\Psi}{a} = \frac{\langle\sigma v\rangle_{22\to11}}{m_{\chi_2}\bar{\rho}_{\chi_2}}\frac{\bar{\rho}^2_{\chi_2,\mathrm{eq}}}{\bar{\rho}^2_{\chi_1,\mathrm{eq}}}\bar{\rho}^2_{\chi_1}\left(\theta_{\chi_1} - \theta_{\chi_2}\right)$$

$$c^2_{s,\chi_2}\frac{\nabla^2\delta_{\chi_2}}{a} \to 0$$

We neglect the sound speed of χ_2

$T_{\chi_2} \simeq 0$ (same as CDM)

- And two independent Einstein equations.

Perturbed Boltzmann Equations

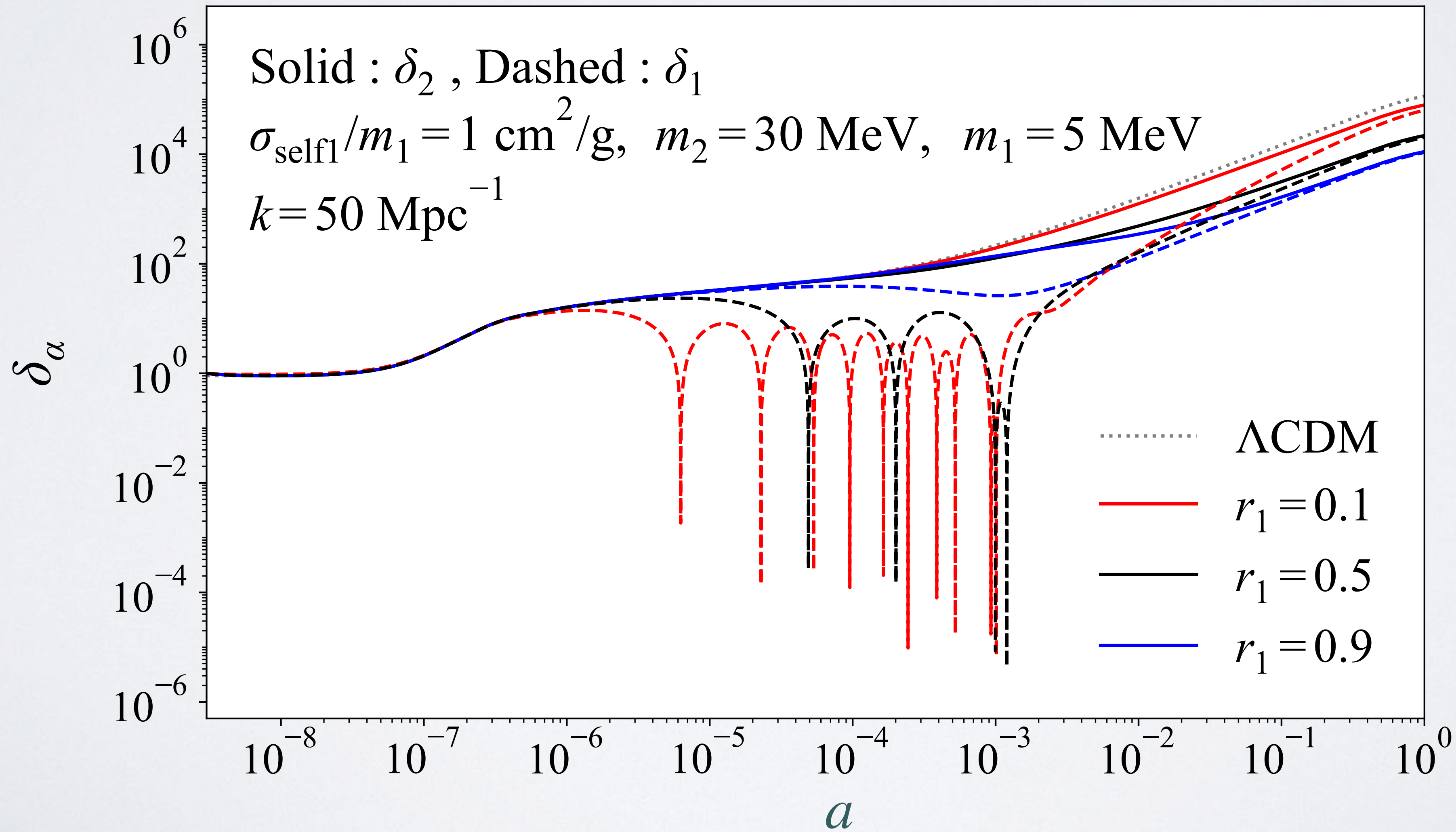
Sehwan Lim, **J. H. Kim**, K.C. Kong, J. Park [2023]

- When $T \ll m_{\chi_i}$ (at around matter-dominated era)

$$\frac{d^2\delta_2}{dt^2} + \left(2H + \frac{\langle \sigma v \rangle_{22\to 11}}{m_2}\bar{\rho}_2\right)\frac{d\delta_2}{dt} - \left(\frac{\langle \sigma v \rangle_{22\to 11}}{m_2}H + 4\pi G\right)\bar{\rho}_2\delta_2 = \left(\text{terms of gravity}\right) + \left(\text{coupled terms with } \delta_1\right)$$

Friction caused by χ_2 annihilation

Negative (δ_{χ_2} grows)



Perturbed Boltzmann Equations

- When $T \ll m_{\chi_i}$ (at around matter-dominated era)

Sehwan Lim, **J. H. Kim**, K.C. Kong, J. Park [2023]

Friction caused by χ_1 annihilation

$$\frac{d^2\delta_1}{dt^2} + \left(2H + 2\frac{\langle\sigma v\rangle_{22\to11}}{m_2}\frac{\bar\rho_2^2}{\bar\rho_1} + \frac{\langle\sigma v\rangle_{11\to \mathrm{SMSM}}}{m_1}\bar\rho_1\right)\frac{d\delta_1}{dt} - \left(\frac{\langle\sigma v\rangle_{22\to11}}{m_2}\frac{\bar\rho_2^2}{\bar\rho_1}H + \frac{\langle\sigma v\rangle_{11\to \mathrm{SMSM}}}{m_1}\bar\rho_1 H + 4\pi G\bar\rho_1 - c_{s,1}^2\frac{k^2}{a^2}\right)\delta_1$$

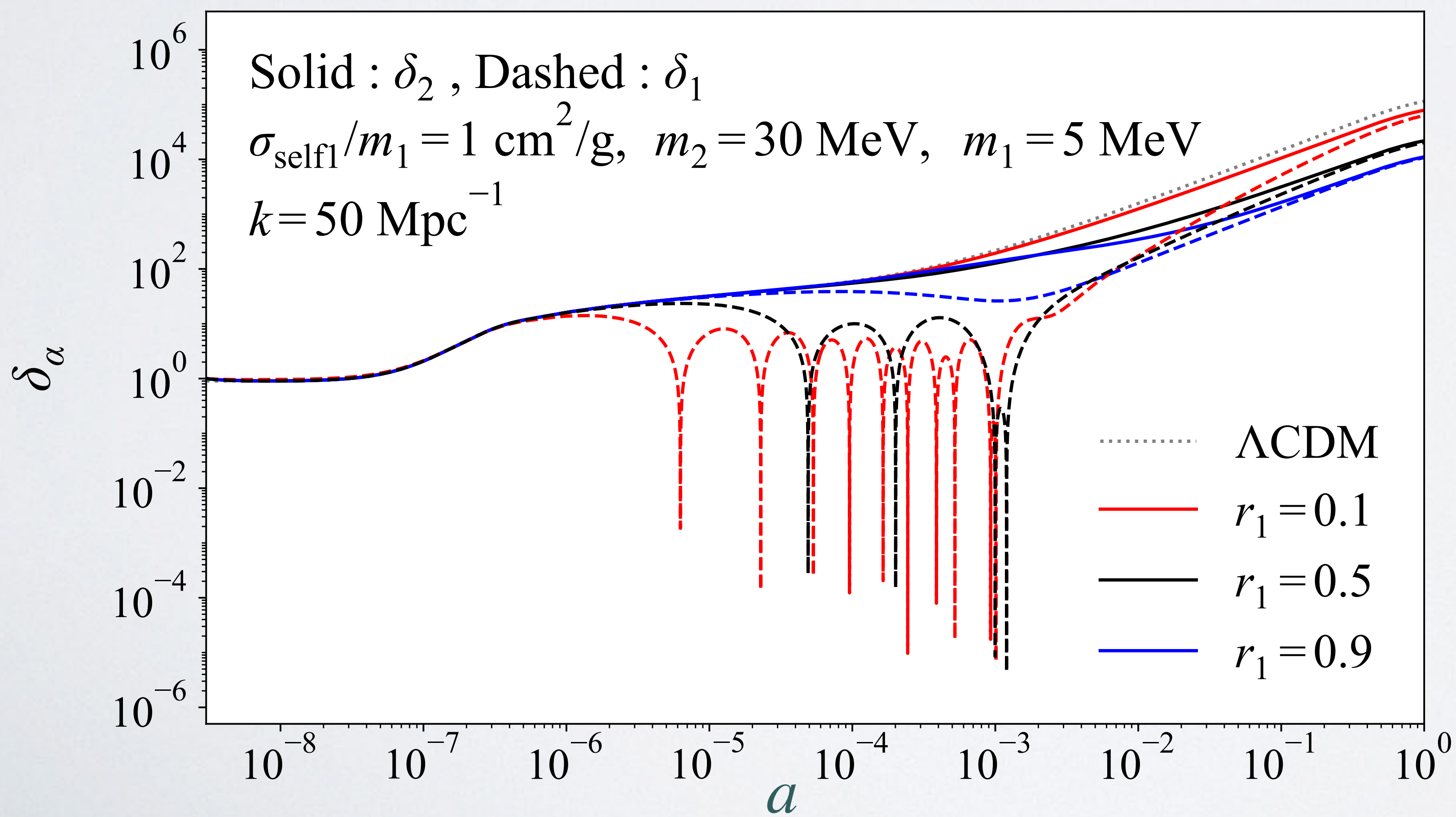
$$= \left(\text{terms of gravity}\right) + \left(\text{coupled terms with } \delta_2\right)$$

Negative: δ_{χ_1} grows

Positive: δ_{χ_1} oscillates

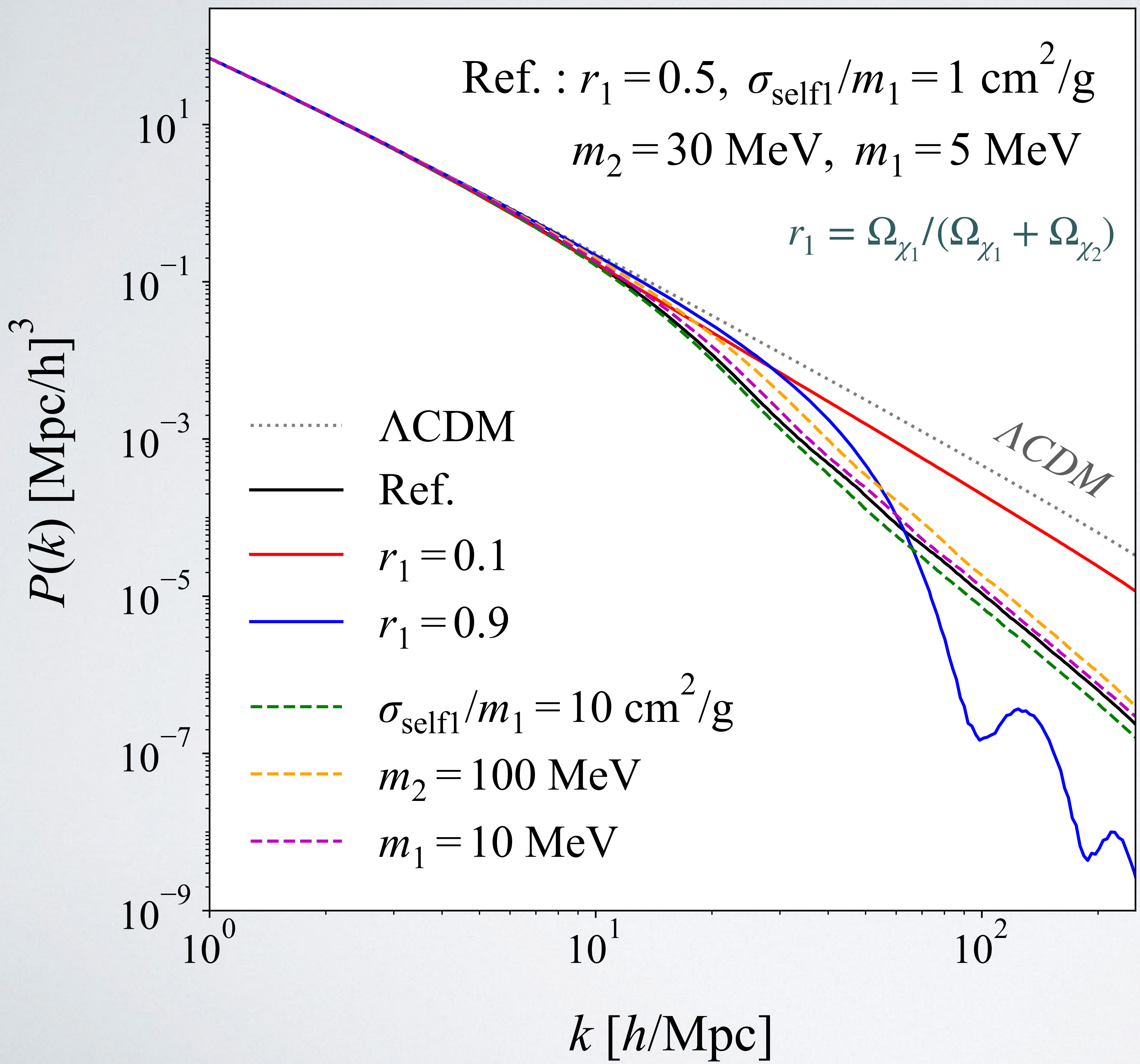
Sound speed of χ_1 resists against the gravity

$$c_{s,\chi_1}^2 = \frac{T_{\chi_1}}{m_{\chi_1}}\left(1 - \frac{1}{3}\frac{\partial \ln T_{\chi_1}}{\partial \ln a}\right)$$



Linear Matter Power Spectrum

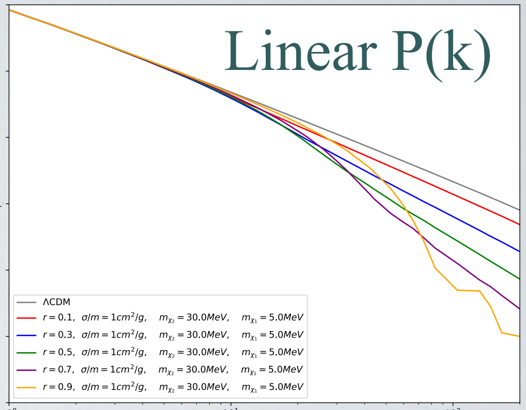
Sehwan Lim, **J. H. Kim**, K.C. Kong, J. Park [2023]



Including Non-Linear Effects

Solve linear Einstein-Boltzmann equations until today

$z = 0$



$z \sim 100$

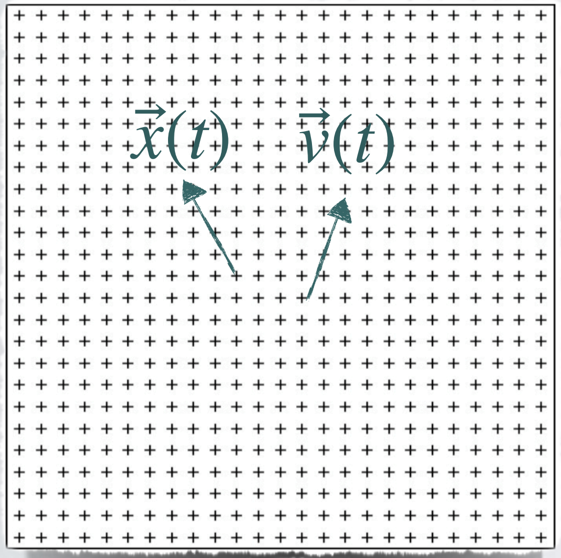
Back-scaling

Newtonian linear growth factor
(Including only background quantities)

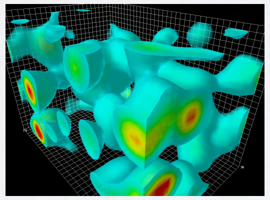
$$\delta_m(z_{\text{start}}) = \delta_m(z=0)\frac{D(z=start)}{D(z=0)}$$

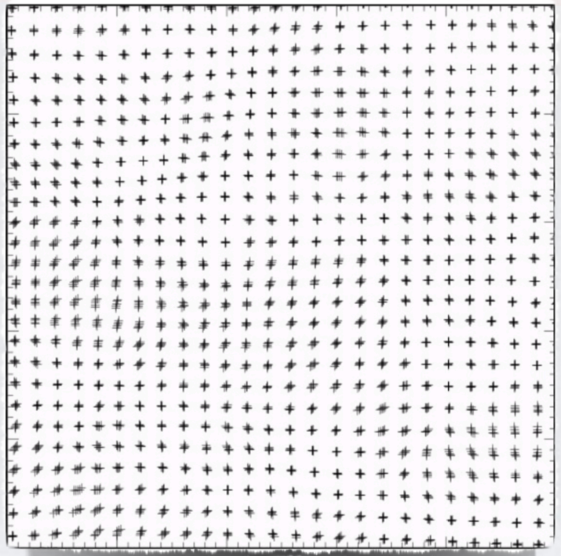
The input of the simulation is the linear P(k) at $z = 0$.

N-body simulations

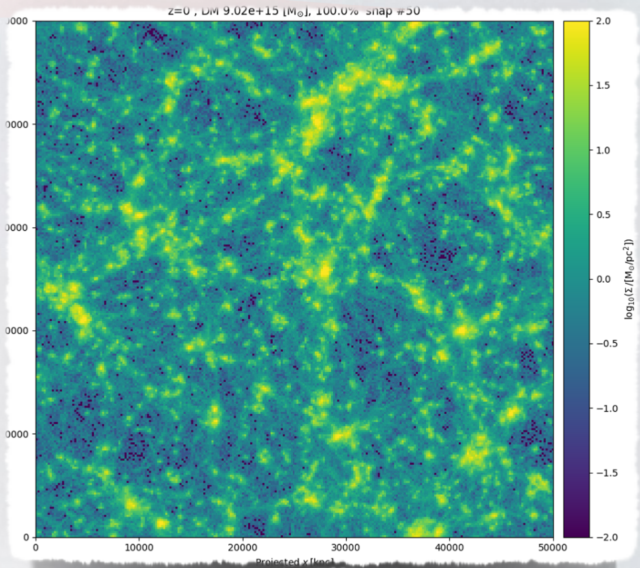


Applying the gaussian initial condition





Simulation starts

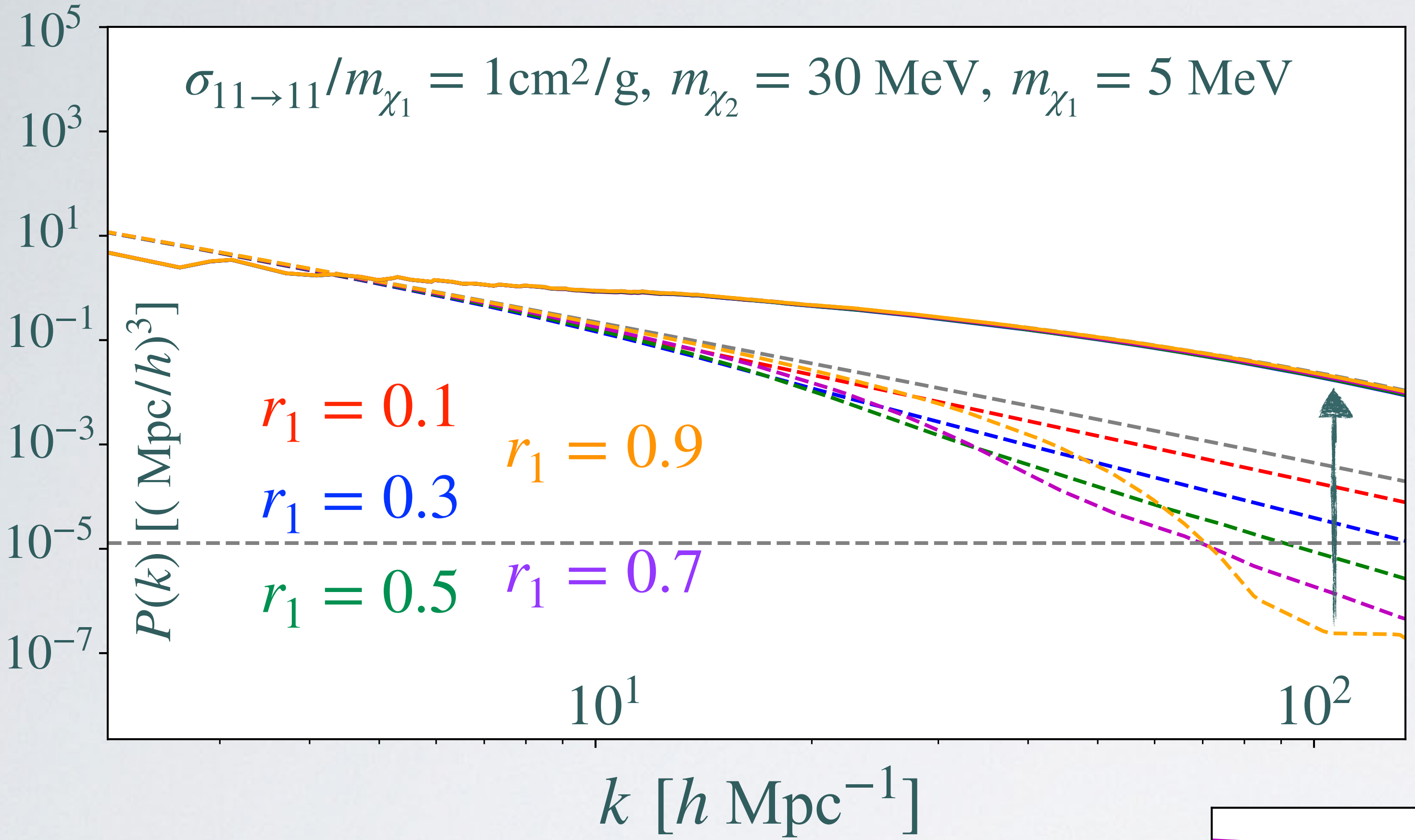


1. $\vec{x}(t) = \vec{q} + \vec{\psi}^{(1)}(t, \vec{q})$
2. $\vec{v}(t) = \dot{x}(t)$

Extracting various small scales observables

Including Non-Linear Effects

Sehwan Lim, **J. H. Kim**, K.C. Kong, J. Park [2023]



- We performed N-body simulations to include non-linear effects.

  Size of a box $= (3\text{Mpc}/h)^3$

  Number of DM particles $= 128^3$

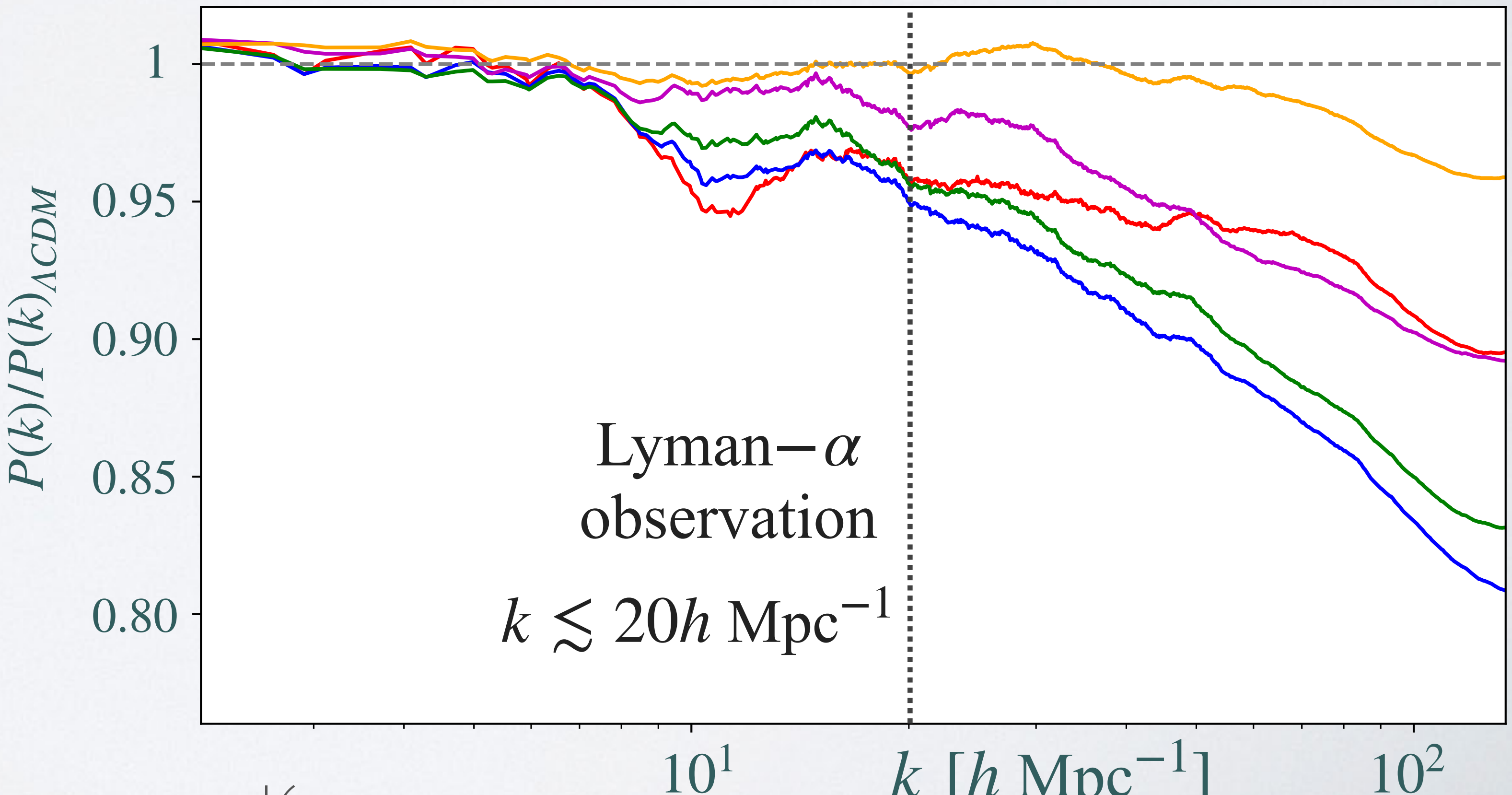
  Starting redshift $z = 100$

  Input $=$ Linear $P(k)$ at $z = 100$

- Non-linear effects can significantly wash out the linear features.

- Nonetheless, there are $5 \sim 20\,\%$ deviations for $k \gtrsim 10h\ \text{Mpc}^{-1}$.

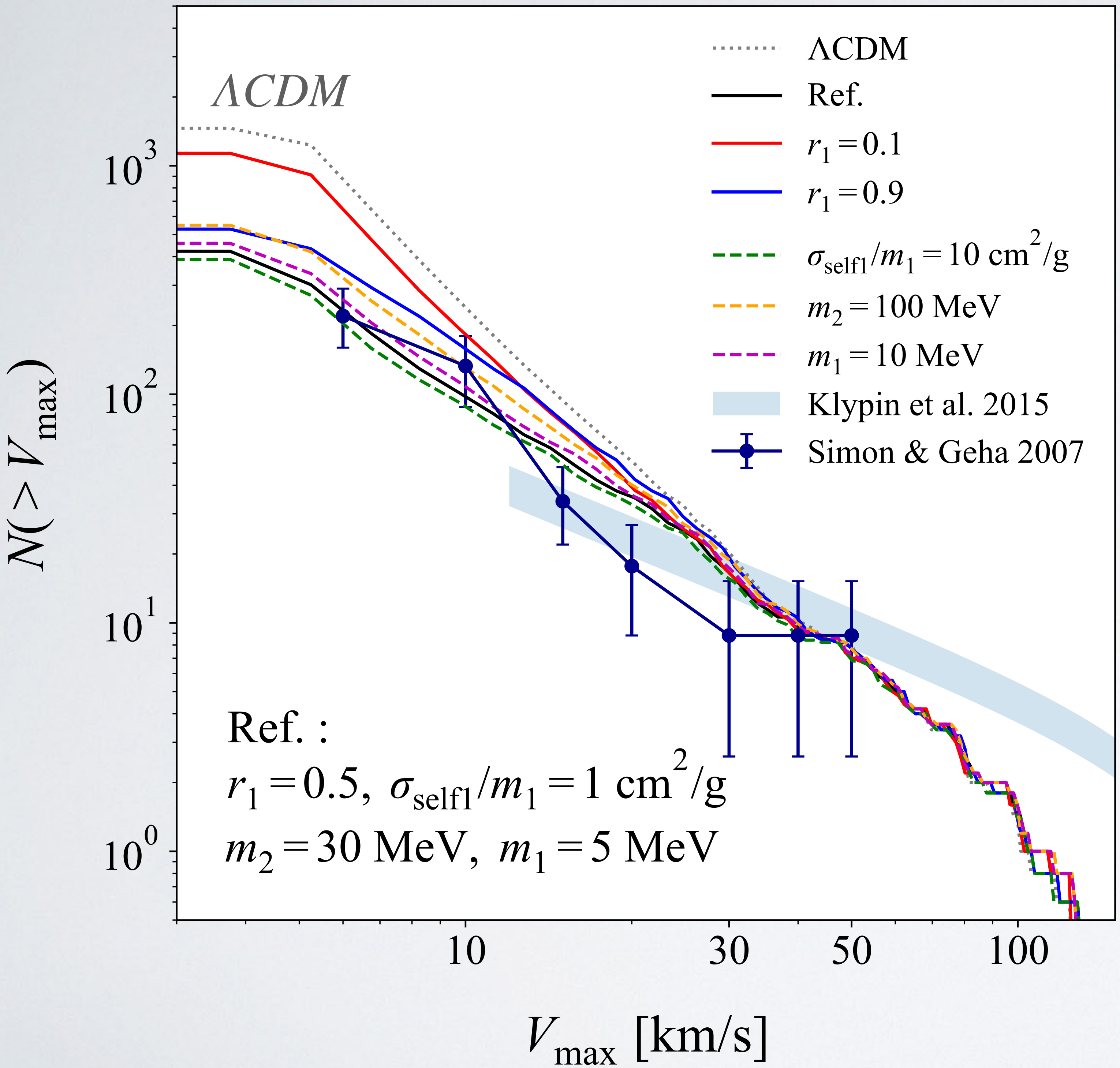
- Lyman$-\alpha$ data can put constraint in the region of $0.5 < k < 20h\ \text{Mpc}^{-1}$.



Observational Constraints

Sehwan Lim, **J. H. Kim**, K.C. Kong, J. Park [2023]

Maximum Circular Velocity Distribution



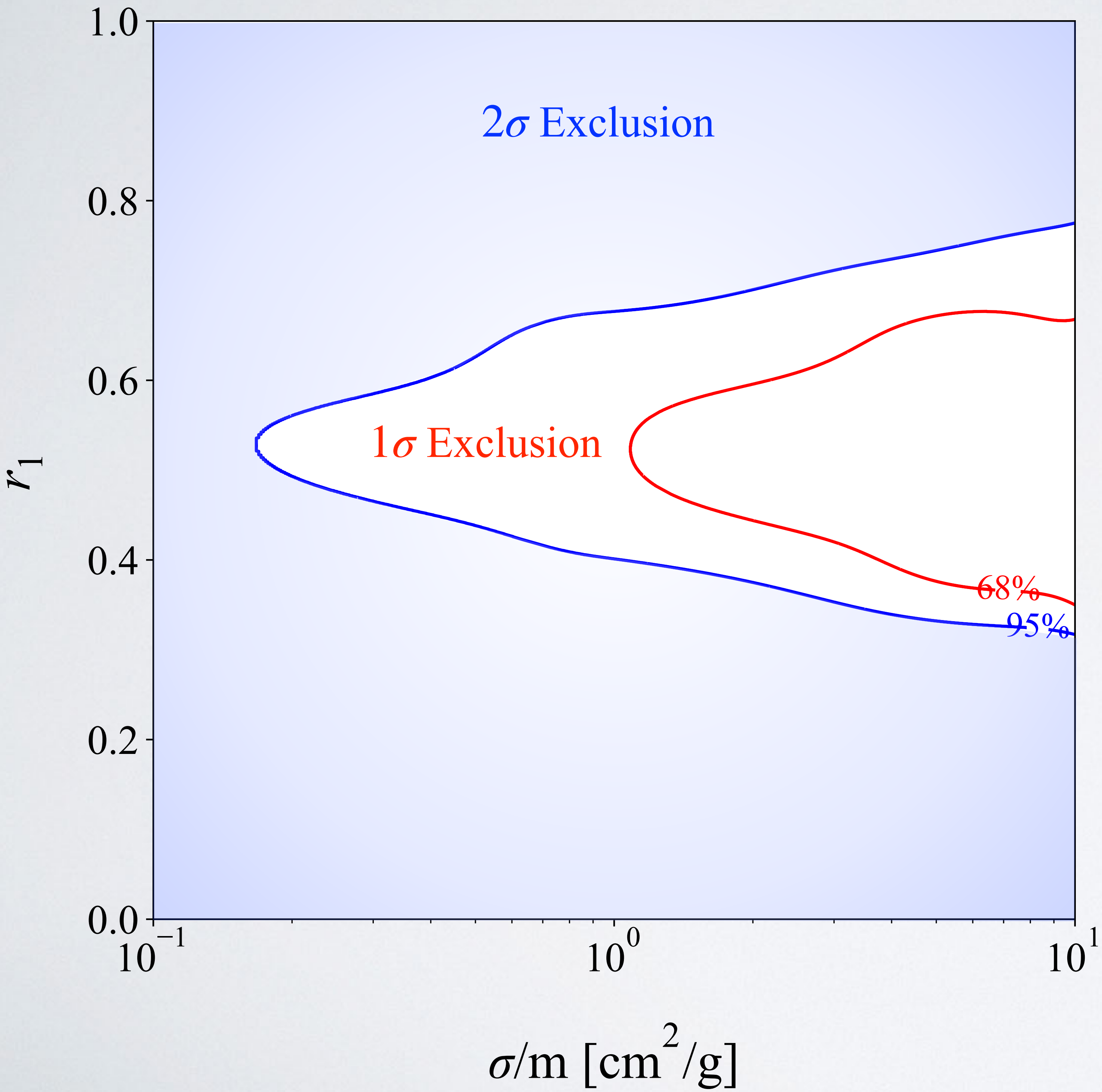
~ mass of subhalos

- The data prefers the Universe with mixed two-component DM.
- The data disfavors large masses m_{χ_1} and m_{χ_2} .
- The data prefers a larger $\sigma_{11\to11}/m_{\chi_1}$.
- *ΛCDM* model is strongly disfavored.

Observational Constraints

Sehwan Lim, **J. H. Kim**, K.C. Kong, J. Park [In Progress]

$m_{\chi_2} = 30$ MeV, $m_{\chi_1} = 5$ MeV

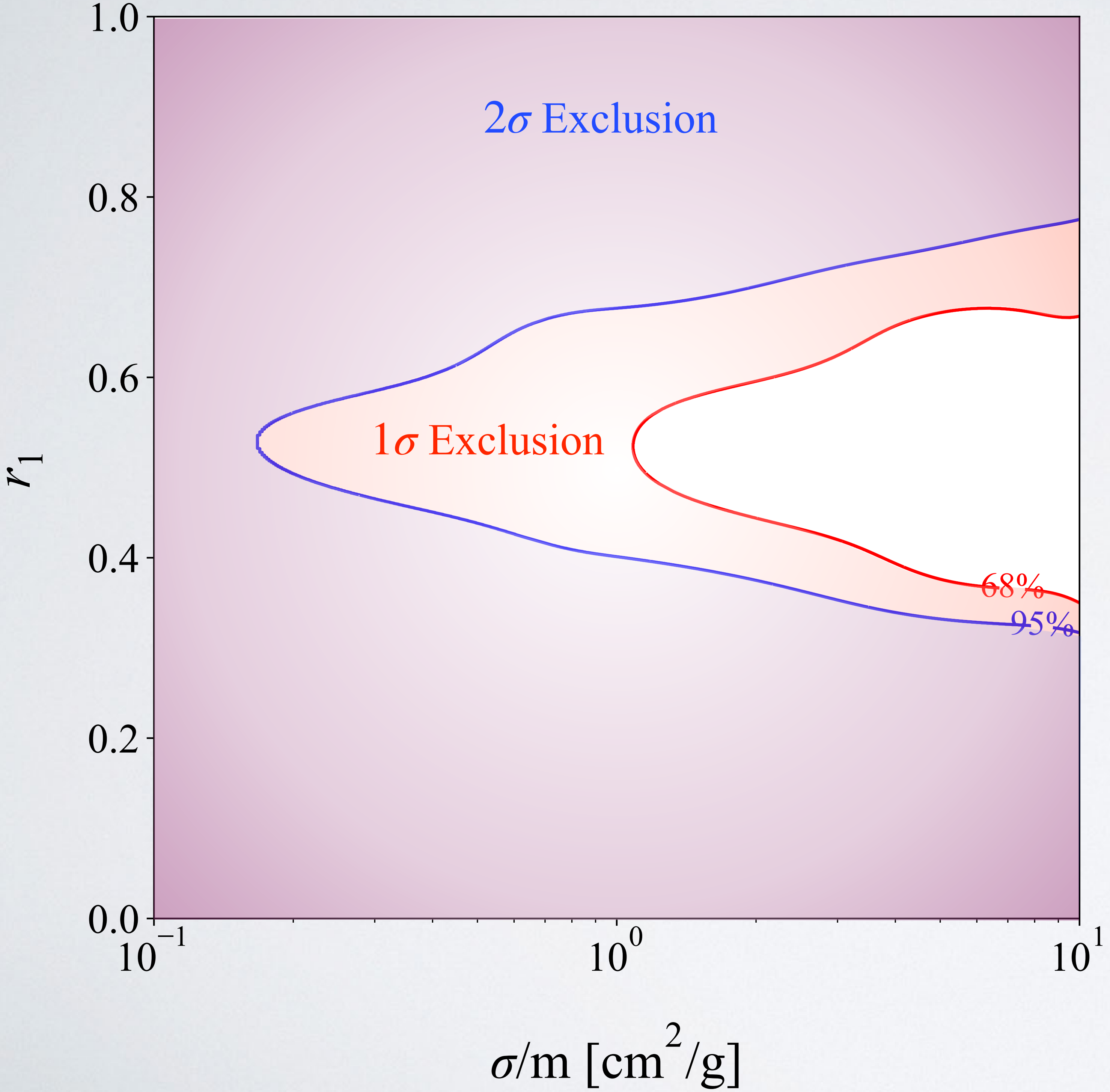


- We perform a chi-square test using the maximum circular velocity distribution
- Single-component limits ($r_1 \sim 1$ or $r_1 \sim 0$) are excluded.
- The data prefers a larger $\sigma_{11\to11}/m_{\chi_1}$.

Observational Constraints

Sehwan Lim, **J. H. Kim**, K.C. Kong, J. Park [2023]

$m_{\chi_2} = 30$ MeV, $m_{\chi_1} = 5$ MeV



- We perform a chi-square test using the maximum circular velocity distribution
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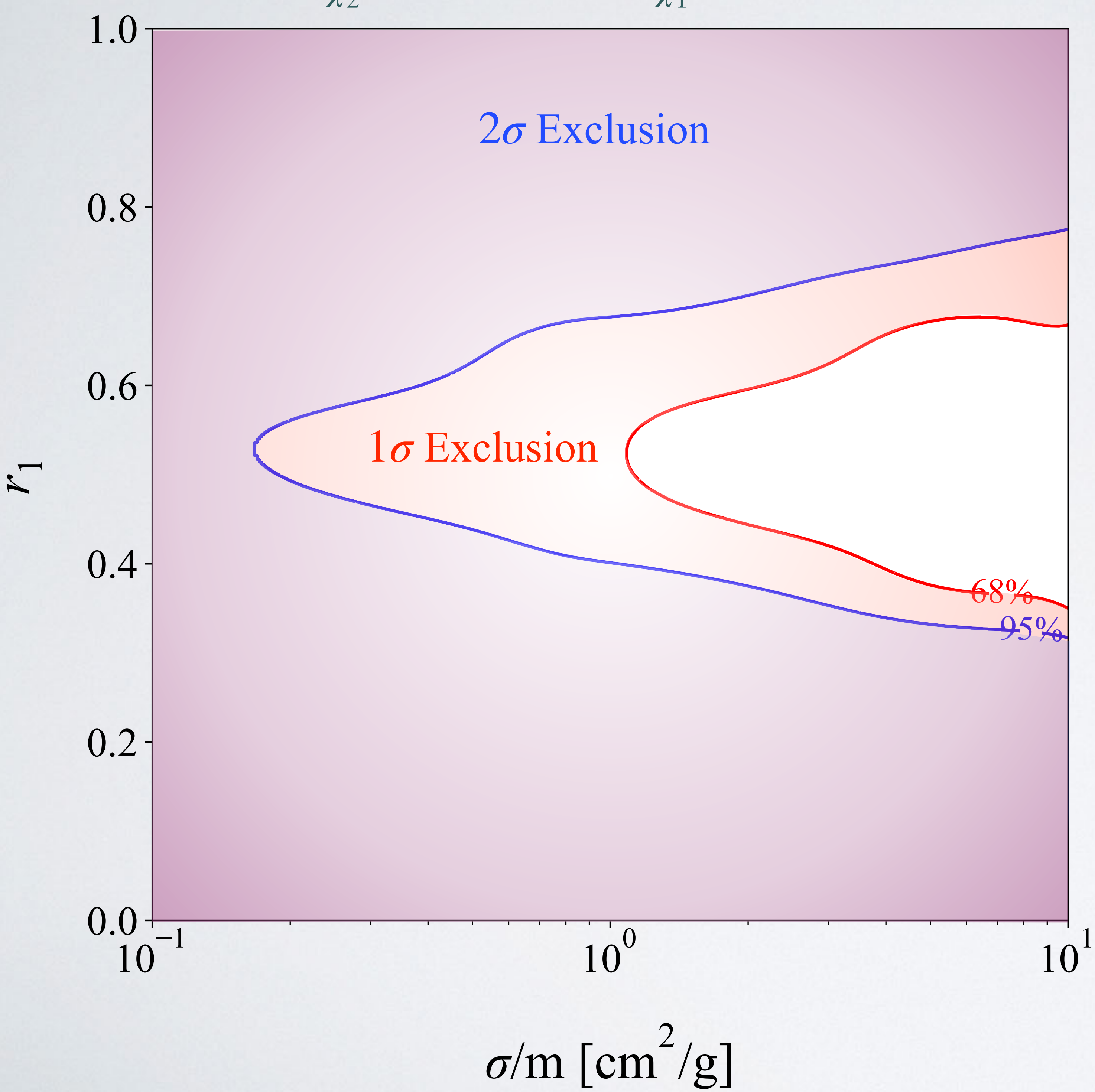
- The data prefers a larger $\sigma_{11\to11}/m_{\chi_1}$.

Future Studies

Sehwan Lim, **J. H. Kim**, K.C. Kong, J. Park [In Progress]

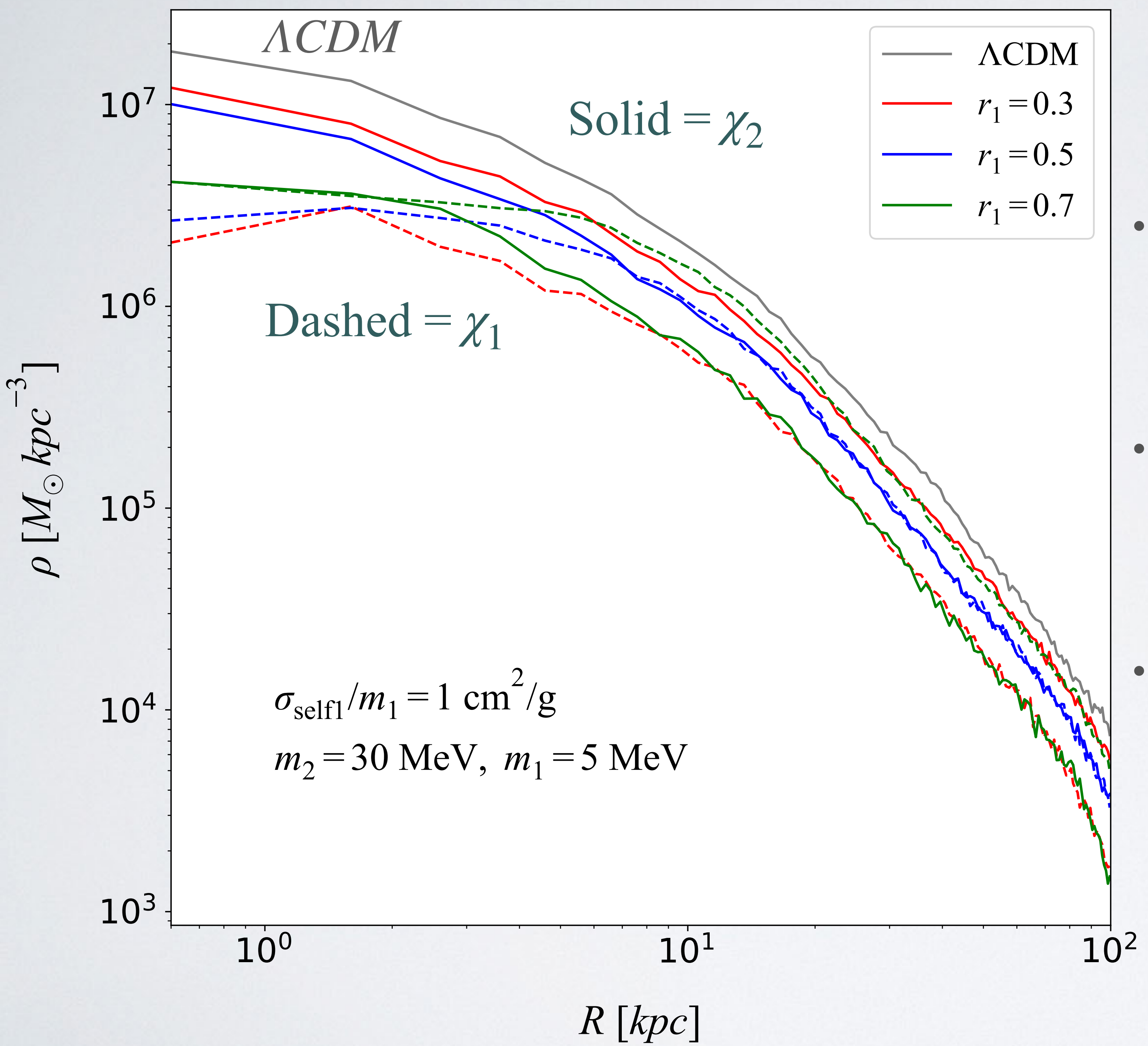
$m_{\chi_2} = 30$ MeV, $m_{\chi_1} = 5$ MeV



- How does the bound change for different masses, m_{χ_1} and m_{χ_2} ?
- How does the bound change if we include the self-interaction of χ_2 ?
- How does the bound change if we include baryons in the simulation ?
- Is the bound compatible with direct detection experiments?
- What are other observables in the small scale structure?

Density Profiles of Halos

Sehwan Lim, **J. H. Kim**, K.C. Kong, J. Park [In Progress]

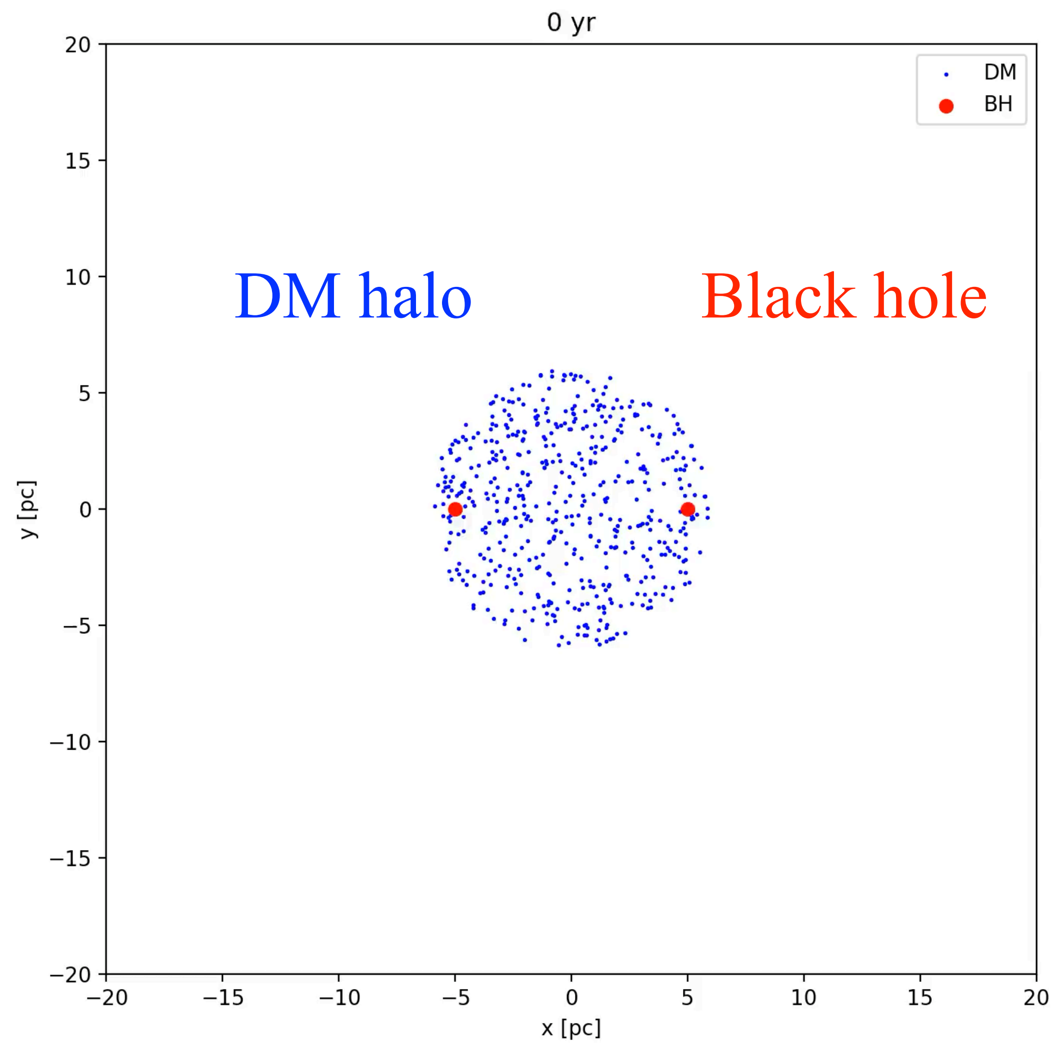


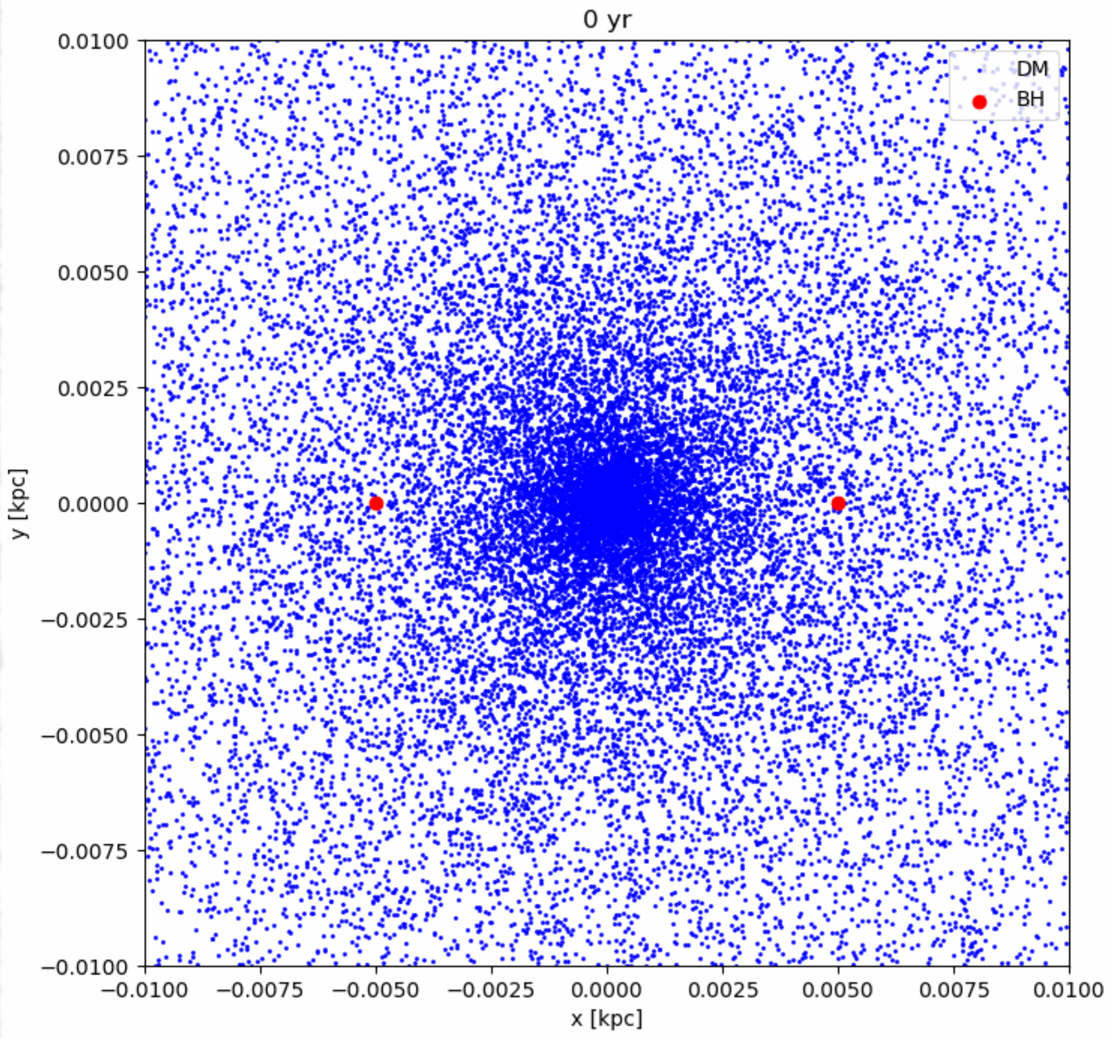
- Heavy χ_2 displays a cusp shape of halo.
- Light χ_1 displays a core shape of halo.
- What are their velocity distributions?

Gravitational Wave Probes

K. Kadota, **J. H. Kim**, Pyungwon Ko, Xing-yu Yang [2306.10828]

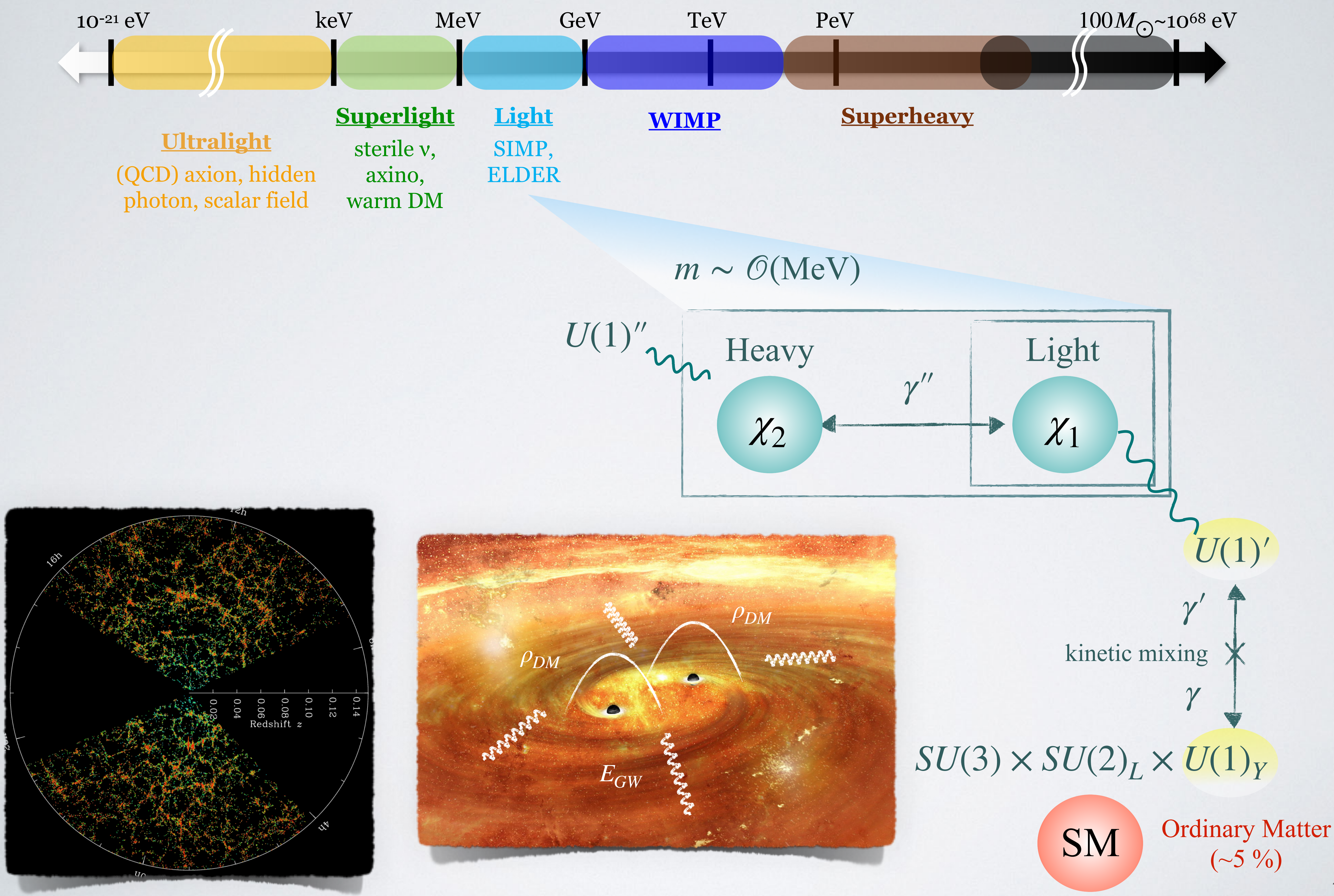
Illustration by Gadget4 Simulation





- The shape of DM overdensities can influence the evolution of a binary system.
- The dense region of DM can lead to the dephasing of GWs which can be detected by a future observation by LISA.

Summary



10^{-21} eV
keV
MeV
GeV
TeV
PeV
$100 M_\odot \sim 10^{68}$ eV
Superlight
sterile ν, axino, warm DM
Light
SIMP, ELDER
WIMP
Superheavy
Ultralight
(QCD) axion, hidden photon, scalar field
$m \sim \mathcal{O}(\text{MeV})$
$U(1)''$
Heavy
Light
γ''
χ_2
χ_1
$U(1)'$
γ'
kinetic mixing
γ
$SU(3) \times SU(2)_L \times U(1)_Y$
SM
Ordinary Matter (~5 %)
Redshift z
ρ_{DM}
ρ_{DM}
E_{GW}

Back-up

Coupled Background Boltzmann Equations

A. Kamada, H. Kim, J. Park, S. Shin [2021]

- Cosmological background evolutions are governed by coupled Boltzmann equations for χ_1 and χ_2 .

$\chi_2\bar{\chi}_2 \to \chi_1\bar{\chi}_1$

1. $$\frac{d\rho_{\chi_2}}{dt} + 3H\rho_{\chi_2} = -\frac{\langle \sigma v \rangle_{22\to 11}}{m_{\chi_2}}\left(\rho_{\chi_2}^2 - \frac{\rho_{\chi_2,\mathrm{eq}}^2}{\rho_{\chi_1,\mathrm{eq}}^2}\rho_{\chi_1}^2\right)$$

(where $\langle \sigma v \rangle_{22\to 11} \simeq 0.2 \left(\frac{5\times 10^{-26}\mathrm{cm}^3/\mathrm{s}}{\Omega_{\chi_2}}\right)$)

Hubble friction — collision terms — χ_2 relic abundance

$\chi_1\bar{\chi}_1 \to$ SM SM

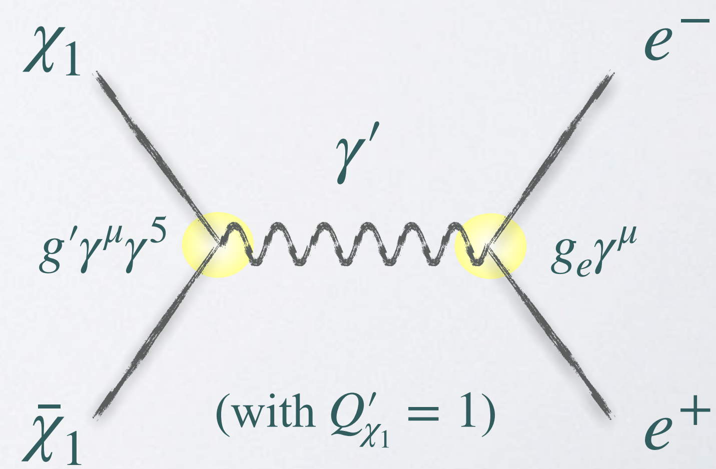
2. $$\frac{d\rho_{\chi_1}}{dt} + 3H\rho_{\chi_1} = -\frac{\langle \sigma v \rangle_{11\to \mathrm{SM\,SM}}}{m_{\chi_1}}\left(\rho_{\chi_1}^2 - \rho_{\chi_1,\mathrm{eq}}^2\right) + \frac{m_{\chi_1}}{m_{\chi_2}}\frac{\langle \sigma v \rangle_{22\to 11}}{m_{\chi_2}}\left(\rho_{\chi_2}^2 - \frac{\rho_{\chi_2,\mathrm{eq}}^2}{\rho_{\chi_1,\mathrm{eq}}^2}\rho_{\chi_1}^2\right)$$

- Here, SM = $e^-, e^+, \gamma, \cdots$ denotes relativistic particles.

- We consider the p-wave cross section $\chi_1\bar{\chi}_1 \to$ SM SM (not to screw CMB, BAO, …).

$$\langle \sigma v \rangle_{11\to \mathrm{SM\,SM}} = \frac{g'^2 g_e^2 \left(2m_{\chi_1}^2 + m_e^2\right)\sqrt{m_{\chi_1}^2 - m_e^2}}{6m_{\chi_1}\left(m_{\gamma'}^2 - 4m_{\chi_1}^2\right)^2\pi}\, v^2 + \mathcal{O}(v^3)$$

Dark photon mass



Coupled Background Boltzmann Equations

A. Kamada, H. Kim, J. Park, S. Shin [2021]

- Large $\langle \sigma v \rangle_{11 \to \mathrm{SM\,SM}}$ can significantly affect the CMB at the 0^{th}-order.

If SM particles are relativistic

$\chi_1 \bar{\chi}_1 \to$ SM SM

$$3. \quad \frac{d\rho_{\mathrm{SM}}}{dt} + 4H\rho_{\mathrm{SM}} = \frac{\langle \sigma v \rangle_{11 \to \mathrm{SM\,SM}}}{m_{\chi_1}} \left(\rho_{\chi_1}^2 - \rho_{\chi_1,\mathrm{eq}}^2 \right)$$

Neglected in this work

(with SM = $e^-, e^+, \gamma, \cdots$)

- The energy injection to the SM plasma can change the ionization history, Compton scattering, …

D. Green, P.D. Meerburg, J. Meyers [2018]
N. Padmanabhan, D.P. Finkbeiner [2005]
…

- With the *p*-wave cross section $\langle \sigma v \rangle_{11 \to \mathrm{SM\,SM}}$, we can evade this constraint.

See also other way around, P.J. Fitzpatrick, H. Liu, T.R. Slatyer, Y.D. Tsai [2011]

- In this work, we focus on the evolution of DM matter densities, and neglect the effect of “3” in the structure formation of the Universe.

(future study)

Initial Conditions

◎ Initial Conditions

Recall For adiabatic perturbations, the fluctuations in all components are related by

$$\delta_\gamma = \delta_\nu = \frac{4}{3}\delta_{CDM} = \frac{4}{3}\delta_b = -2\Phi_i \quad (\text{with } \Phi_i \simeq \Psi_i)$$

where Φ_i is the primordial potential which is given by

$$\Phi_i = \frac{2}{3}R_i$$

Where R_i is the gauge-invariant curvature perturbation. It connects between the era of end of inflation and a deep radiation-dominated era.

Remark From the above initial conditions, we are able to write down the photon and matter fluctuations as (δ_m denotes a matter density contrast)

$$\therefore \delta_\gamma = \frac{4}{3}\delta_m = -2\Phi_i = -\frac{4}{3}R_i$$

Recall The curvature perturbation R_i is determined by

$$\Delta^2_R(k) \equiv \frac{k^3}{2\pi^2}|R_i(k)|^2 = A_s\left(\frac{k}{k_*}\right)^{n_s-1}$$

$\cdots$ scalar amplitude $A_s = \frac{1}{8\pi^2}\frac{1}{\epsilon_*}\frac{H_*^2}{M_{Pl}^2}$

$\cdots$ spectral index $n_s = 1 - 2\epsilon_* - \kappa_*$

} All these quantities are computed at the time of horizon exit

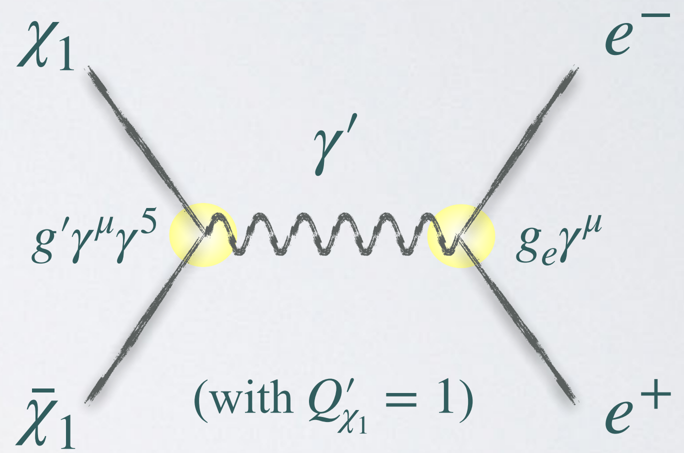
Cross Sections

$$\left\langle \sigma v \right\rangle_{\chi_1,X} = \frac{c_a^2 e_v^2 m_{\chi_1} m_e (3m_{\chi_1}^2 + 2m_{\chi_1} m_e + m_e^2)}{2(m_{\chi_1} + m_e)^2 m_{\gamma'}^4 \pi}$$

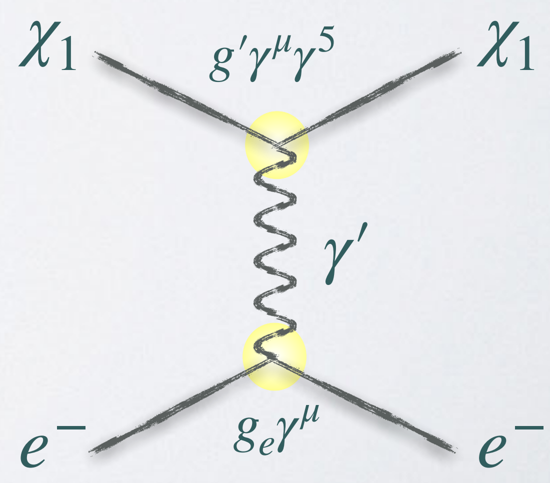
$$\gamma_{\chi_1 \mathrm{sm}} = \frac{\delta E}{T} n_{\mathrm{sm}} \left\langle \sigma v \right\rangle_{\chi_1,sm}$$

p-wave annihilation

$$\left\langle \sigma v \right\rangle_{11 \to \mathrm{SM\ SM}} = \frac{g'^2 g_e^2 \left(2m_{\chi_1}^2 + m_e^2\right) \sqrt{m_{\chi_1}^2 - m_e^2}}{6m_{\chi_1} \left(m_{\gamma'}^2 - 4m_{\chi_1}^2\right)^2 \pi} v^2 + \mathcal{O}(v^3)$$



$$\left\langle \sigma v \right\rangle_{\chi_1,\mathrm{SM} \to \chi_1,\mathrm{SM}} = \frac{3g'^2 g_e^2 m_{\chi_1}^2 m_e^2}{\pi m_{\gamma'}^4 \left(m_{\chi_1} + m_e\right)^2 \pi} v + \mathcal{O}(v^3)$$



Connecting Simulations with Perturbation Theory

$z \sim 100$ $z = 0$

Forward approach

Linear Einstein-Boltzmann → N-body

- It includes a non-linear evolution of matter density
- It includes the effects of radiation in the background expansion, but radiation perturbations are not included (which can suppress the growth of structure)

Back-scaling method

Linear Einstein-Boltzmann

- It includes the effects of linear radiation perturbations

Back-scaling

Newtonian linear growth factor
(Neglecting radiation perturbations)

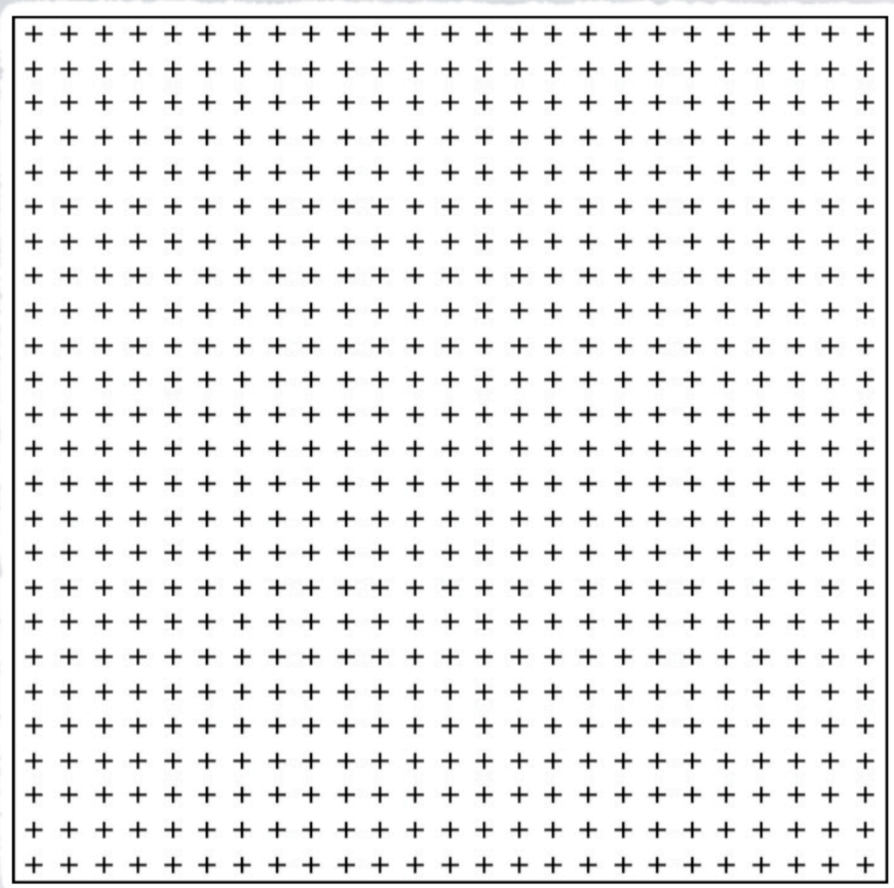
$$\delta_m(z_{\rm start}) = \delta_m(z=0)\frac{D(z=start)}{D(z=0)}$$

N-body

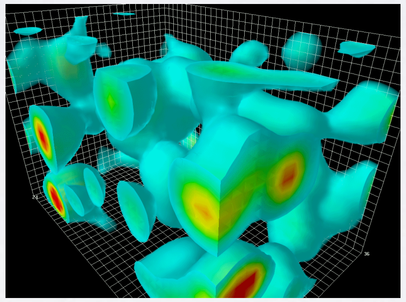
- Back-scaling simulates an artificial radiation-free Universe that is designed to mimi our Universe on large scales and at the present time.
- Small scales are assumed to be well-described in the Newtonian theory so that they should remain unaffected.

Connecting Simulations with Perturbation Theory

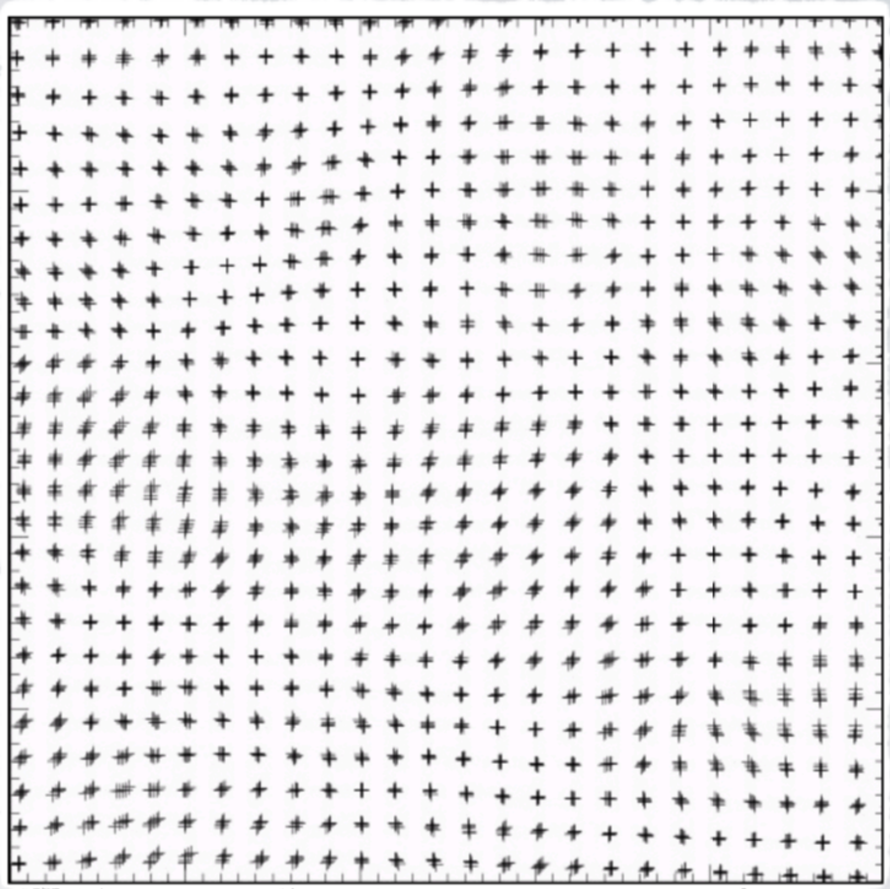
$z = 200$



How to apply primordial curvature perturbations?



$z = 200$



- How to include primordial curvature perturbations in *N*-body simulations?
- It is the task to utilize the linear perturbation theory to set up initial conditions of the simulations.
- Then the gravity interaction will take care of the rest of simulation.

Connecting Simulations with Perturbation Theory

- The dynamics of non-relativistic matters dominated by gravity can be considered as fluids.

- Three master equations to describe the fluid dynamics:

$\rho = \bar{\rho} + \delta\rho = \bar{\rho}(1+\delta)$

Continuity equation (= mass conservation)

1. $\dot{\delta} = -\frac{1}{a}\nabla \cdot \vec{v}$

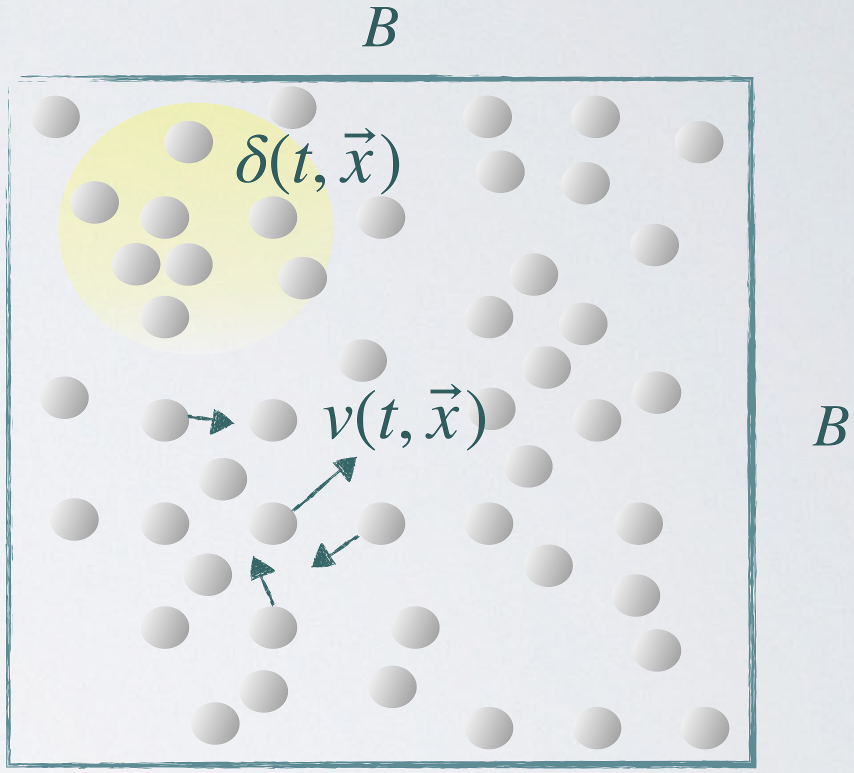
metric perturbation

Euler equation (= momentum conservation)

2. $\dot{\vec{v}} + H\vec{v} = -\frac{1}{a}\nabla\delta\Phi$

Poisson equation

3. $\nabla^2\delta\Phi = 4\pi G a^2 \bar{\rho}\delta$



(neglect pressure $P \ll \rho$ term)

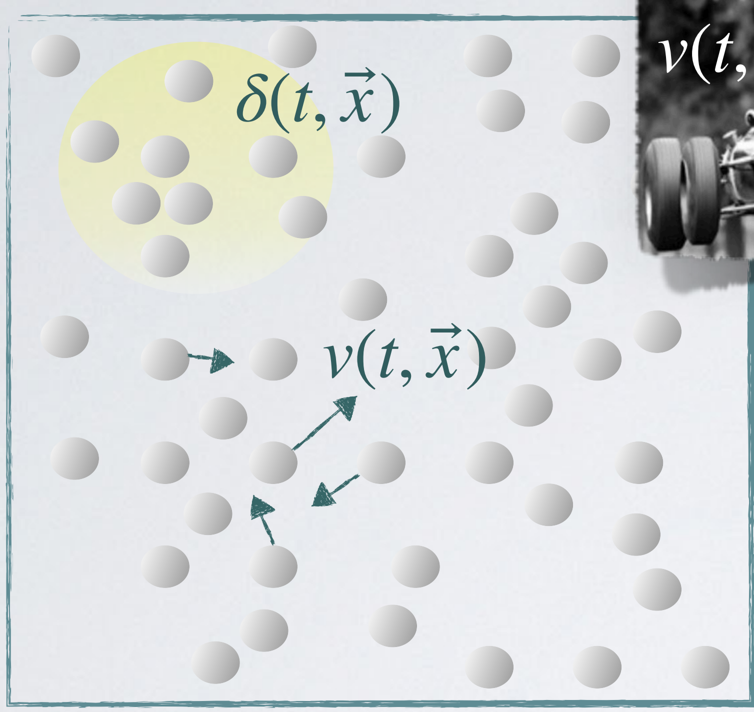
- Combining the equations gives the evolution for the density contrast δ .

$$\ddot{\delta} + 2H\dot{\delta} = 4\pi G\bar{\rho}\delta \longrightarrow \delta \sim a \quad \text{(growing mode)}$$

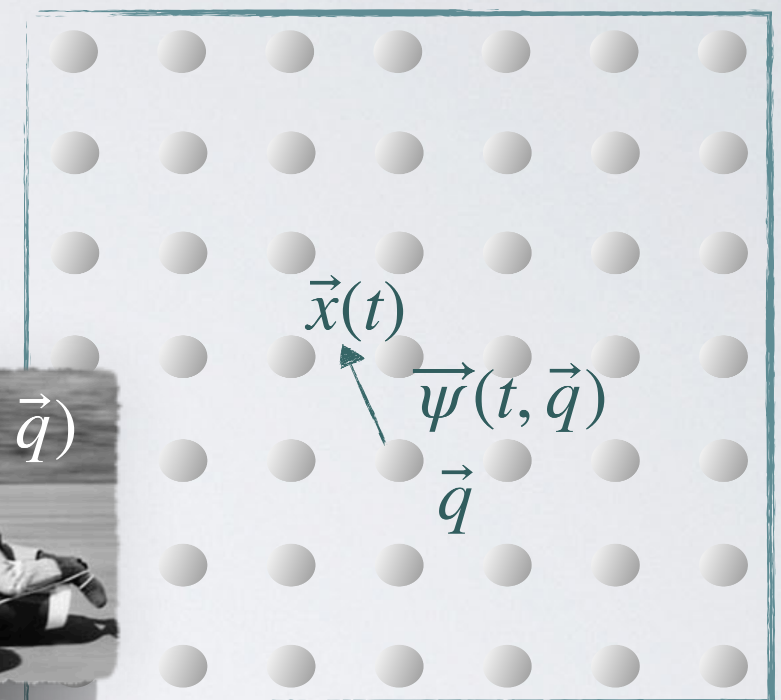
Connecting Simulations with Perturbation Theory

- Two frameworks describe the same physics in different point of views.

Eulerian Framework



Lagrangian Framework



- Fundamental variables: $\delta(t,\vec{x})$ and $\vec{v}(t,\vec{x})$

1. $\dot{\delta} = -\frac{1}{a}\nabla\cdot\vec{v}$

2. $\dot{\vec{v}} + H\vec{v} = -\frac{1}{a}\nabla\delta\Phi$

(· denotes ∂_η)

- Fundamental variables: $\vec{\psi}(t,\vec{q})$ and $\dot{\vec{\psi}}(t,\vec{q})$

1. $\vec{x}(t) = \vec{q} + \vec{\psi}(t,\vec{q})$

Final position — Initial position — Displacement vector

2. $\ddot{\vec{\psi}} + H\dot{\vec{\psi}} = -\frac{1}{a}\nabla\delta\Phi$

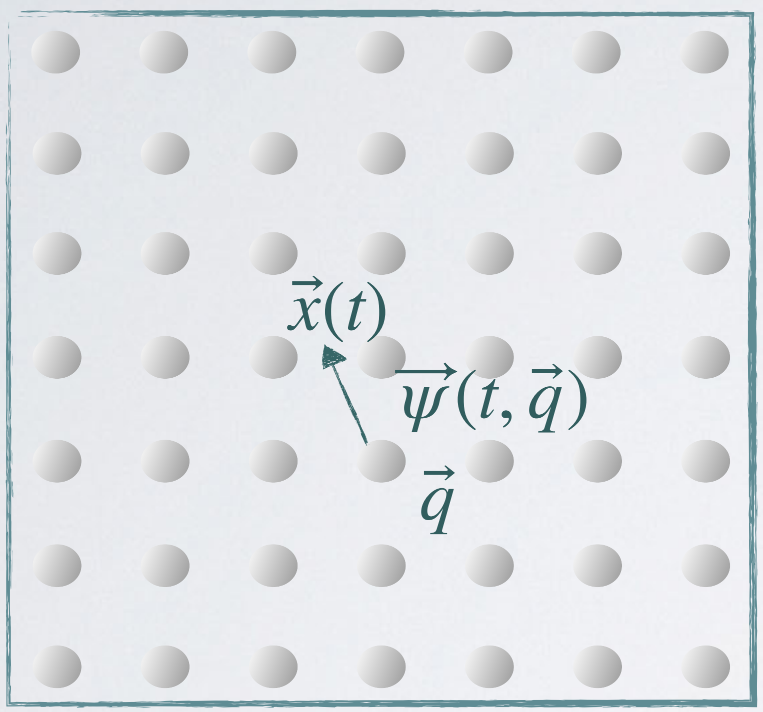
(Equation of motion)

- Common

$$\nabla^2\delta\Phi = 4\pi G a^2\bar{\rho}\delta$$

Connecting Simulations with Perturbation Theory

Lagrangian Picture



- Typically, we solve the equation perturbatively (= series solution)

$$\vec{\psi}(t, \vec{q}) = \vec{\psi}^{(1)}(t, \vec{q}) + \vec{\psi}^{(2)}(t, \vec{q}) + \dots$$

- A final position of a particle can be written as

$$\vec{x}(t) \;=\; \vec{q} \;+\; \vec{\psi}^{(1)}(t, \vec{q}) + \dots$$

Higher-order Lagrangian perturbation theory (LPT)

Zel'dovich approximation

- At the first-order, a solution can be simply written in terms of the density contrast that we know of

$$\nabla \cdot \vec{\psi}^{(1)} = -\,\delta$$

Connecting Simulations with Perturbation Theory

- Once we know the power spectrum at the starting redshift, we can get the displacement vector.

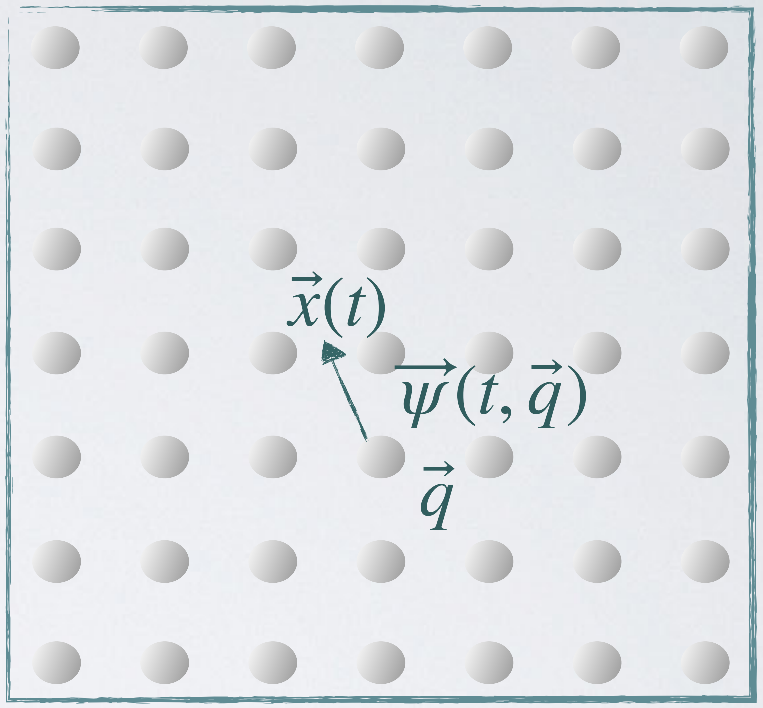
$$\vec{x}(t) = \vec{q} + \vec{\psi}^{(1)}(t, \vec{q})$$

$$\cdots \quad \nabla \cdot \vec{\psi}^{(1)} = -\delta$$

$$\cdots \quad \sim \sqrt{P(k)}$$

power spectrum at starting redshift

Lagrangian Picture



- Final velocity of a particle:

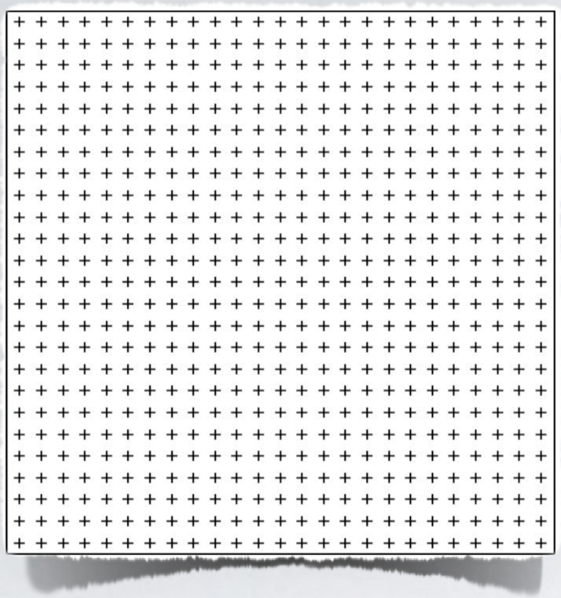
$$\vec{v}(t) = \dot{x}(t)$$

- This will redistribute initial particles and velocities to implement gaussian primordial perturbation.

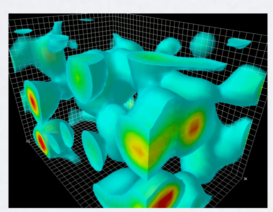
- This is how the initial condition is set.

Connecting Simulations with Perturbation Theory

$z = 200$

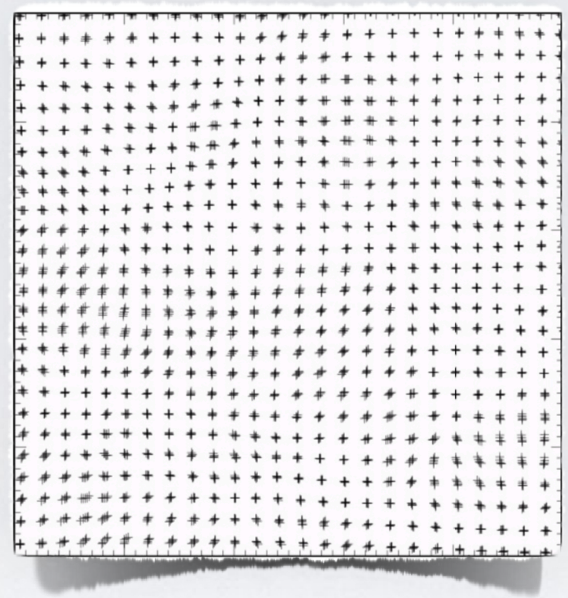


Applying the gaussian initial condition



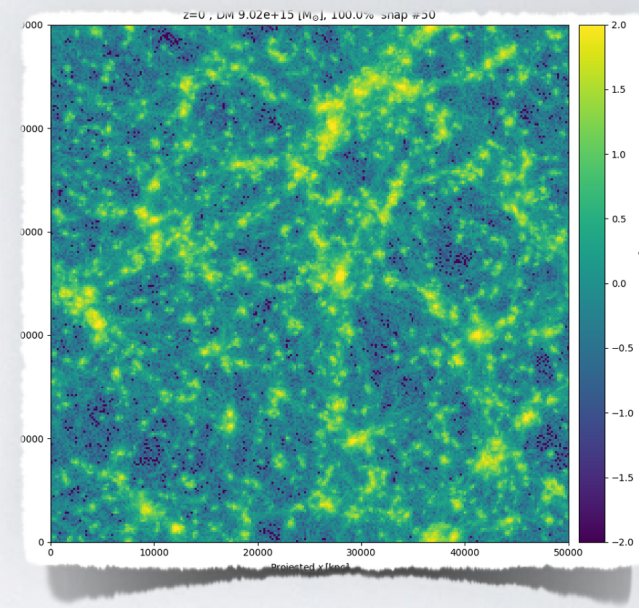
1. $\vec{x}(t) = \vec{q} + \vec{\psi}^{(1)}(t, \vec{q})$
2. $\vec{v}(t) = \dot{x}(t)$

$z = 200$



Simulation starts

$z = 0$ ΛCDM



$z = 0$ ΛCDM + Hotspots