N-body Simulations with Two-Component
Dark Matter — Zoomlandia — 02/06/24

Two-Component Dark Matter

M.V. Medvedev
KU, IAS, Princeton U, MIT

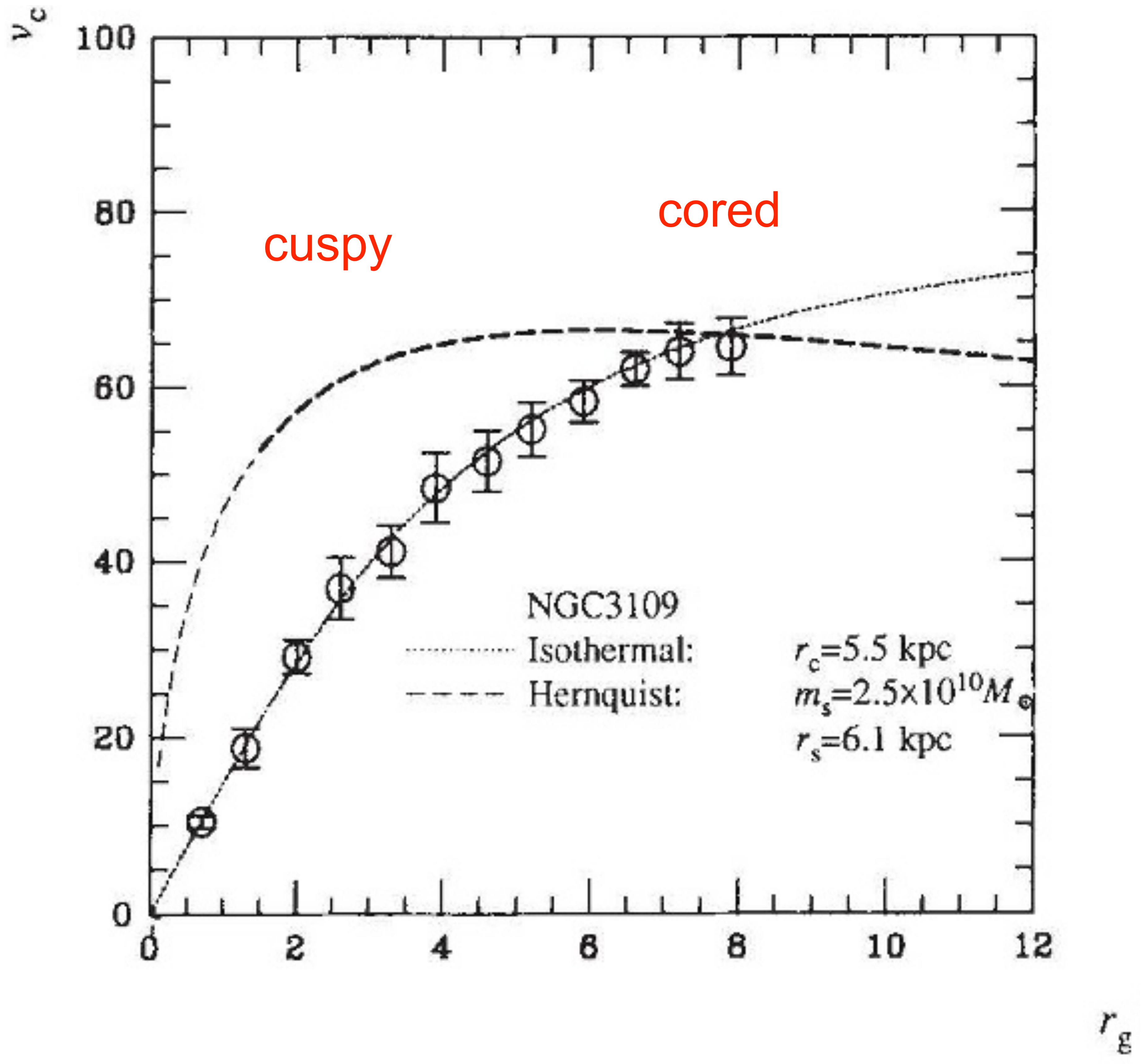
-- Next Decade: A paradigm shift? --

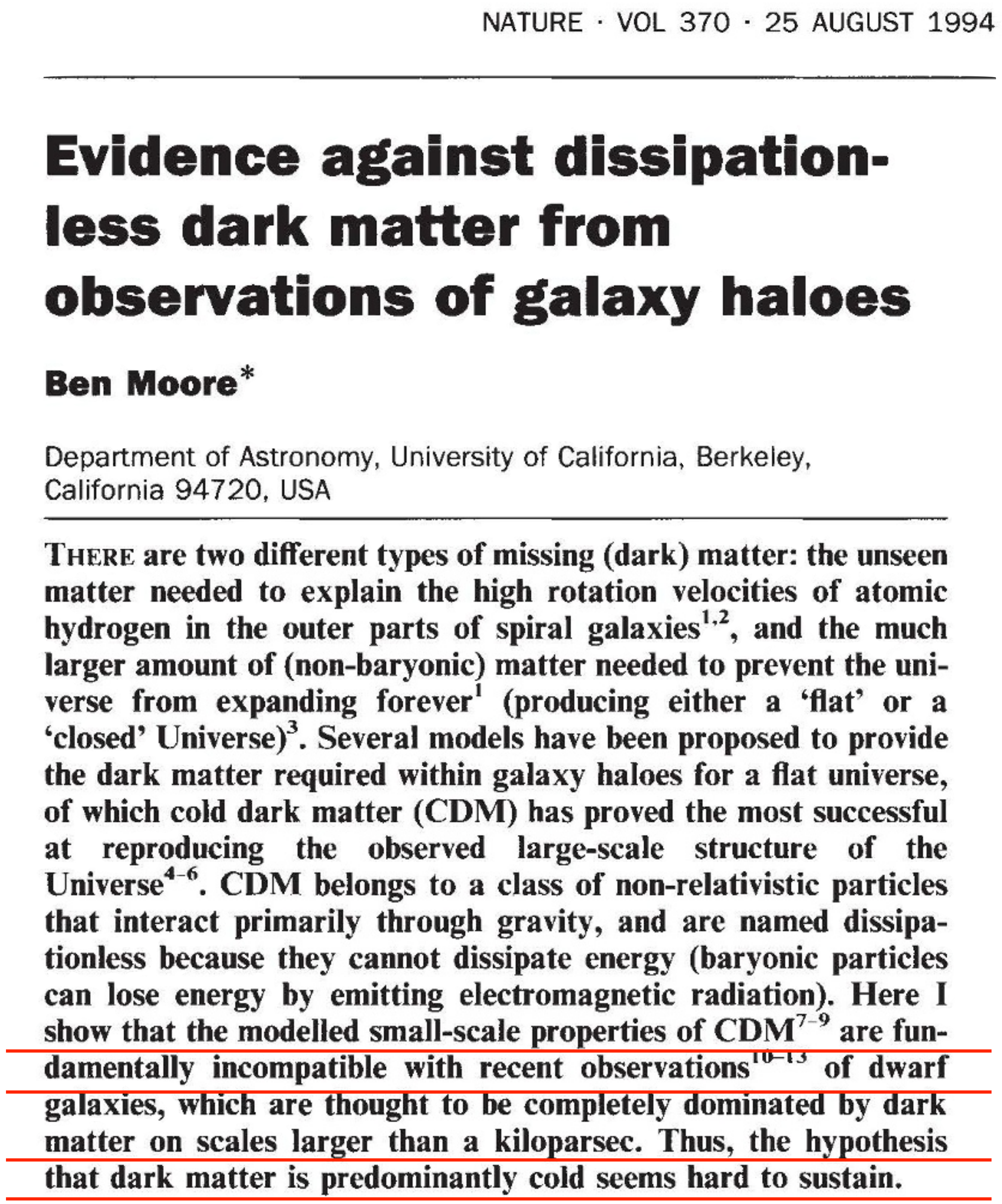
supported by NSF via PHY-2010109

- History & Motivation
- Models (SIDM, etc)
- Overview of 2cDM
  - physics
  - cosmology
  - understanding

Historically first small scale problems

- core/cusp problem





NATURE · VOL 370 · 25 AUGUST 1994

Evidence against dissipationless dark matter from observations of galaxy haloes

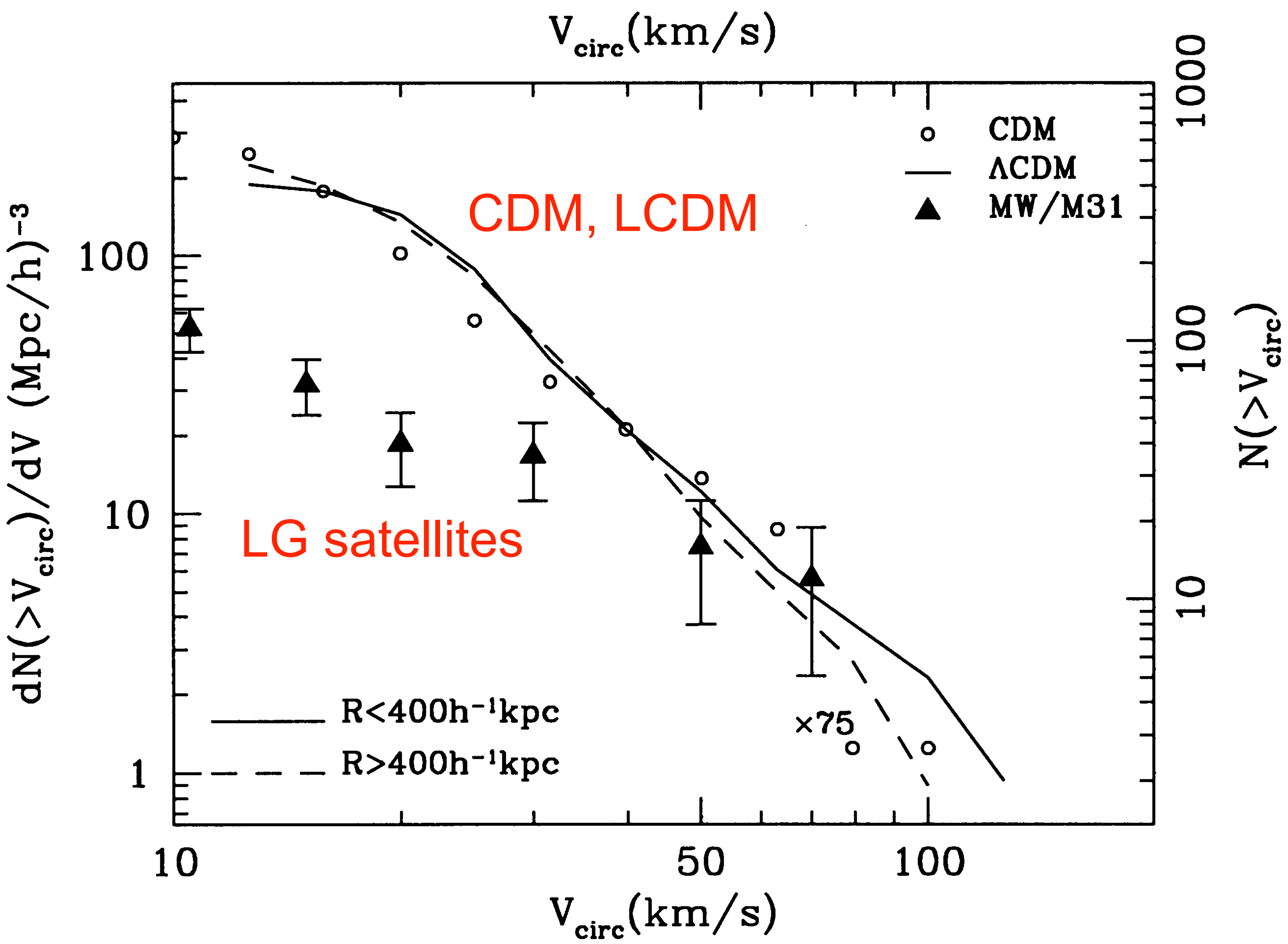
Ben Moore*

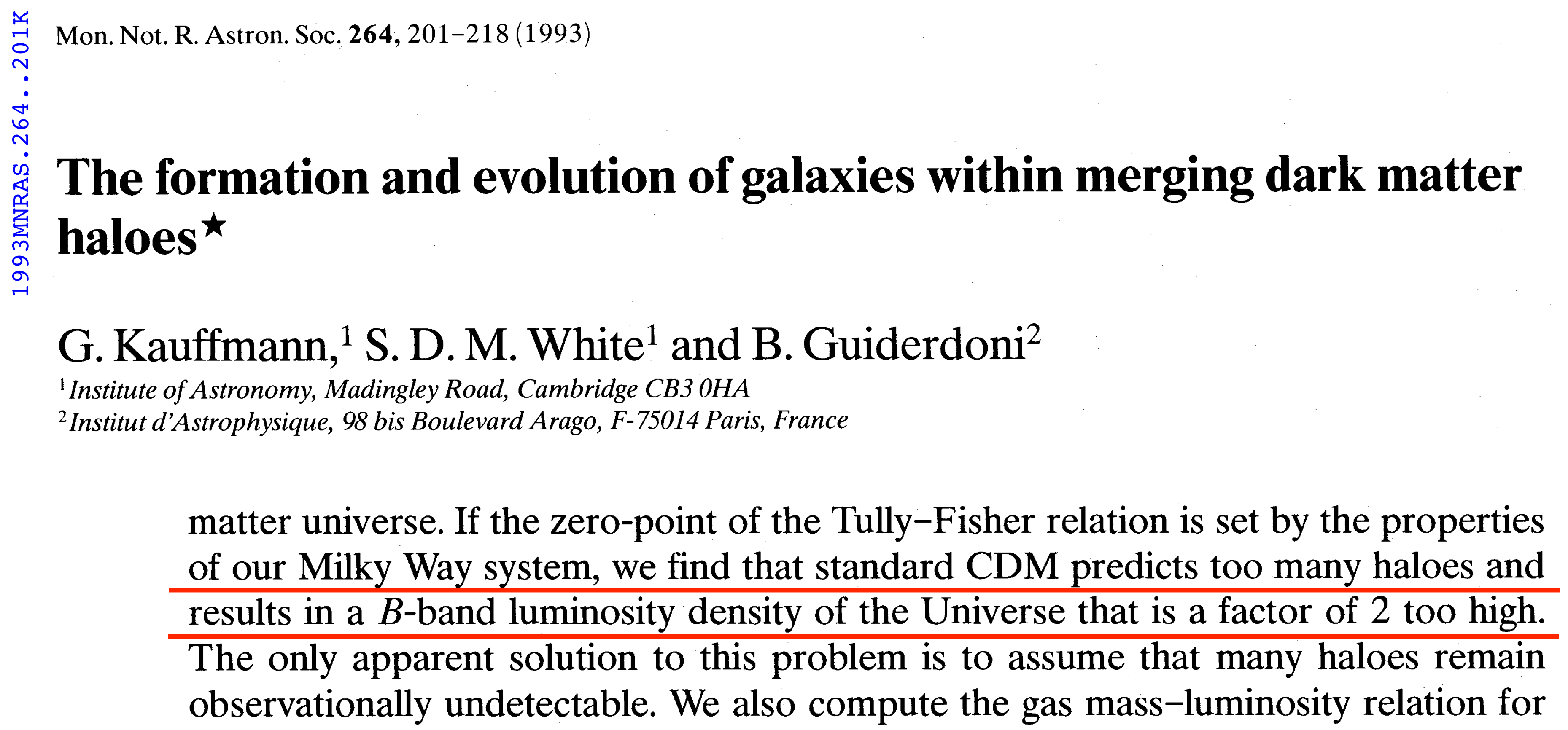
Department of Astronomy, University of California, Berkeley, California 94720, USA

THERE are two different types of missing (dark) matter: the unseen matter needed to explain the high rotation velocities of atomic hydrogen in the outer parts of spiral galaxies[1,2], and the much larger amount of (non-baryonic) matter needed to prevent the universe from expanding forever[1] (producing either a 'flat' or a 'closed' Universe)[3]. Several models have been proposed to provide the dark matter required within galaxy haloes for a flat universe, of which cold dark matter (CDM) has proved the most successful at reproducing the observed large-scale structure of the Universe[4–6]. CDM belongs to a class of non-relativistic particles that interact primarily through gravity, and are named dissipationless because they cannot dissipate energy (baryonic particles can lose energy by emitting electromagnetic radiation). Here I show that the modelled small-scale properties of CDM[7–9] are fundamentally incompatible with recent observations[10–13] of dwarf galaxies, which are thought to be completely dominated by dark matter on scales larger than a kiloparsec. Thus, the hypothesis that dark matter is predominantly cold seems hard to sustain.

Historically first small scale problems

- substructure problem (missing satellites)





Mon. Not. R. Astron. Soc. **264**, 201–218 (1993)

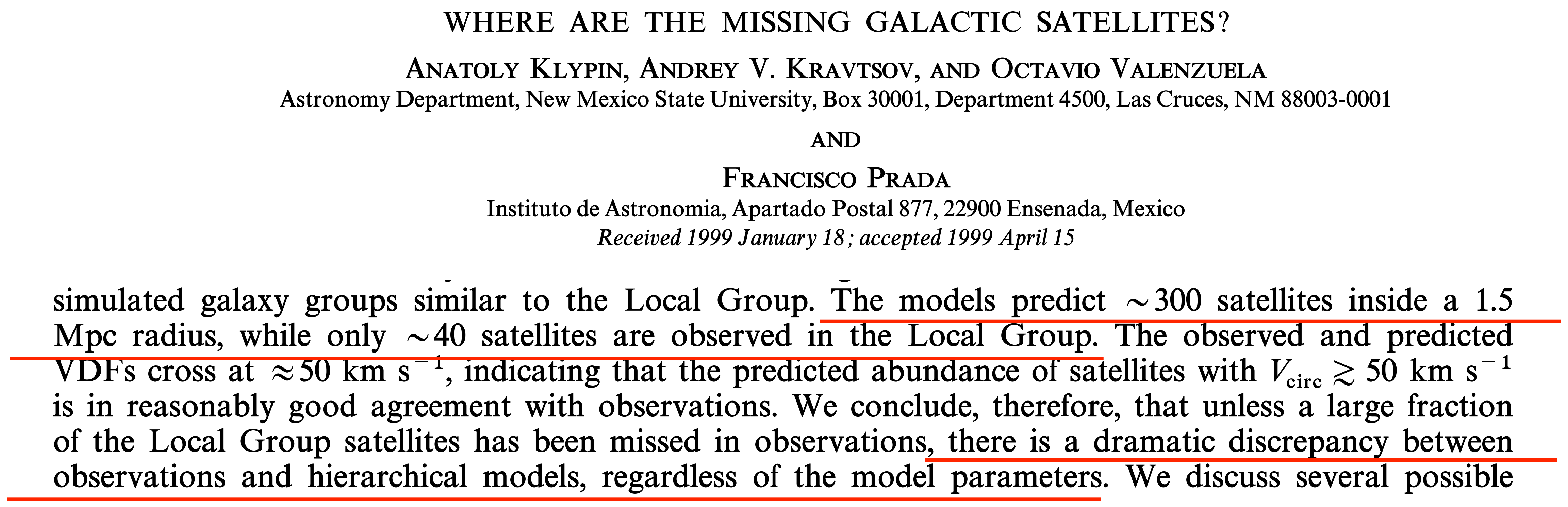
The formation and evolution of galaxies within merging dark matter haloes★

G. Kauffmann,[1] S. D. M. White[1] and B. Guiderdoni[2]

[1] *Institute of Astronomy, Madingley Road, Cambridge CB3 0HA*

[2] *Institut d'Astrophysique, 98 bis Boulevard Arago, F-75014 Paris, France*

matter universe. If the zero-point of the Tully–Fisher relation is set by the properties of our Milky Way system, we find that standard CDM predicts too many haloes and results in a *B*-band luminosity density of the Universe that is a factor of 2 too high. The only apparent solution to this problem is to assume that many haloes remain observationally undetectable. We also compute the gas mass–luminosity relation for


WHERE ARE THE MISSING GALACTIC SATELLITES?

Anatoly Klypin, Andrey V. Kravtsov, and Octavio Valenzuela
Astronomy Department, New Mexico State University, Box 30001, Department 4500, Las Cruces, NM 88003-0001

and

Francisco Prada
Instituto de Astronomia, Apartado Postal 877, 22900 Ensenada, Mexico

Received 1999 January 18; accepted 1999 April 15

simulated galaxy groups similar to the Local Group. The models predict ∼300 satellites inside a 1.5 Mpc radius, while only ∼40 satellites are observed in the Local Group. The observed and predicted VDFs cross at ≈50 km s^{-1}, indicating that the predicted abundance of satellites with $V_{\rm circ} \gtrsim 50$ km s^{-1} is in reasonably good agreement with observations. We conclude, therefore, that unless a large fraction of the Local Group satellites has been missed in observations, there is a dramatic discrepancy between observations and hierarchical models, regardless of the model parameters. We discuss several possible

Historically first small scale problems

- too-big-to-fail problem

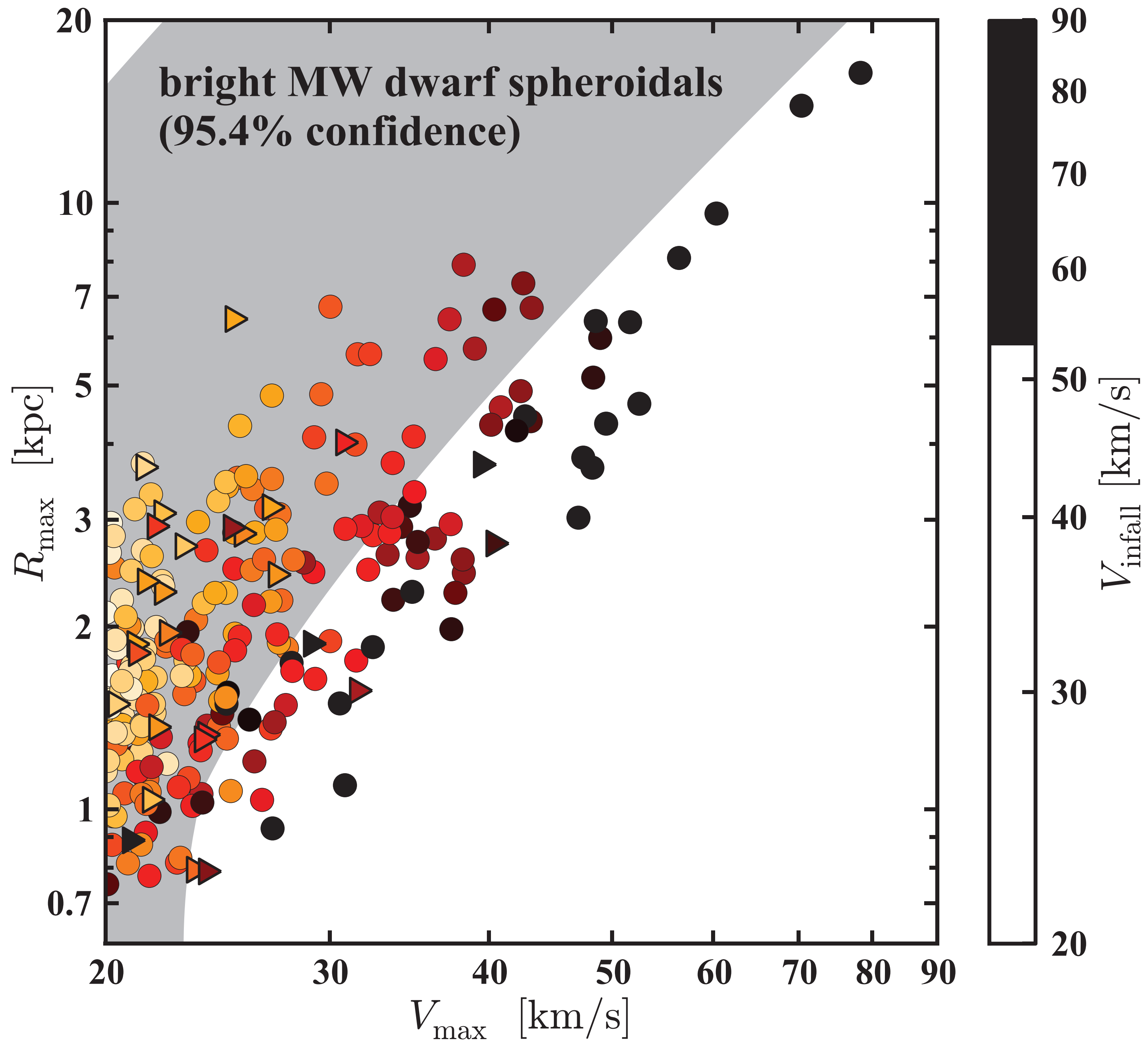
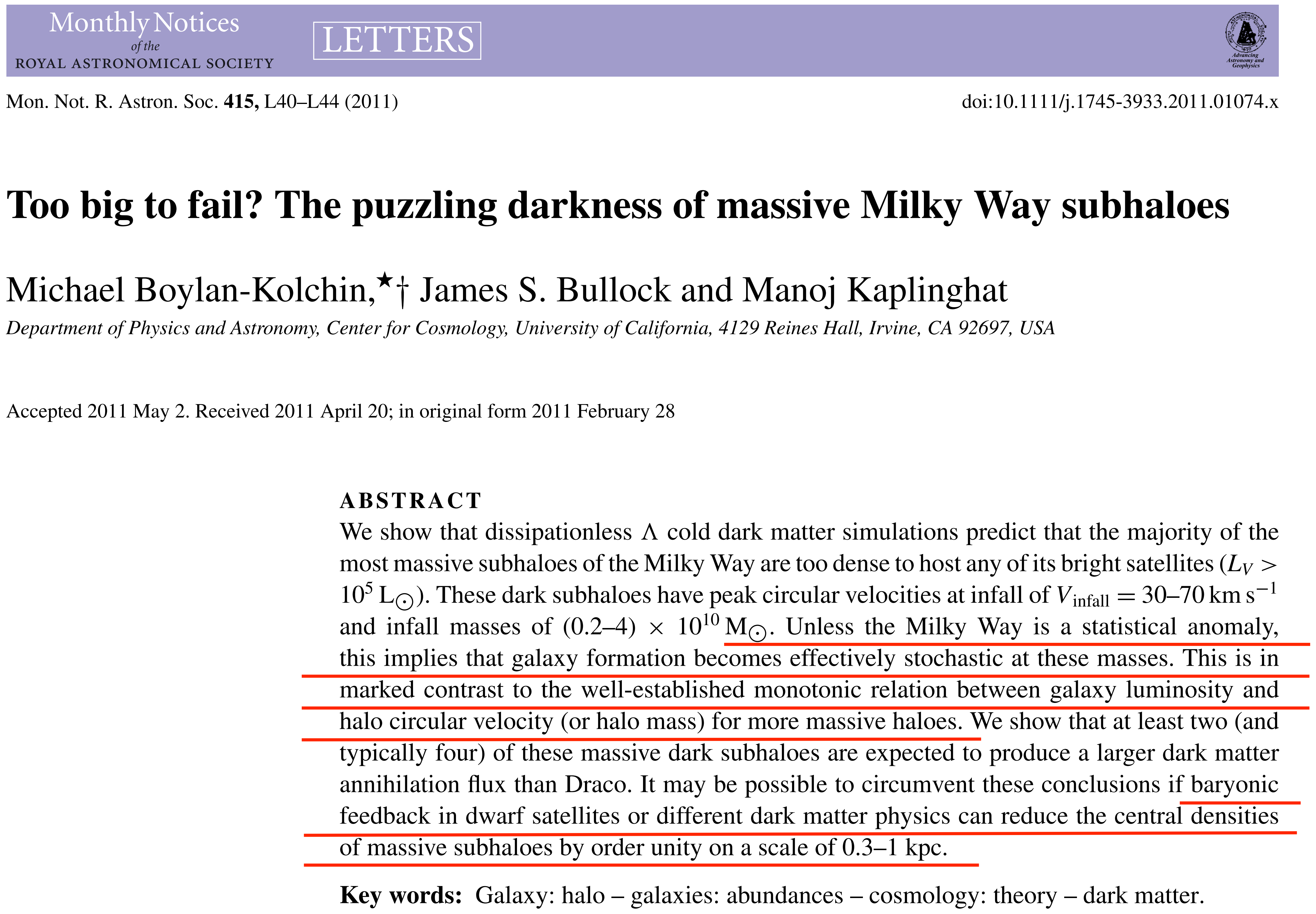


Figure 2. Subhaloes from all six Aquarius simulations (circles) and VL-II (triangles), colour-coded according to $V_{\rm infall}$. The grey-shaded region shows the 2σ confidence interval for possible hosts of the bright MW dwarf spheroidals (see Fig. 1).



Monthly Notices of the ROYAL ASTRONOMICAL SOCIETY LETTERS

Mon. Not. R. Astron. Soc. **415,** L40–L44 (2011) doi:10.1111/j.1745-3933.2011.01074.x

Too big to fail? The puzzling darkness of massive Milky Way subhaloes

Michael Boylan-Kolchin,[★][†] James S. Bullock and Manoj Kaplinghat

Department of Physics and Astronomy, Center for Cosmology, University of California, 4129 Reines Hall, Irvine, CA 92697, USA

Accepted 2011 May 2. Received 2011 April 20; in original form 2011 February 28

ABSTRACT

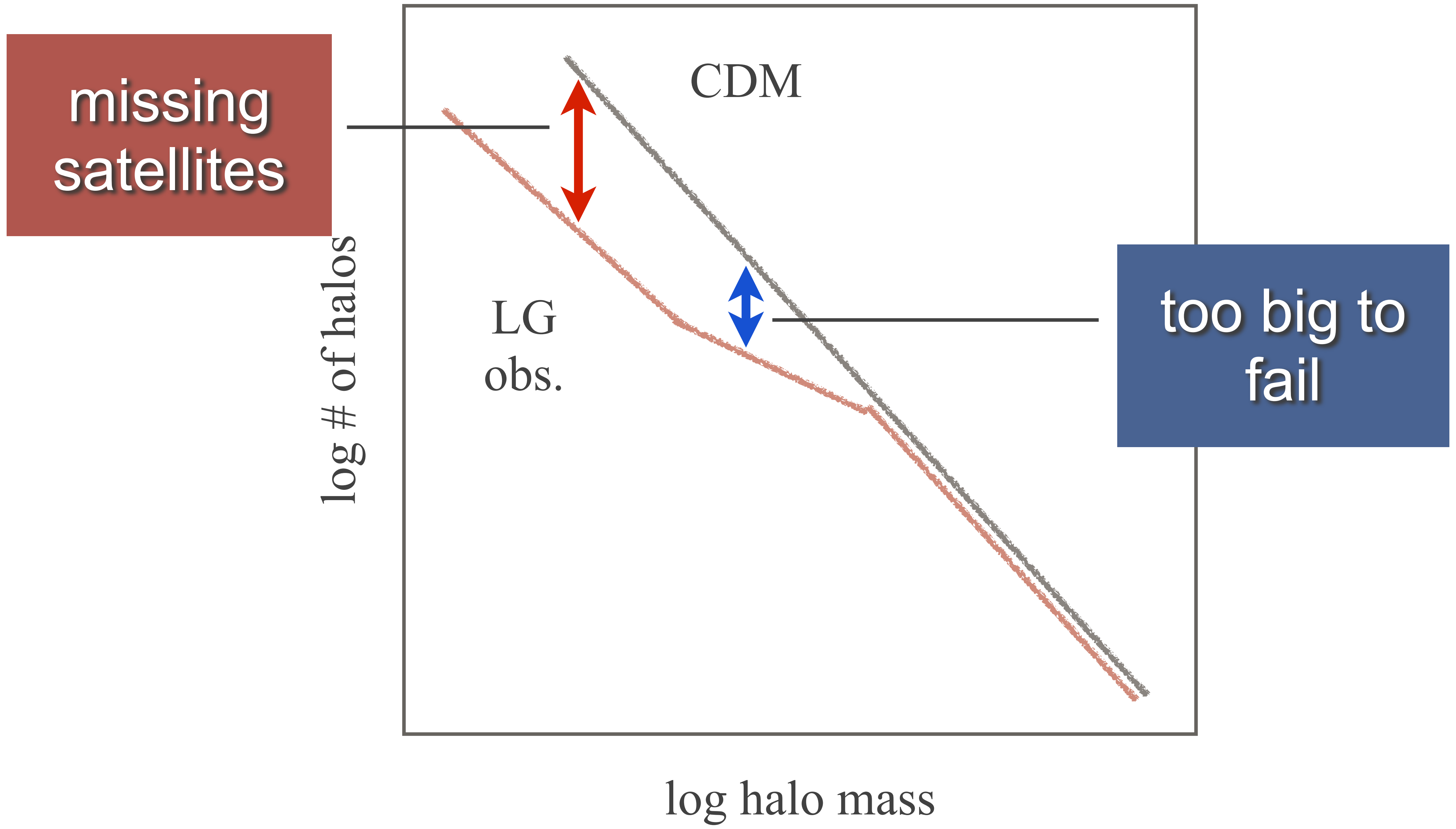
We show that dissipationless Λ cold dark matter simulations predict that the majority of the most massive subhaloes of the Milky Way are too dense to host any of its bright satellites ($L_V > 10^5\,{\rm L}_\odot$). These dark subhaloes have peak circular velocities at infall of $V_{\rm infall} = 30$–$70\,{\rm km\,s^{-1}}$ and infall masses of $(0.2$–$4) \times 10^{10}\,{\rm M}_\odot$. Unless the Milky Way is a statistical anomaly, this implies that galaxy formation becomes effectively stochastic at these masses. This is in marked contrast to the well-established monotonic relation between galaxy luminosity and halo circular velocity (or halo mass) for more massive haloes. We show that at least two (and typically four) of these massive dark subhaloes are expected to produce a larger dark matter annihilation flux than Draco. It may be possible to circumvent these conclusions if baryonic feedback in dwarf satellites or different dark matter physics can reduce the central densities of massive subhaloes by order unity on a scale of 0.3–1 kpc.

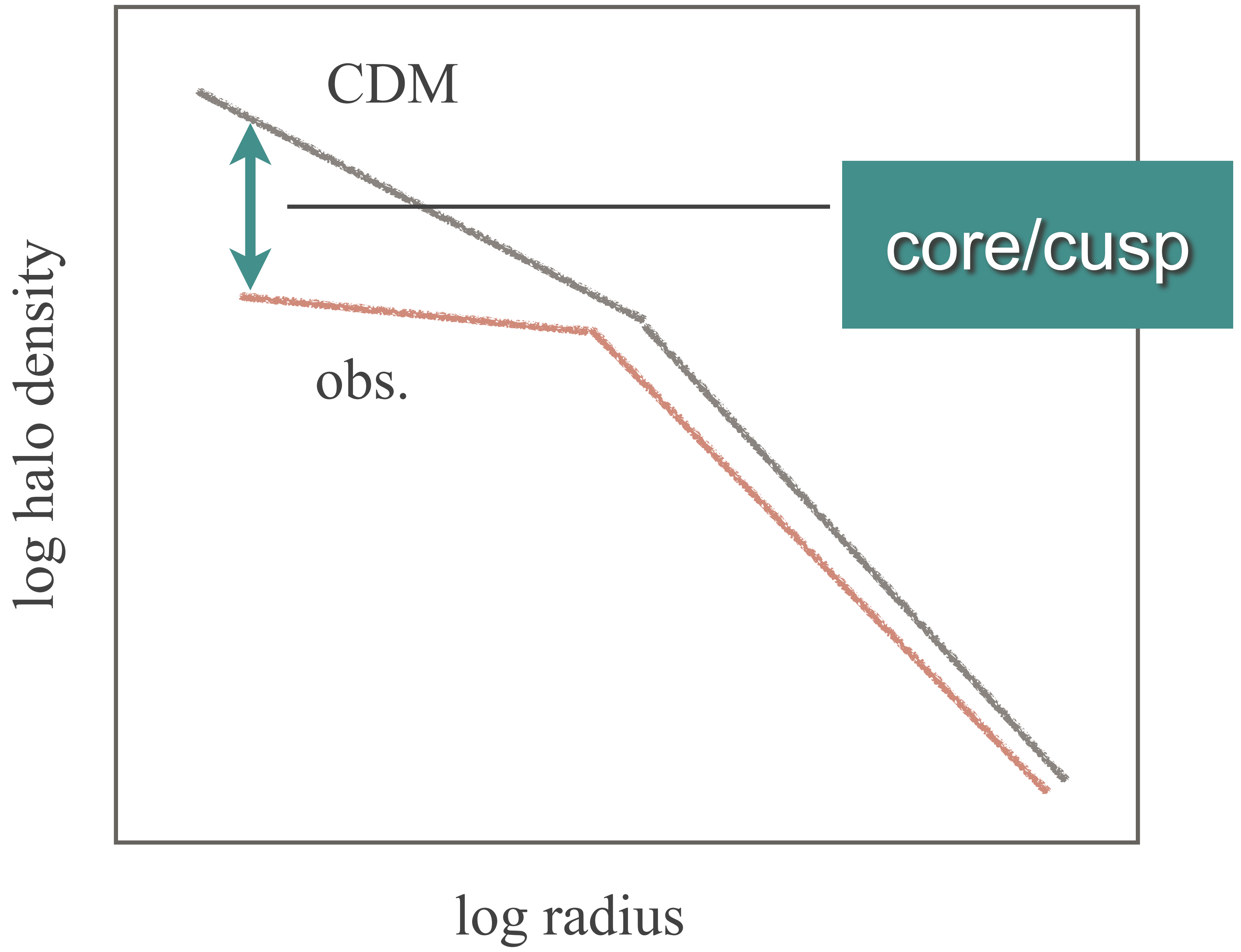
Key words: Galaxy: halo – galaxies: abundances – cosmology: theory – dark matter.

10^7. Many of the subhaloes lie in the range that is consistent at the 2σ level with the dwarfs, but there are a large number of subhaloes that do not. These subhaloes all have central densities that are too high to host any of the bright MW dwarf spheroidals; they also have higher values of both $V_{\rm max}$ and $V_{\rm infall}$, on average.

Historically first small scale problems

- core/cusp problem
- substructure problem (missing satellites)
- too-big-to-fail problem





Current tensions

core-cusp	need collisions	*single species SIDM is enough*
missing satellites	may be gone	*diversity -> baryons (?)*
too big to fail	seems still be present	*multicomponent DM is needed to change the halo abundance*
satellite planes	???, perhaps "initial conditions" at high z	*if so: need strong collisions (fluid) +cooling = naturally multicomponent*
galactic bars	need shallow potential	*cores/heating can do*
profile diversity	grav collapse can do	*collapse takes time to occur, mergers reset the clock*
SMBH formation	need collisional + dissipative DM	*naturally multicomponent*
WIMP miracle	gone(?) (blame: direct detection exp)	$\Omega \propto \frac{1}{\langle \sigma v \rangle} \propto \left(\frac{g_h^4}{m_h^2} \right)^{-1}$ ❌

- History & Motivation
- Models (SIDM, etc)
- Overview of 2cDM
  - physics
  - cosmology
  - understanding

Early DM models

Self-Interacting Dark Matter (SIDM)	*elastic scattering in the dark sector*	*Spergel & Steinhardt, 1999*
Two-component DM (2cDM) with flavor mixing	*works simultaneously for Core/Cusp & Missing satellites*	*MVM, 2000*
"Fuzzy" Dark Matter	*de Broglie wave length ~ 1 kpc core, m~ 1e-22 eV*	*Hu, et al, 1999*
ETHOS	*changes the initial power spectrum and late structure formation*	*Vogelsberger, 2015*
Annihilating Dark Matter	*changes late structure formation*	*Kamionkowski, et al, 2008*
Boosted Dark Matter	*"Lorenz-boosted"*	*Necib, et al, 2017*

Other DM models

Warm Dark Matter

changes initial power spectrum

Atomic DM

many inelastic/multi-component DM models were originally motivated by desire to reconcile DAMA/Libra data with other Direct Detection experiments (Edelweiss, XENON,...)

Axion-like particle DM *(a la fuzzy DM)*

Dark massive photon DM

inelastic, multicomponent models are almost equivalent from the point of view of simulations (except for fluid DM)

Inelastic DM

Excited DM

Exothermic/Endothermic DM

Two-component DM (but not 2cDM)

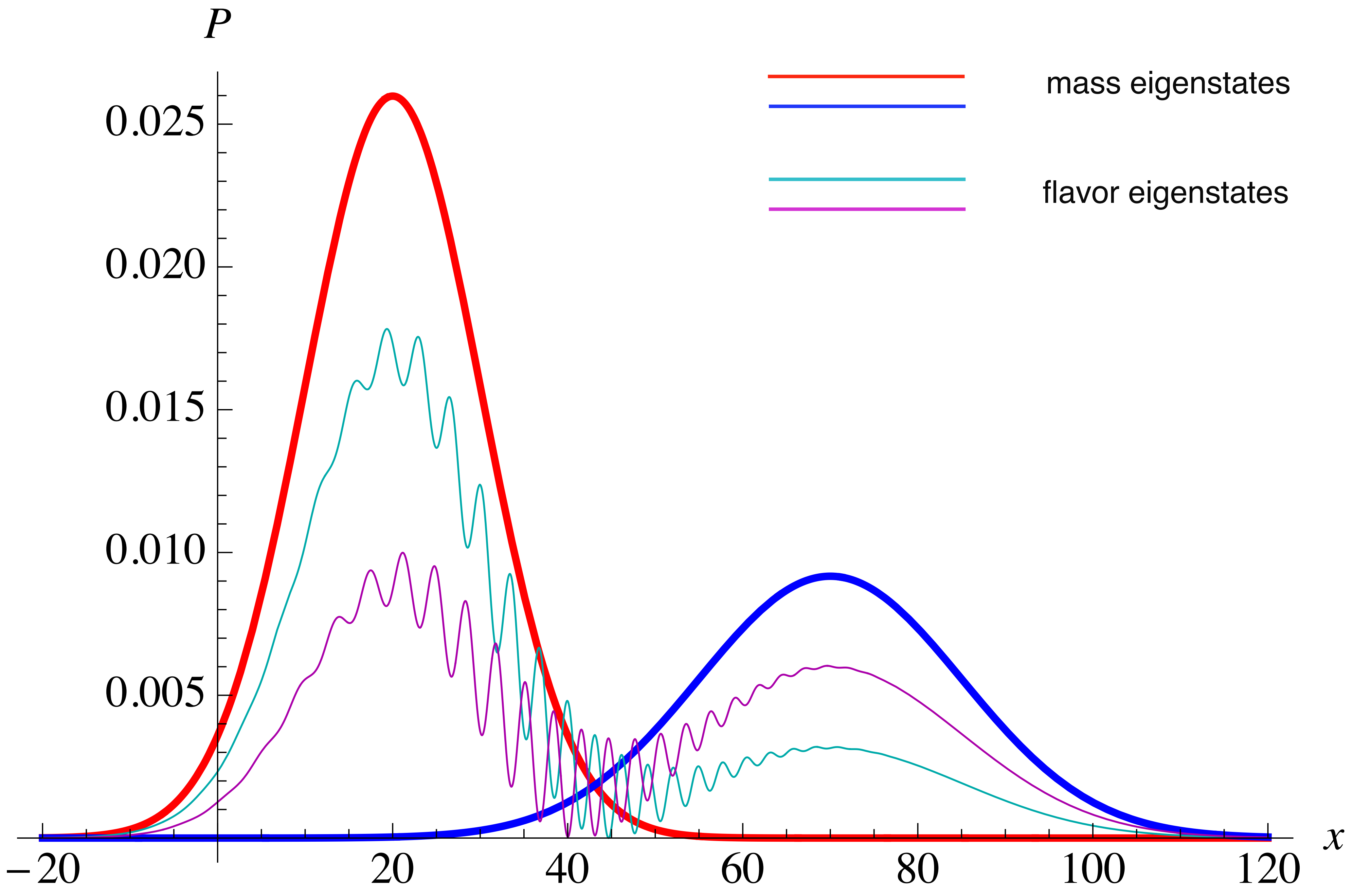
postulate two or more states
"heavy+light" or
"excited+ground states"
and inelastic interactions between them

...more...

- History & Motivation
- Models (SIDM, etc)
- Overview of 2cDM
    - physics
    - cosmology
    - understanding

A flavor-mixed particle

Interactions do not care about propagation (mass) eigenstates;
Propagation does not care about interaction (flavor) eigenstates.



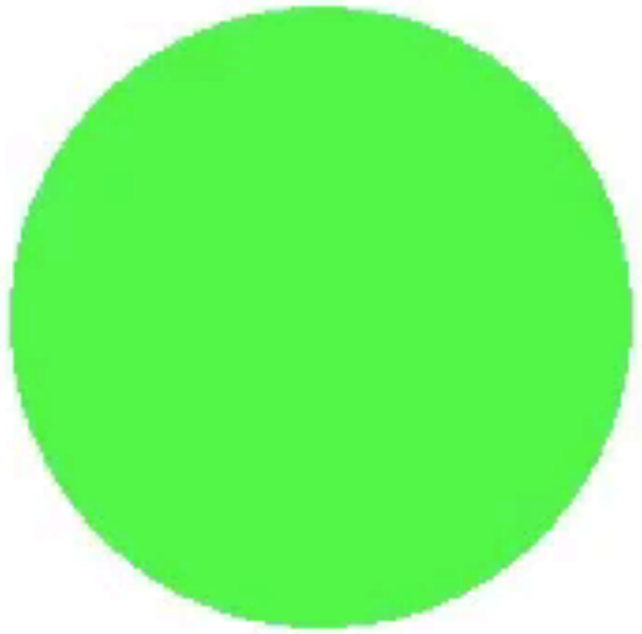
Бруно Понтекорво

$$\begin{pmatrix} |\mathrm{flavor}_1\rangle \\ |\mathrm{flavor}_2\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |\mathrm{mass}_{\mathrm{heavy}}\rangle \\ |\mathrm{mass}_{\mathrm{light}}\rangle \end{pmatrix}$$

B. Pontekorvo
Zh. Teor. Exp Fiz (1957); Soviet JETP (1958)

A flavor-mixed particle

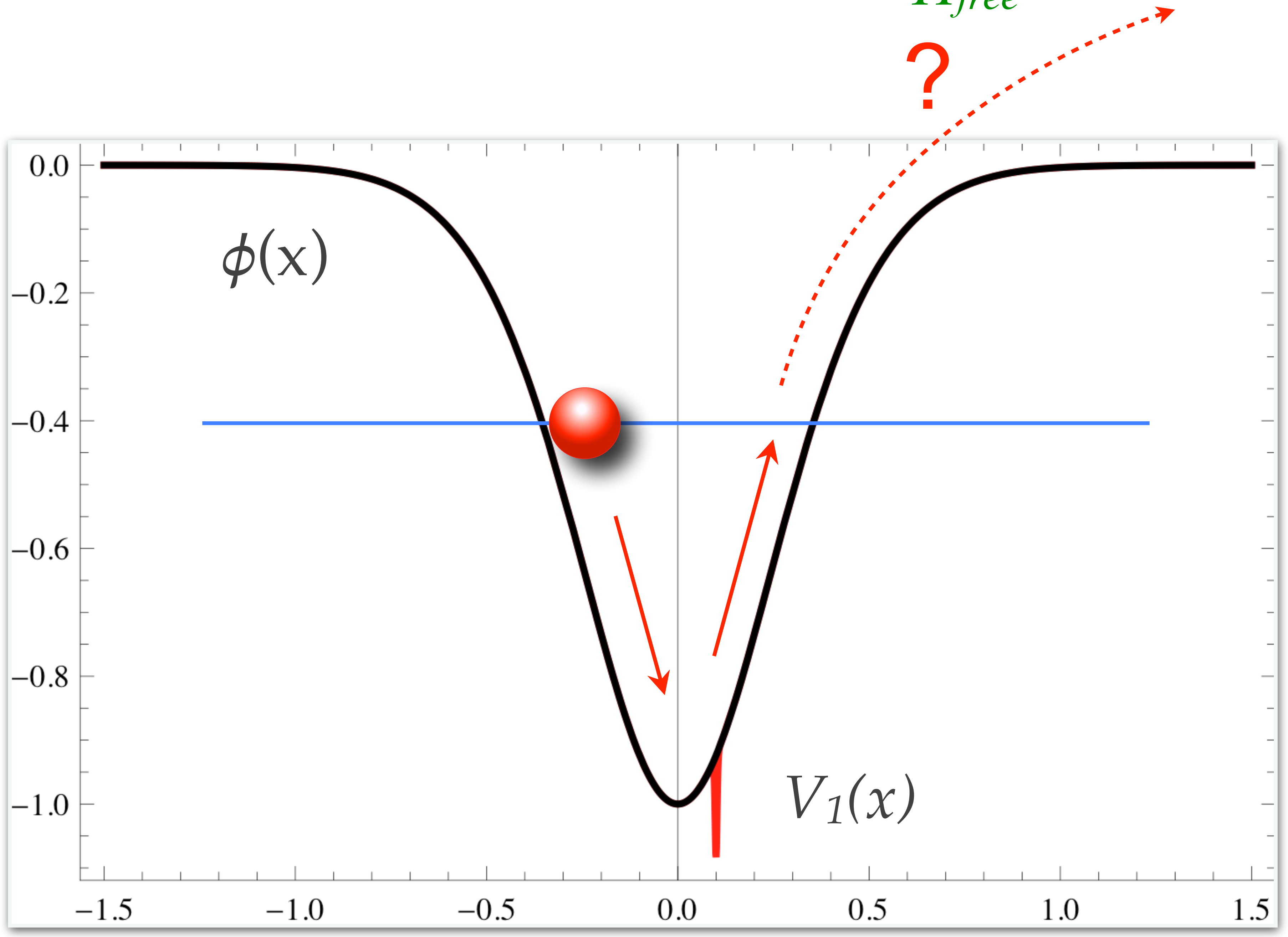
Flavor is a quantum property that allows a particle to have several masses altogether, at the same time and vice versa



Illustrative model

Schrödinger equation

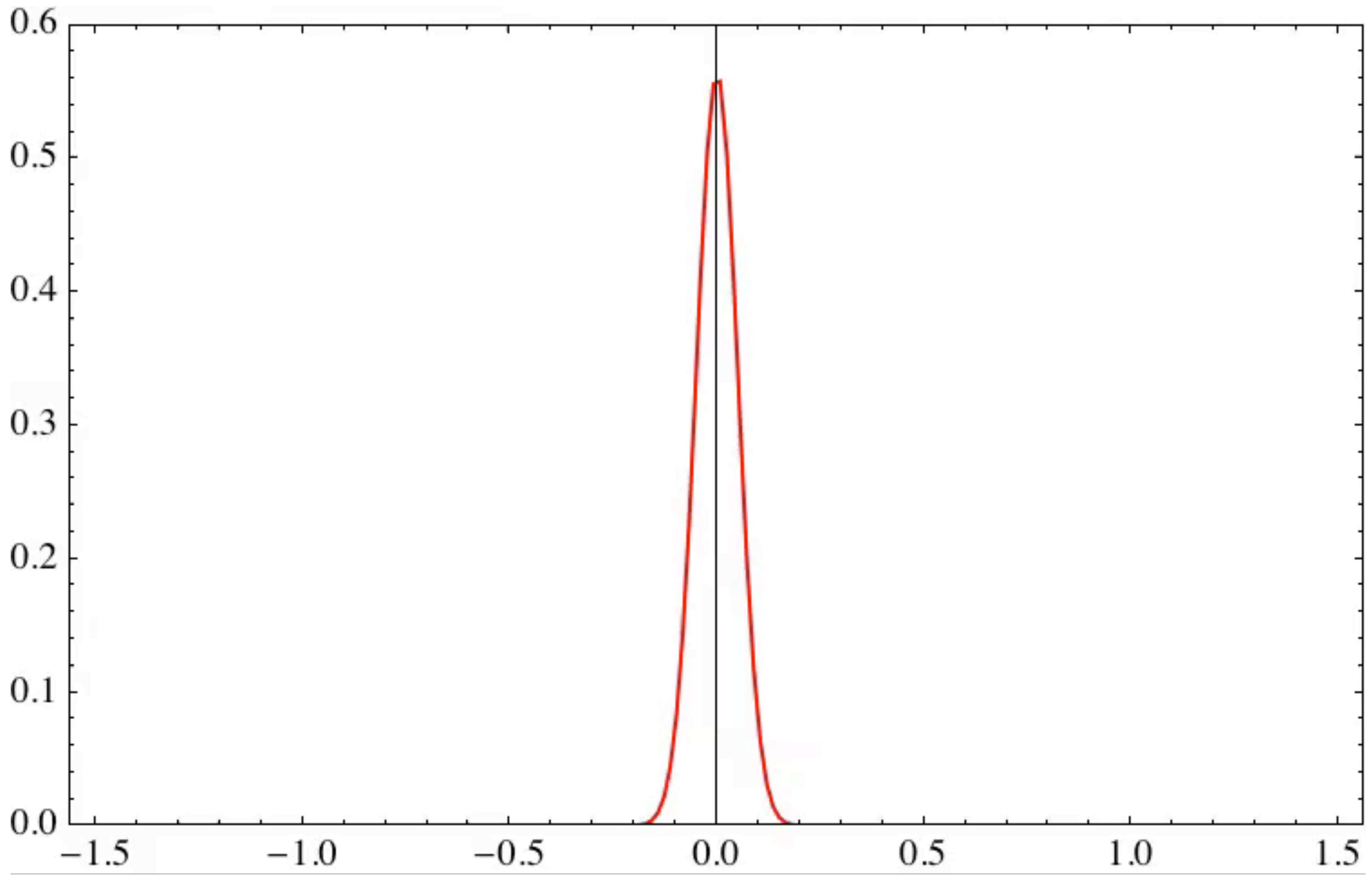
$$i\partial_t \begin{pmatrix} m_h(x,t)\\ m_l(x,t)\end{pmatrix} = \left[\underbrace{\begin{pmatrix} -\partial^2_{xx}/2m_h & 0\\ 0 & -\partial^2_{xx}/2m_l - \Delta m\end{pmatrix}}_{H_{free}} + \underbrace{\begin{pmatrix} m_h\phi(x) & 0\\ 0 & m_l\phi(x)\end{pmatrix}}_{H_{grav}} + \underbrace{\begin{pmatrix} V_{hh} & V_{hl}\\ V_{lh} & V_{ll}\end{pmatrix}}_{V}\right]\begin{pmatrix} m_h(x,t)\\ m_l(x,t)\end{pmatrix}$$

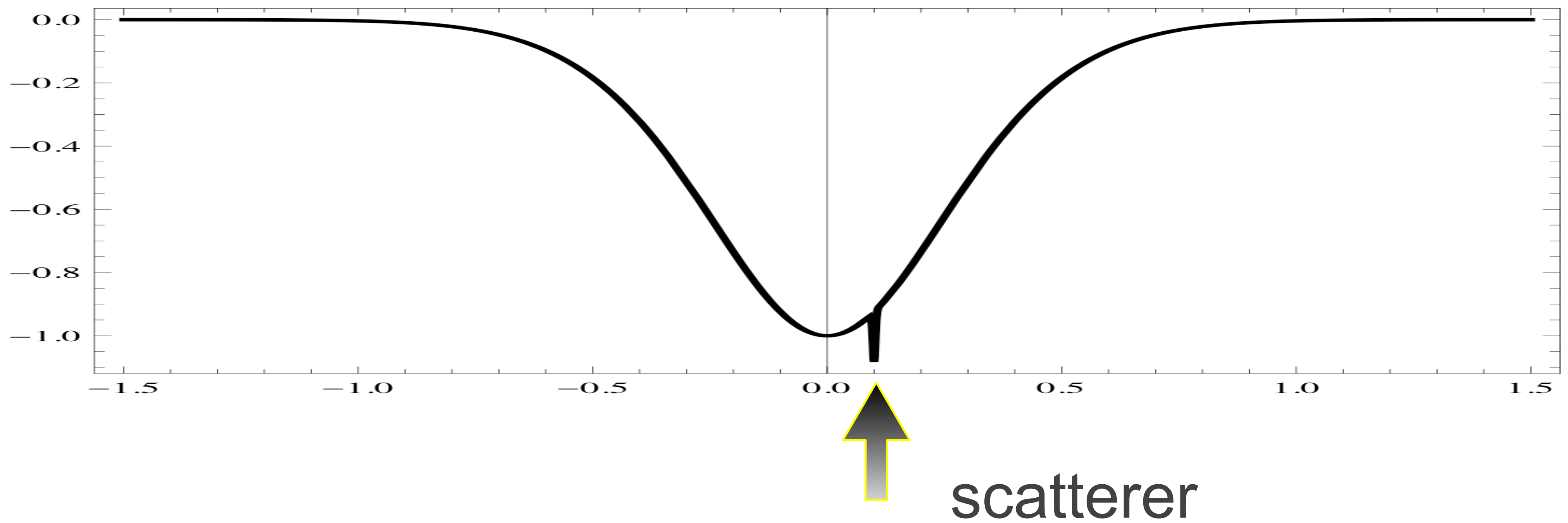


$$\begin{pmatrix} V_{hh} & V_{hl}\\ V_{lh} & V_{ll}\end{pmatrix} = U\begin{pmatrix} V_1 & 0\\ 0 & 0\end{pmatrix}U^\dagger$$

MVM, J Phys A 2010

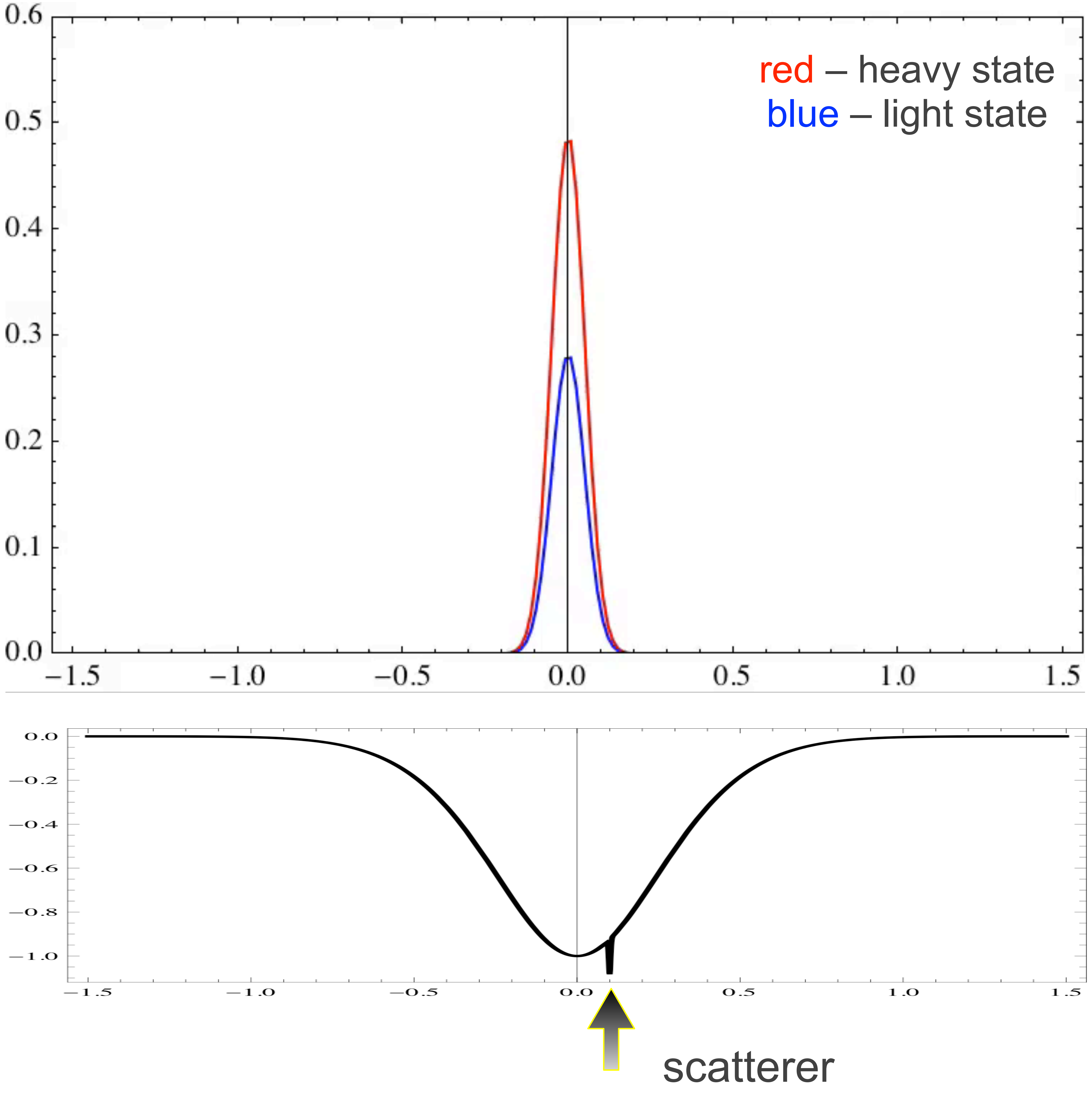
No flavor mixing case





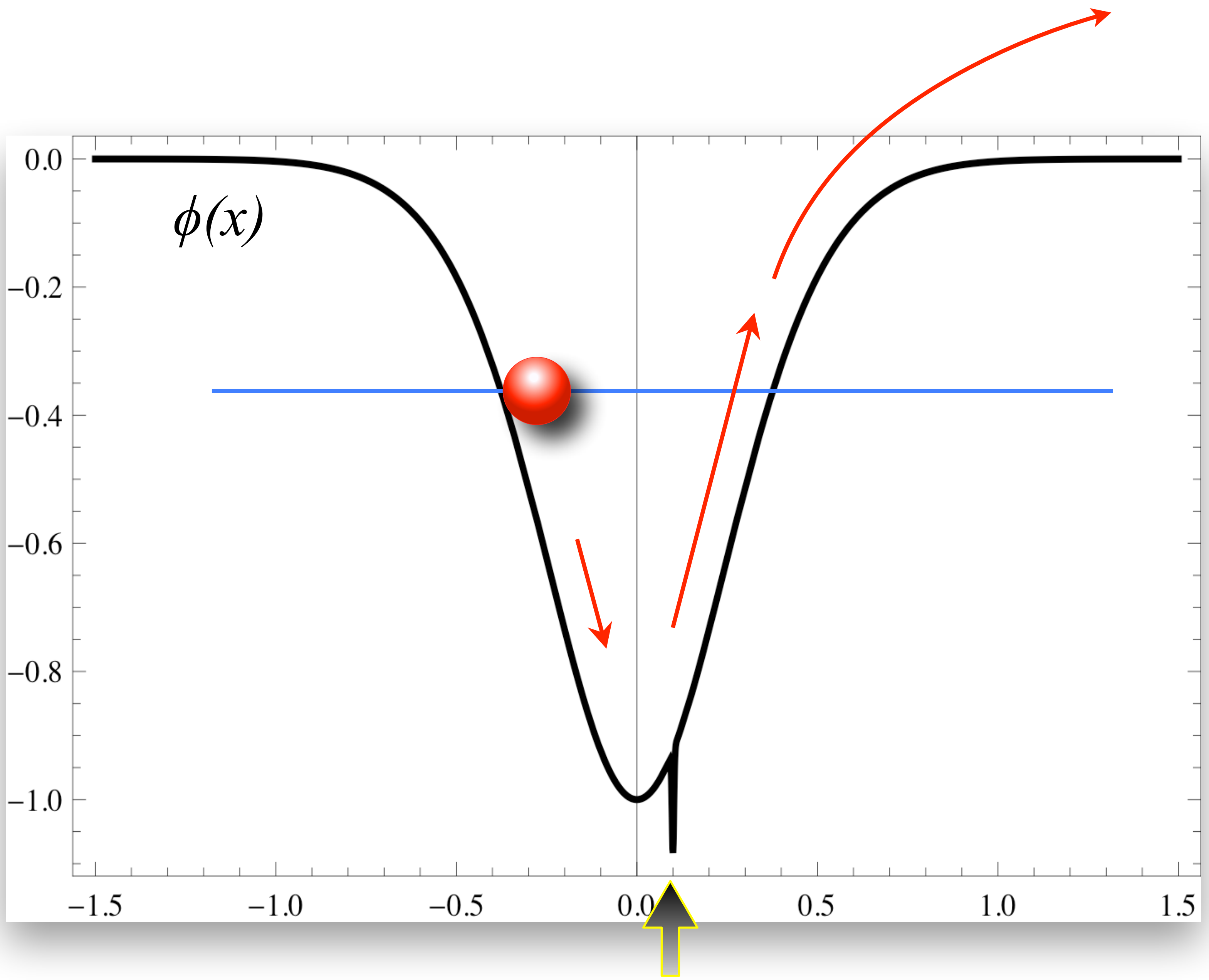
scatterer

With flavor mixing

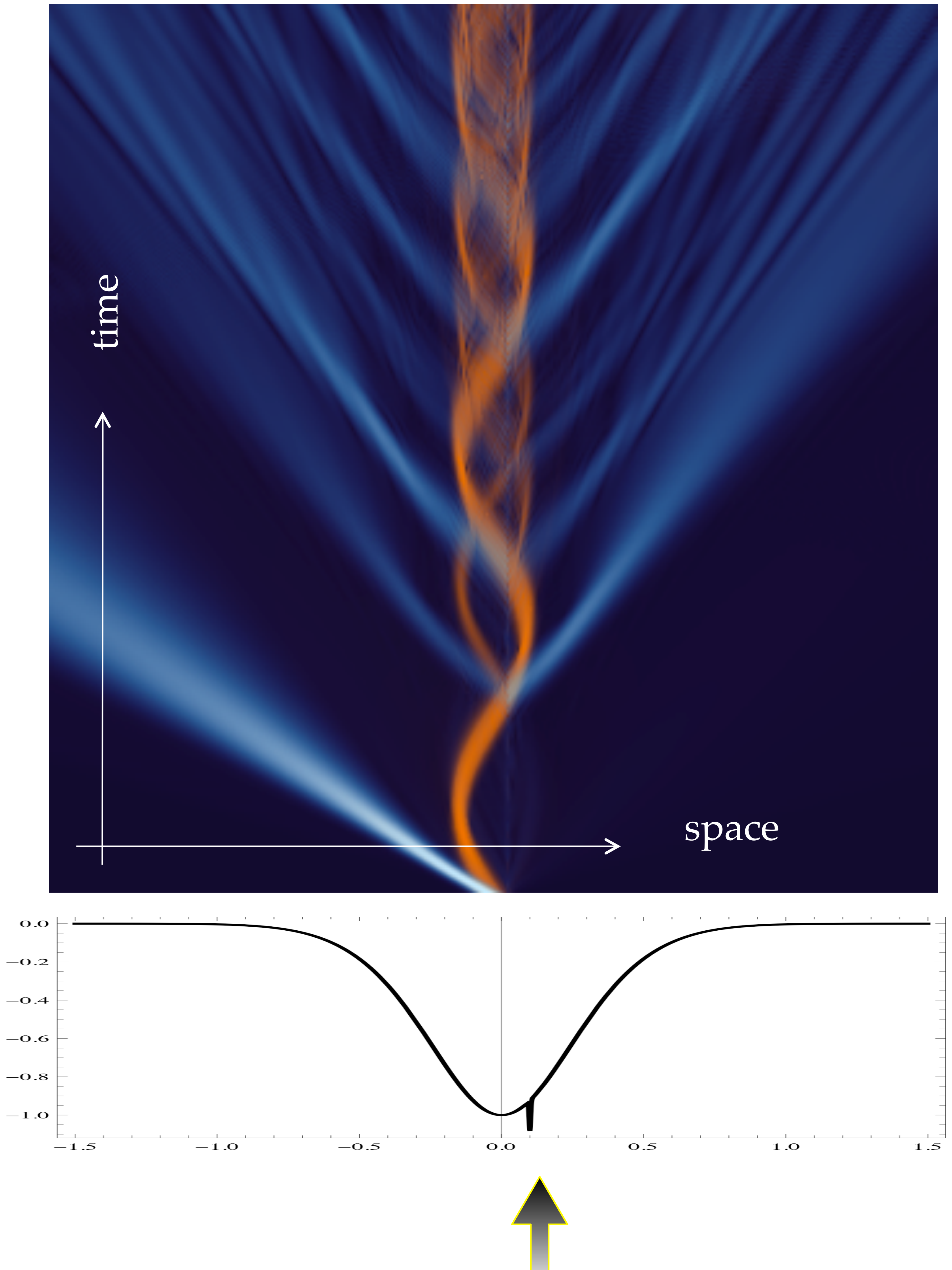


Quantum evaporation

Particle gradual escape from a gravitational potential (in "elastic" collisions) without changing particle's identity



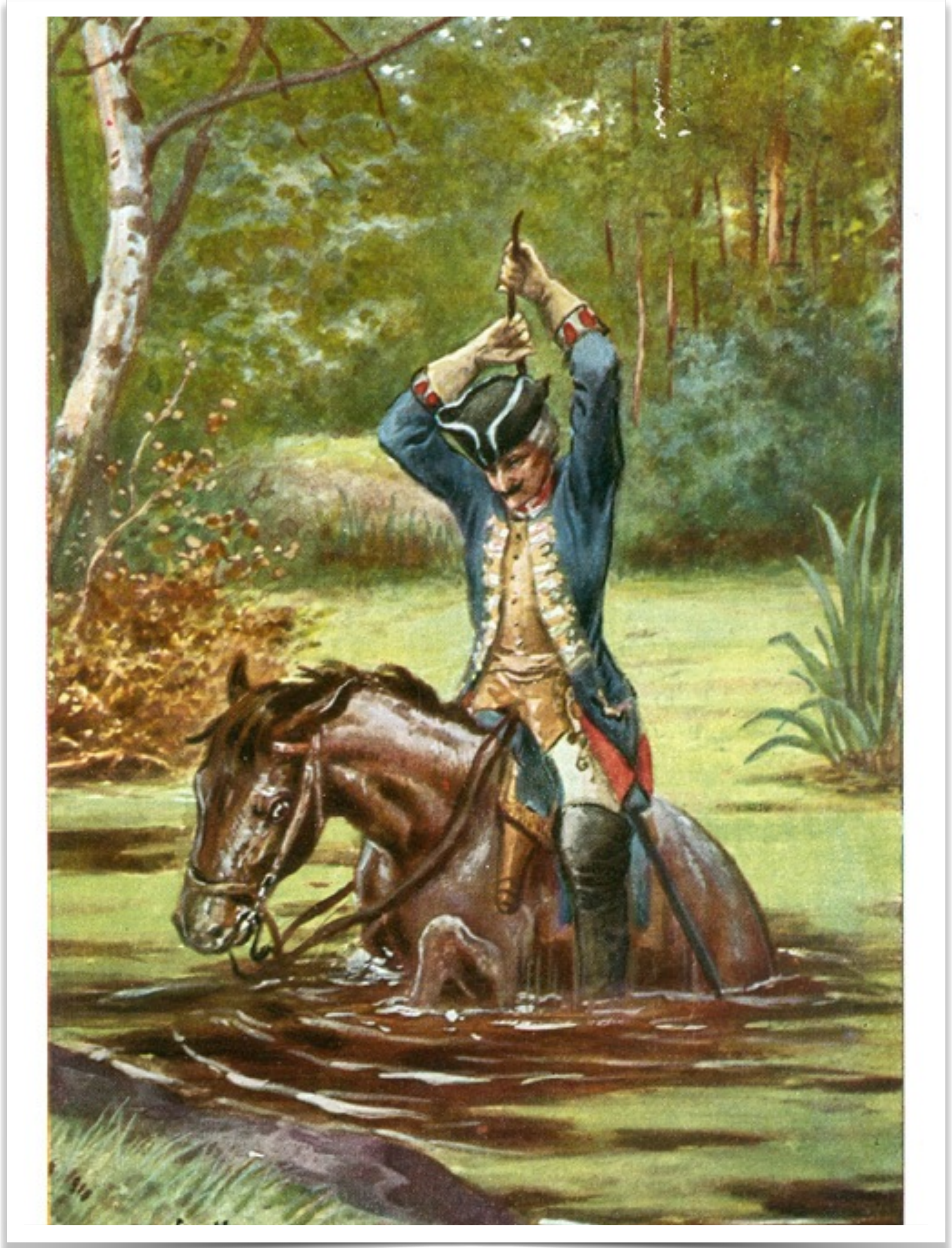
scatterer



scatterer

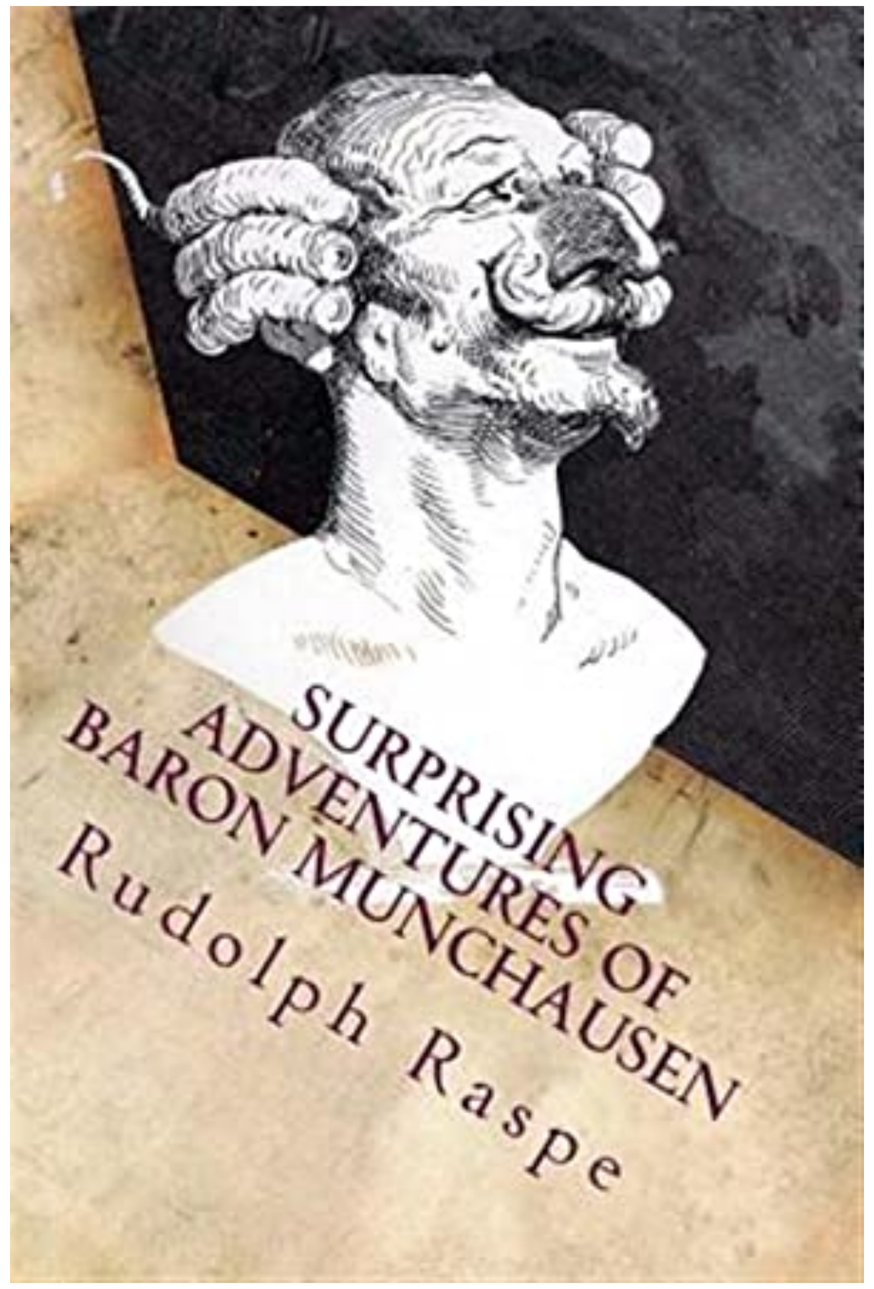
MVM, J Phys A 2010; JCAP 2014

“Munchausen effect”



Baron von Munchausen lifted himself (and his horse) out of the mud by pulling on his own pigtail.

It is one of the “true” stories from “*The Surprising Adventures of Baron Munchausen*” by Rudolph Raspe



Technical: Interaction of 2-comp particles

Wave-functions

$$|ff\rangle \equiv \begin{pmatrix}\alpha\alpha\\ \alpha\beta\\ \beta\alpha\\ \beta\beta\end{pmatrix} \equiv \begin{pmatrix}\alpha_1\alpha_2(\mathbf{x}_1,\mathbf{x}_2,t)\\ \alpha_1\beta_2(\mathbf{x}_1,\mathbf{x}_2,t)\\ \beta_1\alpha_2(\mathbf{x}_1,\mathbf{x}_2,t)\\ \beta_1\beta_2(\mathbf{x}_1,\mathbf{x}_2,t)\end{pmatrix} \qquad |mm\rangle \equiv \begin{pmatrix}hh\\ hl\\ lh\\ ll\end{pmatrix} \equiv \begin{pmatrix}h_1h_2(\mathbf{x}_1,\mathbf{x}_2,t)\\ h_1l_2(\mathbf{x}_1,\mathbf{x}_2,t)\\ l_1h_2(\mathbf{x}_1,\mathbf{x}_2,t)\\ l_1l_2(\mathbf{x}_1,\mathbf{x}_2,t)\end{pmatrix}$$

Mixing

$$\boxed{|ff\rangle = U_2\,|mm\rangle}$$

$$U_2 \equiv U \otimes U = \begin{pmatrix} \cos^2\theta & -\cos\theta\,\sin\theta & -\cos\theta\,\sin\theta & \sin^2\theta\\ \cos\theta\,\sin\theta & \cos^2\theta & -\sin^2\theta & -\cos\theta\,\sin\theta\\ \cos\theta\,\sin\theta & -\sin^2\theta & \cos^2\theta & -\cos\theta\,\sin\theta\\ \sin^2\theta & \cos\theta\,\sin\theta & \cos\theta\,\sin\theta & \cos^2\theta \end{pmatrix}$$

Interaction

$$\tilde{V} = \begin{pmatrix} V_{\alpha\alpha} & 0 & 0 & 0\\ 0 & V_{\alpha\beta} & 0 & 0\\ 0 & 0 & V_{\beta\alpha} & 0\\ 0 & 0 & 0 & V_{\beta\beta}\end{pmatrix}$$

$$\boxed{V = U_2^\dagger \tilde{V} U_2 = \begin{pmatrix} A & E & E & D\\ E & C & D & F\\ E & D & C & F\\ D & F & F & B\end{pmatrix}}$$

$$\boxed{\begin{aligned}
A &= \tfrac{1}{8}\left[3V_{\alpha\alpha} + 2V_{\alpha\beta} + 3V_{\beta\beta} + 4(V_{\alpha\alpha} - V_{\beta\beta})\cos 2\theta + (V_{\alpha\alpha} - 2V_{\alpha\beta} + V_{\beta\beta})\cos 4\theta\right],\\
B &= \tfrac{1}{8}\left[3V_{\alpha\alpha} + 2V_{\alpha\beta} + 3V_{\beta\beta} - 4(V_{\alpha\alpha} - V_{\beta\beta})\cos 2\theta + (V_{\alpha\alpha} - 2V_{\alpha\beta} + V_{\beta\beta})\cos 4\theta\right],\\
C &= \tfrac{1}{8}\left[V_{\alpha\alpha} + 6V_{\alpha\beta} + V_{\beta\beta} - (V_{\alpha\alpha} - 2V_{\alpha\beta} + V_{\beta\beta})\cos 4\theta\right],\\
D &= \tfrac{1}{4}\left[V_{\alpha\alpha} - 2V_{\alpha\beta} + V_{\beta\beta}\right]\sin^2 2\theta,\\
E &= -\tfrac{1}{4}\left[V_{\alpha\alpha} - V_{\beta\beta} + (V_{\alpha\alpha} - 2V_{\alpha\beta} + V_{\beta\beta})\cos 2\theta\right]\sin 2\theta,\\
F &= -\tfrac{1}{4}\left[V_{\alpha\alpha} - V_{\beta\beta} - (V_{\alpha\alpha} - 2V_{\alpha\beta} + V_{\beta\beta})\cos 2\theta\right]\sin 2\theta,
\end{aligned}}$$

(MM, JCAP 2014)

Technical: 2-comp 2-particle dynamics

Schrödinger equation

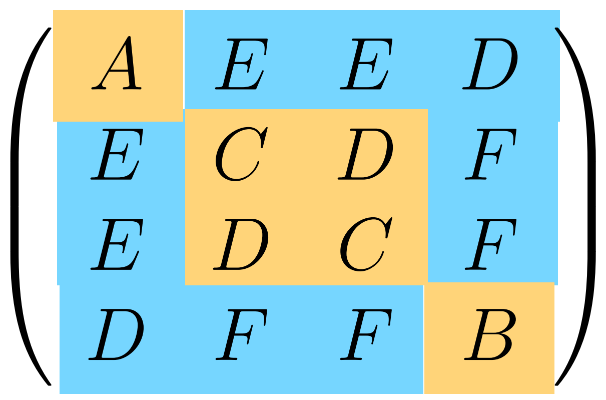
$$\boxed{i\hbar\partial_t\,|mm(x_1,x_2,t)\rangle = (H^{\text{free}}+H^{\text{grav}}+V)\,|mm(x_1,x_2,t)\rangle}$$

$$H^{\text{free}} = \begin{pmatrix} H^{\text{free}}_{hh} & 0 & 0 & 0 \\ 0 & H^{\text{free}}_{hl} & 0 & 0 \\ 0 & 0 & H^{\text{free}}_{lh} & 0 \\ 0 & 0 & 0 & H^{\text{free}}_{ll} \end{pmatrix}$$

$$\begin{aligned} H^{\text{free}}_{hh} &= -\partial^2_{x_1x_1}/2m_h - \partial^2_{x_2x_2}/2m_h, \\ H^{\text{free}}_{hl} &= -\partial^2_{x_1x_1}/2m_h - \partial^2_{x_2x_2}/2m_l - \Delta m, \\ H^{\text{free}}_{lh} &= -\partial^2_{x_1x_1}/2m_l - \partial^2_{x_2x_2}/2m_h - \Delta m, \\ H^{\text{free}}_{ll} &= -\partial^2_{x_1x_1}/2m_l - \partial^2_{x_2x_2}/2m_l - 2\Delta m. \end{aligned}$$

$$H^{\text{grav}} = \begin{pmatrix} H^{\text{grav}}_{hh} & 0 & 0 & 0 \\ 0 & H^{\text{grav}}_{hl} & 0 & 0 \\ 0 & 0 & H^{\text{grav}}_{lh} & 0 \\ 0 & 0 & 0 & H^{\text{grav}}_{ll} \end{pmatrix}$$

$$\begin{aligned} H^{\text{grav}}_{hh} &= m_h\phi(x_1) + m_h\phi(x_2), \\ H^{\text{grav}}_{hl} &= m_h\phi(x_1) + m_l\phi(x_2), \\ H^{\text{grav}}_{lh} &= m_l\phi(x_1) + m_h\phi(x_2), \\ H^{\text{grav}}_{ll} &= m_l\phi(x_1) + m_l\phi(x_2), \end{aligned}$$

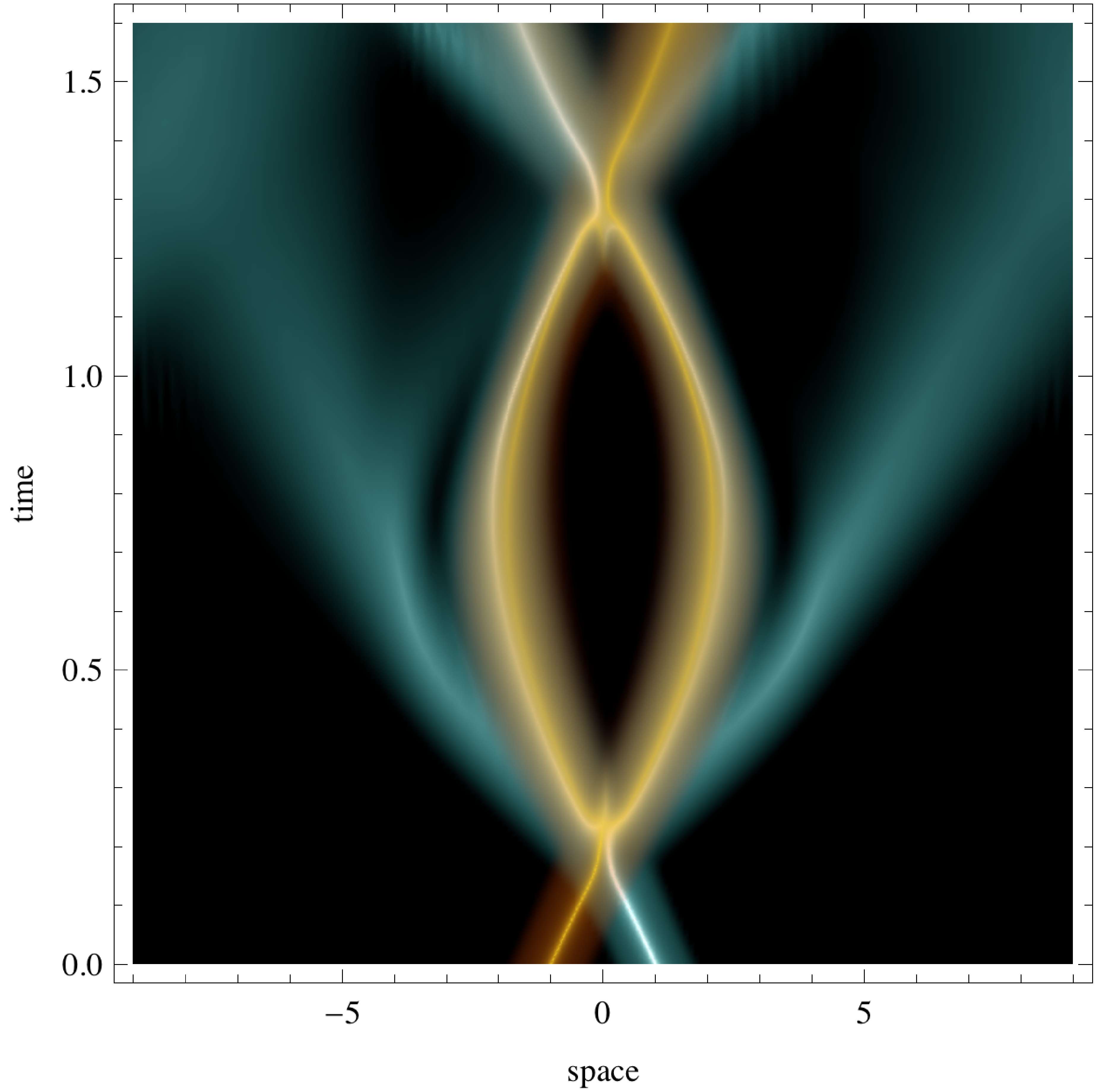


$$V = U_2^\dagger \tilde{V} U_2 = \begin{pmatrix} A & E & E & D \\ E & C & D & F \\ E & D & C & F \\ D & F & F & B \end{pmatrix}$$

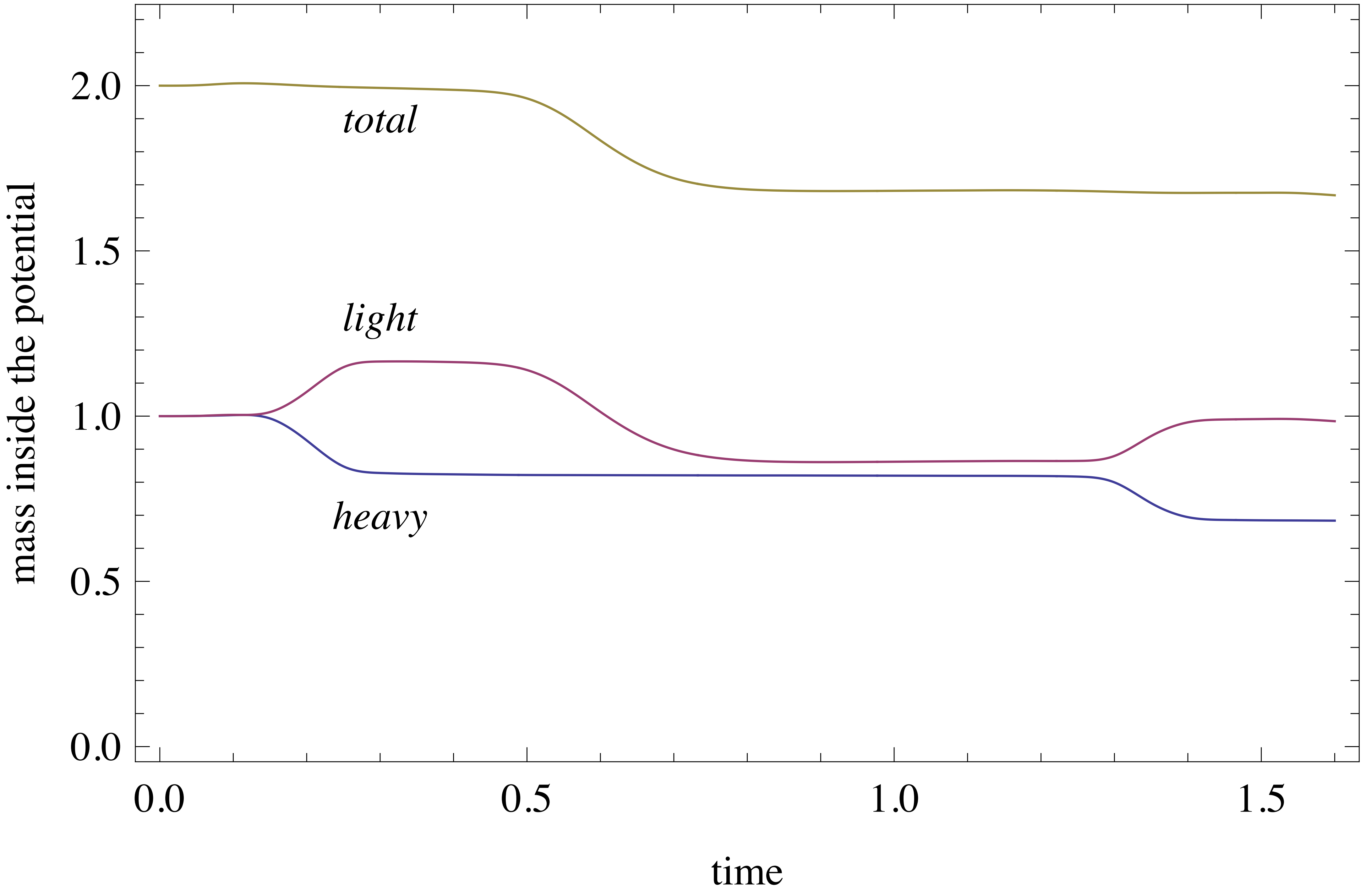
scattering

conversion

Complete evaporation of 2-comp. particles



$$|h\rangle + |l\rangle \rightarrow |l\rangle + |l\rangle$$



(MM, JCAP 2014)

- History & Motivation
- Models (SIDM, etc)
- Overview of 2cDM
  - physics
  - cosmology
  - understanding

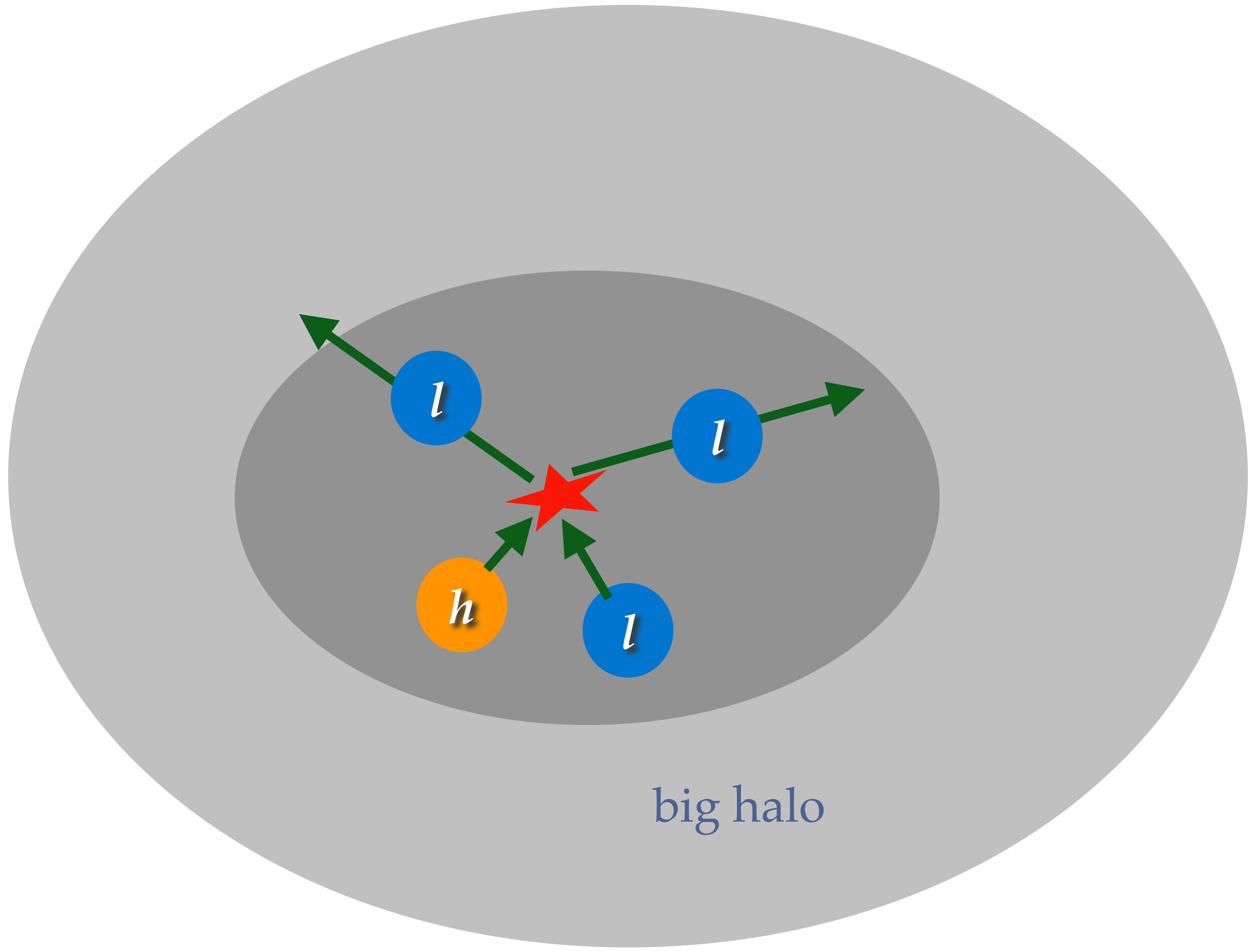
Substructure 2cDM physics

Core heating does not change halo mass.
Thus, SIDM *without baryons* cannot resolve the satellite problem.

inelastic: $|h\rangle + |l\rangle \rightarrow |l\rangle + |l\rangle$

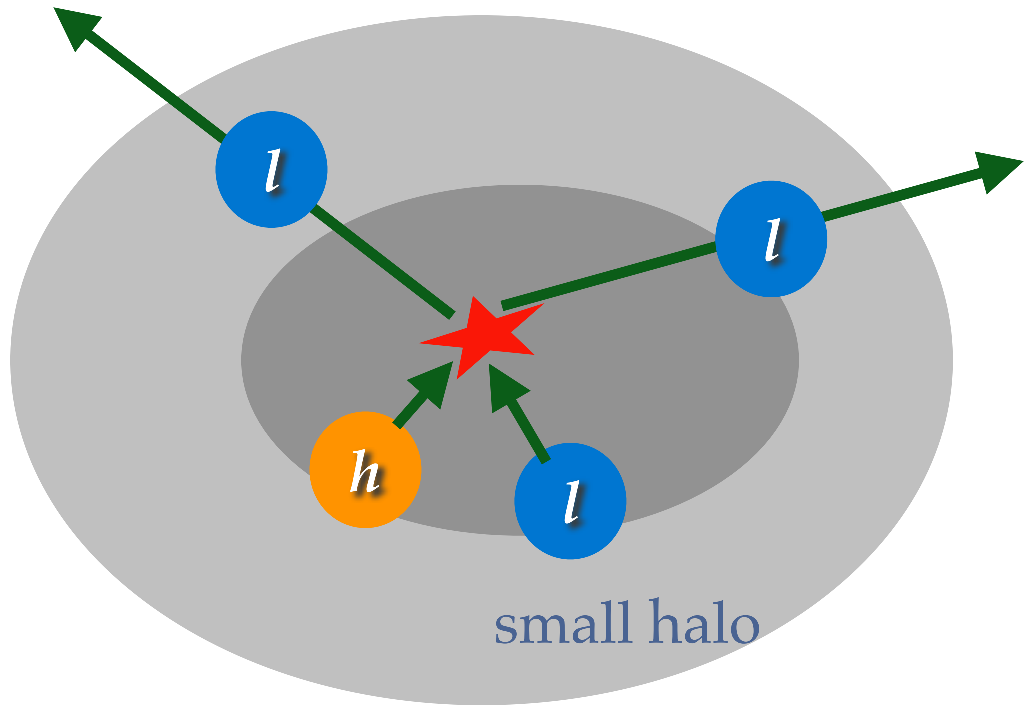
"kick" velocity: $\frac{1}{2}mv^2_{kick} \sim \Delta mc^2$

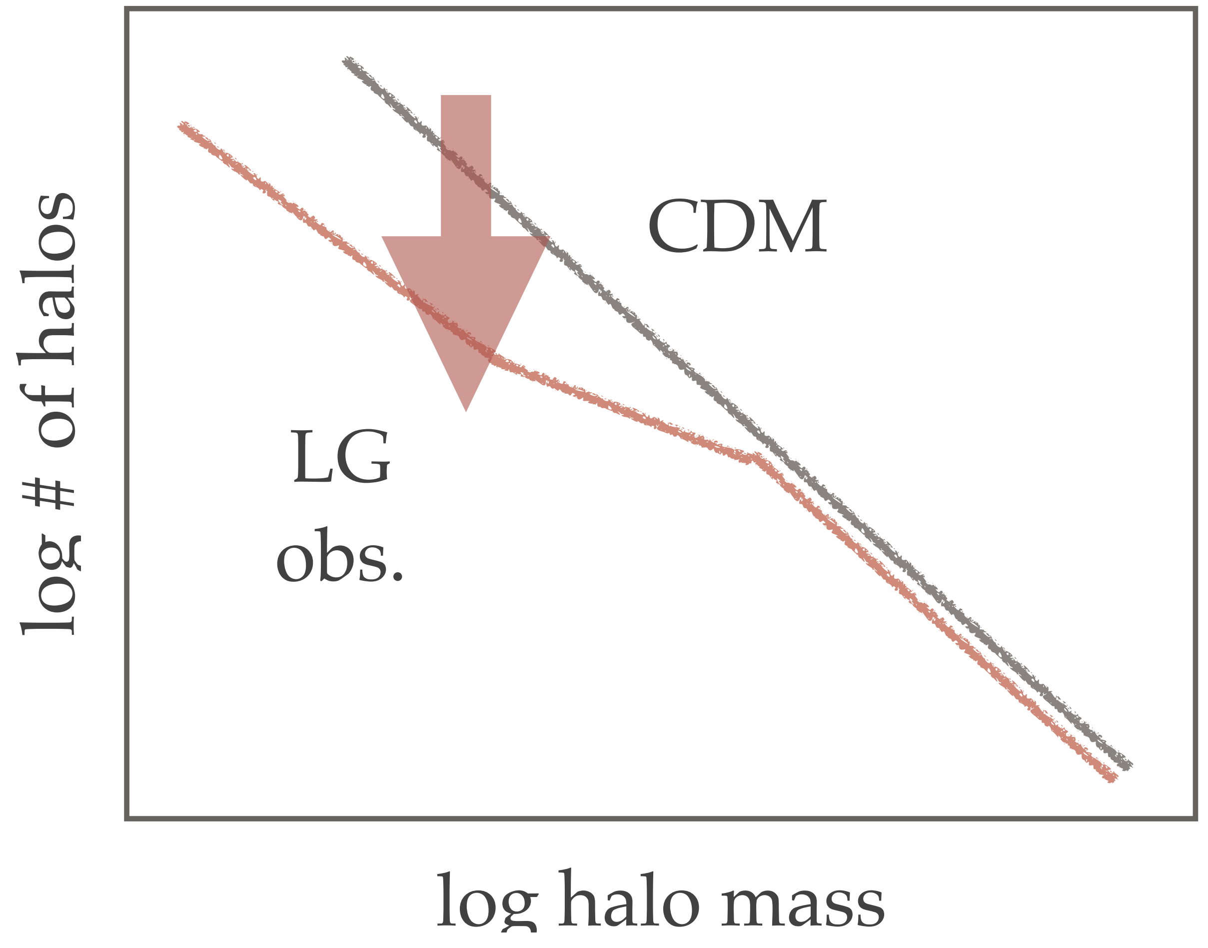
if $v_{kick} \ll v_{escape}$, *central cusps softened*



MVM, ArXiv 2000; J Phys A 2010; JCAP 2014

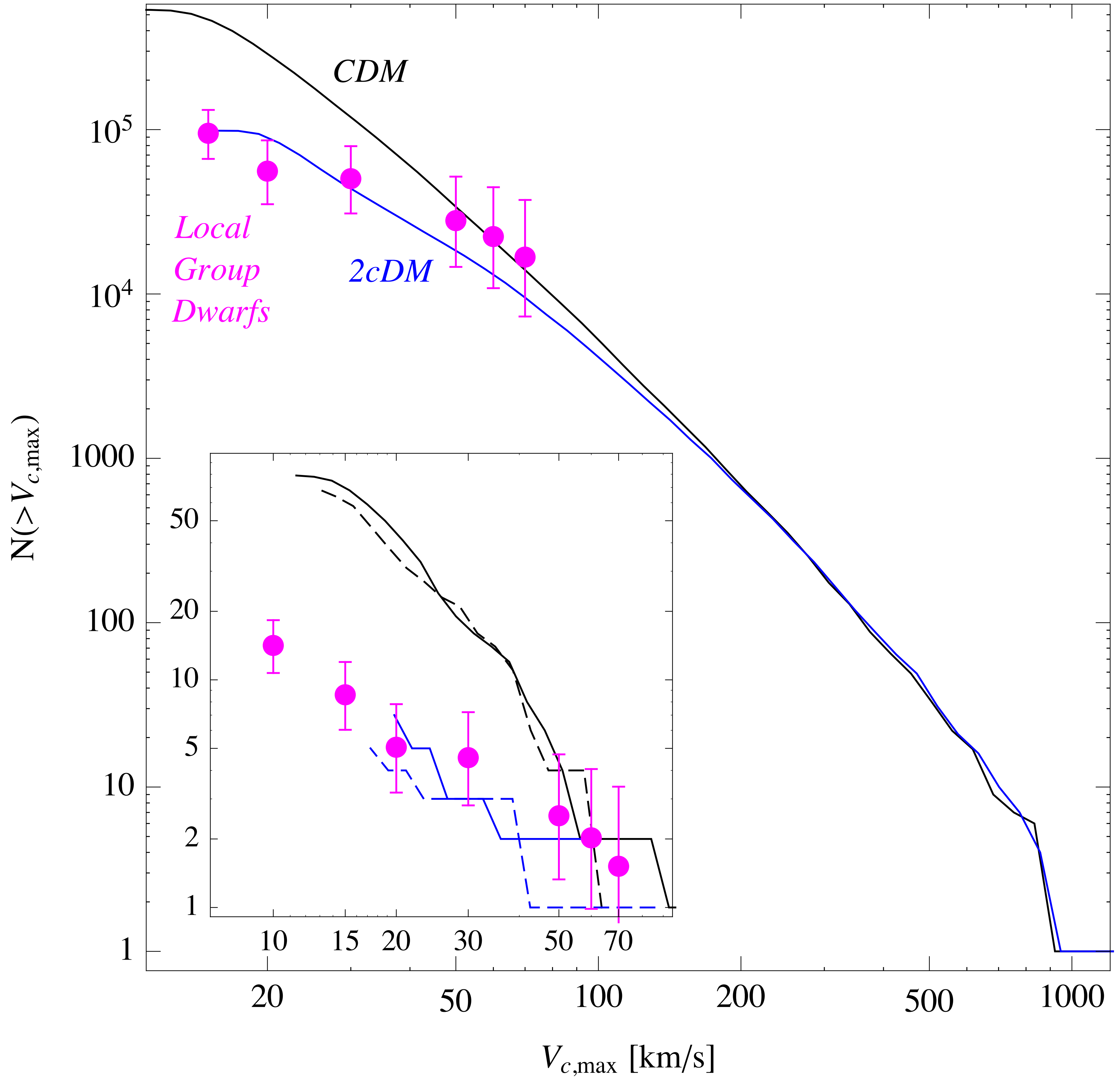
if $v_{kick} \gg v_{escape}$, *dwarf halos destroyed*

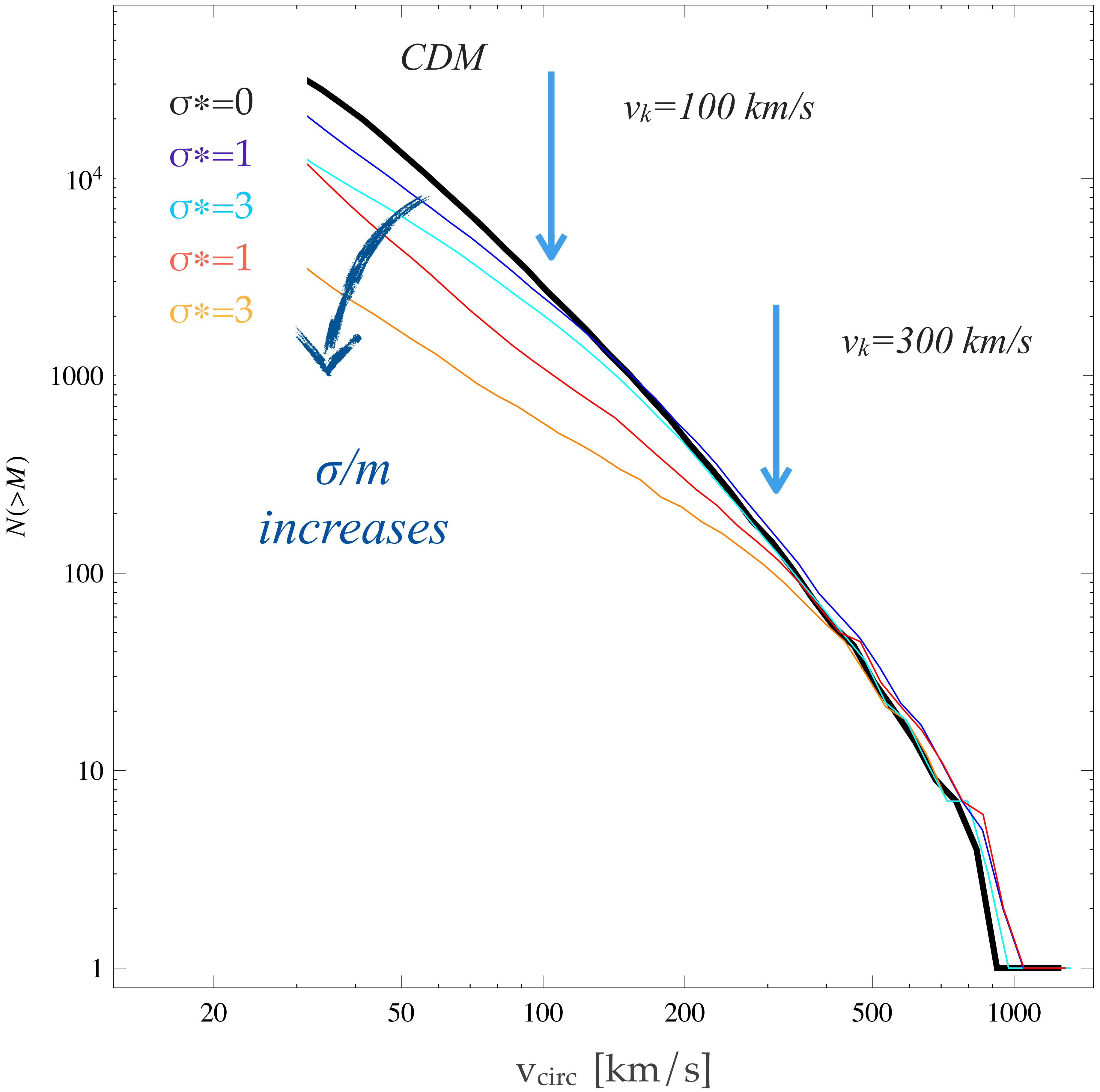




Substructure in simulations

$v_k=c(2\Delta m/m)^{1/2}$
$\sigma_*=\sigma/m$

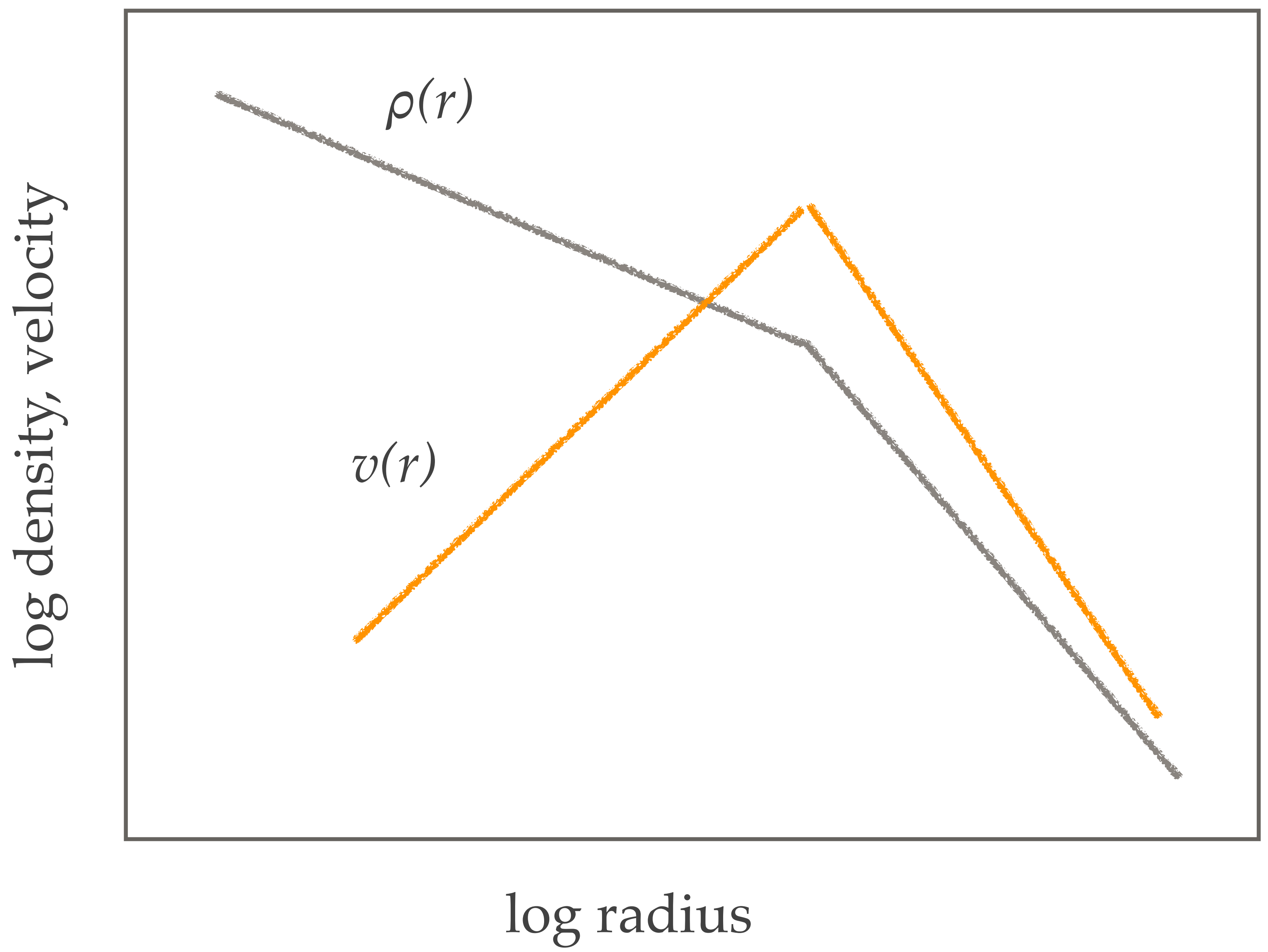




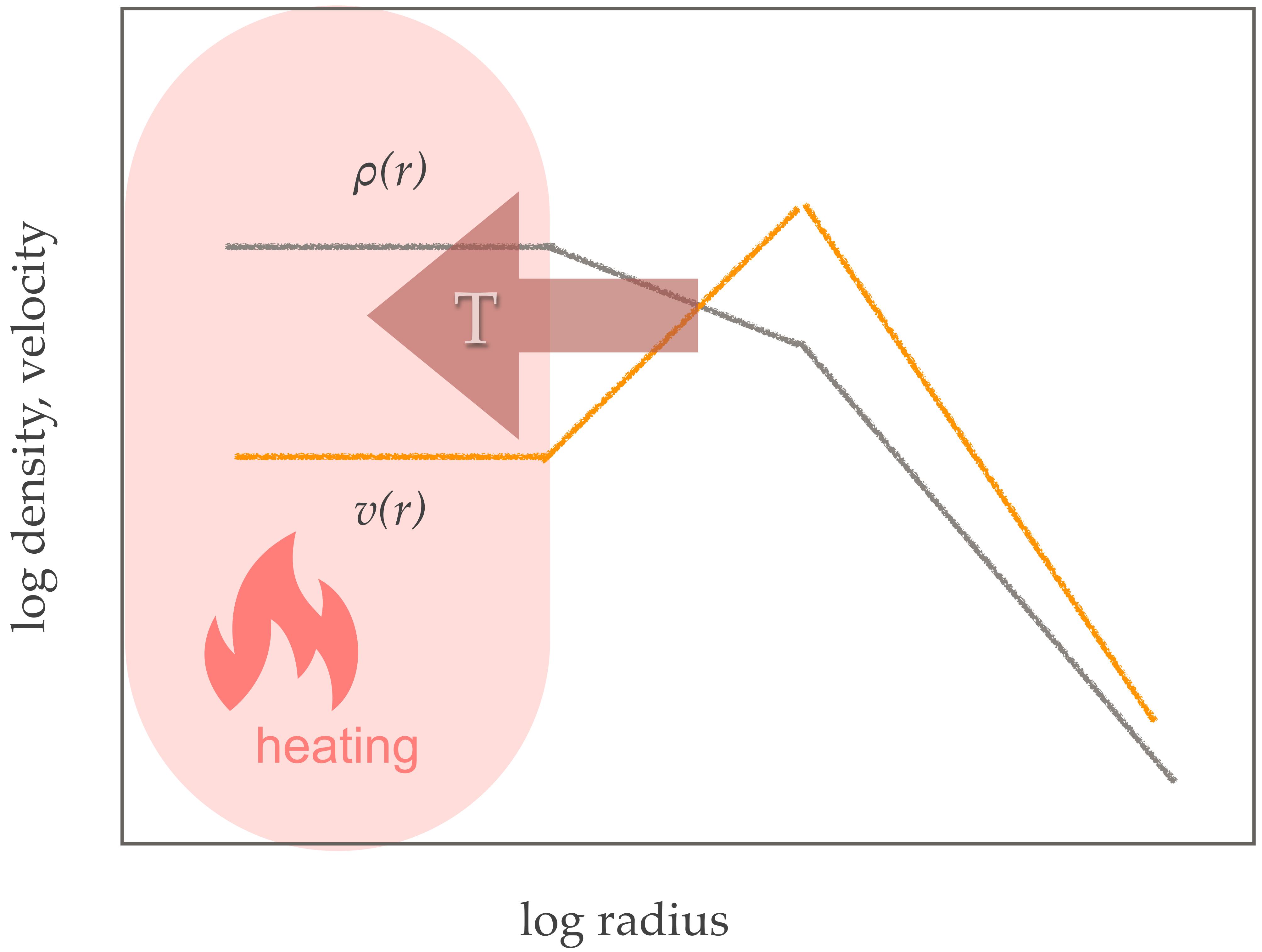
MM, PRL 2014

Core-cusp 2cDM physics

CDM, collisionless halo profile

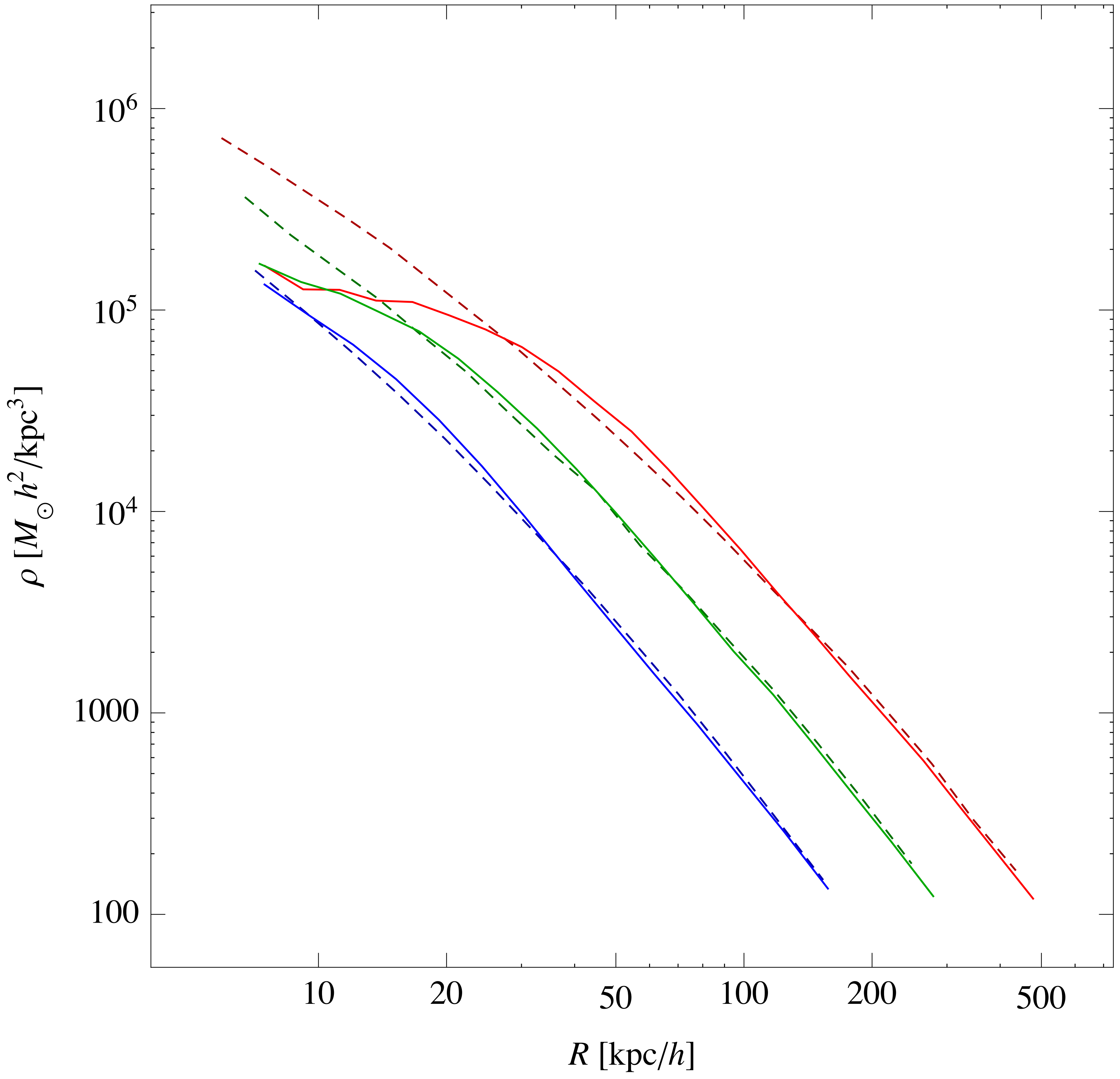


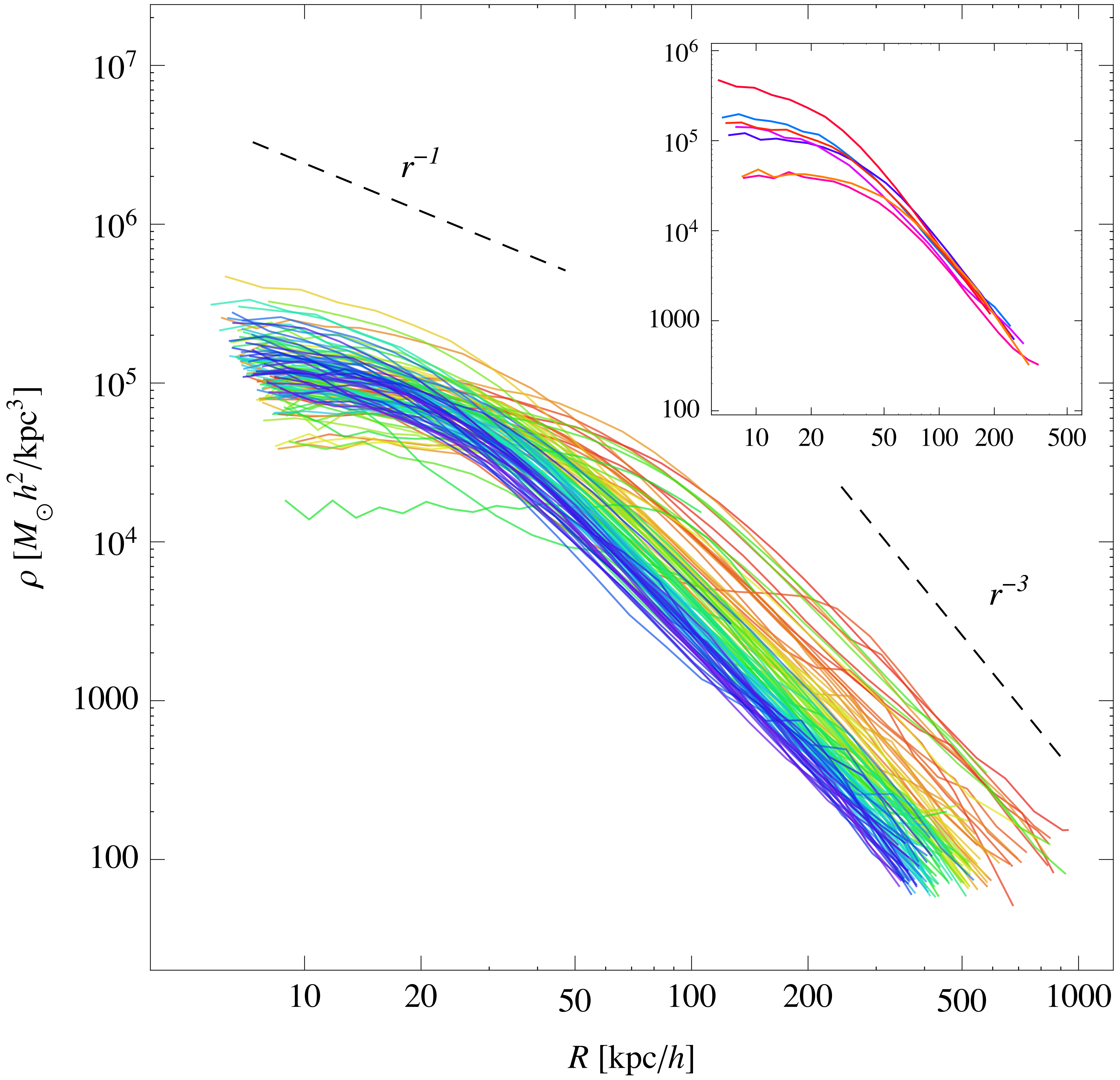
collisional (inelastic) heating
+ temperature equilibration



Core formation via collisions. Self-Interacting DM (SIDM): Spergel & Steinhardt, PRL 1999

Core-cusp in simulations

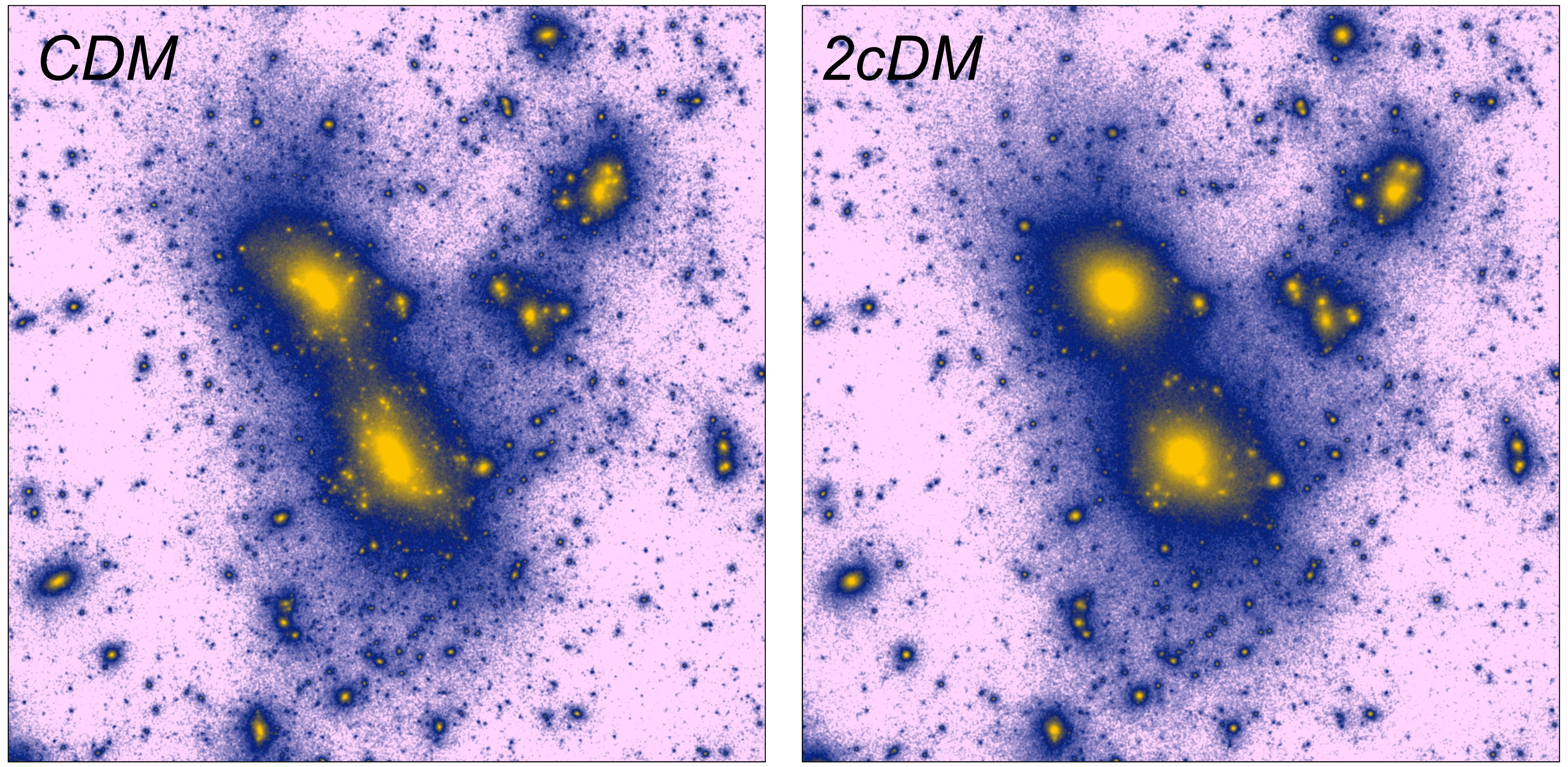




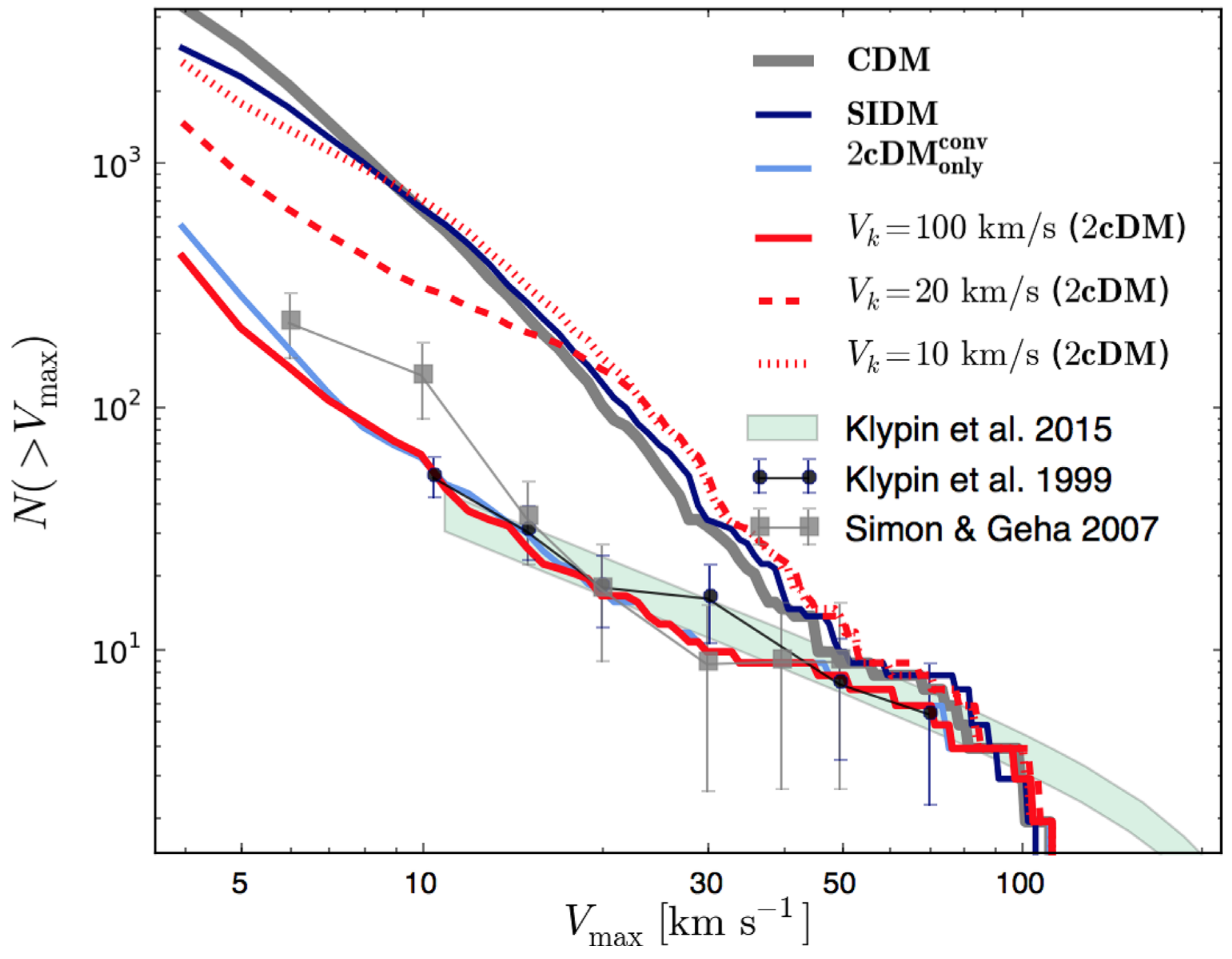
MM, PRL 2014

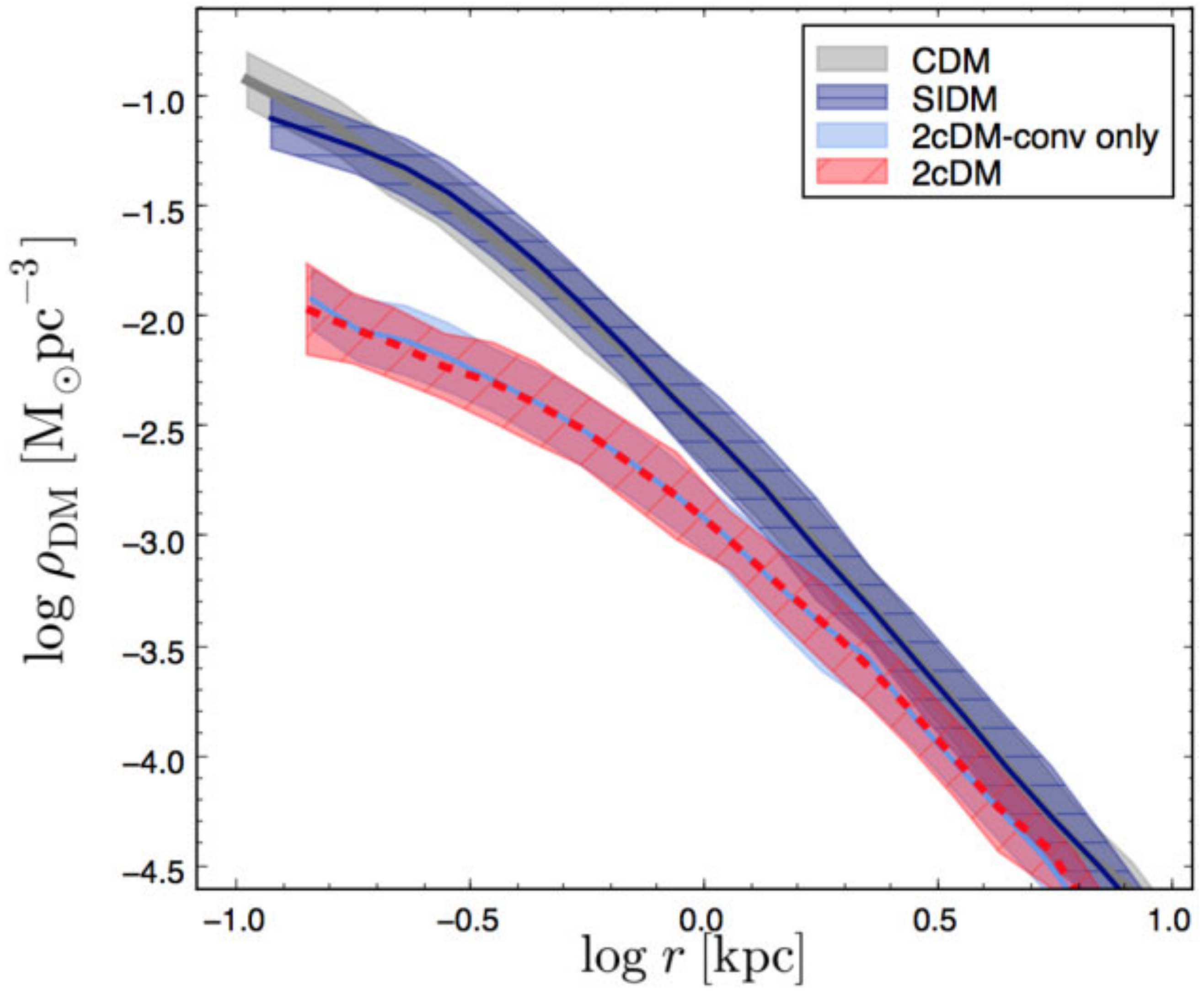
Summary: theory confirmed

MVM+ : **2cDM** simulations



Theory confirmed





MVM+, PRL 2014; MNRAS 2019, 2022

Key: cross-sections

cross-sections

$$\sigma_{(s_it_i)\to(s_it_i)} = \frac{\pi}{k_i^2}\sum_{l=0}^{\infty}(2l+1)\left|1-S^{(l)}_{(s_it_i)(s_it_i)}\right|^2,$$

$$\sigma_{(s_it_i)\to(s_ft_f)} = \frac{\pi}{k_i^2}\sum_{l=0}^{\infty}(2l+1)\left|S^{(l)}_{(s_it_i)(s_ft_f)}\right|^2,$$

parameterize

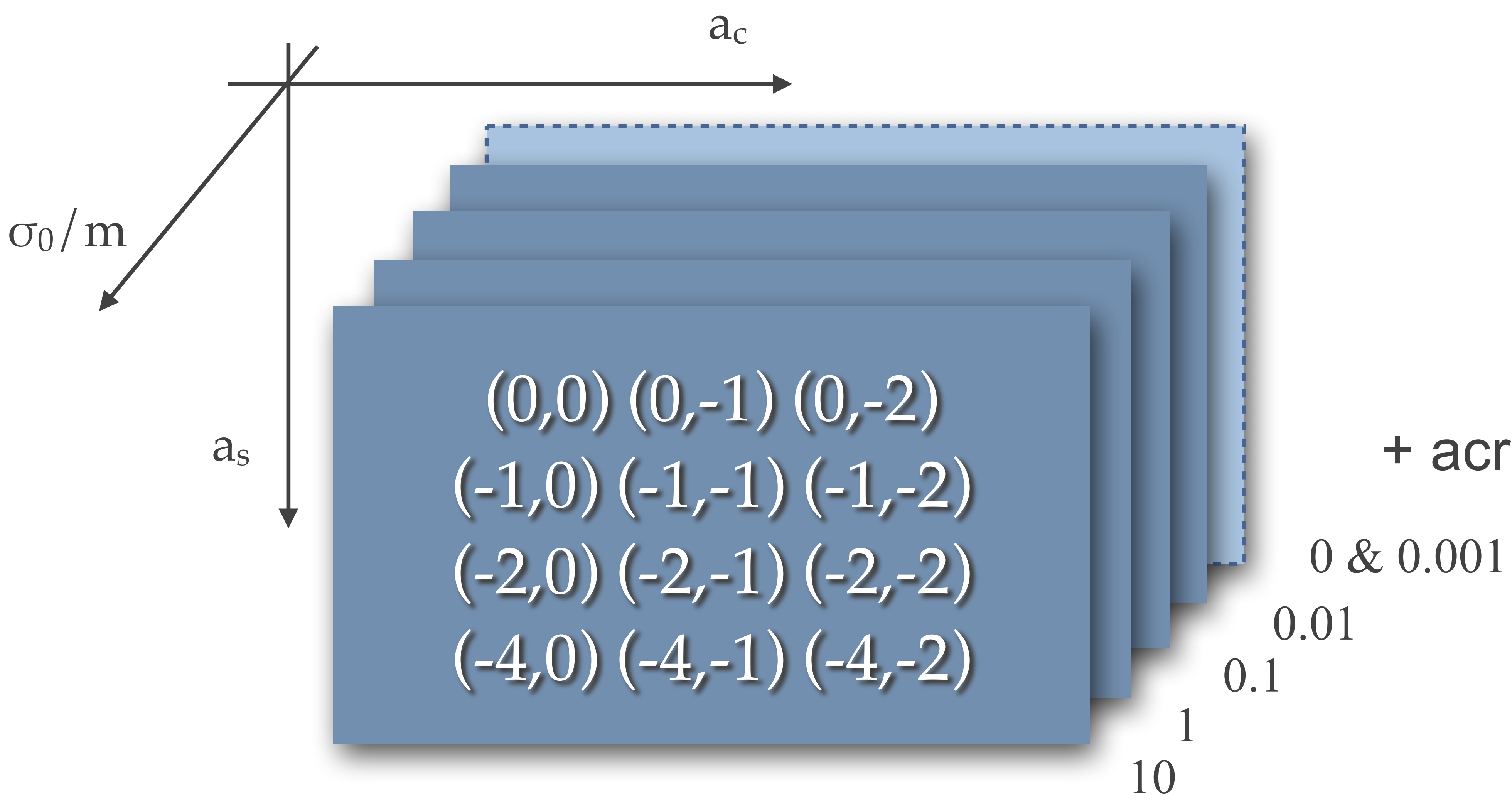
$$\boxed{\sigma_{i\to f}(v) = \begin{cases}\sigma_0(v/v_0)^{a_s} & \text{for scattering,}\\ \sigma_0(p_f/p_i)(v/v_0)^{a_c} & \text{for conversion}\end{cases}}$$

natural: $a_s=a_c$

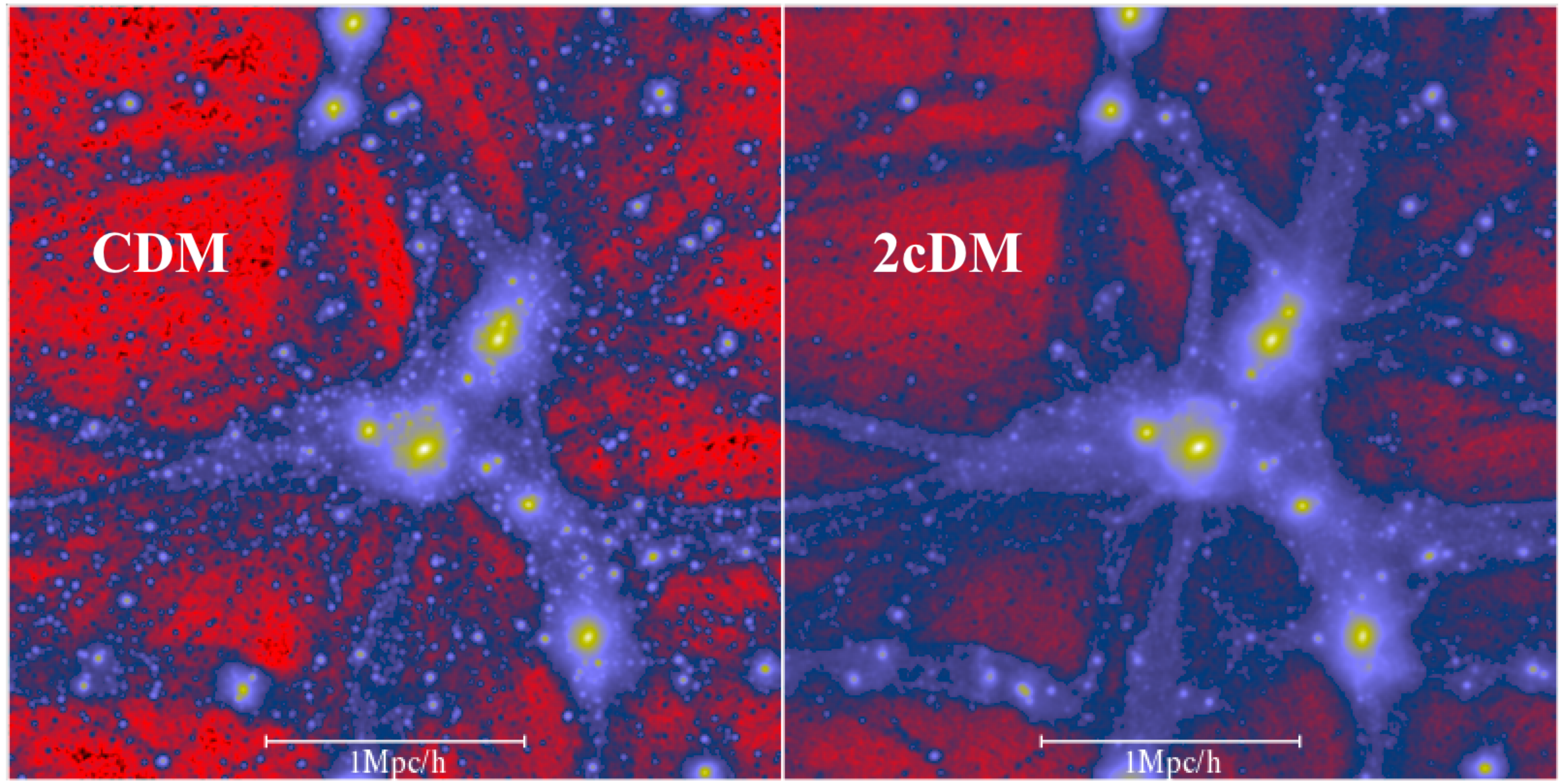
examples:	$a_s = a_c = 0$	"hard spheres" (*s*-wave scattering)
	$a_s = a_c = -1$	annihilation-like
	$a_s = a_c = -2$	maximum conversion probability
	$a_s = -4$	Rutherford-like

2cDM σ(v)-simulations

$$\sigma(v) = \begin{cases} \sigma(v/v_0)^{a_s} & \text{for} \quad \text{scattering} \\ \sigma(v/v_0)^{a_c} & \text{for} \quad \text{conversion} \end{cases}$$



(0,0) (0,-1) (0,-2)
(-1,0) (-1,-1) (-1,-2)
(-2,0) (-2,-1) (-2,-2)
(-4,0) (-4,-1) (-4,-2)



+ across halo mass scales: dwarfs -- clusters

10^7 10^{15}

over 120 different simulations

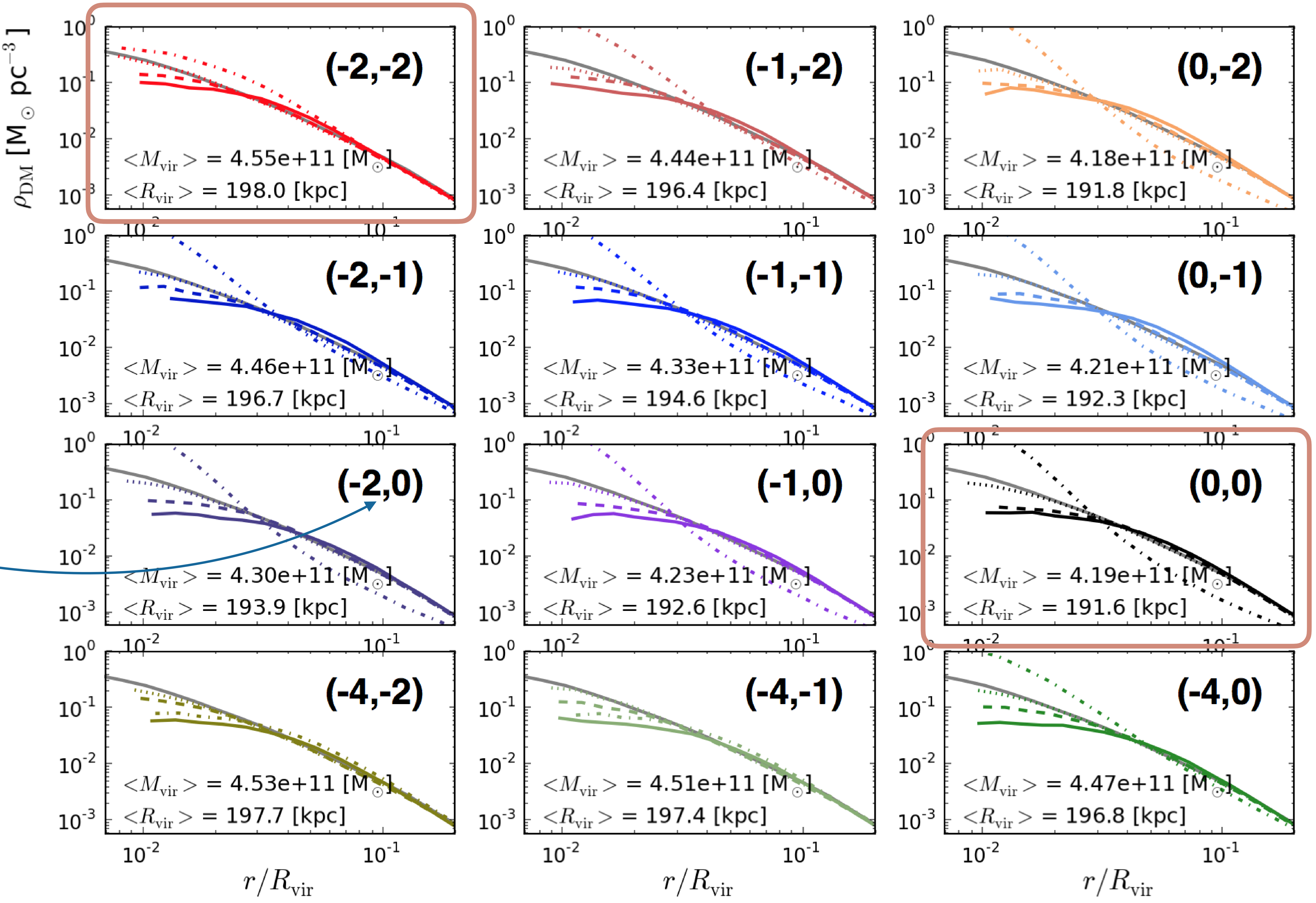
2cDM-σ(v) -- Profiles (MW-like)

$$\sigma(v) = \begin{cases} \sigma(v/v_0)^{a_s} & \text{for} \quad \text{scattering} \\ \sigma(v/v_0)^{a_c} & \text{for} \quad \text{conversion} \end{cases}$$

$\sigma = 0.01, \boxed{0.1, 1}$ and 10,

consistent with observations

(a_s, a_c)



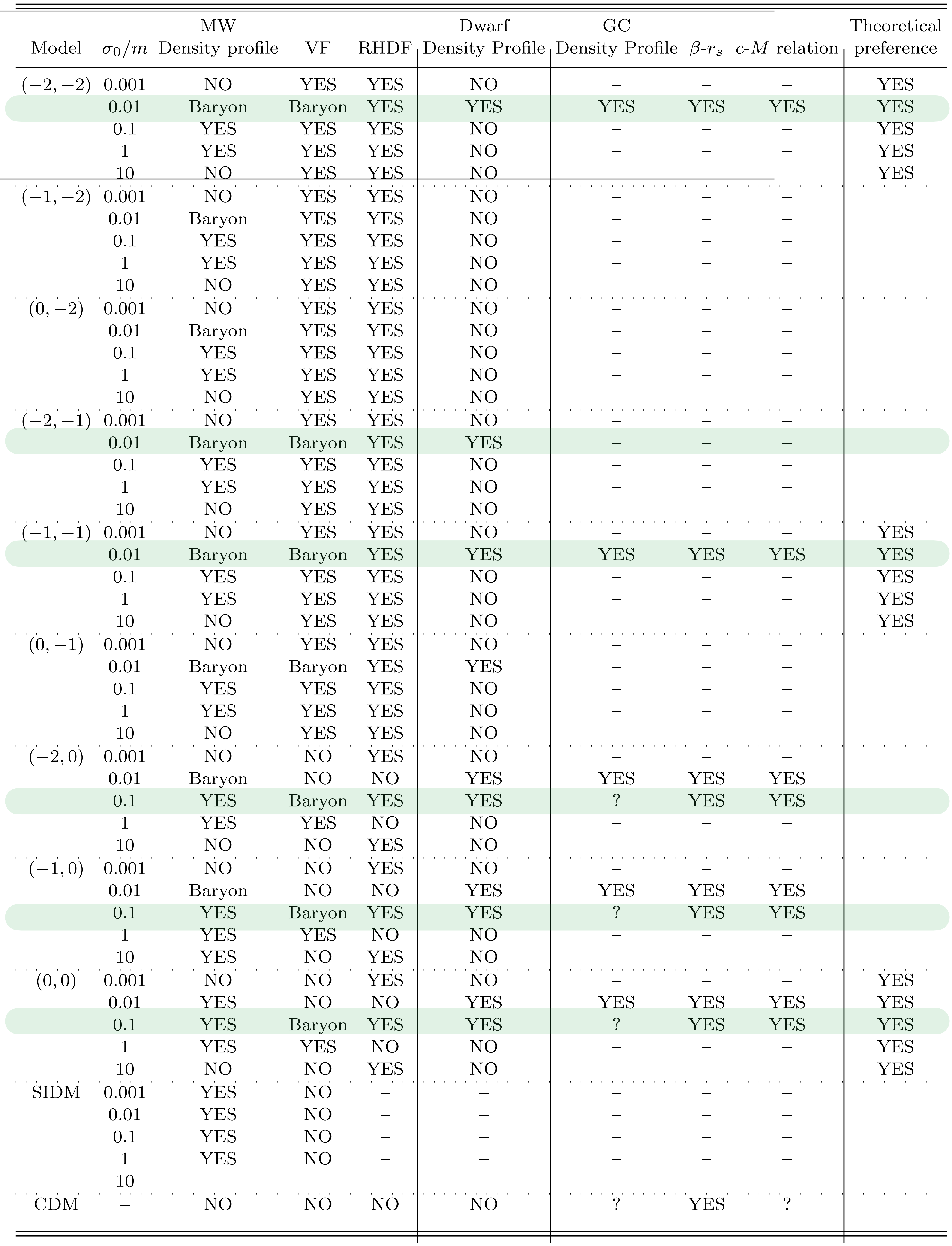
(Todoroki & MM, 2019,2020,2022)

2cDM summary

Some 2cDM models*
simultaneously resolve:

- Substructure Problem
- TBTF problem
- Core/cusp problem across halo mass scales from dwarfs to clusters
- Radial distribution of dwarfs (problem?)

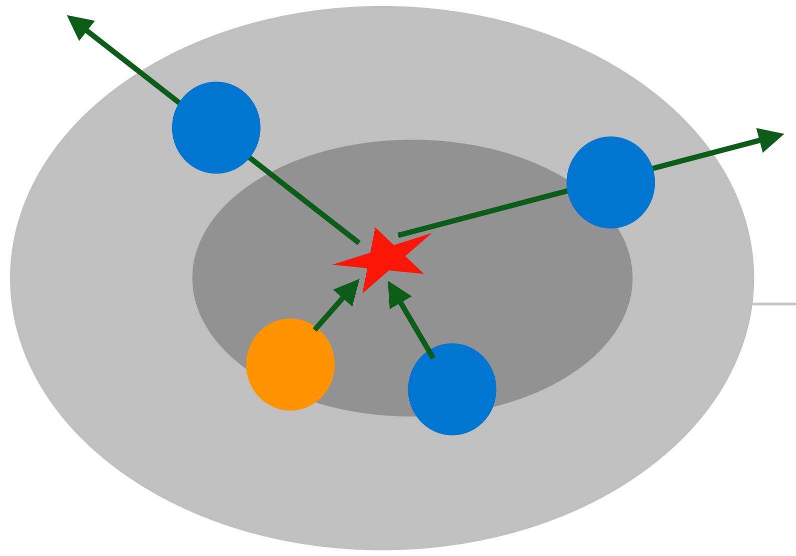
$\sigma(v) \sim 1(?)...0.1...0.01$
$(a_s, a_c) = (0,0), (-2,-2)$ -- natural
$\Delta m/m \sim 10^{-8} \Leftrightarrow v_k \sim 50\text{-}100$ km/s



Model	σ_0/m	MW Density profile	VF	RHDF	Dwarf Density Profile	GC Density Profile	β-r_s	c-M relation	Theoretical preference
$(-2,-2)$	0.001	NO	YES	YES	NO	–	–	–	YES
	0.01	Baryon	Baryon	YES	YES	YES	YES	YES	YES
	0.1	YES	YES	YES	NO	–	–	–	YES
	1	YES	YES	YES	NO	–	–	–	YES
	10	NO	YES	YES	NO	–	–	–	YES
$(-1,-2)$	0.001	NO	YES	YES	NO	–	–	–	
	0.01	Baryon	YES	YES	NO	–	–	–	
	0.1	YES	YES	YES	NO	–	–	–	
	1	YES	YES	YES	NO	–	–	–	
	10	NO	YES	YES	NO	–	–	–	
$(0,-2)$	0.001	NO	YES	YES	NO	–	–	–	
	0.01	Baryon	YES	YES	NO	–	–	–	
	0.1	YES	YES	YES	NO	–	–	–	
	1	YES	YES	YES	NO	–	–	–	
	10	NO	YES	YES	NO	–	–	–	
$(-2,-1)$	0.001	NO	YES	YES	NO	–	–	–	
	0.01	Baryon	Baryon	YES	YES	–	–	–	
	0.1	YES	YES	YES	NO	–	–	–	
	1	YES	YES	YES	NO	–	–	–	
	10	NO	YES	YES	NO	–	–	–	
$(-1,-1)$	0.001	NO	YES	YES	NO	–	–	–	YES
	0.01	Baryon	Baryon	YES	YES	YES	YES	YES	YES
	0.1	YES	YES	YES	NO	–	–	–	YES
	1	YES	YES	YES	NO	–	–	–	YES
	10	NO	YES	YES	NO	–	–	–	YES
$(0,-1)$	0.001	NO	YES	YES	NO	–	–	–	
	0.01	Baryon	Baryon	YES	YES	–	–	–	
	0.1	YES	YES	YES	NO	–	–	–	
	1	YES	YES	YES	NO	–	–	–	
	10	NO	YES	YES	NO	–	–	–	
$(-2,0)$	0.001	NO	NO	YES	NO	–	–	–	
	0.01	Baryon	NO	NO	YES	YES	YES	YES	
	0.1	YES	Baryon	YES	YES	?	YES	YES	
	1	YES	YES	NO	NO	–	–	–	
	10	NO	NO	YES	NO	–	–	–	
$(-1,0)$	0.001	NO	NO	YES	NO	–	–	–	
	0.01	Baryon	NO	NO	YES	YES	YES	YES	
	0.1	YES	Baryon	YES	YES	?	YES	YES	
	1	YES	YES	NO	NO	–	–	–	
	10	YES	NO	YES	NO	–	–	–	
$(0,0)$	0.001	NO	NO	YES	NO	–	–	–	YES
	0.01	YES	NO	NO	YES	YES	YES	YES	YES
	0.1	YES	Baryon	YES	YES	?	YES	YES	YES
	1	YES	YES	NO	NO	–	–	–	YES
	10	NO	NO	YES	NO	–	–	–	YES
SIDM	0.001	YES	NO	–	–	–	–	–	
	0.01	YES	NO	–	–	–	–	–	
	0.1	YES	NO	–	–	–	–	–	
	1	YES	NO	–	–	–	–	–	
	10	–	–	–	–	–	–	–	
CDM	–	NO	NO	NO	NO	?	YES	?	

(Todoroki & MM, 2019,2020,2022)

- History & Motivation
- Models (SIDM, etc)
- Overview of 2cDM
  - physics
  - cosmology
  - understanding



Substructure evaporation

assume profile

$$\rho(r) = \rho_0 \left(\frac{r}{R}\right)^{-\beta}$$

hydrostatic balance yields

$$v_{th}^2 = \frac{4\pi G \rho_0 R^\beta}{\beta(3-\beta)} r^{2-\beta}$$

mass-loss per radius

$$\frac{d\dot{M}}{dr} = 4\pi r^2 \dot{\rho} = 4\pi r^2 \dot{\rho}_0 \left(\frac{r}{R}\right)^{\boldsymbol{\lambda} = 1 - \frac{5}{2}\beta + a(1 - \frac{\beta}{2})}$$

$$\dot{\rho} = -(n\sigma v)\rho = -\rho^2 \left(\frac{\sigma}{m}\right) v = \rho_0 \left(\frac{r}{R}\right)^{-2\beta} \frac{\sigma_0}{m} \left(\frac{v}{v_0}\right)^a v$$

integrate to yield the total halo mass-loss

$$\dot{M} = \underbrace{\frac{3-\beta}{\lambda+3}\frac{\sigma_0 v_0}{m}\left[\frac{G}{v_0\beta}\left(\frac{4\pi\rho_0}{3-\beta}\right)^{1/3}\right]^{a+1}}_{\textit{just a constant}} \underbrace{\left(\frac{r_c}{R}\right)^{\lambda+3}}_{\textit{approximately constant}} M^{1+\frac{2}{3}(a+1)}$$

indep. of halo shape (beta)

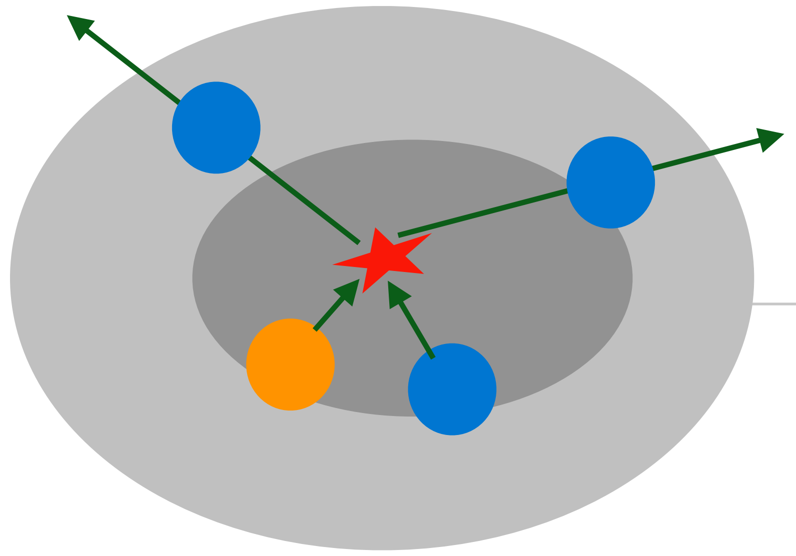
solution

$$\dot{M} = -|A| M^\xi$$

$$M_0 = \left[(1-\xi)At + M^{1-\xi}\right]^{1/(1-\xi)}$$

initial halo mass

final halo mass



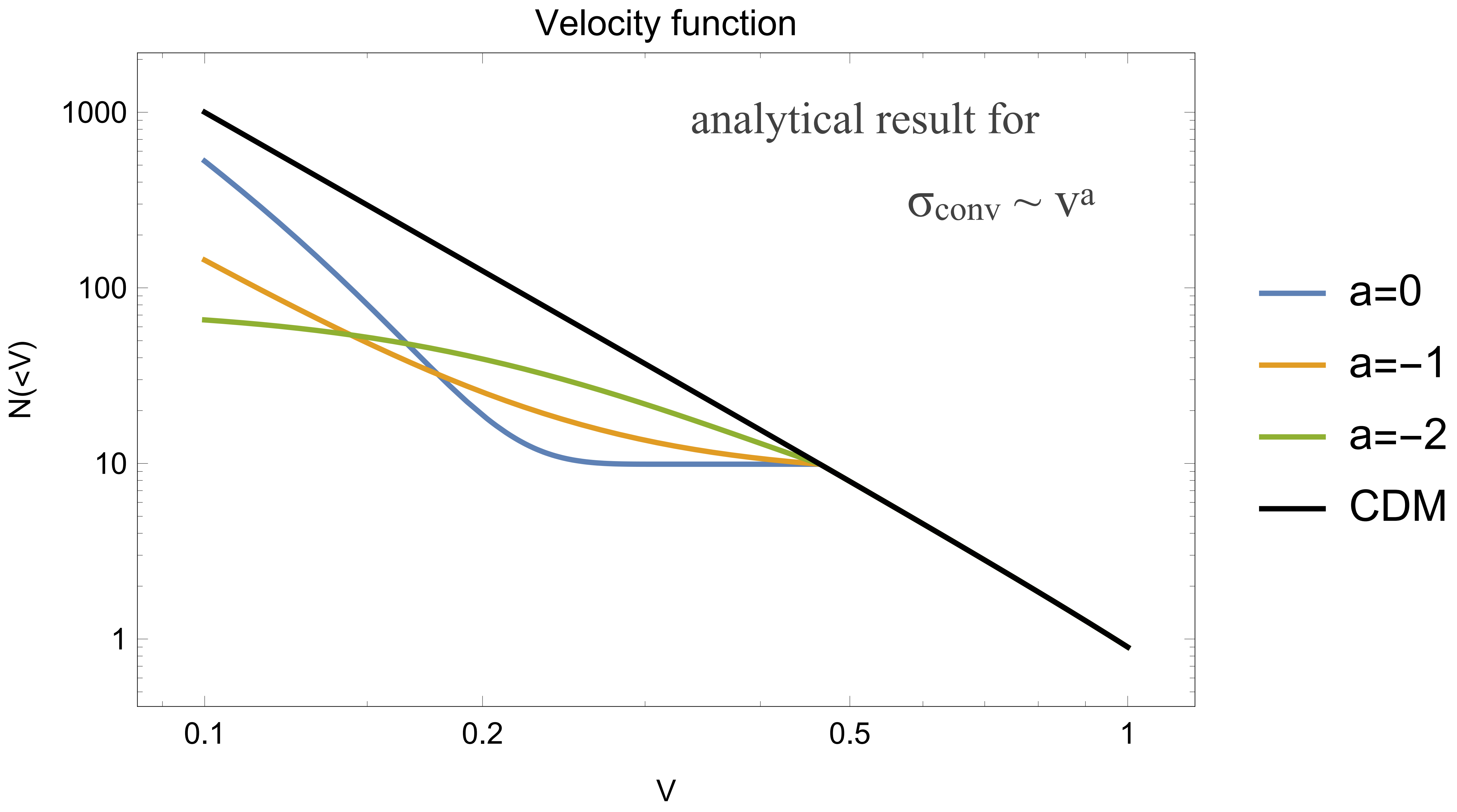
Substructure evaporation

mapping of old to new

$$M_0 = \begin{cases} \left(M^{-2/3} - \frac{2}{3}At\right)^{-3/2}, & a = 0 \\ Me^{At}, & a = -1 \\ \left(M^{2/3} + \frac{2}{3}At\right)^{3/2}, & a = -2 \end{cases}$$

New mass function given the old one $f(M_0)$ is $f(M_0) = f(M_0(M,t)) \equiv f(M,t)$

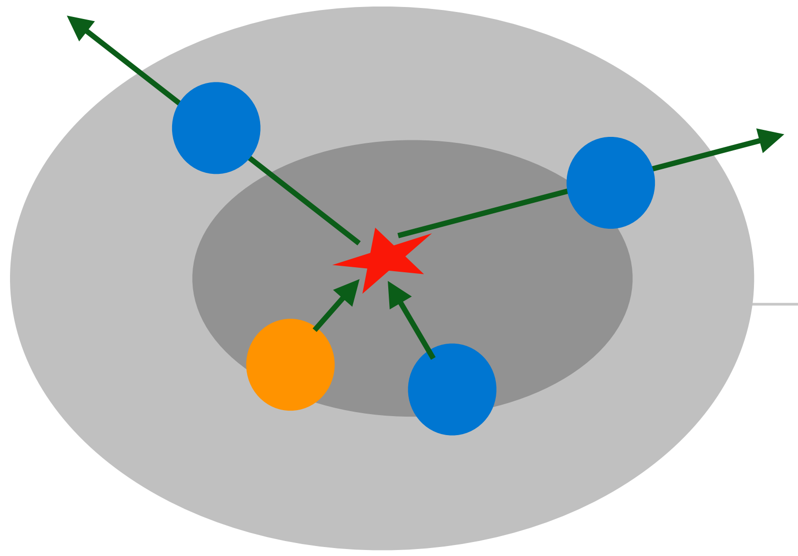
and similarly for the velocity function



analytical result for

$\sigma_{\mathrm{conv}} \sim v^a$

Evaporation resolves substructure & TBTF problems
Shape of mass function tells:
index a_c (conversion) and σ_0/m



Do halos evaporate completely?

abundance evolution eqns.
$$\dot n_h = -(\sigma_{hh}v)\, n_h^2 - (\sigma_{hl}v)\, n_h n_l,$$
$$\dot n_l = -(\sigma_{hl}v)\, n_h n_l,$$

then $\dfrac{d\,n_h}{d\,n_l} = \dfrac{\sigma_{hh}\, n_h}{\sigma_{hl}\, n_l} + 1$

solution $\dfrac{n_h(t)}{n_{h,0}} = \left(\dfrac{n_{l,0}/n_{h,0}}{1-R}\right)\left(\dfrac{n_l(t)}{n_{l,0}}\right) + \left(1 - \dfrac{n_{l,0}/n_{h,0}}{1-R}\right)\left(\dfrac{n_l(t)}{n_{l,0}}\right)^R$

$R = \sigma_{hh}/\sigma_{hl}$

asymptotically $n_h(\infty) \to 0,\ n_l(\infty) \to n_{l,\infty}$

$$\frac{n_{l,\infty}}{n_{l,0}} = \left[1 - \frac{n_{h,0}}{n_{l,0}}(1-R)\right]^{\frac{1}{1-R}}$$

complete evaporation is possible when

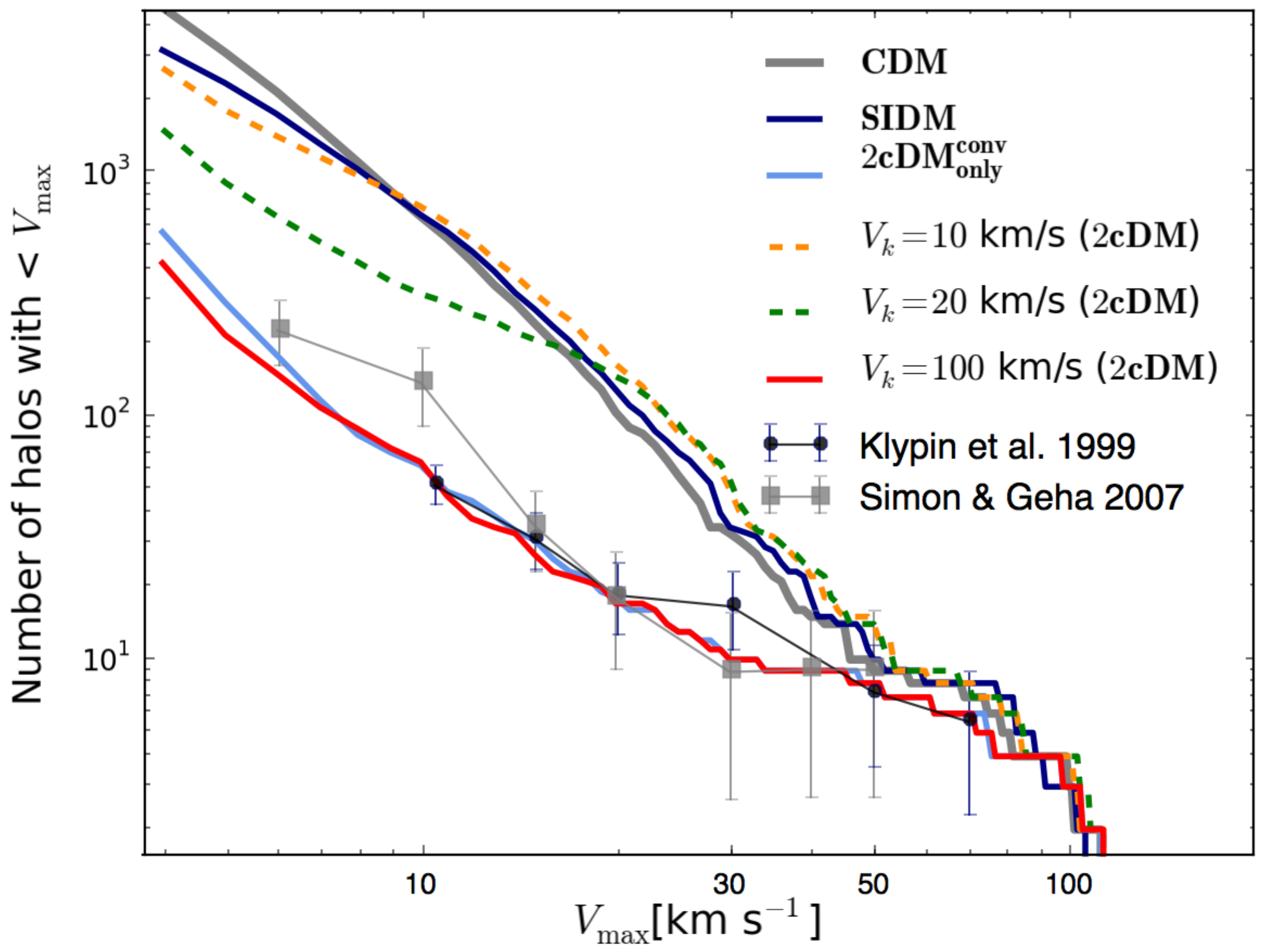
$$\boxed{\frac{n_{l,0}}{n_{h,0}} \le 1 - \frac{\sigma_{hh}}{\sigma_{hl}}}$$

MM, J Phys A 2010; JCAP 2014

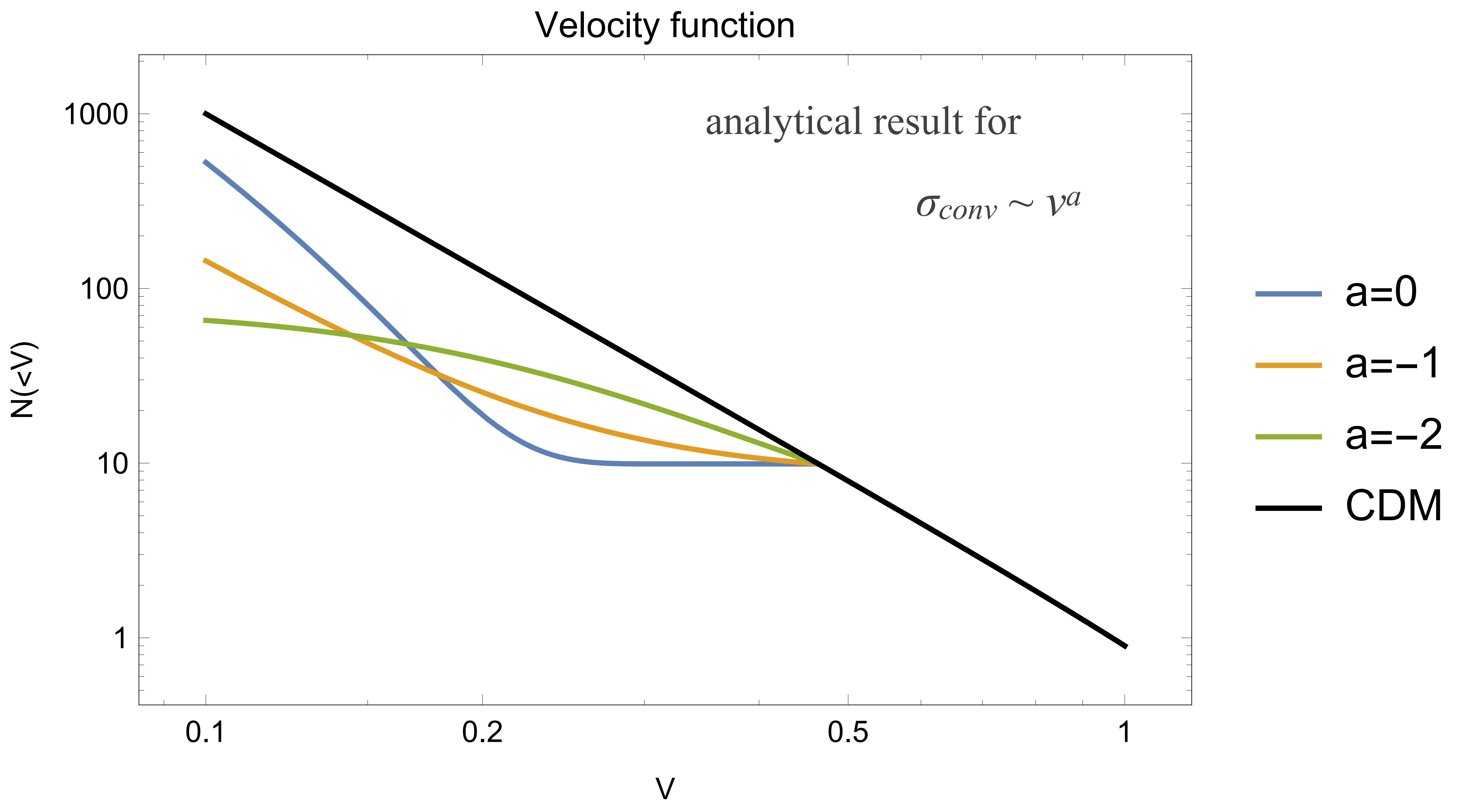
Message 1

SUBSTRUCTURE

simulations vs data - fit well

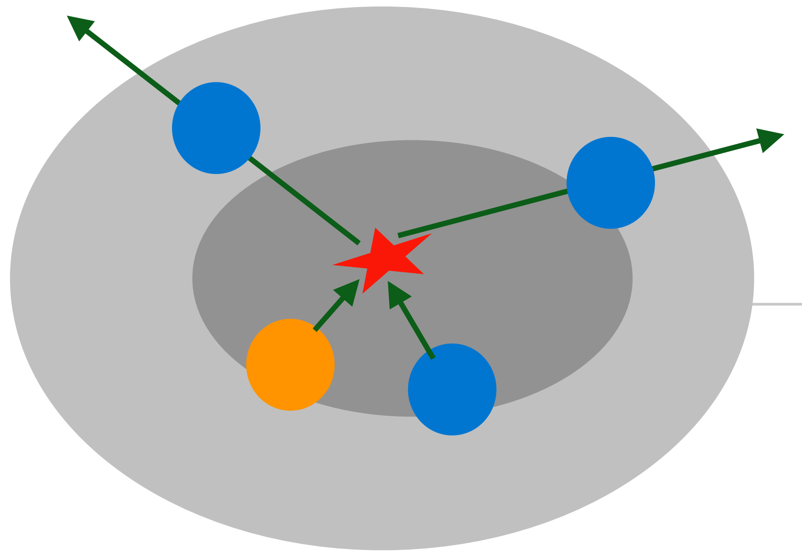


theory with σ(v) can explains TBTF and *missing* and *non-missing* satellites

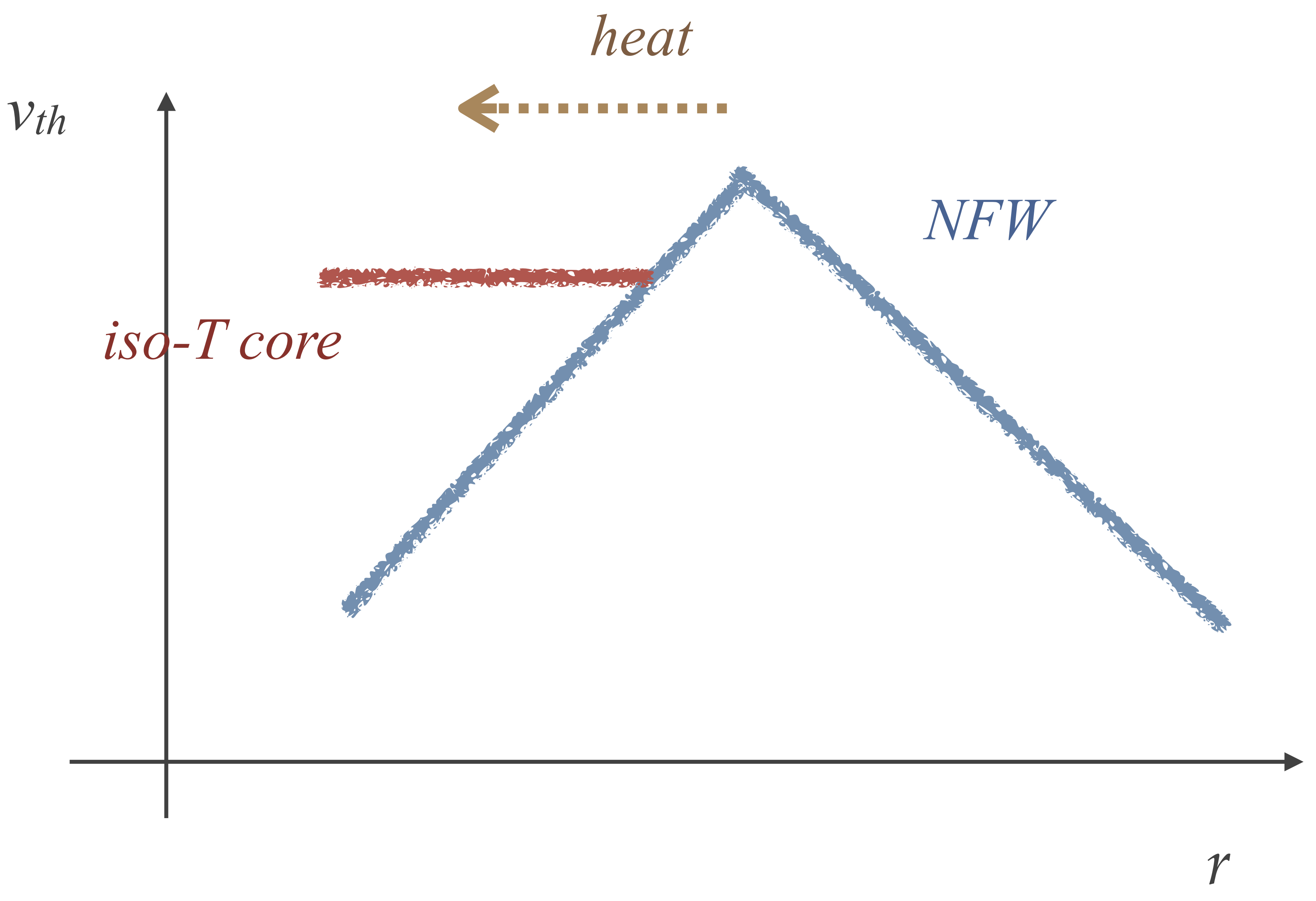


Wide parameter region allowed:
σ(v) ~ 1...0.1...0.01 – consistent with all constraints
$\Delta m/m \sim 10^{-8}$ ⇔ $v_k \sim$ 50-100 km/s

(Todoroki & MM, 2019,2020,2022)



Cusp softening



number of interactions per particle

$$N_{int} = n\sigma v t_H = \rho_{\rm vir}\frac{\sigma_0}{m} t_{\rm H} v_0 \left(\frac{V_{\rm vir}}{v_0}\right)^{a+1} \left(\frac{r_c}{R_{\rm vir}}\right)^{a+1-\frac{\beta}{2}(a+3)}$$

$$\rho_{\rm vir} = \frac{(3-\beta)M_{\rm vir}}{4\pi R_{\rm vir}^3}$$

$$V_{\rm vir}^2 = GM_{\rm vir}/R_{\rm vir}$$

$$N_{\rm vir} \equiv \rho_{\rm vir}\frac{\sigma_0}{m} V_{\rm vir} t_{\rm H}$$

core radius

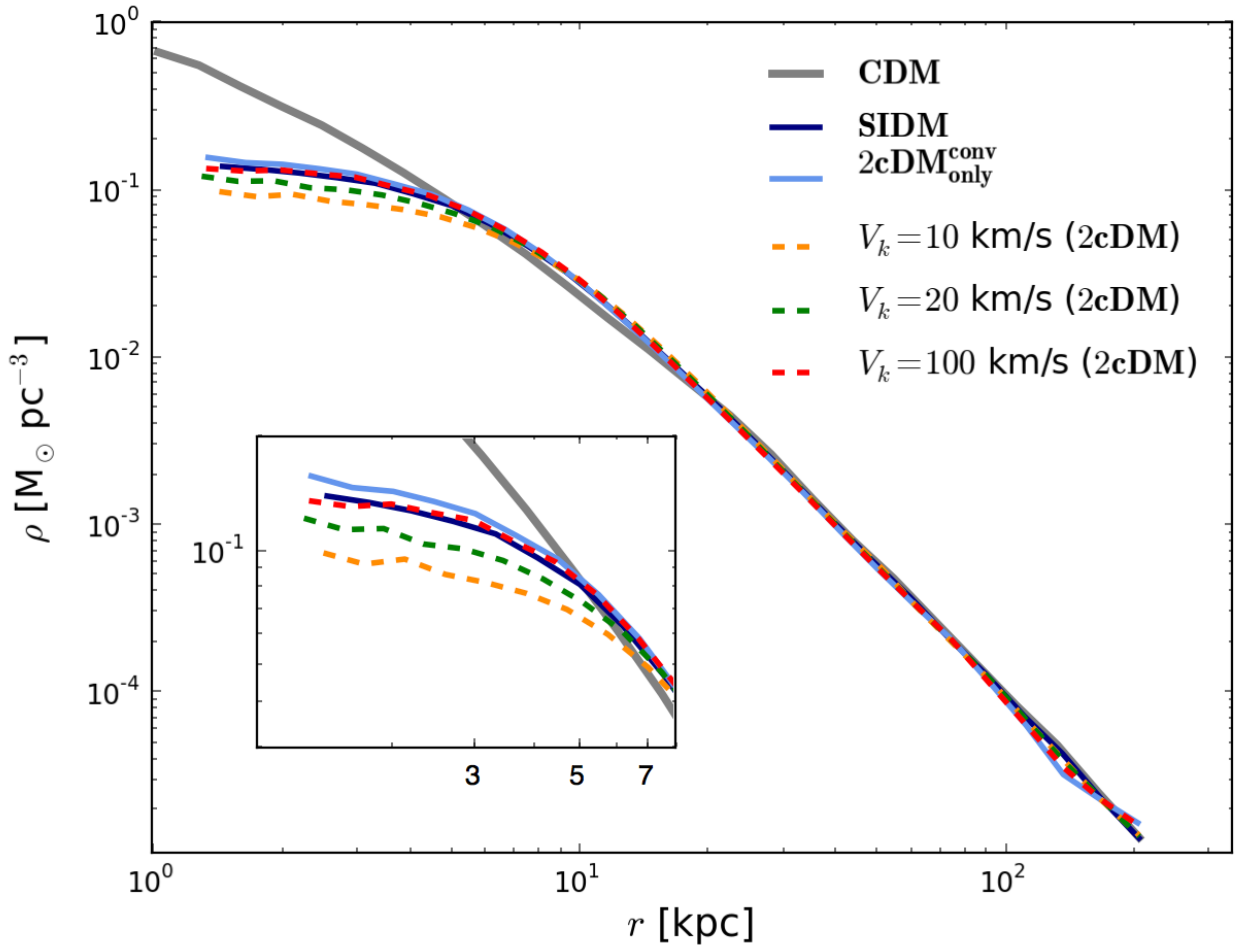
$$\boxed{\frac{r_c}{R_{\rm vir}} \simeq \left[\left(\frac{\text{a few}}{N_{\rm vir}}\right)\left(\frac{V_{\rm vir}}{v_0}\right)^{-a}\right]^{-\xi} \propto \sigma_0^{\xi}}$$

$$\xi = \frac{2}{\beta(a+3)-2(a+1)}$$

Scattering resolves core-cusp problem
Core size tells:
σ_0/m and index a_s (scattering)

Message 2

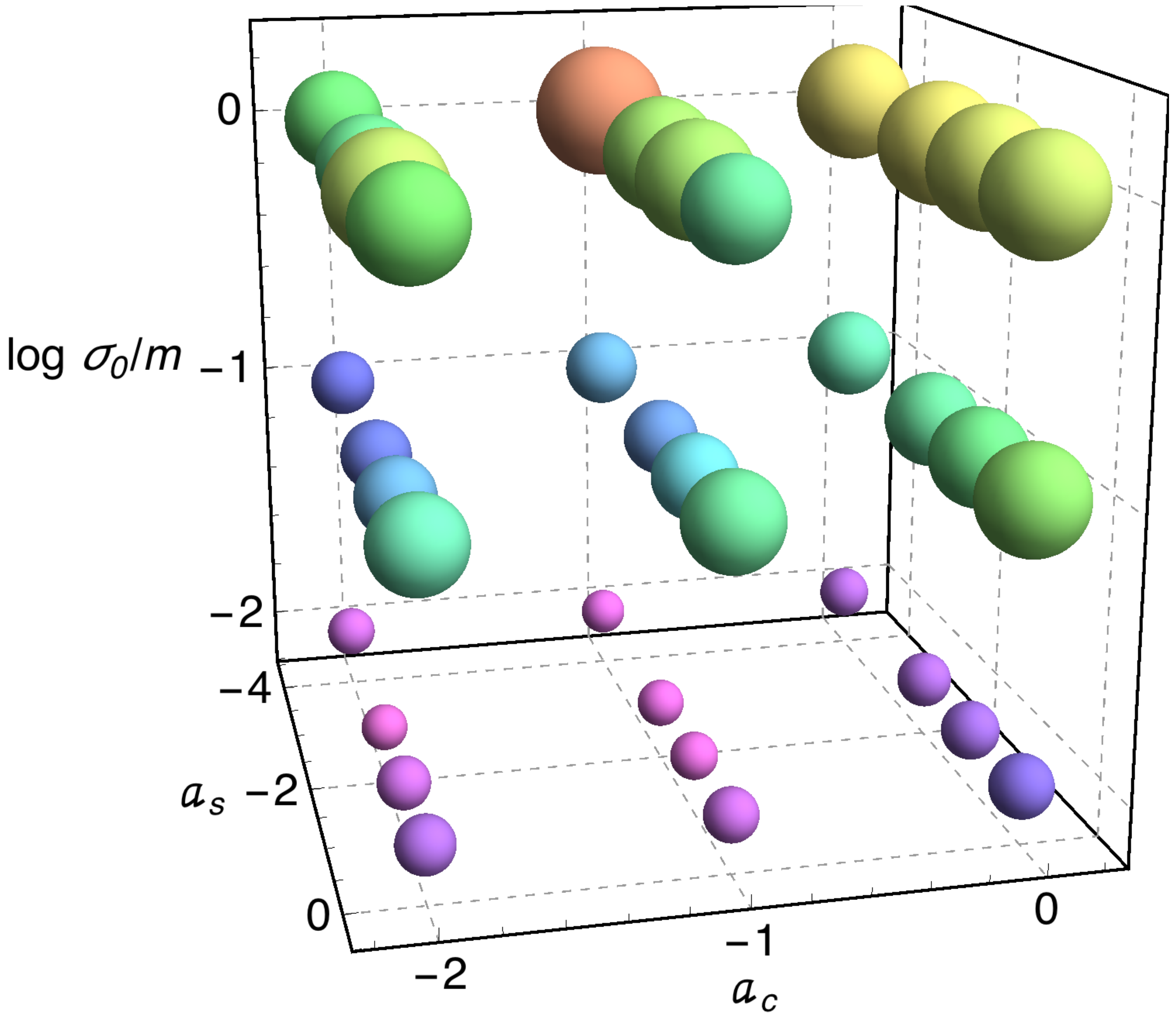
PROFILES



Wide parameter region allowed:
σ(v) ~ 1...0.1...0.01 – consistent with all constraints
Δm/m ~ 10^{-8} ⇔ v_k ~ 50-100 km/s

Resolves core-cusp problem.
Core size tells:
σ_0/m and indexes a_s , a_c

core sizes from fits to simulated halos

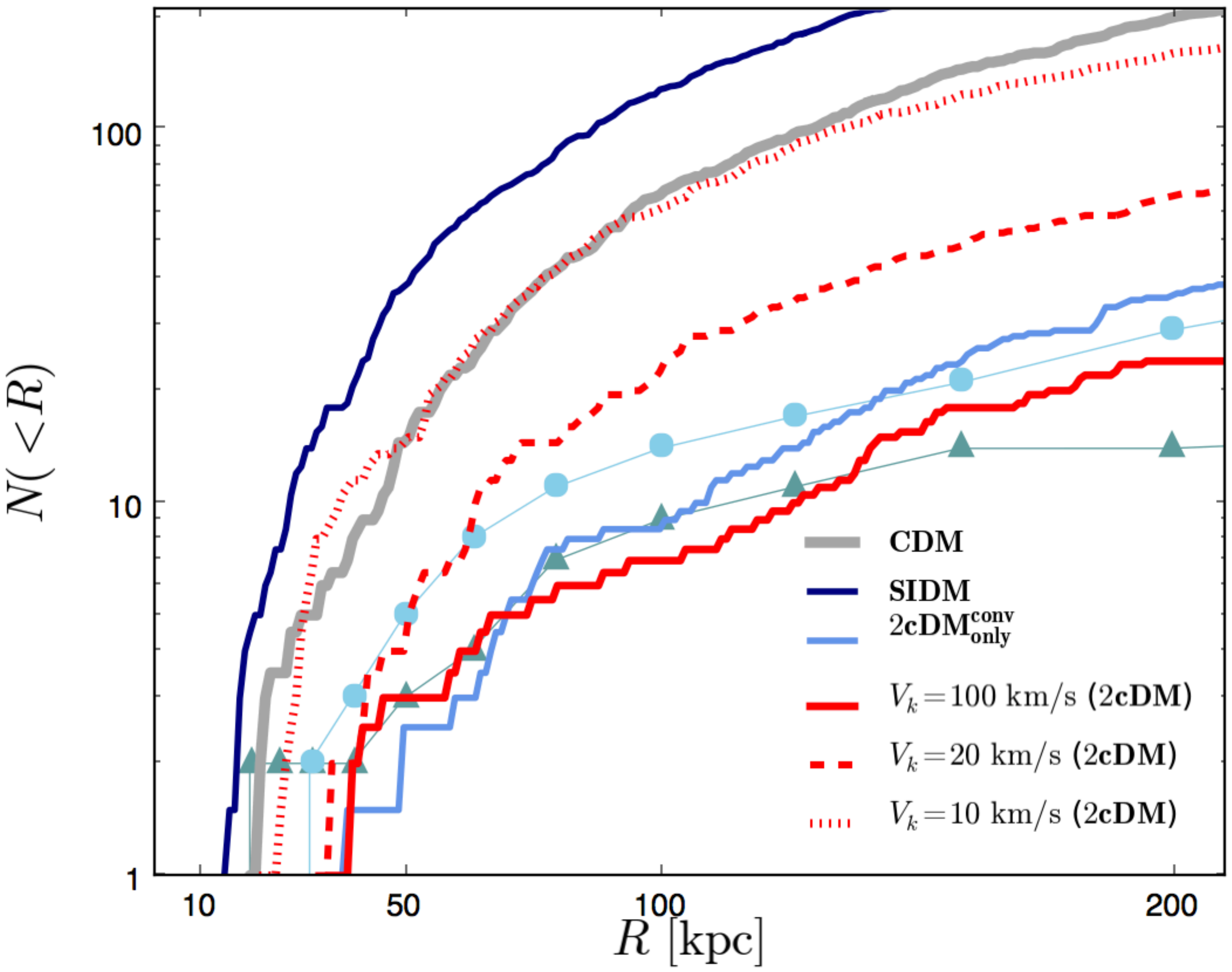


(Todoroki & MM, 2019,2020,2022)

Message 3

DISTRIBUTION of SATELLITES

Resolves substructure radial distribution
Shape of function depends on all parameters



Wide parameter region allowed:
$\sigma(v) \sim 1...0.1...0.01$ – consistent with all constraints
$\Delta m/m \sim 10^{-8}$ ⇔ $v_k \sim$ 50-100 km/s

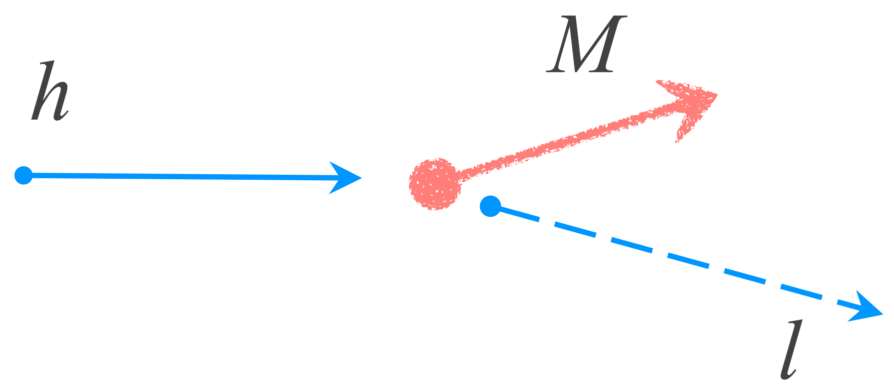
(Todoroki & MM, 2019,2020,2022)

2cDM predictions

(MVM, JPhysA 2010; JCAP, 2014; PRL 2014)

direct detection

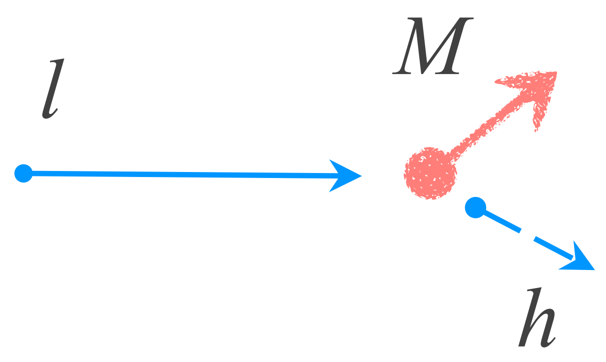
"inelastic recoil"



$+\Delta mc^2$

$\Delta m/m \sim 10^{-7} \ldots 10^{-8}$

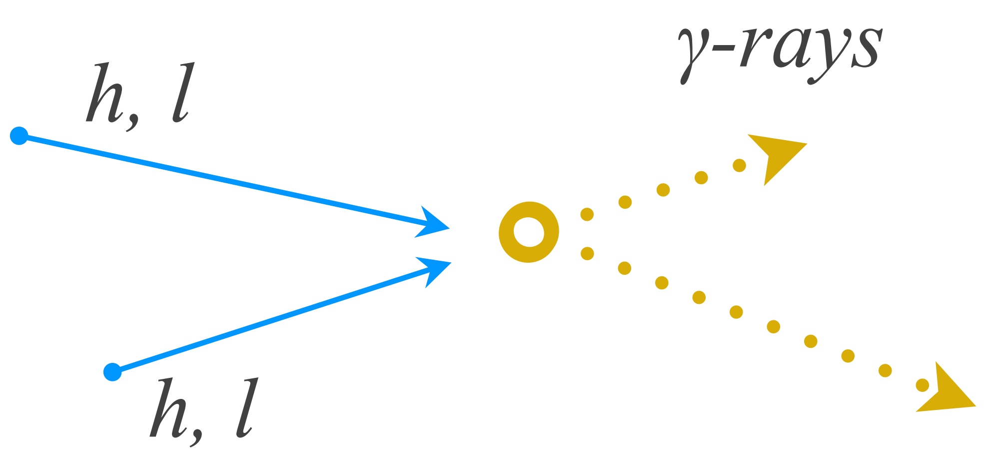
example, m ~ tens GeV, ΔE ~ few keV



$-\Delta mc^2$

indirect detection

"γ-ray annihilation
line triplet"



$E_\gamma = mc^2 \pm \frac{1}{2}\Delta mc^2$

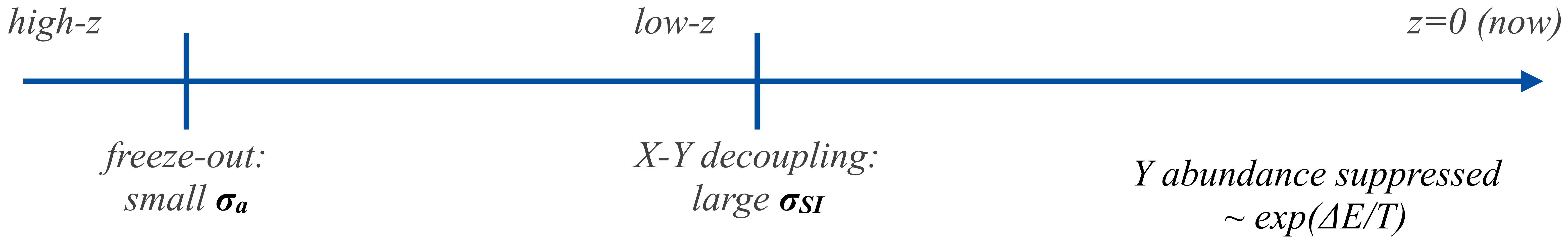
example, 100 GeV ± ~ keV

2cDM vs other inelastic

2cDM looks like any multi-species/composite DM -- allows reactions Y→X

excited, inelastic, exothermal, dark photon, boosted...

early universe "catastrophe"



high-z

low-z

z=0 (now)

freeze-out:
small $\boldsymbol{\sigma_a}$

X-Y decoupling:
large $\boldsymbol{\sigma_{SI}}$

Y abundance suppressed
$\sim exp(\Delta E/T)$

Not a problem for 2cDM: conversions do not occur before structure formation starts (needed to separate mass states)

$$\boxed{\sigma^{\rm fst}_{\rm conv} \sim (\Delta m/m)^4 \sigma_{\rm conv}}$$

before structure formation

after structure formation

(MM, JCAP 2014)

Outcomes

some fine tuning of σ to Hubble time

what about σ(v) ?

too few
collisions -
uninteresting

$0.1 < \sigma/m <$ few cm^2/g

too many collisions -
gravithermal
collapse

maybe it is needed?

large $\sigma/m \sim 1\ \mathrm{cm}^2/\mathrm{g} \sim 1$ barn/GeV

a fifth force?

high degeneracy $\Delta m/m \sim 10^{-7.5}$

possibly needs light mediator

What about low-degeneracy?
Boosted DM =>
See recent KC, et al paper.

small mass splitting:
Y. Zhang, Phys. Dark Univ. 15 (2017)
K. Schutz, T.R. Slatyer, JCAP 01 (2015) 021
J. Kopp et al. JHEP 12 (2016) 033
M. Baumgart et al. JHEP 0904:014,2009
....

2cDM model parameters

Theory

matrix elements	$V_{\alpha\alpha}\ ,\ V_{\alpha\beta}\ ,\ V_{\beta\beta}$
mixing angle	θ
masses of eigenstates	$m_1\ ,\ m_2$
initial composition	*set by mixing*

Simulations

mass degeneracy	$\Delta m/m$
cross-sections	$\sigma_e/m\ ,\sigma_i/m$
what about σ(v) ?	$\sigma_0(v/v_0)^{a_e},\ \sigma_0(v/v_0)^{a_i}$

one can simplify by makings assumptions:

$$\boxed{V = U_2^\dagger \tilde{V} U_2 = \begin{pmatrix} A & E & E & D \\ E & C & D & F \\ E & D & C & F \\ D & F & F & B \end{pmatrix}}$$

$$V_{\alpha\beta} = V_{\beta\alpha} = 0,$$

$$V_{\beta\beta} = -V_{\alpha\alpha}$$

$$\theta = \pi/4.$$

then, we can have *the most "minimal" model*

$$V = V_{\alpha\alpha}\begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

Next decade:

A paradigm shift?

CDM

- Collisionless
- Single species
- Cold

Inelastic DM

- Self-interacting
- Multi-species
- Exothermal/Endothermal
- "Non-minimal"
- LSS evolution with z
- Ly-alpha forest imprint
- Early universe (?)

Need:

- Data!
  - Cosmological observations
  - Indirect detection
  - Direct detection
- Comprehensive realistic simulations w. baryons
- Theory


FOR ATTENTION WE THANK
THE END OF PRESENTATION THIS IS

end of presentation