Heavy flavor in high-energy nuclear collisions

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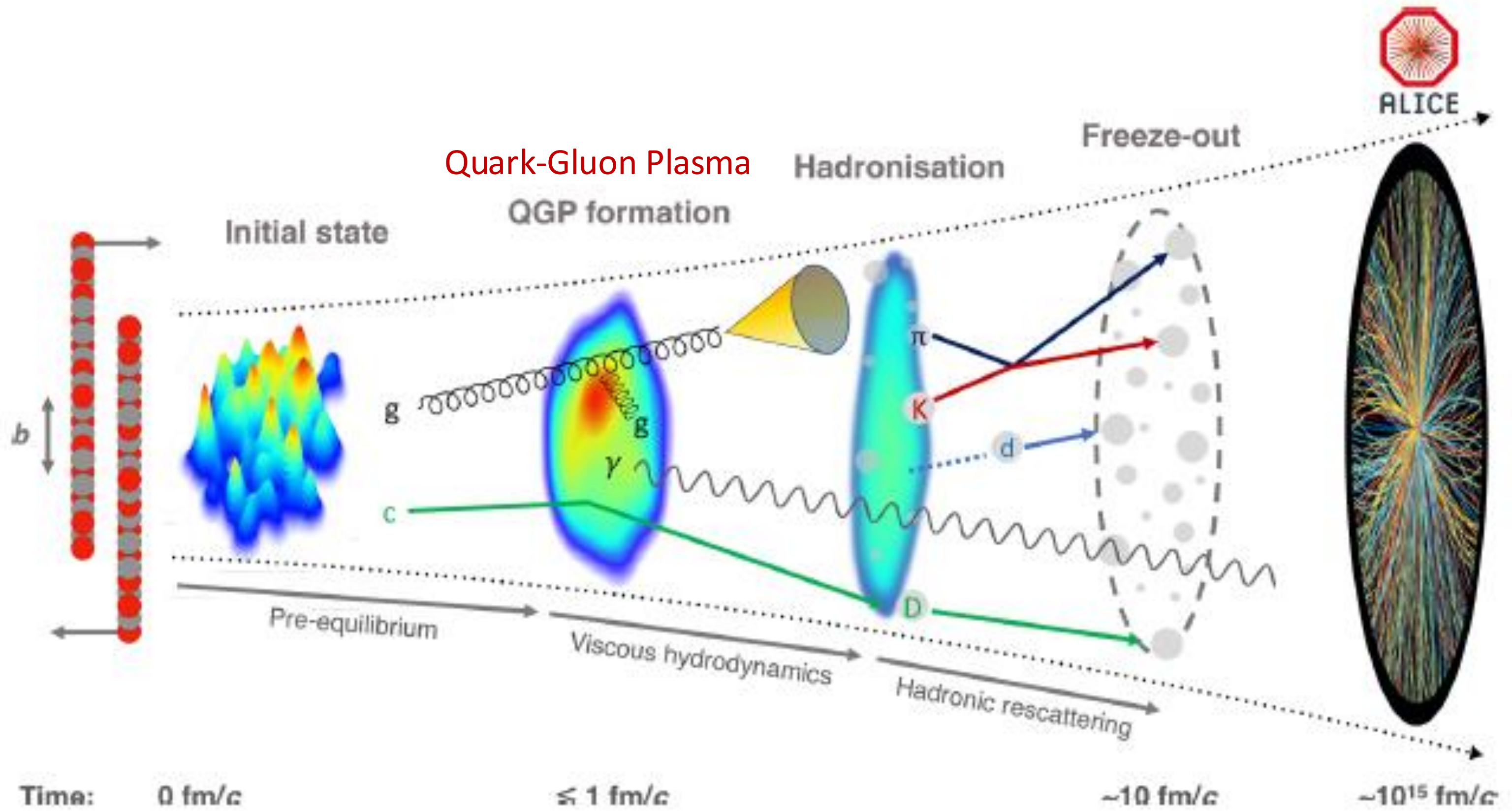
Relativistic heavy-ion collisions



ALICE (2024)

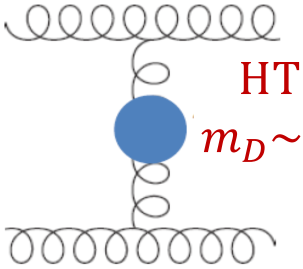
- Transport properties and interactions in hot QCD matter
- Heavy quarks($m \gg T$) created in hard processes at the beginning, conserved
- The relaxation time is longer by $\frac{m}{T}$: no thermal equilibrium in QGP

Effective kinetic theory

- High-temperature weakly coupled QCD plasmas: $g \ll 1 \quad (\alpha_s = \frac{g^2}{4\pi})$
- Hard particle(p$\sim T$) carry most stress-energy tensor: main contribution to thermodynamics
- Soft field (p$\sim gT \ll T$)
- Hard-thermal-loop(HTL) perturbative theory
- Leading-order effective kinetic theory for partons:
  Boltzmann equation for a single-particle phase-space distribution

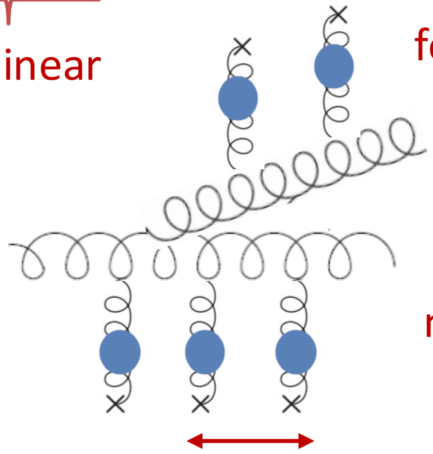
$$\left(\frac{\partial}{\partial t} + \boldsymbol{v} \cdot \frac{\partial}{\partial \boldsymbol{x}}\right) f(\boldsymbol{p}) = \underbrace{C_{2\leftrightarrow 2}[f]}_{\text{elastic}} + \underbrace{C_{1\leftrightarrow 2}[f]}_{\text{collinear}}$$

Arnold, Moore, Yaffe (2003)



HTL

$m_D \sim gT$



formation time

$t_f \sim \frac{1}{g^2 T}$

multiple soft scatterings
coherence(LPM) effect

$\sim \frac{1}{g^2 T}$ mean free path

Outline

- Heavy quark transport
  - collisional/radiative energy loss
  - phenomenology

- Quarkonium

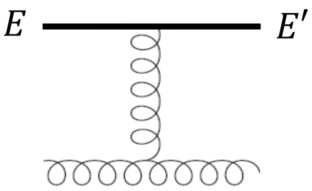
- Pre-equilibrium initial stage

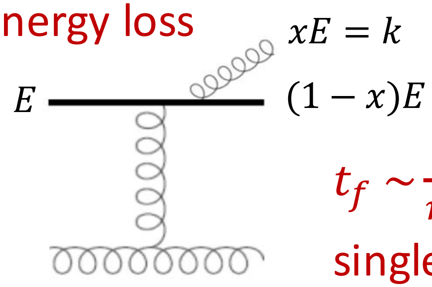
- Small collision systems

Heavy quark transport

- Heavy quark Boltzmann equation:

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v} \cdot \frac{\partial}{\partial \boldsymbol{x}}\right) f(\boldsymbol{p}) = \underbrace{C_{col}[f]}_{\text{collisional}} + \underbrace{C_{rad}[f]}_{\text{radiative energy loss}}$$





$$t_f \sim \frac{k}{m^2x^2 + k_T^2} \lesssim \text{mfp}$$

single scattering?

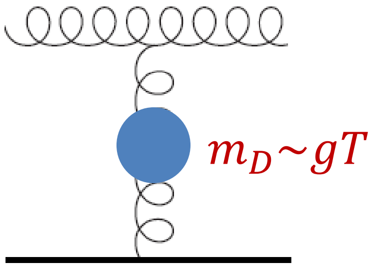
Collisional energy loss

Heavy quark diffusion

- Heavy quark with $m, p \gg T$
- Elastic scattering
- For soft($q \sim gT$) gluon exchange:

$$C_{col}[f] \sim \int d^3q \left[\underbrace{\frac{d\Gamma(p+q)}{d^3q} f(p+q)}_{\displaystyle \frac{d\Gamma}{d^3q} f + \left(q \cdot \frac{\partial}{\partial p} + \frac{1}{2} q_i q_j \frac{\partial^2}{\partial p_i \partial p_j} \right) \left(\frac{d\Gamma}{d^3q} f \right)} - \frac{d\Gamma(p)}{d^3q} f(p) \right]$$

Svetitsky (1988)



- Fokker-Planck equation: $C_{\rm col}[f] = \dfrac{\partial}{\partial p^i} \big[\underbrace{\eta(\boldsymbol{p})}_{\text{drag}} p^i f(\boldsymbol{p}) \big] + \dfrac{1}{2} \dfrac{\partial^2}{\partial p^i \partial p^j} \big[\underbrace{\kappa^{ij}(\boldsymbol{p})}_{\text{diffusion}} f(\boldsymbol{p}) \big]$
- Collision rate in Coulomb gauge:

$$\begin{aligned} C(\boldsymbol{q}) &\equiv (2\pi)^3 \frac{d\Gamma(\boldsymbol{q})}{d^3\boldsymbol{q}}\,, \\ &= \frac{\pi}{2} g^2 C_F m_D^2 \int d\omega\, \delta(\omega - \boldsymbol{q} \cdot \boldsymbol{v}) \frac{T}{q} \left[\underbrace{\frac{2}{|\boldsymbol{q}^2 + \Pi_L(Q)|^2} + \frac{(q^2 - \omega^2)(q^2 v^2 - \omega^2)}{q^4 |\boldsymbol{q}^2 - \omega^2 + \Pi_T(Q)|^2}}_{\text{HTL resummations}} \right] \end{aligned}$$

Fokker-Planck equation

Langevin equation: $\frac{dp_i}{dt} = \xi_i(t) - \eta p_i$

$\langle \xi_i(t)\xi_j(t')\rangle = \kappa\delta_{ij}\delta(t-t')$

stochastic noise

- Fokker-Planck equation:

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v}\cdot\frac{\partial}{\partial \boldsymbol{x}}\right) f(\boldsymbol{p}) = \frac{\partial}{\partial p^i}\left[\eta(\boldsymbol{p})p^i f(\boldsymbol{p})\right] + \frac{1}{2}\frac{\partial^2}{\partial p^i \partial p^j}\left[\kappa^{ij}(\boldsymbol{p})f(\boldsymbol{p})\right]$$

- Momentum diffusion: $\kappa^{ij}(\boldsymbol{p}) = \kappa_L(p)\hat{p}^i\hat{p}^j + \kappa_T(p)(\delta^{ij} - \hat{p}^i\hat{p}^j)$

- HQ moving in $\hat{z}$:

$$\kappa_L(p) = \int d^3\boldsymbol{q}\frac{d\Gamma(\boldsymbol{q})}{d^3\boldsymbol{q}}\boldsymbol{q}_z^2\,,$$

$$\kappa_T(p) = \frac{1}{2}\int d^3\boldsymbol{q}\frac{d\Gamma(\boldsymbol{q})}{d^3\boldsymbol{q}}\boldsymbol{q}_T^2$$

- At leading-log: $\kappa_{L,T}(p=0) \sim \alpha_s^2 T^3 \ln\frac{T}{m_D}$

- t→∞: approach the thermal equilibrium $f(p) \propto e^{-E/T}$

$$\eta(p) = \frac{\kappa_L(p)}{2TE} \quad \text{up to } O\left(\frac{T}{E}\right)$$

→ HQ transport coefficient κ

- Collisional energy loss: $-\frac{dE}{dz} = p\eta(p)$

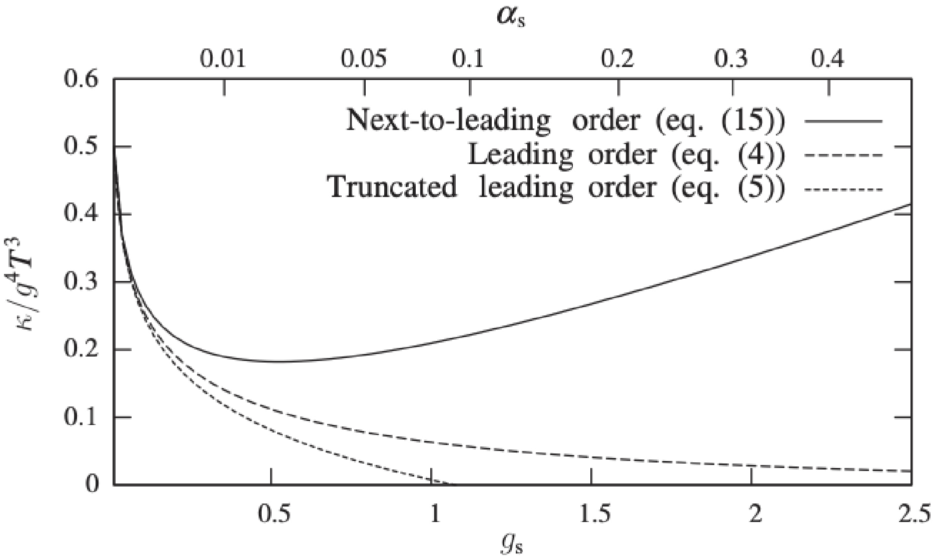
Heavy quark diffusion coefficient

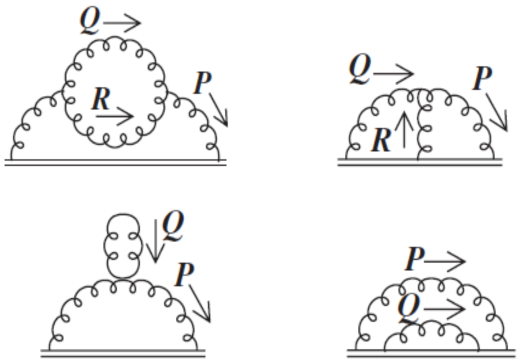
At weak coupling

- LO: $\kappa_{LO} \sim \alpha_s^2 T^3$
- NLO: $\kappa_{NLO} \sim g\alpha_s^2 T^3$

Bose−enhancement with soft$(k\sim gT)$ gluons

$$\alpha_s n_B(k) \sim \alpha_s \frac{1}{e^{k/T}-1} \sim \frac{g^2}{4\pi}\frac{T}{k} \sim g$$





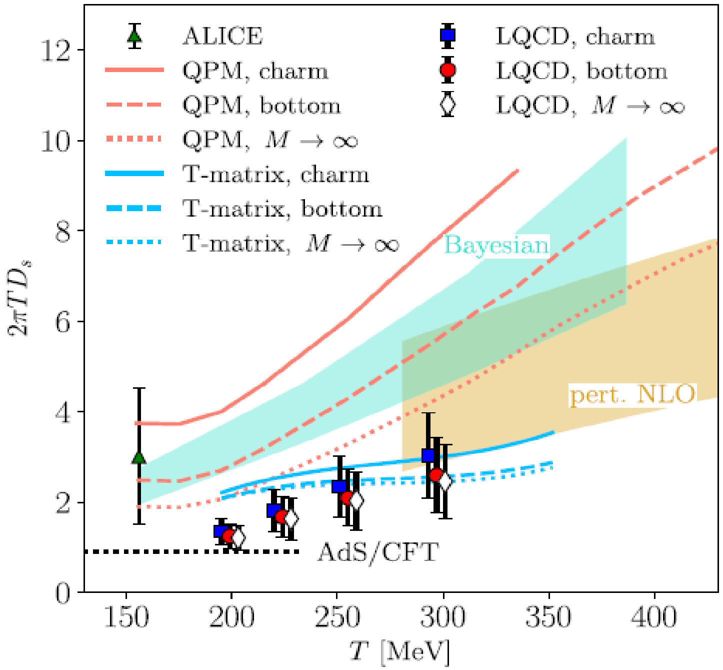
Caron-Huot, Moore (2008)

- For a realistic value of strong coupling $\alpha_s \sim 0.3$, poor convergence

Nonperturbative determination of κ

- Lattice QCD, phenomenological estimate from experimental data

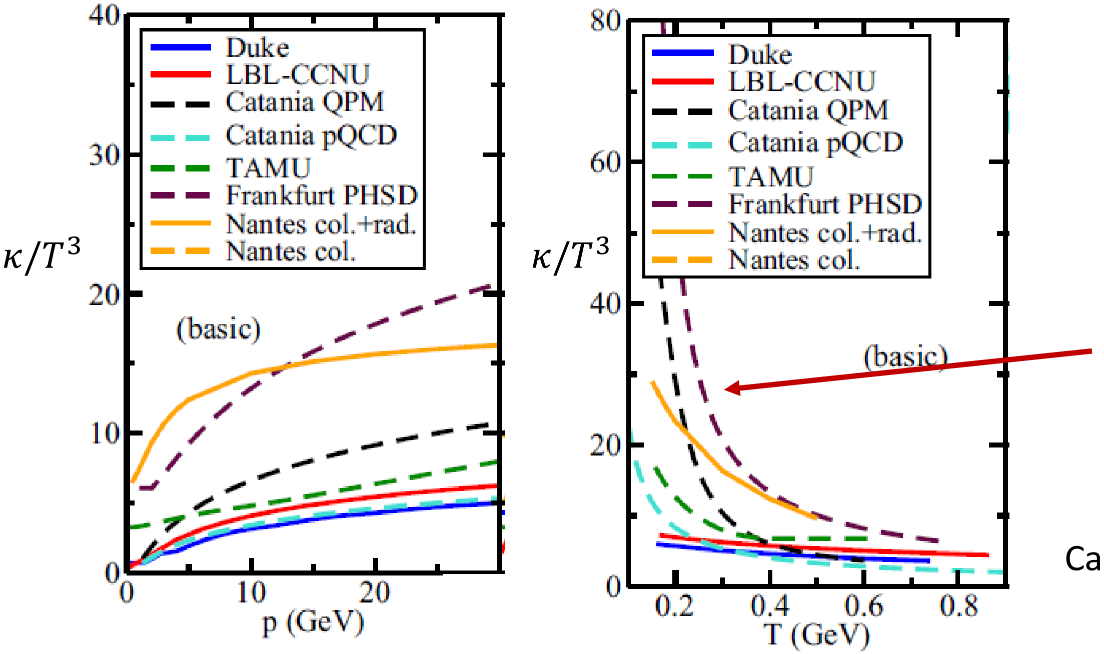
- Spatial diffusion: $D_s = \frac{2T^2}{\kappa}$



HotQCD (2024)

- D_s increases with T: running coupling

Momentum/temperature dependence of κ



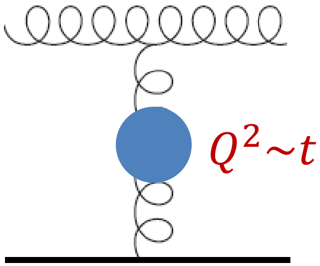
nonperturbative

Cao et al. (2019)

- Transport coefficients vary due to large differences between models
- κ increases with heavy quark momentum
- κ/T^3 decreases with temperature due to running coupling

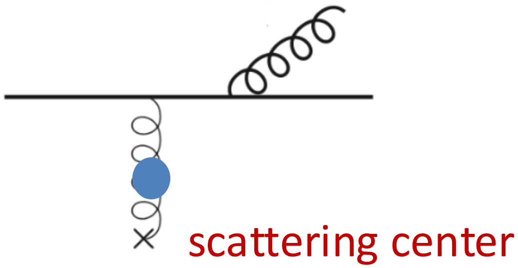
$$\alpha_s(Q^2) = \frac{12\pi}{(11N_c - 2N_f)\ln(\frac{Q^2}{\Lambda_{QCD}^2})}$$

$m_D^2 \lesssim Q^2 \lesssim ET$



Radiative energy loss

Medium-induced gluon radiation



- In the collinear limit,

$$M_{rad} \sim M_{el} \cdot (gluon\ emission\ factor)$$

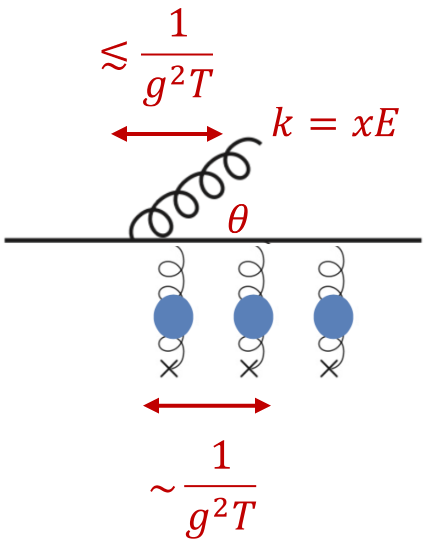
- α_s expansion but $\mathcal{O}(1/g^2)$ enhancement for soft gluon exchange and collinear emission

- For ultra-relativistic partons, elastic scattering and gluon-bremsstrahlung at LO

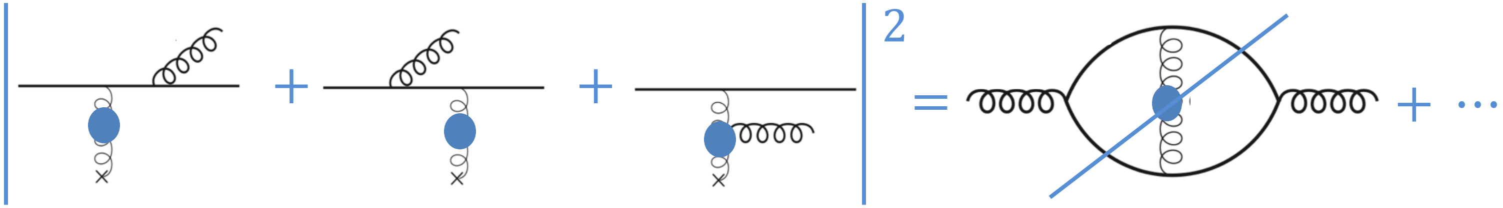
- Formation time: $t_f \sim \frac{k}{m^2x^2+k_T^2} \lesssim mfp$

- Dead-cone for $\theta \sim \frac{k_T}{k} < \frac{m}{E}$ Dokshitzer, Kharzeev (1998)

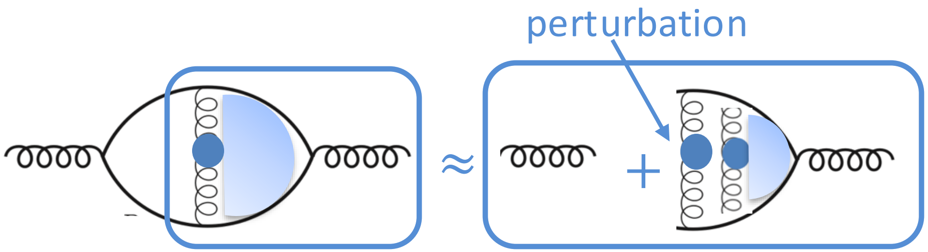
- Depending on HQ momentum, single scattering or multiple soft scatterings with LPM effect



Gluon emission rate



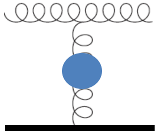
- Bethe-Salpeter equation:



- Integral equation summing ladder diagrams:

Arnold, Moore, Yaffe (2002)

$$2\boldsymbol{p}_T = i\delta E\, F(\boldsymbol{p}_T) + \int \frac{d^3\boldsymbol{q}}{(2\pi)^3}\underbrace{C(\boldsymbol{q})}_{\text{same collision kernel as diffusion}}\left[F(\boldsymbol{p}_T) - F(\boldsymbol{p}_T + \boldsymbol{q}_T)\right]$$



- Gluon emission rate:

$$\frac{d\Gamma(E_{\boldsymbol{p}}, k)}{dk} = \overbrace{\frac{g^2 C_F}{8\pi k^3}[1 + n_B(k)][1 - n_F(E_{\boldsymbol{p}-\boldsymbol{k}})]\frac{(1-x)^2 + 1}{(1-x)^2}}^{\text{splitting } HQ \to HQ + g}\int \frac{d^2\boldsymbol{p}_T}{(2\pi)^2}\boldsymbol{p}_T \cdot \mathrm{Re}\, F(\boldsymbol{p}_T)$$

- Eikonal approximation: $C_{\mathrm{rad}}[f] = \int dk \left[f((p+k)\hat{\boldsymbol{p}})\frac{d\Gamma(E_{(p+k)\hat{\boldsymbol{p}}}, k)}{dk} - f(\boldsymbol{p})\frac{d\Gamma(E_{\boldsymbol{p}}, k)}{dk}\right]$

HQ Phenomenology

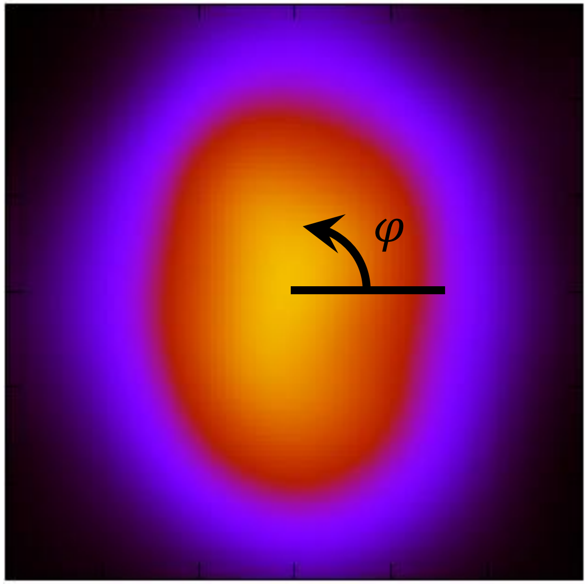
Observables

- Nuclear modification factor: $R_{AA} = \dfrac{\frac{dN_{AA}}{d^2p_T}}{N_{coll}\frac{dN_{pp}}{d^2p_T}}$ azimuthally averaged energy loss
- Collective flow: Fourier components of the azimuthal angle distribution

$$\frac{dN}{d^2p_T} = \frac{1}{2\pi}\frac{dN}{p_T dp_T}\left[1 + 2\textstyle\sum_n v_n \cos n\varphi\right]$$

- Elliptic flow: $v_2 = \dfrac{\int d\varphi \frac{dN}{d^2p_T}\cos 2\varphi}{\int d\varphi \frac{dN}{d^2p_T}}$

  - HQ distribution in the transverse plane
  - the asymmetric energy loss (path-length difference depending on φ)
  - HQ momentum anisotropy through coupling to the collective motion of media



Heavy flavor phenomenology

- HQ initial production, HQ interaction, medium evolution, hadronization, hadronic re-scattering

- Heavy quark interaction with QGP: heavy quark transport coefficient
  heavy quark energy loss as a main contribution to HQ suppression

- Expanding thermal background media: hydrodynamics

Hydrodynamics

- Describe the evolution of thermal media in relativistic heavy-ion collisions
- Conservation of the energy-momentum tensor:

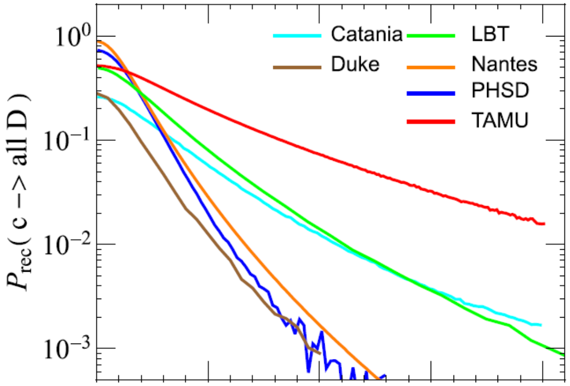
$$\partial_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = \underbrace{(e+\mathcal{P})u^\mu\, u^\nu + \mathcal{P} g^{\mu\nu}}_{\text{ideal}} - \underbrace{\overset{\text{shear viscosity}}{\eta}\left(\nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}\partial_\lambda u^\lambda\right)}_{\text{1st order}} + (2nd\ order)$$

- (3+1)D relativistic hydrodynamic codes provide the space-time distributions of temperature and flow velocity fields
- The initial spatial anisotropy due to geometric deformation transfer to the momentum space through the pressure gradients

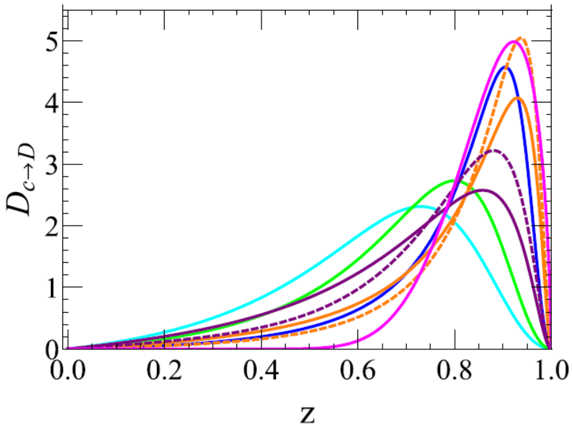
Hadronization

- Coalescence (c+$\overset{\text{QGP}}{q}$⟶D)



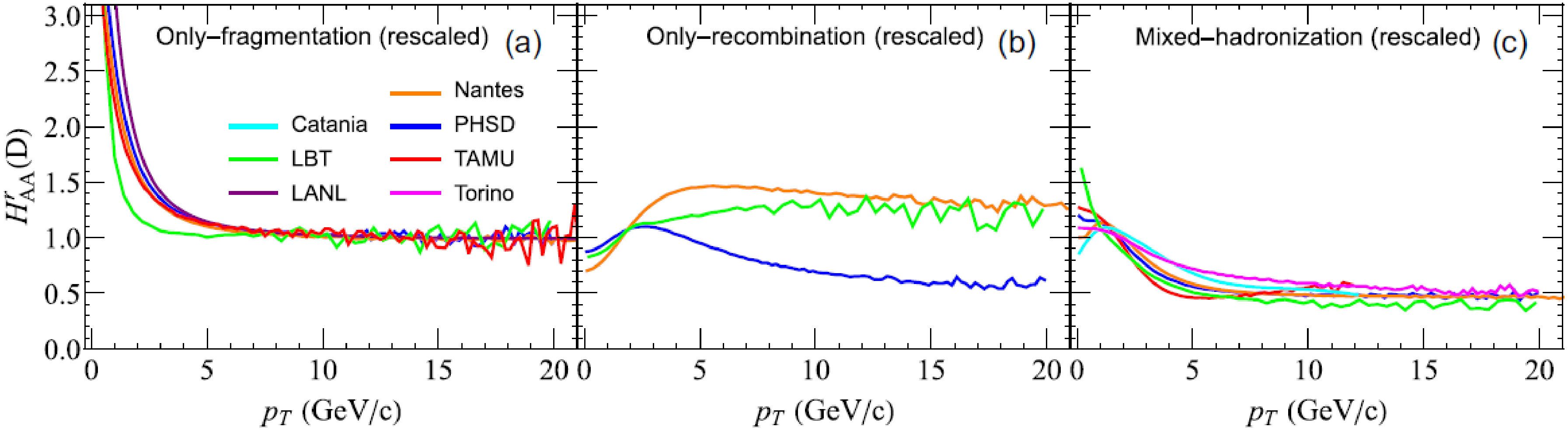
p_T^c(GeV)

- Fragmentation (c⟶D)

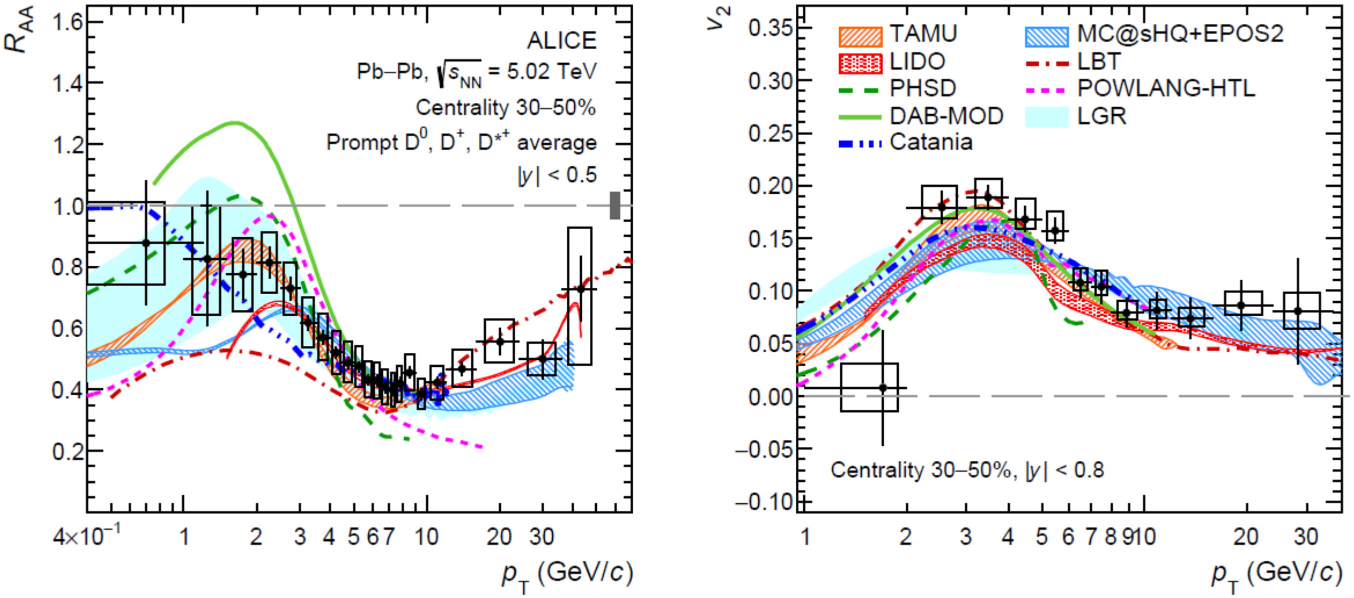


- $H_{AA} = \dfrac{\frac{dN_D}{dp_T}}{\frac{dN_c}{dp_T}}$: hadronization effects

Zhao et al. (2024)



Heavy quark suppression and elliptic flow



ALICE (2022)

Collisional energy loss only? or collisional+radiative?

- Collisional/radiative energy loss is dominant at low/high momentum:

transition at intermediate momentum?

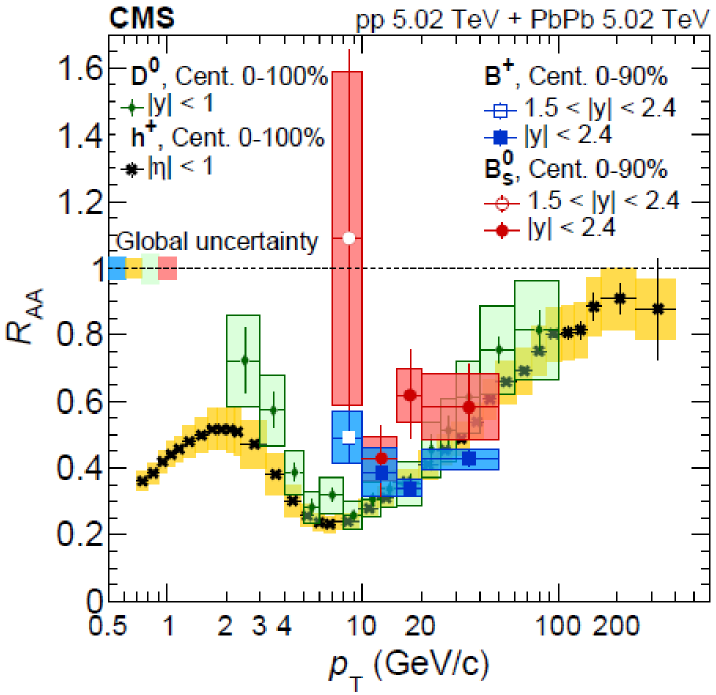
$t_f \lesssim \mathrm{mfp}$

multiple scatterings?

- Different transport coefficients, hadronization, and medium evolution
- Simultaneous description of R_{AA} and v_2:
  with adjustment of model parameters, most models can describe data

Bottom quark energy loss

- Less energy loss than charm:



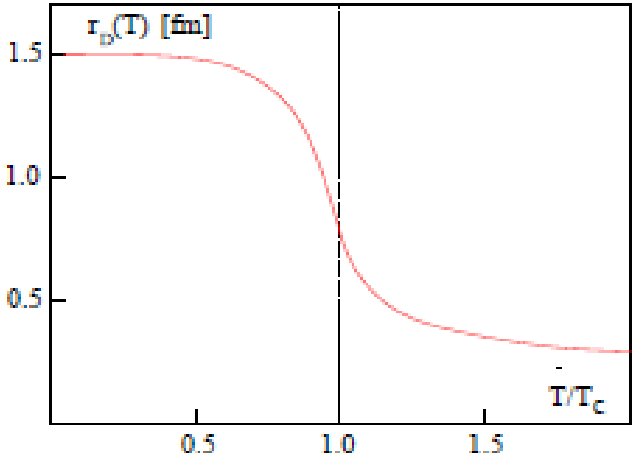
CMS (2024)

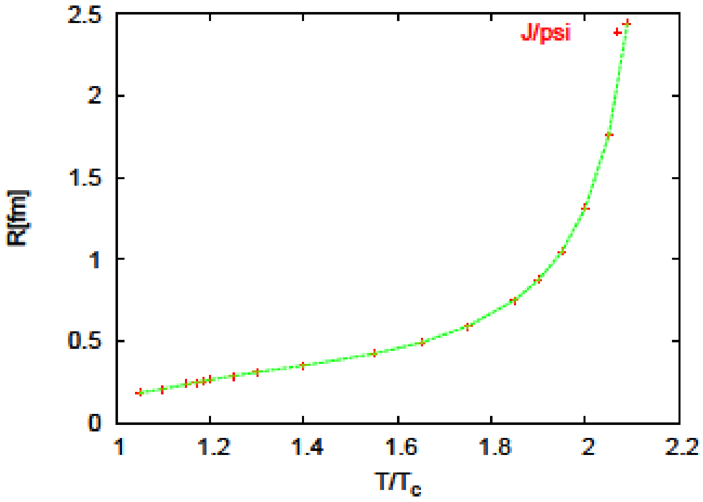
- Theoretically more precise in the heavy quark approximation
- Less cold matter effect

Quarkonium

Quarkonium

- Bound states of heavy quark and antiquark
- Color screening: Interaction length ↔ bound state radius





Satz (2006)

- Melting at T_{diss}
- Signal of QGP formation
- Thermal properties of QGP

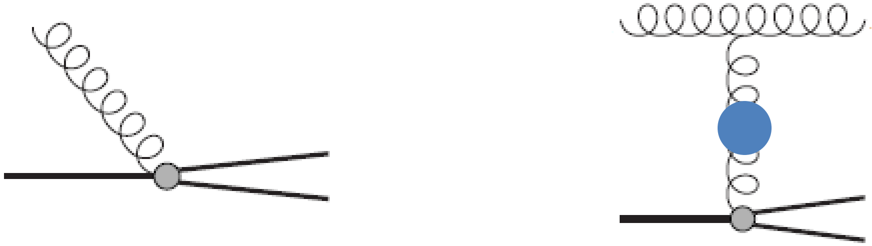
State	T_{diss} (MeV)
ϒ(1S)	593
ϒ(2S)	228
ϒ(3S)	< 192
X_{b1}	265
X_{b2}	< 192

Mocsy, Petreczky, Strickland (2013)

Quarkonium transport

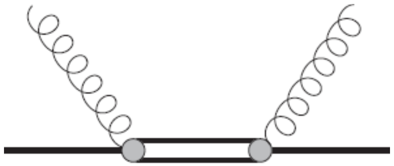
- Dynamics: $\left(\frac{\partial}{\partial t} + \boldsymbol{v} \cdot \frac{\partial}{\partial \boldsymbol{x}}\right) f(\boldsymbol{p}) = C_{diss}[f] + C_{reg}[f] + C_{diff}[f]$ Yao, Muller (2019)

- Dissociation: gluo-dissociation, inelastic parton scattering



- Regeneration: inverse processes, using HQ spectrum

- Diffusion: energy loss by elastic scattering

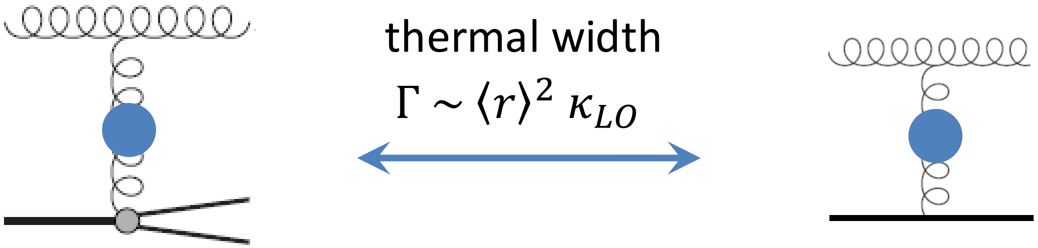


Effective field theory

- Potential Non-Relativistic QCD
- $m \gg \frac{1}{r} \sim p_{rel} \gg \underbrace{T \gtrsim m_D}_{\text{strong coupling?}}, \; E_b$

Brambilla et al. (2008,2013)

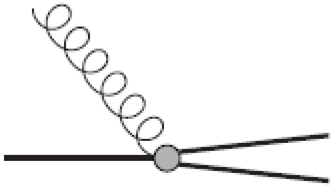
- Thermal corrections to the singlet static potential: $\delta m - i\frac{\Gamma}{2}$
- High T: $m_D \gg E_b$, inelastic parton scattering



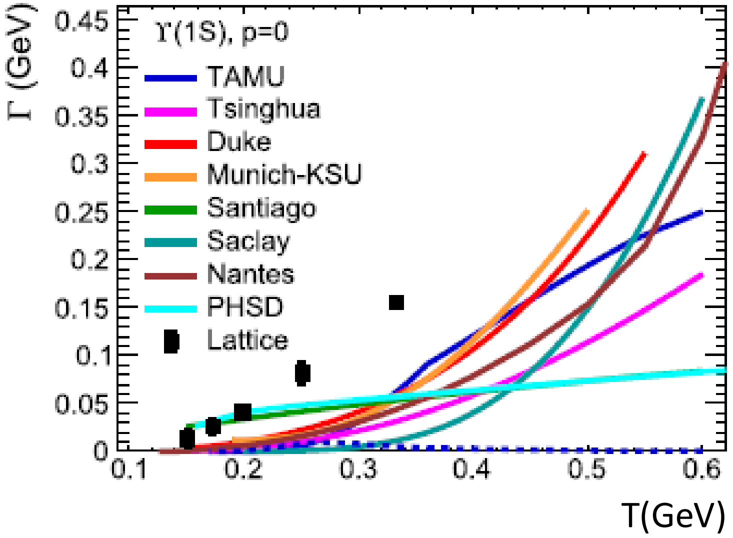
thermal width

$\Gamma \sim \langle r \rangle^2 \, \kappa_{LO}$

- Near T_c: $E_b \gg m_D$, gluo-dissociation



Quarkonium thermal width



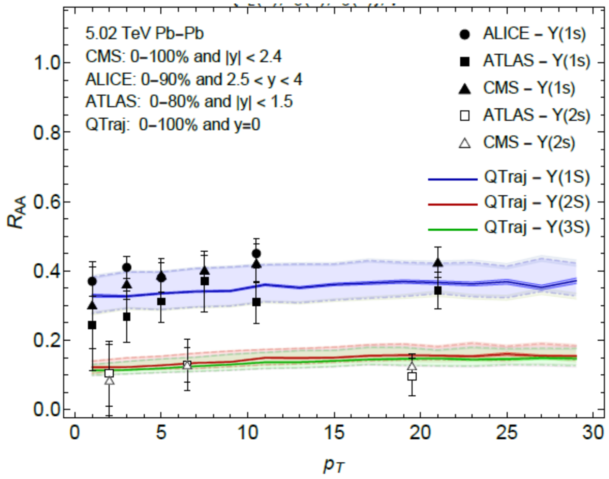
Andronic et al. (2024)

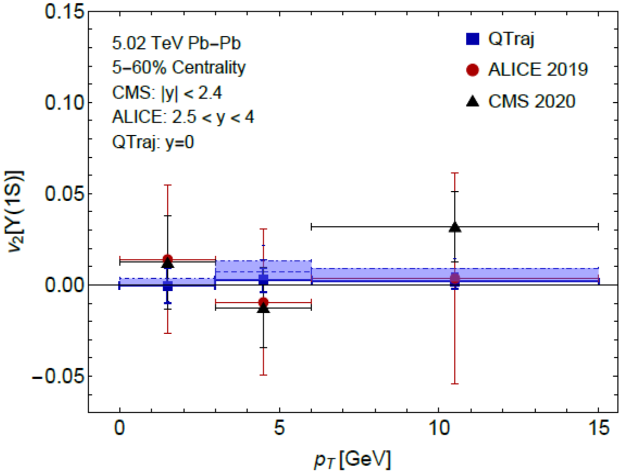
- Large differences between models:
  - heavy quark-antiquark potential (lattice QCD constraints)
  - binding energy
  - medium evolution
  - HQ transport
  - feed-down

Open quantum system

- System(heavy quark-antiquark pair) + environment(QGP)
- Hamiltonian: $H = H_S + H_E + H_I$
- Time evolution of the density matrix: $\frac{d\rho(t)}{dt} = -i[H, \rho(t)]$
- Quantum Brownian motion: $\tau_R \gg \tau_E, \quad \tau_S \gg \tau_E$

  $\tau_R \sim \frac{1}{r^2 T^3}$ (relaxation time), $\tau_S \sim \frac{1}{E_b}$ (system intrinsic time), $\tau_E \sim \frac{1}{T}$ (environment correlation time)



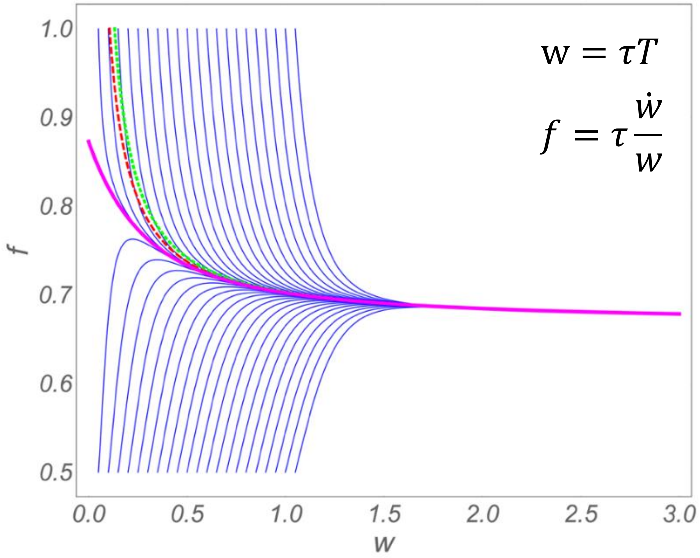


Brambilla et al. (2021)

- Open quantum system approach for heavy quark diffusion
- Open quantum system approach for medium-induced gluon radiation?

Initial stages

Pre-equilibrium



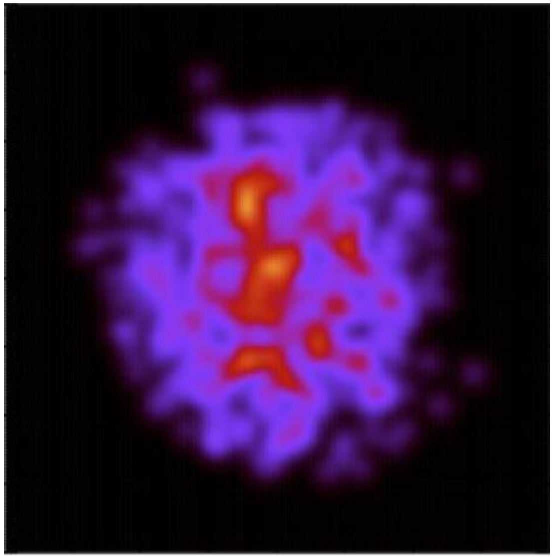
Heller, Spalinski (2015)

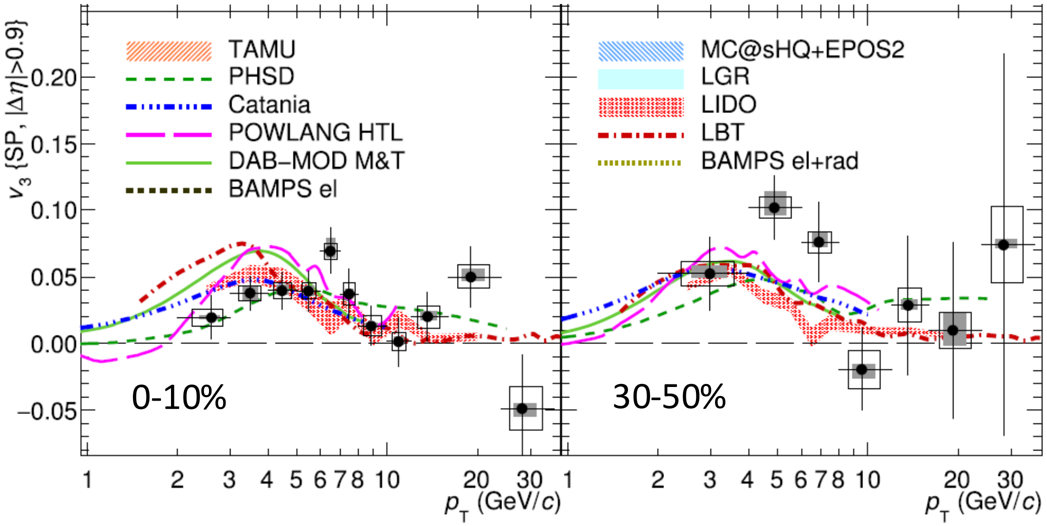
- Attractor solution: final evolution is fluid dynamical
  even if the initial state is far from equilibrium

- Models based on effective kinetic theory: approach to fluid dynamics

Initial state fluctuations

- Initial conditions for hydrodynamics: large uncertainty
- Fluctuations of nuclear positions, quantum fluctuations of color charges
- Triangular flow: $v_3 = \dfrac{\int d\varphi \, \dfrac{dN}{d^2p_T} \cos 3\varphi}{\int d\varphi \, \dfrac{dN}{d^2p_T}}$

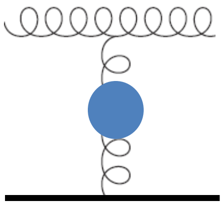




ALICE (2021)

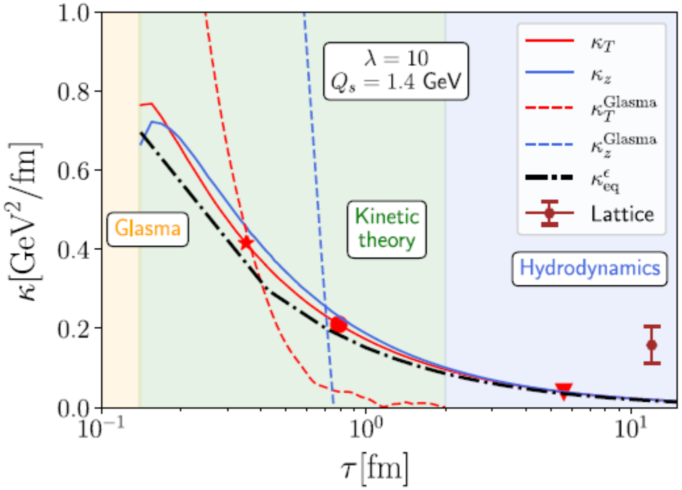
Non-equilibrium transport coefficient

- Equilibrium diffusion coefficient: $f(\mathrm{p}) = \dfrac{1}{e^{p/T} - 1}$



- Non-equilibrium diffusion coefficient:
  non-equilibrium parton distribution

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v} \cdot \frac{\partial}{\partial \boldsymbol{x}}\right) f(\boldsymbol{p}) = C_{2\leftrightarrow 2}[f] + C_{1\leftrightarrow 2}[f]$$

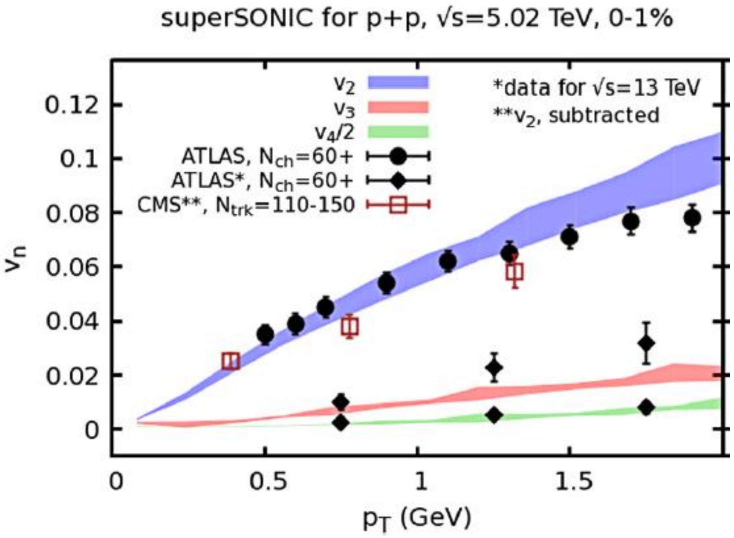


Boguslavski et al. (2021)

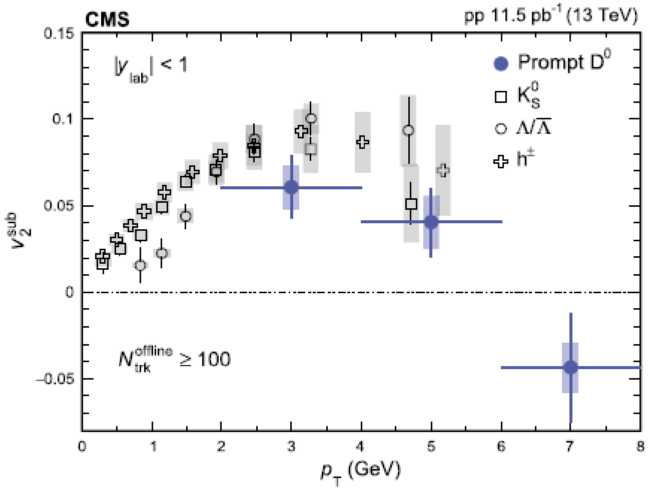
- HQ transport employing non-equilibrium $\kappa_{T,L}(t)$:
  impact of pre-equilibrium stages in HQ and quarkonium phenomenology

Small collision systems

Collectivity in small systems



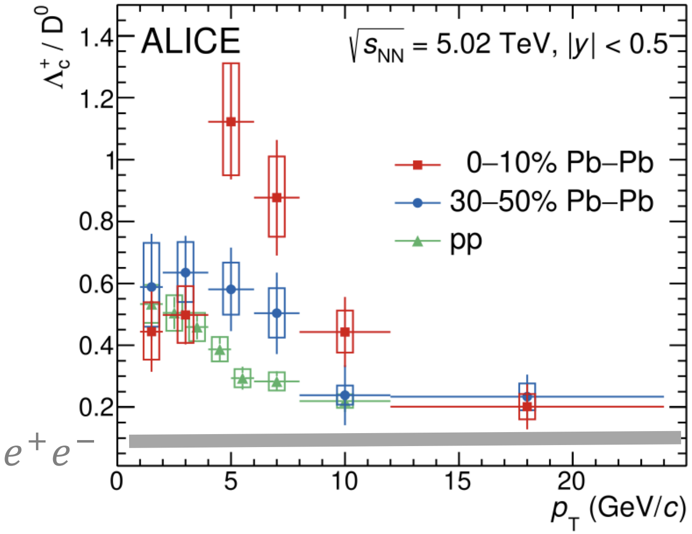
Weller, Romatschke (2017)



CMS (2021)

- Collective flow described by hydrodynamics: small QGP in pp collisions?
- Heavy flavor v_2 in small systems
- HQ interaction with QGP and hadronization in pp as in AA?
- Small systems: medium-size, path-length dependent energy loss
  cold nuclear matter effect

Heavy baryon enhancement



ALICE (2023)

- $\frac{\Lambda_c^+}{D^0}$ enhancement in heavy-ion collisions:
  incompatible with hadronization in e^+e^- collisions (~constant ratio)
- Enhancement even in pp collisions: factorization broken
- Fragmentation function in pp not same as in e^+e^-?
- In-medium hadronization: coalescence+fragmentation using HQ spectrum
- Enhancement at low p_T: interaction with QGP,
  more baryons in heavy flavor hadronization (recombination with diquarks?)

Summary & outlook

- Heavy quark transport by collisional and radiative energy loss
  - Transition between diffusion and radiation at intermediate momentum
  - Nonperturbative effects near T_c

Employing HQ spectra,

- Quarkonium dissociation, regeneration, and energy loss
  - Heavy quark diffusion and radiation in an open quantum system approach

- Pre-equilibrium initial stages
  - Impact of non-equilibrium effects in heavy flavor phenomenology

- Small collision systems
  - HQ interaction with possible QGP and in-medium hadronization in pp/pA collisions

More quantitative study in future

Thank you for your attention