
# LCSR application to $p \to e^{+}\gamma$

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(Based on: AB, Namit Mahajan (2112.XXXX) )

# Outline

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- Form Factors in LCSR
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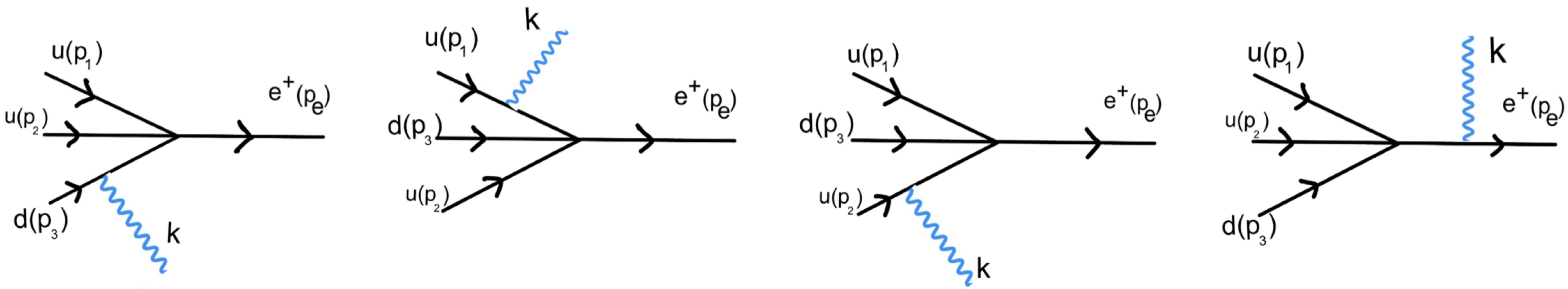
- Summary & Conclusions

# Motivation

- Proton decay : Forbidden in the Standard Model
  $\Longrightarrow$ A clear signal of physics beyond the standard model.
- Prominent decay mode: $p \rightarrow e^{+}\pi^{0}$ (Experimental contraints : $\tau_p > 10^{34}$ years).
- Has been studied using various methods and models (Bag Model, Lattice QCD,etc.)
- A recent analysis using LCSR. [Haisch et al, JHEP 05 (2021) 258]
- Though it is prominent, it has not been observed experimentally yet.
- Detailed study of other modes is important.

- $p \to e^+\gamma$ is $\mathcal{O}(\alpha_{em})$ suppressed compared to mesonic mode.
- But, $p \to e^+\gamma$ is free from nuclear absorption and complications due to strong interactions compared to the mesonic modes.
- Hence, a cleaner channel for experimental analysis.
- Experimental partial mean life $>6.7 \times 10^{32}$ year. [PDG]
- Helpful in understanding the structure of Proton.
- Can help in constraining the parameter space of various BSM models.
- Gained very less attention in the past, only one paper by Silverman et. Al. [PLB, Vol. 100, n.2 (1981)]
- A careful analysis of the form factors is still missing.

# Introduction




- Proton decay is possible via baryon number violating dim-6 operator.

$$\mathcal{O}_{\Gamma\Gamma'} = \epsilon^{abc}(\bar{d}^c_a P_\Gamma u_b)(\bar{e}^c P_{\Gamma'} u_c)$$

Projection operators

[Weinberg, (PRL, Vol. 43, 21 (1979))]

- The amplitude for the process is:

$$\mathscr{A}(p \to e^+\gamma) = \sum_{\Gamma,\Gamma'} c_{\Gamma\Gamma'} \left\langle e^+(p_e)\gamma(k) \,|\, \mathcal{O}_{\Gamma\Gamma'} \,|\, p(p_p) \right\rangle = \sum_{\Gamma,\Gamma'} c_{\Gamma\Gamma'} \bar{v}_e^c(p_e) H_{\Gamma\Gamma'}(p_p, p_e) u_p(p_p)$$

- All the flavour effects are absorbed in Wilson coefficients , $c_{\Gamma\Gamma'}$.
- Using gauge invariance, it can be written in terms of two form factors (to be calculated using LCSR):

$$\mathscr{A}(p \to e^+\gamma) = \sum_{\Gamma,\Gamma'} c_{\Gamma\Gamma'} \left\{ i\sigma^{\alpha\beta} k_\beta \epsilon_\alpha^* \left( A_{\Gamma\Gamma'} + B_{\Gamma\Gamma'}\gamma_5 \right) \right\}$$

- Becaue the parity is conserved in QCD,

$A_{LL} = A_{RR}$, $A_{LR} = A_{RL}$, $B_{LL} = B_{RR}$, and $B_{LR} = B_{RL}$

# Light Cone Sum Rules

**TOOLS TO DERIVE SUM RULES**

- **Idea**: *To compute hadronic parameters using the analytic properties of the correlation function (treated in the framework of OPE).*

**Dispersion Relation**
**(relates real part of correlation function to its imaginary part)**

**Operator Product Expansion**
**(Enables one to write correlation function as a product of short distance and long distance physics)**

**Quark Hadron Duality**
**(Relates the non-perturbative spectral function to the perturbatively calculated amplitude function)**

**Borel Transformation**
**(To supress the effect of continuum and higher resonances)**

# Light Cone Sum Rules for $p \to e^+\gamma$

- The hadronic matrix element to be calculated is,

$$H_{\Gamma\Gamma'} u_p(p_p) = \left\langle \gamma(k) \middle| \epsilon^{abc}(d_a^T C P_\Gamma u_b) P_{\Gamma'} u_c \middle| p(p_p) \right\rangle$$

- Two possibilities:
  1. Interpolating proton current and using photon distribution amplitudes.
  2. Interpolating electromagnetic current and using proton distribution amplitudes.

***Distibution amplitude:*** The probability amplitude for finding meson (baryon) as a two (three) quark state with the momentum factions $u$ and $(1-u)$ $(\alpha_1,\ \alpha_2,$ and $\alpha_3)$.

# Case-1: Using proton DAs

- Interpolation the photon current:

$$\bar{v}_e^c H_{\Gamma\Gamma'} u_p(p_p) = -ie\epsilon^{*\alpha}\int d^4x\, e^{ik.x}\left\langle e^+ \left| T\{j_{\alpha^{em}}\epsilon^{abc}(d_a^T C P_\Gamma u_b)(e^T C P_{\Gamma'} u_c\}\right| p(p_p)\right\rangle$$

- The generalised Fierz transformations: [arXiv: hep-ph/0306087]

$$e_s(1234) = \frac{1}{4}\left(e_S(31^c4^c2) - e_V(31^c4^c2) - e_T(31^c4^c2) - e_A(31^c4^c2) + e_P(31^c4^c2)\right)$$

$$\Rightarrow \quad (d_a^T C P_L u_b)(e^T C P_L u_C) = \frac{1}{4}\left(2(e^T C P_L d_a)(u_c^T C P_L u_b) - (e^T C \sigma_{\mu\nu} P_L d_a)(u_c^T C \sigma_{\mu\nu} P_L u_b)\right)$$

- Leading twist (twist-3) nucleon DA, [arXiv:hep-ph/0007279]

$$4\left\langle 0 \left| \epsilon^{abc} u_\alpha^a(a_1 n) u_\beta^b(a_2 n) d_\gamma^c(a_3 n)\right| P(p_P)\right\rangle = f_P\Big[(p_P . \gamma C)_{\alpha\beta}(\gamma_5 u_P)V_1(a_i n . p_P) + (p_P . \gamma C)_{\alpha\beta}(\gamma_5 u_P)V_1(a_i n . p_P) + (p_P . \gamma\gamma_5 C)_{\alpha\beta}(u_P)A_1(a_i n . p_P)$$

$$+ (i\sigma_{\rho\sigma} p_P^\sigma C)_{\alpha\beta}(\gamma_\rho\gamma_5 u_P)T_1(a_i n . p_P)\Big]$$

- In QCD,

$$A_{LL}^{QCD} = \frac{-em_P\epsilon^{*\alpha}}{2} f_P \int \mathcal{D}\alpha_i T_1(\alpha_i) \left[ \frac{Q_d}{2} \frac{1+2\alpha_3}{(k-\alpha_3 p_P)^2} + Q_u \frac{5\alpha_1 - 2}{(k-\alpha_1 p_P)^2} \right]$$

- Dispersion relation (by sturating the sum rule with $\frac{1}{2}^+$ and $\frac{1}{2}^-$ states.),

$$\epsilon^{*\alpha} \left\langle 0 \left| O_{LL}(0) j_\alpha^{em}(x) \right| P(p_P) \right\rangle \sim \epsilon^{*\alpha} \underbrace{\left\langle 0 \left| O_{LL} \right| P(p_P - k) \right\rangle}_{\sim \lambda_P m_P u_P(p_P - k)} \underbrace{\left\langle P(p_P - k) \left| j_\alpha^{em}(x) \right| P(p_P) \right\rangle}_{\bar{u}_P(p_P - k) \left[ \gamma_\alpha F_1(Q^2) - i \frac{\sigma_{\mu\nu} k^\nu}{2m_P} F_2(Q^2) \right] u_P(p_P)} + \dots$$

$\sim \lambda_P m_P u_P(p_P - k)$

$\bar{u}_P(p_P - k) \left[ \gamma_\alpha F_1(Q^2) - i \frac{\sigma_{\mu\nu} k^\nu}{2m_P} F_2(Q^2) \right] u_P(p_P)$

Ellipses implies the negative parity states, higher resonances and continuam.

- The final sum rule turns out be,

$$\lambda_P F_2^{LL}(Q^2) = ef_P \int \mathscr{D}\alpha_i T_1(\alpha_i) \left[ \frac{Q_d}{2} \frac{1+2\alpha_3}{\alpha_3} e^{-(1-\alpha_3)\frac{m_P^2}{M^2}} + Q_u \frac{5\alpha_1 - 2}{\alpha_1} e^{-(1-\alpha_1)\frac{m_P^2}{M^2}} \right] e^{\frac{m_P^2}{M^2}}$$

$M$ is the Borel parameter, and

$$T_1(\alpha_i) = 120\alpha_1\alpha_2\alpha_3 \left[ 1 + \frac{1}{2}(\tilde{\phi}_3^- - \tilde{\phi}_3^+)(\mu)(1 - 3\alpha_3) \right]$$

- The form factor $A_{LL}(Q^2)$ is,

$$A_{LL}(Q^2) = \frac{-m_P}{Q^2 - m_P^2 - i\epsilon} \lambda_P F_2^{LL}(Q^2)$$

# Case-2: Using photon DAs

- The leading twist (twist-2) DA for photon is:

$$\left\langle \gamma(k) \middle| \bar{q}(x)\sigma_{\mu\nu}q(0) \middle| 0 \right\rangle = -ie_q \langle \bar{q}q \rangle (\epsilon_\mu^* k_\nu - \epsilon_\nu^* k_\mu) \int_0^1 du e^{iq.x\bar{u}} \chi \phi_\gamma(u)$$

Quark Condensate

Magnetic Susceptibility

Twist-2 DA

$$\phi_\gamma(u) = 6u\bar{u} \left( 1 + \sum_{n=2,4,..}^{\infty} L^{(\gamma_n - \gamma_0)/b} \phi_n c_n^{3/2}(u - \bar{u}) \right)$$

- The proton interpolation current is chosen to be,

$$\eta(x) = 2\epsilon^{abc}(u_a^T(x) C\gamma_5 d_b(x)) u_c(x)$$

Such that $\left\langle 0 \middle| \eta(0) \middle| p(p_p) \right\rangle = m_p \lambda_p u_p(p_p)$.

# Form Factors in QCD

- In QCD,

Charge Conjugation matrix

$$\Pi_{\Gamma\Gamma'} = i\int d^4x e^{ip_e.x}\left\langle \gamma(k)\left| T\left\{\epsilon^{abc}(d_A^T(x)CP_\Gamma u_b(x))P_{\Gamma'}u_c(x)\times 2\epsilon^{ijk}\bar{u}_c(0)(u_a^T(0)C\gamma_5 d_b(x))\right\}\right| 0\right\rangle$$

$$-\frac{1}{2}\epsilon_{ijk}\epsilon_{abc}P_{\Gamma'}\{(\bar{u}^a(0)\Gamma_A u^i(x))\Big(s_u^{kc}(x)\gamma_5\tilde{s}_d^{jb}(x)P_\Gamma\Gamma^A + s_u^{kc}(x)Tr(\Gamma^A\gamma_5\tilde{s}_d^{jb}(x)P_\Gamma) + \Gamma^A\gamma_5\tilde{s}_d^{jb}(x)P_\Gamma s_u^{kc}(x) + \Gamma^A Tr(s_u^{kc}\gamma_5\tilde{s}_d^{jb}(x)P_\Gamma)\Big)$$
$$+(\bar{d}^a(0)\Gamma_A d^i(x))\Big(s_u^{kc}(x)\gamma_5\tilde{\Gamma}^A P_\Gamma s_u^{jb}(x) + s_u^{kc}(x)Tr(s_u^{jb}(x)\gamma_5\tilde{\Gamma}^A P_\Gamma)\Big)\}$$

$$\Gamma_A = \left\{1,\gamma_5,\gamma^\rho,i\gamma_\rho\gamma_5,\frac{1}{\sqrt{2}}\sigma^{\rho\sigma}\right\}$$

$$s^{ij}(x) = \frac{ix_\mu\gamma^\mu}{2\pi^2x^4}$$

$$\tilde{\Gamma}_A = C\Gamma_A C$$

- At leading twist, only $\Gamma_A = \frac{1}{\sqrt{2}}\sigma^{\rho\sigma}$ will contribute.

- In QCD,

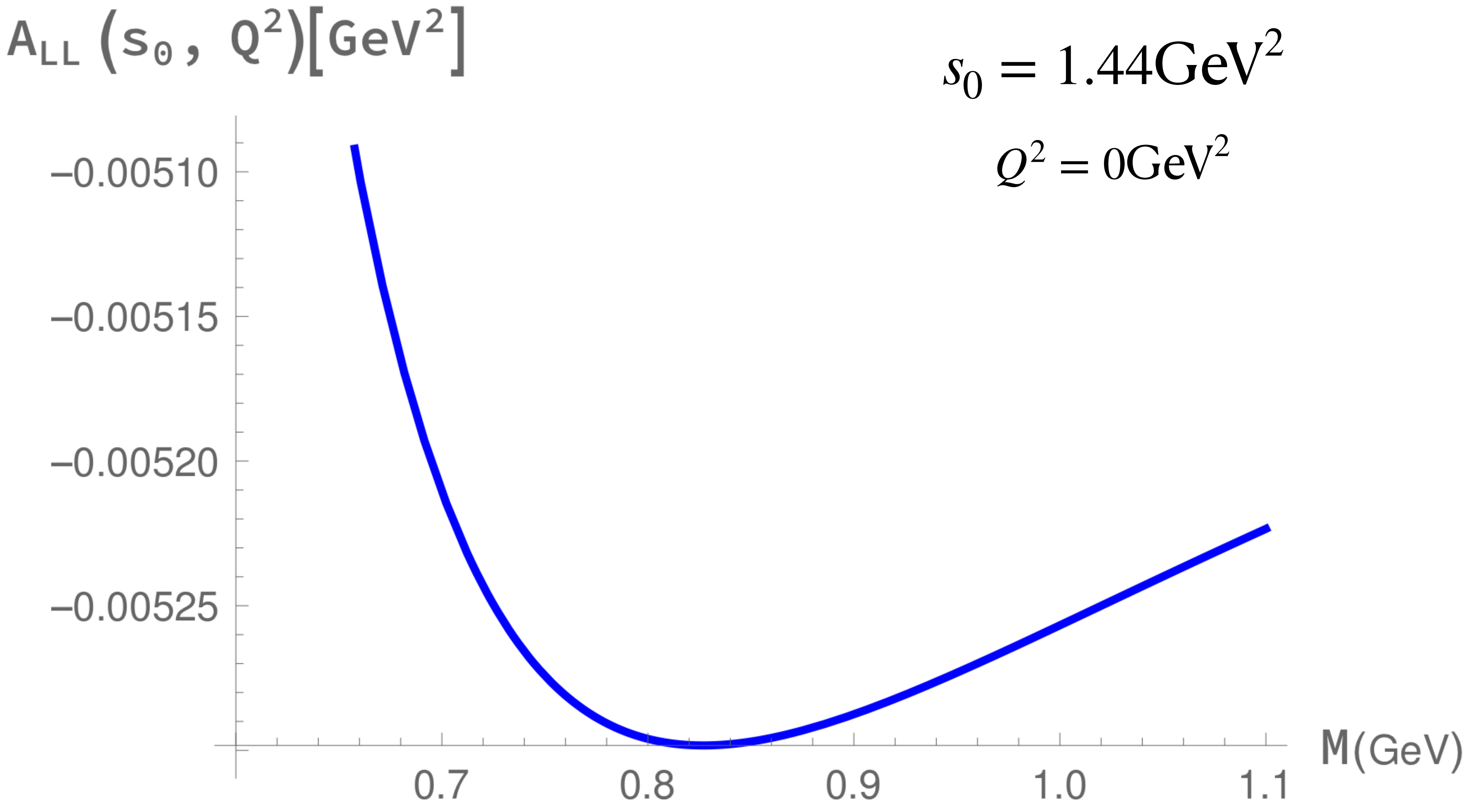
$$A_{LL}^{QCD}(Q^2) = \frac{-630\chi\langle\bar{q}q\rangle}{576\pi^2}\int_0^1 \frac{d\alpha}{36}P^2 ln(-P^2)\phi_\gamma(\alpha)$$

where, $P^2 = (p_e + \alpha k)^2 = -\alpha P_p^2 - \bar{\alpha}Q^2$ and $Q^2 = -p_e^2$

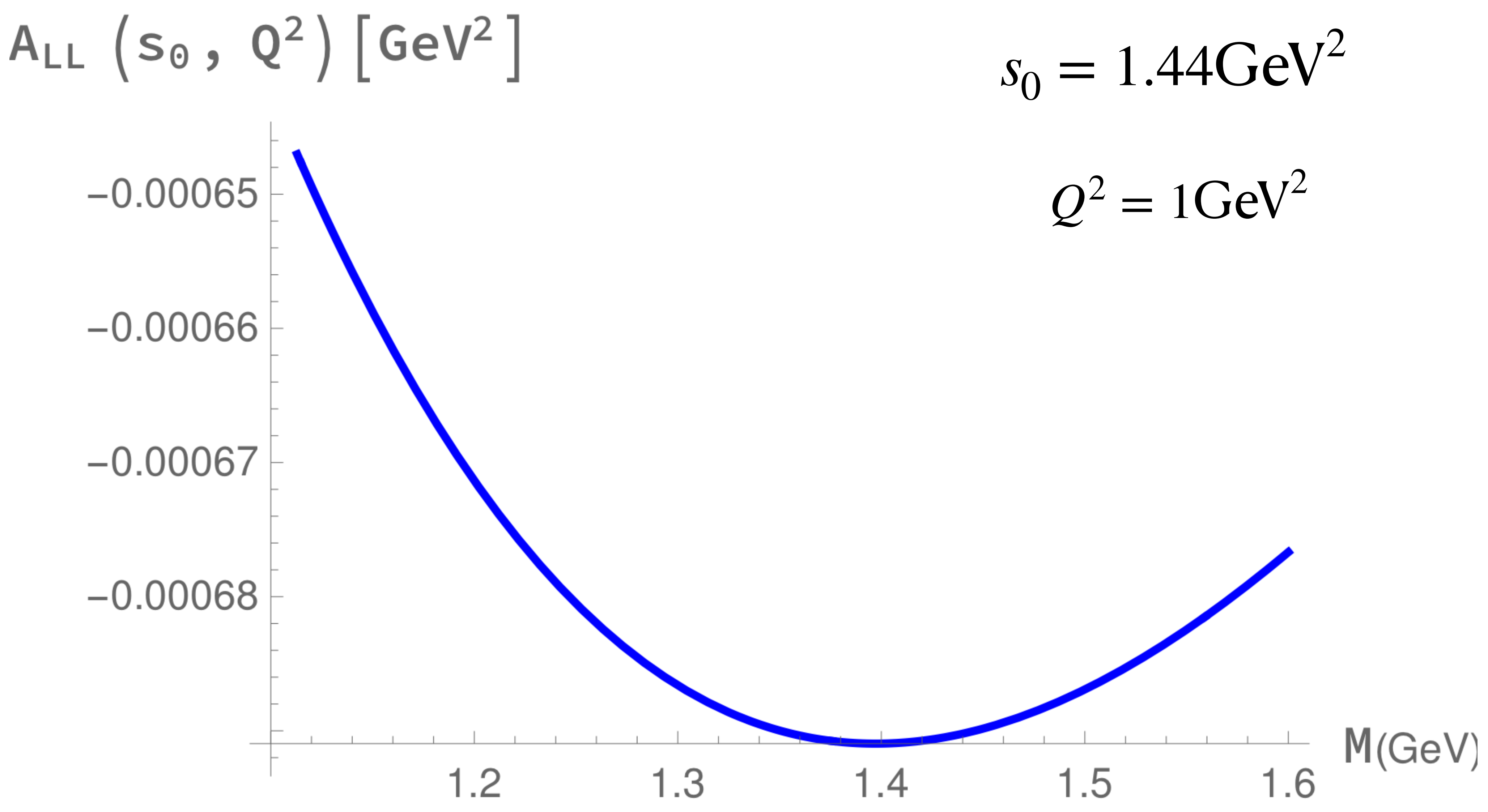
- For dispersion relation, we again saturate the sum rule with proton.
- The higher resonances and continuam contribution can be related to QCD result using quark hadron duality.
- The final sum rule turns out to be,

$$i\lambda_p m_p^2 e^{\frac{-m_p^2}{M^2}} A_{LL}(s_0, Q^2) = \frac{1}{\pi}\int_0^{s_0} ds e^{\frac{-s}{M^2}} Im\left(A_{LL}^{QCD}(s, Q^2)\right)$$

# Results




Using proton DAs




Using photon DAs

# Summary & Conclusions

- $p \to e^{+}\gamma$ decay involves 2 FFs: calculated in the fracmework of LCSR.
- FFs presented upto leading twist accuracy using photon DA (twist-2) and proton decay (twist-3).
- At this accuracy, the numerical estimates for form factors using different DAs seems different.
- Higher twist three-particel DAs are required to get better estimates for the form factor involved.
- If the differences persists than it might give deeper insight to the structure of proton.

# Thank You