

Neutron Star Equations of State and Gravitational Waves

Chang-Hwan Lee / Pusan National University

in collaboration with

Young-Min Kim, Kyujin Kwak (UNIST)

Yeunhwan Lim (Darmstadt)

Chang Ho Hyun (Daegu)

Dense Nuclear & Stellar Matter Studies

for **RAON** New Rare Isotope Accelerator & **MMA** Multi-Messenger Astrophysics

BUD² Collaboration

Busan (**CHL**, Myungkuk KIM)

Ulsan (**Kyujin KWAK**, Young-Min KIM)

Daegu (Chang Ho HYUN)

Daejeon (Youngman KIM)

Montreal (Sangyong JEON, McGill)



Astro-Hadron Physics in Korea *my personal point of view*

Hadron Physics

NS EoS with **Effective Field Theories**
(with D.P.Min, **M.Rho** & G.E.Brown)

Science-Business-Belt Project
initiated by **D.P. Min**

RAON project was approved

New Transport DJBUU
Nuclear Structure DRHBc
New EDF KIDS
.....

First run of RAON
Symmetry Energy (later)

Astrophysics

NS Binary as a source of GW
(with **G.E.Brown**@Stony Brook)

2003 Korean Gravitational Wave Group
Nuclear physics + Astrophysics +
Mathematics + Artificial Intelligence

2009 KGWG joined LIGO Scientific Collab.

2017 GW from NS-NS mergers
(Multi-messenger Astrophysics)
Tidal deformability of NS

2021

Astro-Hadron Physics

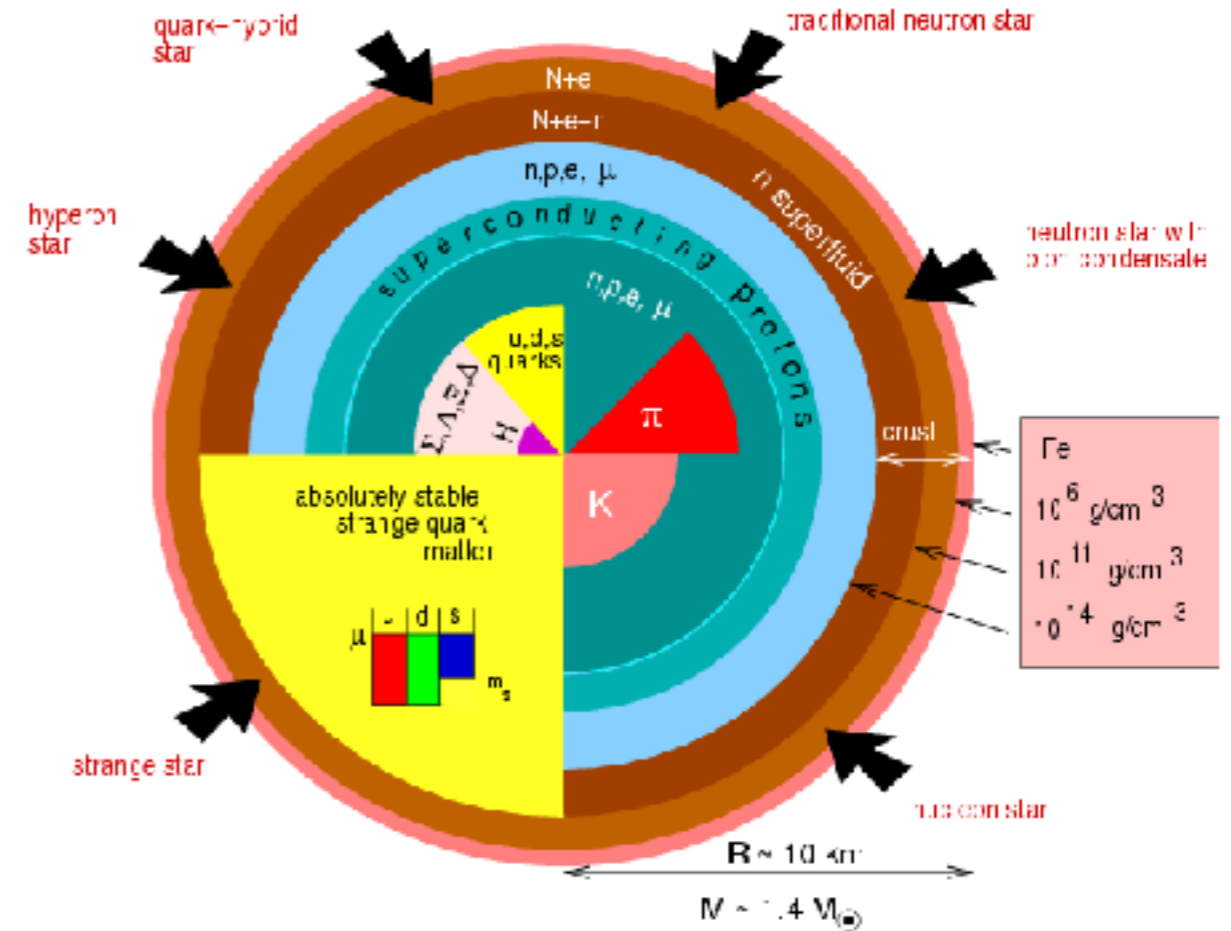
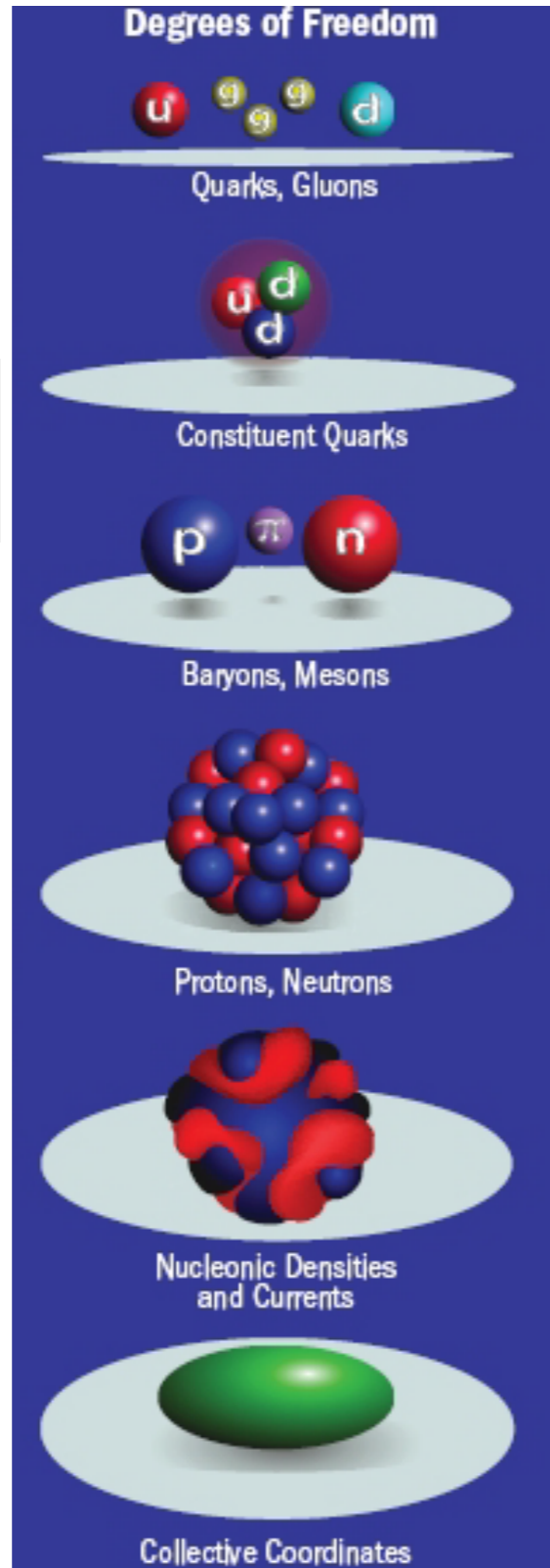
QCD

$$E > \Lambda_{\text{QCD}}$$

$$\Lambda_{\text{QCD}}$$

$$E < \Lambda_{\text{QCD}}$$

Effective Theories



QCD + Effective Theories



Neutron Star

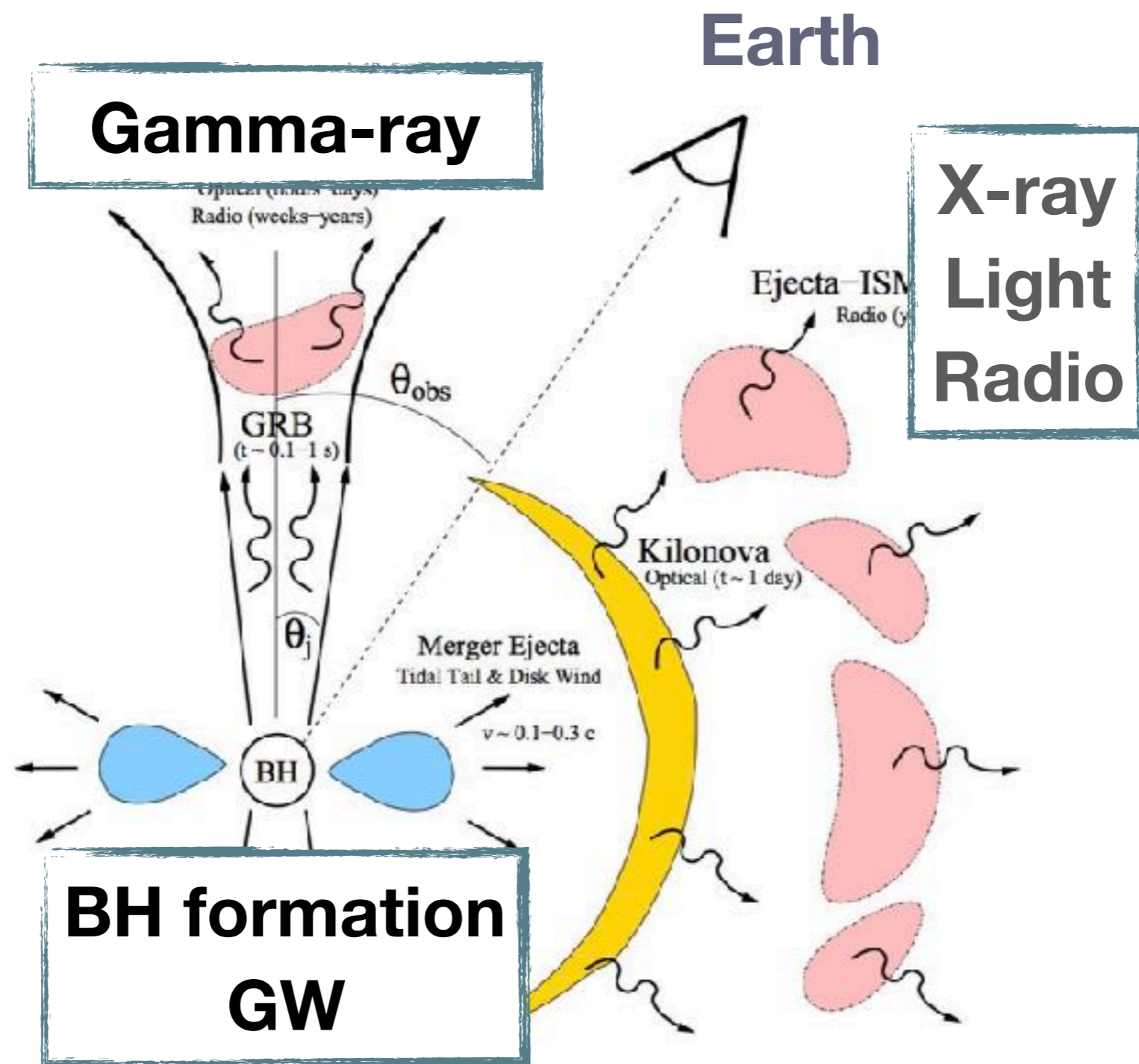
Figure: taken from G.F. Bertsch, D.J. Dean, and W. Nazarewicz, SciDAC Review 6, 42 (2007)

GW 170817 ($d=40$ Mpc)

GRB 170817A by Fermi-GBM

Kilonova/X-ray/Optical Afterglows

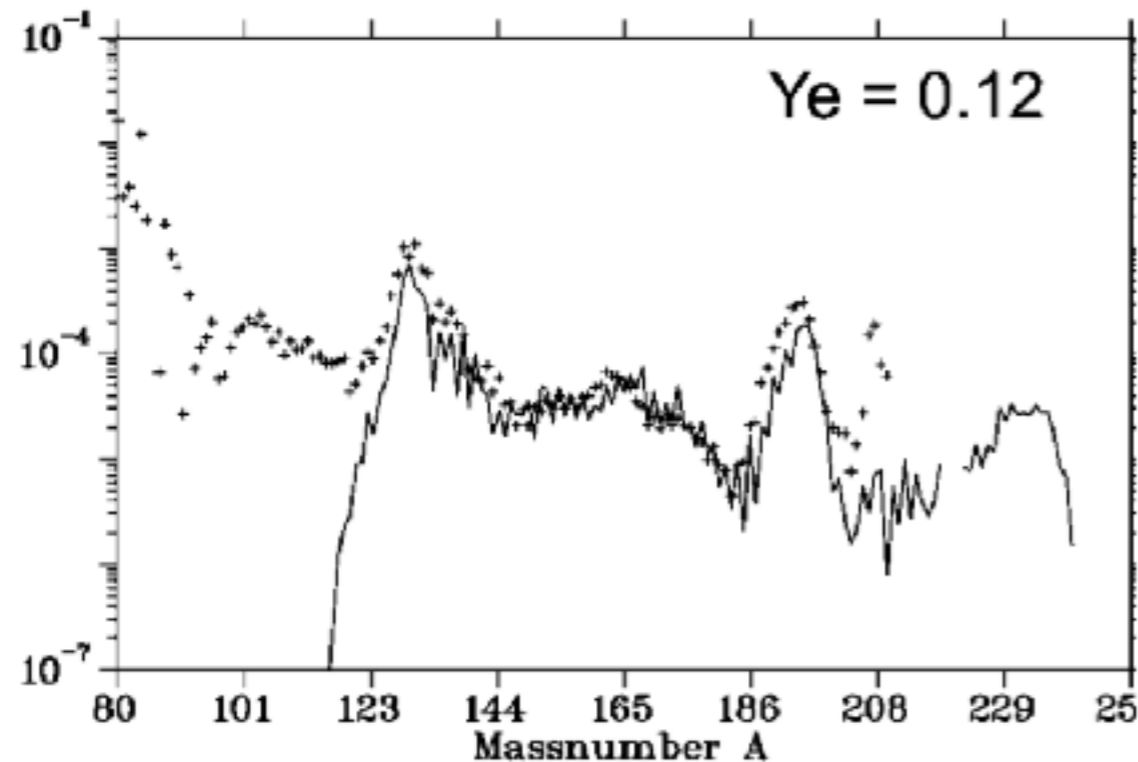
NS binary merger



Heavy Elements from NS mergers

Sources of Heavy Elements

S Rosswog 2015



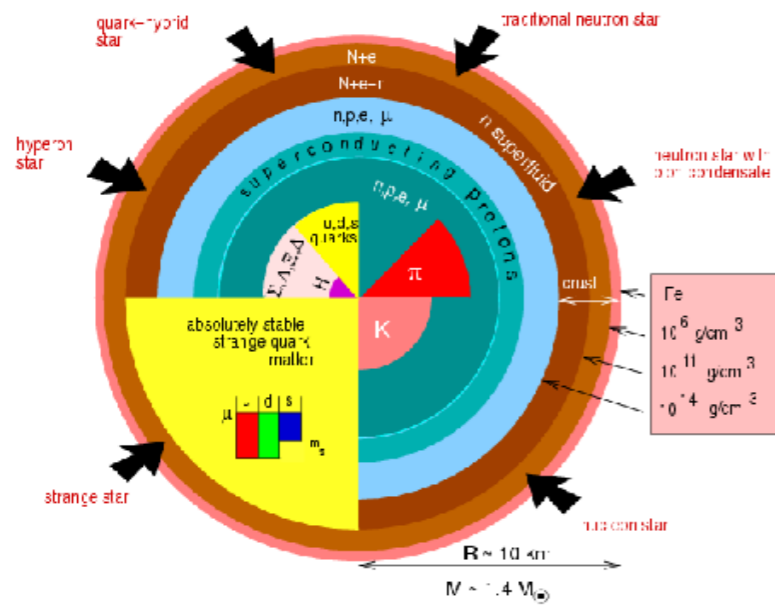
solar pattern vs NS-merger

- **Supernovae:**
neutrino-driven wind
r-process **peak at $A \sim 130$**
- **NS mergers:**
r-process **peak at $A \sim 195$**

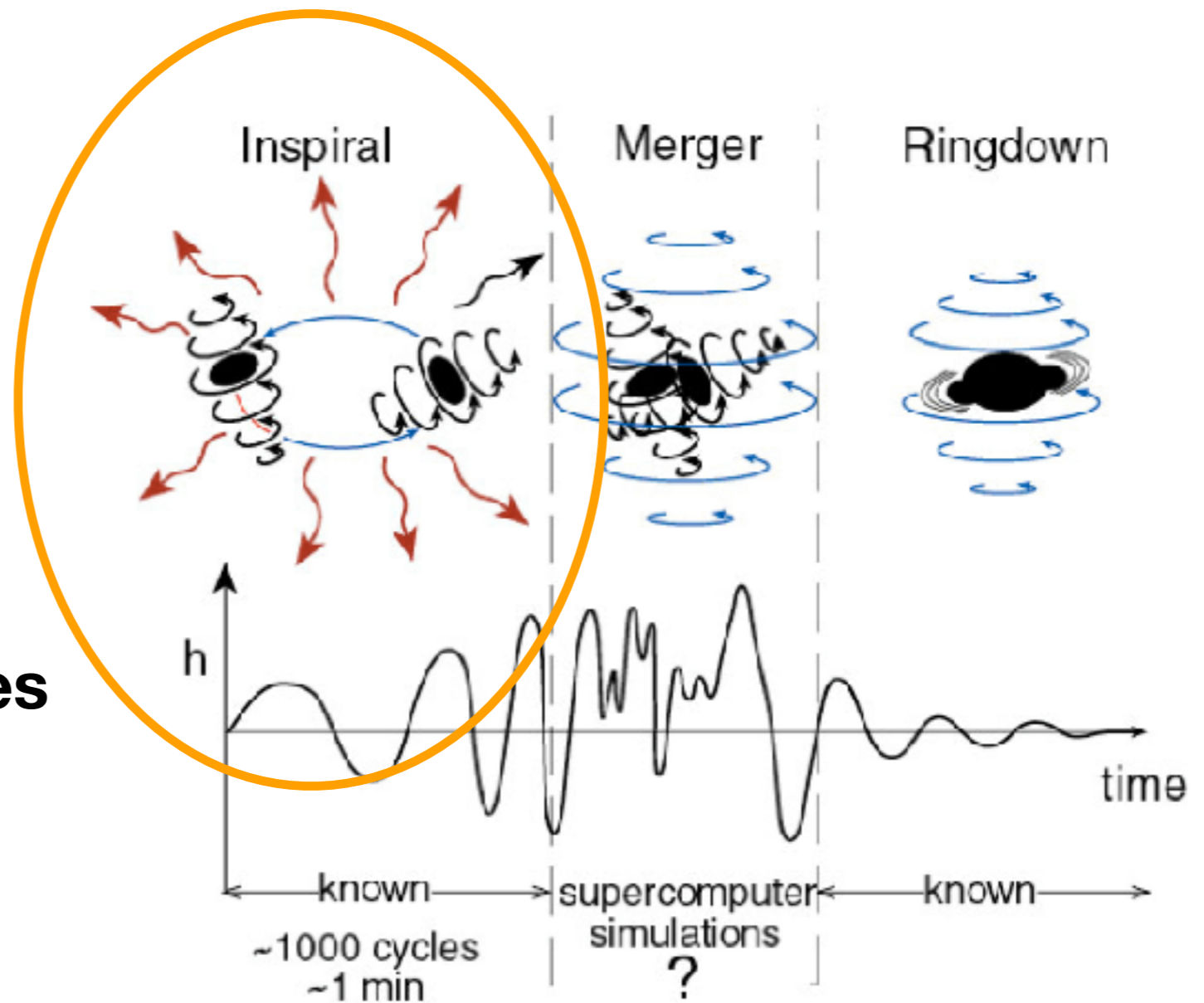
NS in new era of GW & multi-messenger astronomy

Tidal Love number & Deformability

Response of NS to GW during Inspiral



perturbative approaches



Tidal deformability & Love number

Selected references

- **A.E.H. Love** (1909) - The Yielding of the Earth to Disturbing Forces
- **K.S. Thorne** & A. Campolattaro (1967) - non-radial pulsation of NS
- J.B. Hartle & **K.S. Thorne** (1969) - stability of rotating NS
-
- **K.S. Thorne** (1998) - Tidal stabilization of rigid rotating, fully relativistic neutron star
-



Tidal deformability & Love number

$$-\frac{(1 + g_{tt})}{2} = -\frac{m}{r} - \frac{3Q_{ij}}{2r^3} \left(n^i n^j - \frac{1}{3} \delta^{ij} \right) + \mathcal{O} \left(\frac{1}{r^3} \right) + \frac{\mathcal{E}_{ij}}{2} r^2 n^i n^j + \mathcal{O}(r^3)$$

\mathcal{E}_{ij} : external quadrupole tidal field

Q_{ij} : quadrupole moment of NS

λ : Tidal deformability

$$Q_{ij} = -\lambda \mathcal{E}_{ij}$$

$$Q_{ij} = \int d^3x \delta\rho(x) \left(x_i x_j - \frac{1}{3} r^2 \delta_{ij} \right)$$

$$n^i = \frac{x^i}{r}$$

dimensionless parameter

k_2 : $l = 2$ Tidal Love number

$$k_2 = \frac{3}{2} G \lambda R^{-5}$$

Hinderer et al. PRD 81 (2010)

Systematic Parameter Errors in Inspiring Neutron Star Binaries

Marc Favata*

$$\tilde{h}_T(f) = \mathcal{A} f^{-7/6} e^{i\Psi_T(f)}$$

$$\Psi_T(f) = \varphi_c + 2\pi f t_c + \frac{3}{128\eta v^5} (\Delta\Psi_{3.5\text{PN}}^{\text{pp}} + \Delta\Psi_{3\text{PN}}^{\text{spin}} + \Delta\Psi_{2\text{PN}}^{\text{ecc.}} + \Delta\Psi_{6\text{PN}}^{\text{tidal}} + \Delta\Psi_{6\text{PN}}^{\text{tm}})$$

$$v = (\pi f M)^{1/3}$$

$$v/c = (GM\pi f/c^3)^{1/3}$$

$$\Delta\Psi_{6\text{PN}}^{\text{tidal}} = -\frac{39}{2} \tilde{\Lambda} v^{10} + v^{12} \left(\frac{6595}{364} \delta\tilde{\Lambda} - \frac{3115}{64} \tilde{\Lambda} \right)$$

5PN

6PN

$$\tilde{\Lambda} \equiv 32 \frac{\tilde{\lambda}}{M^5} = \frac{8}{13} [(1 + 7\eta - 31\eta^2)(\hat{\lambda}_1 + \hat{\lambda}_2) - \sqrt{1 - 4\eta(1 + 9\eta - 11\eta^2)}(\hat{\lambda}_1 - \hat{\lambda}_2)].$$

phase shift vs deformability

$$\left. \frac{d\Psi_T}{dx} \right|_{\text{tidal,5PN}} = -\frac{195}{8} \frac{x^{3/2}}{\eta} \frac{\tilde{\lambda}}{M^5} \propto \frac{\tilde{\lambda}}{M^5}$$

$$x = (\omega M)^{2/3} \Rightarrow \left(\omega \frac{GM}{c^3} \right)^{2/3}$$

$$\eta = m_1 m_2 / M^2$$

dimensionless

$$\Lambda = \frac{\lambda}{m^5} \Rightarrow G\lambda \left(\frac{c^2}{Gm} \right)^5 \approx 950.5 \left(\frac{m_\odot}{m} \right)^5 \left(\frac{\lambda}{10^{36} \text{ g cm}^2 \text{ s}^2} \right)$$

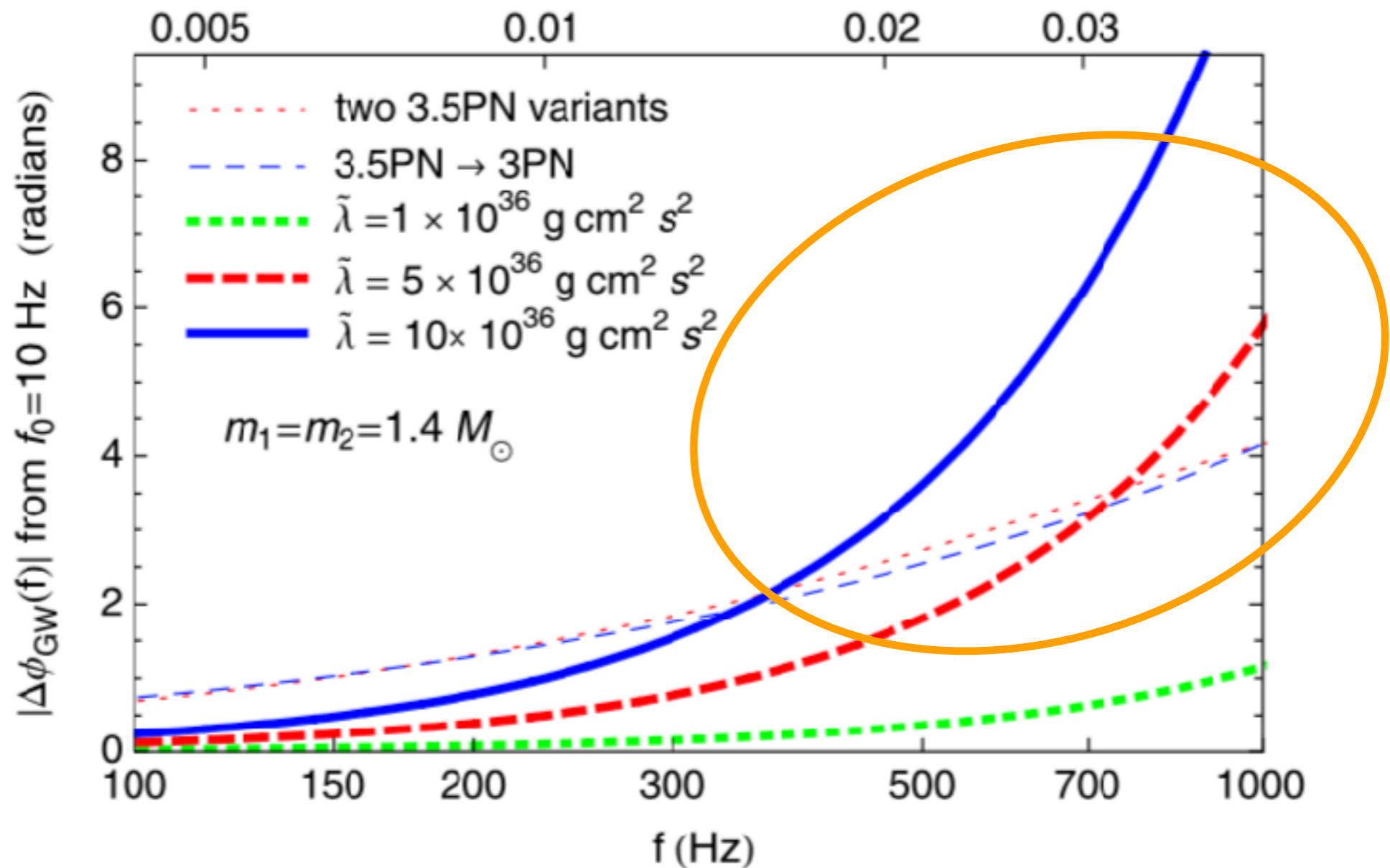
$$\Lambda = G \left(\frac{c^2}{Gm} \right)^5 \times \frac{2}{3} \frac{R^5}{G} k_2 = \frac{2}{3} \left(\frac{Rc^2}{Gm} \right)^5 k_2 \approx 9495 \left(\frac{R_{10\text{km}}}{m_{M_\odot}} \right)^5 k_2$$

Tidal deformability of neutron stars with realistic equations of state and their gravitational wave signatures in binary inspiral

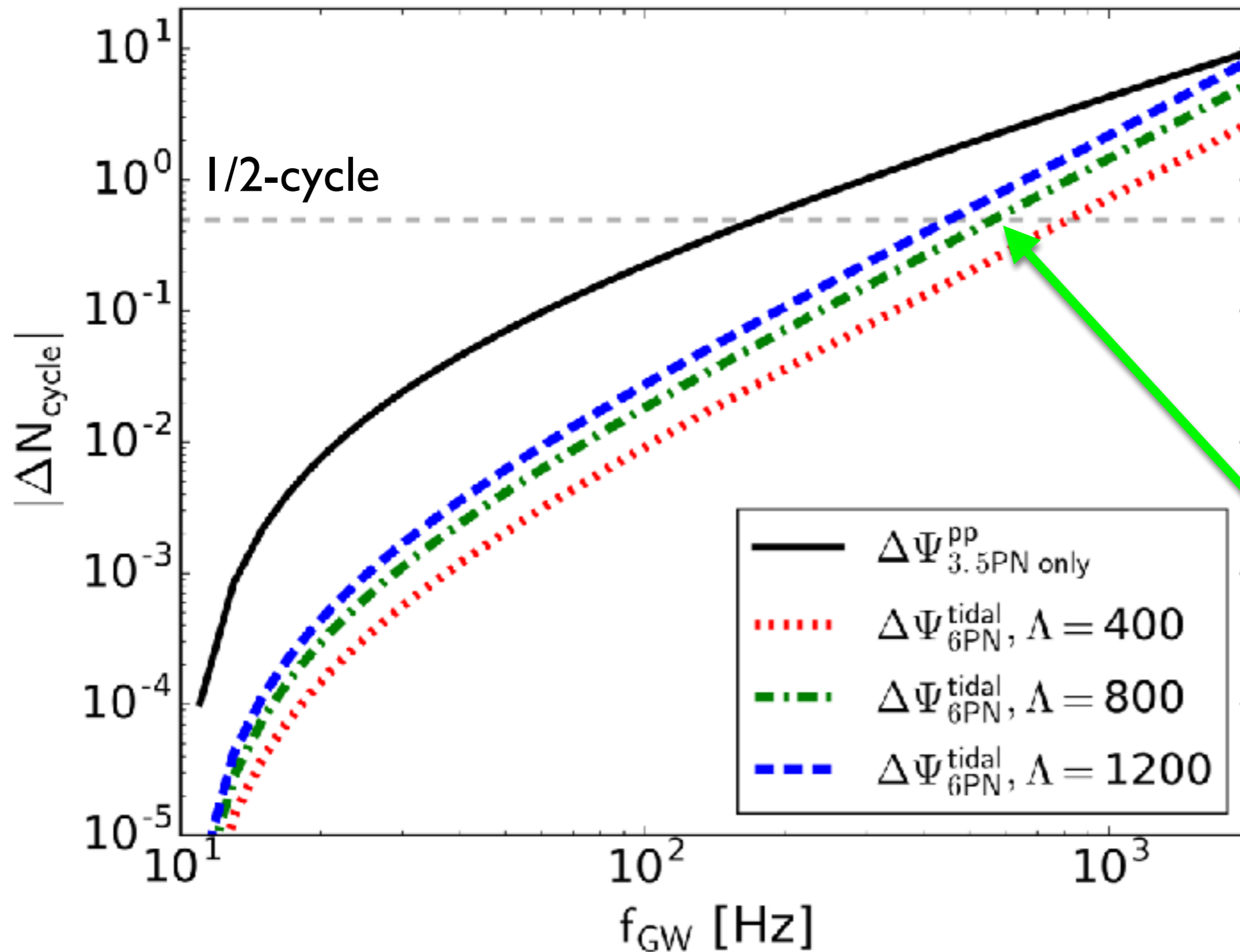
Tanja Hinderer,¹ Benjamin D. Lackey,² Ryan N. Lang,^{3,4} and Jocelyn S. Read⁵

accumulated GW phase

$$|\Delta\phi_{\text{GW}}(f)| = \left| \Psi(f)_{\text{pp}(3.5\text{PN})} - \Psi(f)_{\text{pp}(3.5\text{PN})+\text{tidal}(5\text{PN})} \right|$$



accumulated GW phase



waveform model:
TaylorF2(SPA)

$M_{\text{ch}} = 1.188 M_{\odot}$

$M_1 = M_2 = 1.365 M_{\odot}$

~ 600 Hz



GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral

TABLE I. Source properties for GW170817: we give ranges encompassing the 90% credible intervals for different assumptions of the waveform model to bound systematic uncertainty. The mass values are quoted in the frame of the source, accounting for uncertainty in the source redshift.

	Low-spin priors ($ \chi \leq 0.05$)	High-spin priors ($ \chi \leq 0.89$)
Primary mass m_1	$1.36\text{--}1.60 M_\odot$	$1.36\text{--}2.26 M_\odot$
Secondary mass m_2	$1.17\text{--}1.36 M_\odot$	$0.86\text{--}1.36 M_\odot$
Chirp mass \mathcal{M}	$1.188^{+0.004}_{-0.002} M_\odot$	$1.188^{+0.004}_{-0.002} M_\odot$
Mass ratio m_2/m_1	$0.7\text{--}1.0$	$0.4\text{--}1.0$
Total mass m_{tot}	$2.74^{+0.04}_{-0.01} M_\odot$	$2.82^{+0.47}_{-0.09} M_\odot$
Radiated energy E_{rad}	$> 0.025 M_\odot c^2$	$> 0.025 M_\odot c^2$
Luminosity distance D_L	40^{+8}_{-14} Mpc	40^{+8}_{-14} Mpc
Viewing angle Θ	$\leq 55^\circ$	$\leq 56^\circ$
Using NGC 4993 location	$\leq 28^\circ$	$\leq 28^\circ$
Combined dimensionless tidal deformability $\tilde{\Lambda}$	≤ 800	≤ 700
Dimensionless tidal deformability $\Lambda(1.4M_\odot)$	≤ 800	≤ 1400

GW170817
Information of Neutron Star Structure
has been revealed by Gravitational Waves

(revised) properties of GW170817

Abbott et al. (LSC and Virgo), arxiv:1805.11579

	Low-spin prior ($\chi \leq 0.05$)	High-spin prior ($\chi \leq 0.89$)
Binary inclination θ_{JN}	146_{-27}^{+25} deg	152_{-27}^{+21} deg
Binary inclination θ_{JN} using EM distance constraint [104]	151_{-11}^{+15} deg	153_{-11}^{+15} deg
Detector frame chirp mass \mathcal{M}^{det}	$1.1975_{-0.0001}^{+0.0001} M_{\odot}$	$1.1976_{-0.0002}^{+0.0004} M_{\odot}$
Chirp mass \mathcal{M}	$1.186_{-0.001}^{+0.001} M_{\odot}$	$1.186_{-0.001}^{+0.001} M_{\odot}$
Primary mass m_1	(1.36, 1.60) M_{\odot}	(1.36, 1.89) M_{\odot}
Secondary mass m_2	(1.16, 1.36) M_{\odot}	(1.00, 1.36) M_{\odot}
Total mass m	$2.73_{-0.01}^{+0.04} M_{\odot}$	$2.77_{-0.05}^{+0.22} M_{\odot}$
Mass ratio q	(0.73, 1.00)	(0.53, 1.00)
Effective spin χ_{eff}	$0.00_{-0.01}^{+0.02}$	$0.02_{-0.02}^{+0.08}$
Primary dimensionless spin χ_1	(0.00, 0.04)	(0.00, 0.50)
Secondary dimensionless spin χ_2	(0.00, 0.04)	(0.00, 0.61)
Tidal deformability $\tilde{\Lambda}$ with flat prior	300_{-190}^{+500} (symmetric) / 300_{-230}^{+420} (HPD)	(0, 630)

300_{-190}^{+500} (symmetric) / 300_{-230}^{+420} (HPD)

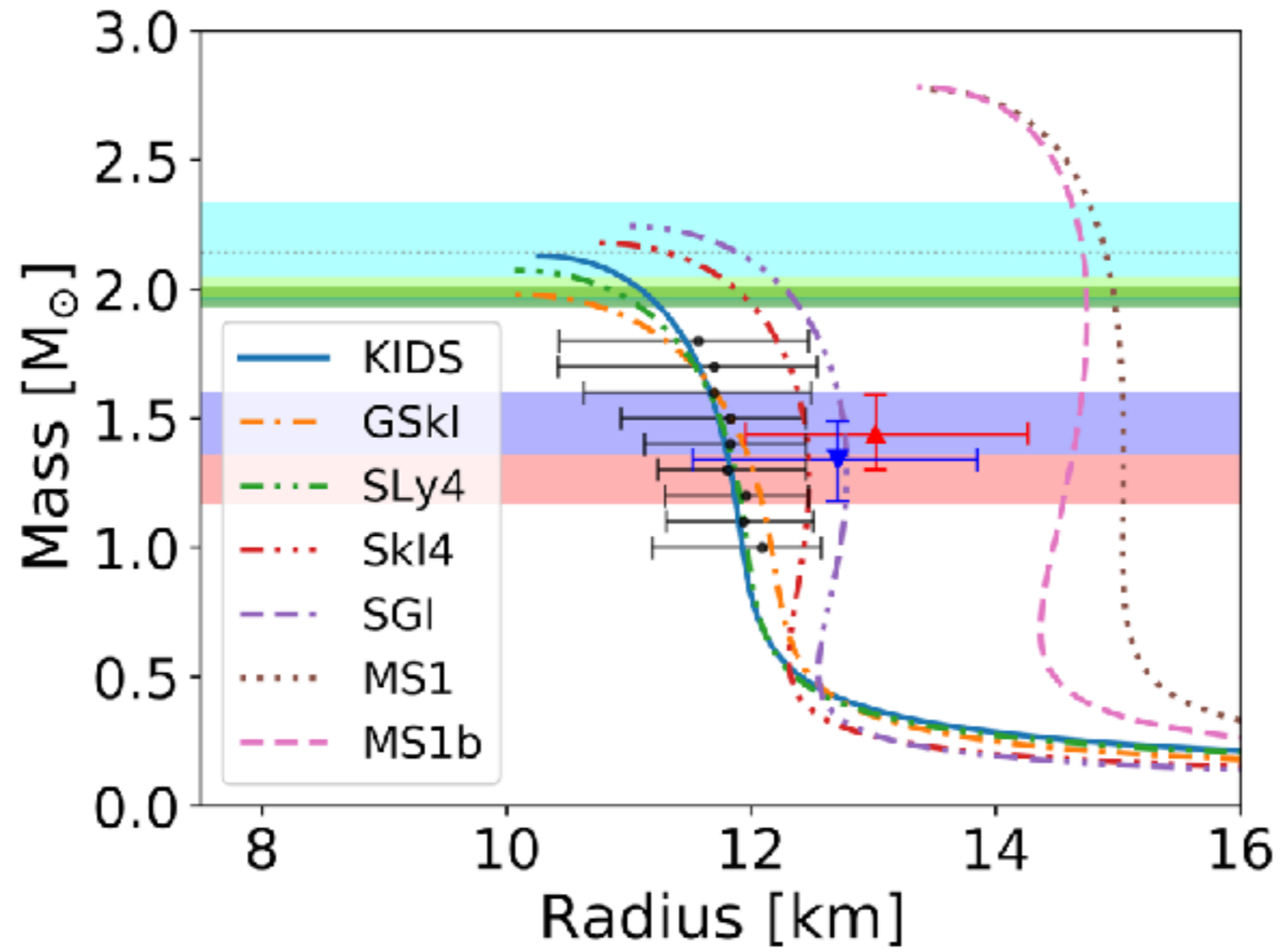
(0, 630)

GW190425

	Low-spin prior ($\chi < 0.05$)	High-spin prior ($\chi < 0.89$)
Primary mass m_1	$1.62 - 1.88 M_\odot$	$1.61 - 2.52 M_\odot$
Secondary mass m_2	$1.45 - 1.69 M_\odot$	$1.12 - 1.68 M_\odot$
Chirp mass \mathcal{M}	$1.44^{+0.02}_{-0.02} M_\odot$	$1.44^{+0.02}_{-0.02} M_\odot$
Detector-frame chirp mass	$1.4868^{+0.0003}_{-0.0003} M_\odot$	$1.4873^{+0.0008}_{-0.0006} M_\odot$
Mass ratio m_2/m_1	$0.8 - 1.0$	$0.4 - 1.0$
Total mass m_{tot}	$3.3^{+0.1}_{-0.1} M_\odot$	$3.4^{+0.3}_{-0.1} M_\odot$
Effective inspiral spin parameter χ_{eff}	$0.013^{+0.01}_{-0.01}$	$0.058^{+0.11}_{-0.05}$
Luminosity distance D_L	$161^{+67}_{-73} \text{ Mpc}$	$159^{+69}_{-71} \text{ Mpc}$
Combined dimensionless tidal deformability $\bar{\Lambda}$	≤ 600	≤ 1100

GW190425

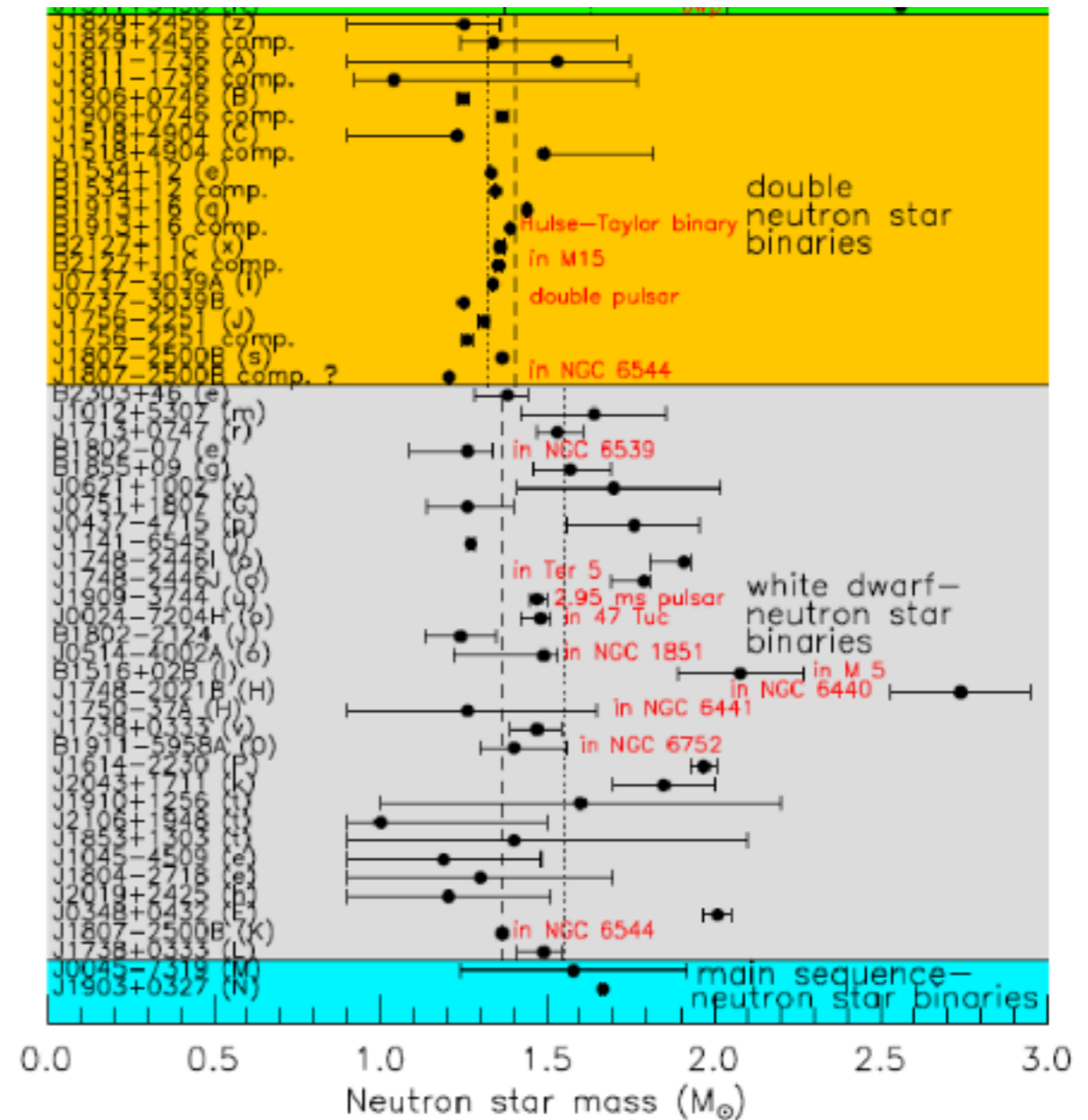
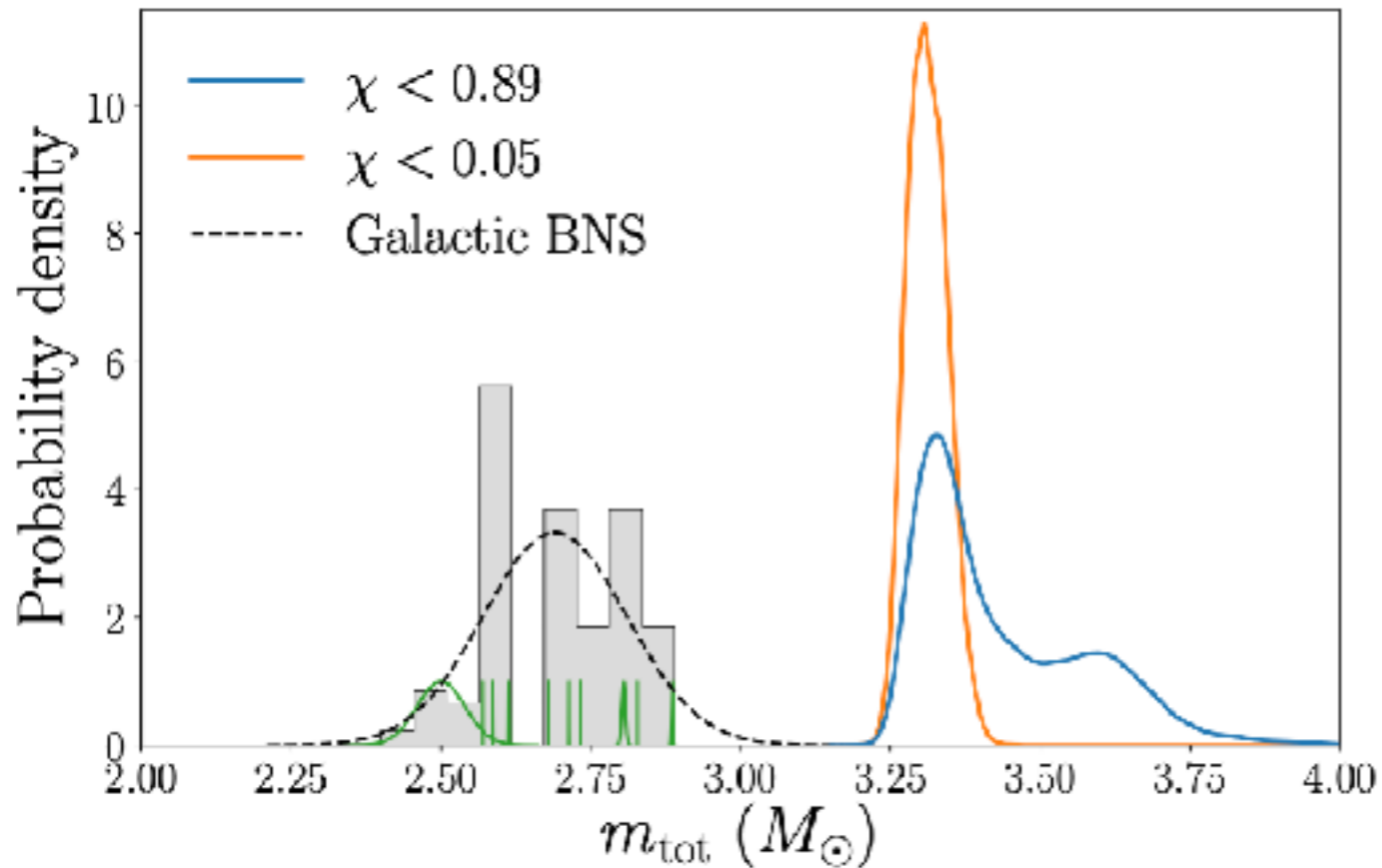
	Low-spin prior ($\chi < 0.05$)	High-spin prior ($\chi < 0.89$)
Primary mass m_1	$1.62 - 1.88 M_\odot$	$1.61 - 2.52 M_\odot$
Secondary mass m_2	$1.45 - 1.69 M_\odot$	$1.12 - 1.68 M_\odot$



$$2.14^{+0.20}_{-0.18} M_\odot$$

H. T. Cromartie, *et al.*,
 Nature Astronomy (2019).

GW190425: Observation of a Compact Binary Coalescence with Total Mass $\sim 3.4 M_{\odot}$



PHYSICAL REVIEW C **98**, 065805 (2018)

Tidal deformability of neutron stars with realistic nuclear energy density functionals

Young-Min Kim,¹ Yeunhwan Lim,² Kyujin Kwak,¹ Chang Ho Hyun,³ and Chang-Hwan Lee⁴

Neutron star equation of state and tidal deformability with nuclear energy density functionals

Young-Min Kim¹, Kyujin Kwak¹, Chang Ho Hyun², and Chang-Hwan Lee^{3a}

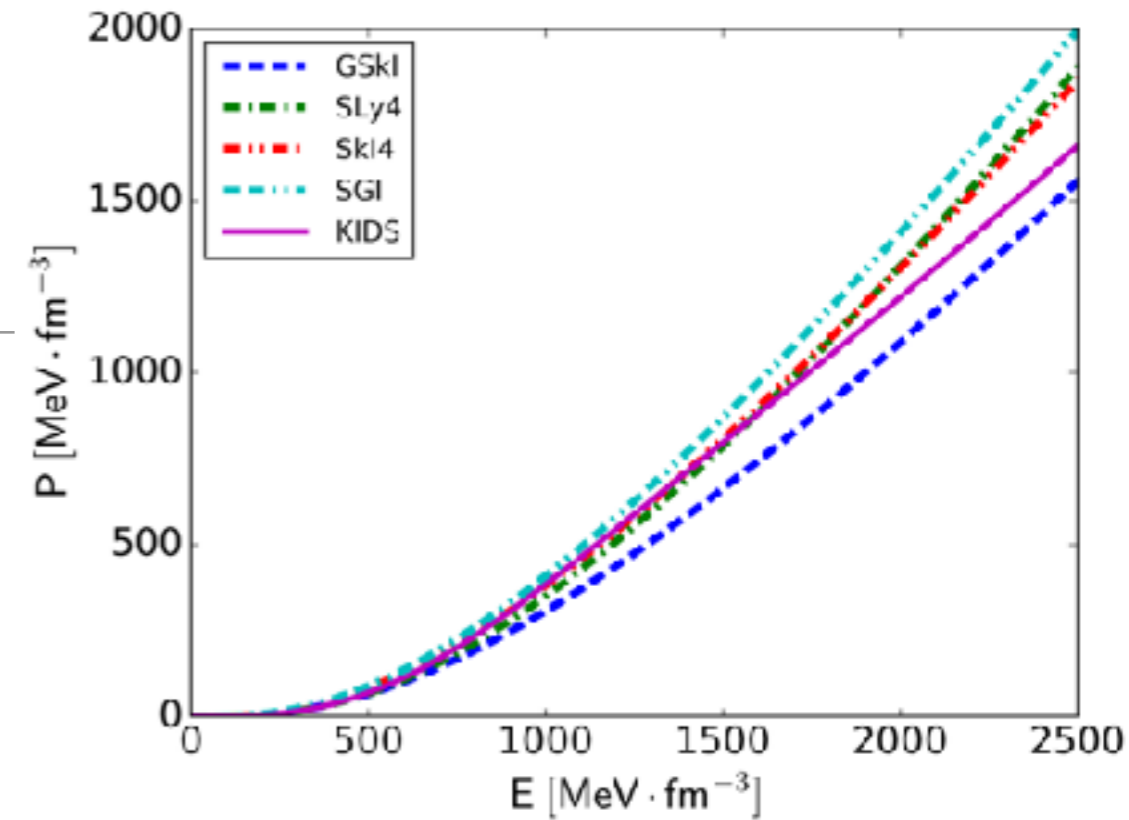
Constraints on Nuclear EoS

- Nuclear data: hundreds of models (Skyrme force, RMF, ...)
- Neutron star maximum mass
 $1.97 \pm 0.04 M_{\odot}$ [Nature 467, 1081 (2010)]
 $2.01 \pm 0.04 M_{\odot}$ [Science 340, 448 (2013)]
- II experimental/empirical data for nuclear matter around saturation density [Phys.Rev. C 85, 035201 (2012)]

Constraint	Quantity	Eq.	Density Region	Range of constraint exp/emp	Range of constraint from CSkP	Ref.
SM1	K_{\circ}	(7), (15)	ρ_{\circ} (fm ⁻³)	200 – 260 MeV	202.0 – 240.3 MeV	[64]
SM2	$K' = -Q_{\circ}$	(8), (16)	ρ_{\circ} (fm ⁻³)	200 – 1200 MeV	362.5 – 425.6 MeV	[65]
SM3	$P(\rho)$	(6)	$2 < \frac{\rho}{\rho_{\circ}} < 3$	Band Region	see Fig. 1	[78]
SM4	$P(\rho)$	(6)	$1.2 < \frac{\rho}{\rho_{\circ}} < 2.2$	Band Region	see Fig. 2	[80]
PNM1	$\frac{E_{PNM}}{E_{PNM}^{\circ}}$	(31)	$0.014 < \frac{\rho}{\rho_{\circ}} < 0.106$	Band Region	see Fig. 3	[39, 40]
PNM2	$P(\rho)$	(6)	$2 < \frac{\rho}{\rho_{\circ}} < 3$	Band Region	see Fig. 5	[78]
MIX1	J	(9)	ρ_{\circ} (fm ⁻³)	30 – 35 MeV	30.0 – 35.5 MeV	[44]
MIX2	L	(10)	ρ_{\circ} (fm ⁻³)	40 – 76 MeV	48.6 – 67.1 MeV	[101]
MIX3	$K_{\tau, \nu}$	(21)	ρ_{\circ} (fm ⁻³)	-760 – -372 MeV	-407.1 – -360.1 MeV	[107]
MIX4	$\frac{S(\rho_{\circ}/2)}{J}$	-	ρ_{\circ} (fm ⁻³)	0.57 – 0.86	0.61 – 0.67	[110]
MIX5	$\frac{3P_{PNM}}{L\rho_{\circ}}$	(41)	ρ_{\circ} (fm ⁻³)	0.90 – 1.10	1.02 – 1.10	[112]

Selected EoSs

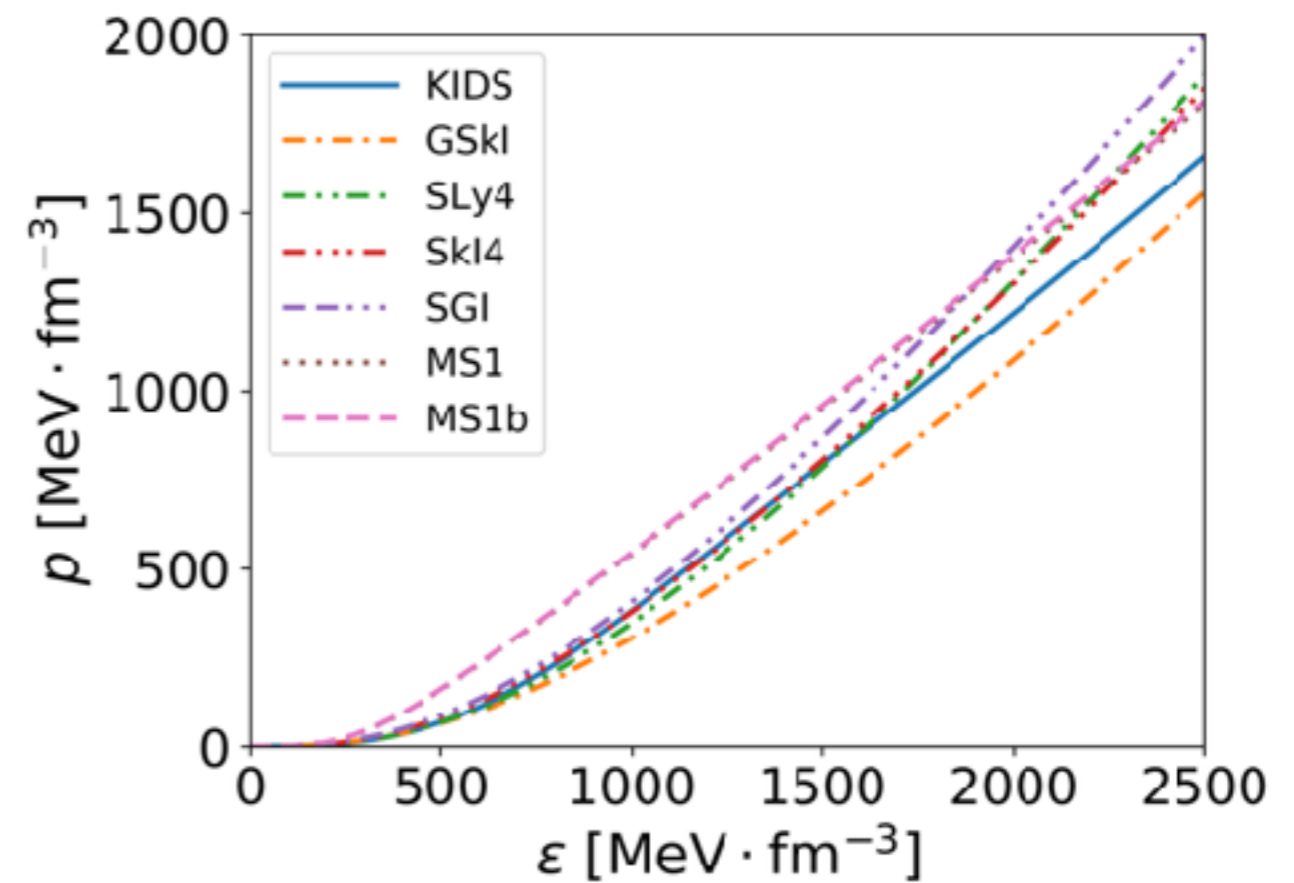
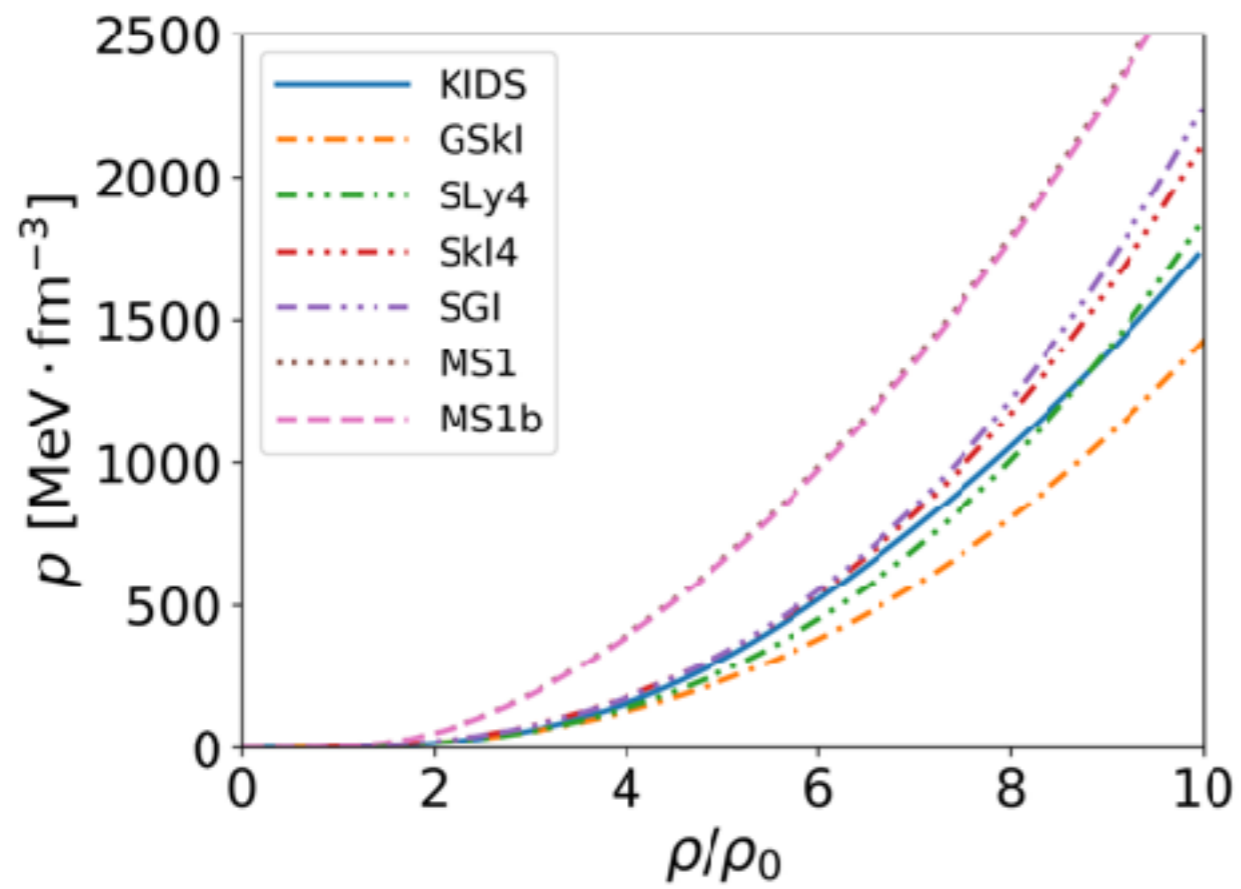
- Skyrme force models
- Basically fitted to properties of well-known nuclei
- Good saturation properties
- M_{\max} more than $2M_{\text{sun}}$



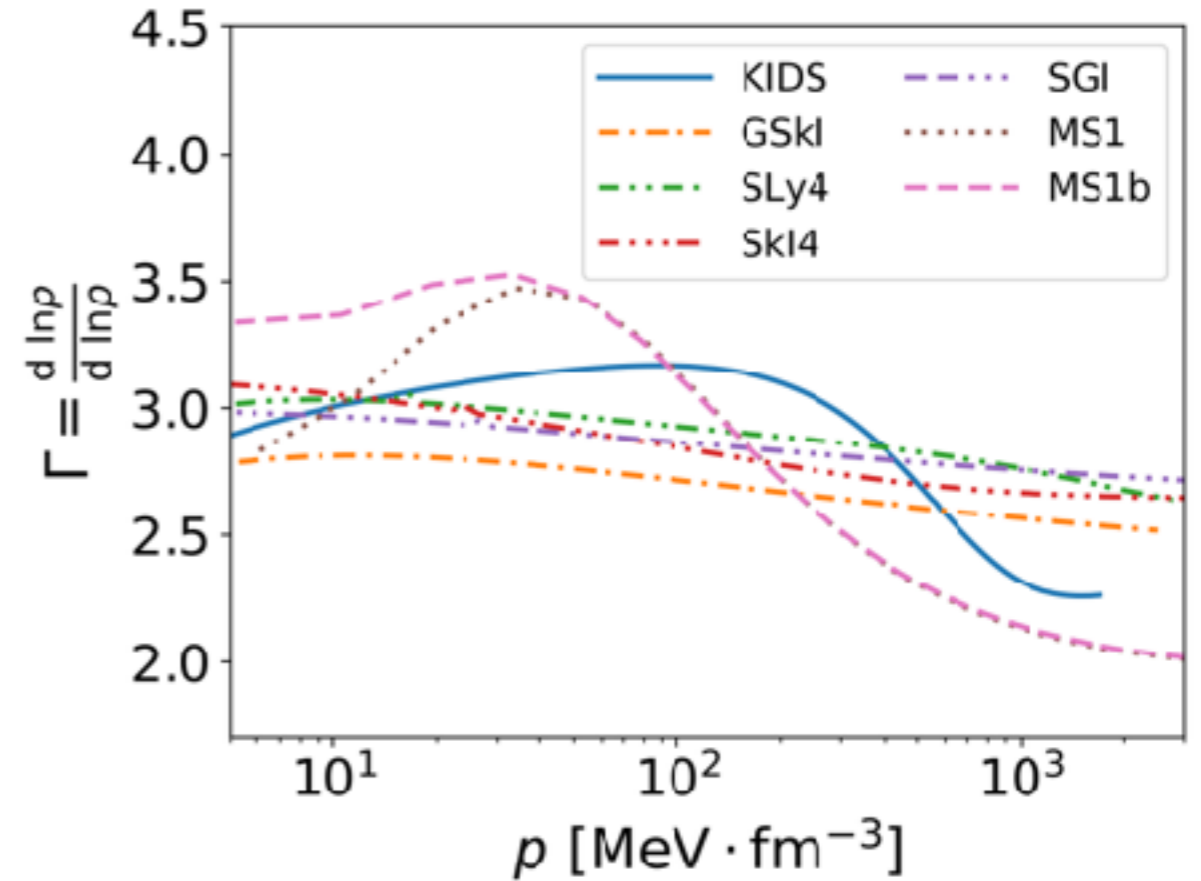
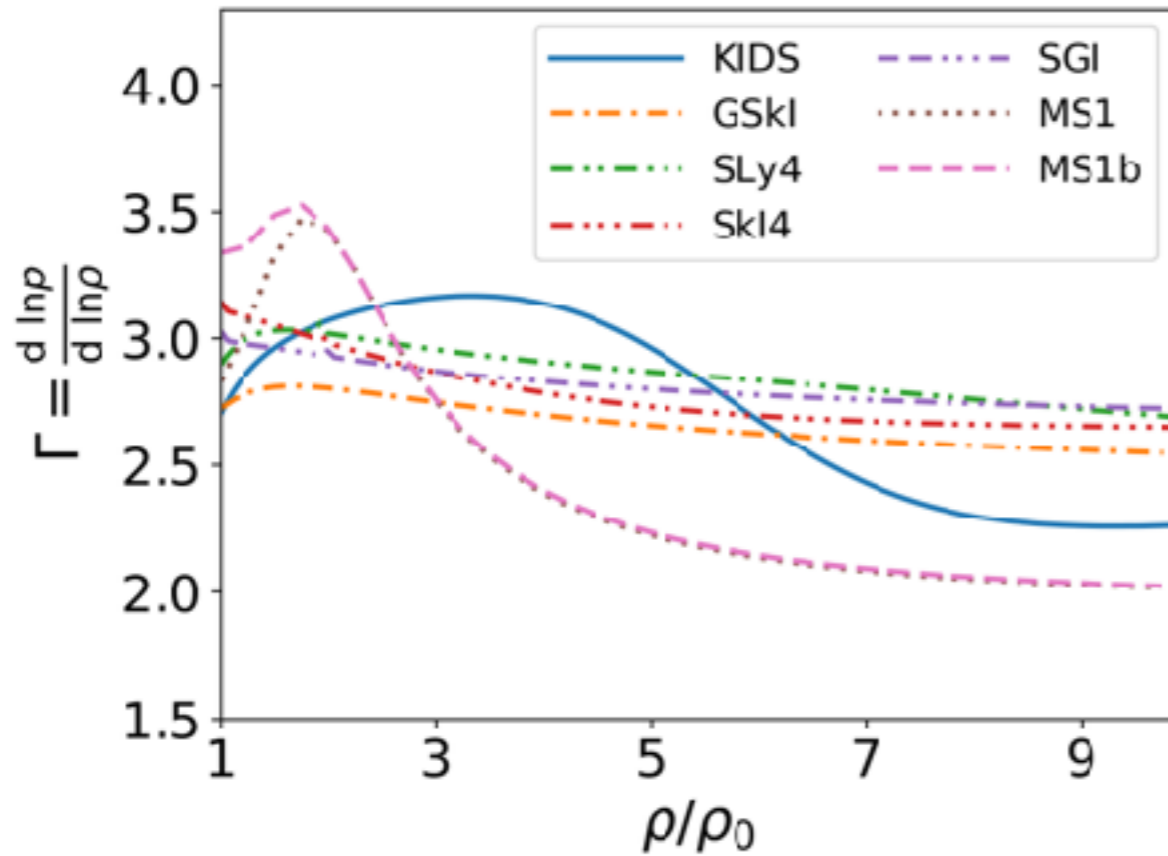
Model	ρ_0	E_0	K_0	$-Q_0$	J	L	$-K_\tau$	M_{\max}
Exp/Emp	$\simeq 0.16$	$\simeq 16.0$	200 ~ 260	200 ~ 1200	30 ~ 35	40 ~ 76	372 ~ 760	$\geq 1.93 \sim 2.05$
CSkP	-	-	202.0 ~ 240.3	362.5 ~ 425.6	30.0 ~ 35.5	48.6 ~ 67.1	360.1 ~ 407.1	-
GSkI	0.159	16.02	230.2	405.6	32.0	63.5	364.2	1.98
SLy4	0.160	15.97	229.9	363.1	32.0	45.9	322.8	2.07
SkI4	0.160	15.95	248.0	331.2	29.5	60.4	322.2	2.19
SGI	0.154	15.89	261.8	297.9	28.3	63.9	362.5	2.25
KIDS	0.160	16.00	240.0	372.7	32.8	49.1	375.1	2.14

KIDS (Korea: IBS-Daegu-Sungkyunkwan): A new systematic expansion scheme for nuclear EDF
 [Phys. Rev. C 97, 014312 (2018)]

Pressure

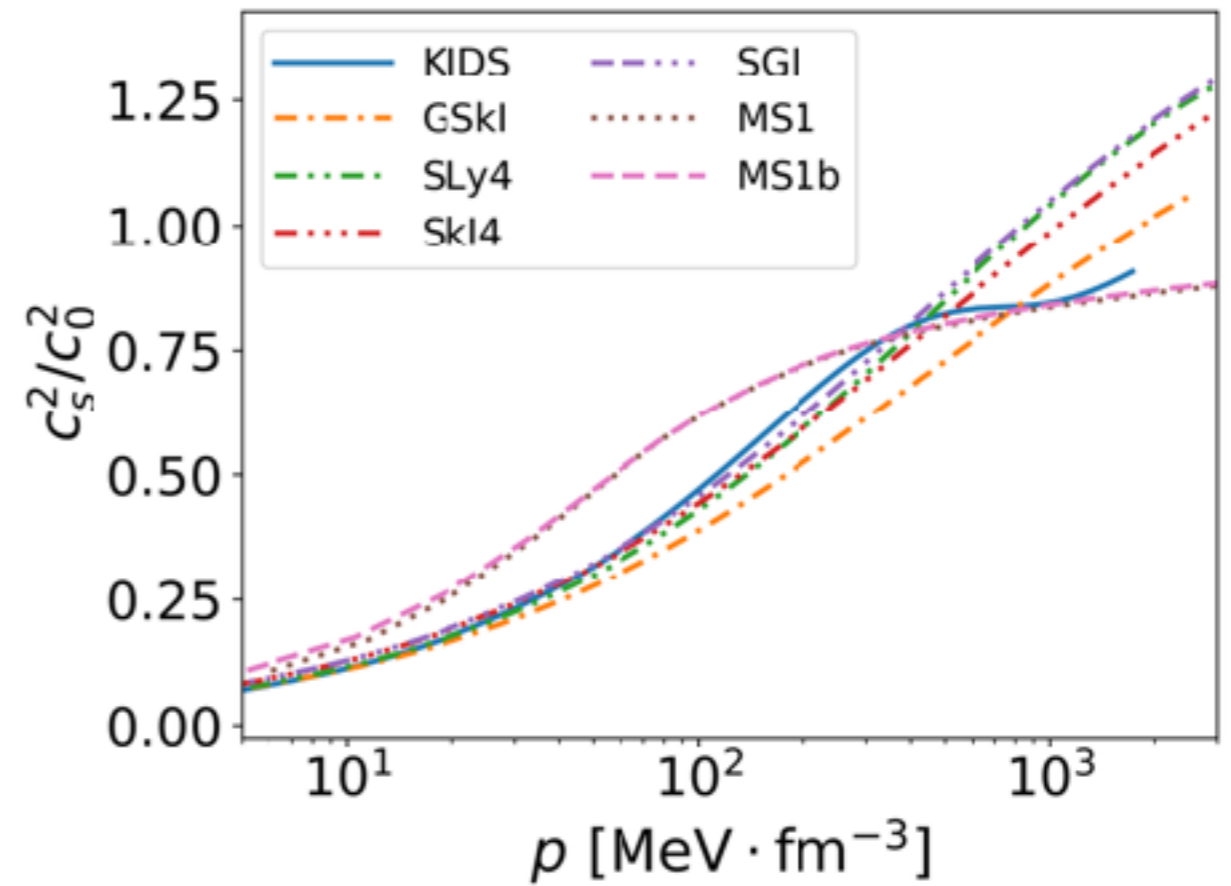
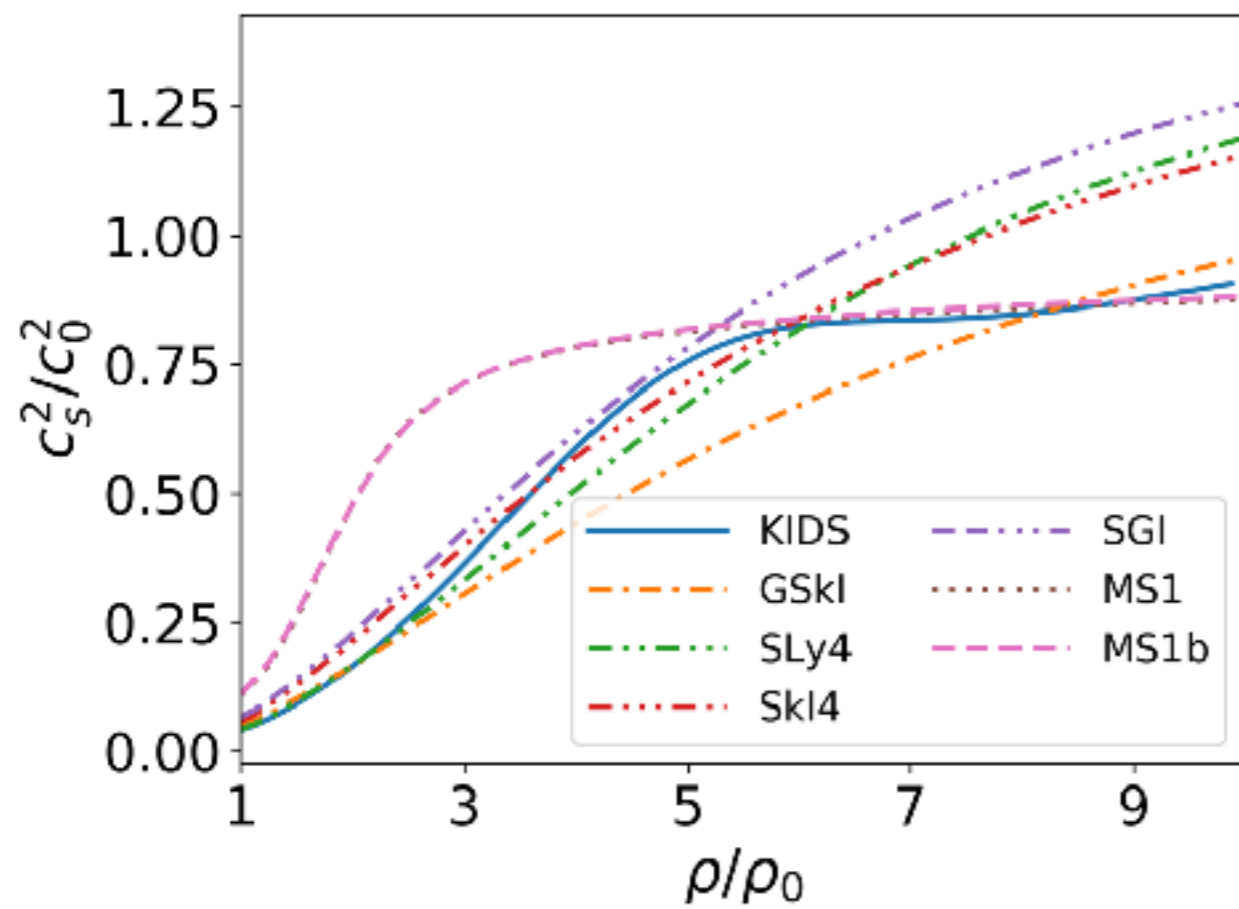


Adiabatic index

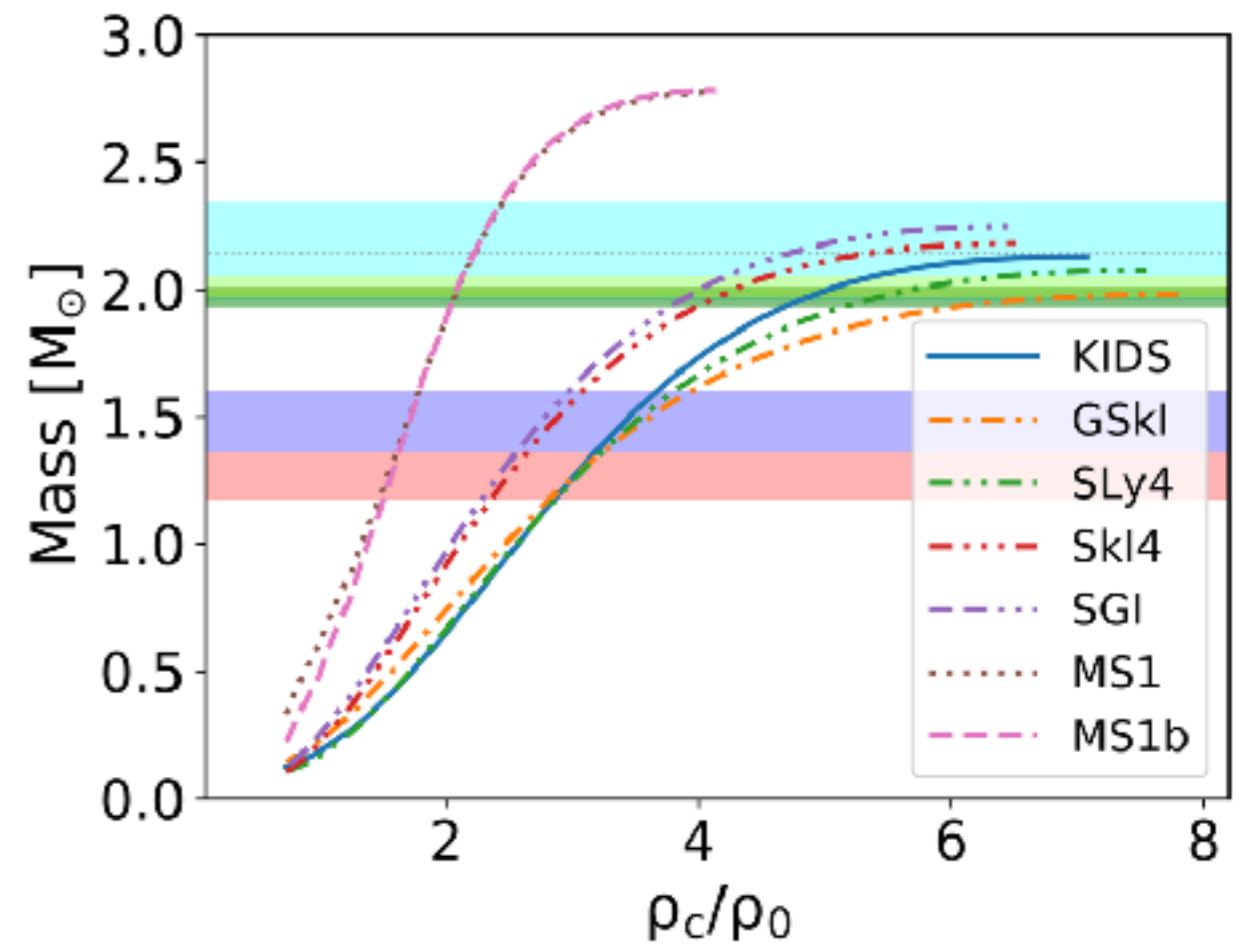
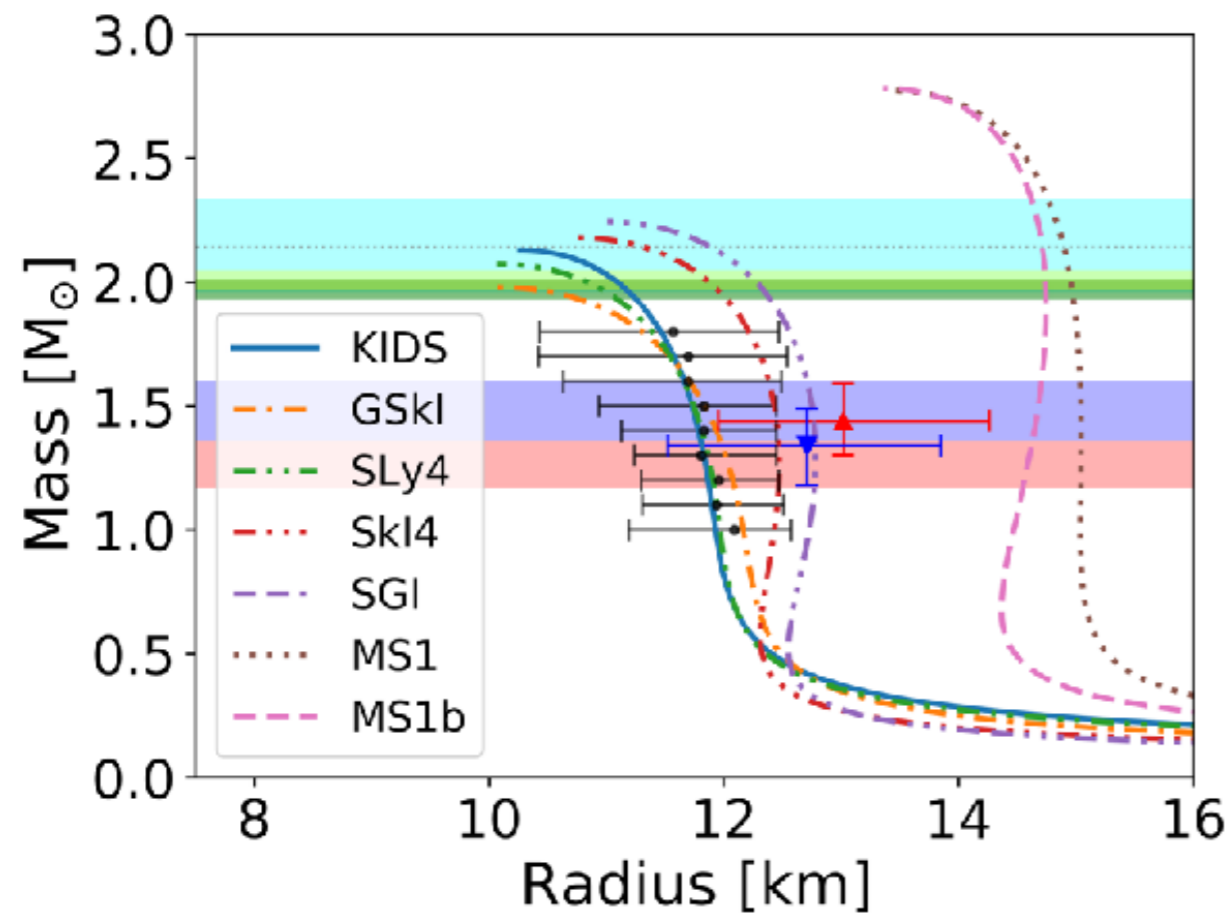


$$\Gamma(p) \equiv \frac{d(\ln p)}{d(\ln \rho)}$$

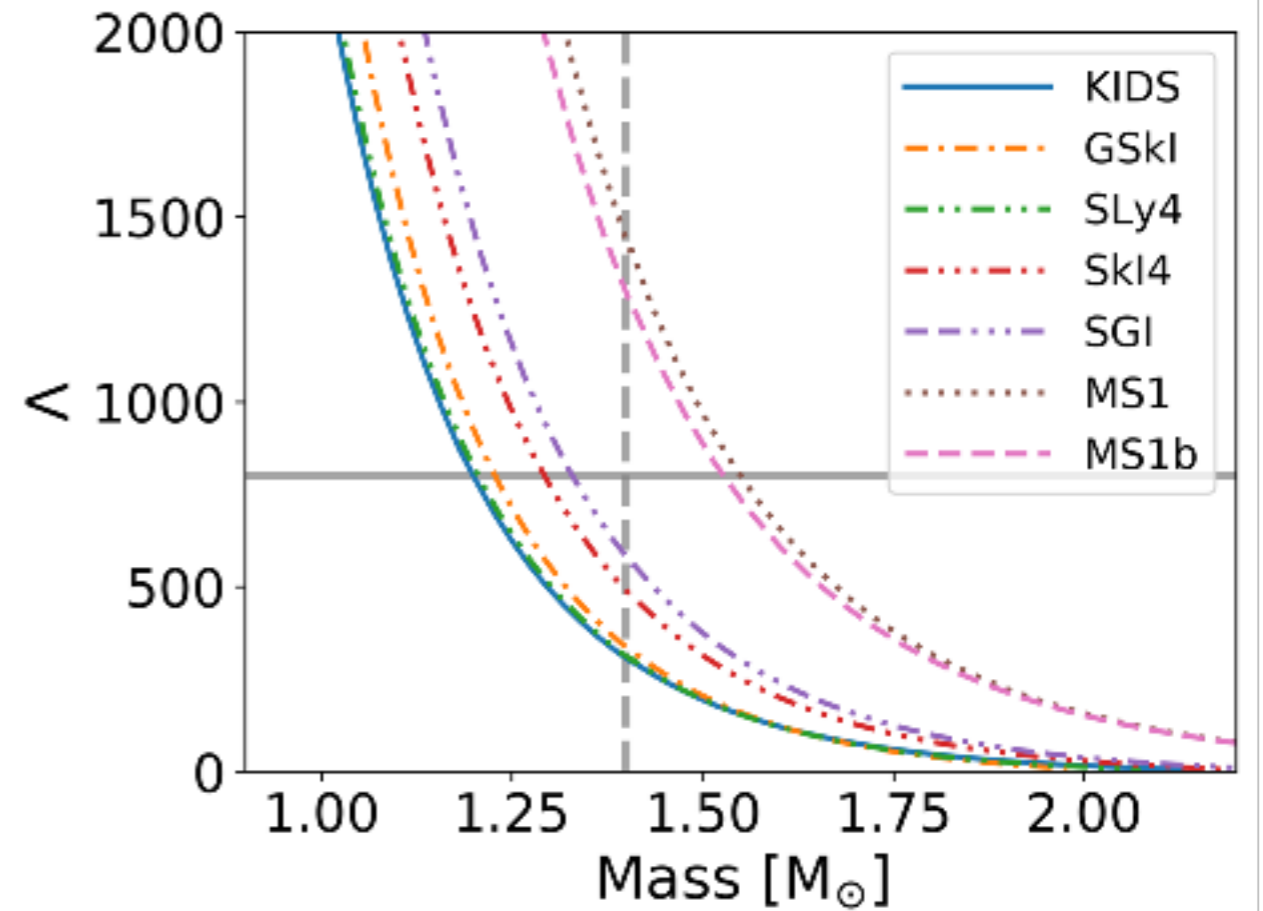
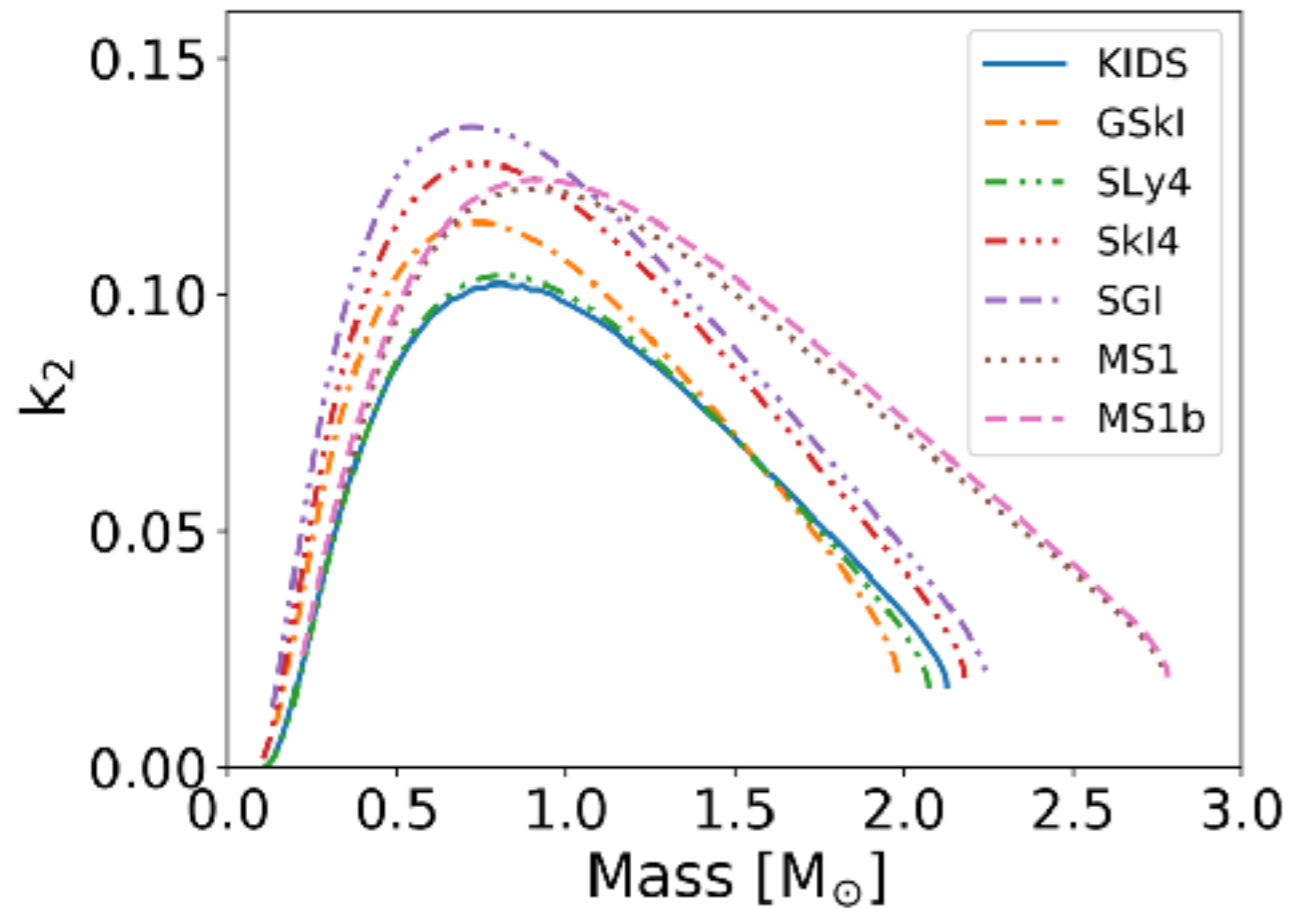
Sound speed



Neutron star properties



Tidal deformability

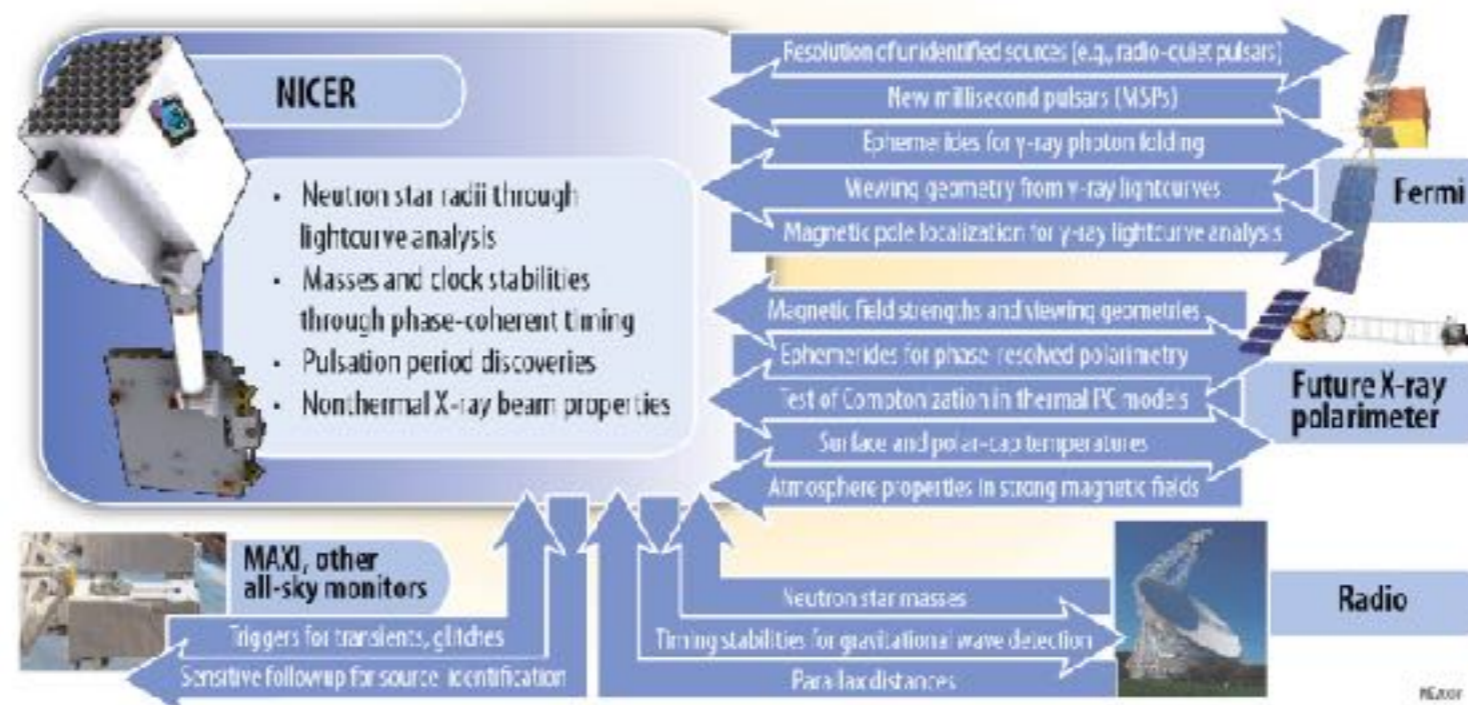


$$Q_{ij} = -\Lambda M^5 \mathcal{E}_{ij}$$

$$\Lambda = \frac{2}{3} \left(\frac{M}{R} \right)^{-5} k_2$$

NICER Neutron star Interior Composition ExploreR

- **launch:** early 2017, SpaceX
- **platform:** ISS ELC (ExPRESS Logistics Carrier)
- **instrument:** X-ray (0.2-12 keV)
- **objective**
 - **structure:** neutron star radii to 5%, cooling timescales
 - **dynamics:** stability of pulsars as clocks, properties of outbursts, oscillations, and precession
 - **energetics:** intrinsic radiation patterns, spectra, and luminosities



1. Mass

- $1.34^{+0.15}_{-0.16}$ Msun vs. $1.44^{+0.15}_{-0.14}$ Msun

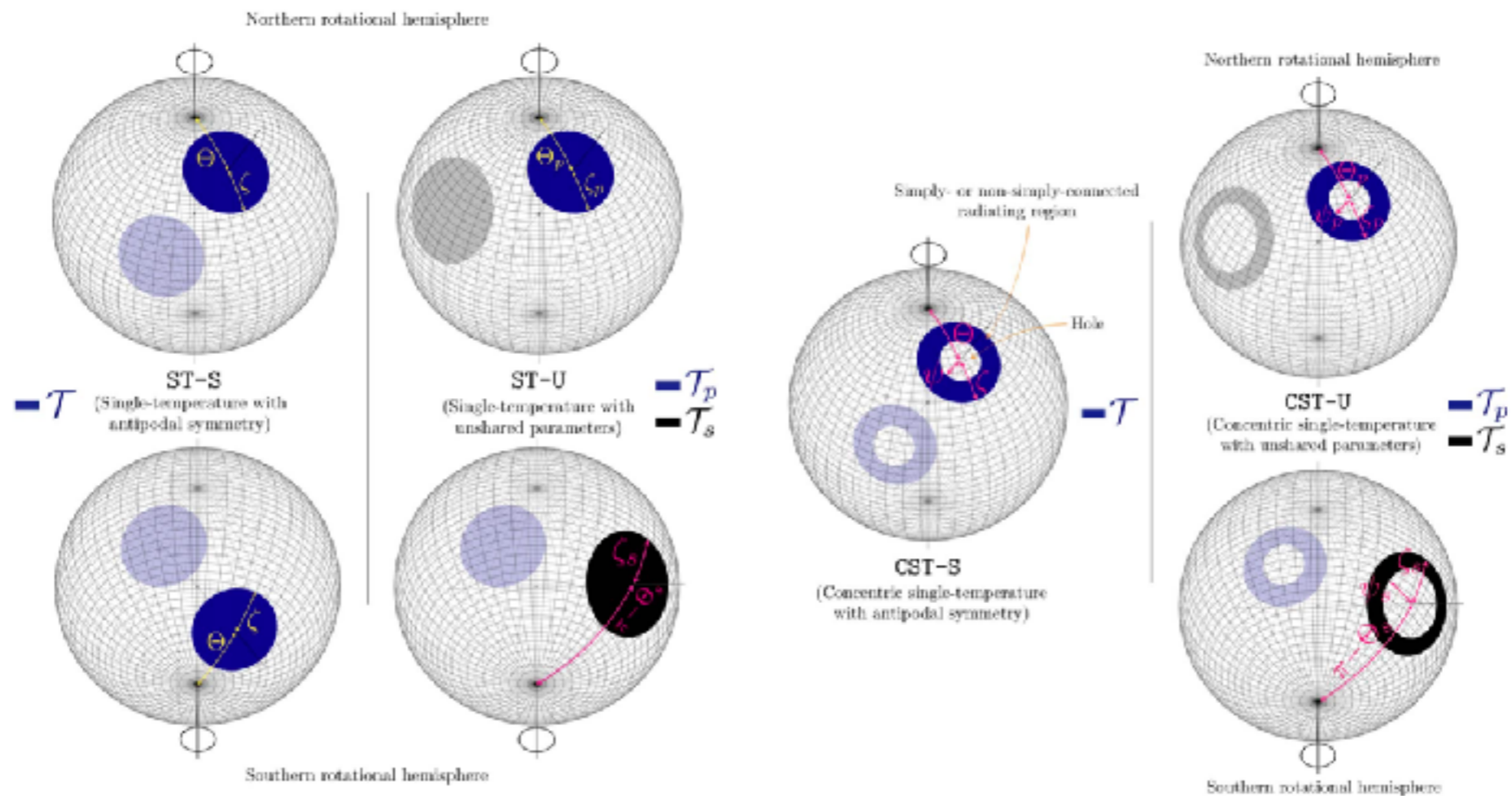
2. Radius

- $12.71^{+1.14}_{-1.19}$ Msun vs. $13.02^{+1.24}_{-1.06}$ Msun

3. Methods

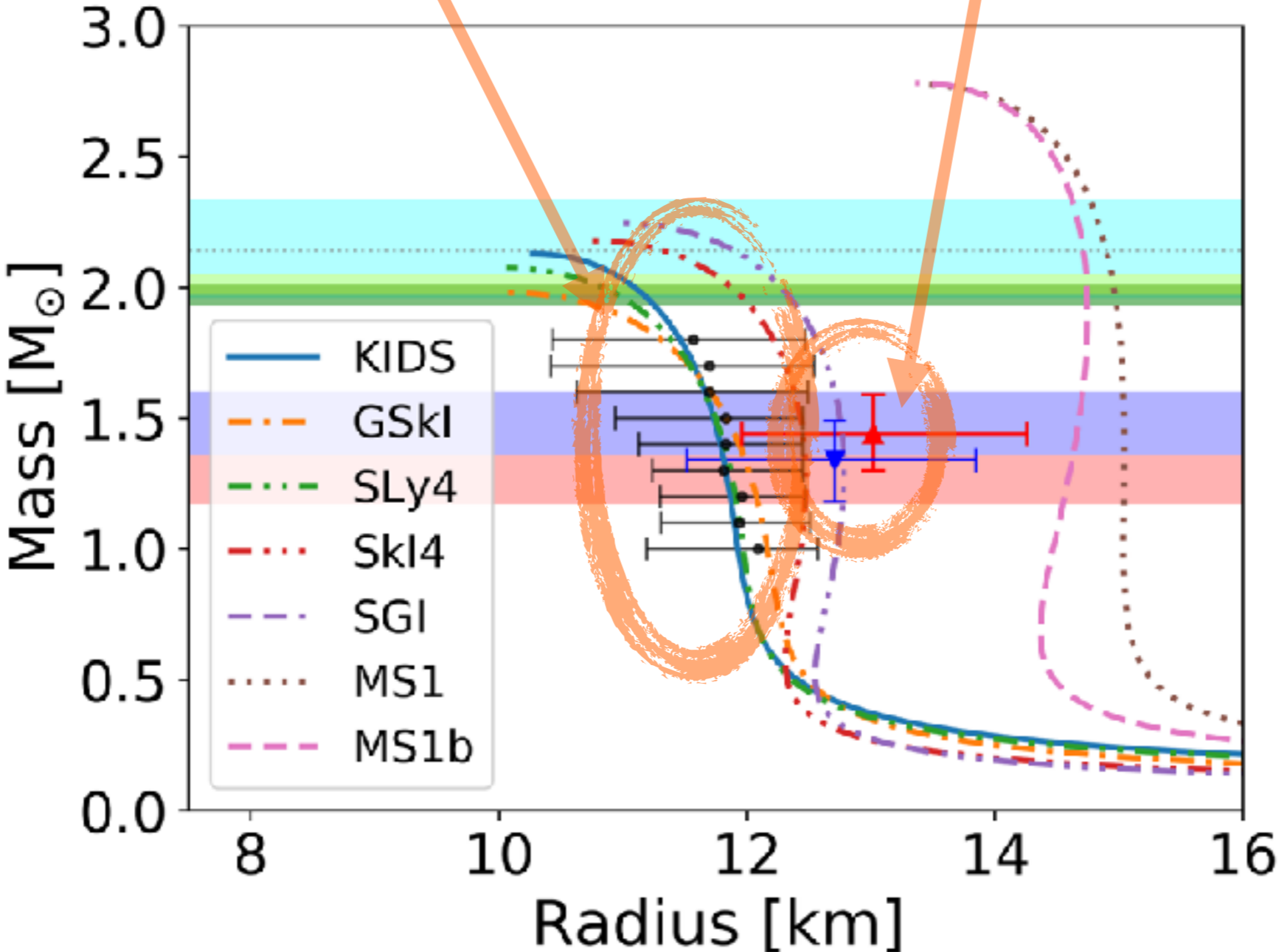
- MultiNest vs. MultNest & emcee (MCMC)
- X-PSI bayesian code vs. Miller's own code
- Different heated regions
- Pulse profile model vs. Pulse waveform model

Hot spot region model of Riley 2019



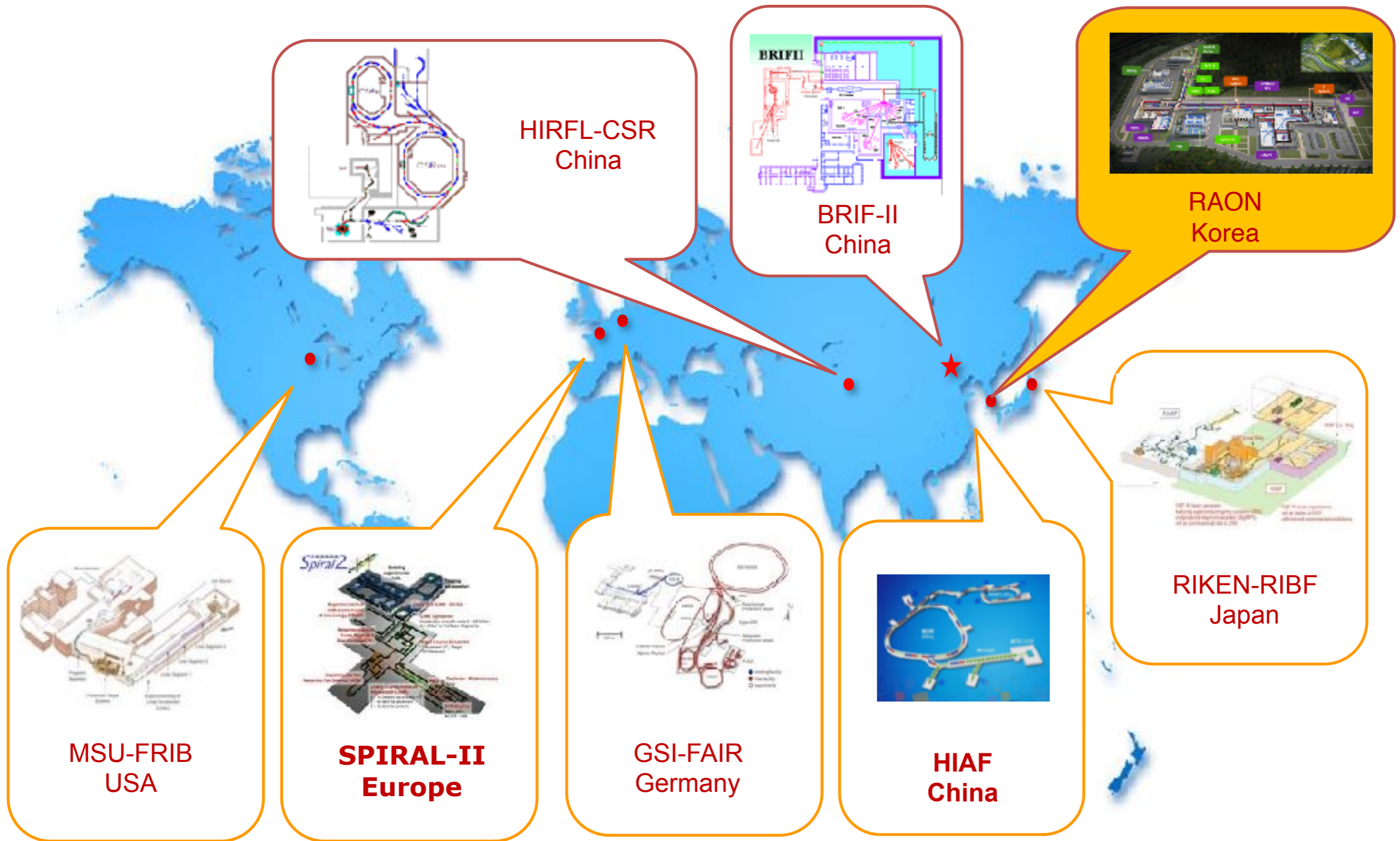
Single NS (better constraint)

Low-mass X-ray binary (NS binary)



$$2.14^{+0.20}_{-0.18} M_{\odot}$$

Facilities for Rare Isotope Beams



Nuclear Equation of States

$$x = \frac{\rho_p}{\rho_p + \rho_n}$$



$$E(\rho, x) = -B + \frac{K_0}{18} \left(\frac{\rho}{\rho_0} - 1 \right)^2 + \frac{K'_0}{162} \left(\frac{\rho}{\rho_0} - 1 \right)^3 + E_{\text{sym}}(\rho)(1 - 2x)^2 + \dots$$

Incompressibility

$$K_0 \simeq 230 \text{ MeV}$$

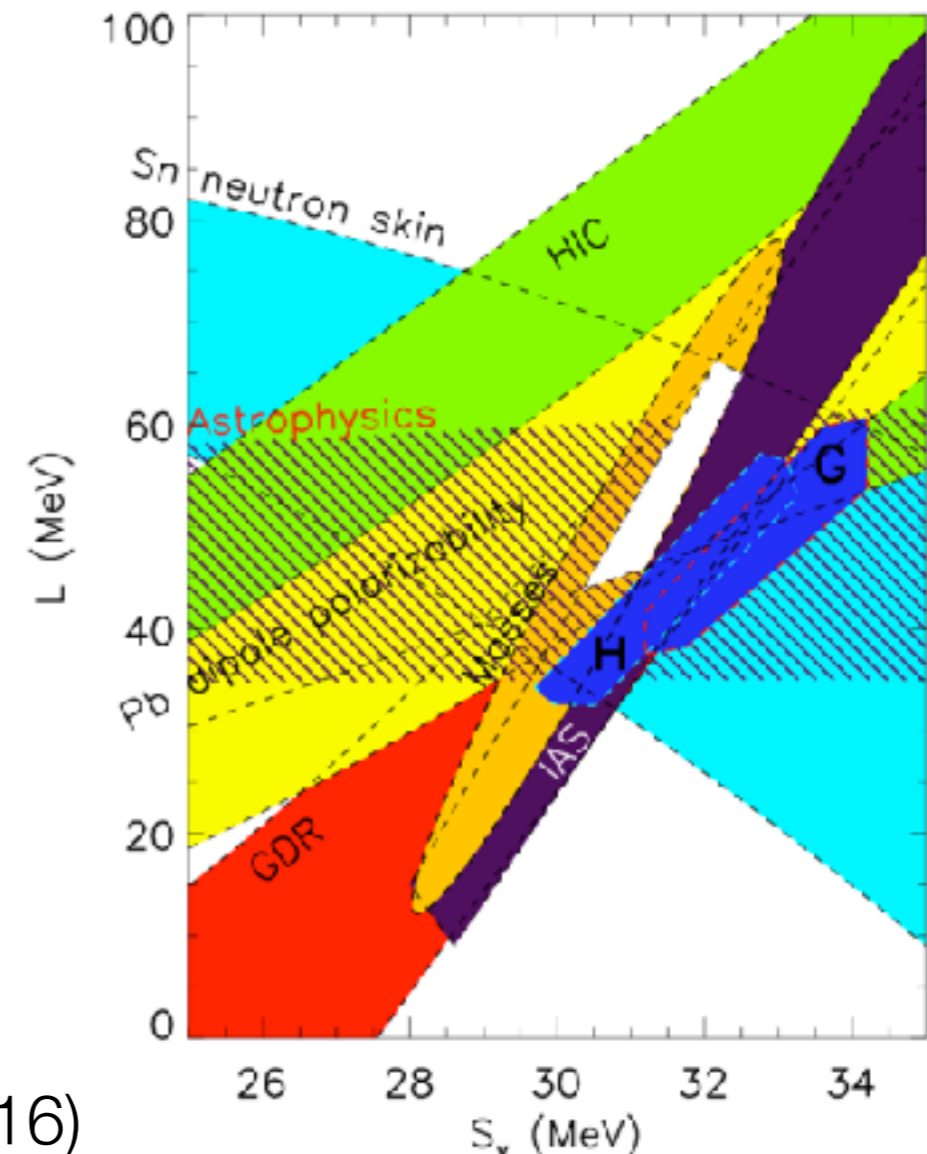
Skewness

$$K'_0 \sim -2000 \text{ MeV}$$

$$S_0 \equiv E_{\text{sym}}(\rho_0)$$

Symmetry Energy

$$L \equiv 3\rho \left. \frac{\partial E_{\text{sym}}}{\partial \rho} \right|_{\rho_0}$$



J.M. Lattimer, Y. Lim (2016)

Prospects

- Both Mass & Tidal Deformability of Neutron Star can be measured simultaneously from Gravitational Waves
- Gravitational Waves & Rare Isotope Experiments can give constrains on the high-density EOS
- Expecting more GW observations and new experimental results from RAON