Numerical studies for accreting matter flow around a black hole

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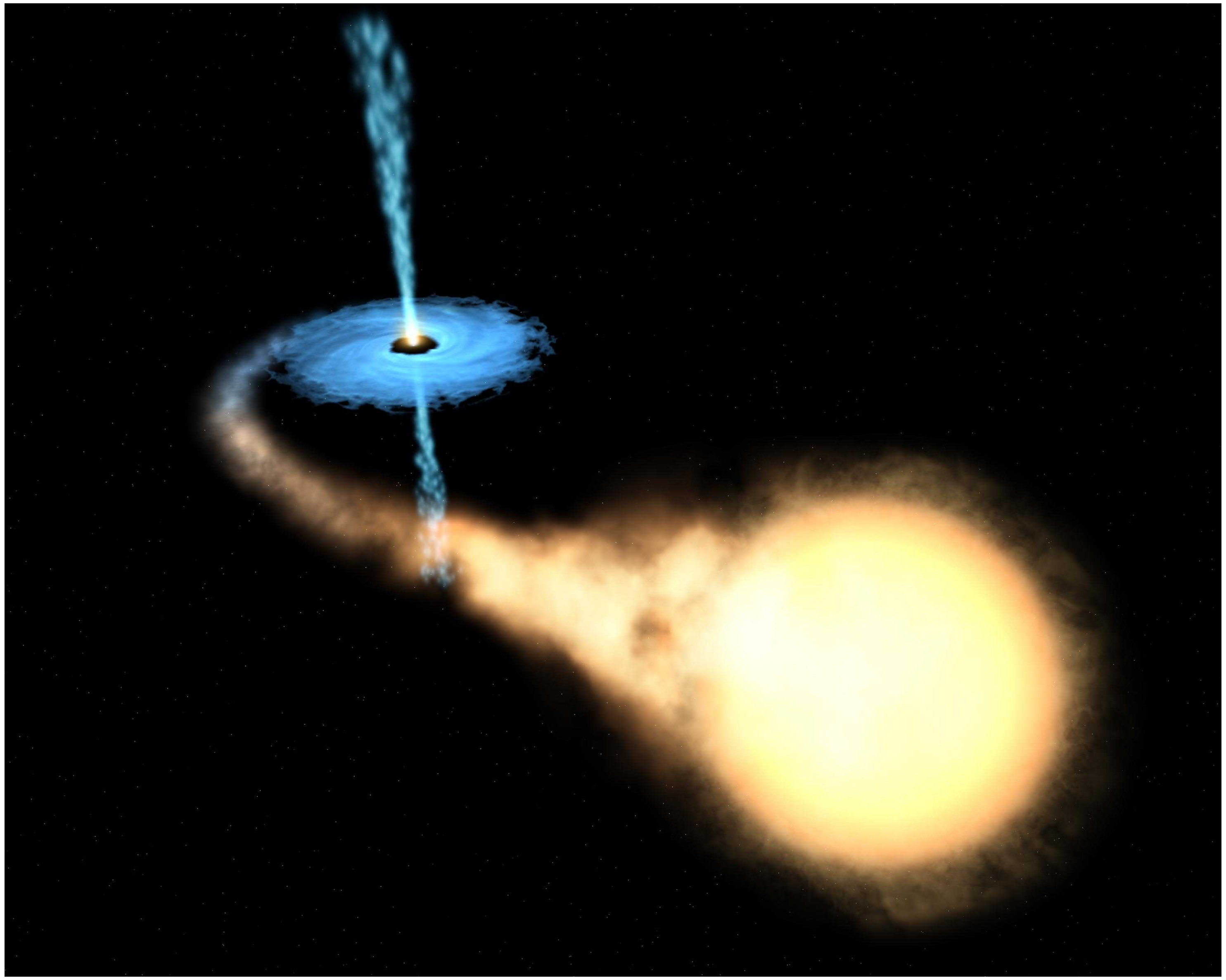
What is an accretion disk?

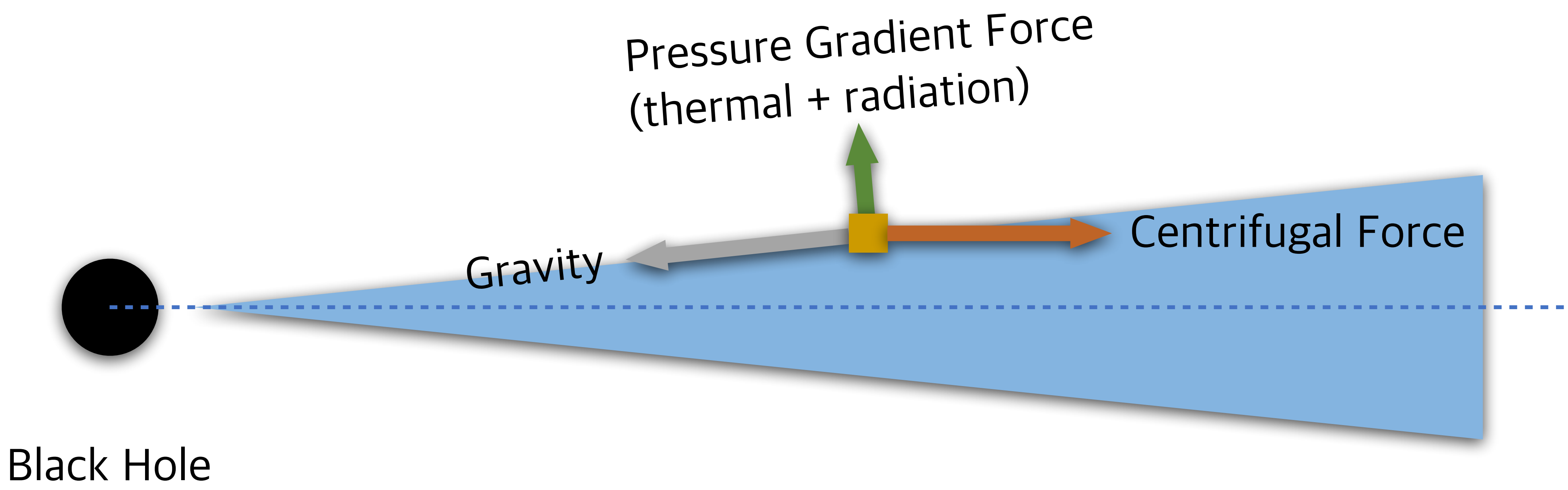
An accretion disk is a structure (often a circumstellar disk) formed by diffused material in orbital motion around a massive central body.



Artist's rendition of a black hole with an orbiting companion star that exceeds its Roche limit.

Image is provided by NASA

Equilibrium (stationary) Structure of Accretion Disks



Continuity Equation: $$\cancelto{0}{\frac{\partial \rho}{\partial t}} + \nabla \cdot \left(\rho \vec{v}\right) = 0$$

Momentum Equation: $$\cancelto{0}{\frac{\partial \vec{v}}{\partial t}} + \vec{v} \cdot \nabla \vec{v} + \nabla P = \vec{f}_{\mathrm{grav}}$$

Energy equation or EOS

Thin Disk Approximation

1. 디스크는 매우 얇고 (R/h<<1) 디스크 물질의 회전축에 대하여 대칭이다.
2. 디스크는 시간적으로 거의 변화가 없다 ($\partial/\partial t$=0).
3. 중력은 가운데 있는 별에 의해서 좌우되고 디스크의 자체중력은 무시할 수 있다.
4. 디스크에 있는 물질은 케플러 운동을 한다.
5. 디스크 물질의 회전 운동이 반경 방향 운동보다 매우 크다 (v_r<<v_φ).
6. 디스크면의 수직방향으로는 정역학적 평형을 이룬다.
7. 디스크면의 수직방향으로는 광학적 깊이가 충분히 크다.
8. Viscosity를 설명하기 위해서는 α-viscosity를 채택한다.

Newtonian Disk Solution

$$h=\left(\frac{2}{3}\alpha\right)^{-\frac{1}{10}}\kappa_{es}^{\frac{1}{10}}\left(\frac{3}{32\pi^2\sigma}\right)^{\frac{1}{10}}G^{-\frac{7}{20}}\left(\frac{k}{\mu m_p}\right)^{\frac{2}{5}}M^{-\frac{7}{20}}\dot{M}^{\frac{1}{5}}R^{\frac{21}{20}}\left[1-\left(\frac{R_{in}}{R}\right)^{\frac{1}{2}}\right]^{\frac{1}{5}}$$

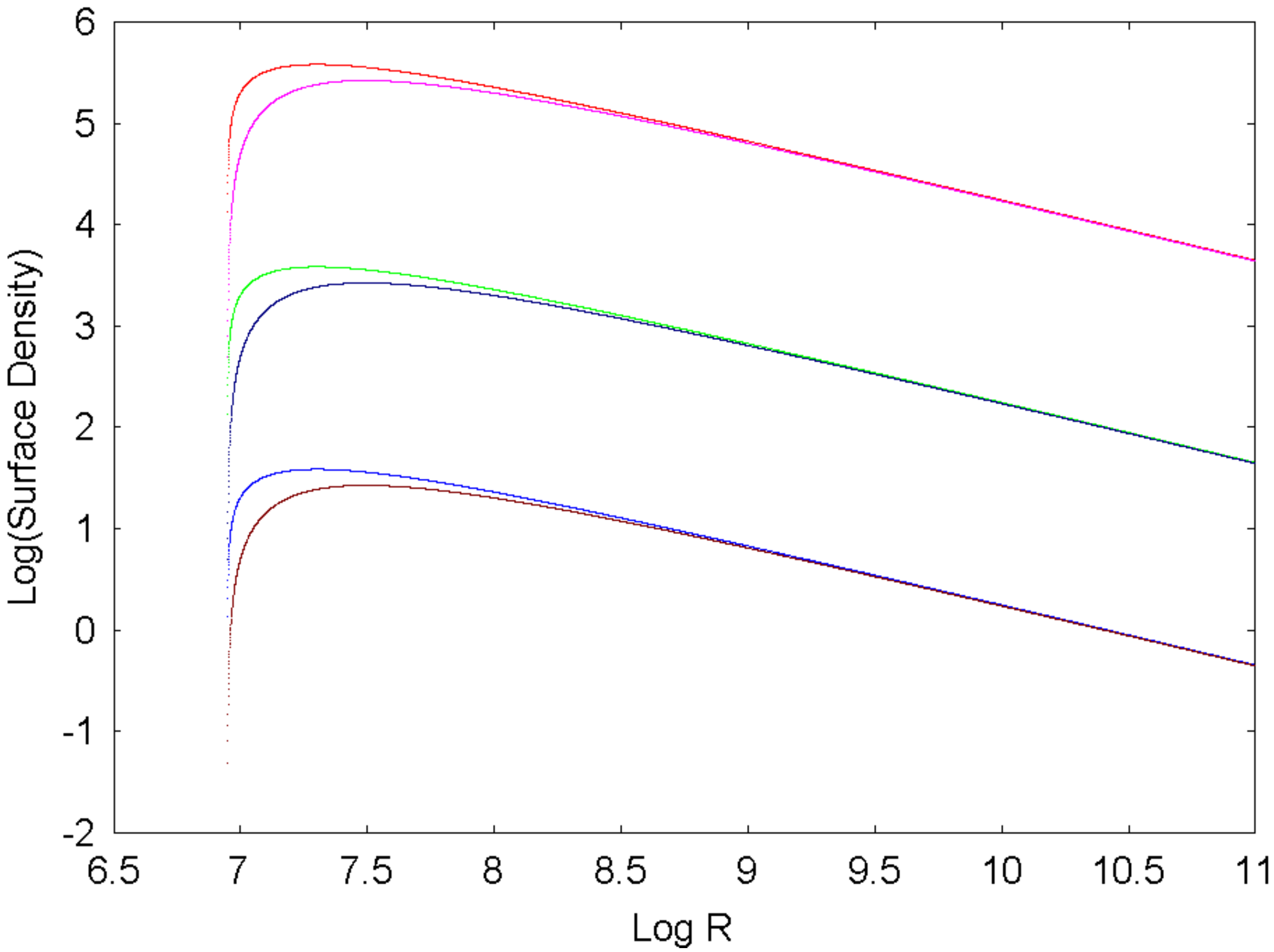
$$\Sigma=\left(\frac{2}{3}\alpha\right)^{-\frac{16}{45}}\kappa_{es}^{-\frac{1}{5}}(3\pi)^{-\frac{16}{45}}\left(\frac{9}{32\pi\sigma}\right)^{-\frac{1}{5}}G^{\frac{1}{5}}\left(\frac{k}{\mu m_p}\right)^{-\frac{4}{5}}M^{\frac{1}{5}}\dot{M}^{\frac{3}{5}}R^{-\frac{3}{5}}\left[1-\left(\frac{R_{in}}{R}\right)^{\frac{1}{2}}\right]^{\frac{3}{5}}$$

$$\tau=\left(\frac{2}{3}\alpha\right)^{-\frac{16}{45}}\kappa_{es}^{\frac{4}{5}}(3\pi)^{-\frac{16}{45}}\left(\frac{9}{32\pi\sigma}\right)^{-\frac{1}{5}}G^{\frac{1}{5}}\left(\frac{k}{\mu m_p}\right)^{-\frac{4}{5}}M^{\frac{1}{5}}\dot{M}^{\frac{3}{5}}R^{-\frac{3}{5}}\left[1-\left(\frac{R_{in}}{R}\right)^{\frac{1}{2}}\right]^{\frac{3}{5}}$$

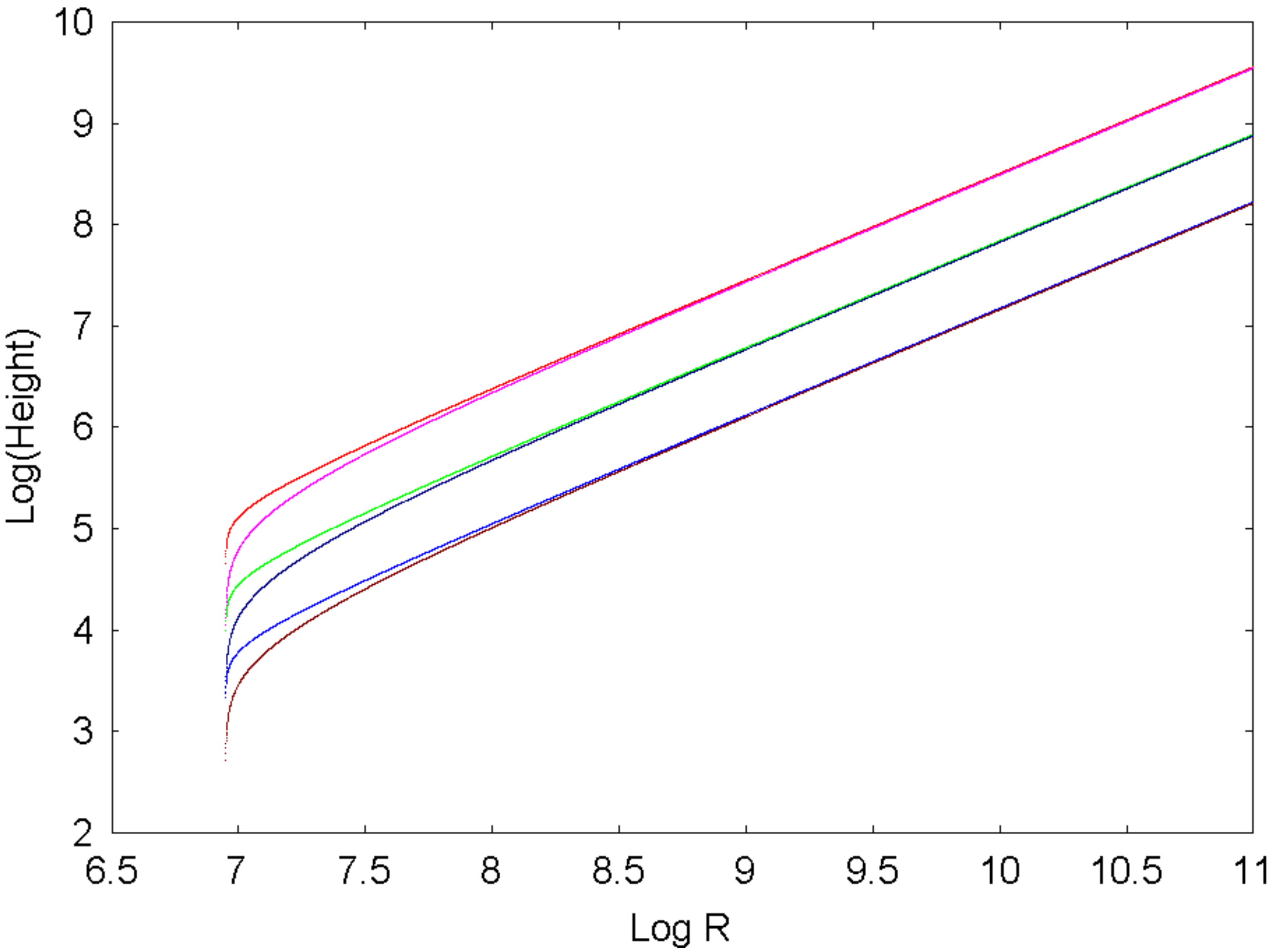
$$T_c=\left(\frac{2}{3}\alpha\right)^{-\frac{1}{5}}\kappa_{es}^{\frac{1}{5}}(3\pi)^{-\frac{1}{5}}\left(\frac{9}{32\pi\sigma}\right)^{\frac{1}{5}}G^{\frac{3}{10}}\left(\frac{k}{\mu m_p}\right)^{-\frac{1}{5}}M^{\frac{3}{10}}\dot{M}^{\frac{2}{5}}R^{-\frac{9}{10}}\left[1-\left(\frac{R_{in}}{R}\right)^{\frac{1}{2}}\right]^{\frac{2}{5}}$$

Disk Structure with $M_{\rm BH} = 10 M_\odot$

Column Density

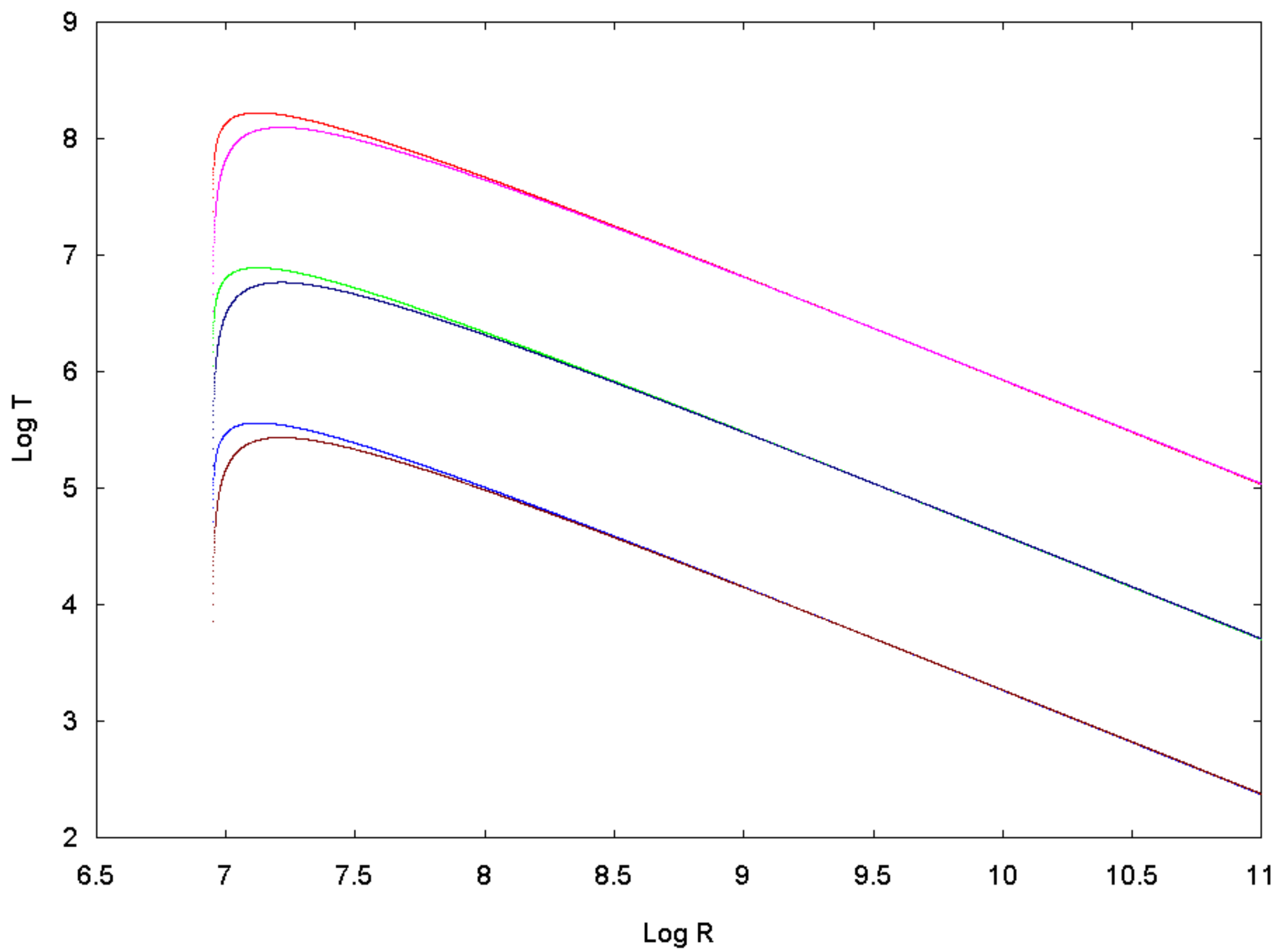


Disk Height

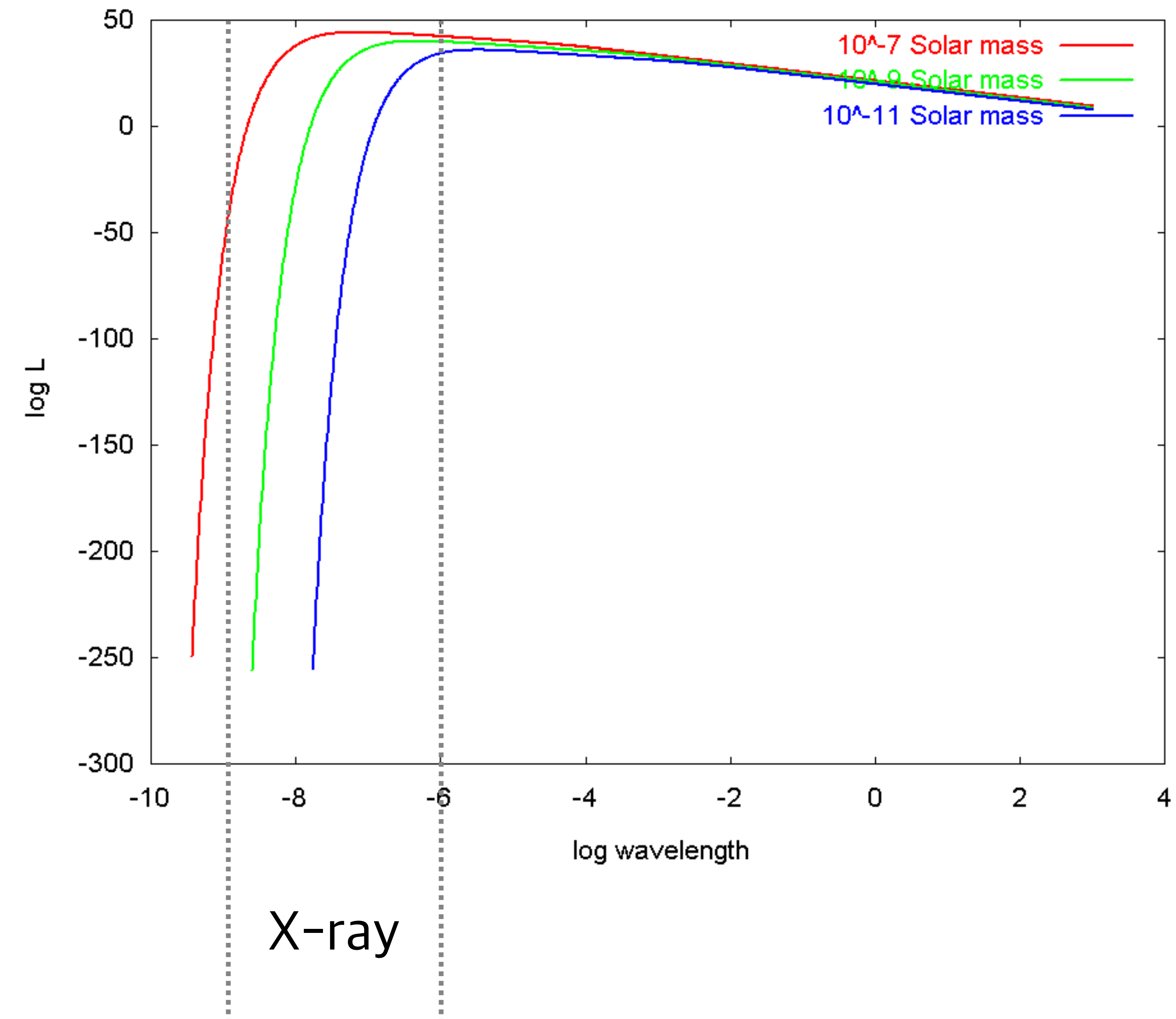


Disk Structure with $M_{\mathrm{BH}} = 10M_{\odot}$

Temperature



Spectrum



Shakura Sunyaev disk

Shakura and Sunyaev (1973) obtained analytic model of the accretion disk using so called α-viscosity: $\nu = \alpha c_s h$

$$H = 1.7 \times 10^{8} \, \alpha^{-1/10} \, \dot{M}_{16}^{3/20} \, m_1^{-3/8} \, R_{10}^{9/8} \, f^{3/5} \ \mathrm{cm}$$

$$T_c = 1.4 \times 10^{4} \, \alpha^{-1/5} \, \dot{M}_{16}^{3/10} \, m_1^{1/4} \, R_{10}^{-3/4} \, f^{6/5} \ \mathrm{K}$$

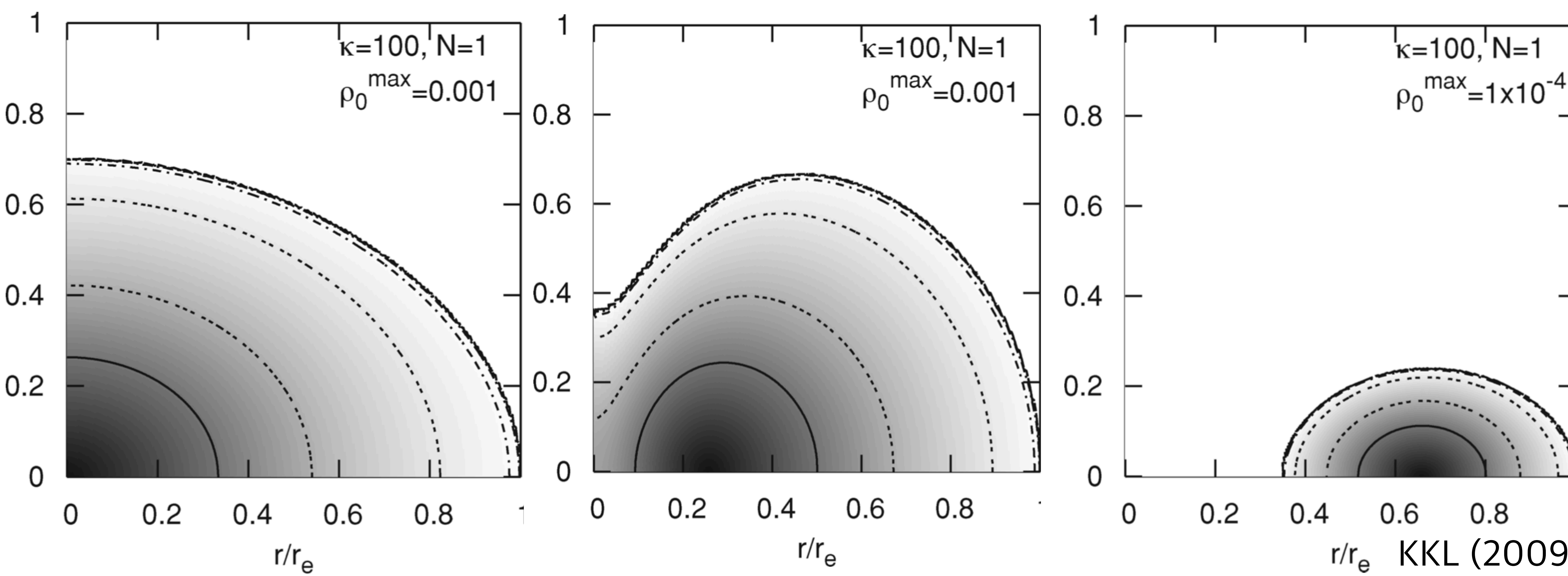
$$\rho = 3.1 \times 10^{-8} \, \alpha^{-7/10} \, \dot{M}_{16}^{11/20} \, m_1^{5/8} \, R_{10}^{-15/8} \, f^{11/5} \ \mathrm{g/cm^3}$$

$$\text{where } f = \left[1 - \left(\frac{R_{in}}{R} \right)^{1/2} \right]^{1/4}$$

Equilibrium models of rotating objects

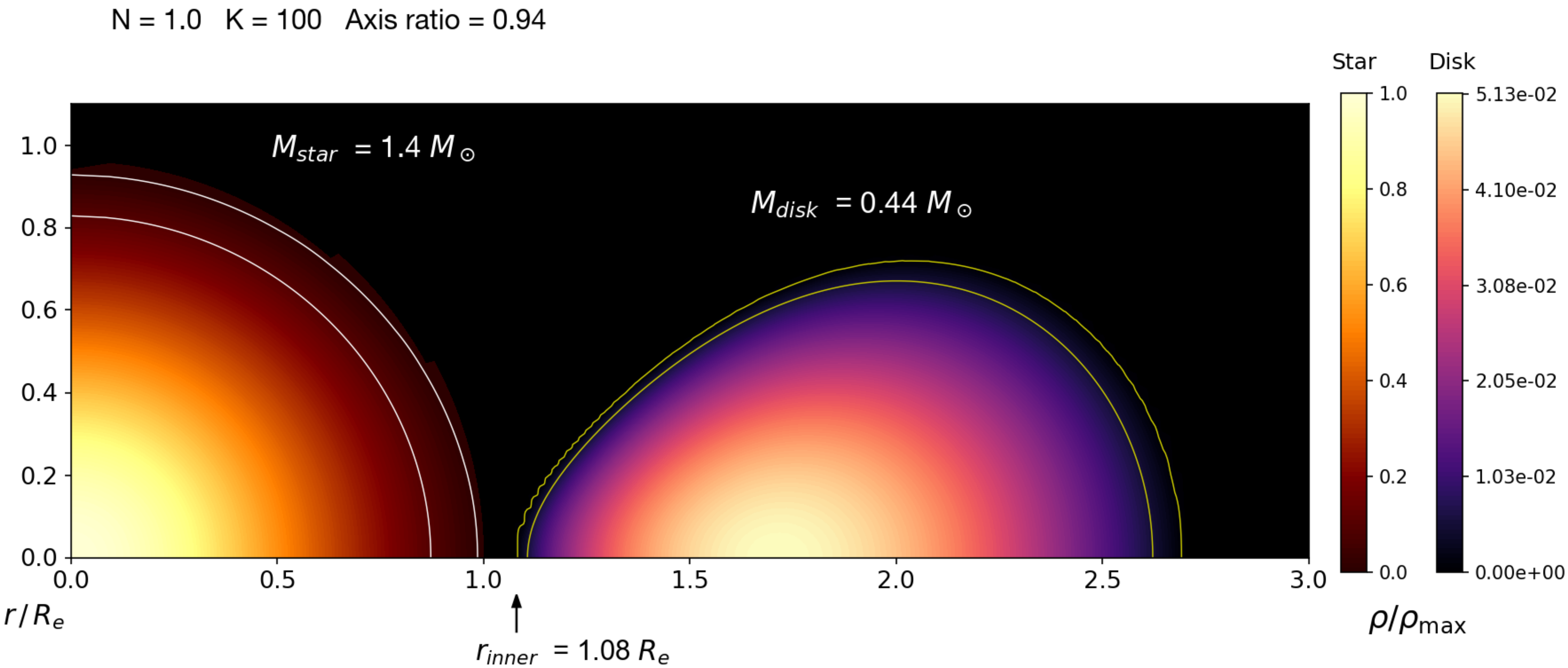
Hachisu Self-Consistent Field (HSCF) Method

- Axis ratio vs. Rotating velocity
- Spheroidal / Toroidal (ring) structure
- Newtonian: Hachisu (1986), GR: KEH (1989), CST(1994), pseudo-Newtonian: Kim, Kim & Lee (2009)



KKL (2009)

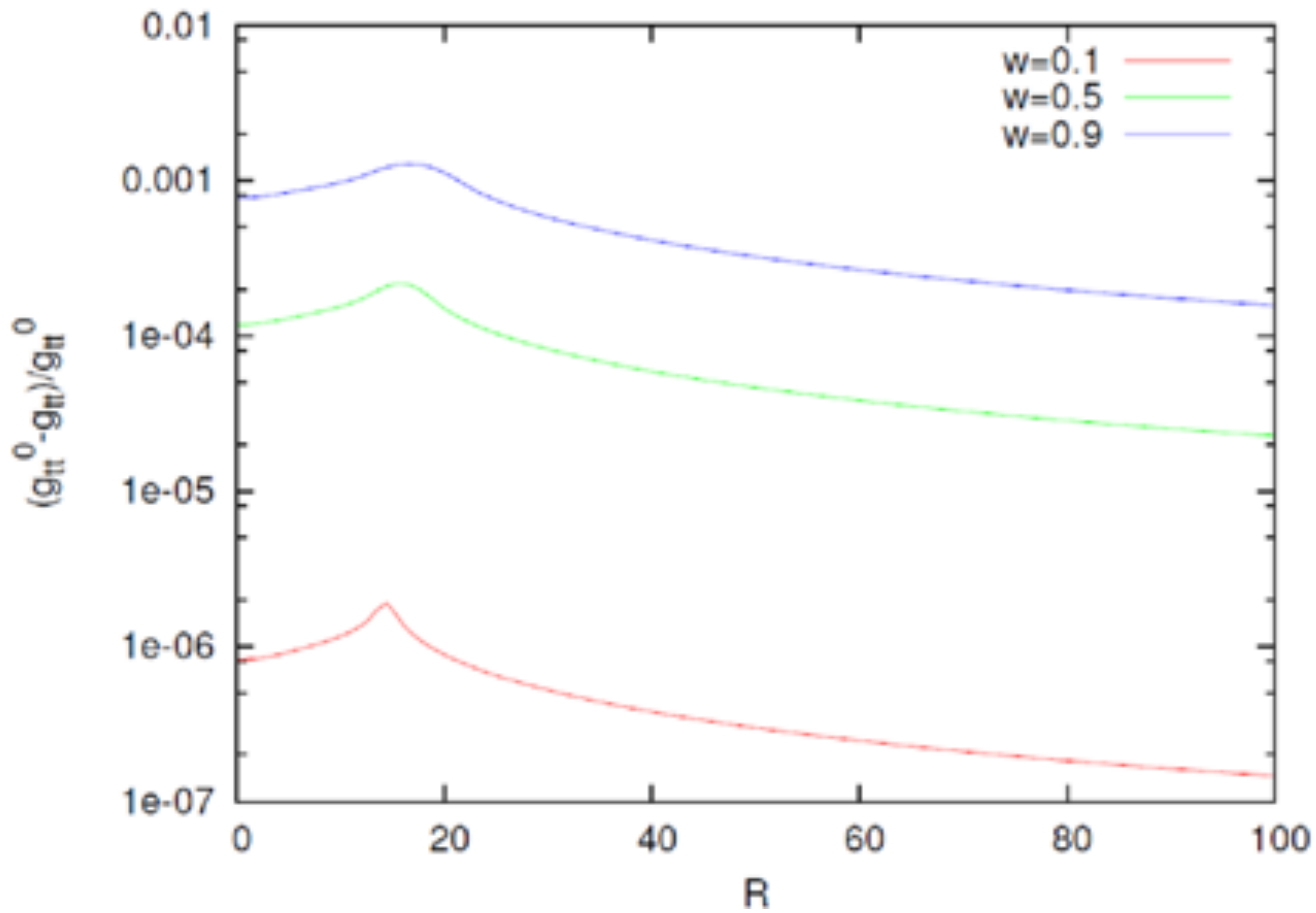
Self gravitating Disk around neutron star



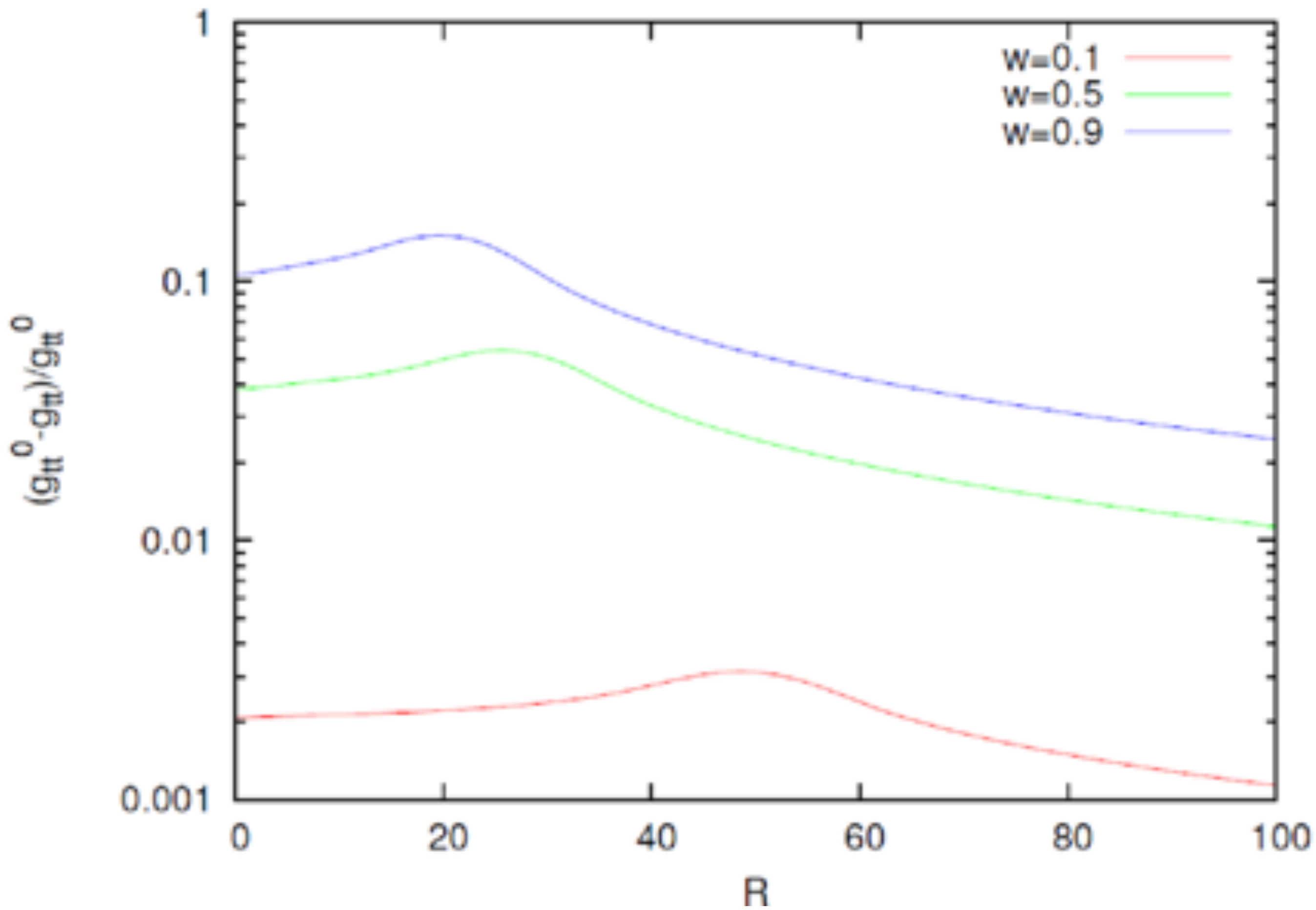
$$M_{\rm disk}/M_{\rm star} = 0.31$$

Courtesy of Yoonsoo Kim

Significance of Self-gravity



$\alpha = 0.25$



$\alpha = 0.45$

Hydrodynamic Simulation

Hydrodynamics in General Relativity

The general relativistic (magneto-)hydrodynamics equations consist of the local conservation laws of the the matter current density and the stress energy tensor (Bianchi identity, energy & momentum conservation).

$$\nabla_\mu J^\mu = 0$$

Baryon number conservation or total mass conservation

$$\nabla_\mu \left(\rho u^\mu\right) = 0$$

$$\nabla_\mu T^{\mu\nu} = 0$$

Spatial projection gives momentum conservation equation

$$\gamma_i^\nu \nabla_\mu T_\nu^\mu = 0$$

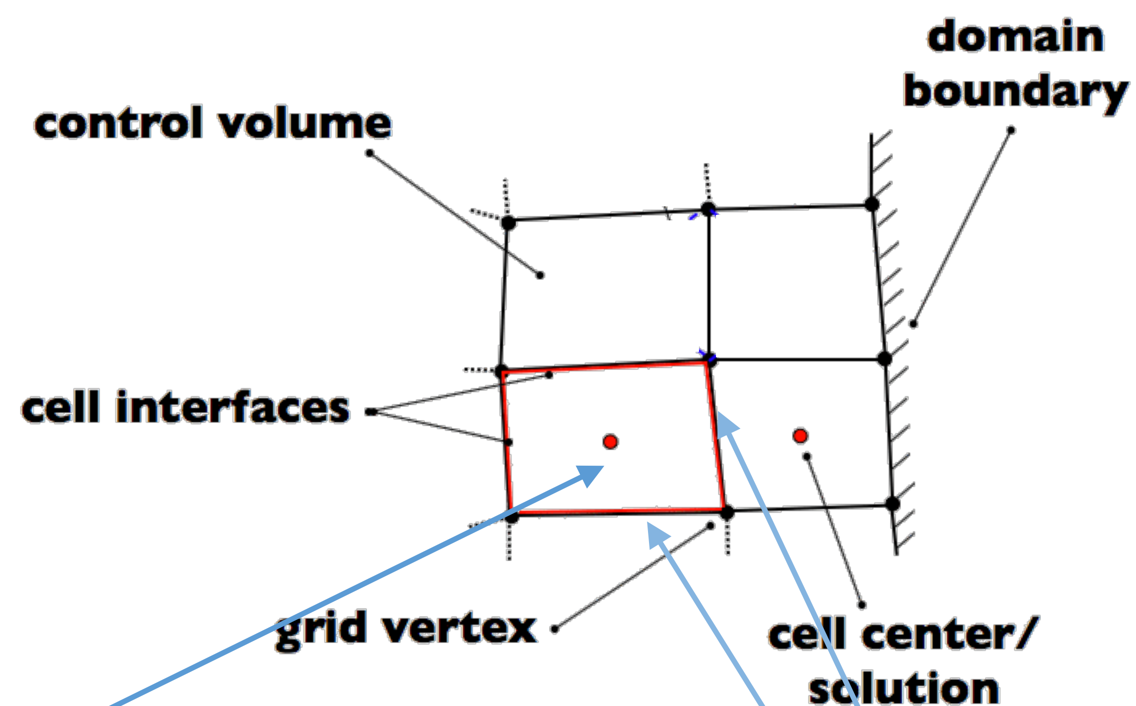
Normal projection gives energy conservation equation

$$n^\nu \nabla_\mu T_\nu^\mu = 0$$

Finite Volume Method

Finite volume method enforces the **local conservation** of the fluid conservative quantities in a control volume.

The conservative quantities on each grid represent the **volume averaged quantities**.



$$\bar{U} = \frac{\int_{\Delta V} \sqrt{\gamma} U d^3x}{\int_{\Delta V} \sqrt{\gamma} d^3x}$$

Fluxes are evaluated on the face of the mesh (interface between the control volume).

$\bar{F}^i$

Solution Strategy

Conservative form of the equation

$$\frac{\partial\left(\sqrt{\gamma}U\right)}{\partial t}+\frac{\partial\left(\sqrt{-g}F^{i}\right)}{\partial x^{i}}=\sqrt{-g}\Sigma$$

Volume integration over time and spatial volume gives:

$$\int_{\Delta V^{(4)}}\frac{\partial\left(\sqrt{\gamma}U\right)}{\partial t}dV^{(4)}+\int_{\Delta V^{(4)}}\frac{\partial\left(\sqrt{-g}F^{i}\right)}{\partial x^{i}}dV^{(4)}=\int_{\Delta V^{(4)}}\sqrt{-g}\Sigma dV^{(4)}$$

Final form of the numerically adapted equation

$$\bar{U}\Delta V^{(3)}\Big|_{x^0+\Delta x^0}\bar{U}\Delta V^{(3)}\Big|_{x^0}=$$

$$-\left(\int_{\Sigma x^1+\frac{\Delta x^1}{2}}\sqrt{-g}F^1dx^0dx^2dx^3-\int_{\Sigma x^1-\frac{\Delta x^1}{2}}\sqrt{-g}F^1dx^0dx^2dx^3\right)$$

$$-\left(\int_{\Sigma x^2+\frac{\Delta x^2}{2}}\sqrt{-g}F^1dx^0dx^1dx^3-\int_{\Sigma x^2-\frac{\Delta x^2}{2}}\sqrt{-g}F^1dx^0dx^1dx^3\right)$$

$$-\left(\int_{\Sigma x^3+\frac{\Delta x^3}{2}}\sqrt{-g}F^1dx^0dx^1dx^2-\int_{\Sigma x^3-\frac{\Delta x^3}{2}}\sqrt{-g}F^1dx^0dx^1dx^2\right)$$

$$+\int_{\Delta V^{(4)}}\sqrt{-g}\Sigma dV^{(4)}$$

Two-component advective flow (TCAF) Model

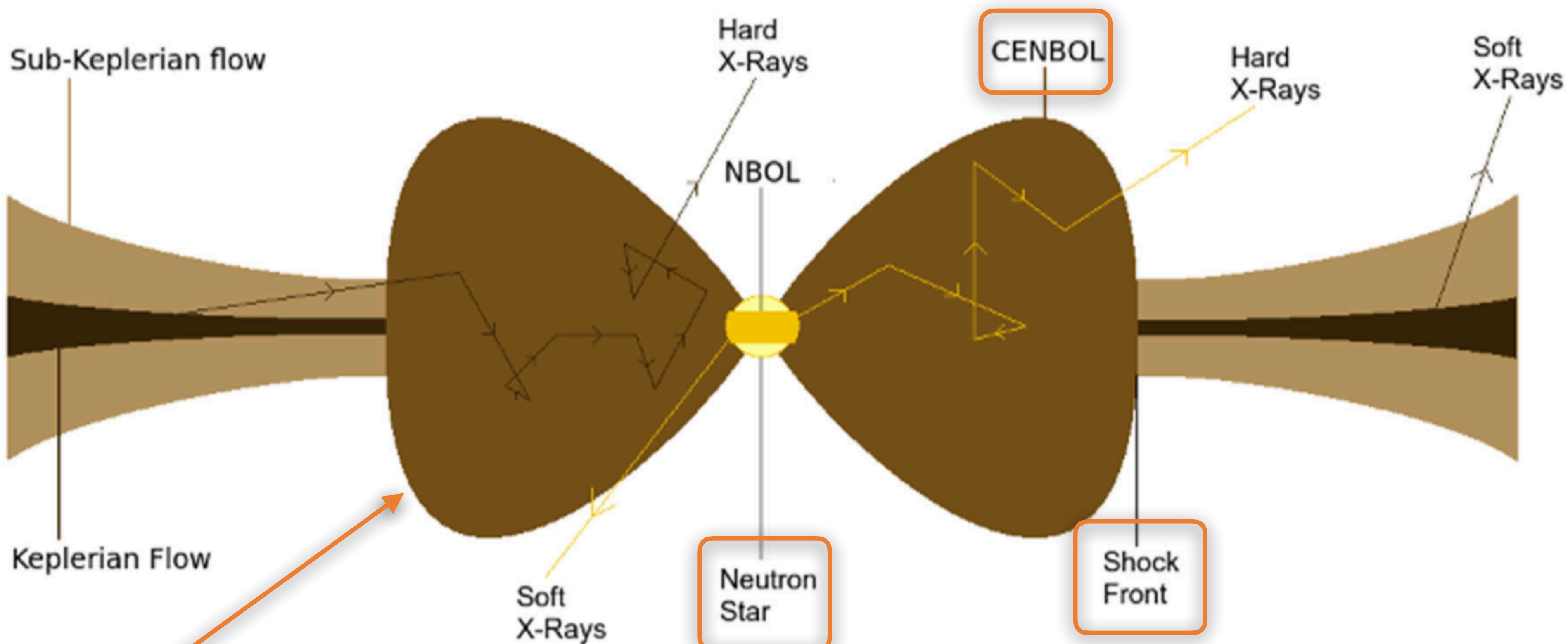
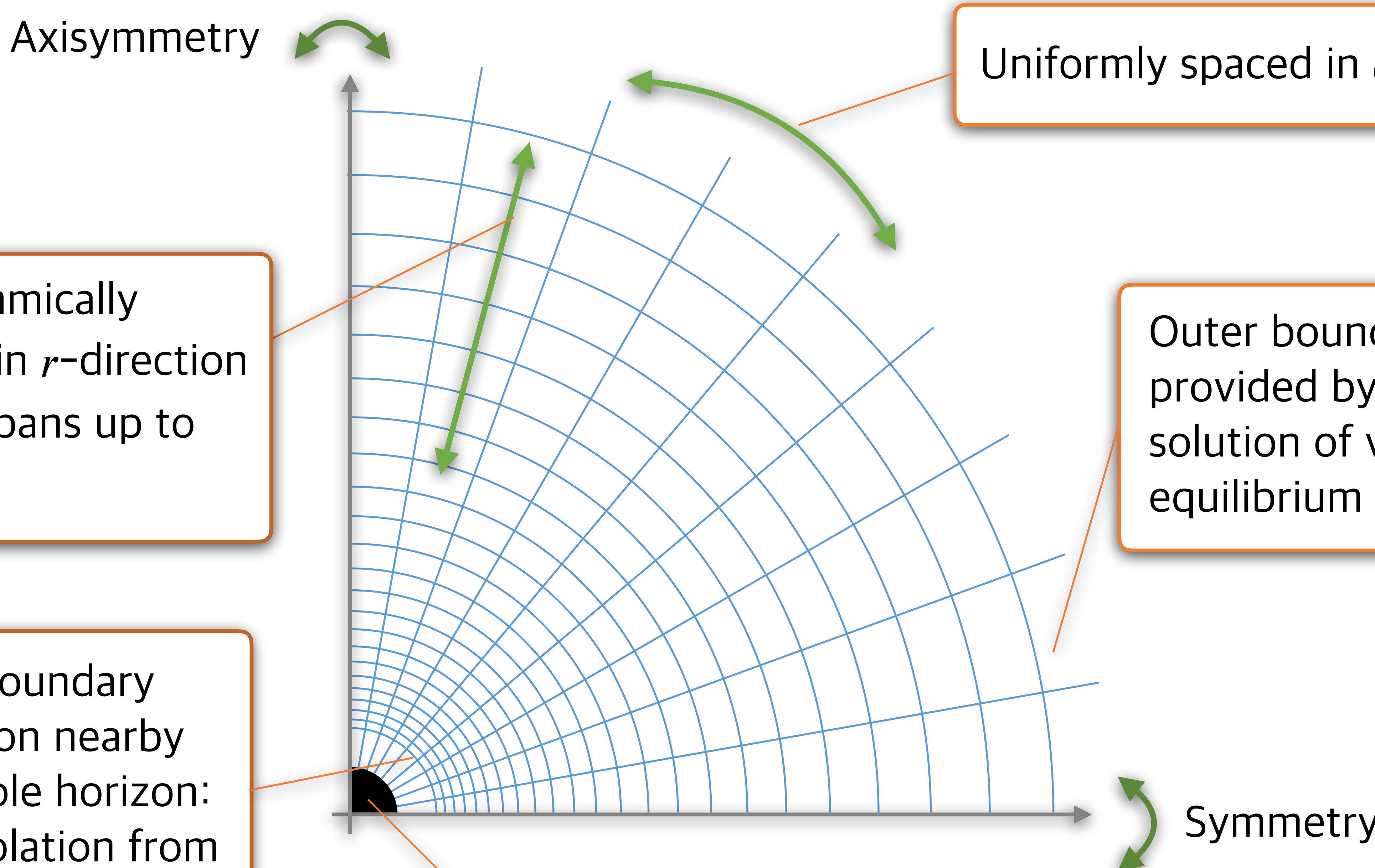


Image from Bhattacharjee & Chakrabarti (2017)

CENtrifugal pressure supported BOundary Layer

Numerical Strategy



Uniformly spaced in θ-direction

Logarithmically spaced in r-direction which spans up to $200r_s$

Outer boundary condition is provided by analytical disk solution of vertical equilibrium thin disk model.

Inner boundary condition nearby blackhole horizon: extrapolation from the outside

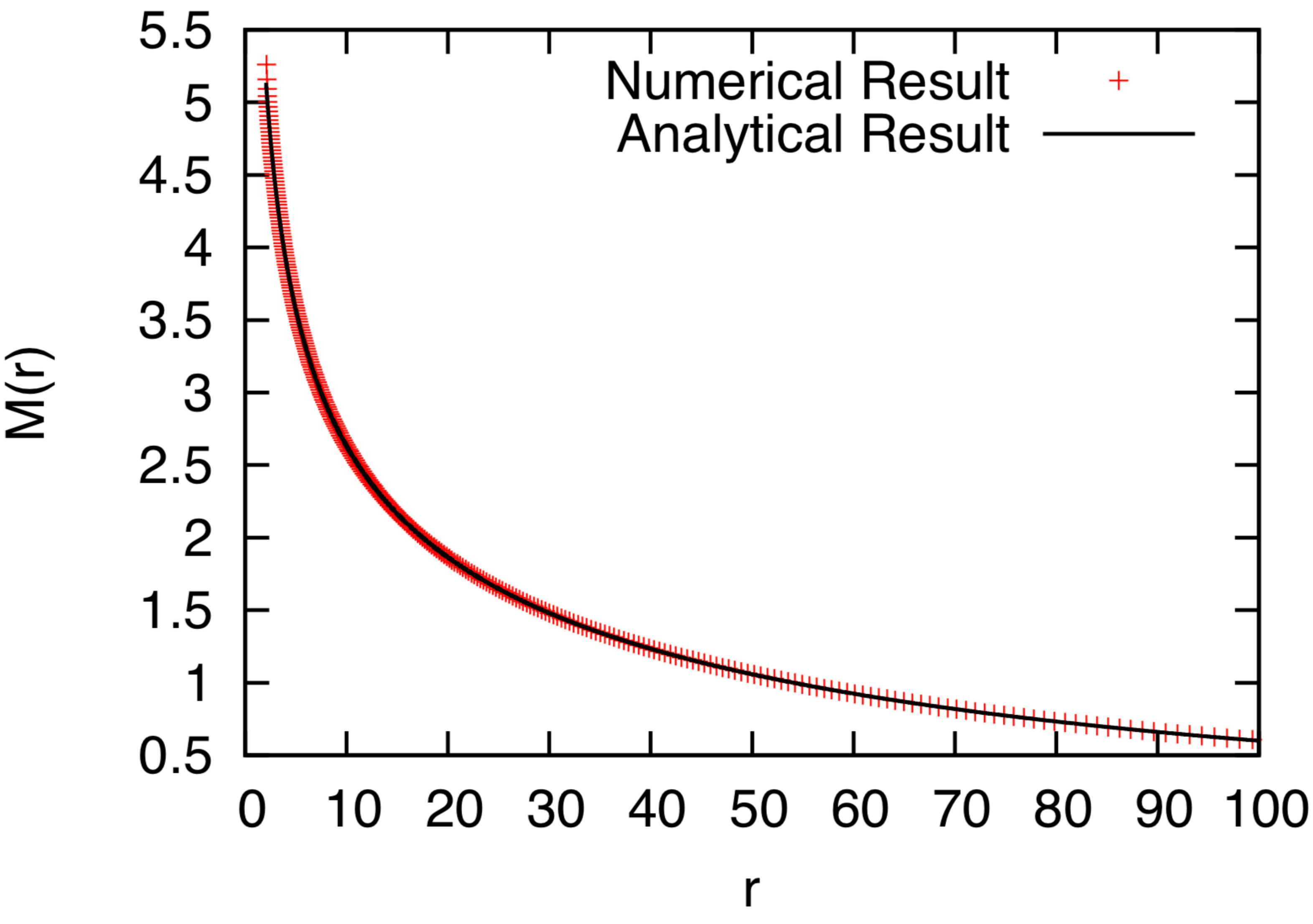
1. Schwarzschild blackhole
2. Kerr back hole in Boyer-Lindquist coordinates

Simulation lasts at least 20 dynamical time

Accretion onto Schwarzschild Blackhole

Code Test 1: Michel Solution

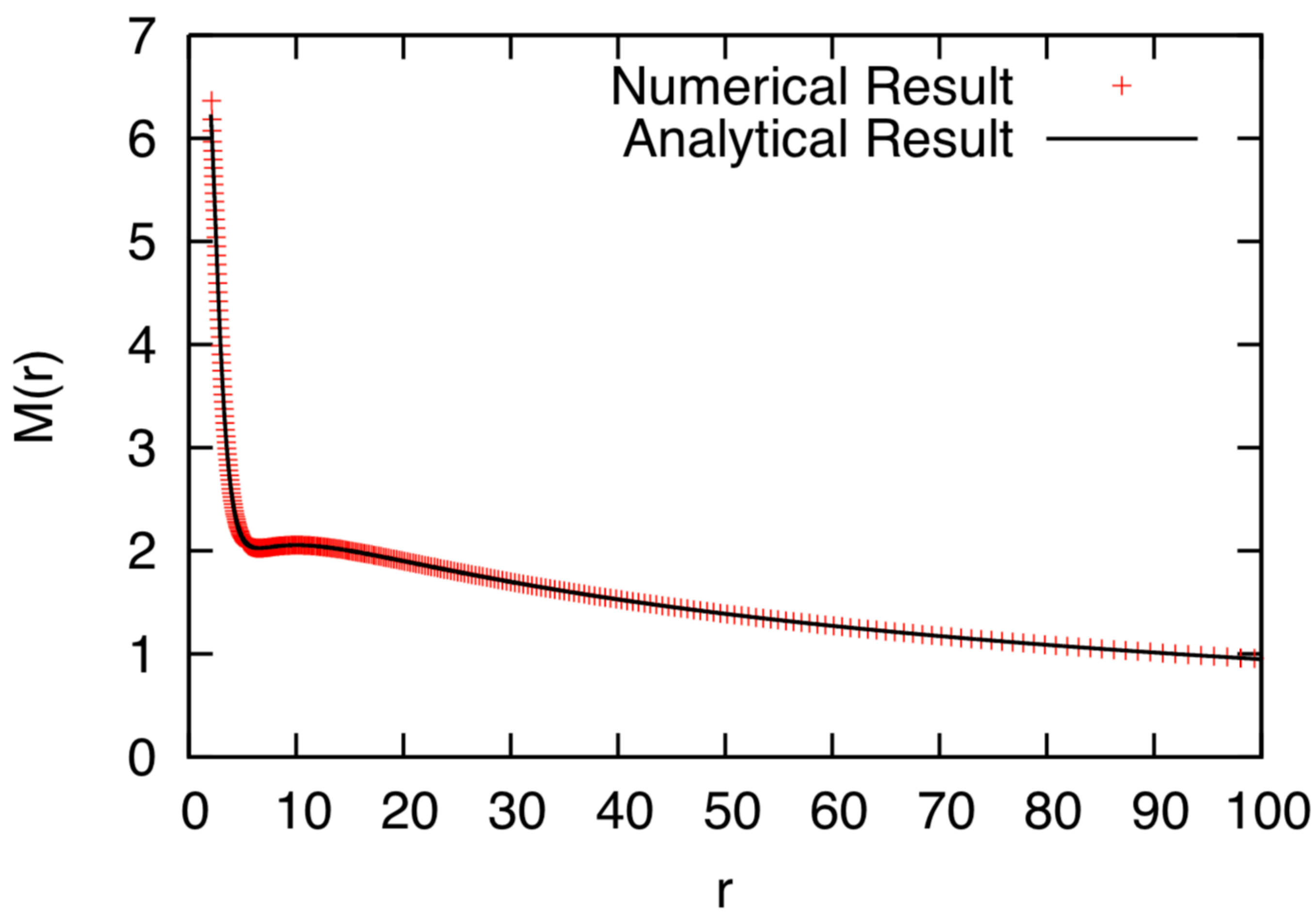
- Michel solution is the spherical accretion solution around Schwarzschild blackhole which is GR counterpart of the Bondi accretion.
- The solution is characterized by the conserved quantities (accretion rate and energy).

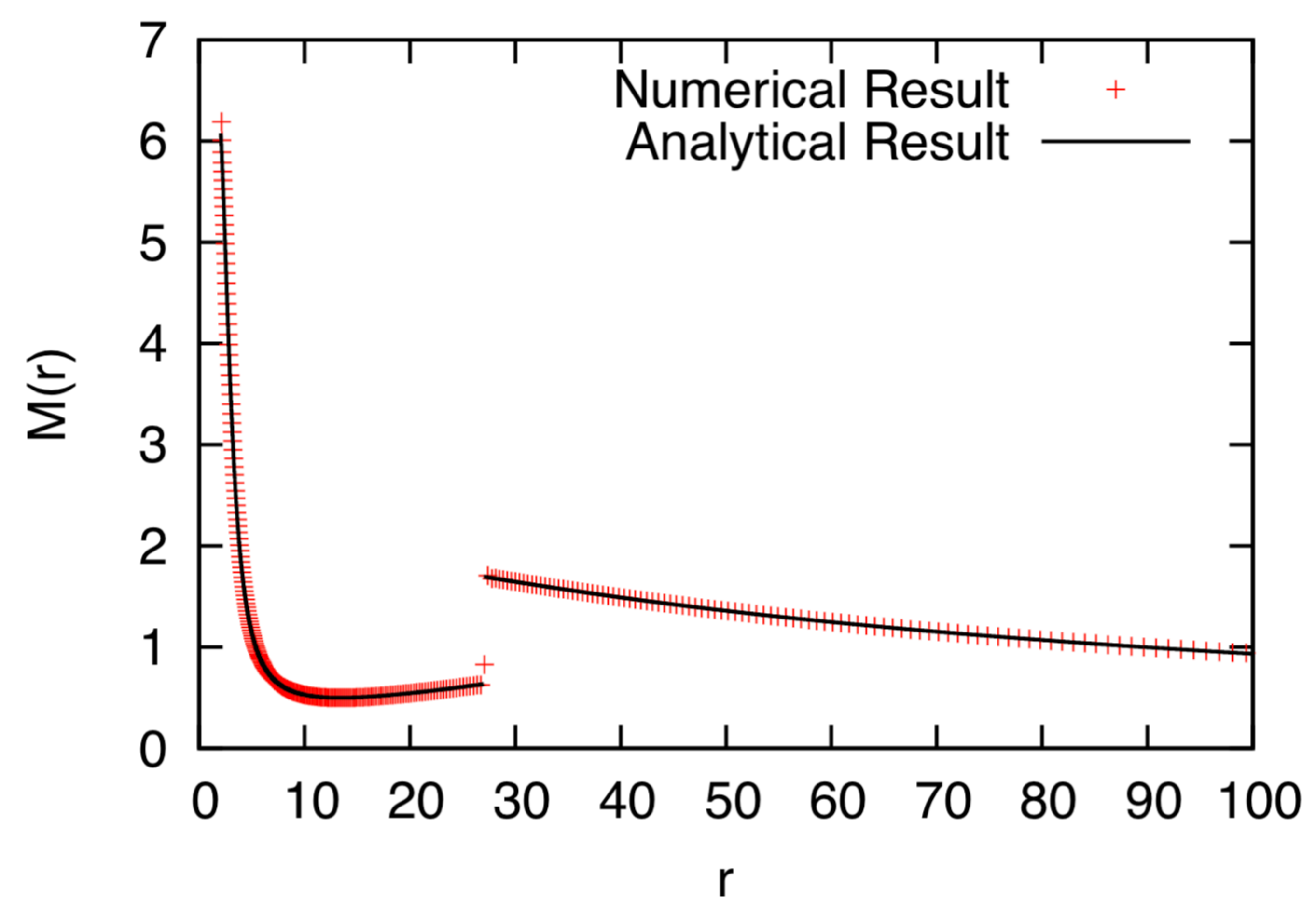


Numerical result shows good agreement with analytic solution!

Code Test2: 1-D Accretion Flow with Angular Momentum

- Vertical equilibrium condition gives one-dimensional solution characterized by the accretion rate, energy and angular momentum.
- There exist shock depending on the choice of conserved quantity values.

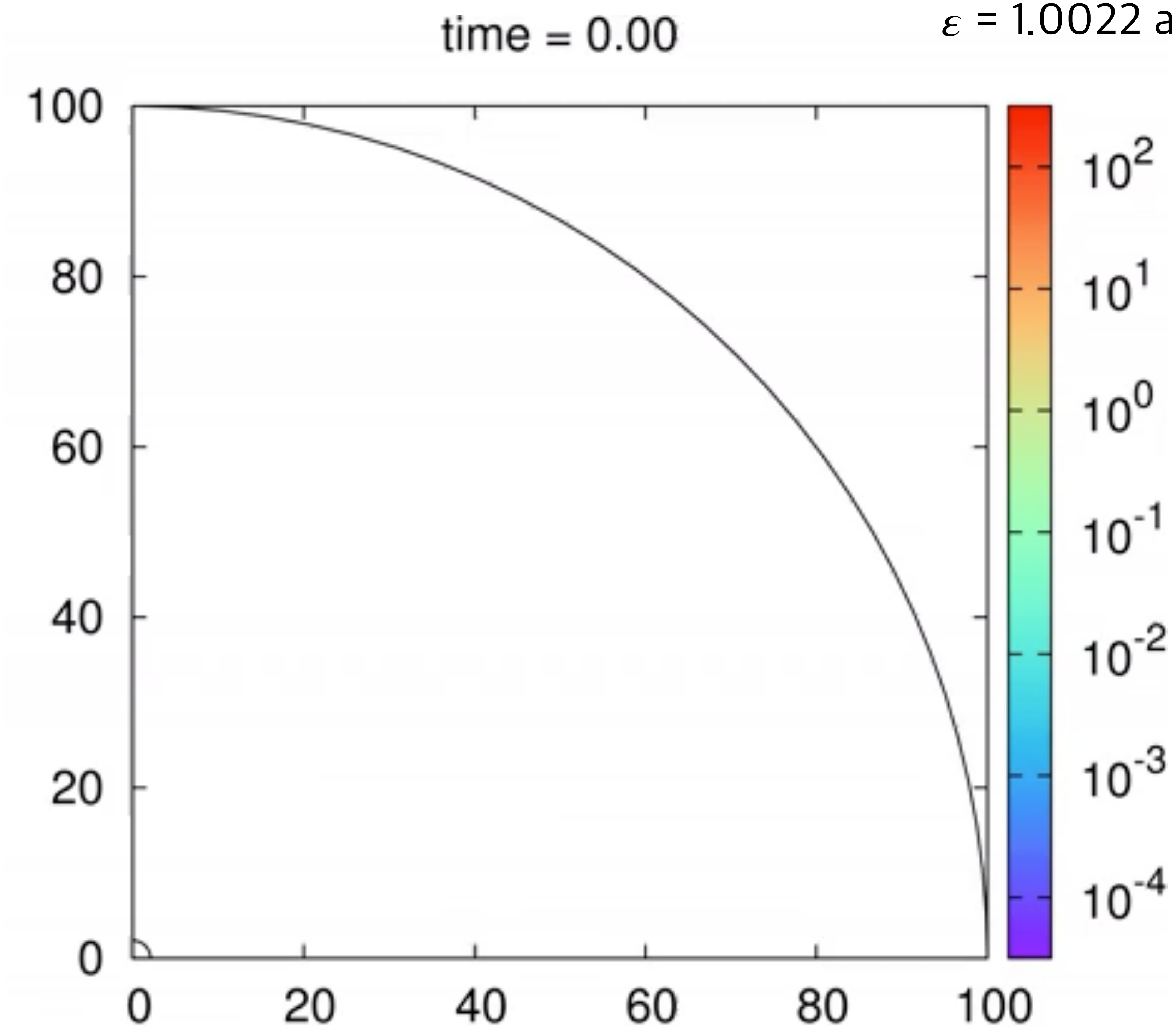




Numerical results show good agreements with analytic solutions.

2-D Simulation

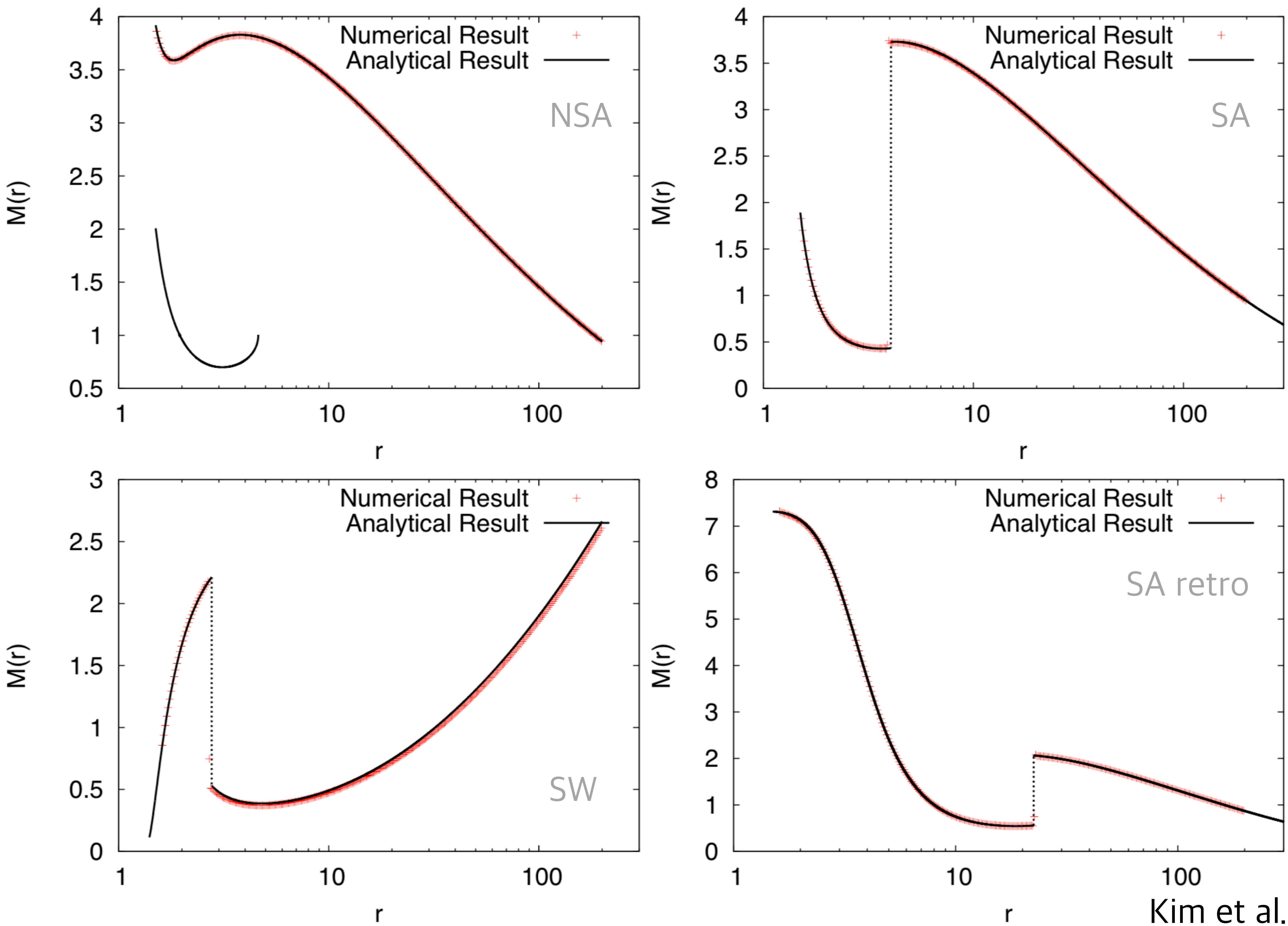
ε = 1.0022 and l = 3.36



Kim et al. (2017)

Accretion onto Rotating Blackhole

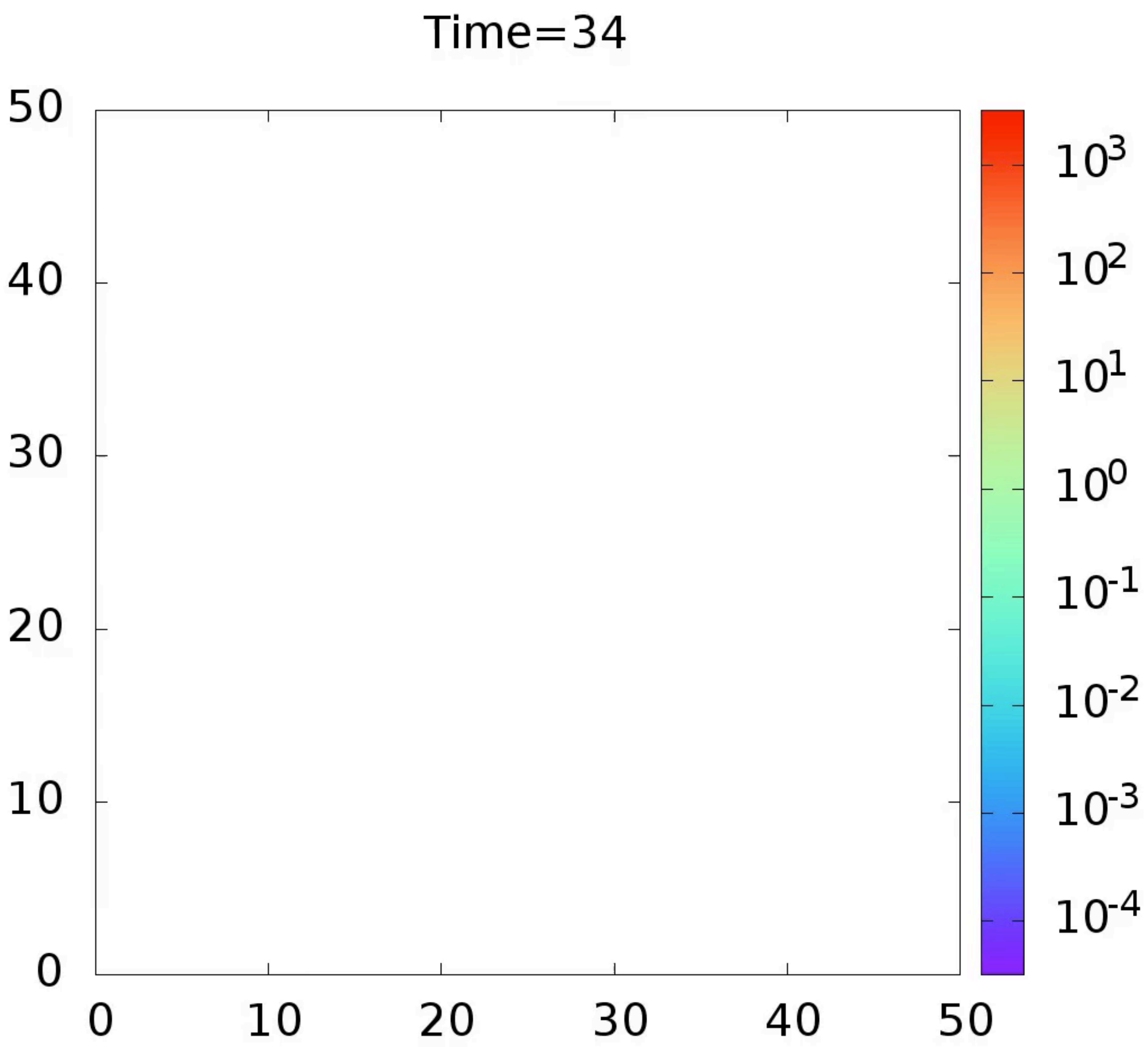
1-D Test with a=0.95

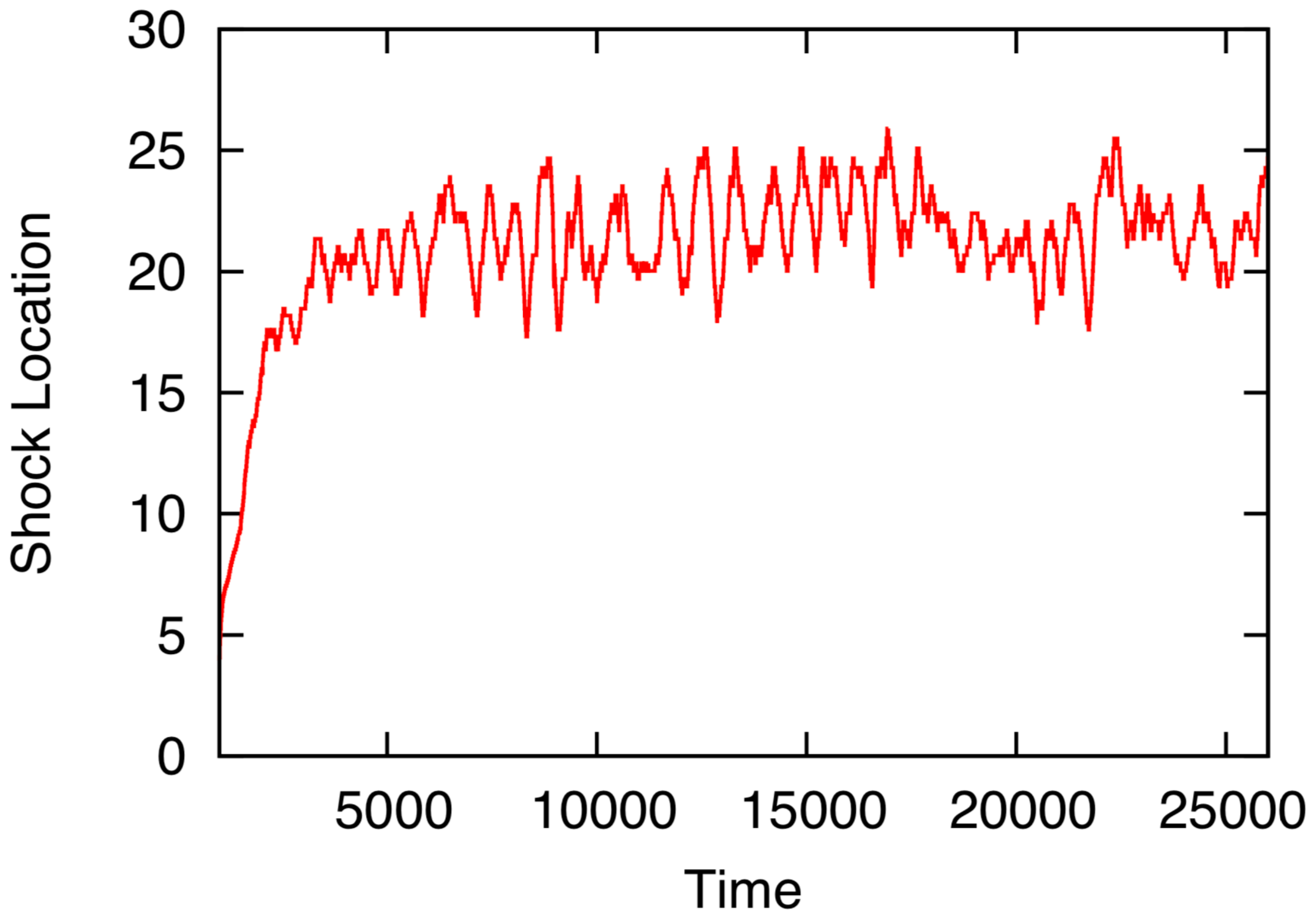


Kim et al. (2019)

2D Simulation of Accretion with Shock

ε = 1.001, l = 2.25 and a=0.95

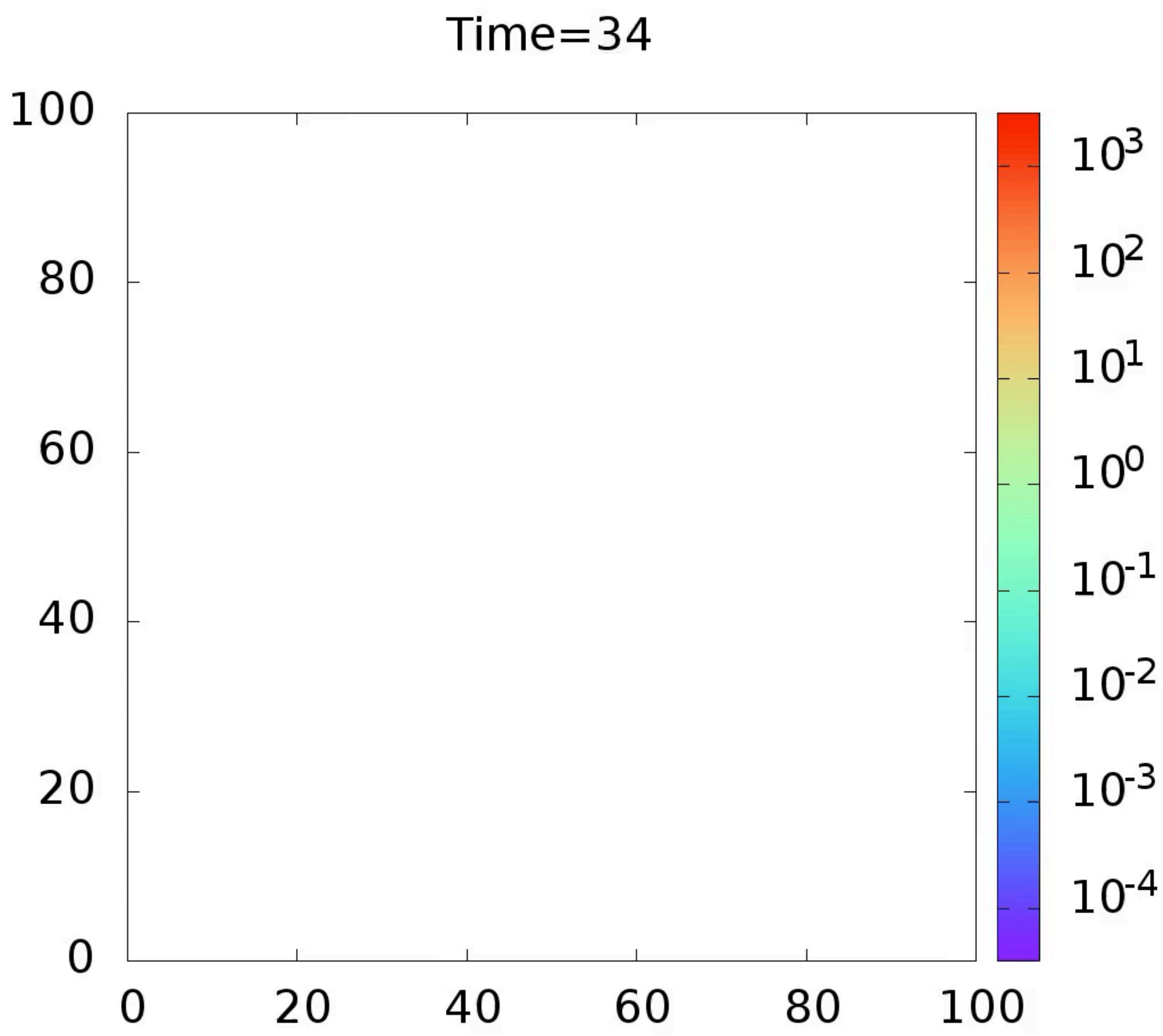


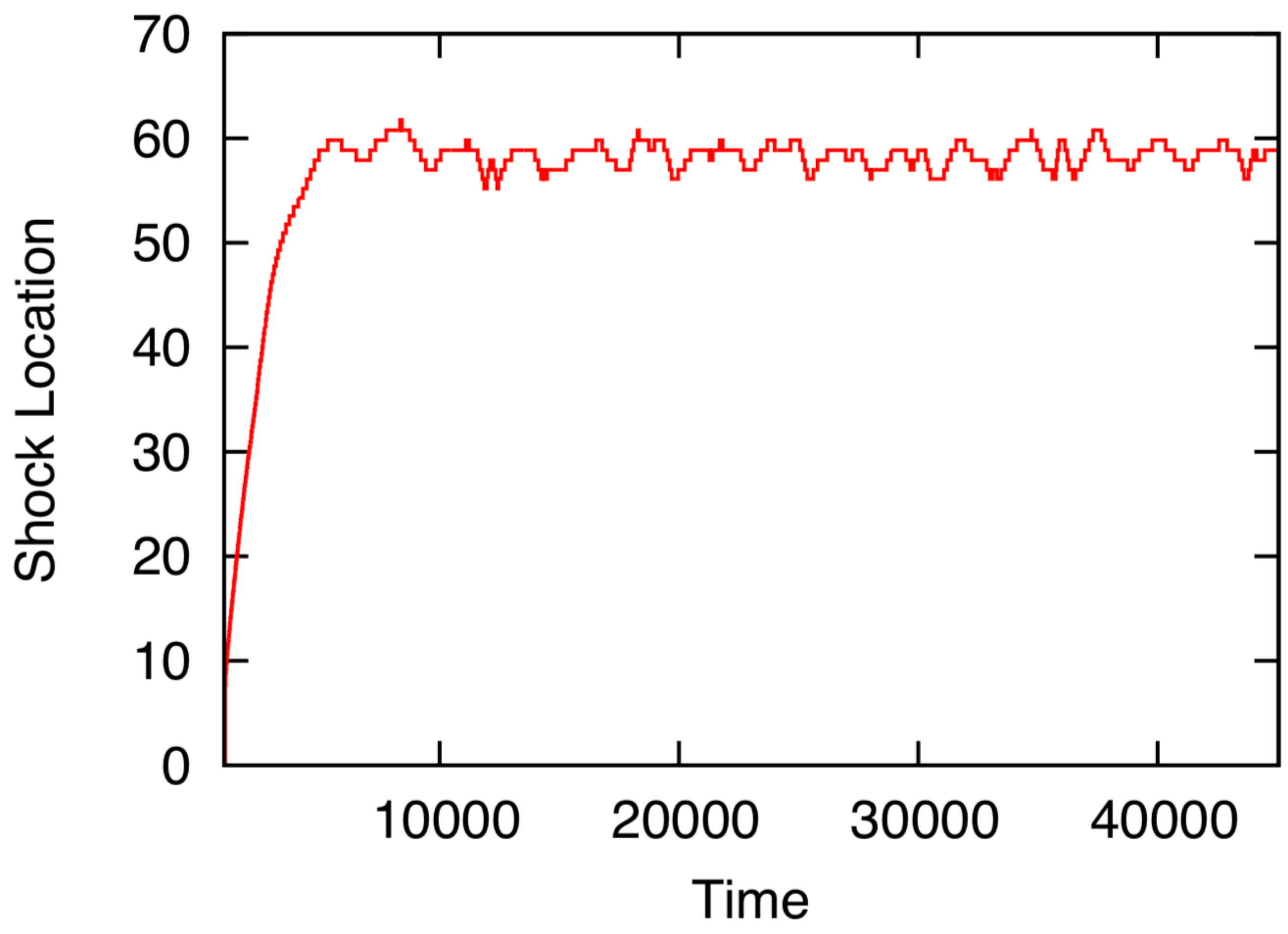


Kim et al. (2019)

2D Simulation of Accretion with Shock – retrograde

ε = 1.001, l = 4.0 and a=-0.95

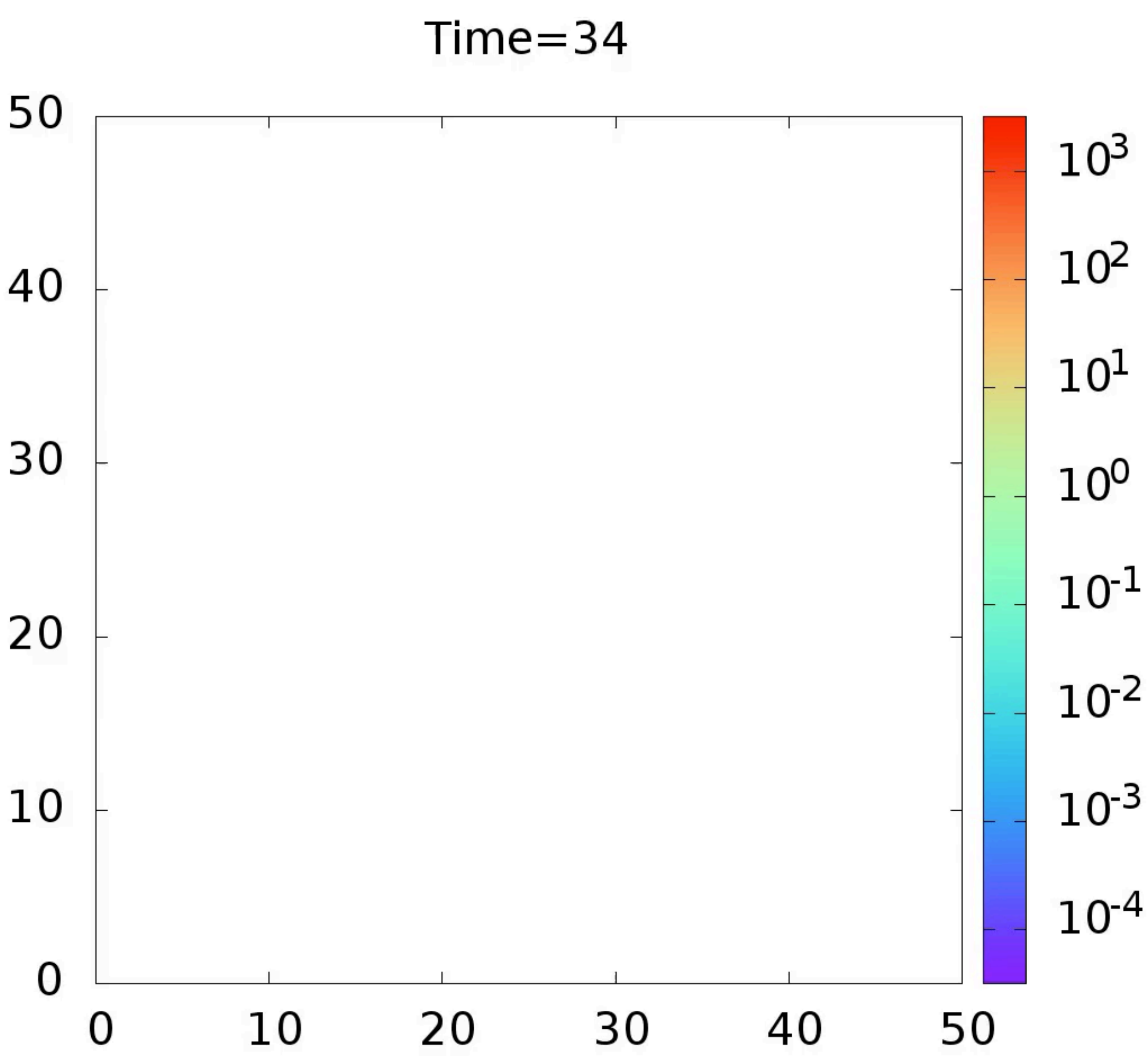


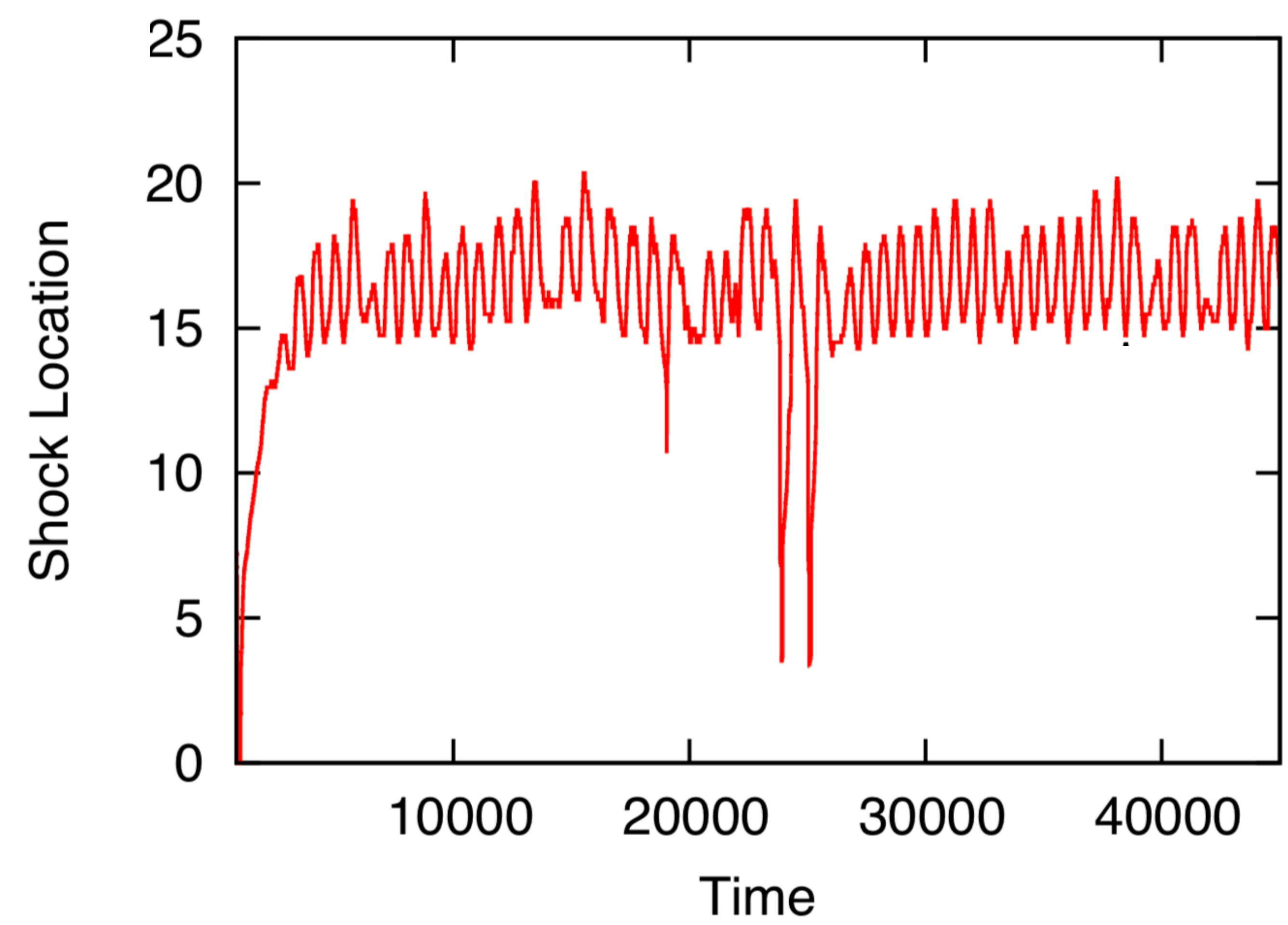


Kim et al. (2019)

2D Simulation of Accretion without Shock

ε = 1.001, l = 2.21 and a=0.95

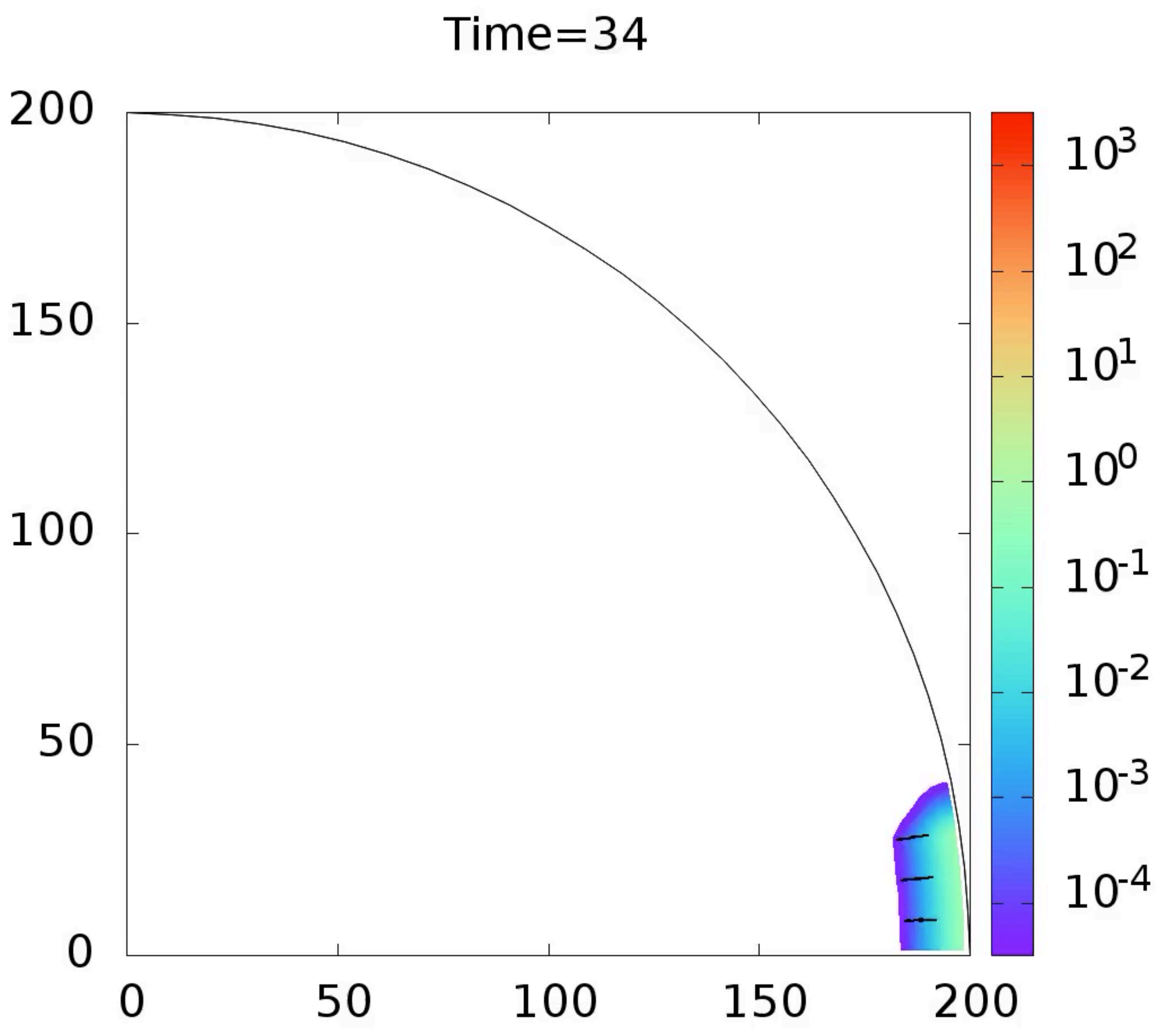


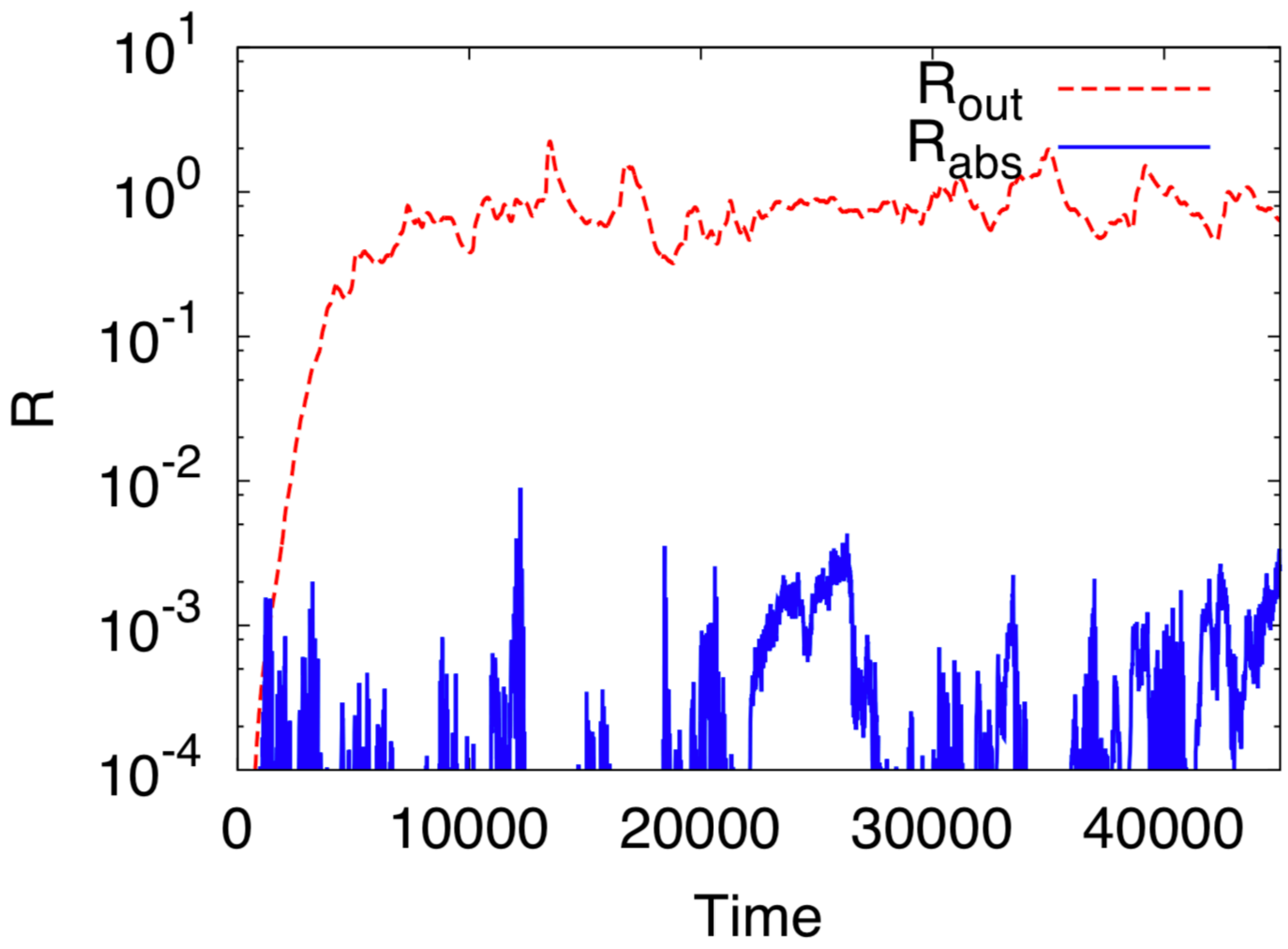


Kim et al. (2019)

2D Simulation of Accretion with High l

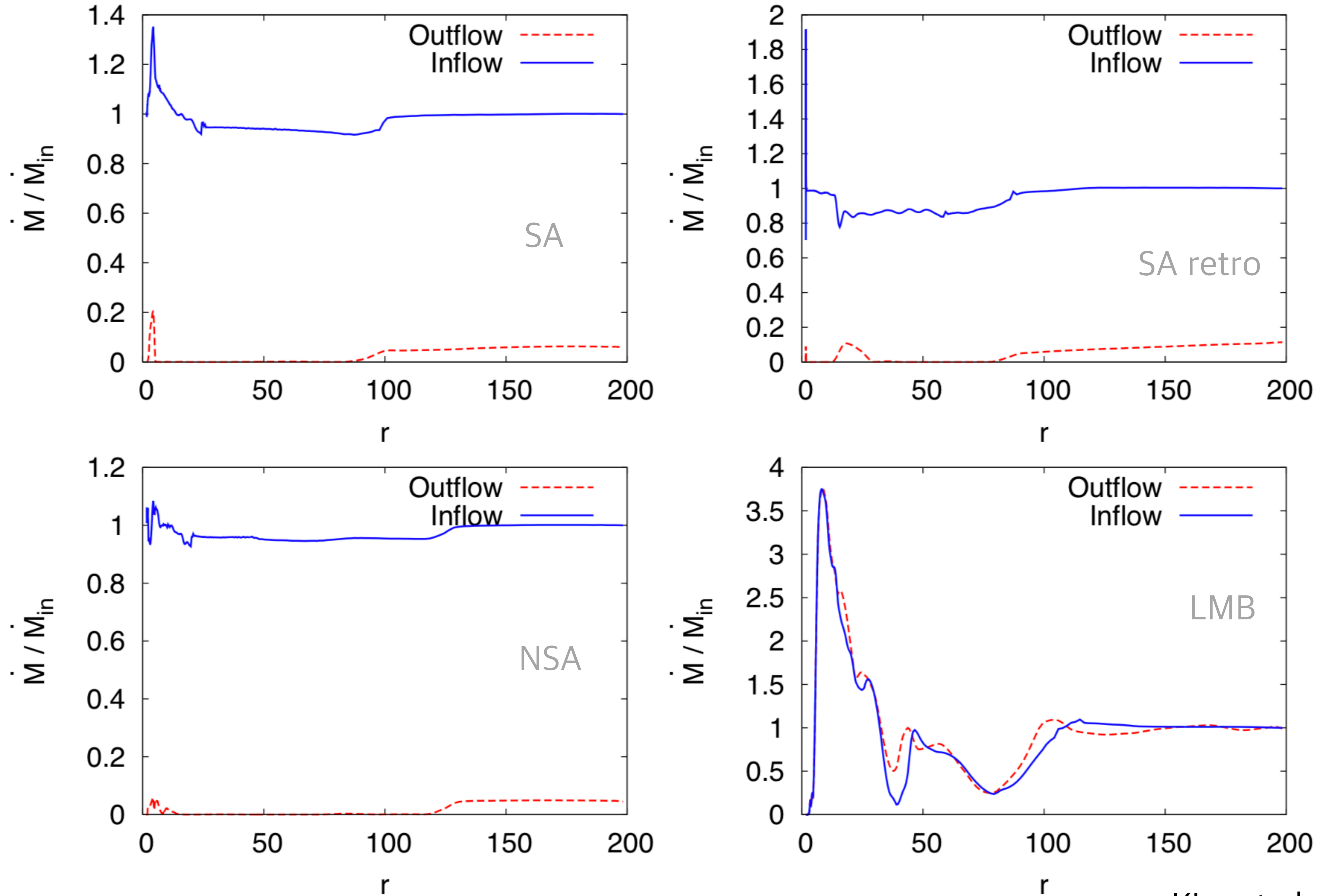
ε = 1.001, l = 2.63 and a=0.95





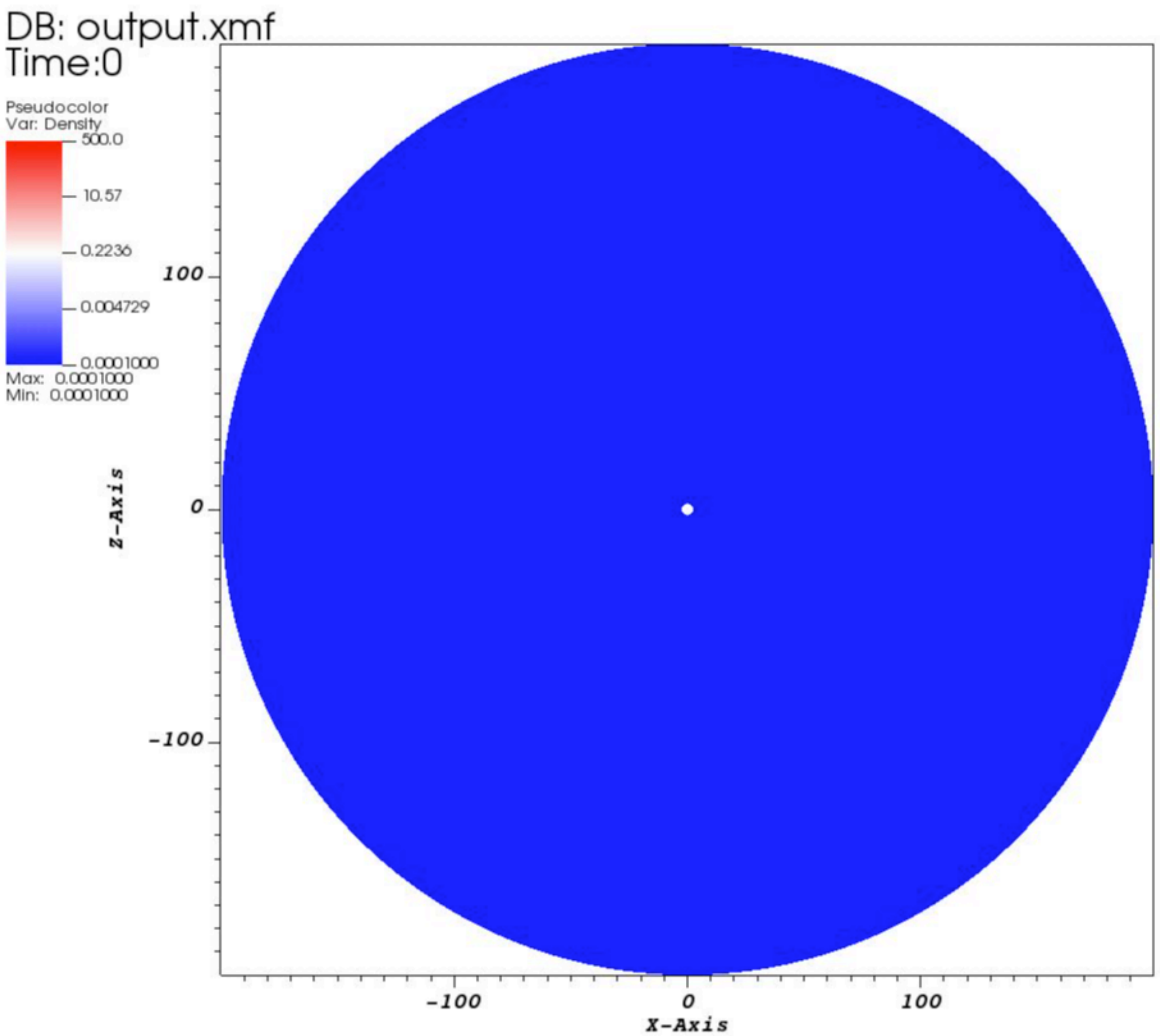
Kim et al. (2019)

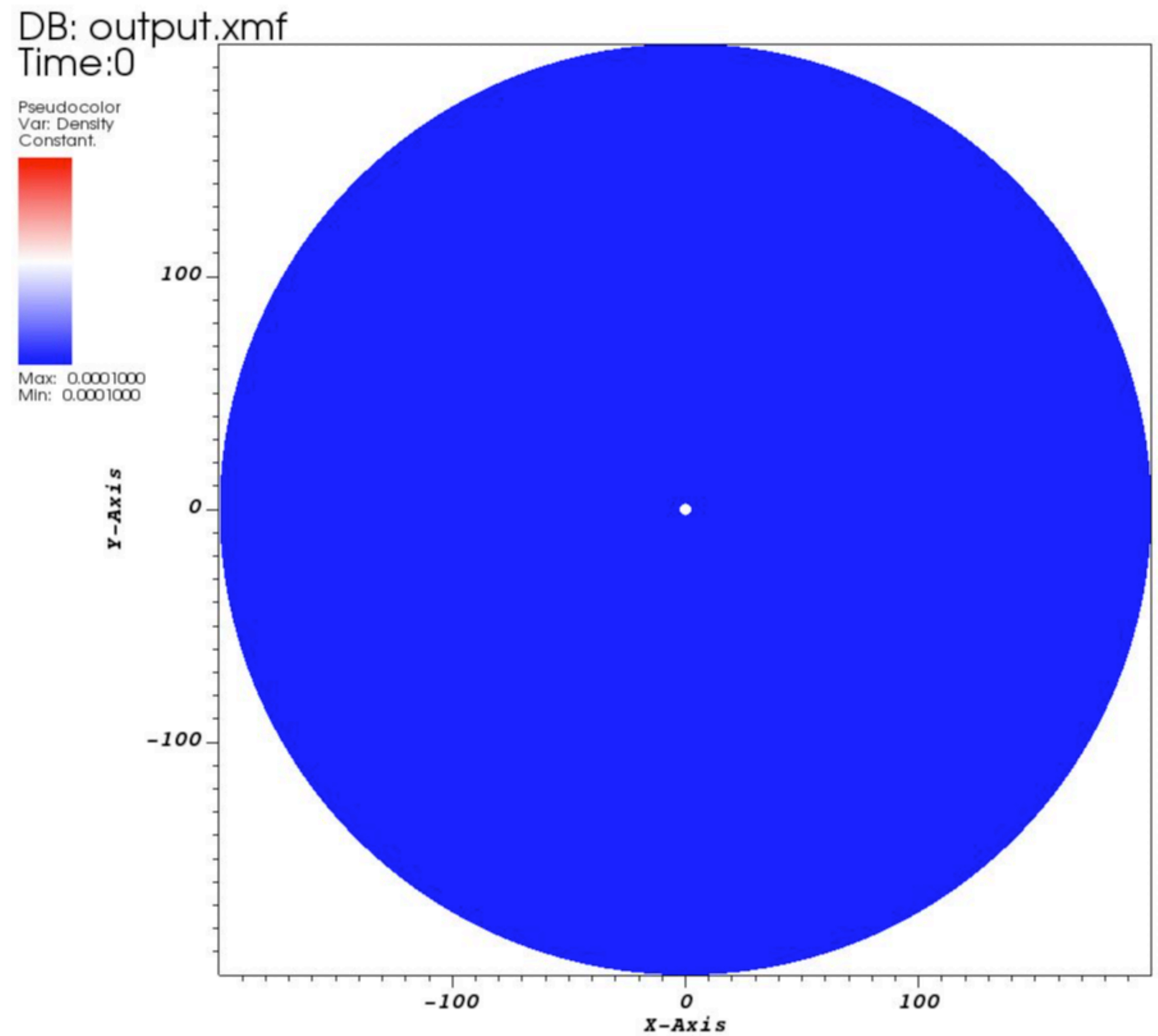
Inflow & Outflow Ratio



Kim et al. (2019)

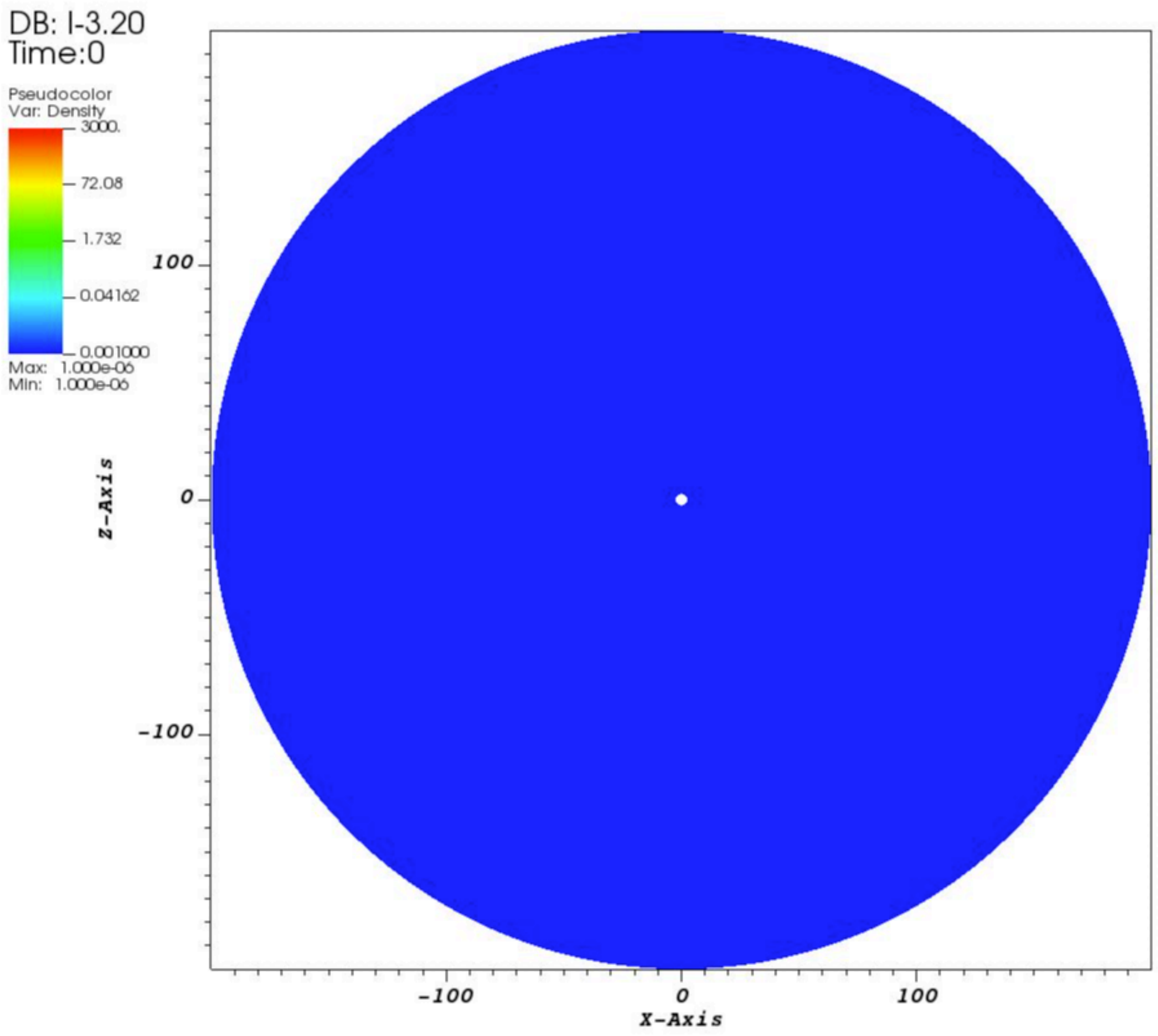
Three dimensional simulation

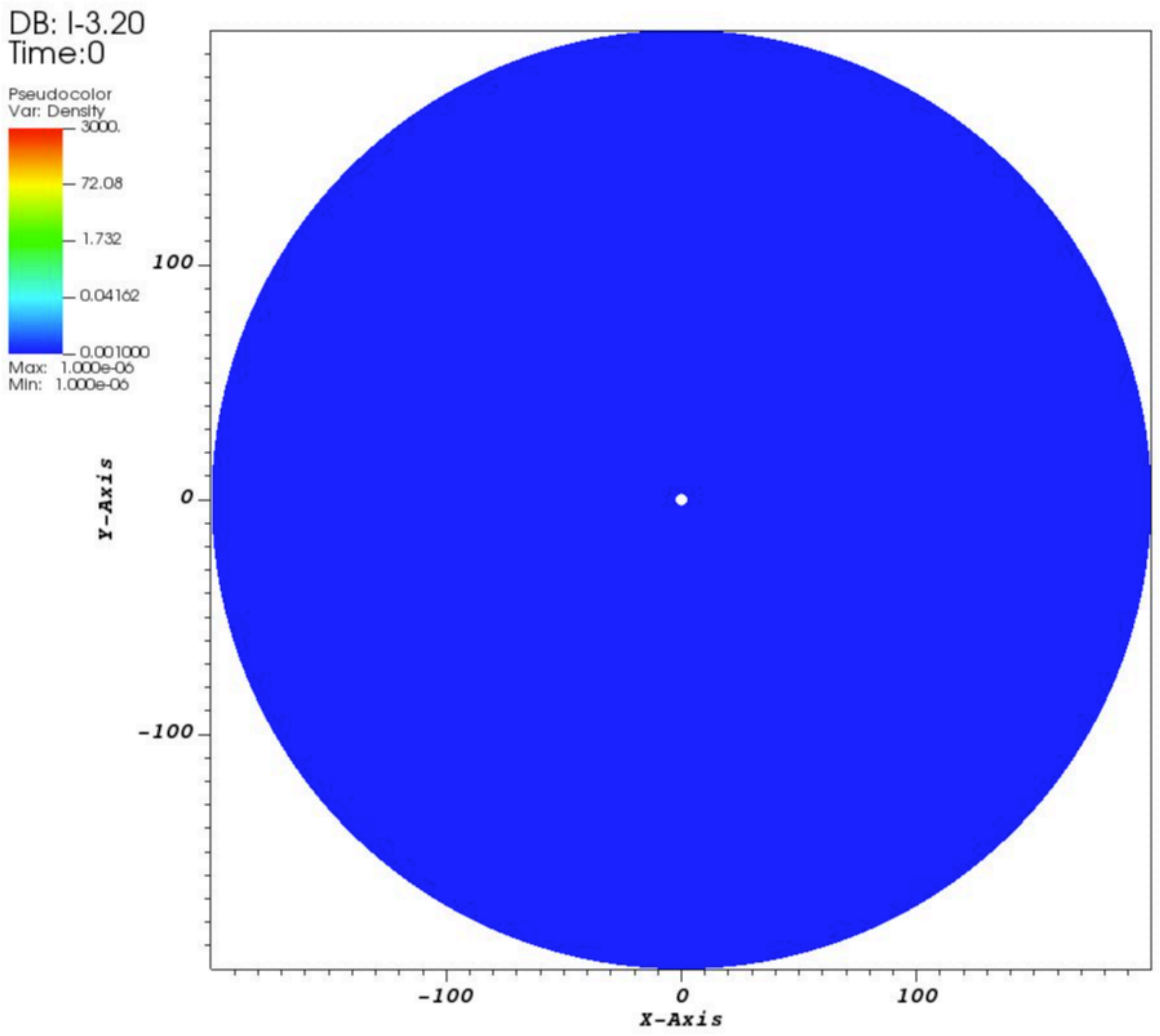




Preliminary result using KISTI Nurion with 1700 cores
(production runs use 28,000 cores)

Three dimensional simulation





Conclusion & Plan

- General relativistic simulation on the accretion disk around (non-)spinning black hole has been carried out.
- This results are consistent with those of Newtonian simulation.

- Non-axisymmetric simulation on the tilted disk using KISTI Nurion is planned to show
  - Precession of the jet
  - Warp of the disk