Analytic fitting Inspiral-Merger-Ringdown Waveform using Genetic Algorithm

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2020.02.07

1. Motivation

- Fast and accurate gravitational wave generators are required for efficient detection and parameter estimations.

- Only Numerical Relativity (NR) can solve full IMR waveform but the others are fitting models of NR either numerically or analytically.

- Each fitting model has its disadvantages, such as 'not accurate enough (Phenom)' , 'not fast enough (EOB)' or 'hard to implement in practical usages (Surrogate models)'.

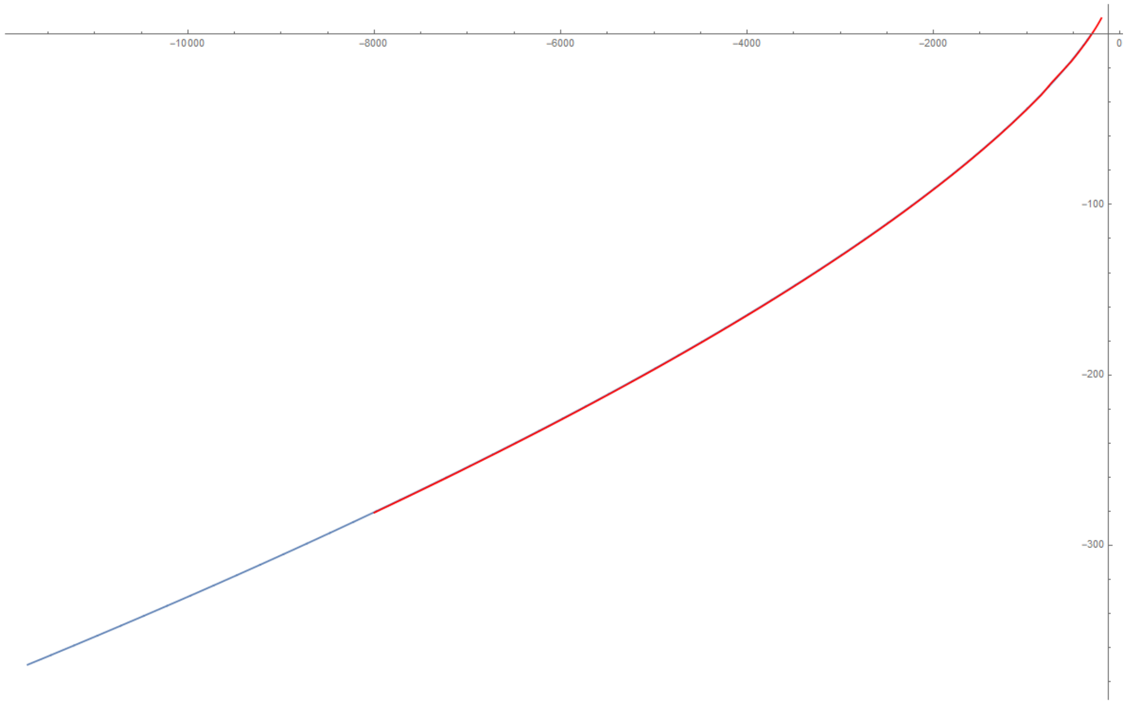
- To solve these problems, I suggest to use 'Genetic algorithm' to get analytic fitting of gravitational waves.

2. Strategies

a) Decompose strain data $h = h_+ + i\, h_x = A\, Exp(i\, \phi + i\, c)$ into **amplitude** and **phase**.

it is because that

a.1) The evolution of amplitude and orbital angular velocity (in the case of quasi-circular orbit) are determined. Thus one procedure of getting analytic expression from a single numerical relativity simulation is enough to produce every phase-different waveforms (it is equivalent with a choice of observers location)



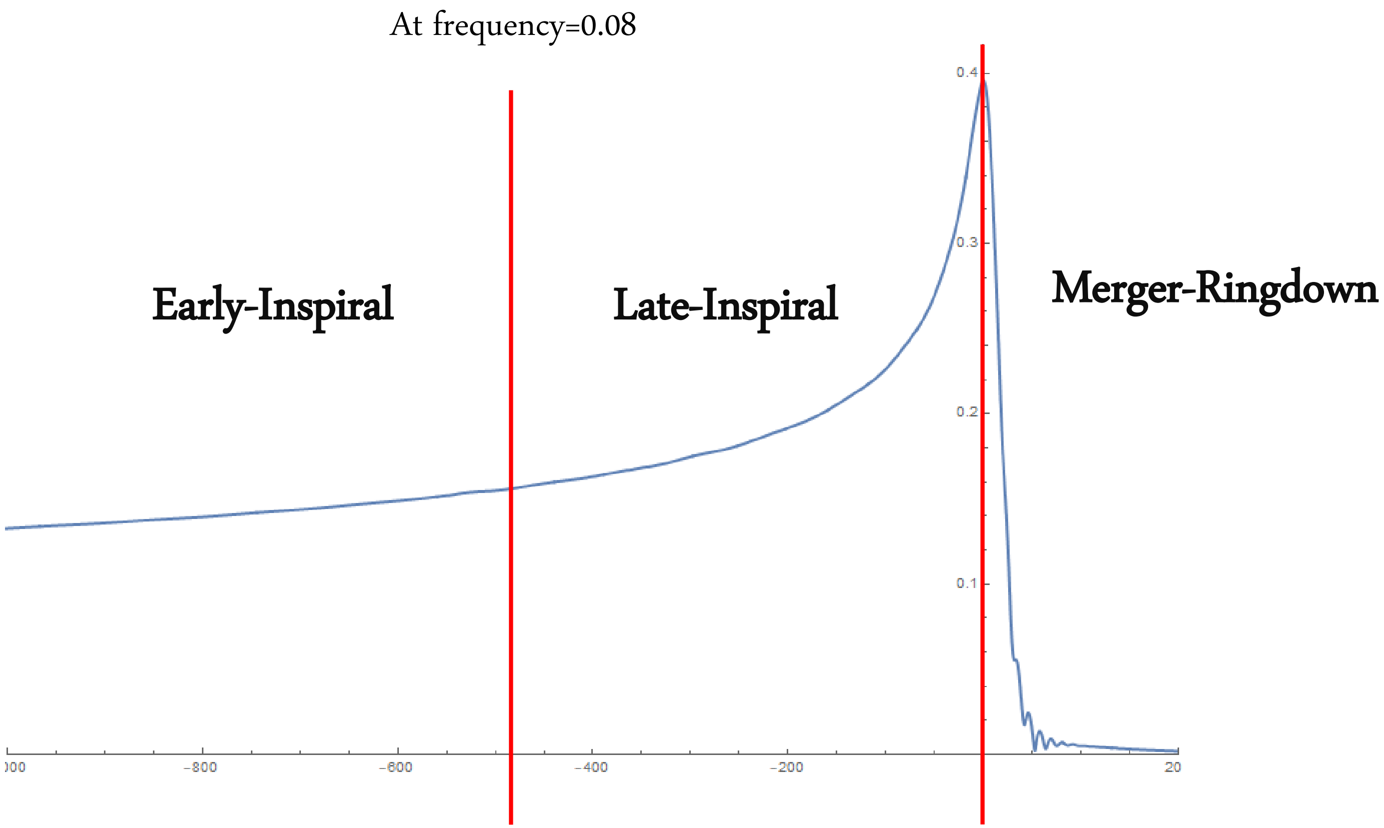
a.2) They have simple forms :

(eliminate wavy features from waves)

2. Strategies

b) Split waveform into Early-Inspiral (E), Late-Inspiarl (L) and Merger-Ringdown(M) stages

Because trivial shapes are fitted simply :



2. Strategies

c) By using Genetic Algorithm to get symbolic expressions for 6 components.

The symmetric mass ratio is restricted within $\eta = 0.0826446\sim \eta = 0.25$, but only $\eta = 0.17284$ case is used for fitting.

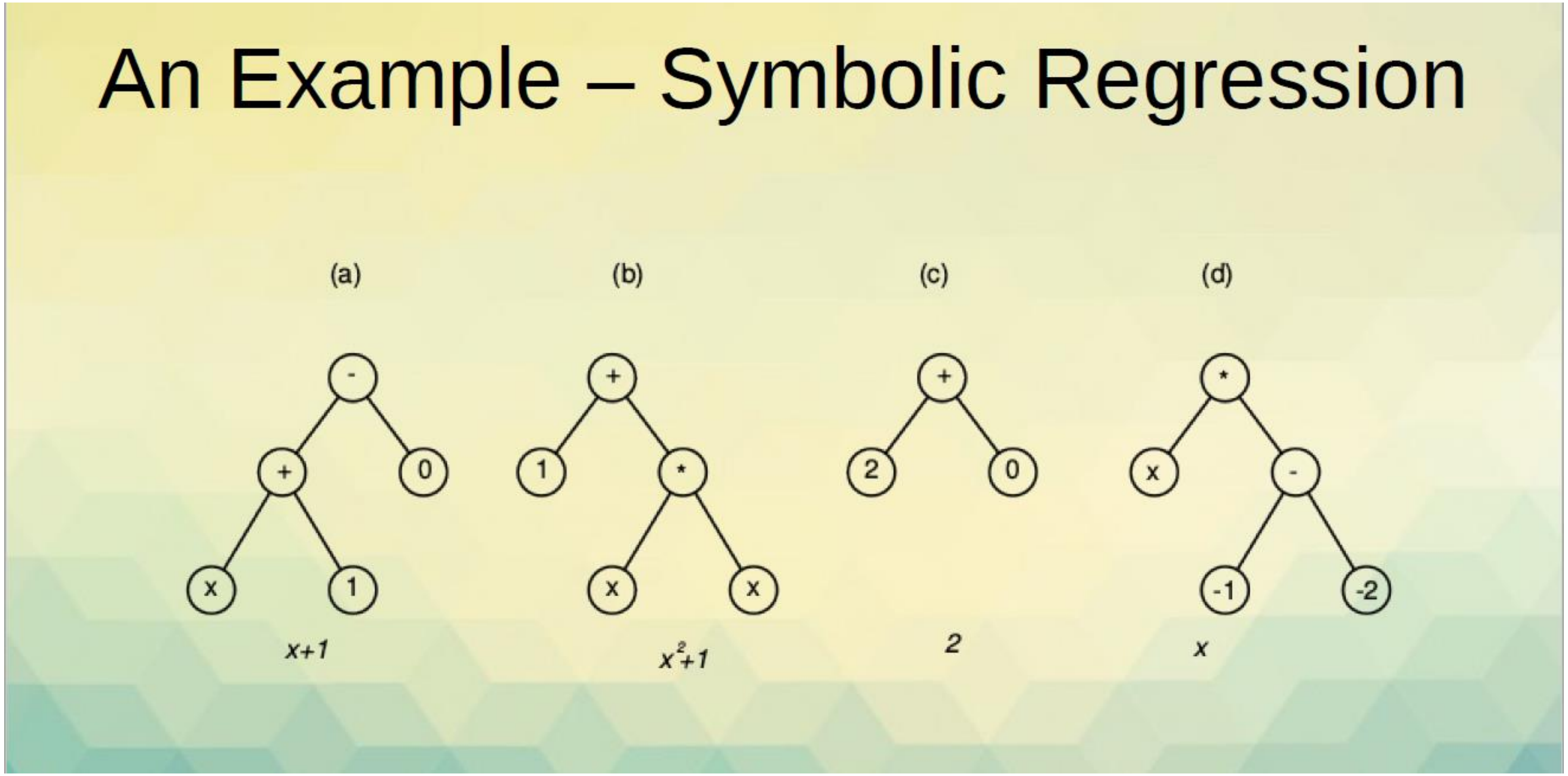
And replace every numerical coefficients into polynomials in η.

To simplicity, I restricted quasi-circular cases and used only 22 mode to make it independent on inclination angle of observers.

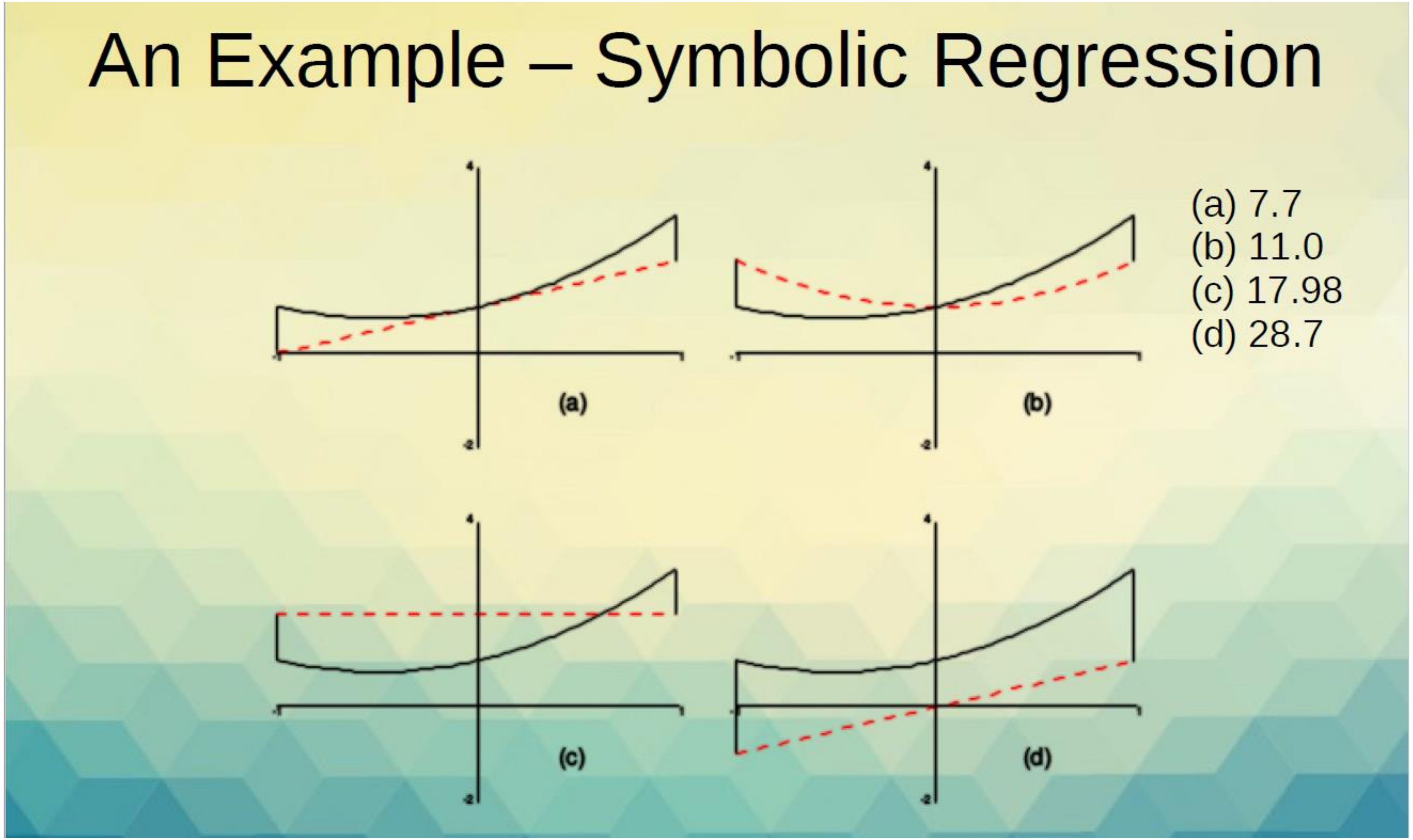
I adopt Geometric unit system (G=c=1) so that this model is independent on the scale of binary systems.

3. Genetic Algorithm

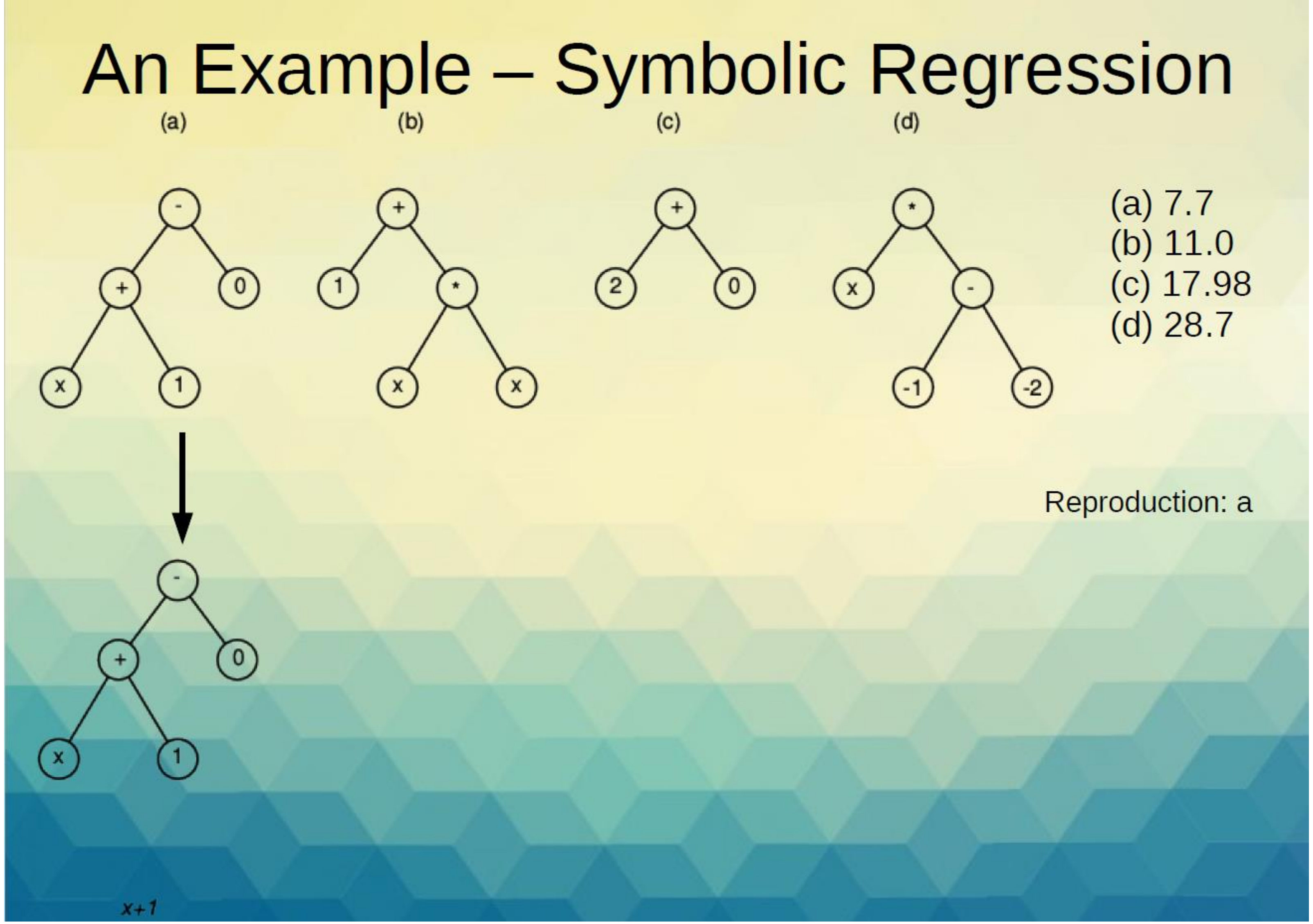
Find $x^2 + x + 1$



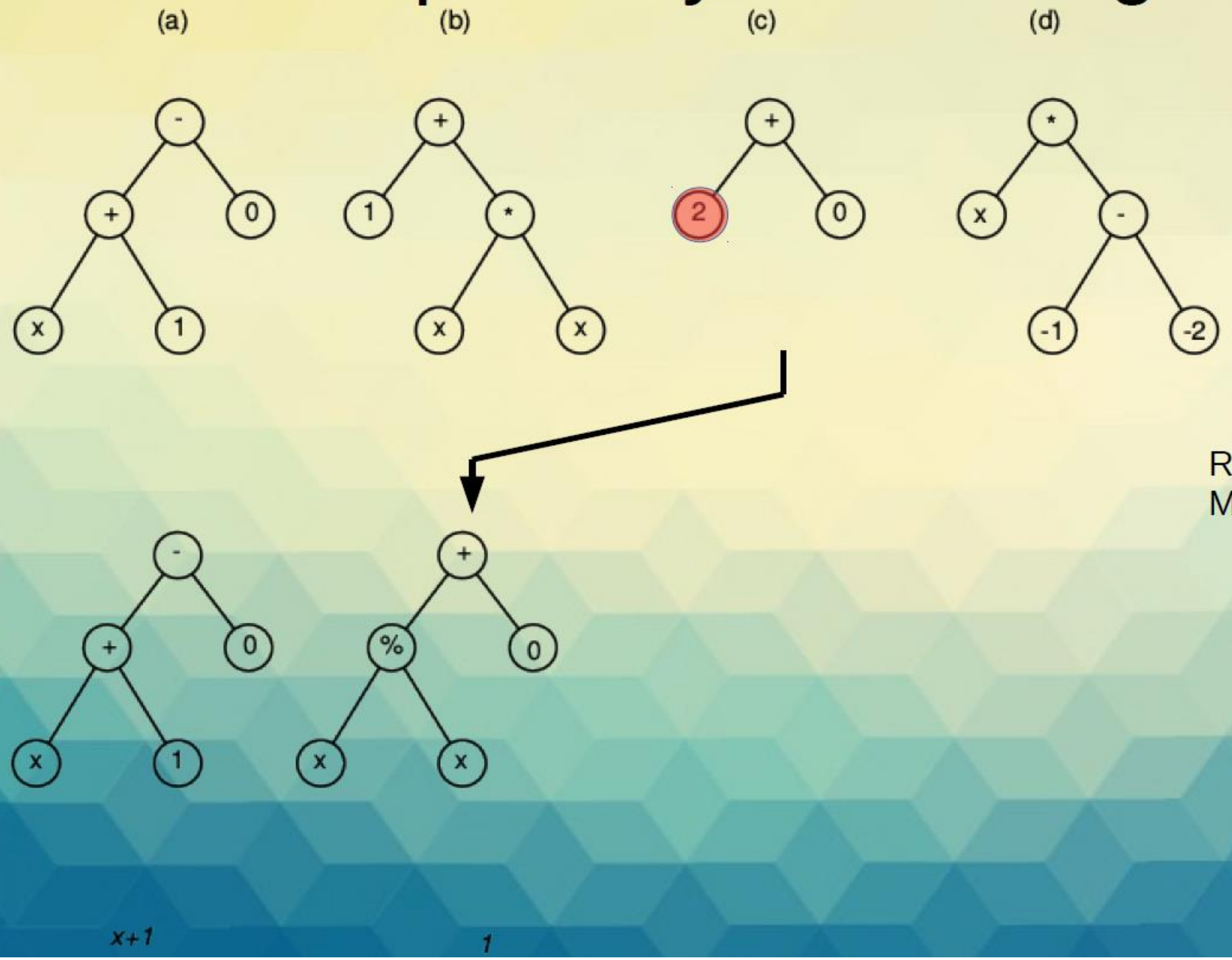
3. Genetic Algorithm



3. Genetic Algorithm



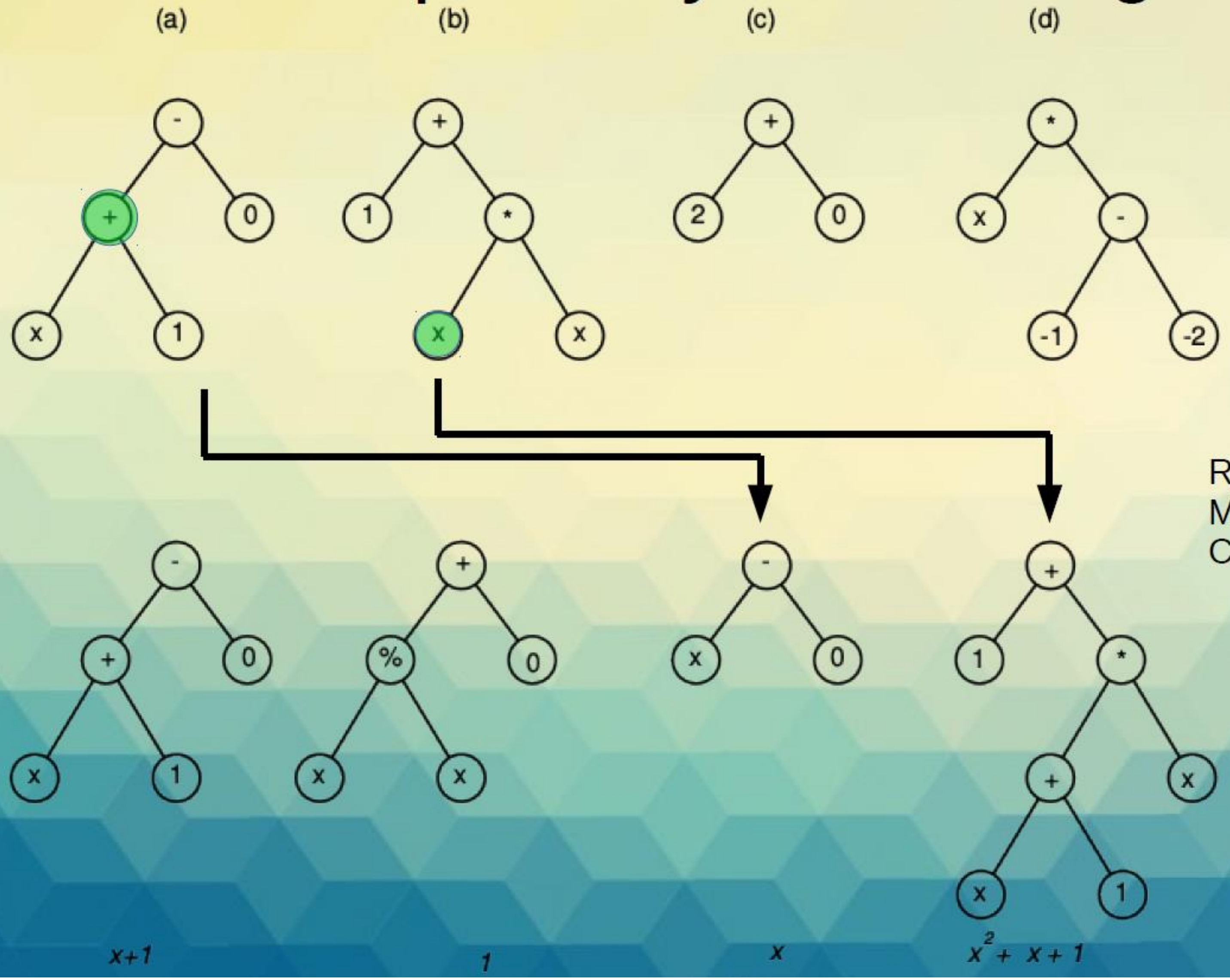
An Example – Symbolic Regression



(a) 7.7
(b) 11.0
(c) 17.98
(d) 28.7

Reproduction: a
Mutation: c

An Example – Symbolic Regression



(a) 7.7
(b) 11.0
(c) 17.98
(d) 28.7

Reproduction: a
Mutation: c
Crossover: a, b

4. Results

- For example, amplitude of late-inspiral has the following form

$$a - d\,\mathrm{Tanh}\left[e\,(-t)^{1/4} - b\,(-t)^{1/3}\right]\,\mathrm{Tanh}[\mathrm{Tanh}[c\,t]]$$

- The phase of late-inspiral has the following form

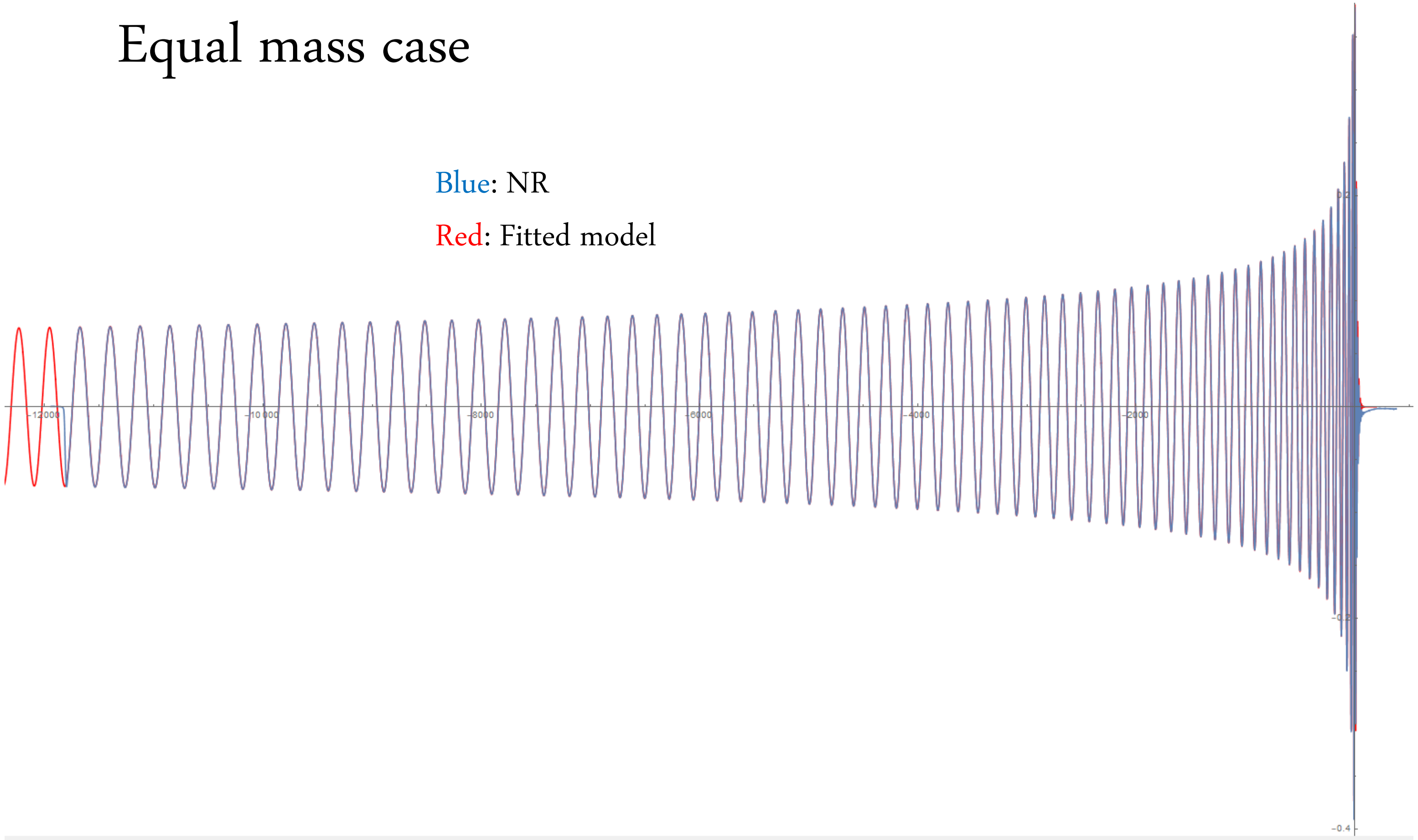
$$\frac{-b}{\mathrm{Log}[c]} - e\,t + \frac{b + a\,t}{\mathrm{Log}[c - t]}$$

4. Results

Equal mass case

Blue: NR

Red: Fitted model

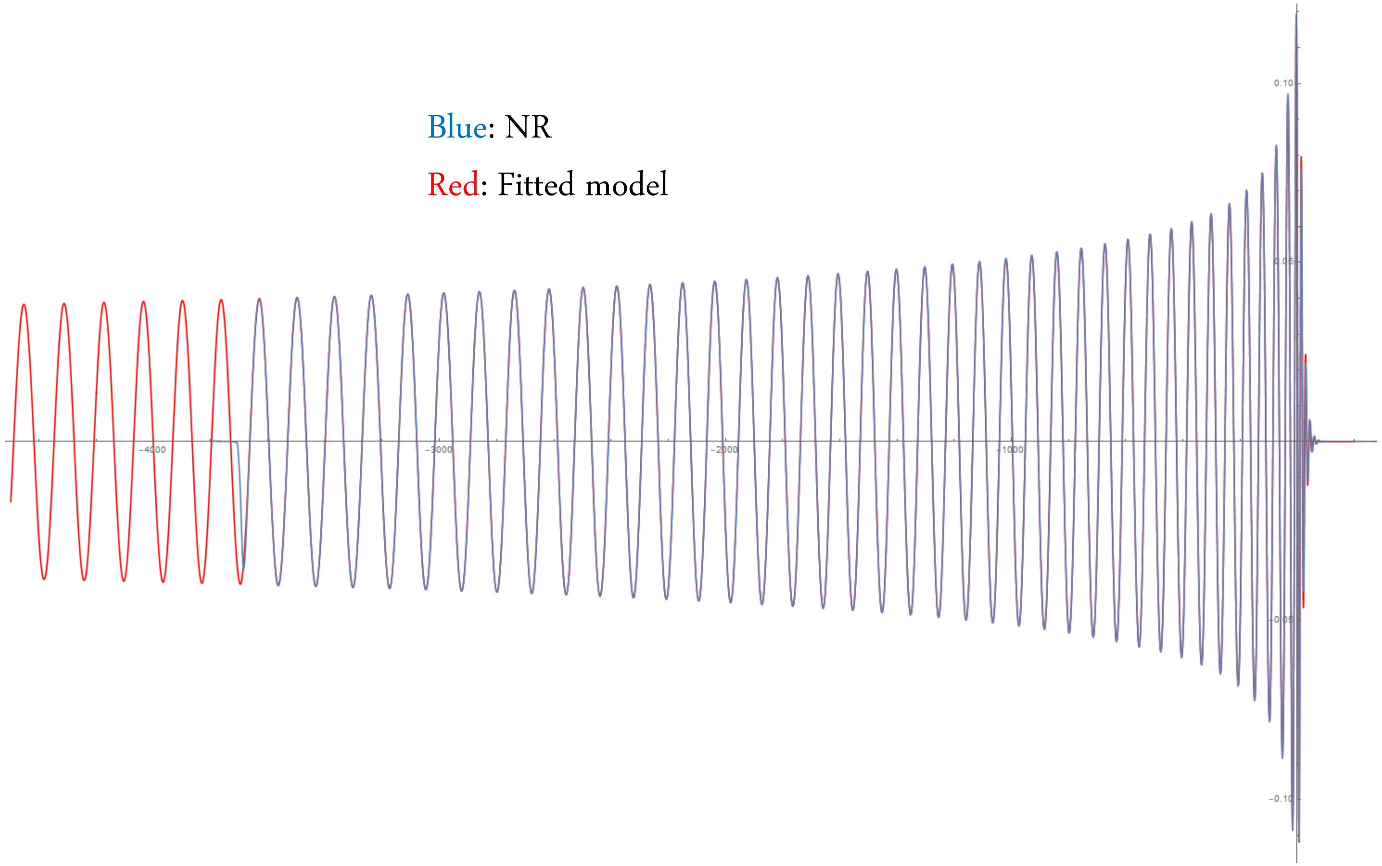


Almost completely overlapped

overlap=0.9994

4. Results

Mass ratio=10



Almost completely overlapped

overlap=0.9989

5. Advantages

- Fast(?) : One waveform needs~0.5 second in Mathematica. (When total mass =100 solar mass, and sample rates is 1/4096 sec starting from 40 hz.) It will be much faster in other language (e.g. C). Target speed is ~10^-3 second.

- Accurate(?) : Overlap ~ 0.999 (both train samples and test samples)

- Easy to implement : You can use this model if you know the technology Ctrl+C and Ctrl+V.

6. Future works

- Using EOB waveform rather than NR waveform : It is because that NR waveforms are noisy, specially in Ringdown and NR waveforms are too short.

- Interpolating to very large mass ratio cases (~10^9)

Thank you