Lucien HEURTIER

NEHOP '25 – Bruxelles

Thermal Imprints of Light Primordial Black Holes

King's College London

UKRI Science and Technology Facilities Council

Three genuine statements about PBHs

1. $\beta \gtrsim 10^{-(10^8)}$ requires quite some new physics

2. New physics has no preferred scale

3. PBHs do not have to be dark matter

PBHs may as well be

very light

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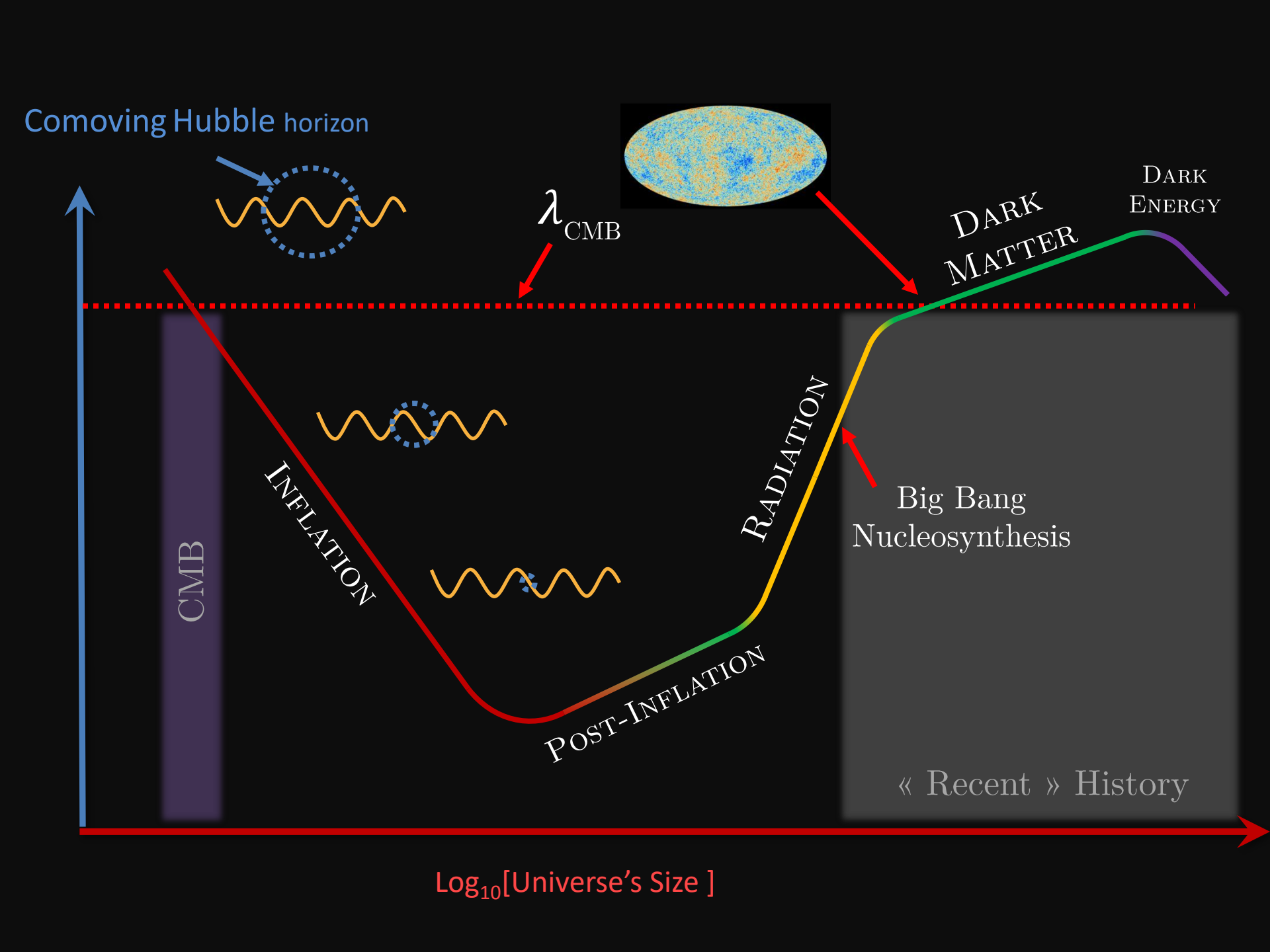
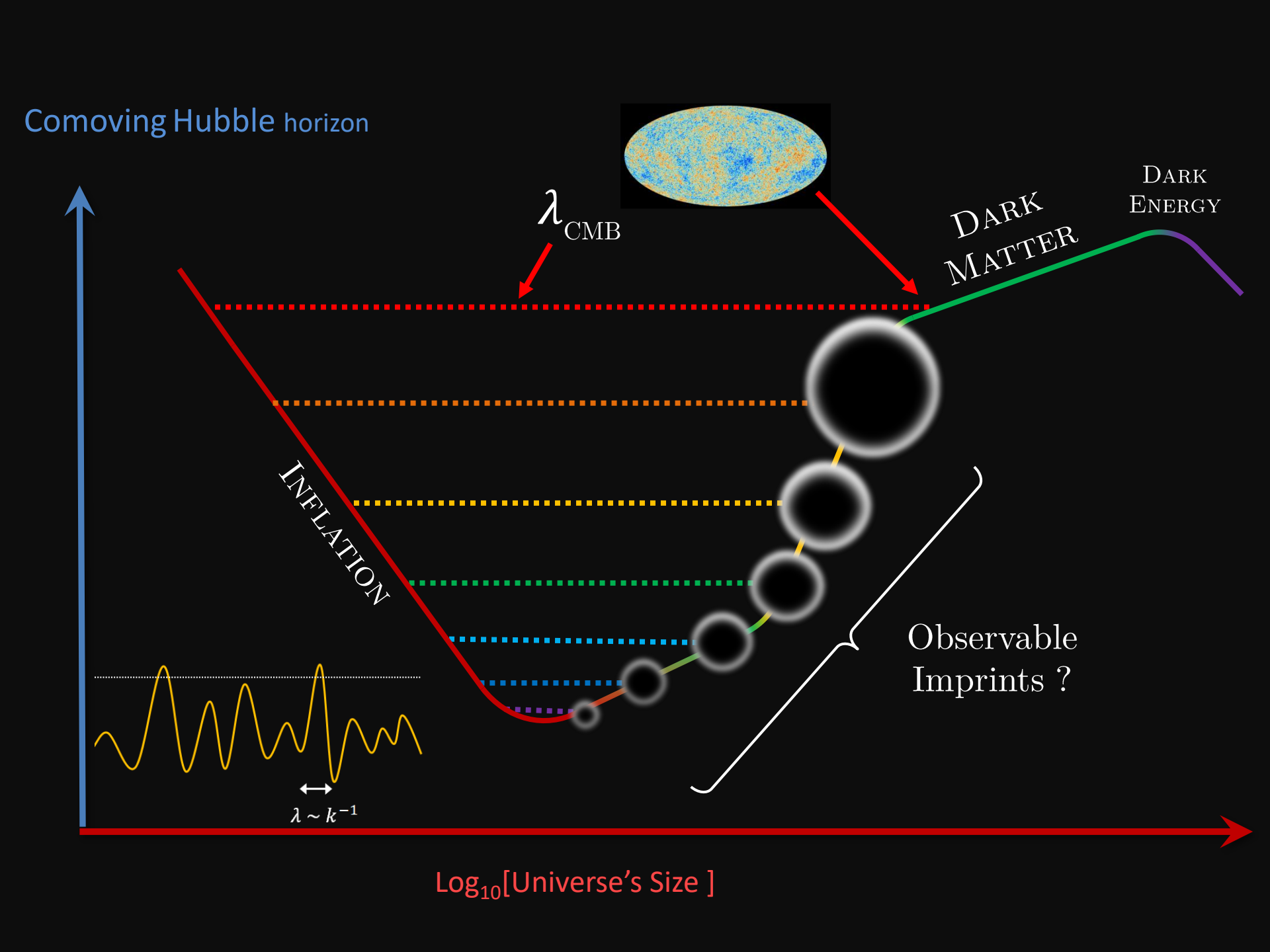

Comoving Hubble horizon
λ_{CMB}
Dark Matter
Dark Energy
Inflation
Radiation
Post-Inflation
CMB
Big Bang Nucleosynthesis
« Recent » History
Log_{10}[Universe's Size]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$\lambda \sim k^{-1}$
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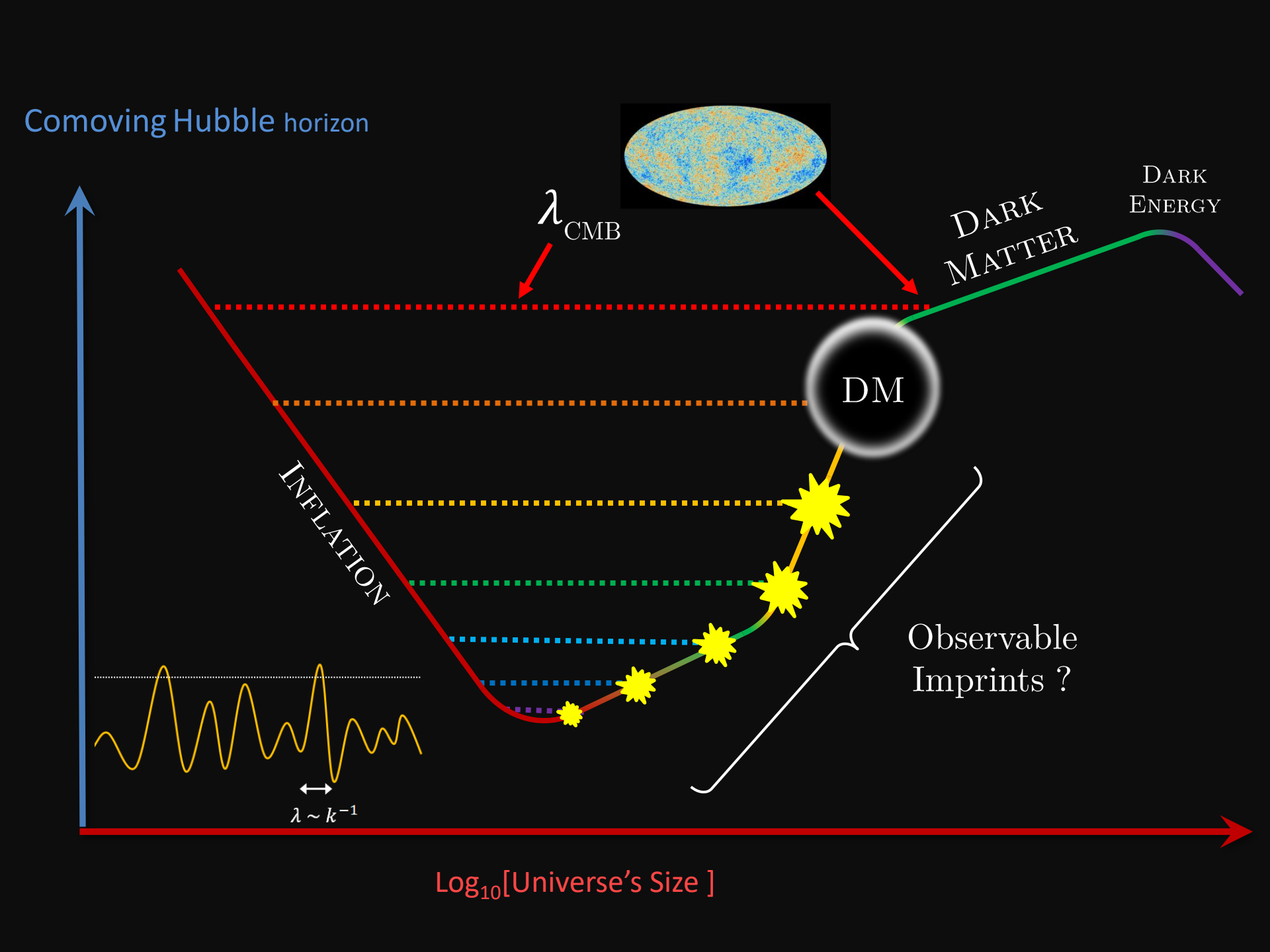
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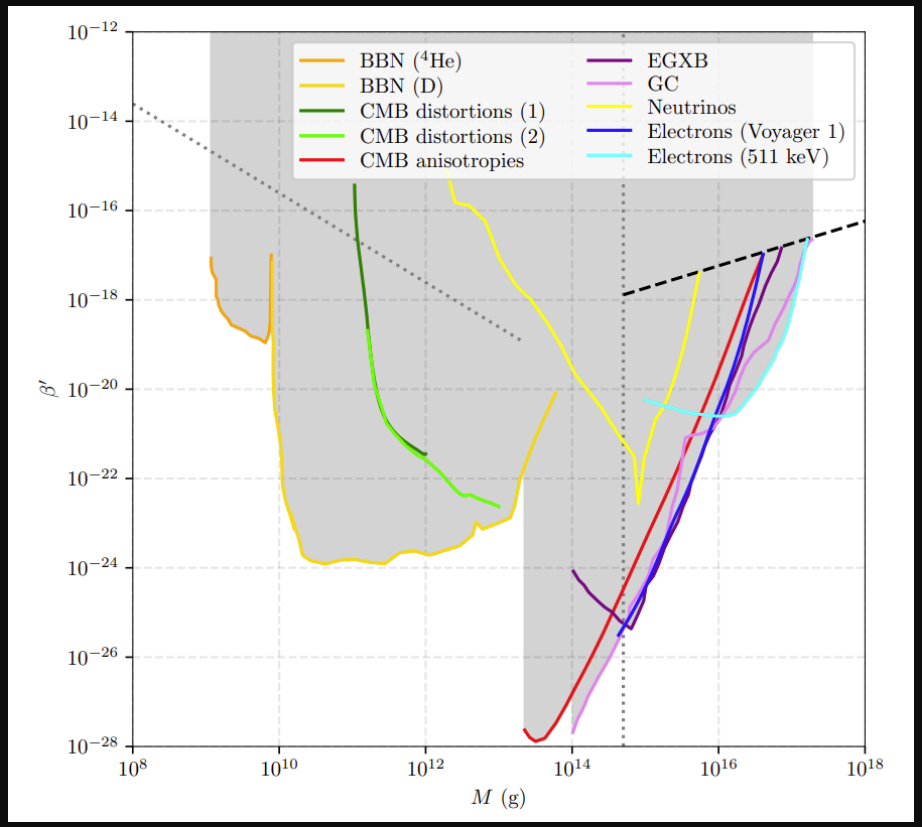
Light PBHs can be constrained...

.. iff what they emit is able to free stream!



[Auffinger 2022]

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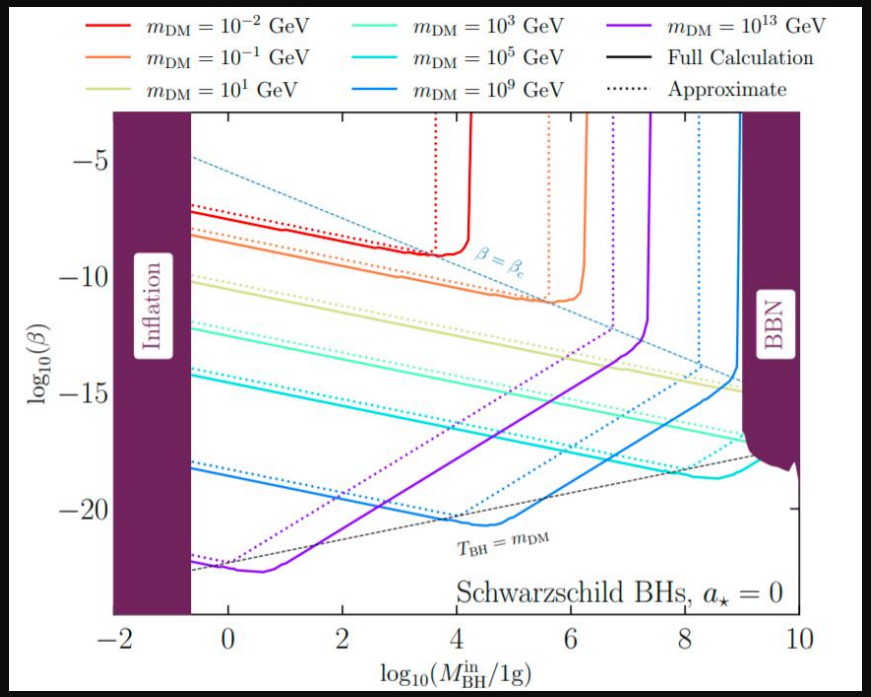

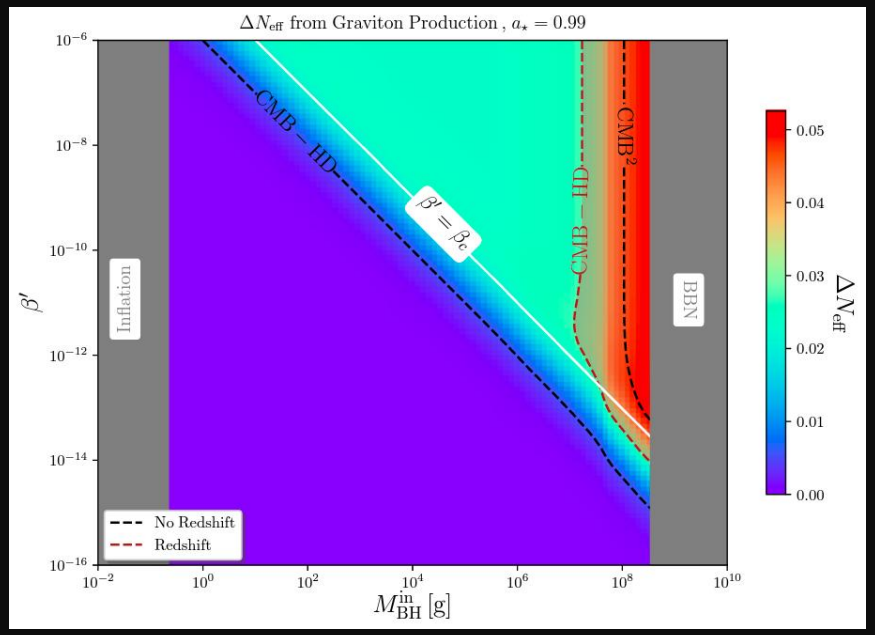

[Cheek, LH, Perez-Gonzalez and Turner]

Code Publicly available: FRISBHEE

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One crucial assumption:

SM background homogeneous at all times

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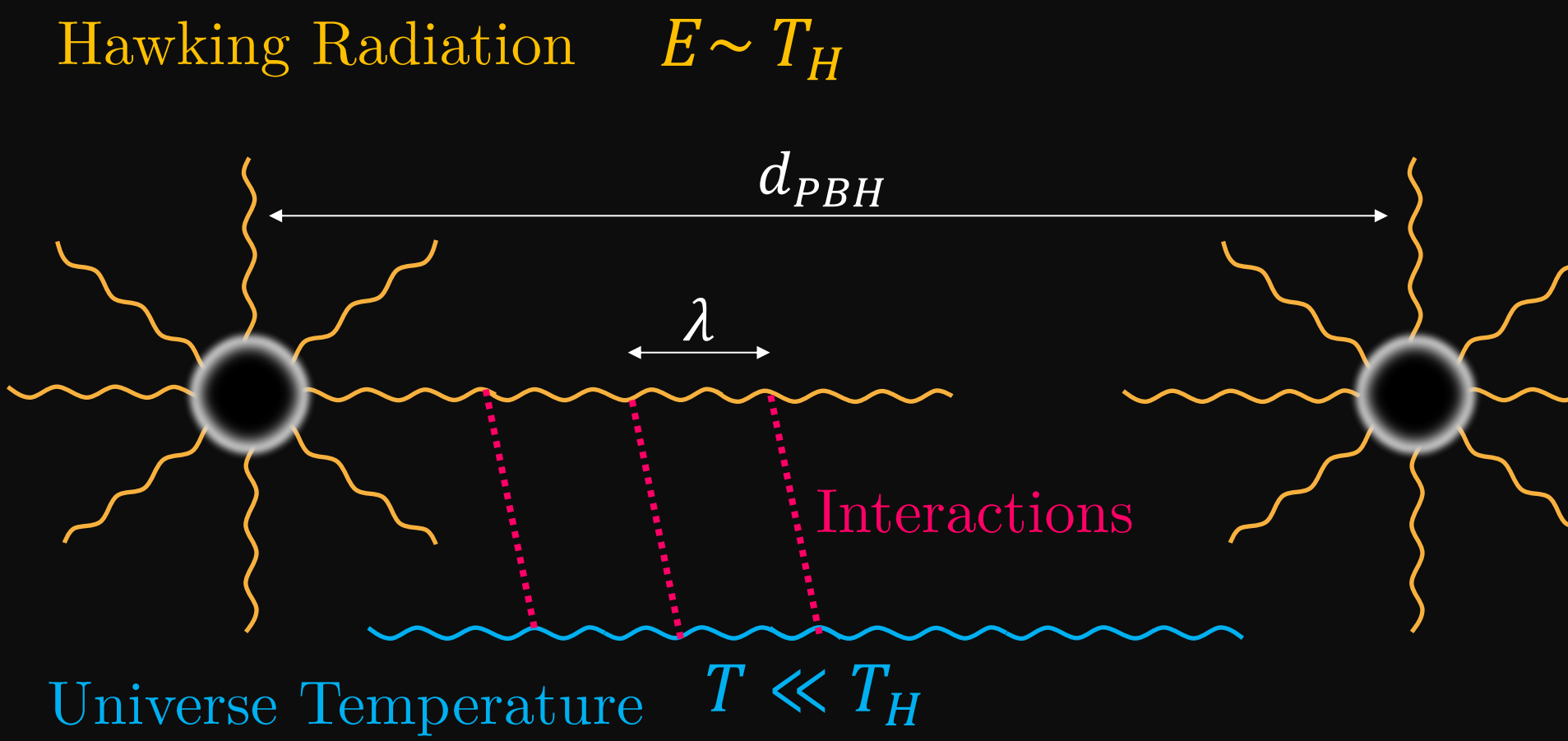


Light PBHs can be constrained...

.. iff what they emit is able to free stream!

If $\lambda > d_{PBH}$: emission can be assumed homogeneous

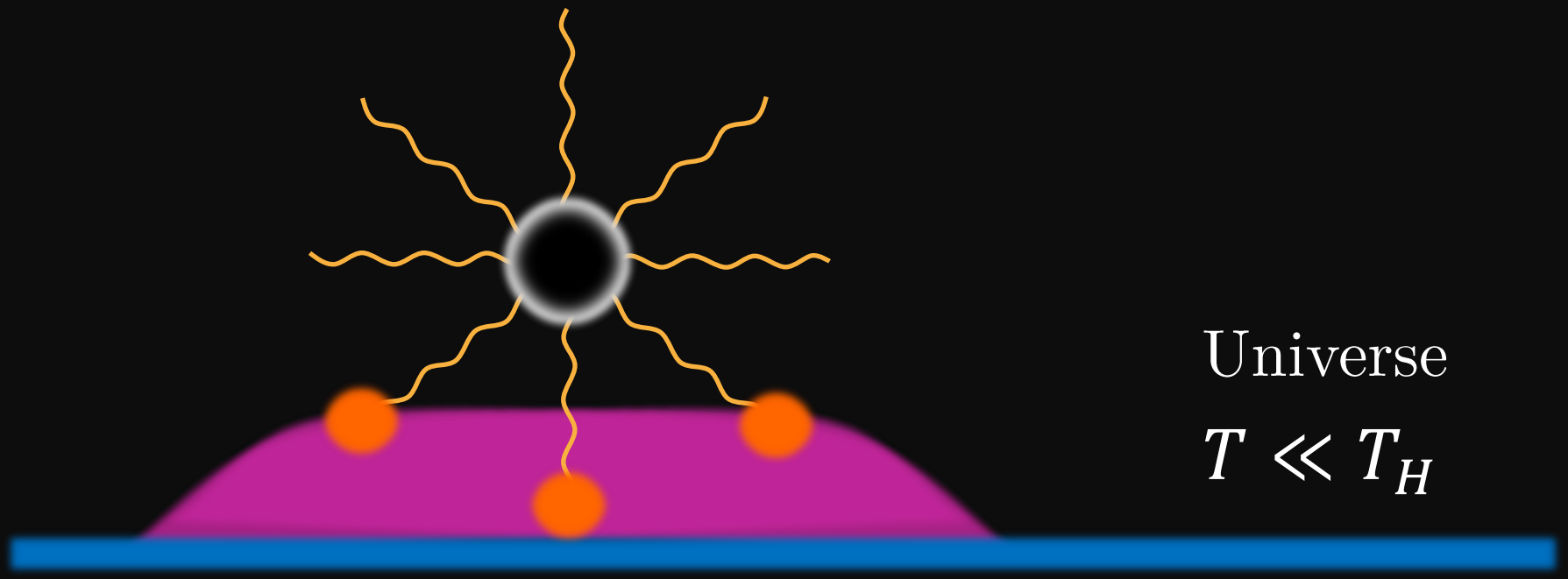
If $\lambda < d_{PBH}$: localisation effects may matter

Problem: λ often estimated in a homogeneous plasma...

In Reality

Hawking Radiation

$E \sim T_H$



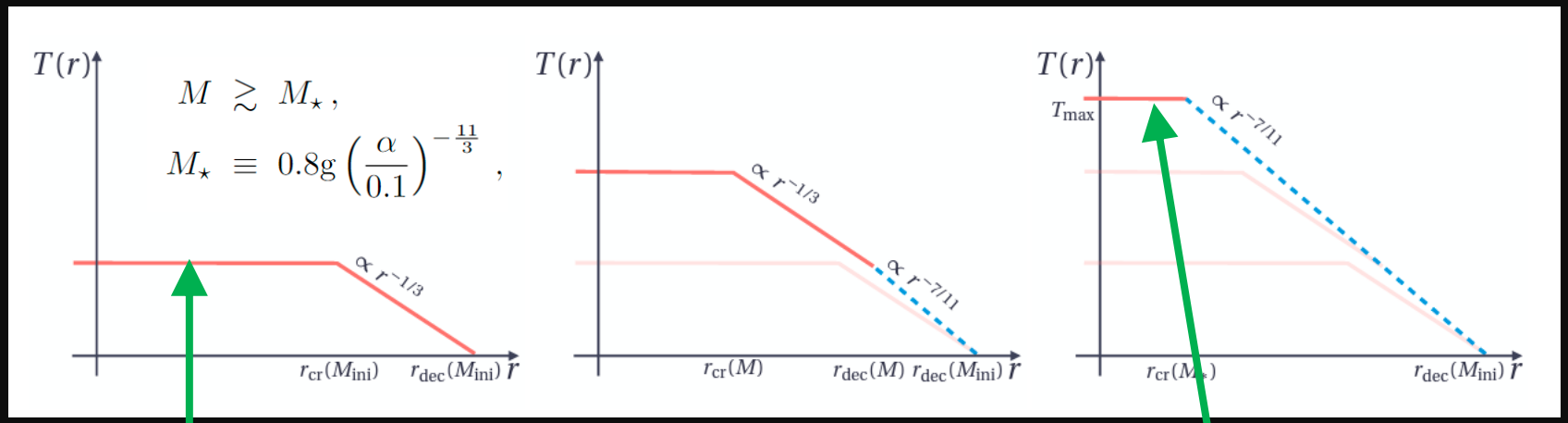
Universe

$T \ll T_H$

In Reality

Hawking Radiation heats the ambient plasma locally

He et al. *JCAP* 01 (2023) 027



$$T_{\rm plateau} \approx 2\times 10^{-4}\left(\frac{\alpha}{0.1}\right)^{\frac{8}{3}} T_H$$

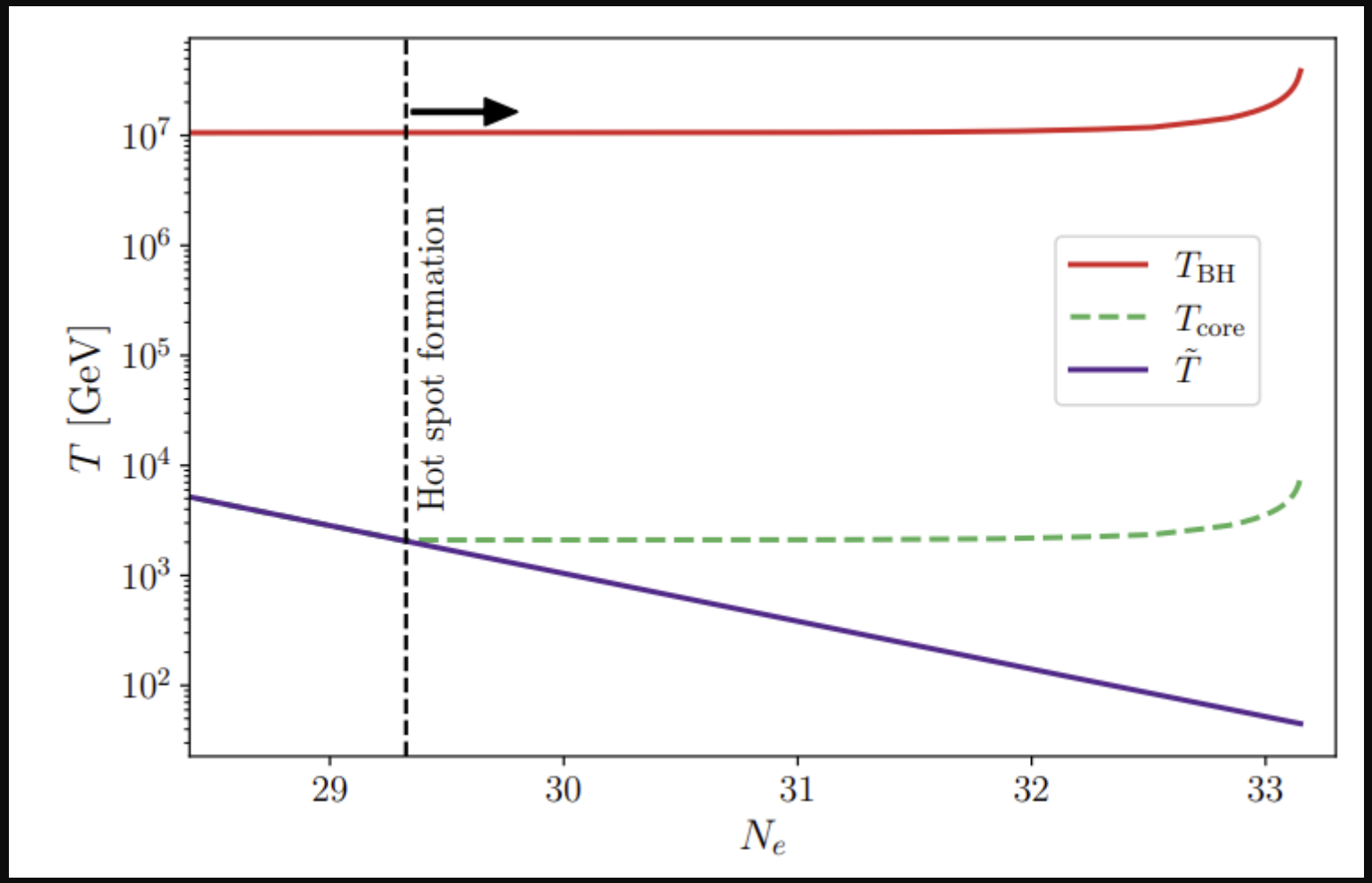
$$r_{\rm plateau} \approx 7\times 10^{8}\left(\frac{\alpha}{0.1}\right)^{-6} r_H$$

$$T_{\rm max} \approx 2\times 10^{9}{\rm GeV}\left(\frac{\alpha}{0.1}\right)^{\frac{19}{3}}$$

$$r_{\rm max} = r_{\rm plateau}\Big|_{T_H=T_{\rm max}}$$

Hot Spot & BSM

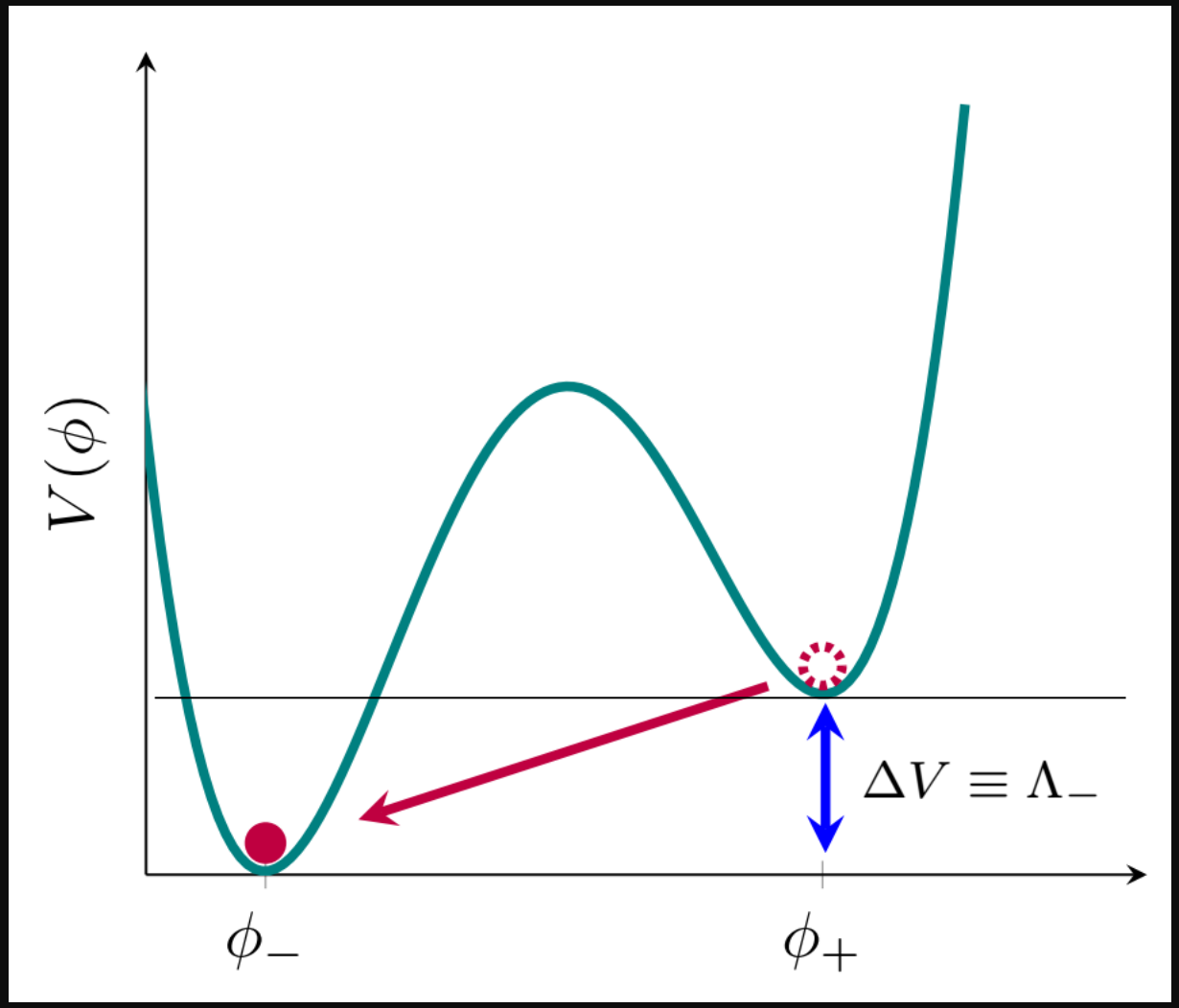
Hot Spots rise while the universe cools down



[J.Gunn, LH, Y.F. Perez-Gonzalez, J. Turner, '24]

Example 1: PBHs and 1st order phase transitions

1st-Order Phase Transitions for Dummies



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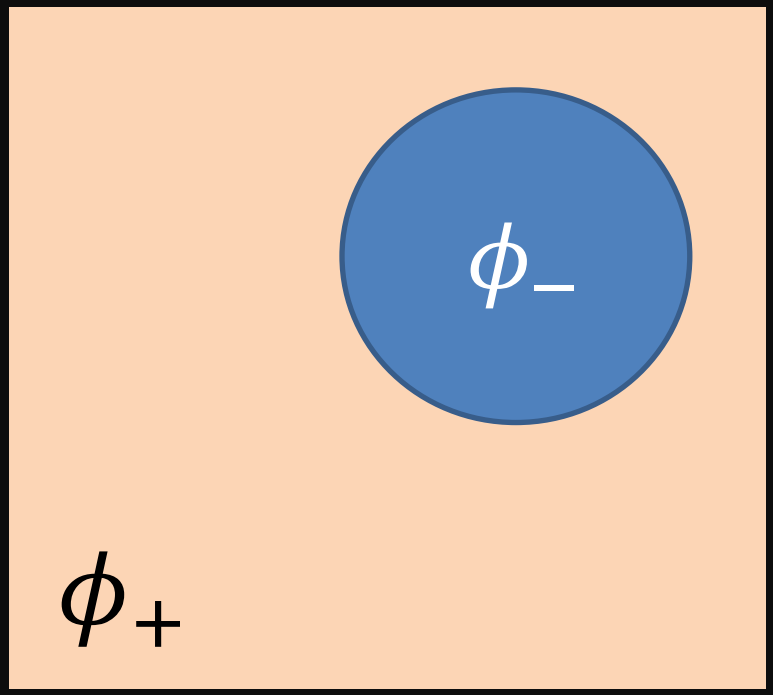


Transition ⟷ Energy Loss

In (cold) GR: Coleman & De Luccia, Phys. Rev. D 21, 3305 (1980).

⟶ The metric and bubble adjust to conserve energy

Energy = Metric Deformation

Question: What happens around a radiating Black Hole?

1ST-Order Phase Transitions for Dummies

Question: What happens around a radiating Black Hole?

So Far: Only considered in very extreme situations…

BH radiating in the vacuum (Unruh vacuum)

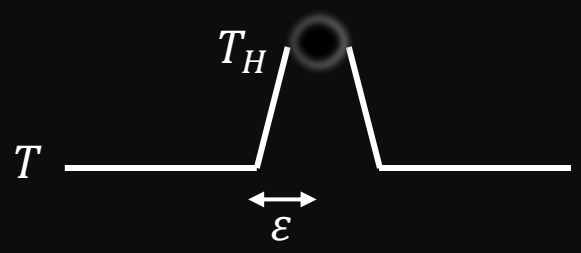
→ No definite answer. Partial results only obtained in 2D.

BH in thermal equilibrium with the plasma (Hartle-Hawking vacuum)

→ The BH and the plasma both behave as thermostats.

$$I_{\rm b}[T] = \beta \int dx^3 \sqrt{-h} \left(-\frac{R}{16\pi G} + \frac{1}{2} h^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + V(\phi) \right)$$

+ Bckgd terms
+ Conical deficit
if $\beta \neq \beta_H$

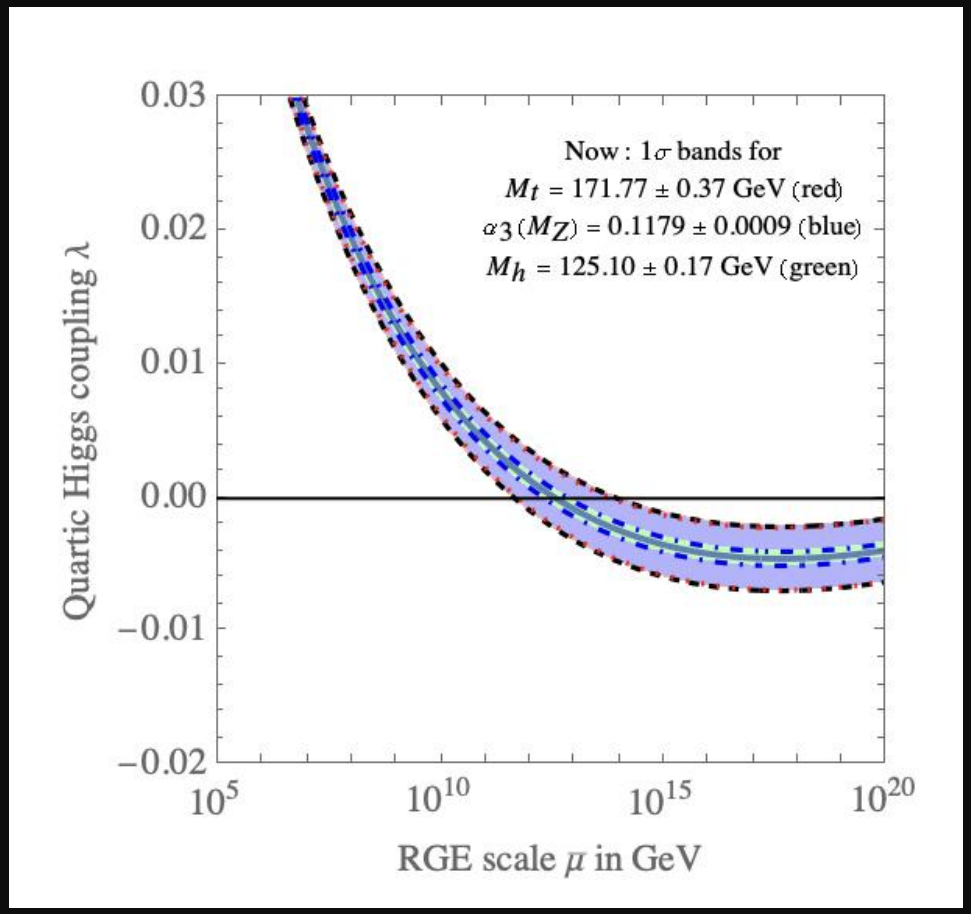


$$I_{\rm b}[T] = \frac{\mathcal{A}_+}{4G} - \frac{\mathcal{A}_-}{4G} = I_{\rm b}[T_H]$$

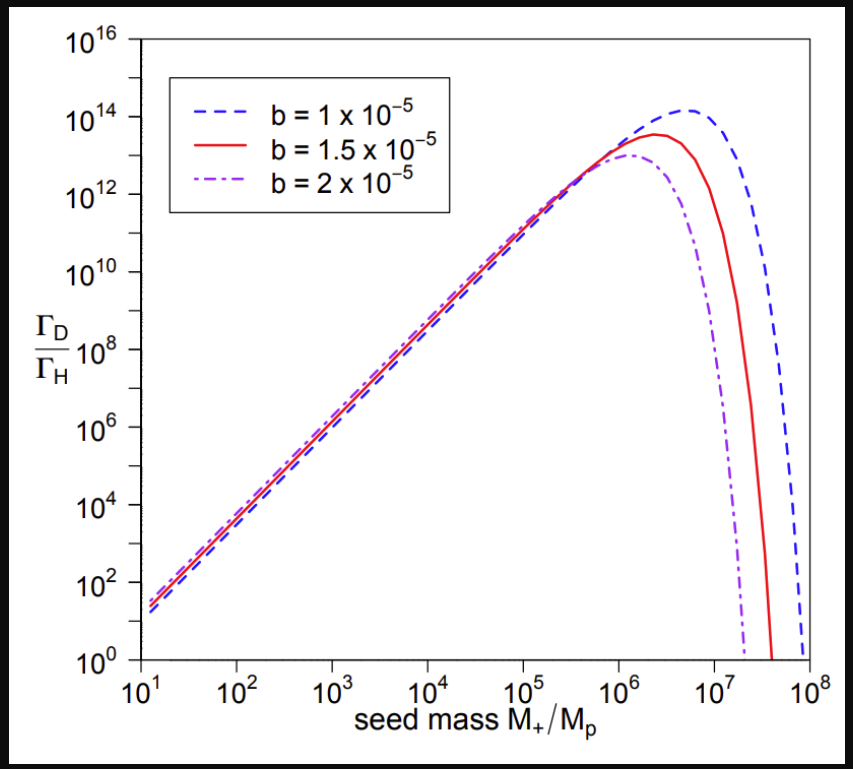
Gregory, Moss, and Withers, JHEP 03, 081(2014)

Only used with Hartle-Hawking so far…

The Higgs Case

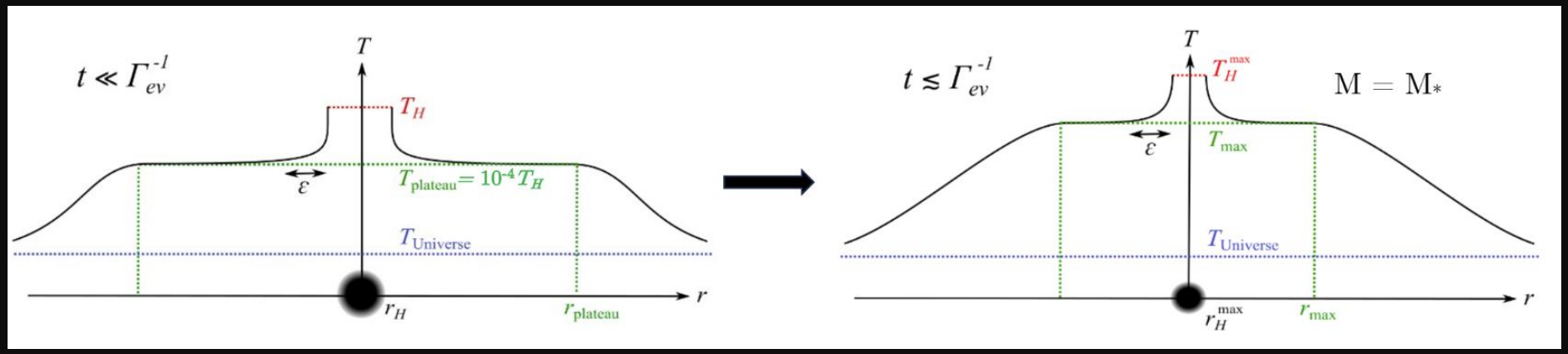


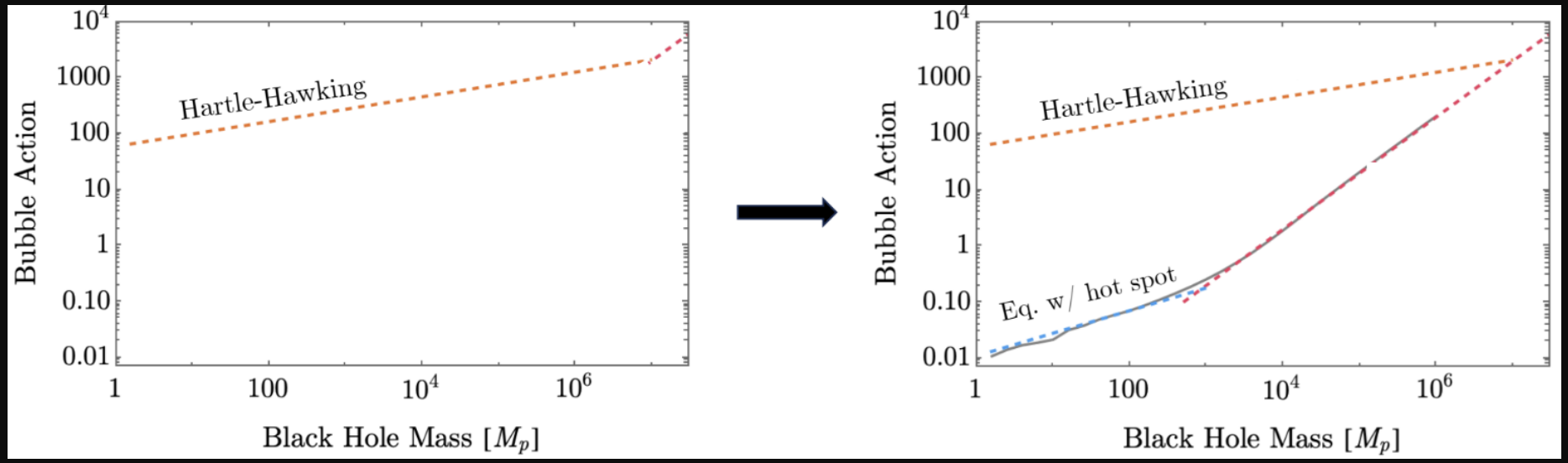
A. Strumia, stolen from X



Gregory, Moss, and Withers, JHEP 03, 081(2014)

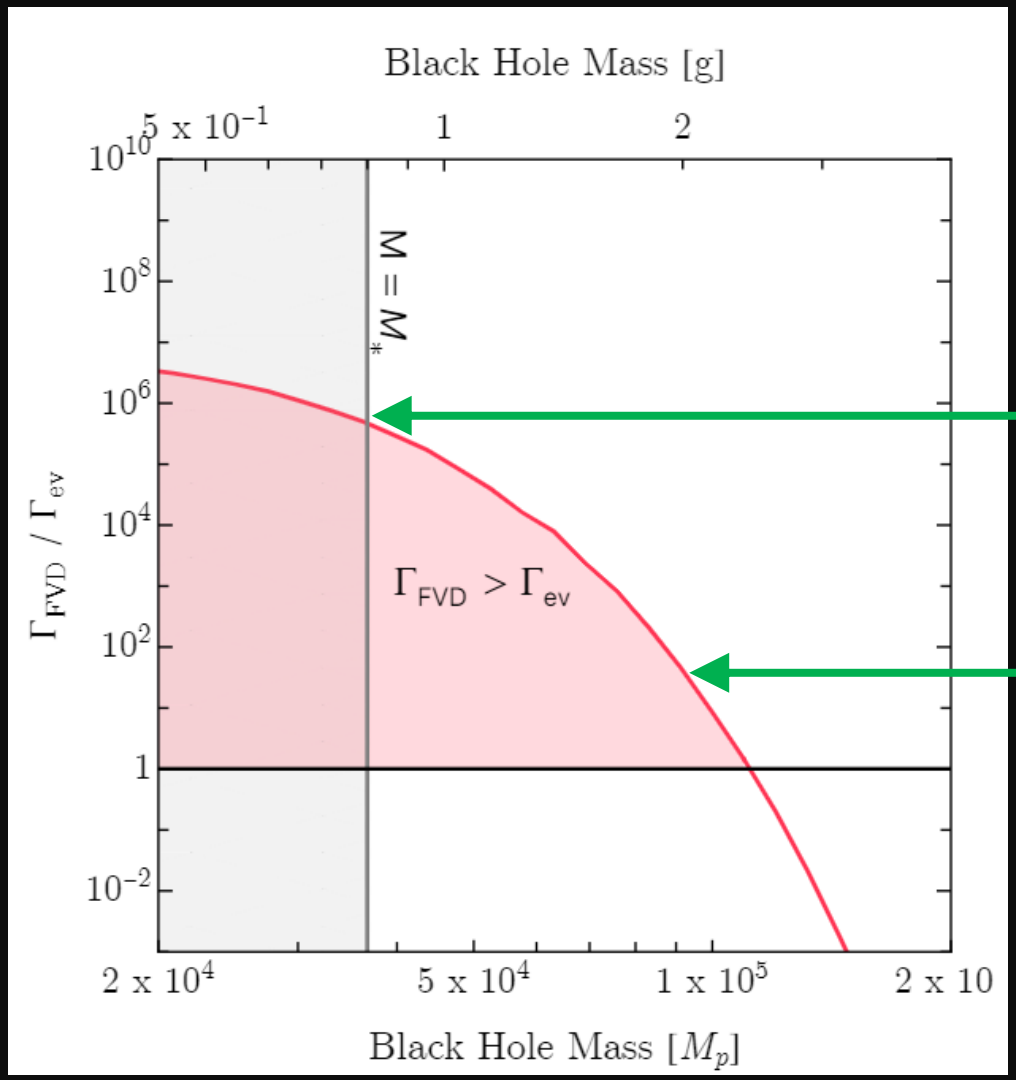
$$\Gamma_{\rm FVD}(T) \approx T\left(\frac{I_{\rm b}[T_H]}{2\pi}\right)^{1/2} \exp\left(-I_{\rm b}[T_H]\right) ,$$





An example: The EW vacuum

Our Universe may be metastable (at ~2σ)



At $M = M_*$, $T = T_{max}$

Using $T_{plateau}(M)$

[Hamaide, LH, Hu, Cheek, ArXiv:2311.01869]

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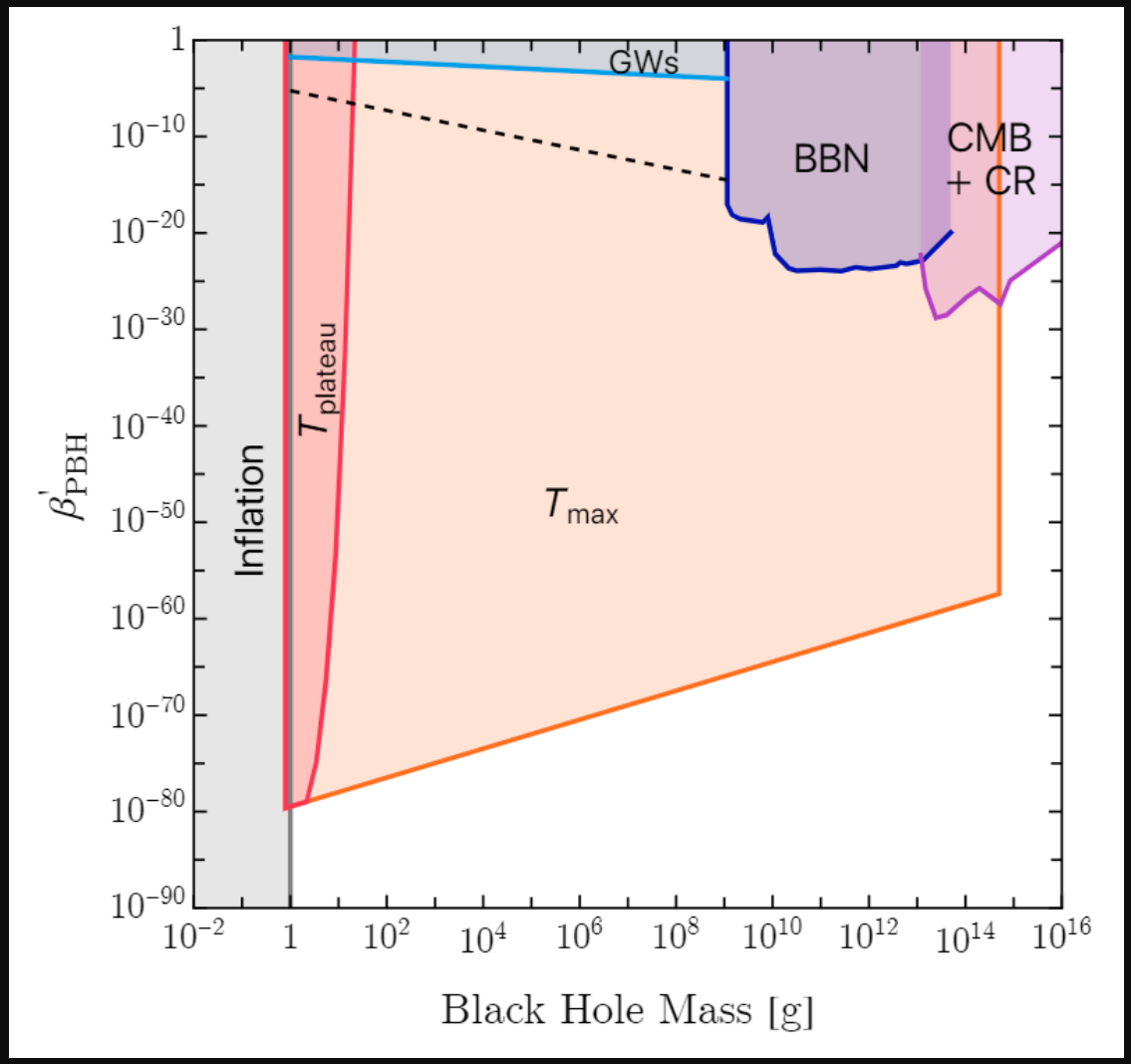
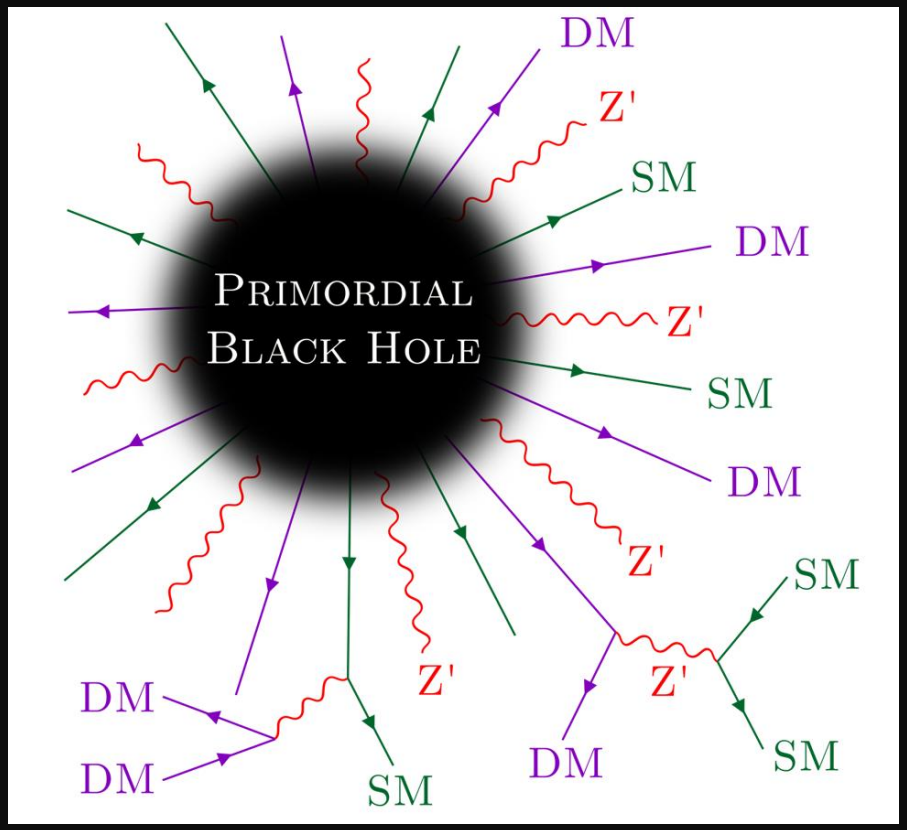


[Hamaide, LH, Hu, Cheek, 2023]

Example 2: Hot Spot & BSM

Interaction rates vary across the hot spot… and so does the mean free path of Hawking radiation!



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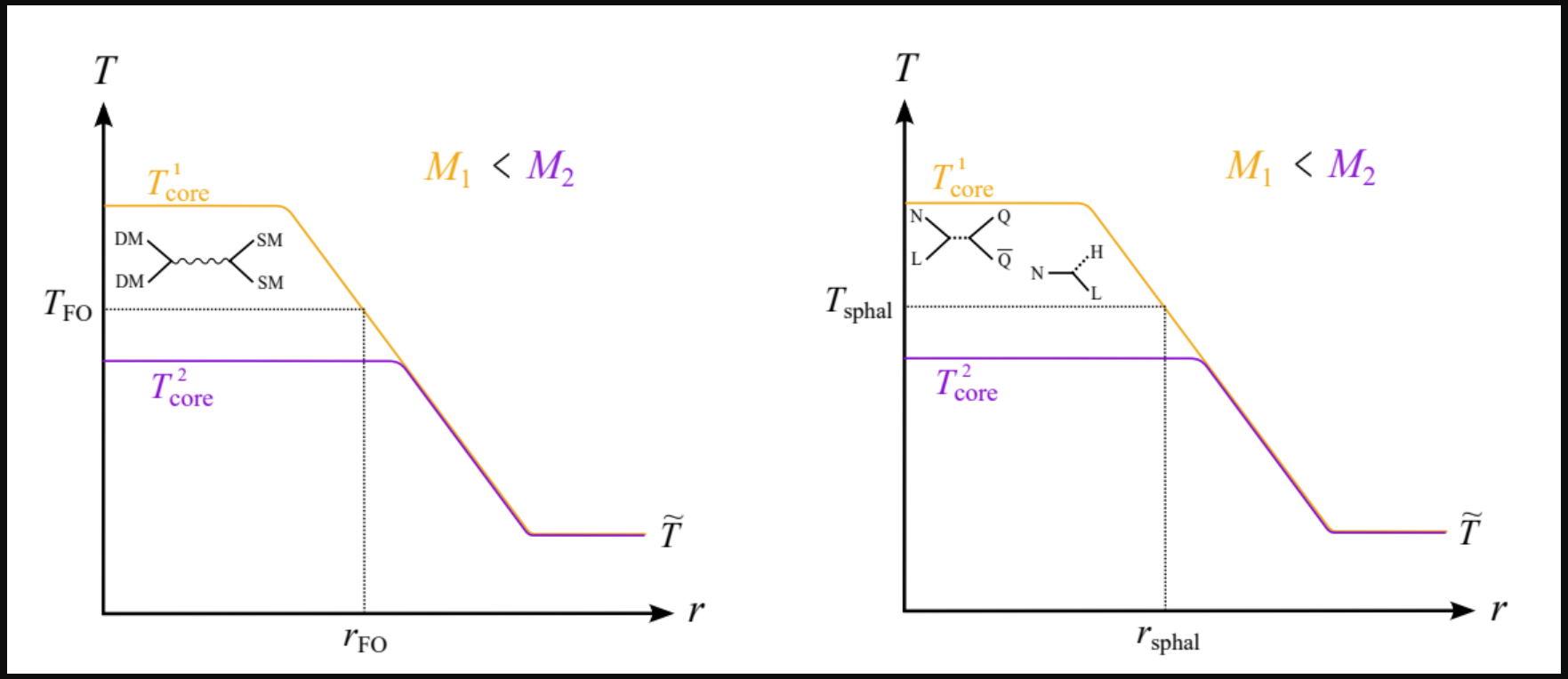

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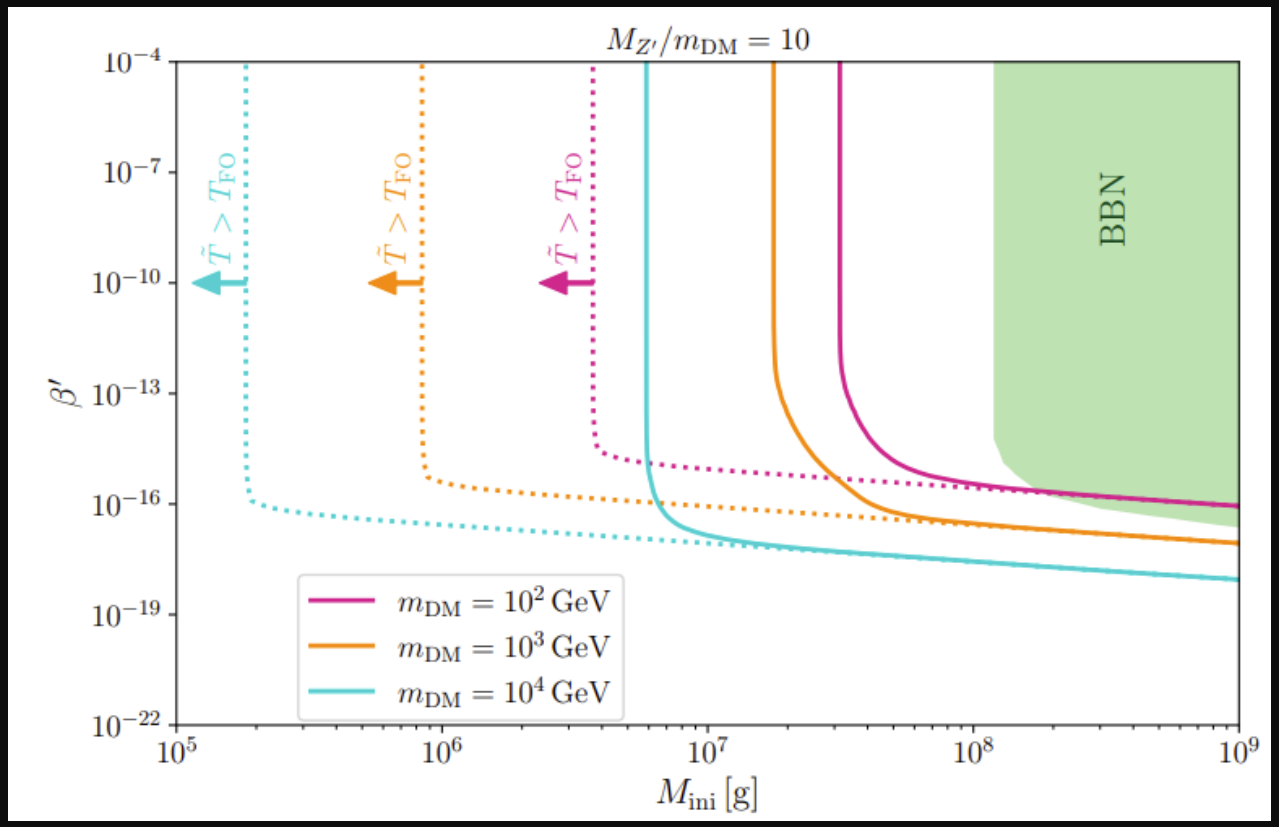


[J.Gunn, LH, Y.F. Perez-Gonzalez, J. Turner, '24]

Example 3:
PBHs and Big Bang Nucleosynthesis

See presentation by

Clelia Altomonte

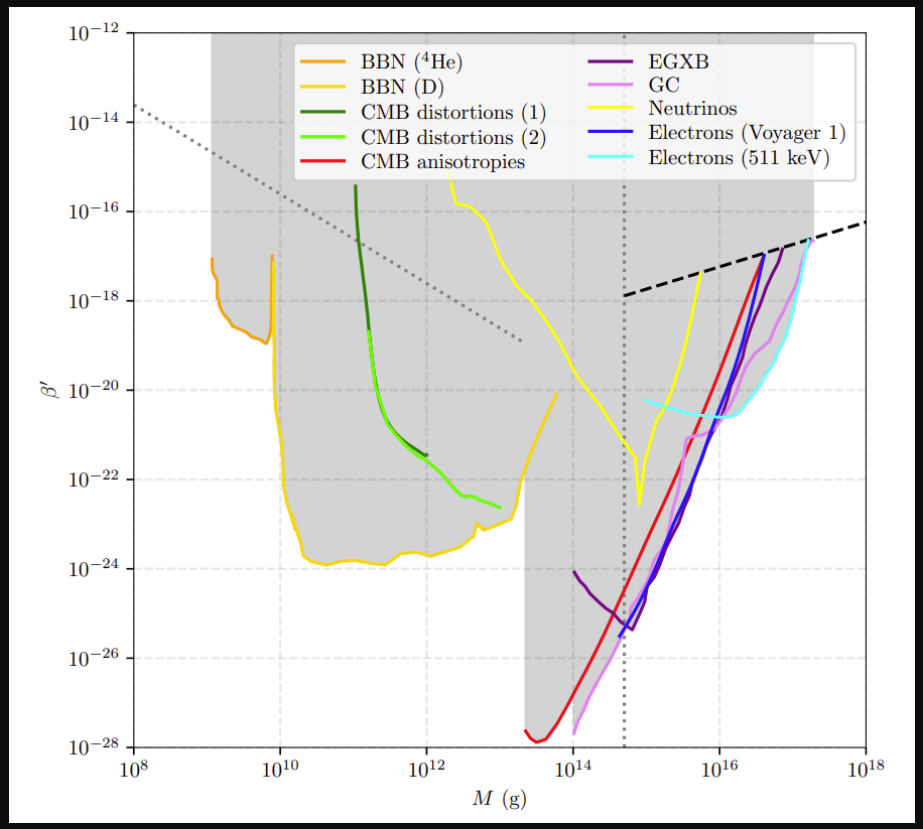
Conclusion

- Light PBHs are as likely to be produced as PBH-DM
- They can produce out of equilibrium particles, leading to modified predictions for particle searches
- PBHs can act as local radiators, forming hot spots
- Hot spots affect particle physics around PBHs. Not just a little…
- PBHs may leave more traces in cosmological data than expected. Stay tuned!

https://github.com/yfperezg/frisbhee

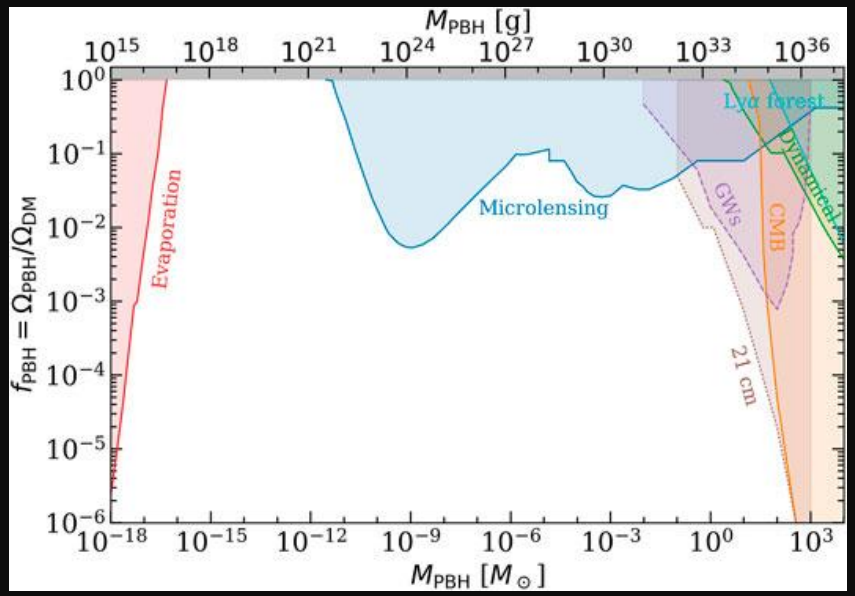
The Parameter Space

Evaporating PBHs

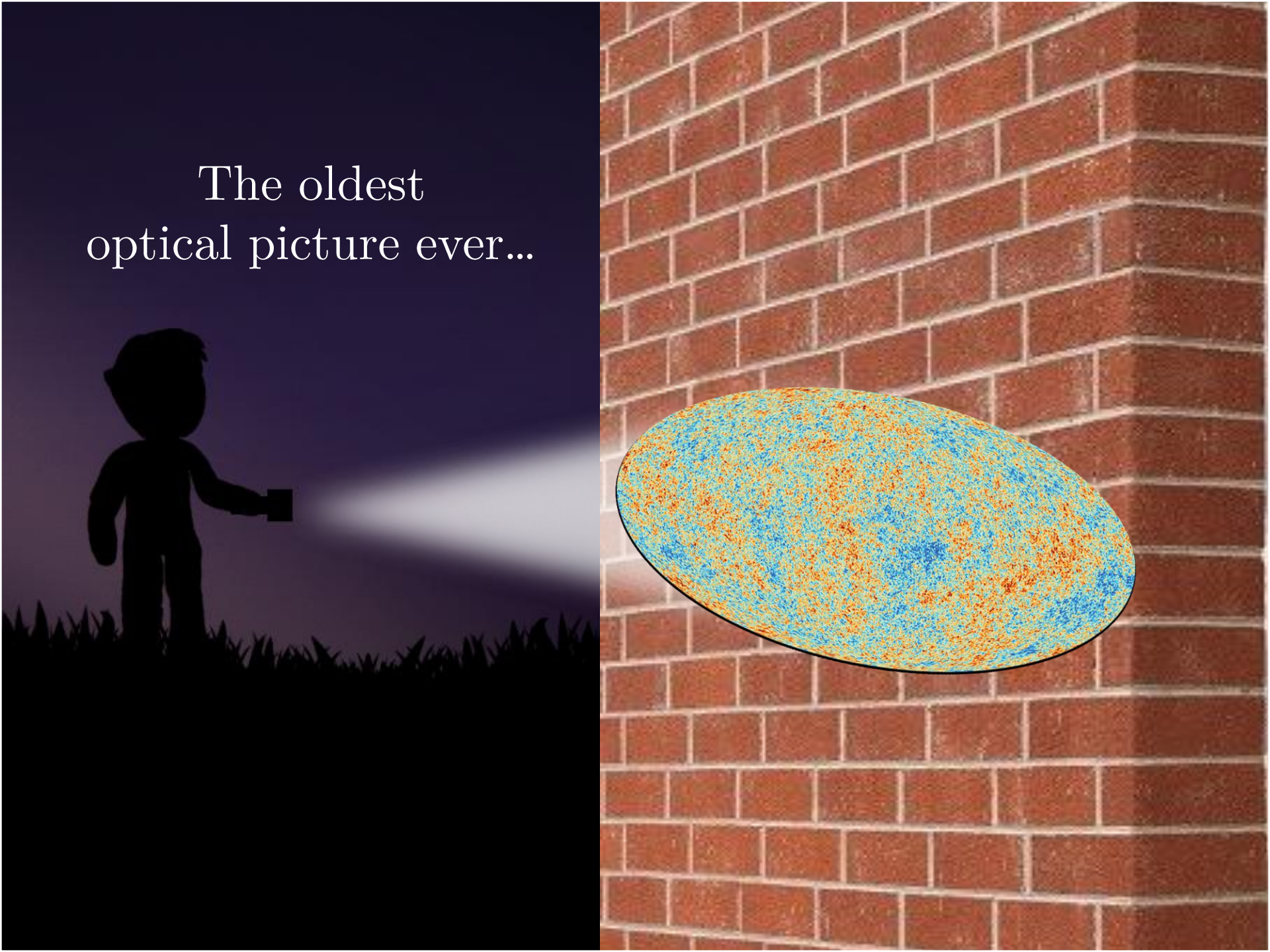


[Auffinger 2022]

PBH Dark Matter



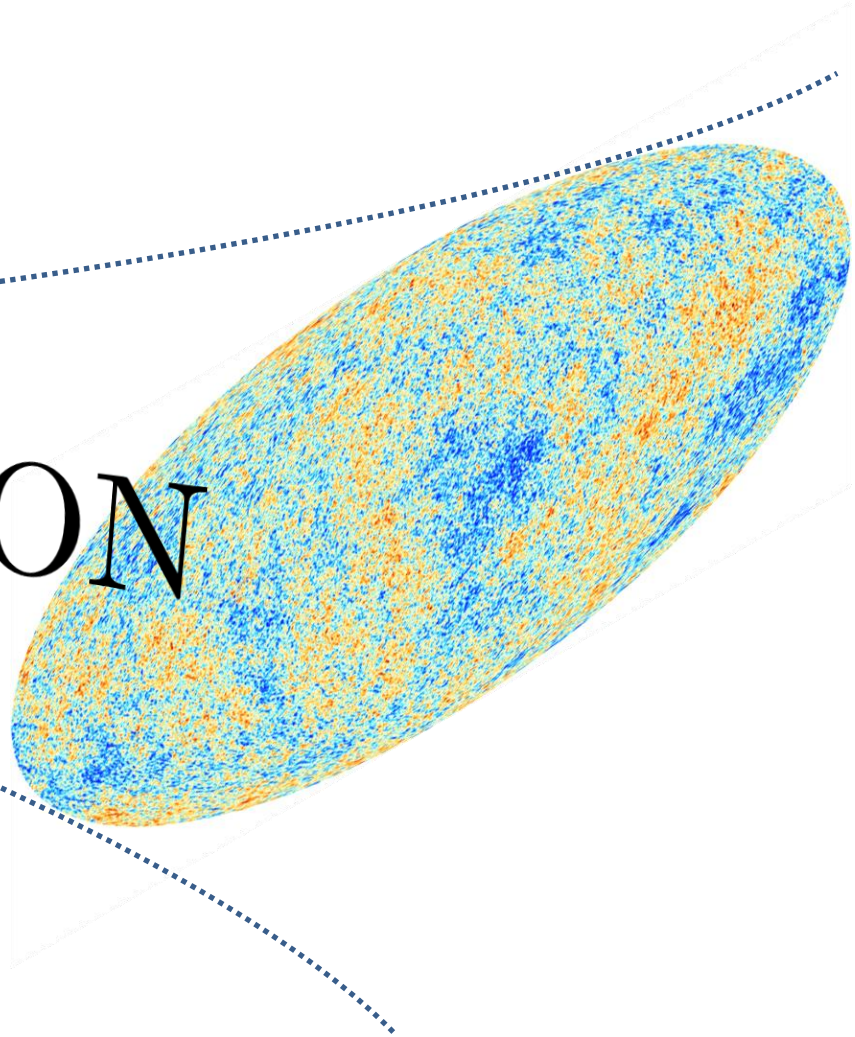
[Villanueva-Domingo, Mena & Palomares-Ruiz]

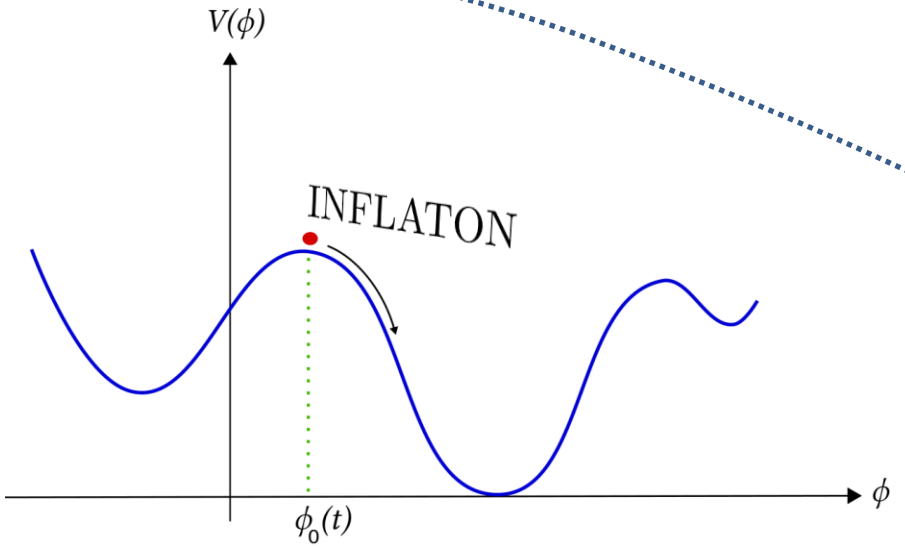

The oldest
optical picture ever...

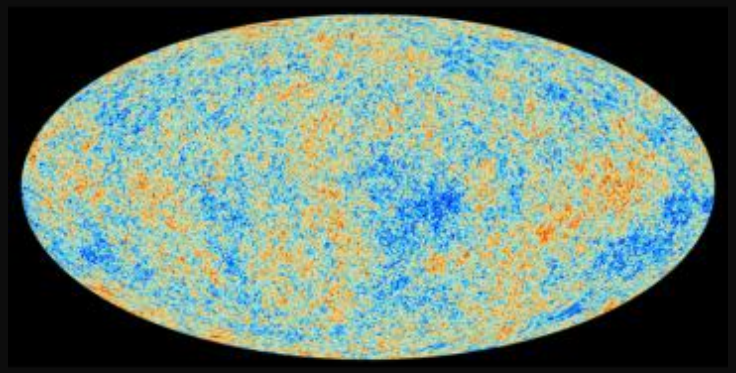
Cosmic Inflation

Primordial Universe

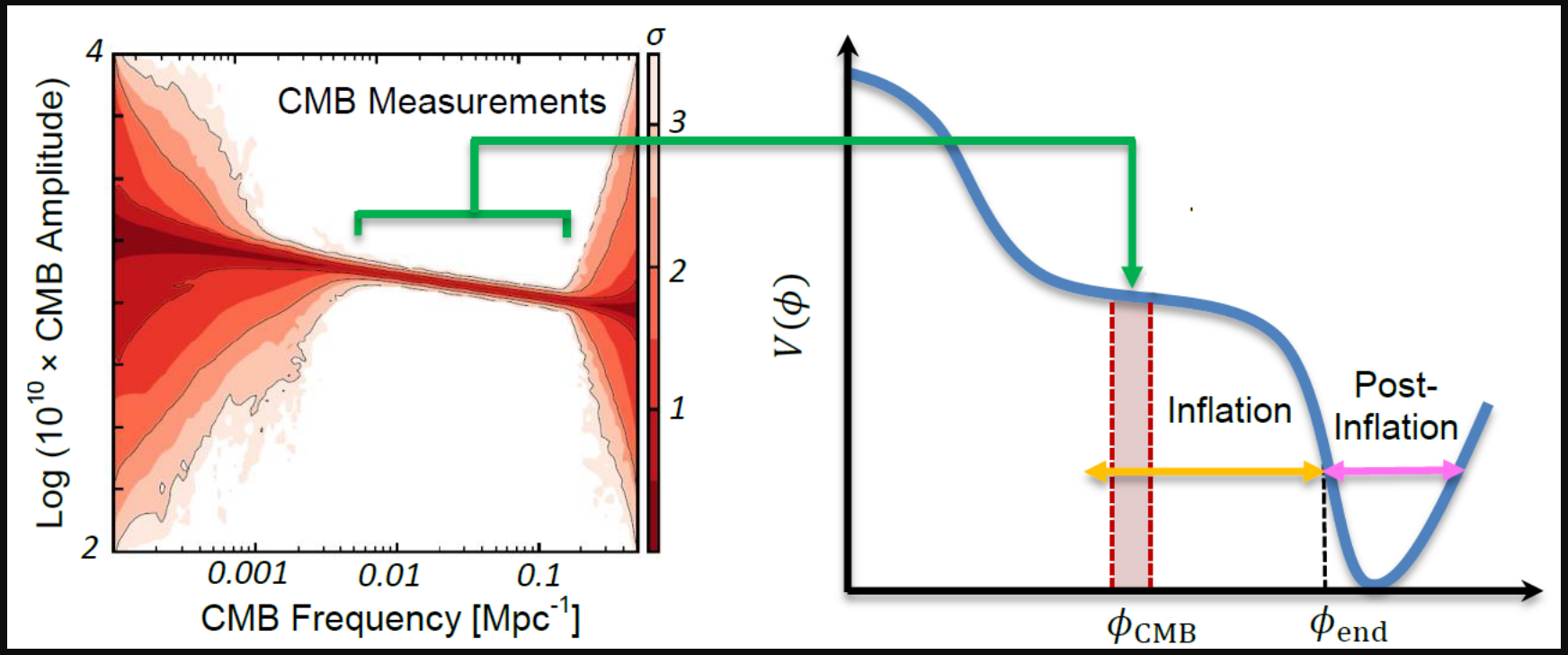
INFLATION



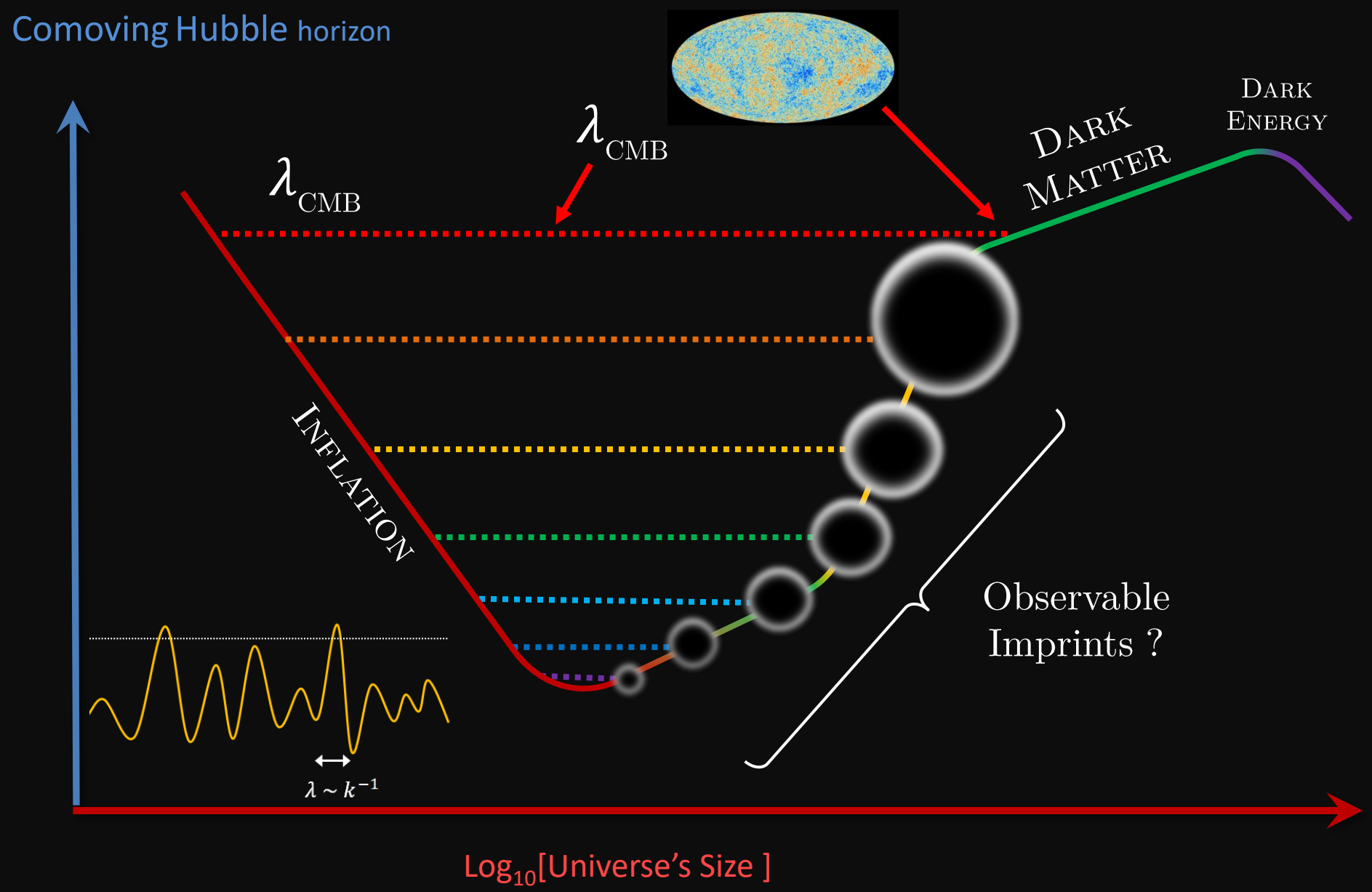




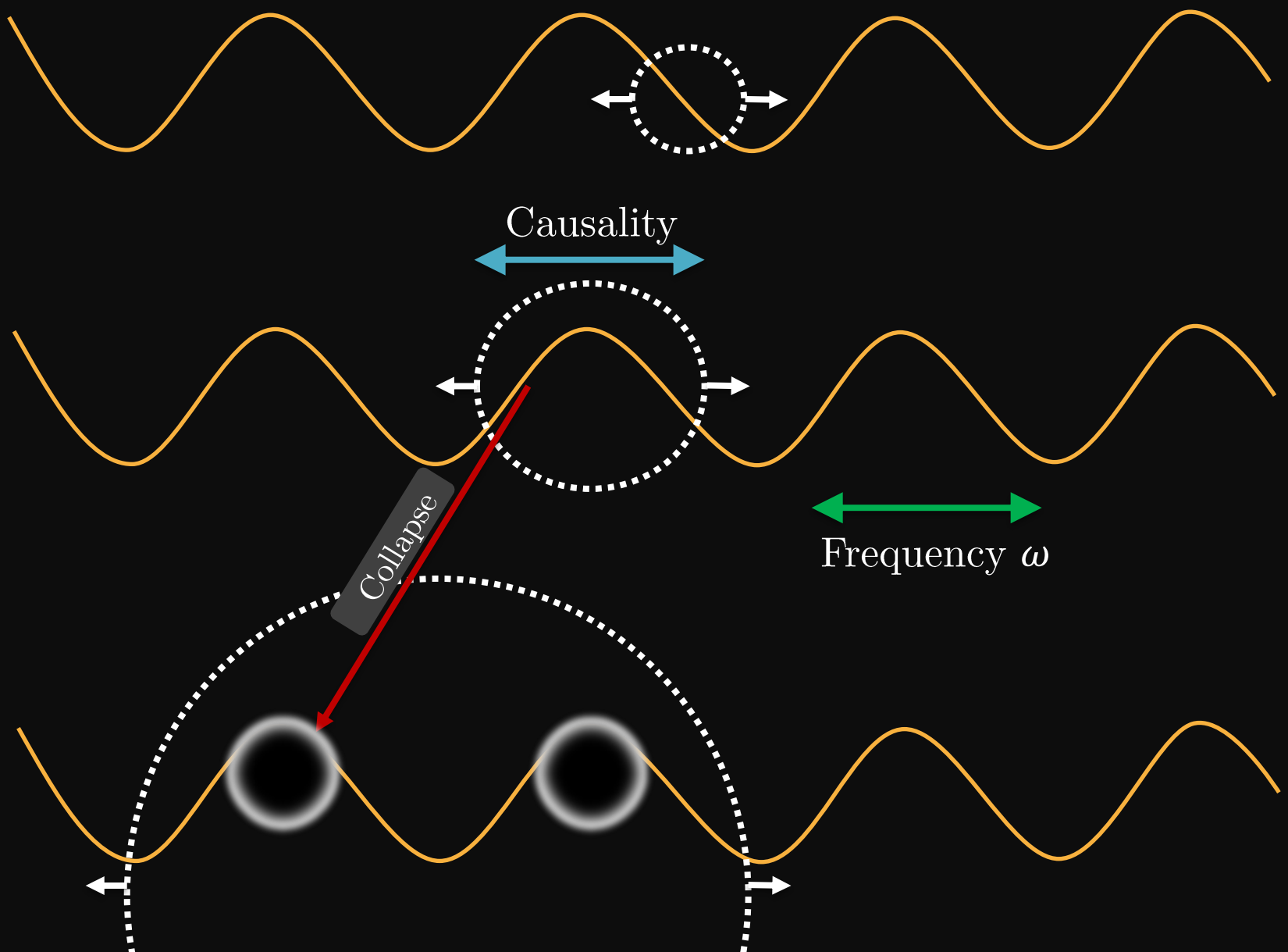
CMB: A small sample of Cosmic Inflation



History of the Universe



Why Primordial Black Holes?



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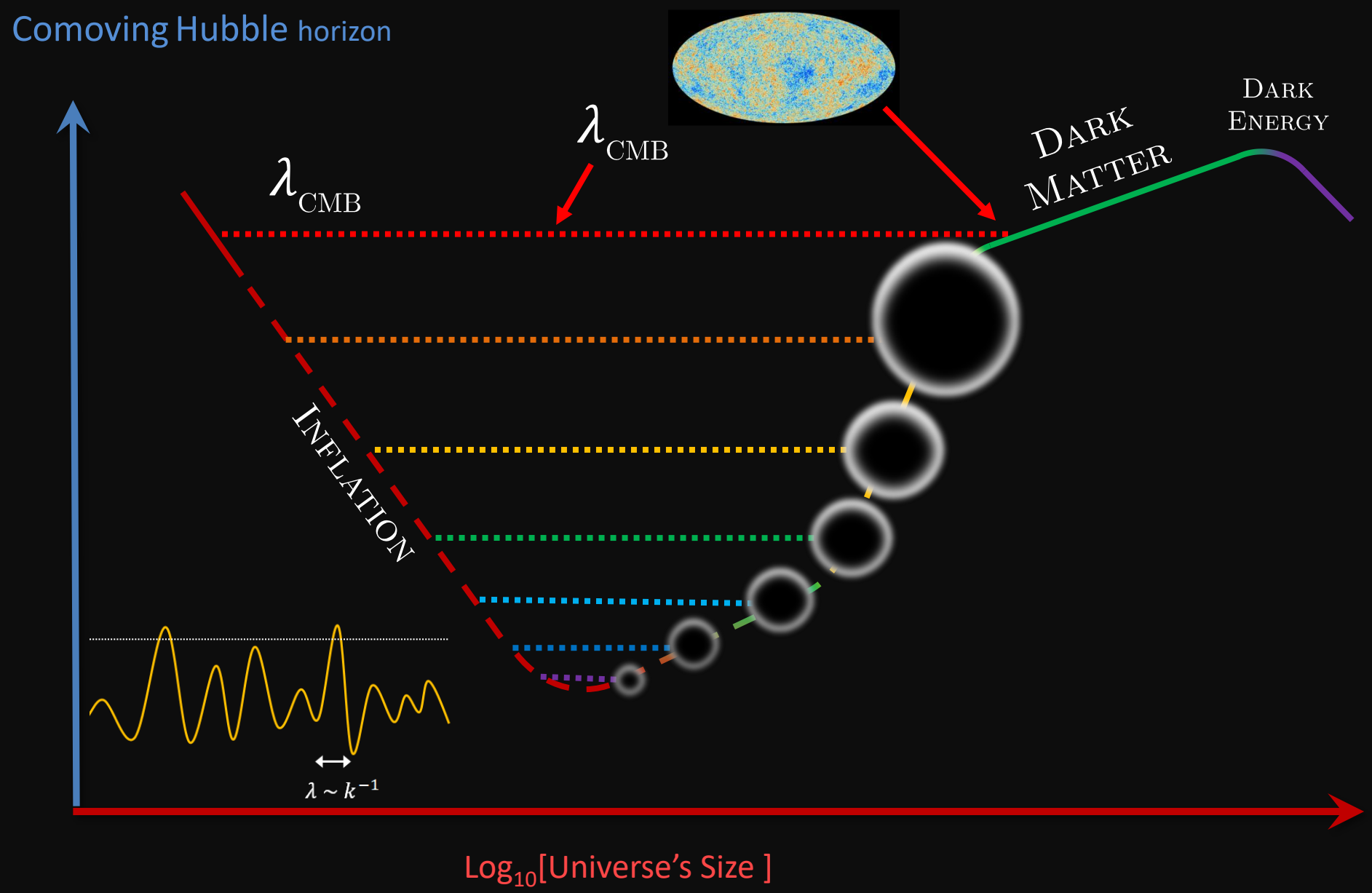
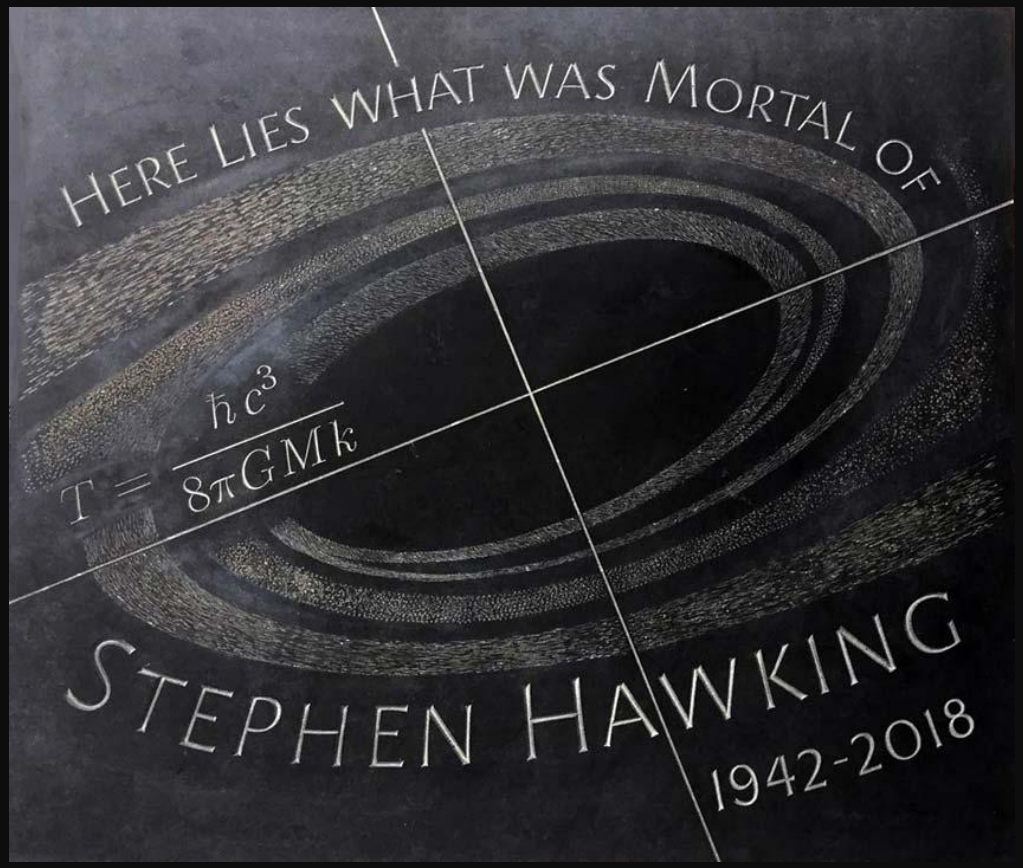


Light PBHs are relics of early universe cosmology!

Black Holes Evaporate

S. Hawking, 1974



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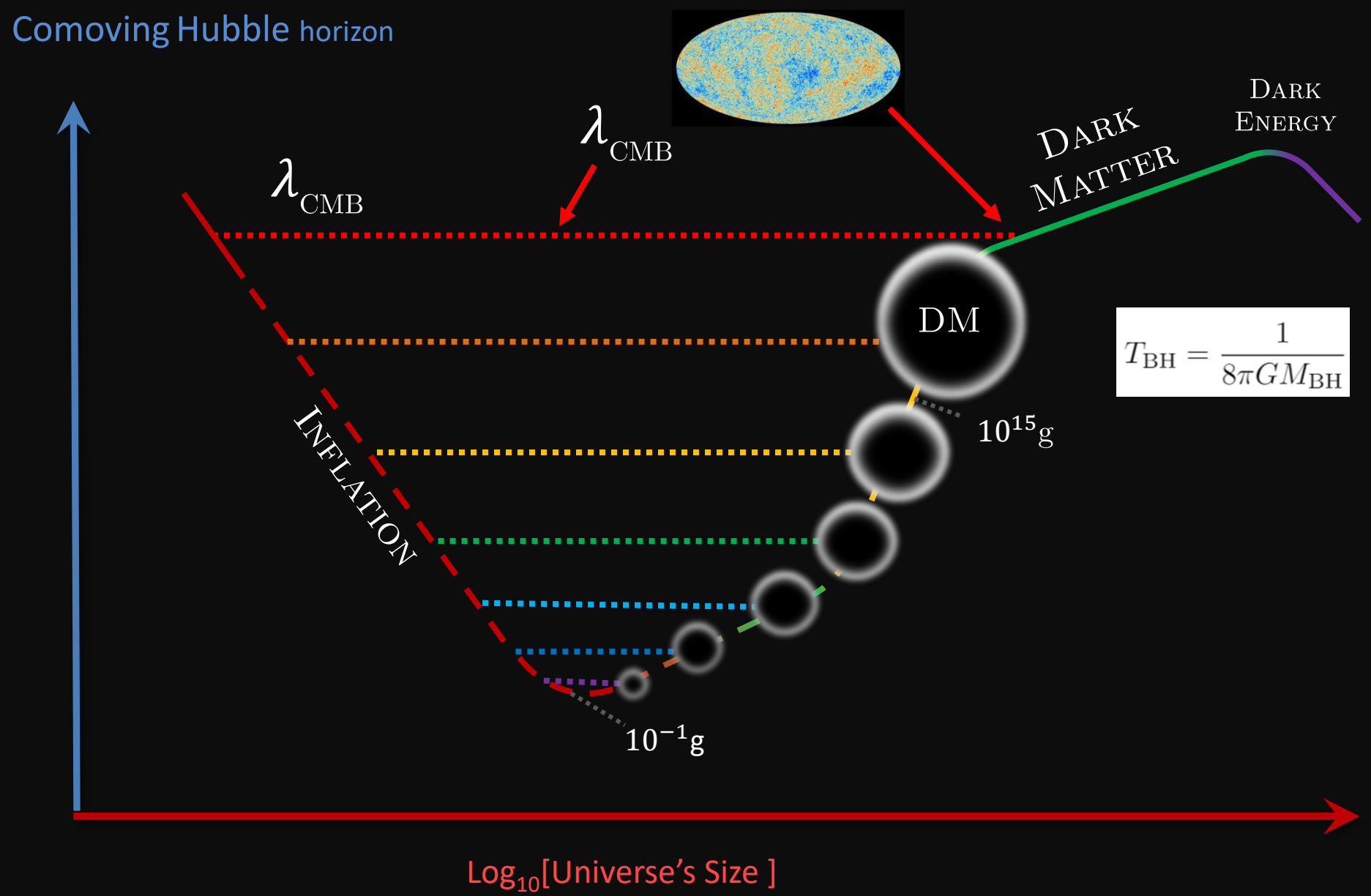
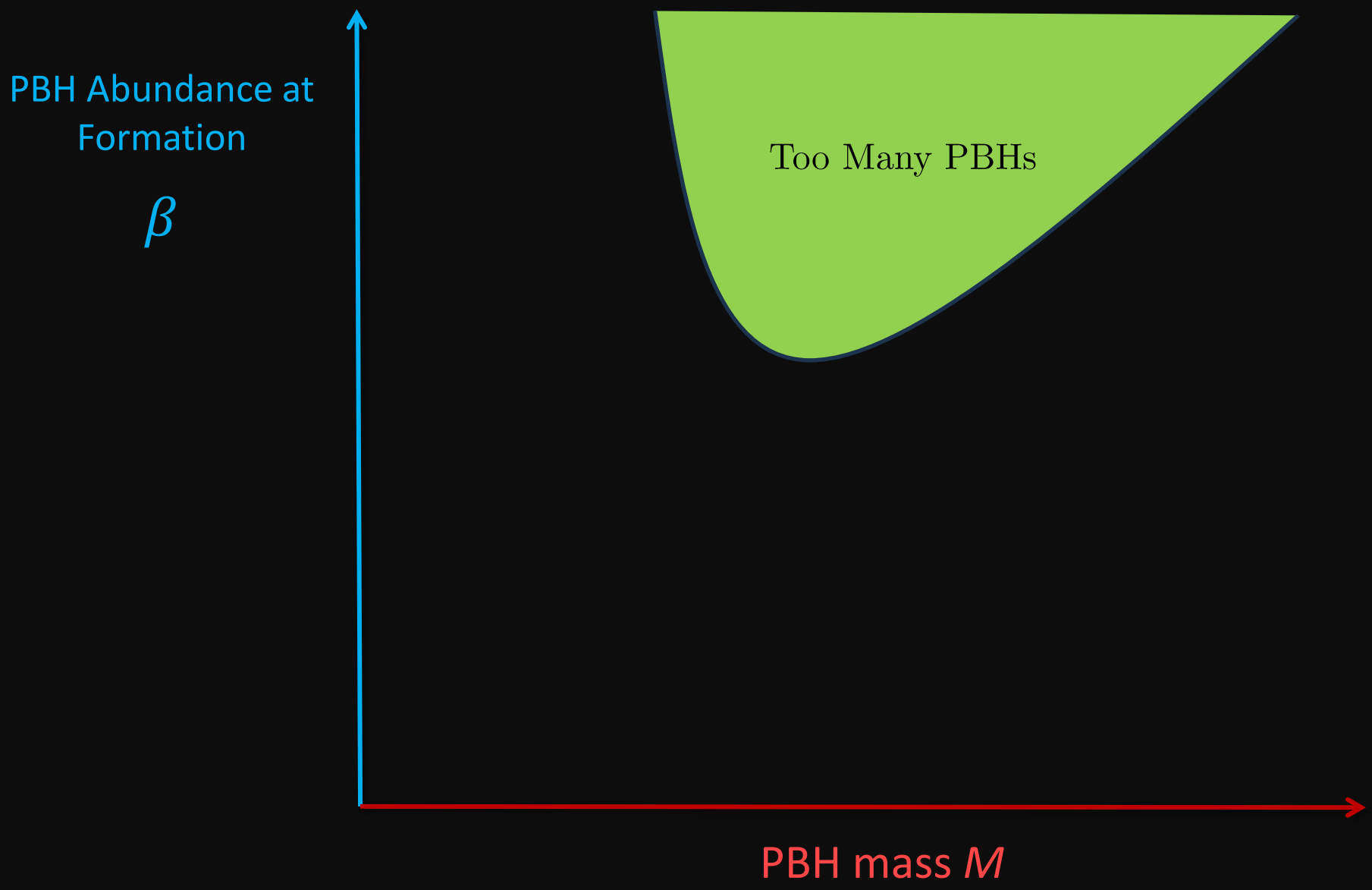


The Parameter Space



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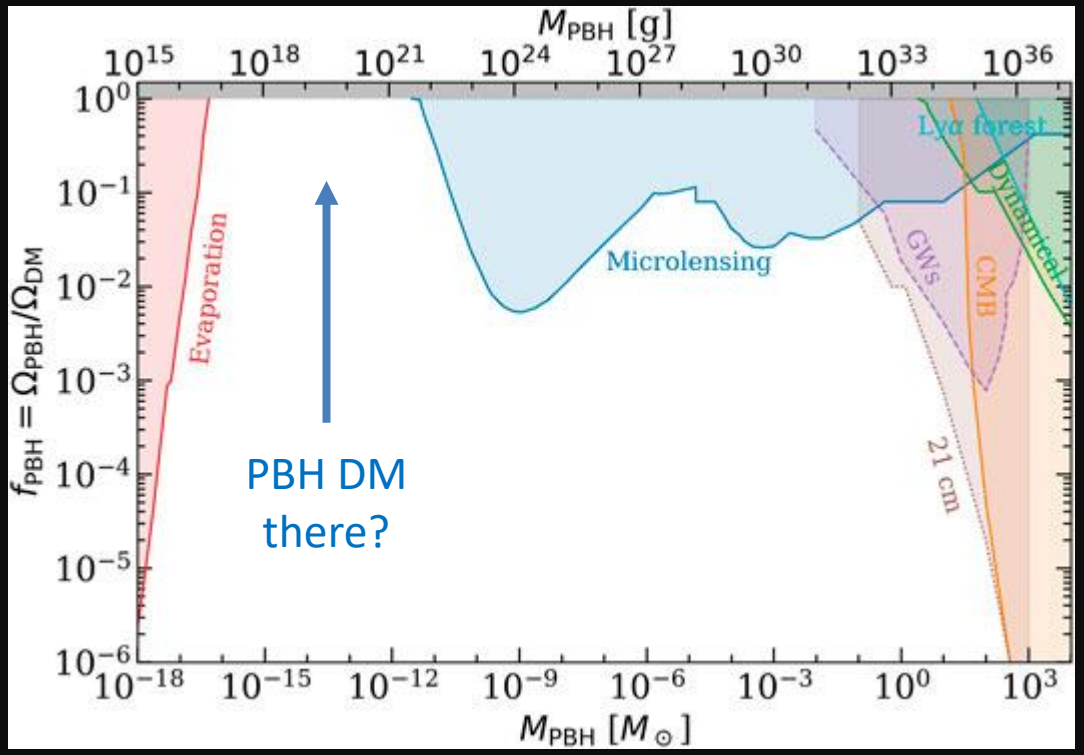

[Villanueva-Domingo, Mena & Palomares-Ruiz]

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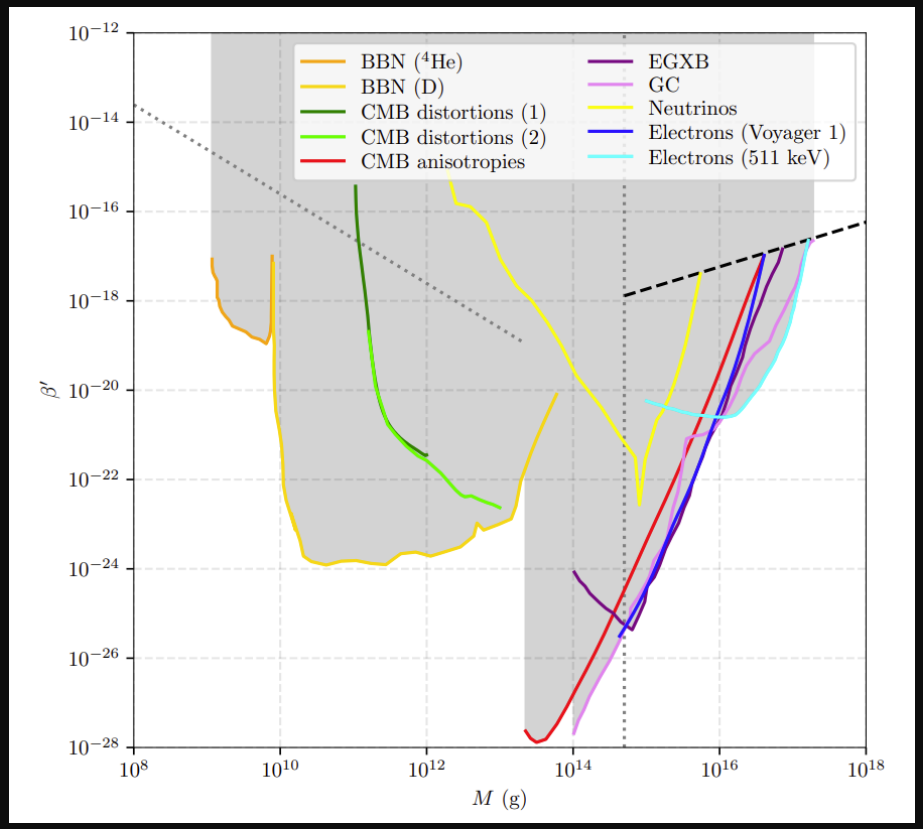

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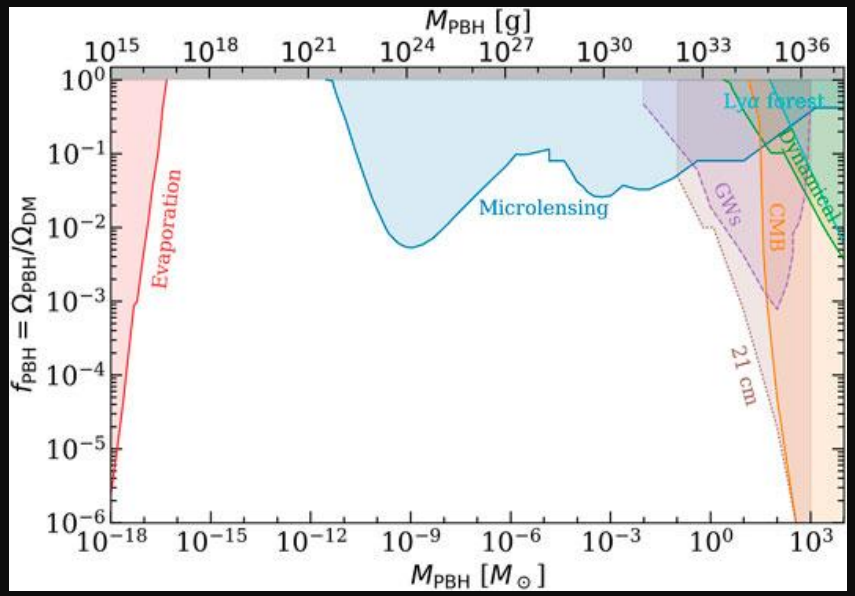

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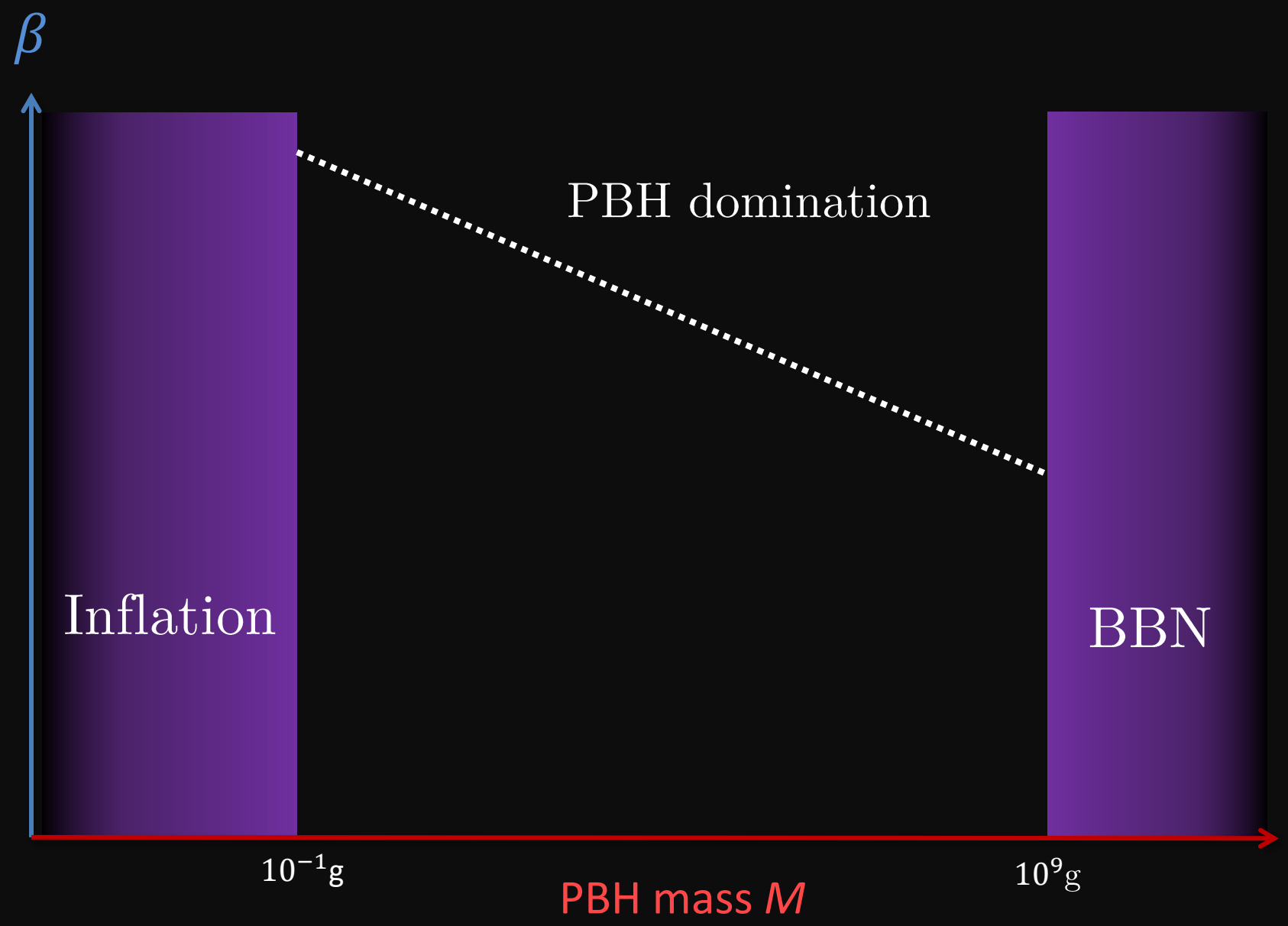


Imprints of PBH Evaporation

1. PBHs produce SM and BSM particles
2. PBHs produce boosted particles
3. PBHs heat the plasma
4. PBHs modify cosmic history

PBH evaporation

$$\frac{\mathrm{d}M_{\mathrm{BH}}}{\mathrm{d}t} \equiv \sum_i \left.\frac{\mathrm{d}M_{\mathrm{BH}}}{\mathrm{d}t}\right|_i = -\sum_i \int_0^\infty E_i \frac{\mathrm{d}^2\mathcal{N}_i}{\mathrm{d}p\mathrm{d}t}\mathrm{d}p \; = -\varepsilon(M_{\mathrm{BH}})\frac{M_p^4}{M_{\mathrm{BH}}^2}$$

$$\frac{\mathrm{d}^2\mathcal{N}_i}{\mathrm{d}p\,\mathrm{d}t} = \frac{g_i}{2\pi^2}\frac{\sigma_{s_i}(M_{\mathrm{BH}},\mu_i,p)}{\exp\left[E_i(p)/T_{\mathrm{BH}}\right]-(-1)^{2s_i}}\frac{p^3}{E_i(p)}$$

$$\varepsilon(M_{\mathrm{BH}}) \equiv \sum_i g_i\varepsilon_i(z_i)$$

$$z_i \;=\; \mu_i/T_{\mathrm{BH}}$$

BSM Contributions?

$$T_{\mathrm{BH}} = \frac{1}{8\pi G M_{\mathrm{BH}}} \sim 1.06\ \mathrm{GeV}\left(\frac{10^{13}\ \mathrm{g}}{M_{\mathrm{BH}}}\right)$$

PBH evaporation

$$\frac{\mathrm{d}^2\mathcal{N}_i}{\mathrm{d}p\,\mathrm{d}t} = \frac{g_i}{2\pi^2}\frac{\sigma_{s_i}(M_{\mathrm{BH}},\mu_i,p)}{\exp\left[E_i(p)/T_{\mathrm{BH}}\right]-(-1)^{2s_i}}\frac{p^3}{E_i(p)}$$

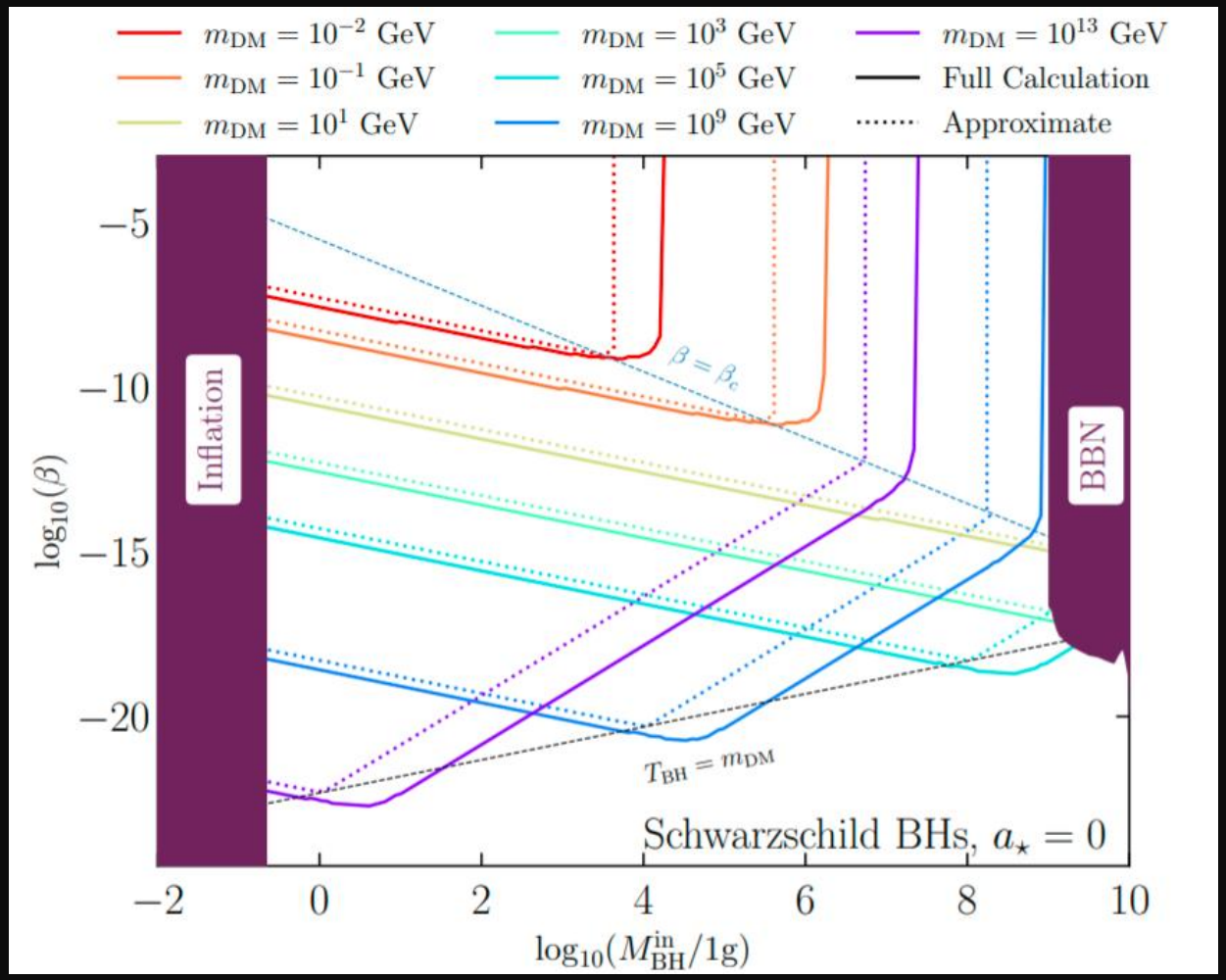
https://github.com/yfperezg/frisbhee

Open-source code, able to:

- Calculate the emission rate for an arbitrary SM+BSM spectrum
- Solve Friedman equations while tracking PBH evaporation
- Solve Boltzmann equations in a homogeneous universe

DM from Evaporation

$$f_{\rm PBH}(M) = \delta(M - M_{\rm PBH})$$

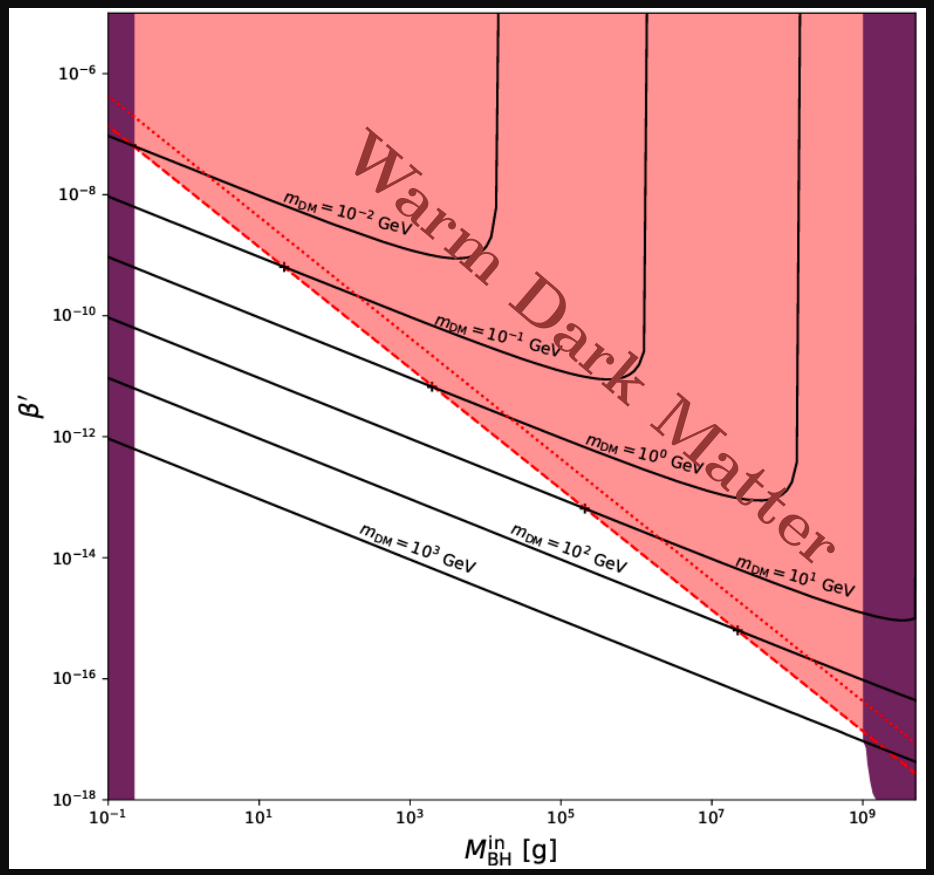


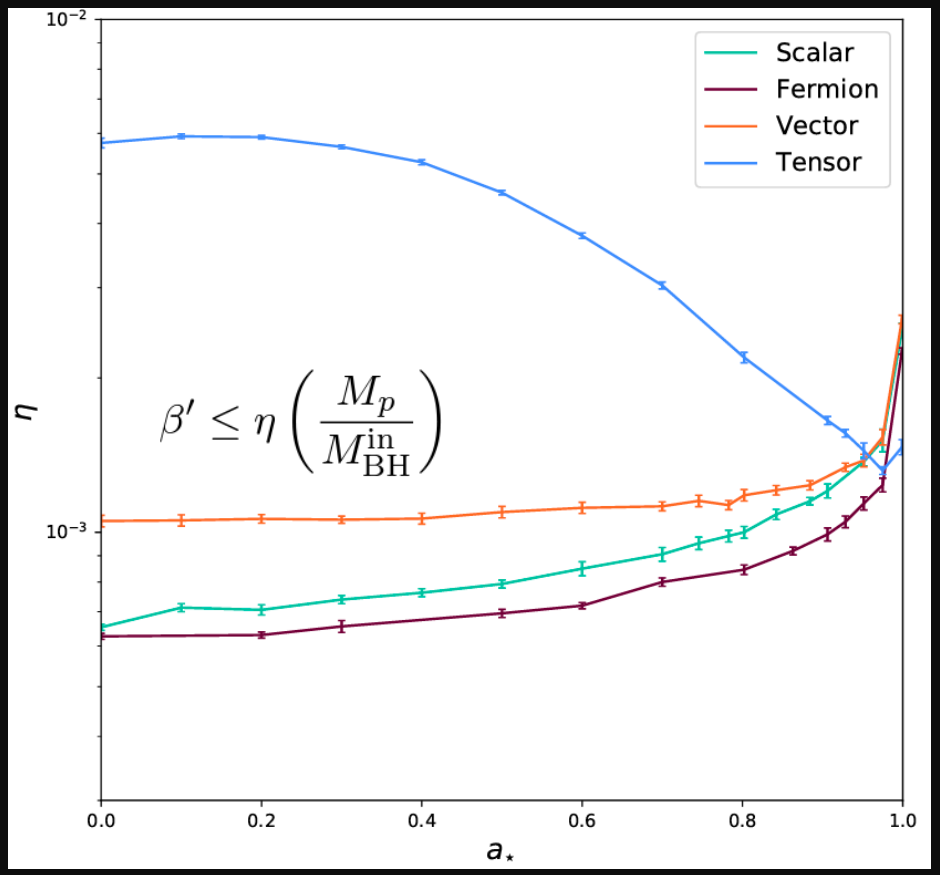
[Cheek, LH, Perez-Gonzalez and Turner '22] **Code Publicly available: FRISBHEE**

Kerr PBHs and Warm Dark Matter

Using CLASS: expected matter power spectrum

$$P(k) = P_{\rm CDM}(k) T^2(k)$$



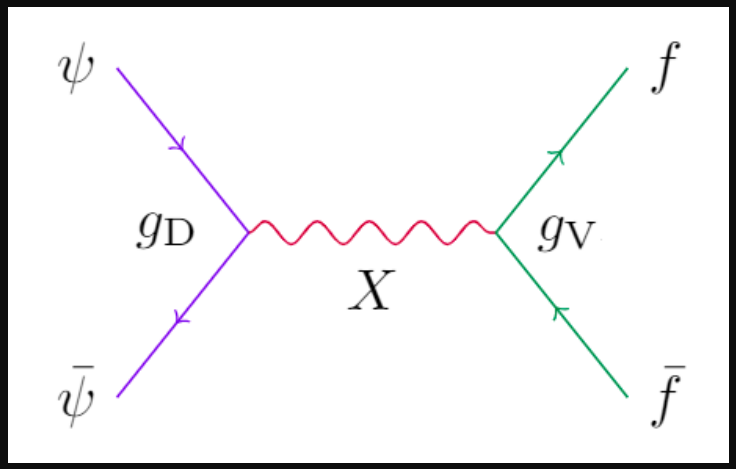


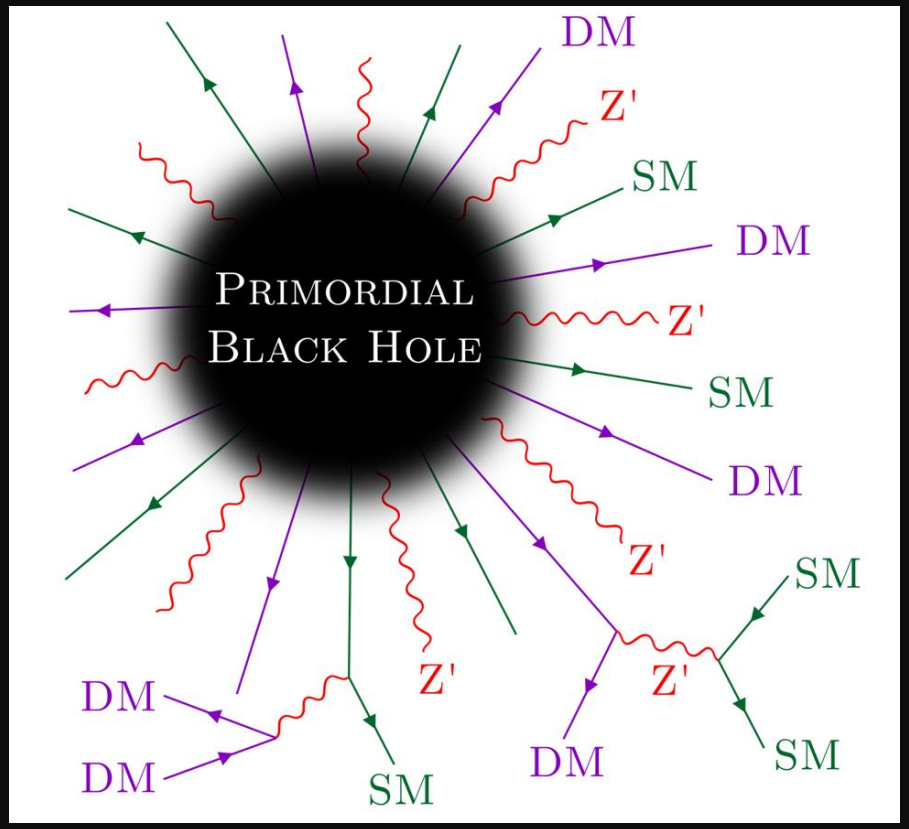
[Cheek, LH, Perez-Gonzalez and Turner '23]

Code Publicly available: FRISBHEE

Thermal Production of DM

- DM may interact with SM particles and be produced in the early universe through thermal processes…
- Freeze-In or Freeze-Out





Thermal Production of DM

DM Annihilation, X decay — PBH evaporation

$$\dot{n}_{\rm DM} + 3Hn_{\rm DM} = g_{\rm DM}\int C[f_{\rm DM}]\frac{{\rm d}^3p}{(2\pi)^3} + \left.\frac{{\rm d}n_{\rm DM}}{{\rm d}t}\right|_{\rm BH},$$

$$\dot{n}_X + 3Hn_X = g_X\int C[f_X]\frac{{\rm d}^3p}{(2\pi)^3} + \left.\frac{{\rm d}n_X}{{\rm d}t}\right|_{\rm BH},$$

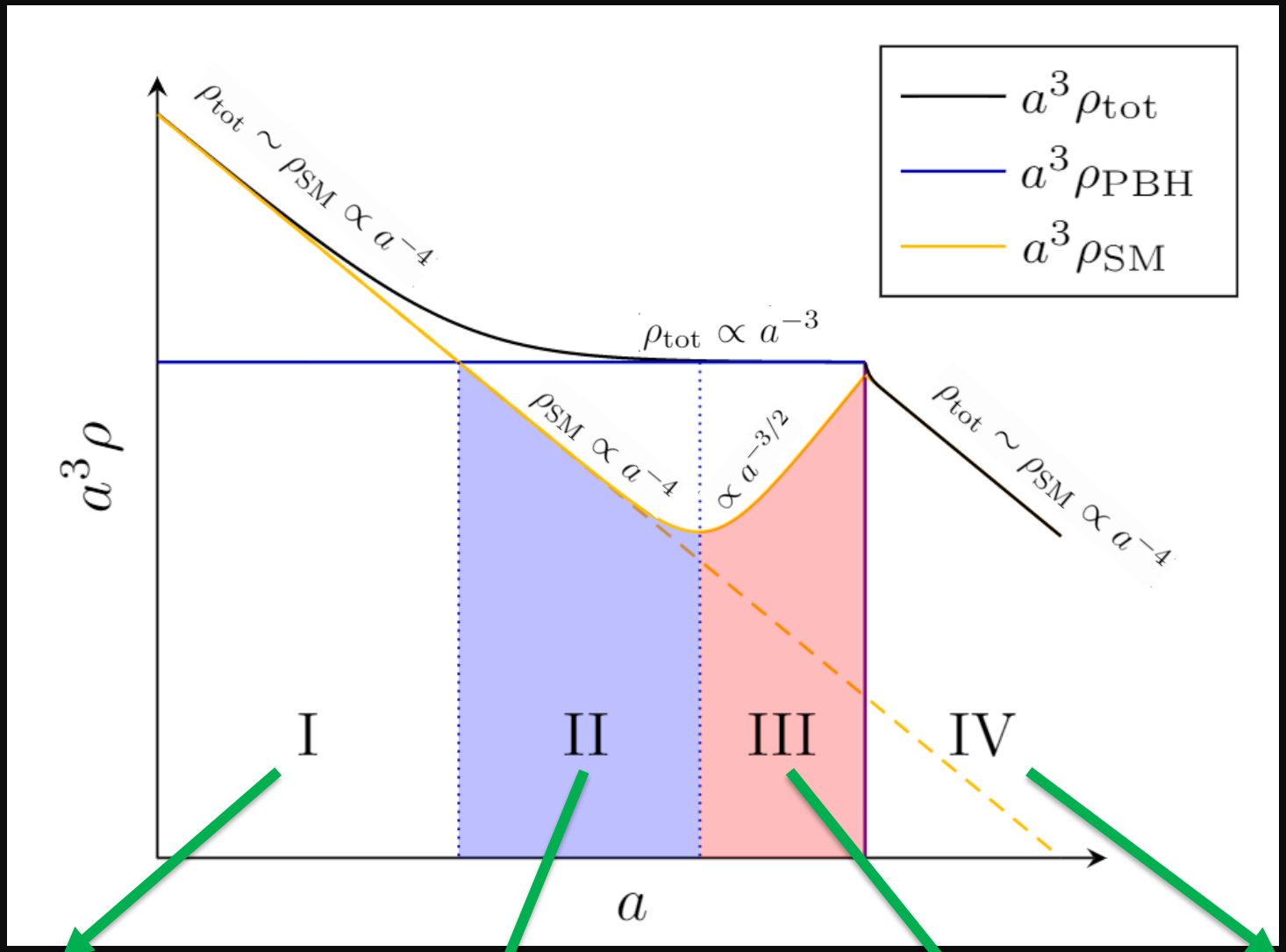
$$\dot{\rho}_{\rm SM} + 4H\rho_{\rm SM} = \left.\frac{{\rm d}M}{{\rm d}t}\right|_{\rm SM}.$$

PBHs evaporate **non-trivial distributions** of DM and X particles

Non-trivial evolution of the full distributions $f_{\rm X}(p)$ and $f_{\rm DM}(p)$

$$\left.\frac{{\rm d}n_i}{{\rm d}t}\right|_{\rm BH} = n_{\rm BH}g_i\int\left.\frac{\partial f_i}{\partial t}\right|_{\rm BH}\frac{p^2{\rm d}p}{2\pi^2}$$

Modified Cosmology



FI/FO + entropy dilution

Regular FI/FO

Matter-Dominated FI/FO

FI/FO during entropy injection

Results

Freeze-In

[Cheek, LH, Perez-Gonzalez and Turner '22]

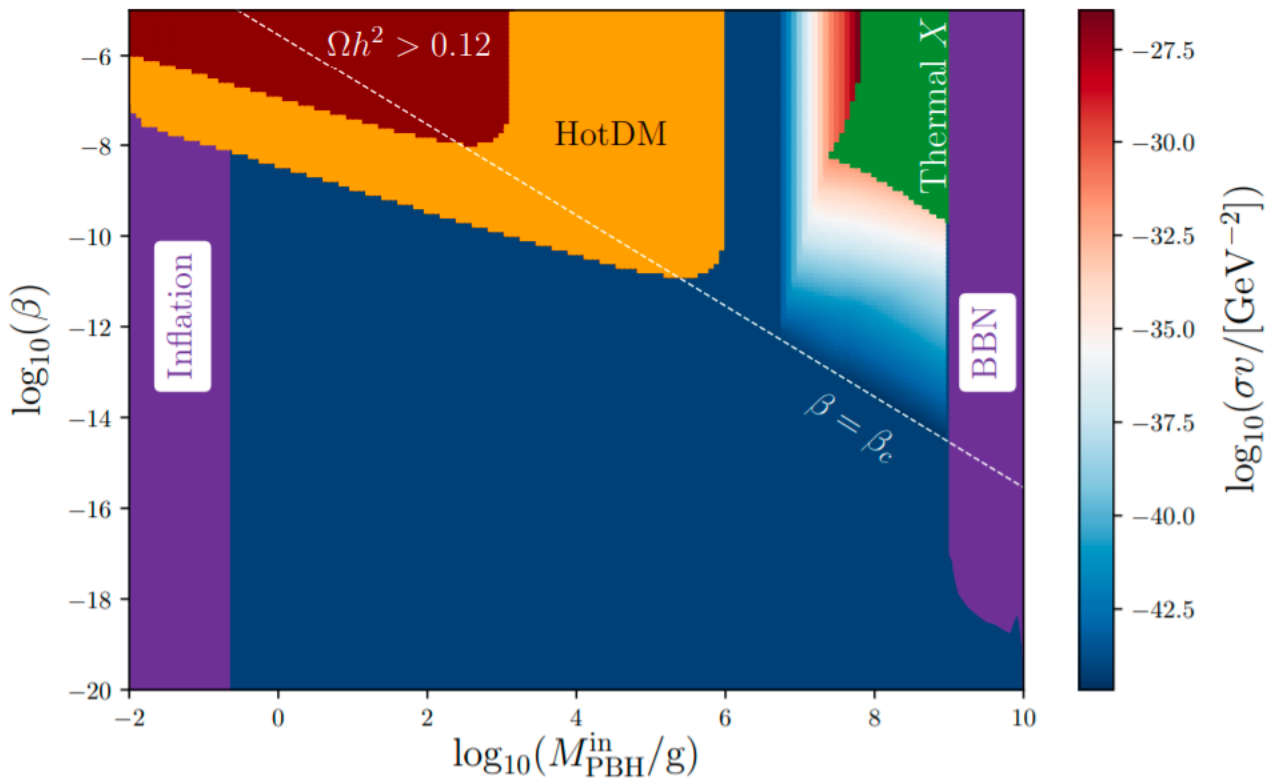


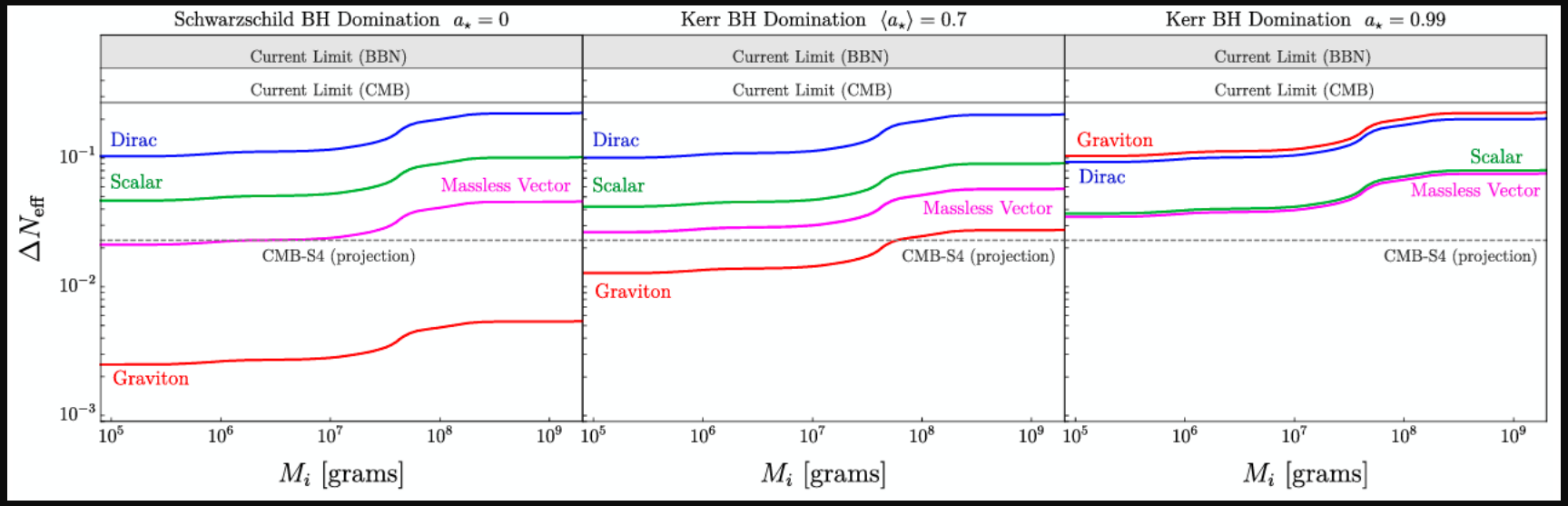
Fig. 11. Two-dimensional scan over the PBH fraction β and mass $M_{\rm BH}$ for a mediator mass $m_X = 1\,{\rm TeV}$, a dark matter mass $m_{\rm DM} = 1\,{\rm MeV}$, and ${\rm Br}(X \to {\rm SM}) = 10^{-7}$. The color map indicates the value of the non-relativistic cross-section of DM annihilation leading to the correct relic abundance in the Freeze-In case. See the main text for a description of the different constraints.

Kerr PBHs and Dark Radiation

Dark particles with small masses can contribute to ΔNeff

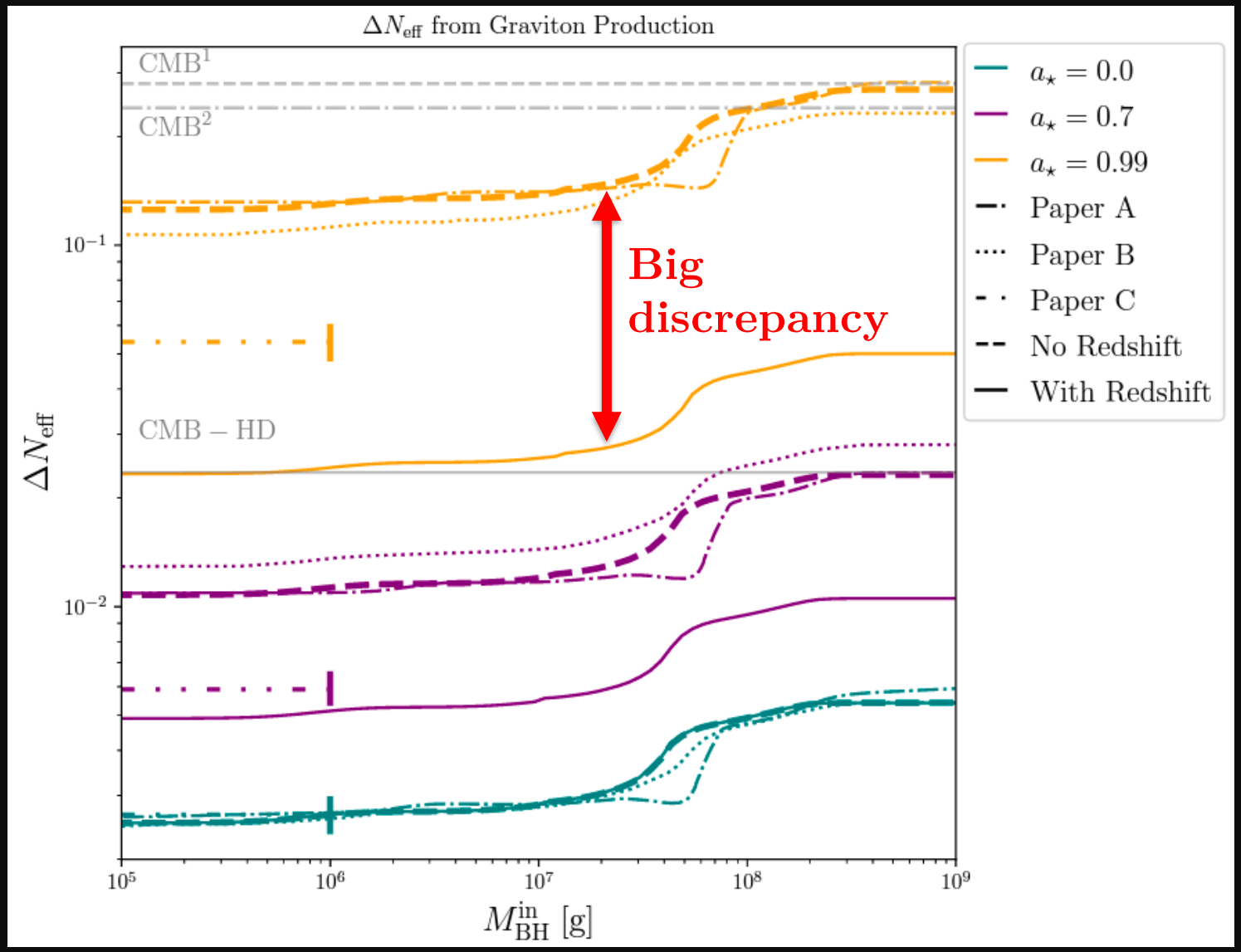
Schwarzschild PBH → Negligible

Kerr PBH → Argued to be critical



[Hooper et al '20]

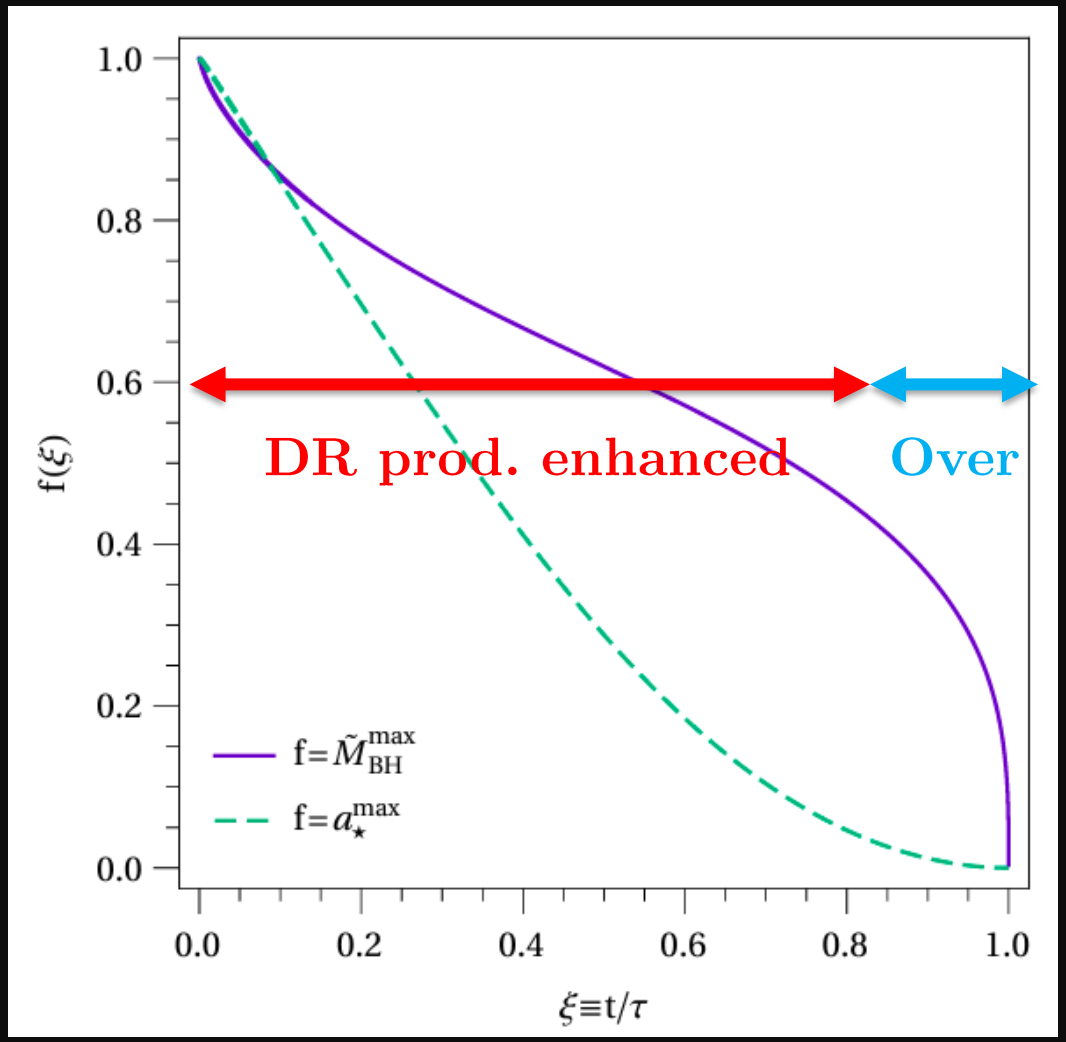
Kerr PBHs and Dark Radiation



[Cheek, LH, Perez-Gonzalez and Turner '22] **Code Publicly available: FRISBHEE**

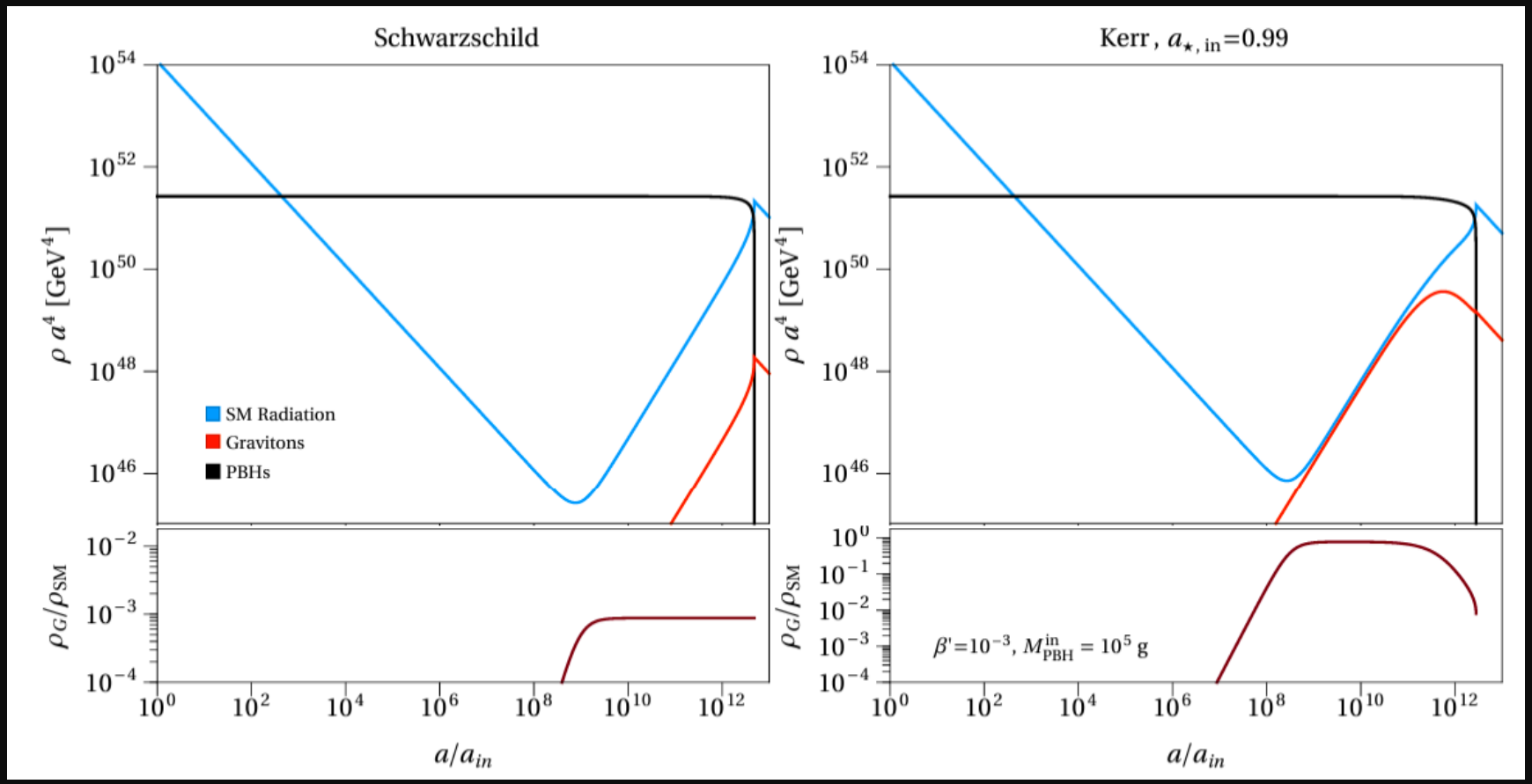
Kerr PBHs and Dark Radiation

Why ?



Kerr PBHs and Dark Radiation

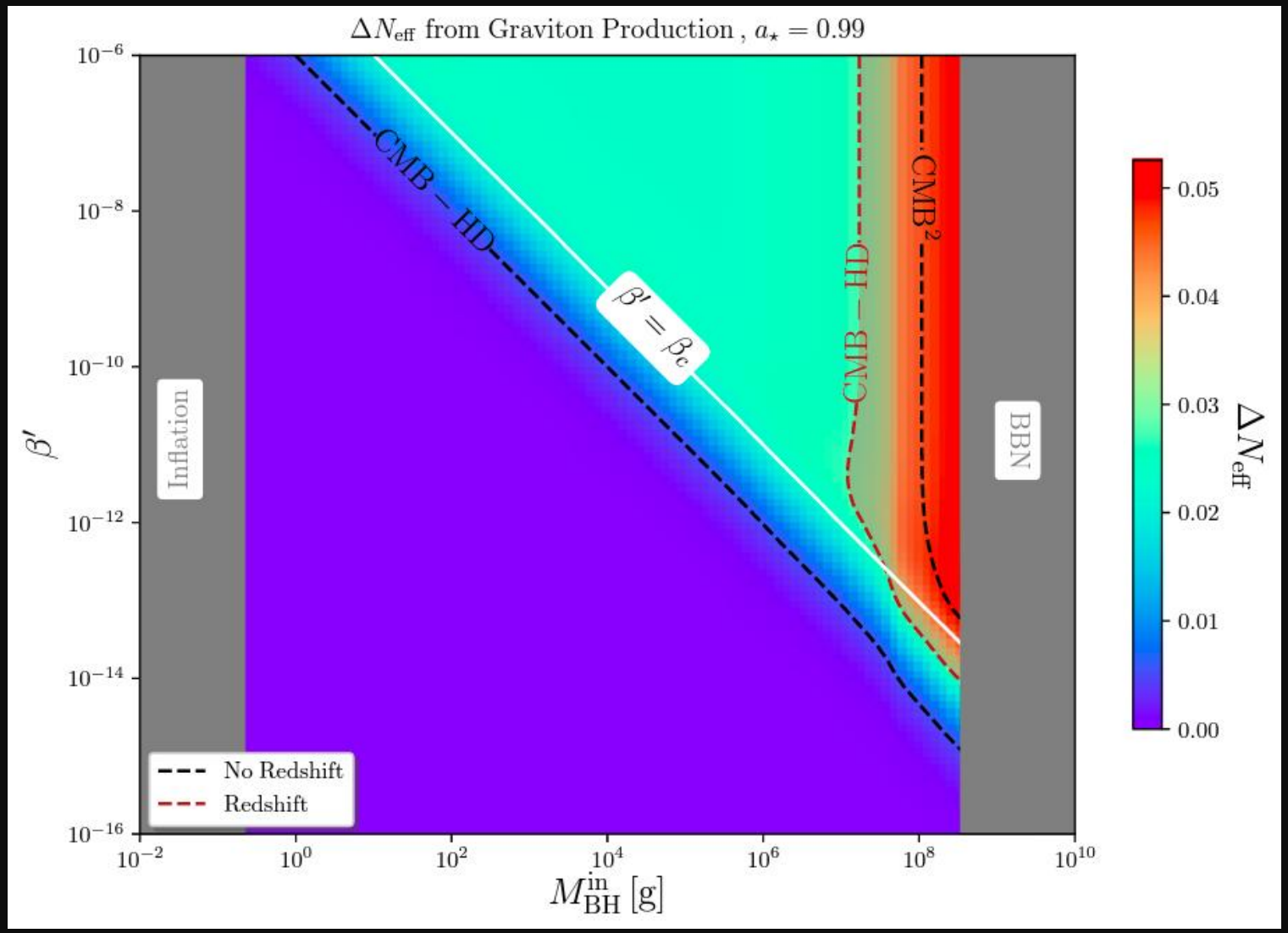
Why ?



[Cheek, LH, Perez-Gonzalez and Turner '22]

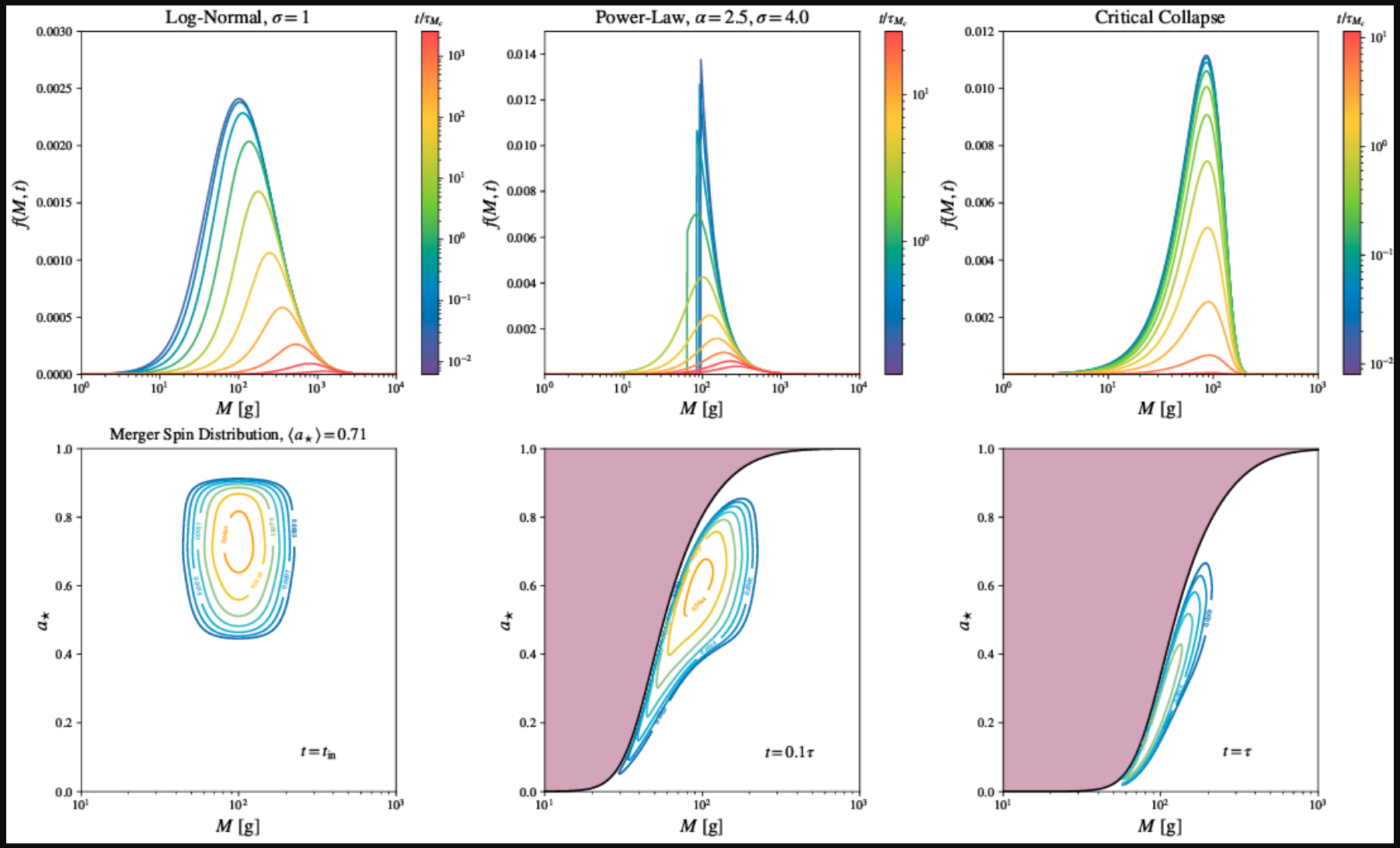
Code Publicly available: FRISBHEE

Kerr PBHs and Dark Radiation



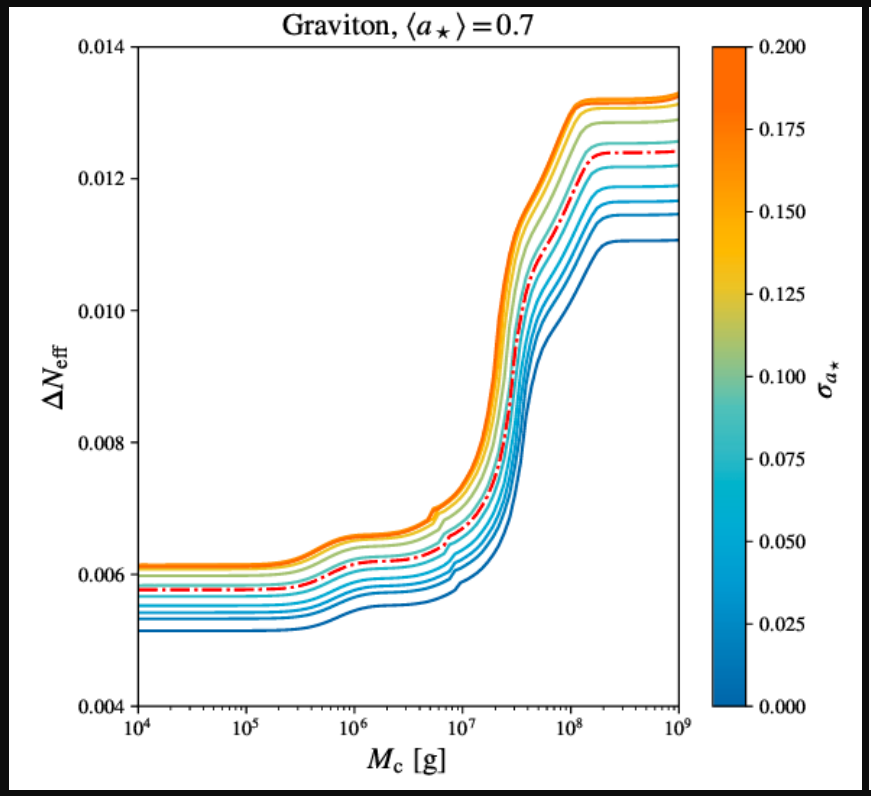
[Cheek, LH, Perez-Gonzalez and Turner '22] **Code Publicly available: FRISBHEE**

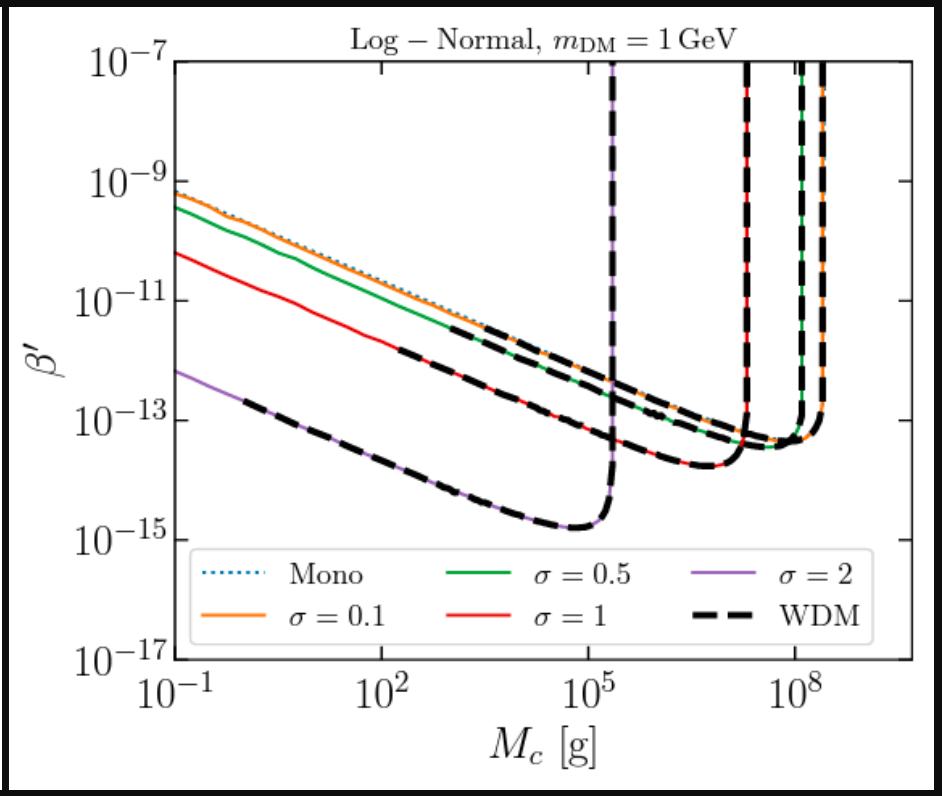
Evaporation of Extended Distributions



[Cheek, LH, Perez-Gonzalez, Turner '23]

Evaporation of Extended Distributions

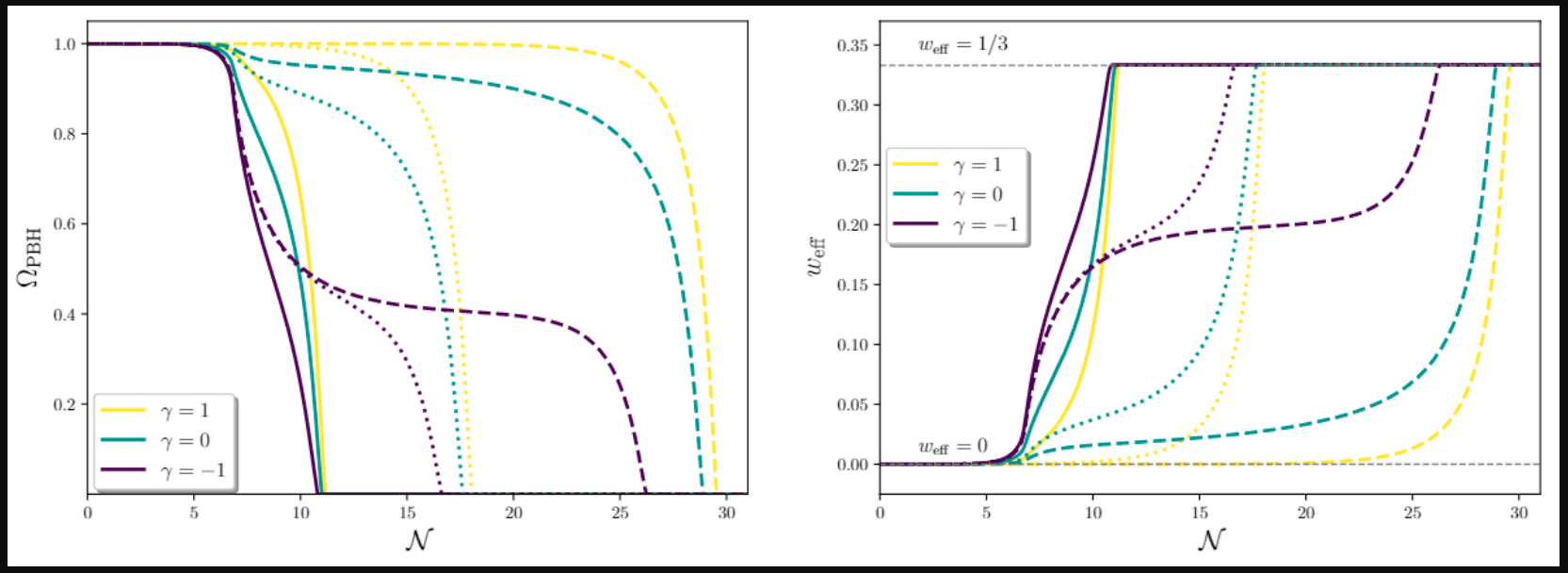




[Cheek, LH, Perez-Gonzalez, Turner '23]

Evaporation of Extended Distributions

$$\frac{\mathrm{d}n}{\mathrm{d}M} \propto M^{-\alpha} \quad \text{with} \quad \alpha = \frac{2\,(1+2\,w)}{1+w}$$



'Stasis' regime reached for $0 < w \leq 1$

[Copeland, Liddle, Barrow '91]
[Dienes, LH, Huang, Kim, Tait, Thomas '22]
[Cheek, LH, Perez-Gonzalez, Turner '23]

PBH Evaporation Beyond Homogeneity

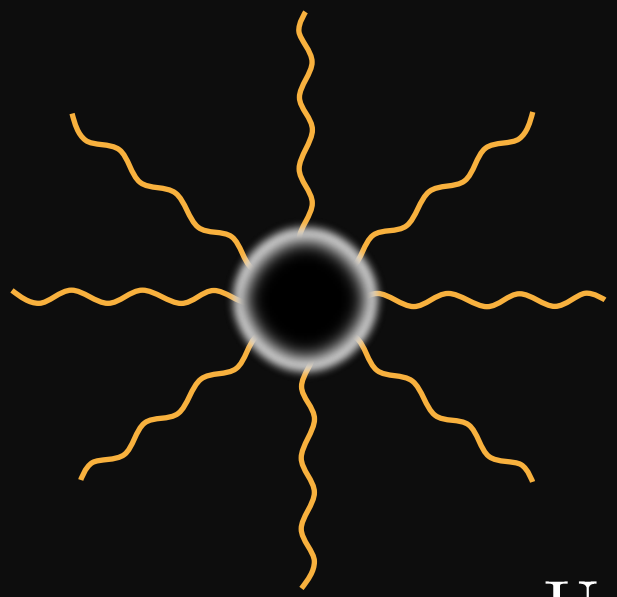
Primordial Black Holes

are

Powerful & Local radiators

in

Cosmology



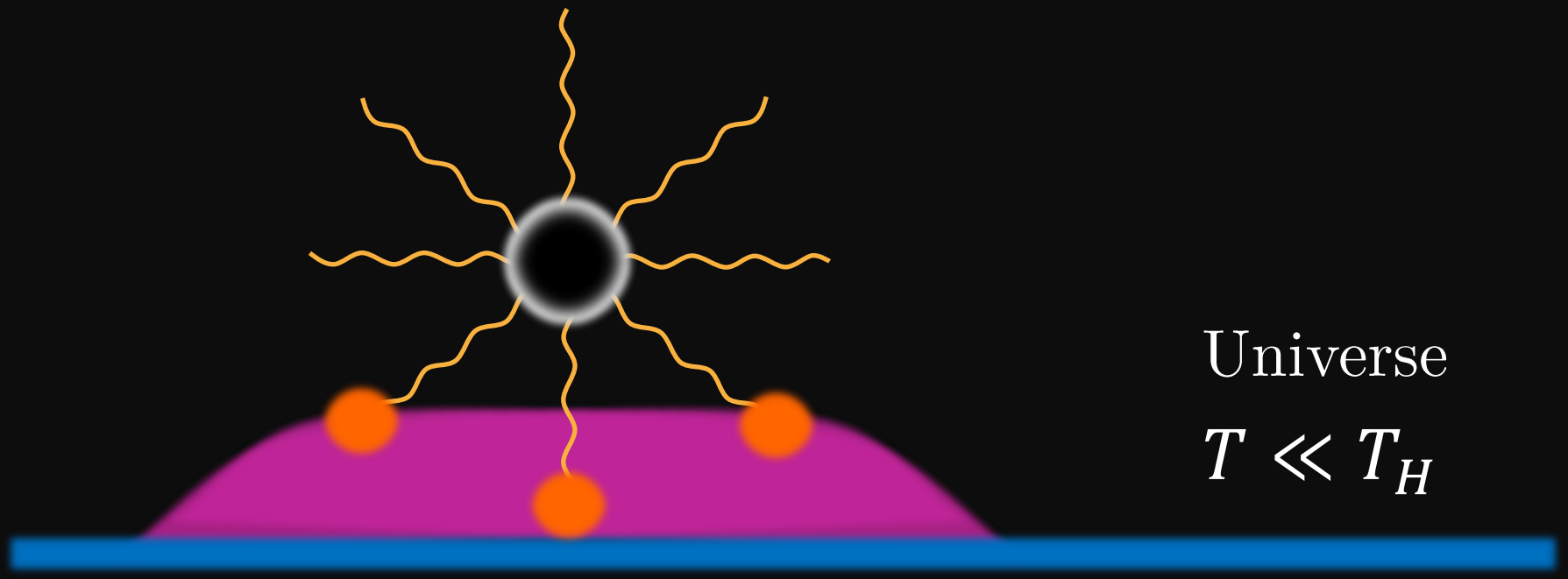
Hawking Radiation $E \sim T_H$

Universe Temperature $T \ll T_H$

In Reality

Hawking Radiation

$E \sim T_H$



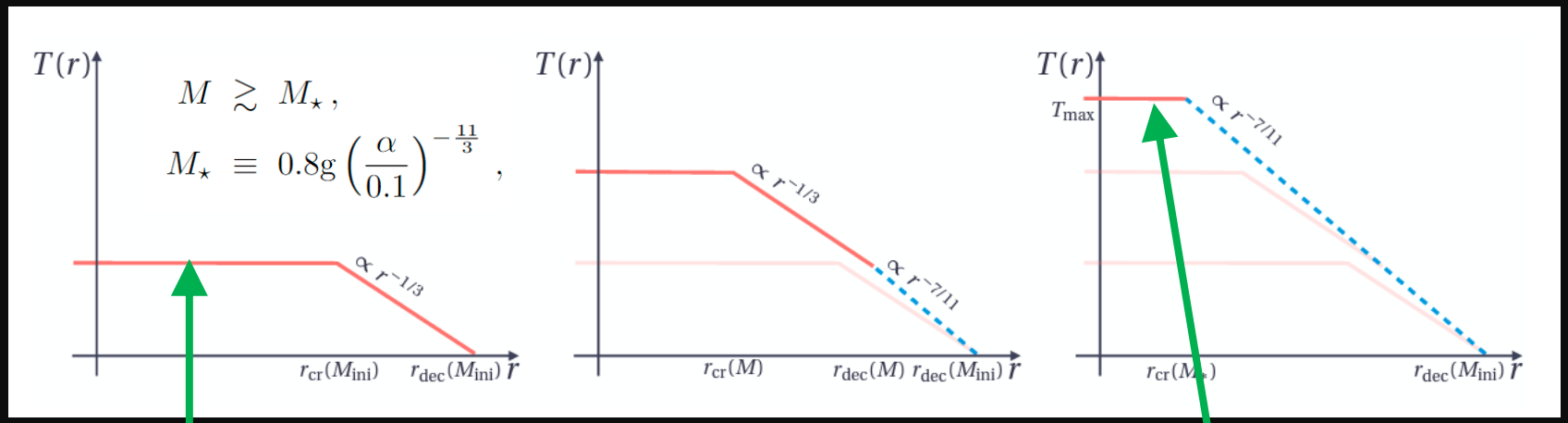
Universe

$T \ll T_H$

In Reality

Hawking Radiation heats the ambient plasma locally

He et al. *JCAP* 01 (2023) 027



$$T_{\rm plateau} \approx 2\times 10^{-4}\left(\frac{\alpha}{0.1}\right)^{\frac{8}{3}} T_H$$

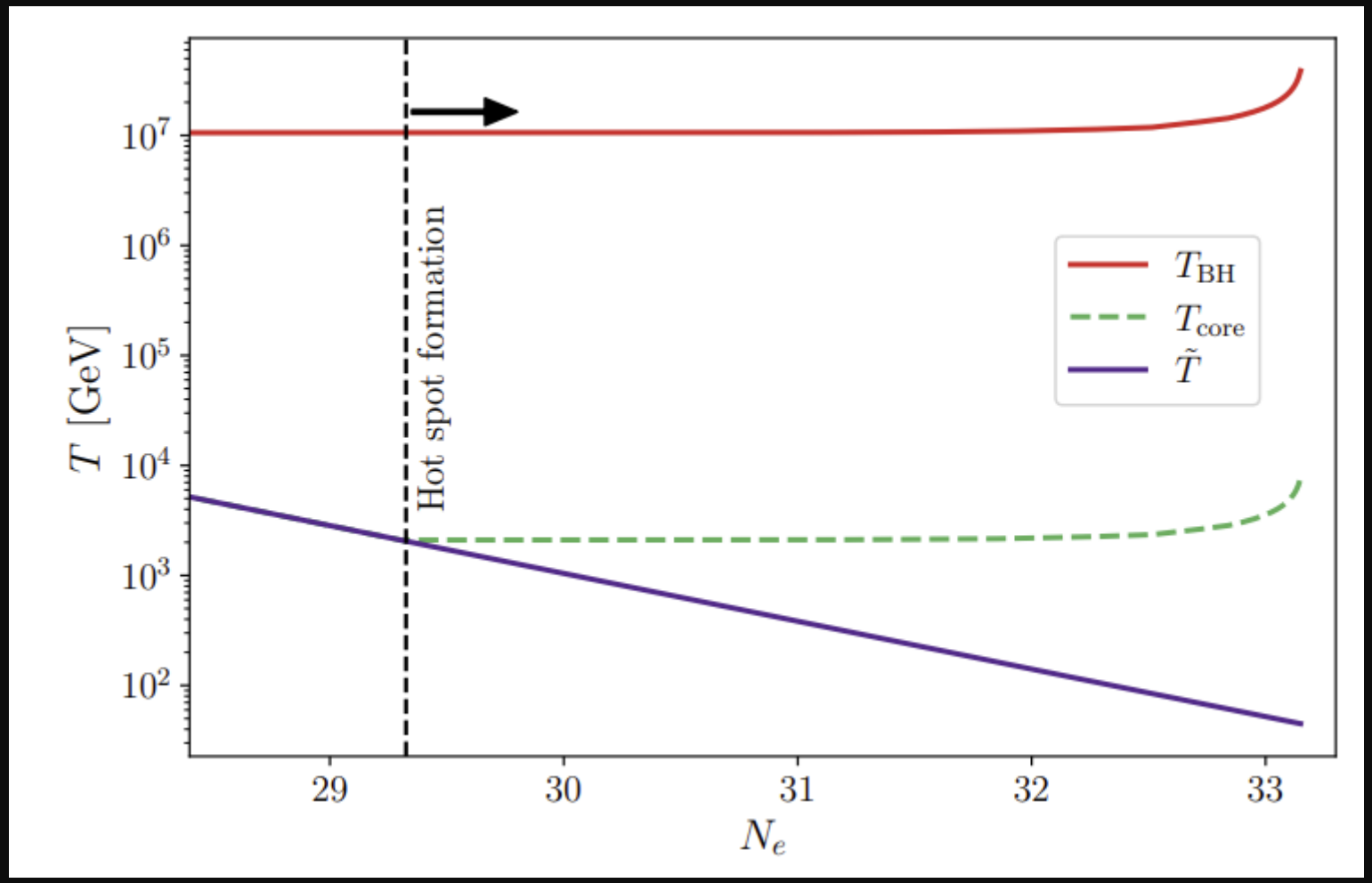
$$r_{\rm plateau} \approx 7\times 10^{8}\left(\frac{\alpha}{0.1}\right)^{-6} r_H$$

$$T_{\rm max} \approx 2\times 10^{9}{\rm GeV}\left(\frac{\alpha}{0.1}\right)^{\frac{19}{3}}$$

$$r_{\rm max} = r_{\rm plateau}\Big|_{T_H=T_{\rm max}}$$

Hot Spot & BSM

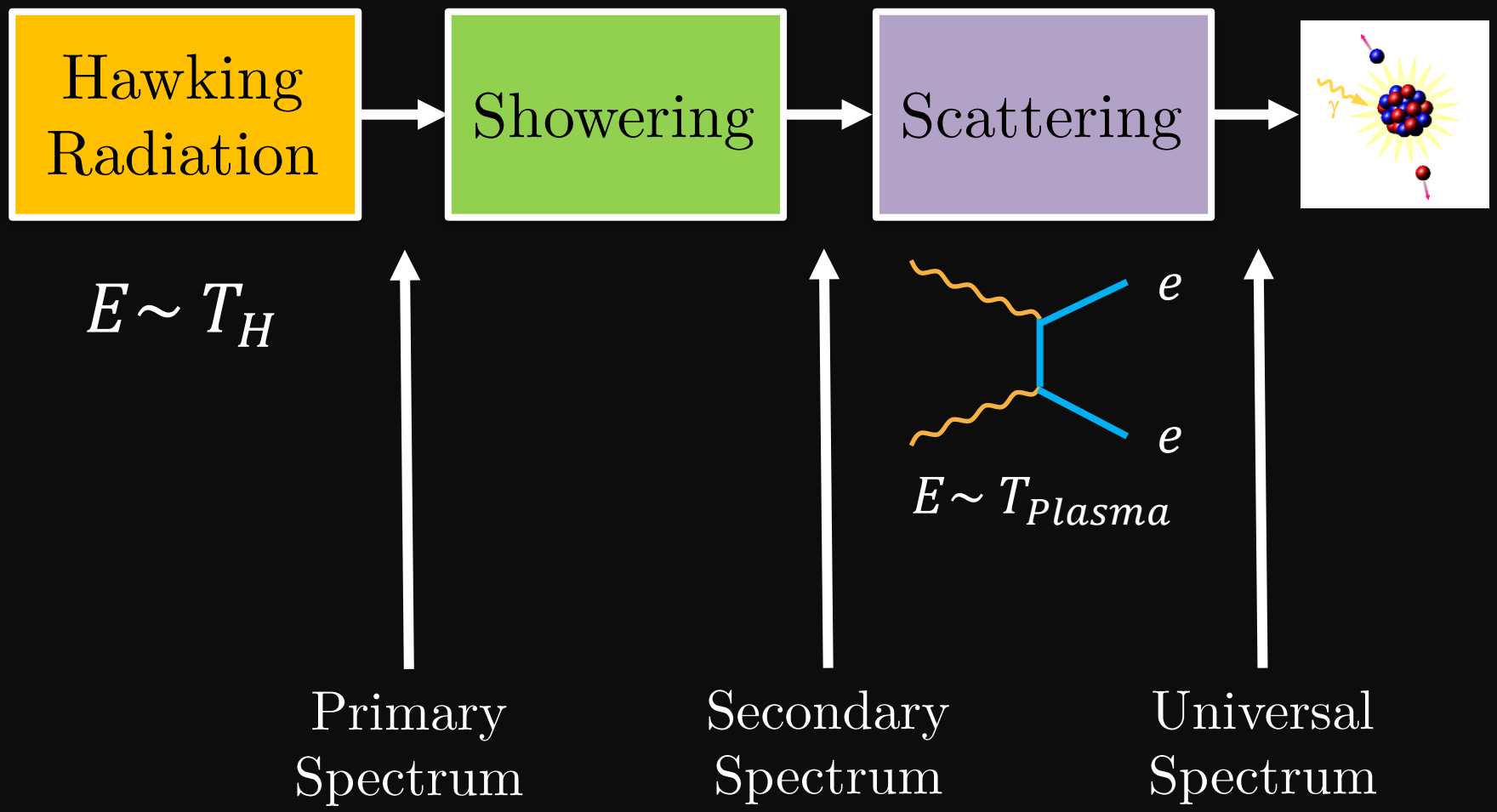
Hot Spots rise while the universe cools down



[J.Gunn, LH, Y.F. Perez-Gonzalez, J. Turner, '24]

Hot Spot & BBN

The case of Photodissociation



Hot Spot & BBN

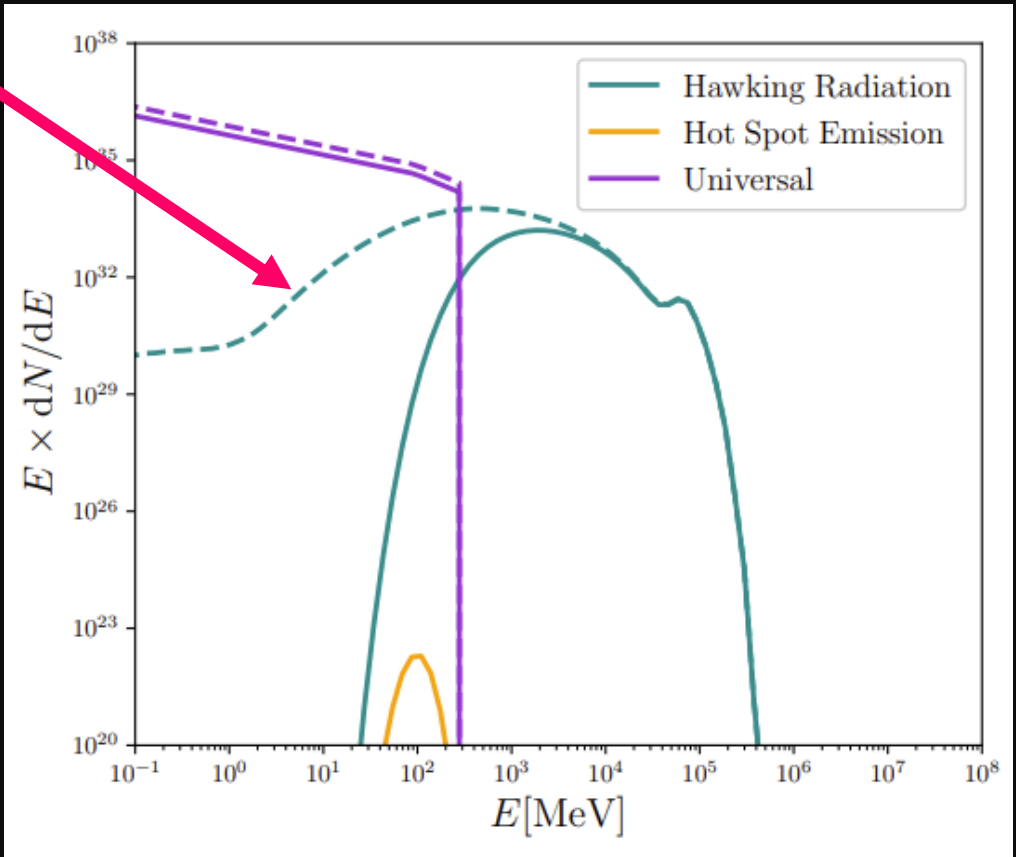
BlackHawk

[Arbey and Auffinger]

$$\mathrm{OD}^{\gamma}(E,r) \equiv \int_{r}^{r_{\mathrm{HS}}} \frac{dr'}{\lambda\left(r', E_{\gamma}\right)}$$

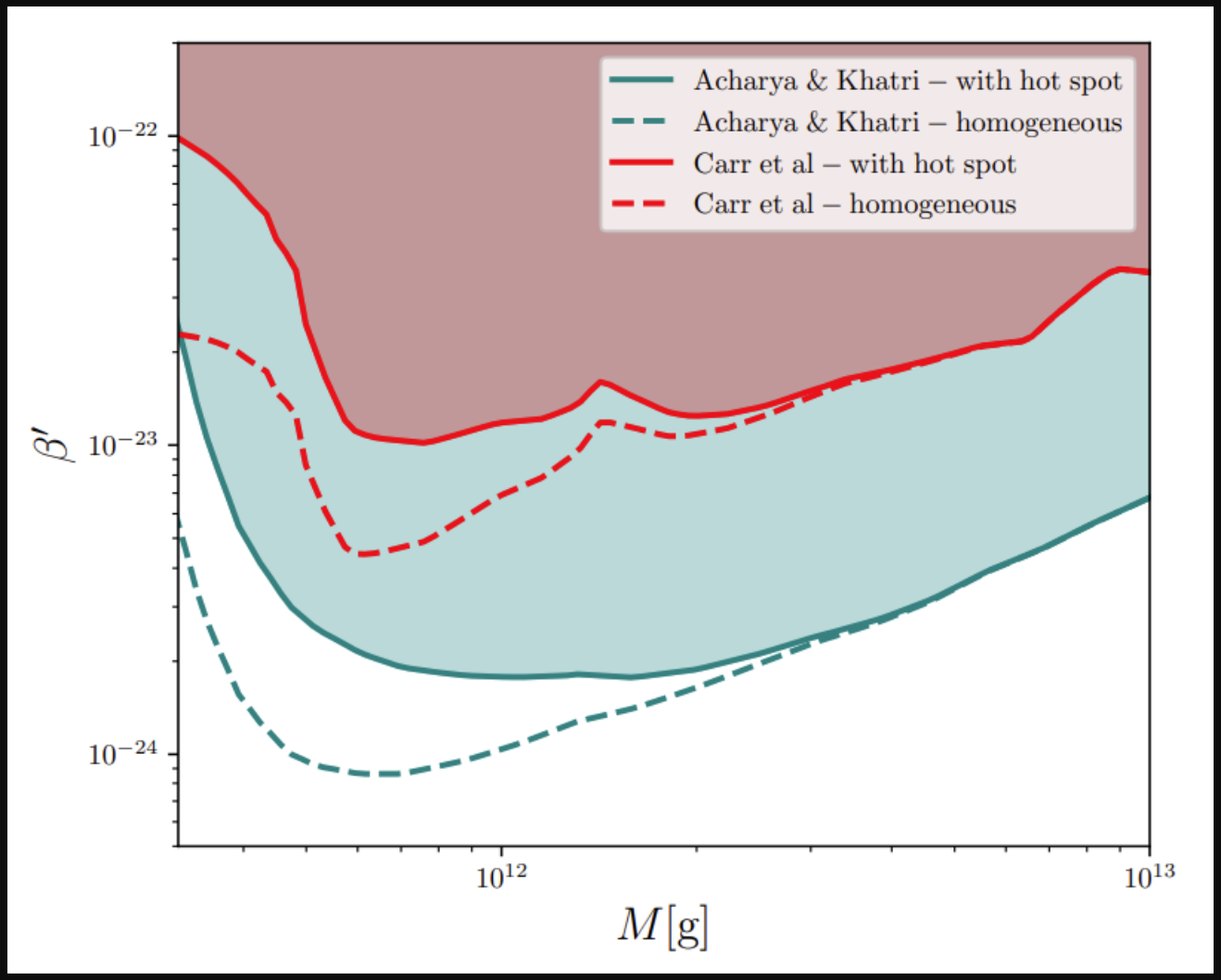
$$P^{\gamma}_{\mathrm{esc}}(E,r) = e^{-\mathrm{OD}^{\gamma}(E,r)}$$

$$\mathcal{T}(M) \equiv \frac{\int \frac{dN}{dEdt}(E_0)\Big|_{\mathrm{secondary}} \times E_0 P^{\gamma}_{\mathrm{esc}}(E_0) dE_0}{\int \frac{dN}{dEdt}(E_0)\Big|_{\mathrm{secondary}} \times E_0 dE_0}.$$



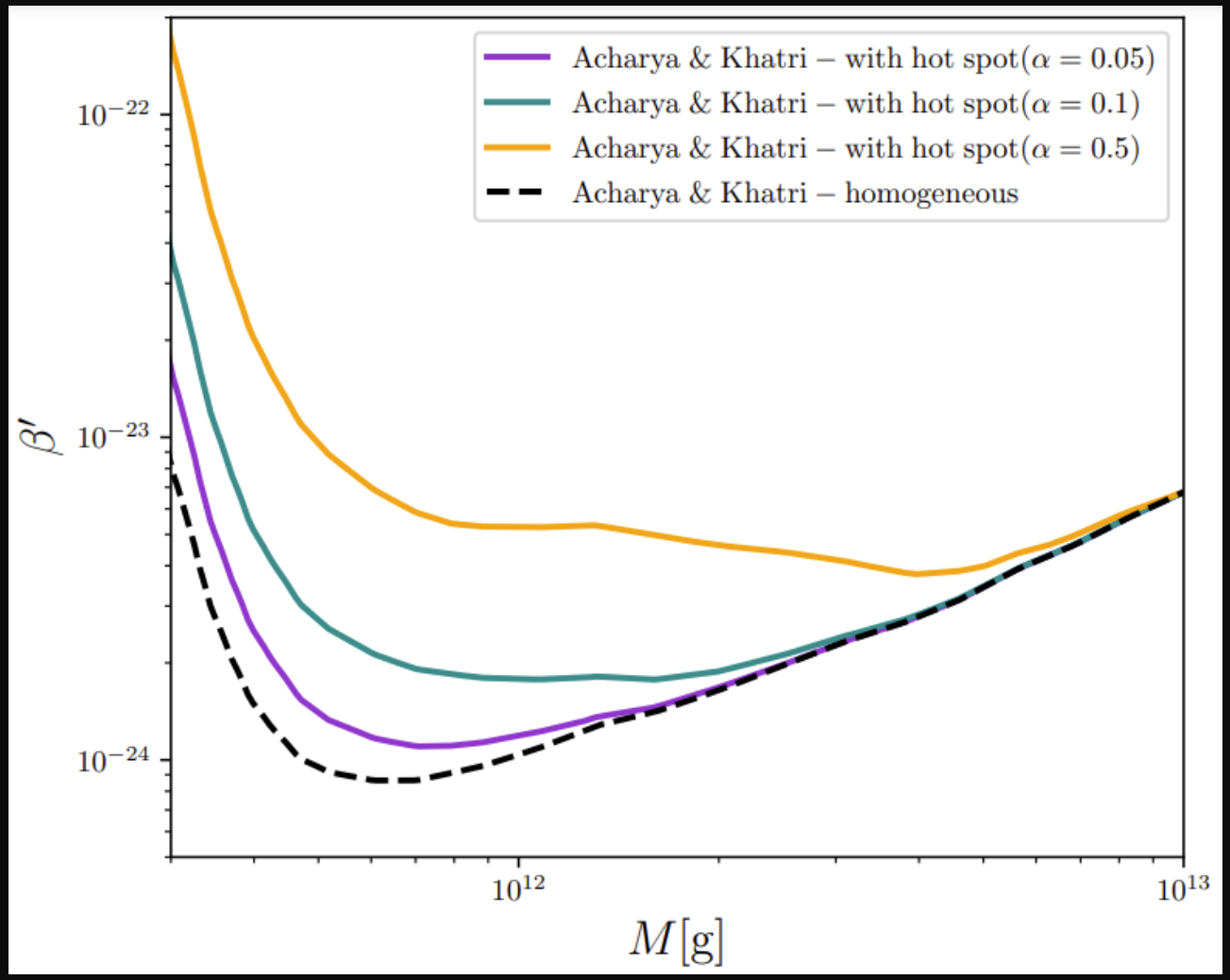
[C.Altomonte, M. Fairbairn, LH, '25]

Hot Spot & BBN



[C.Altomonte, M. Fairbairn, LH, '25]

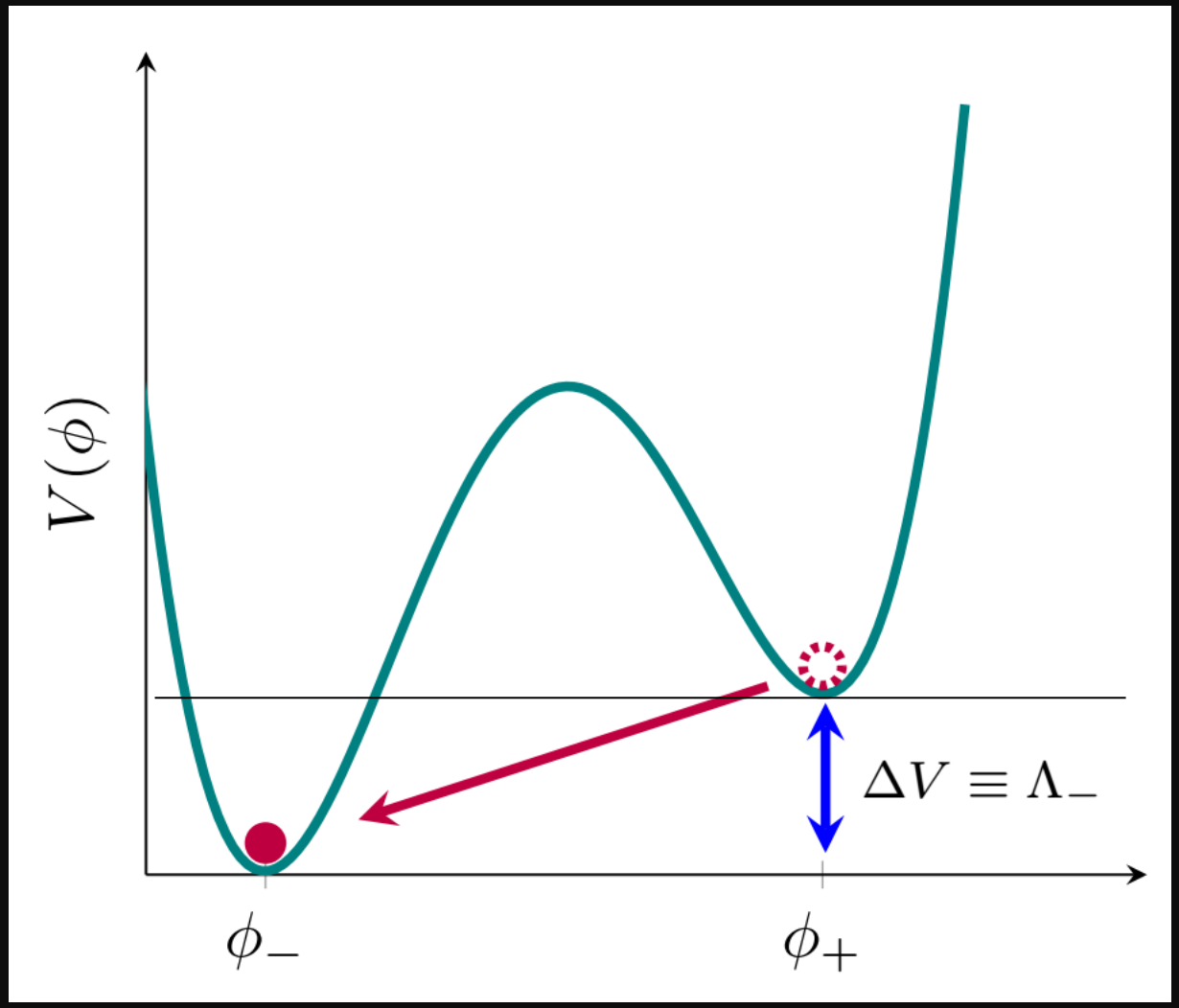
Hot Spot & BBN



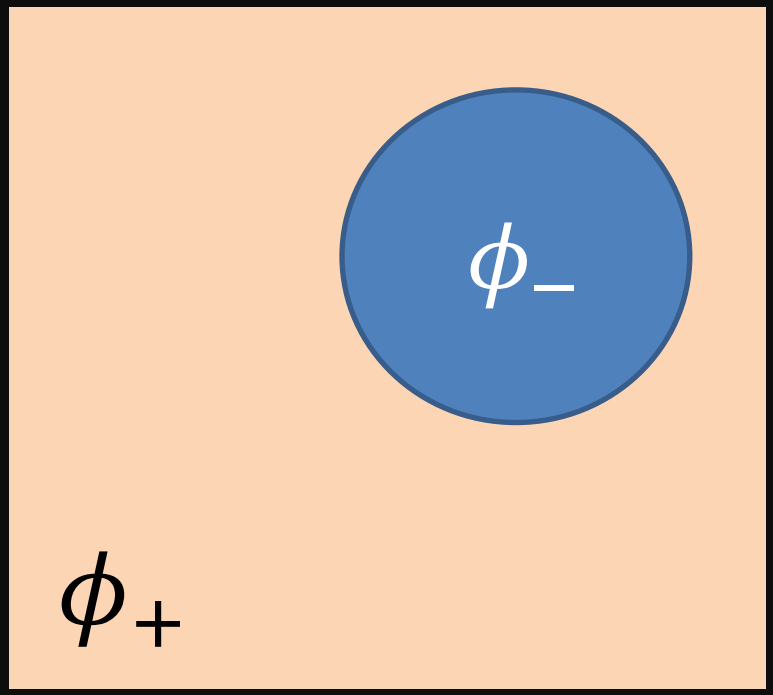
[C.Altomonte, M. Fairbairn, LH, '25]

An old idea: PBHs may trigger 1st order phase transitions

1st-Order Phase Transitions for Dummies



1st-Order Phase Transitions for Dummies



Transition ⟷ Energy Loss

In the vacuum: S. R. Coleman, Phys. Rev. D 15, 2929 (1977)

→ The bubble expands (*O(4)* symmetric bubble)

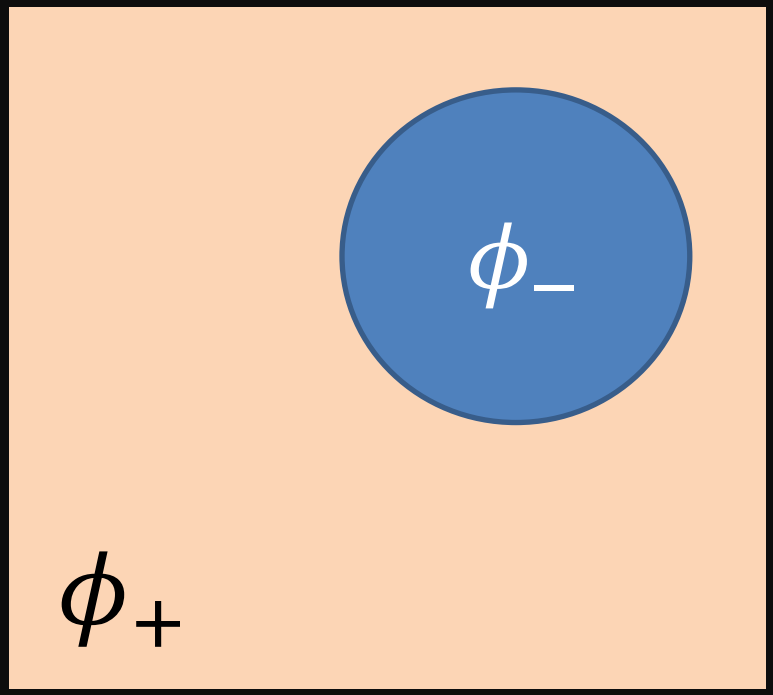
Energy = Kinetic

In a Thermal Bath: A. D. Linde, Phys. Lett. B 100, 37 (1981).

→ The bubble is static (*O(3)* symmetric bubble)

Energy from Thermostat

1st-Order Phase Transitions for Dummies



Transition ⟷ Energy Loss

In (cold) GR: Coleman & De Luccia, Phys. Rev. D 21, 3305 (1980).

⟶ The metric and bubble adjust to conserve energy

Energy = Metric Deformation

Question: What happens around a radiating Black Hole?

1ST-Order Phase Transitions for Dummies

Question: What happens around a radiating Black Hole?

So Far: Only considered in very extreme situations…

BH radiating in the vacuum (Unruh vacuum)

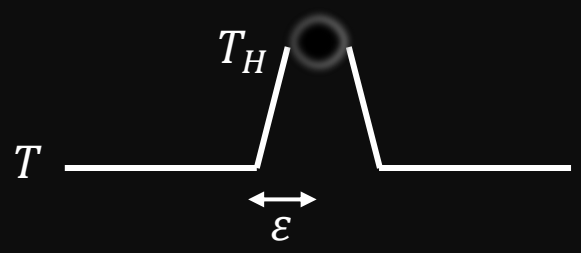
→ No definite answer. Partial results only obtained in 2D.

BH in thermal equilibrium with the plasma (Hartle-Hawking vacuum)

→ The BH and the plasma both behave as thermostats.

$$I_{\rm b}[T] = \beta \int dx^3 \sqrt{-h} \left(-\frac{R}{16\pi G} + \frac{1}{2} h^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + V(\phi) \right)$$

+ Bckgd terms
+ Conical deficit
if $\beta \neq \beta_H$

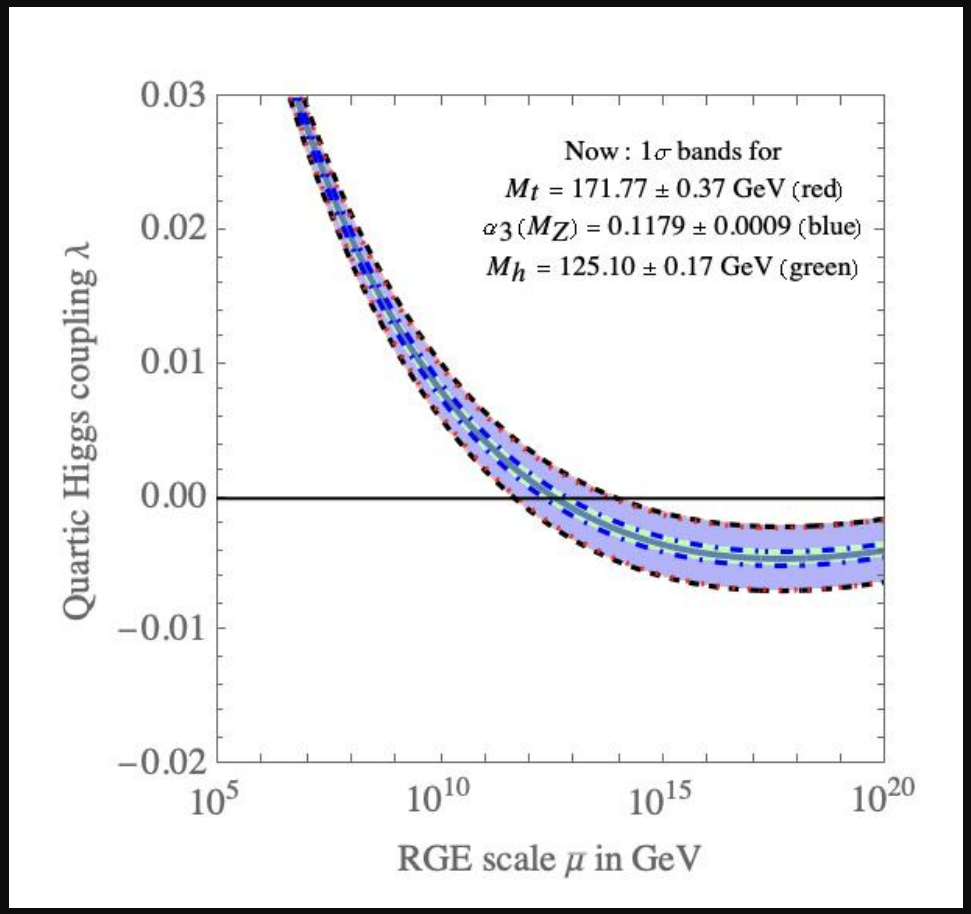


$$I_{\rm b}[T] = \frac{\mathcal{A}_+}{4G} - \frac{\mathcal{A}_-}{4G} = I_{\rm b}[T_H]$$

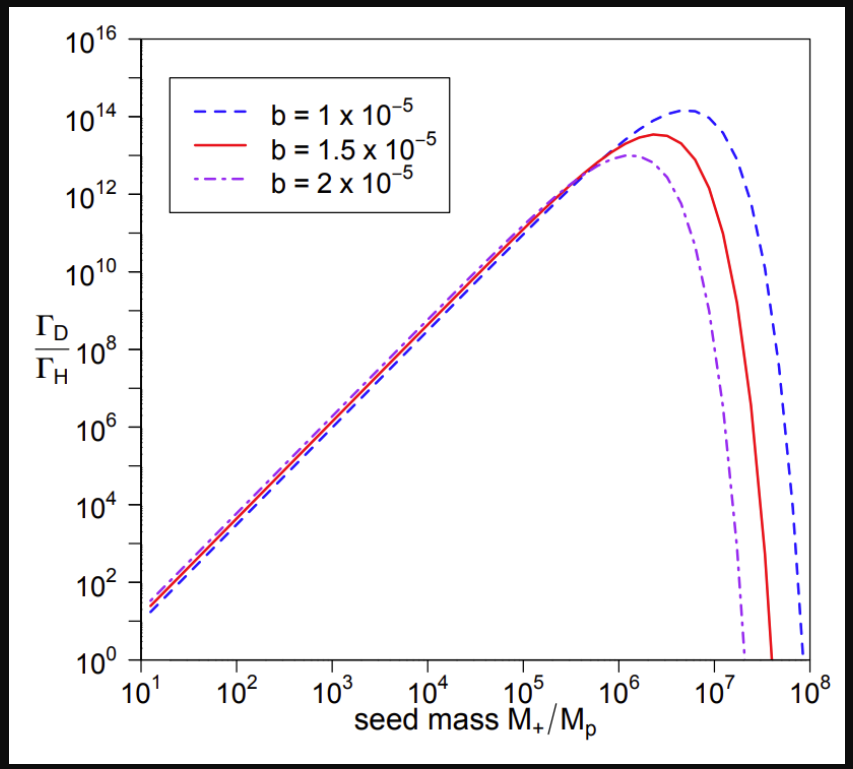
Gregory, Moss, and Withers, JHEP 03, 081(2014)

Only used with Hartle-Hawking so far…

The Higgs Case



A. Strumia, stolen from X



Gregory, Moss, and Withers, JHEP 03, 081(2014)

1st-Order Phase Transitions for Dummies

Idea under attack....

arXiv > hep-ph > arXiv:2209.05504

High Energy Physics - Phenomenology

[Submitted on 12 Sep 2022 (v1), last revised 24 Feb 2023 (this version, v2)]

Black holes don't source fast Higgs vacuum decay

Alessandro Strumia

Our Idea: What if PBHs and the temperature around them are not static?

In Reality

Hawking Radiation heats the ambient plasma locally

→ In the $\varepsilon \to 0$ limit, can use the result

$$I_{\rm b}[T] = \frac{\mathcal{A}_+}{4G} - \frac{\mathcal{A}_-}{4G} = I_{\rm b}[T_H] \quad (*)$$

→ To calculate the rate:

$$\Gamma_{\rm FVD}^{\rm HH} \equiv (GM_+)^{-1} \left(\frac{I_{\rm b}[T_H]}{2\pi}\right)^{1/2} \exp\left(-I_{\rm b}[T_H]\right) .$$

T_H

Linde's result

$$\frac{\Gamma}{V} = T\left(\frac{S_3(\varphi)}{2\pi T}\right)^{3/2} \exp\left[-S_3(\varphi)/T\right]$$

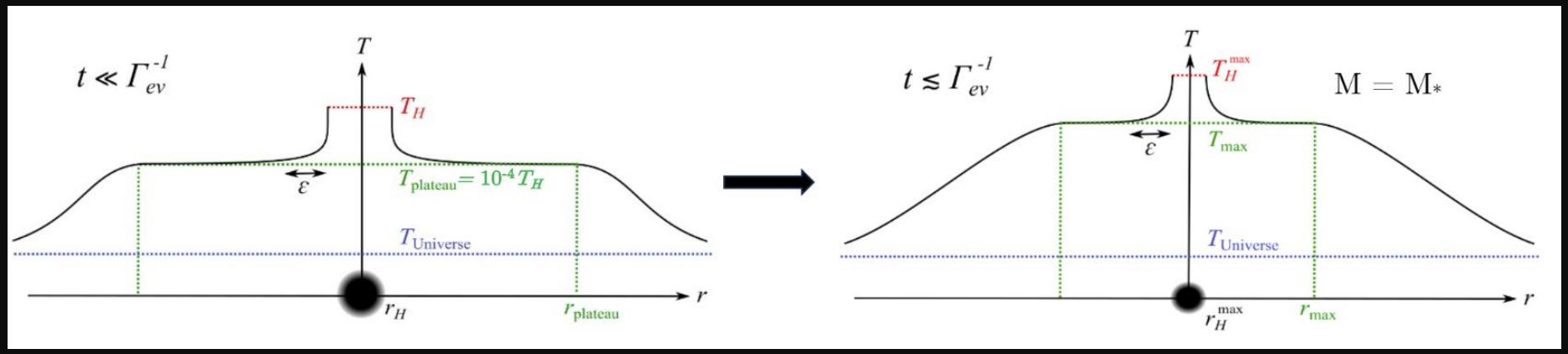
Generalisation to arbitrary T

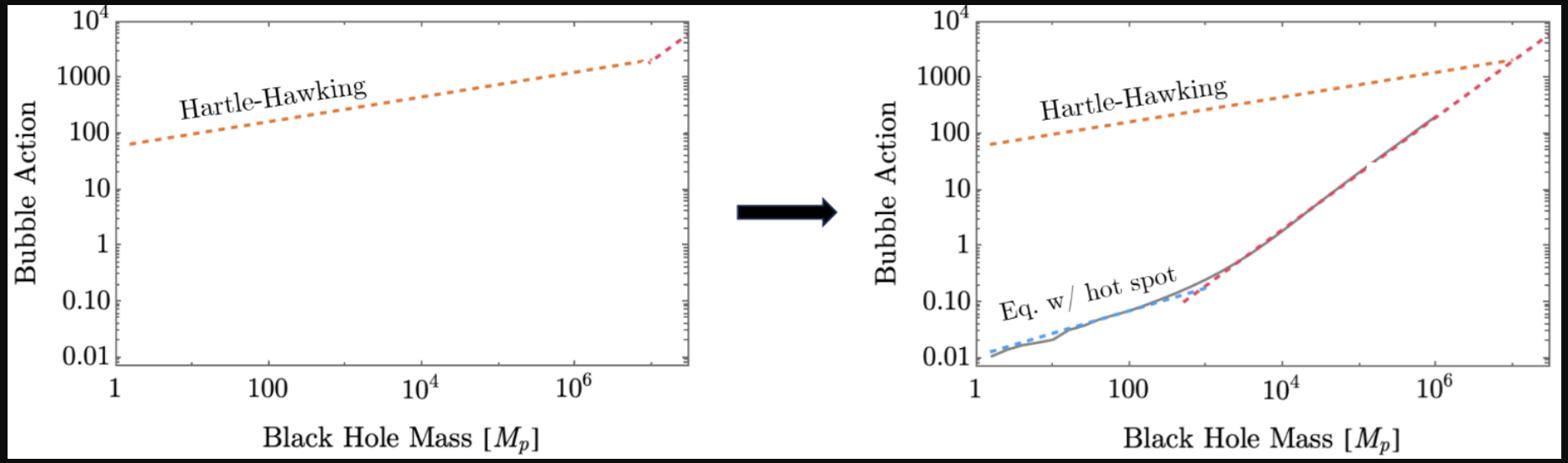
$$\begin{aligned}\Gamma_{\rm FVD}(T) &\approx T\left(\frac{I_{\rm b}[T]}{2\pi}\right)^{1/2} \exp\left(-I_{\rm b}[T]\right) , \\ &\approx T\left(\frac{I_{\rm b}[T_H]}{2\pi}\right)^{1/2} \exp\left(-I_{\rm b}[T_H]\right) ,\end{aligned}$$

(*)

Rate to be compared to the evaporation rate...

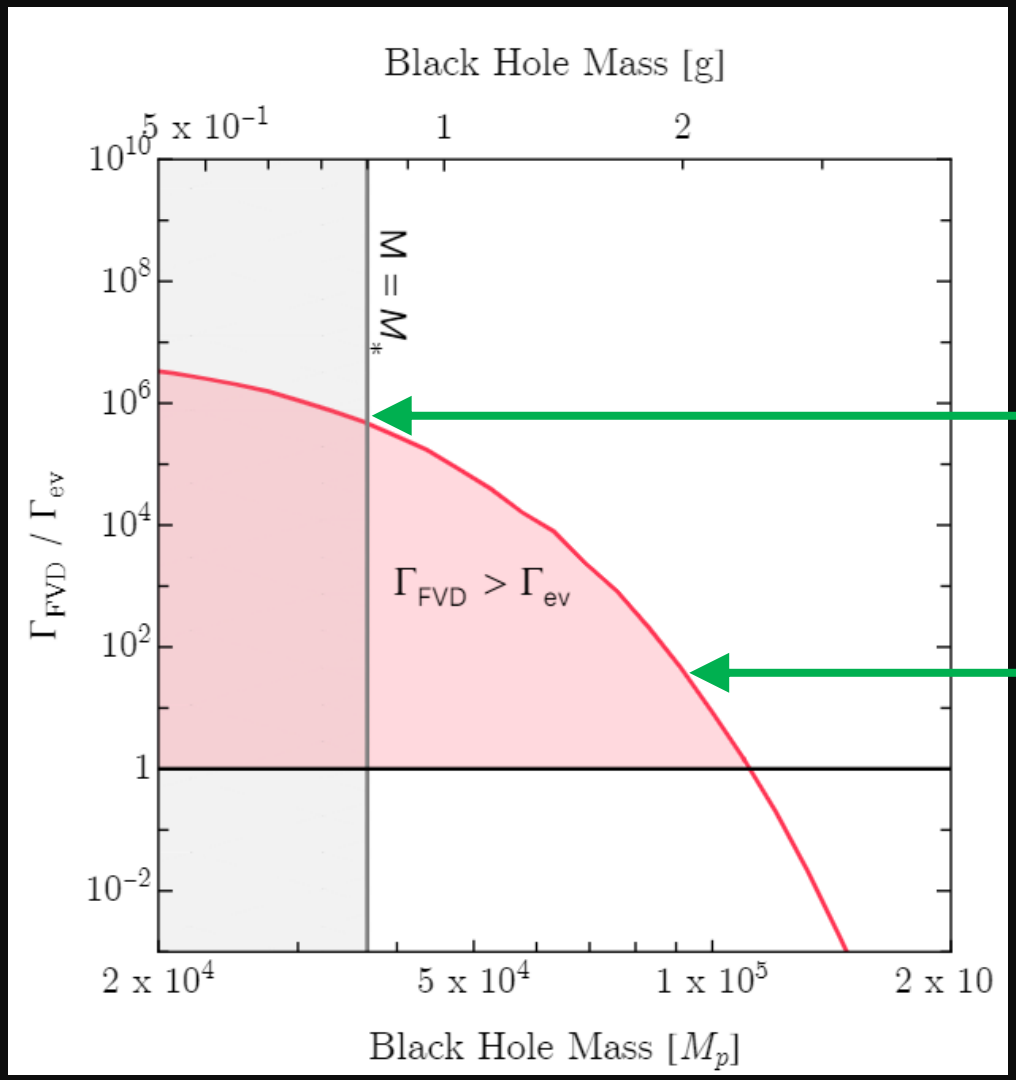
$$\Gamma_{\rm FVD}(T) \approx T\left(\frac{I_{\rm b}[T_H]}{2\pi}\right)^{1/2} \exp\left(-I_{\rm b}[T_H]\right) ,$$





An example: The EW vacuum

Our Universe may be metastable (at ~2σ)



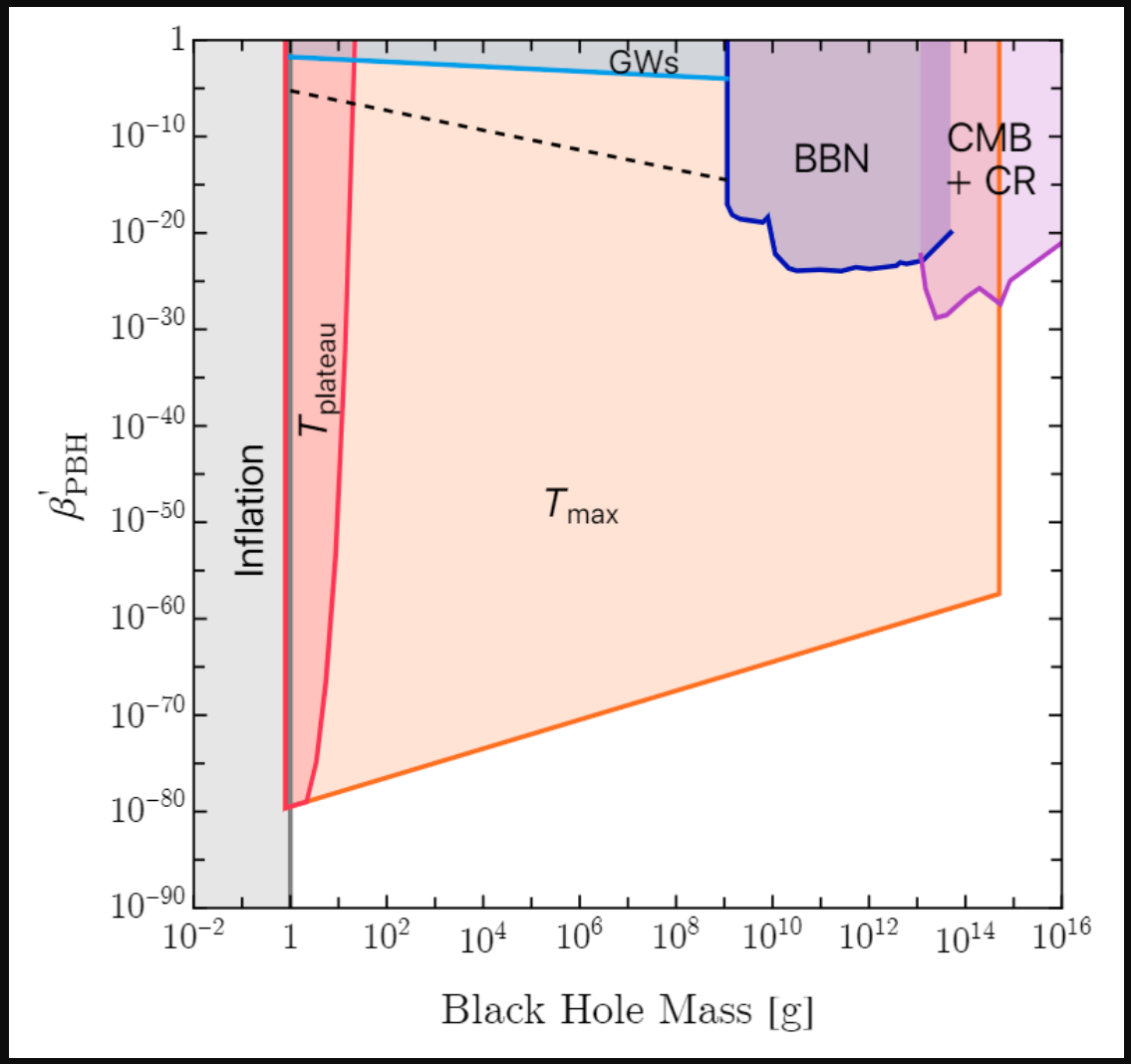
At $M = M_*$, $T = T_{max}$

Using $T_{plateau}(M)$

[Hamaide, LH, Hu, Cheek, ArXiv:2311.01869]

An example: The EW vacuum

Our Universe may be metastable (at ~2σ)



[Hamaide, LH, Hu, Cheek, 2023]

Conclusion

PBHs can leave several imprints in the early Universe

- They modify cosmology
- Produce out of equilibrium particles, leading to modified predictions for particle searches
- PBHs can act as local radiators, forming hot spots
- Hot spots affect particle physics around PBHs. Not just a little…
- PBHs may leave more traces in cosmological data than expected. Stay tuned!

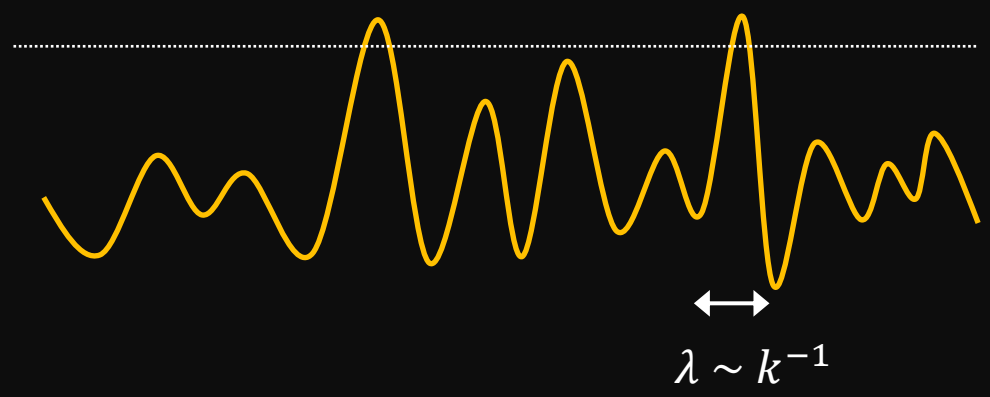
https://github.com/yfperezg/frisbhee

Thank you very much !

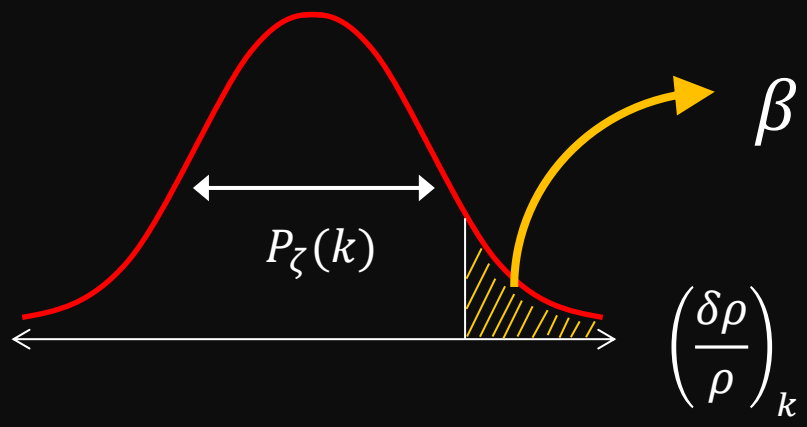
PBH Formation

Collapse of Overdensities

$$\left|\frac{\delta\rho}{\rho}\right| > \rho_c$$



Key Ingredients : 1) Scalar curvature *Power Spectrum* $P_\zeta(k)$



$$\beta(M_k) \sim \exp\left(-\frac{1}{P_\zeta(k)}\right)$$

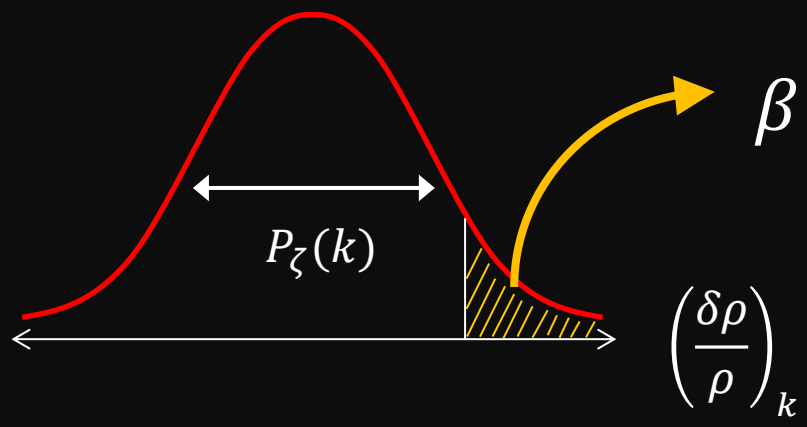
Small Power Spectrum → Small density of PBHs formed

PBH Formation

Key Ingredients : 1) Scalar curvature *Power Spectrum* $P_\zeta(k)$

2) Equation of State in the Universe $\boldsymbol{w}$

$$\delta_c(k) = \frac{3(1+w)}{5+3w} \sin^2\left(\frac{\pi\sqrt{w}}{1+3w}\right)$$



$$\beta(M_k) \sim \exp\left(-\frac{1}{P_\zeta(k)}\right)$$

Small Power Spectrum → Small density of PBHs formed

PBH Formation

Key Ingredients : 1) Scalar curvature *Power Spectrum* $P_\zeta(k)$

During inflation, small perturbations may be generated at scales $k \sim k_{\rm CMB}$

Later, larger perturbations may be sourced at scales $k \gg k_{\rm CMB}$

- Bumpy potentials (Ultra-Slow-Roll period)
- Colliding scalar-field bubbles
- Aborting phase transitions [W.Y. Ai, LH, TH Jung, '24]
- Enhancement due to early matter domination
- …

PBH Formation

Collapse of Small-Scale Density Perturbations during Preheating in Single Field Inflation

Karsten Jedamzik* Martin Lemoine† and Jérôme Martin‡

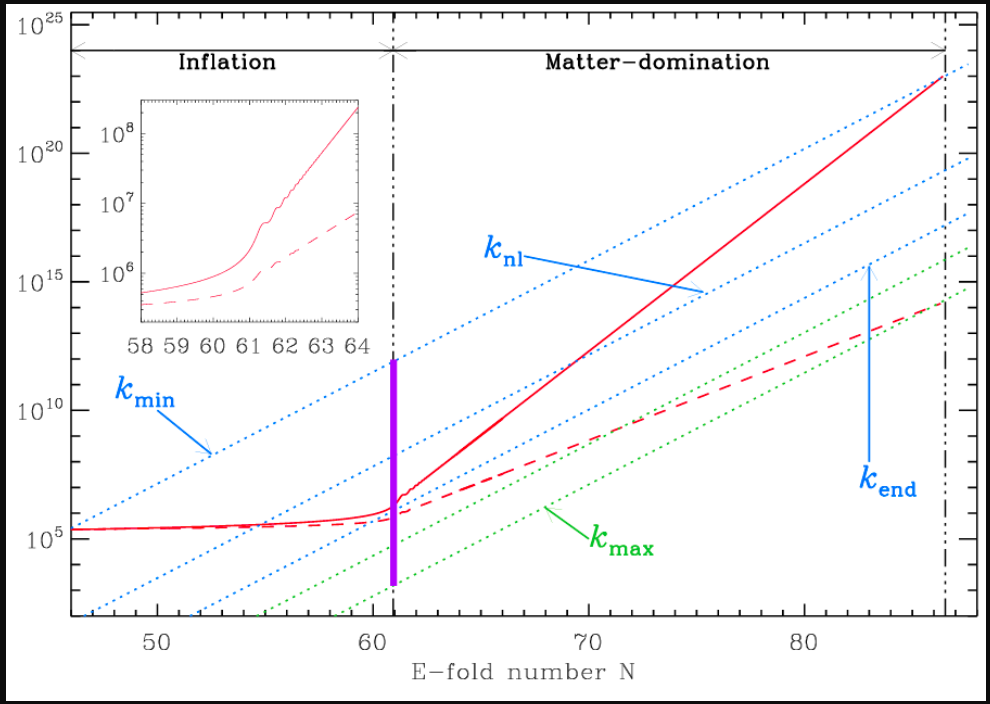
Primordial black holes from the preheating instability in single-field inflation

Jérôme Martin,[a] Theodoros Papanikolaou,[b] Vincent Vennin[b,a]

$$V(\phi) = \frac{m^2}{2}\phi^2.$$

$$m = 2H_{\rm end}\frac{M_{\rm Pl}}{\phi_{\rm end}}.$$

$$\phi(t) \simeq \phi_{\rm end}\left(\frac{a_{\rm end}}{a}\right)^{3/2}\sin(mt)$$



$$\frac{{\rm d}^2\tilde{v}_{\boldsymbol{k}}}{{\rm d}z^2} + \left[1 + \frac{k^2}{m^2a^2} - \sqrt{6\kappa}\phi_{\rm end}\left(\frac{a_{\rm end}}{a}\right)^{3/2}\cos(2z)\right]\tilde{v}_{\boldsymbol{k}} = 0\,,$$

where we have defined $z \equiv mt + \pi/4$. This equation is similar to a Mathieu equation

$$\frac{{\rm d}^2\tilde{v}_{\boldsymbol{k}}}{{\rm d}z^2} + \left[A_{\boldsymbol{k}} - 2q\cos(2z)\right]\tilde{v}_{\boldsymbol{k}} = 0 \qquad (13)$$

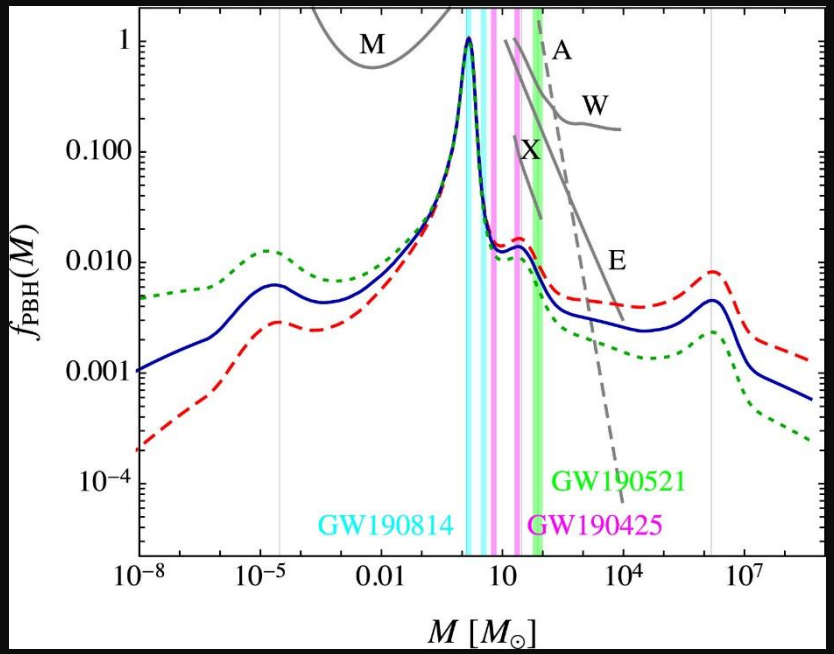
PBH Formation

Key Ingredients : 2) Equation of State in the Universe $\boldsymbol{w}$

Fluctuation of w : During phase transitions (QCD) may fluctuate → variations of $\beta(M)$

Dynamics of w leaves an imprint in the PBH spectrum

Probing its shape → Reading the spectrum pattern



[Byrnes, Hindmarsh, Young, Hawkins '18]
[Carr, Clesse, García-Bellido, Kühnel '20]
[Juan, Serpico, Abellán '22]
[Musco, Jedamzik, Young '23]

PBH Formation

Key Ingredients : 2) Equation of State in the Universe $\boldsymbol{w}$

Cosmological moduli may start to oscillate

→ Early Matter Domination ($\boldsymbol{w} = 0$)

- String Theory compactification
- Transverse directions in SUGRA
- Axion-like particle models
- ...

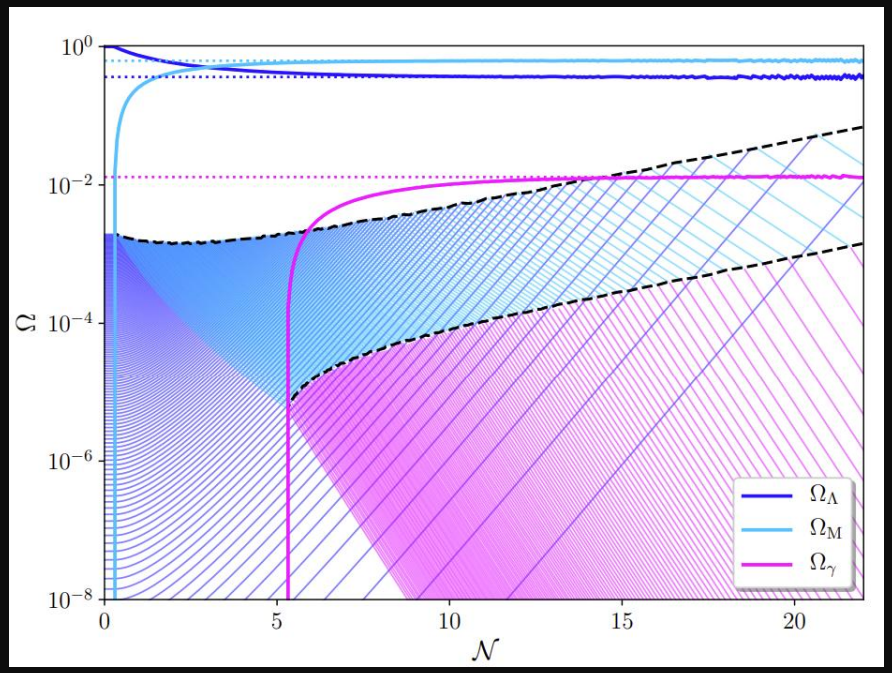
PBH Formation

Key Ingredients : 2) Equation of State in the Universe w

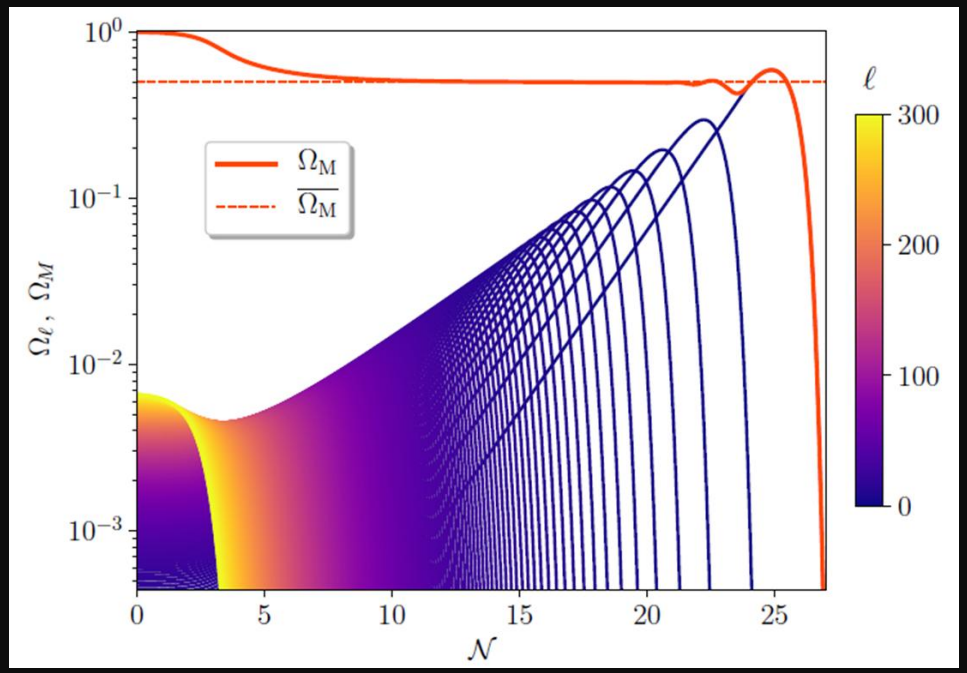
Compact Extra Dimensions may place the Universe in *Stasis*

→ Mixed Matter/Radiation state ($w \in [0, 1/3]$)

[Dienes, LH, Huang, Tait, Thomas '23]



[Dienes, LH, Huang, Kim, Tait, Thomas '21]



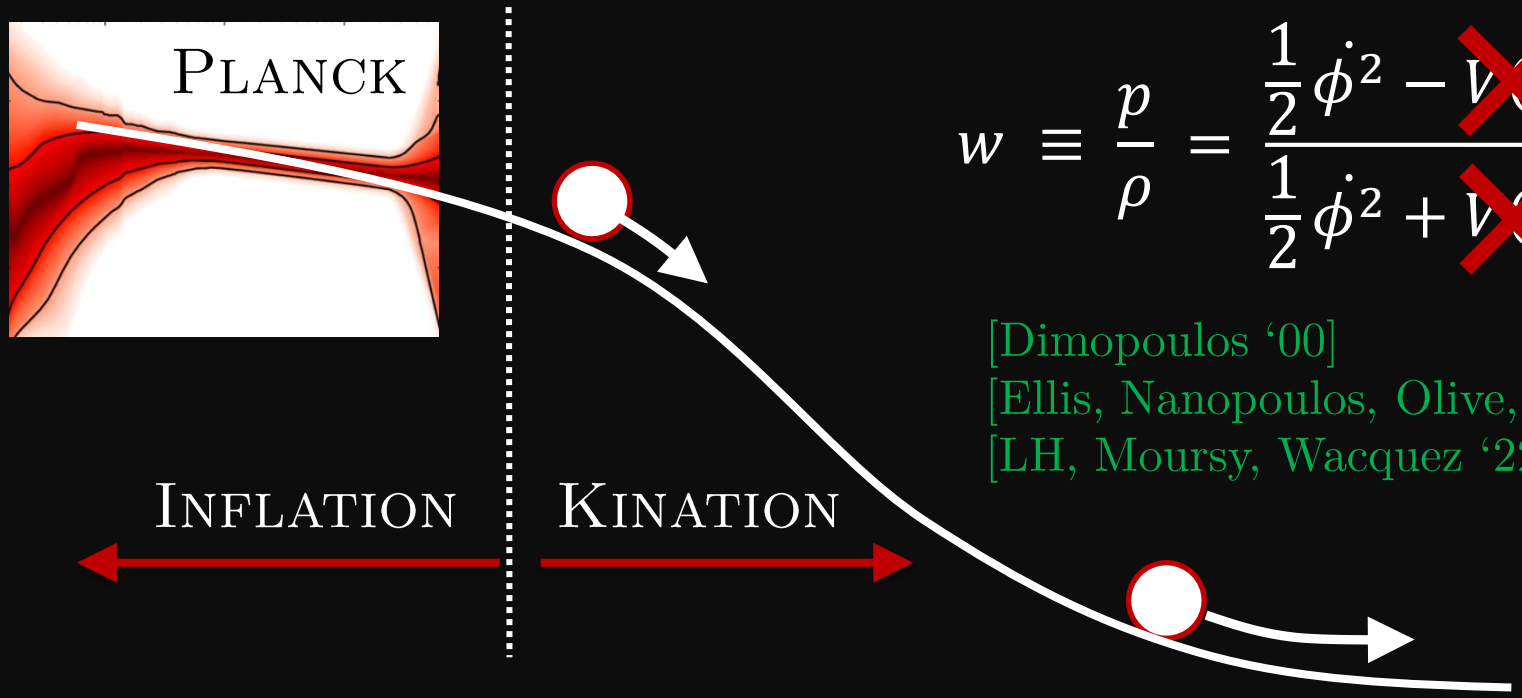
PBH Formation

Key Ingredients : 2) Equation of state in the Universe $\boldsymbol{w}$

The post-inflationary Universe, quèsaco ?

Runaway directions : Quintessential (non-oscillatory) inflation

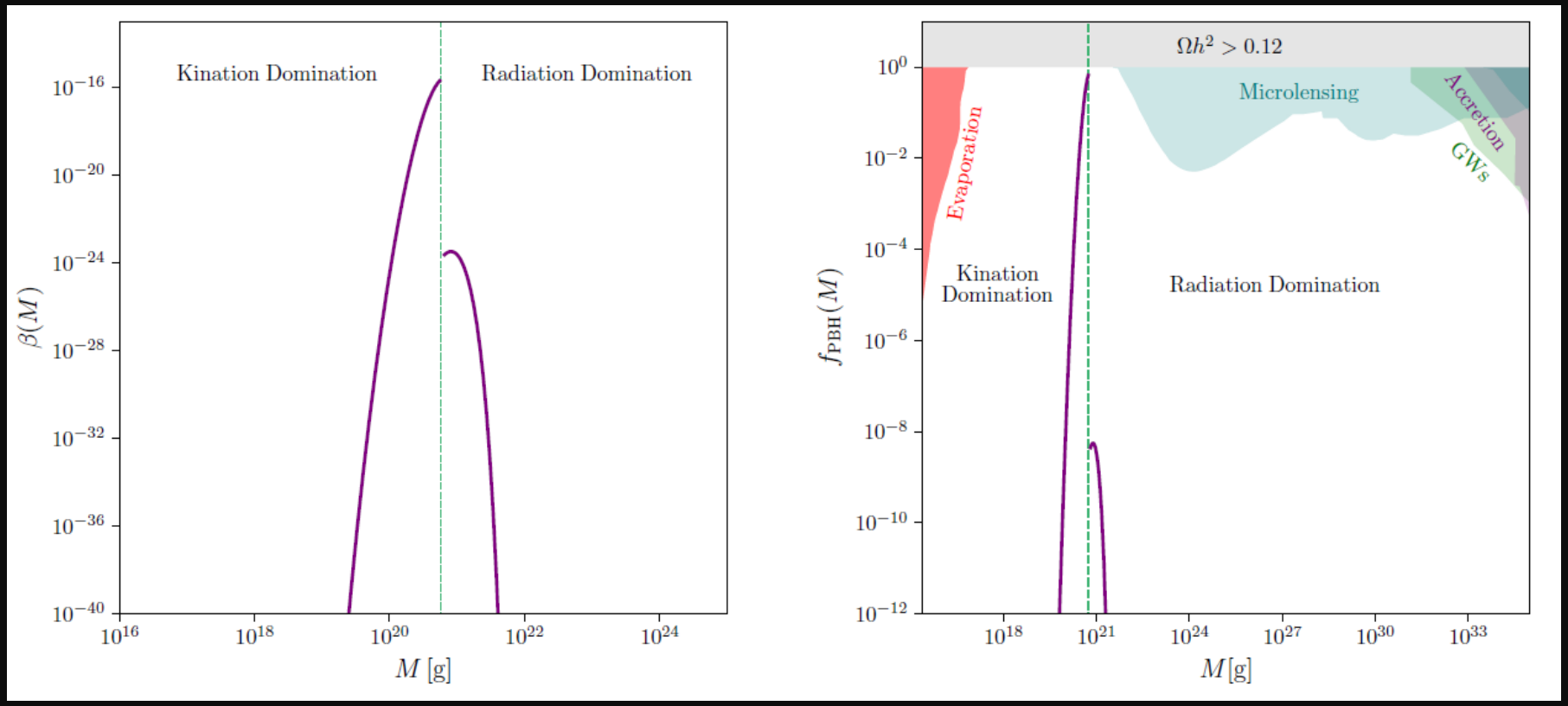
→ Kination ($\boldsymbol{w} = 1$)



$$w \equiv \frac{p}{\rho} = \frac{\frac{1}{2}\dot{\phi}^2 - \cancel{V(\phi)}}{\frac{1}{2}\dot{\phi}^2 + \cancel{V(\phi)}} \approx 1$$

[Dimopoulos '00]
[Ellis, Nanopoulos, Olive, Verner '20]
[LH, Moursy, Wacquez '22] ...

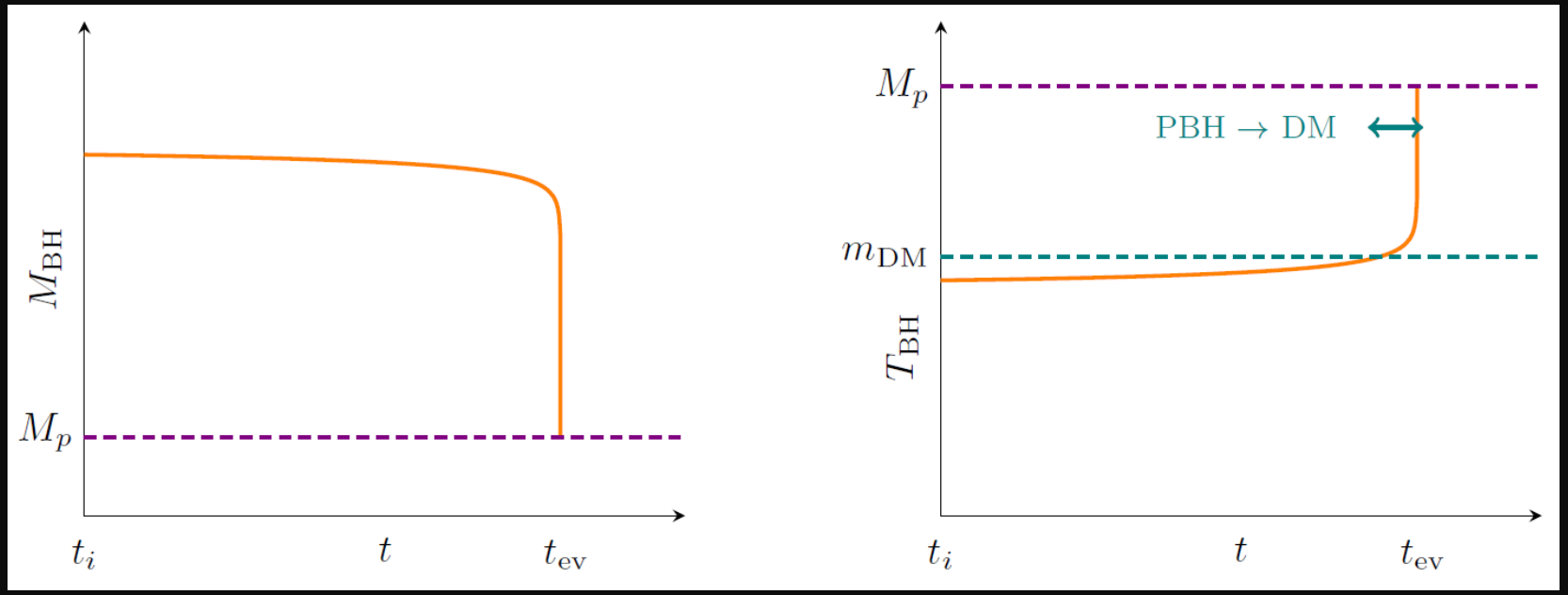
PBH Formation



[LH, Moursy, Wacquez '22]

PBH evaporation

$$T_{\rm BH} = \frac{1}{8\pi G M_{\rm BH}} \sim 1.06\ {\rm GeV} \left(\frac{10^{13}\ {\rm g}}{M_{\rm BH}}\right)$$



→ More and more particles contribute to the evaporation

PBH evaporation

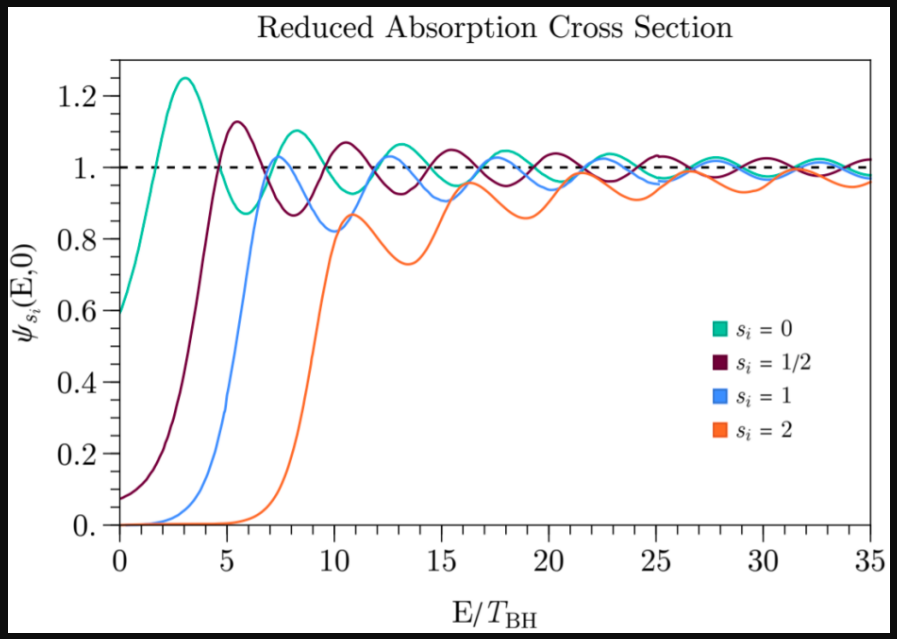
$$\frac{\mathrm{d}^2\mathcal{N}_i}{\mathrm{d}p\,\mathrm{d}t} = \frac{g_i}{2\pi^2}\frac{\sigma_{s_i}(M_{\mathrm{BH}},\mu_i,p)}{\exp\left[E_i(p)/T_{\mathrm{BH}}\right]-(-1)^{2s_i}}\frac{p^3}{E_i(p)}$$

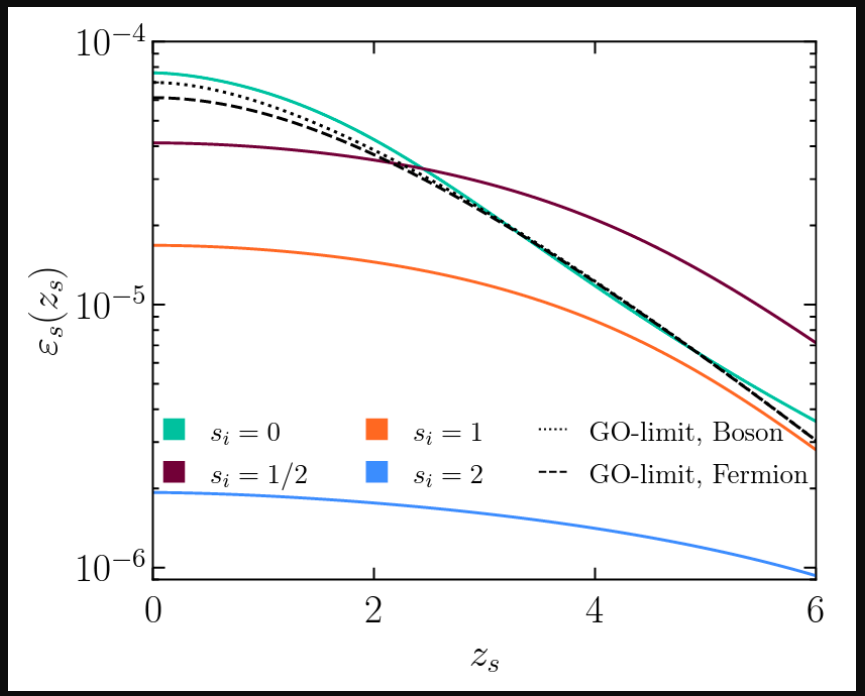
Very bad approximation at (not too)

low momentum…

$$\psi_{s_i}(E,\mu) \equiv \frac{\sigma_{s_i}(E,\mu)}{27\pi G^2 M_{\mathrm{BH}}^2}$$

$$\varepsilon_i(z_i) = \frac{27}{8192\pi^5}\int_{z_i}^{\infty}\frac{\psi_{s_i}(x)(x^2-z_i^2)}{\exp(x)-(-1)^{2s_i}}\,x\,\mathrm{d}x$$





[Cheek, LH, Perez-Gonzalez and Turner '22]

Code Publicly available: FRISBHEE

Analytical Results

Freeze-In contribution

$$\Omega_{\rm I} = \alpha\, m_X^3 \frac{m_{\rm DM}}{\rho_c} \frac{36\sqrt{10}}{\pi\sqrt{g_{\star,\rho}(m_X)}} \frac{g_{\star,s}(T_{\rm eq})}{g_{\star,s}(m_X)} \frac{T_{\rm eq}^3 m_p}{m_X^4} \frac{a_{\rm eq}^3}{a_0^3} G_{1,3}^{2,1}\left({}_{\frac{3}{2},\frac{5}{2},0}^{1} \middle| \frac{m_X}{T_{\rm eq}}, \frac{1}{2}\right),$$

$$\Omega_{\rm II} = \frac{\alpha\, m_X^3}{4} \frac{m_{\rm DM}}{\rho_c} \sqrt{\frac{3m_p^2}{\rho_{\rm PBH}^c}} \left(\frac{a_c}{a_0}\right)^3 T_c \left(\frac{g_{\star,s}(T_c)}{g_{\star,s}(m_X)}\right)^{\frac{1}{3}} G_{1,3}^{2,1}\left({}_{-\frac{1}{2},\frac{1}{2},-\frac{7}{4}}^{-\frac{3}{4}} \middle| \frac{m_X}{2T_c}\left(\frac{g_{\star,s}(m_X)}{g_{\star,s}(T_c)}\right)^{\frac{1}{3}}, \frac{1}{2}\right),$$

$$\Omega_{\rm III} = 2\alpha\, m_X^3 \frac{m_{\rm DM}}{\rho_c} \sqrt{\frac{3m_p^2}{\rho_{\rm PBH}^{\rm ev}}} \left(\frac{a_{\rm ev}}{a_0}\right)^3 T_{\rm ev} G_{1,3}^{2,1}\left({}_{-\frac{1}{2},\frac{1}{2},-\frac{11}{2}}^{-\frac{9}{2}} \middle| \frac{m_X}{2T_{\rm ev}}, \frac{1}{2}\right),$$

$$\Omega_{\rm IV} = \alpha\, m_X^3 \frac{m_{\rm DM}}{\rho_c} \frac{36\sqrt{10}}{\pi\sqrt{g_{\star,\rho}(m_X)}} \frac{g_{\star,s}(T_0)}{g_{\star,s}(m_X)} \frac{T_0^3 m_p}{m_X^4} G_{1,3}^{2,1}\left({}_{\frac{3}{2},\frac{5}{2},0}^{1} \middle| \frac{m_X}{T_0}, \frac{1}{2}\right),$$

Freeze-Out contribution

- Regime I and IV:

$$x_{\rm FO} = \ln\left[\frac{3}{2}\sqrt{\frac{5}{\pi^5 g_\star(T_{\rm FO})}} g_{\rm DM} m_{\rm DM} m_p \langle\sigma v\rangle \sqrt{x_{\rm FO}}\right]$$

- Regime II:

$$x_{\rm FO} = \ln\left[\frac{3}{2}\sqrt{\frac{5}{\pi^5 g_\star(T_{\rm FO})}} \frac{g_{\rm DM} m_{\rm DM} m_p \langle\sigma v\rangle}{\sqrt{\kappa}}\right],$$

- Regime III:

$$x_{\rm FO} = \ln\left[\frac{3}{2}\sqrt{\frac{5}{\pi^5 g_\star(T_{\rm FO})}} \frac{g_{\rm DM} m_p \langle\sigma v\rangle}{m_{\rm DM}} T_{\rm ev}^2 x_{\rm FO}^{5/2}\right].$$

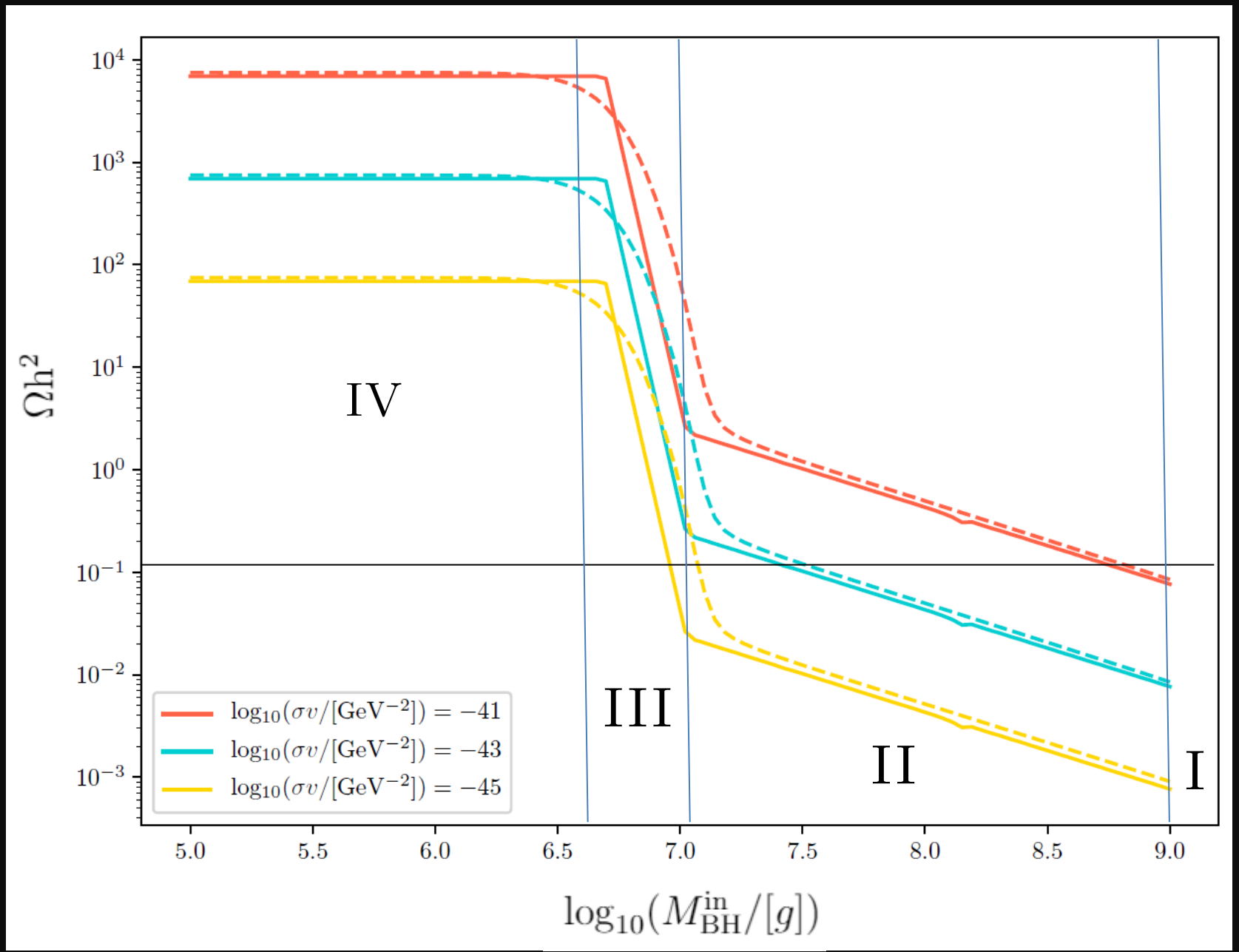
$$\Omega_{\rm I} = \frac{15}{2\pi} \frac{x_{\rm FO}}{\sqrt{10 g_\star(T_{\rm FO})}} \frac{s_{\rm eq}}{m_p \langle\sigma v\rangle \rho_c} \left(\frac{a_{\rm eq}}{a_0}\right)^3,$$

$$\Omega_{\rm II} = \frac{45}{4\pi} \frac{1}{m_{\rm DM} m_p \langle\sigma v\rangle} \sqrt{\frac{\kappa}{10 g_\star(T_{\rm FO}}} x_{\rm FO}^{3/2},$$

$$\Omega_{\rm III} = \frac{\pi}{2} \sqrt{\frac{g_\star(T_{\rm FO})}{10}} \frac{m_{\rm DM}^2}{m_p \langle\sigma v\rangle} \kappa \left(\frac{m_{\rm DM} T_{\rm ev}}{T_{\rm FO}^2}\right)^2,$$

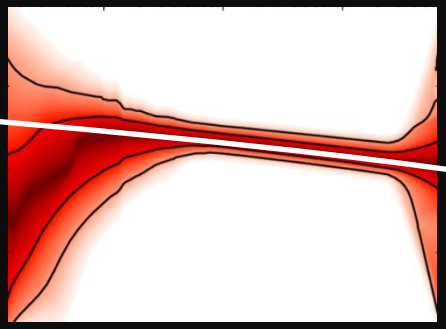
$$\Omega_{\rm IV} = \frac{15}{2\pi} \frac{x_{\rm FO}}{\sqrt{10 g_\star(T_{\rm FO})}} \frac{s_0}{m_p \langle\sigma v\rangle \rho_c},$$

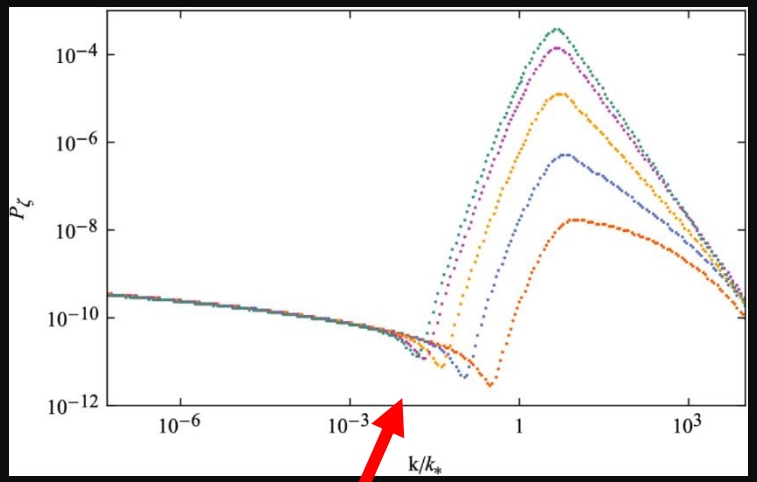
Comparison with Numerics



PBH Formation

Planck



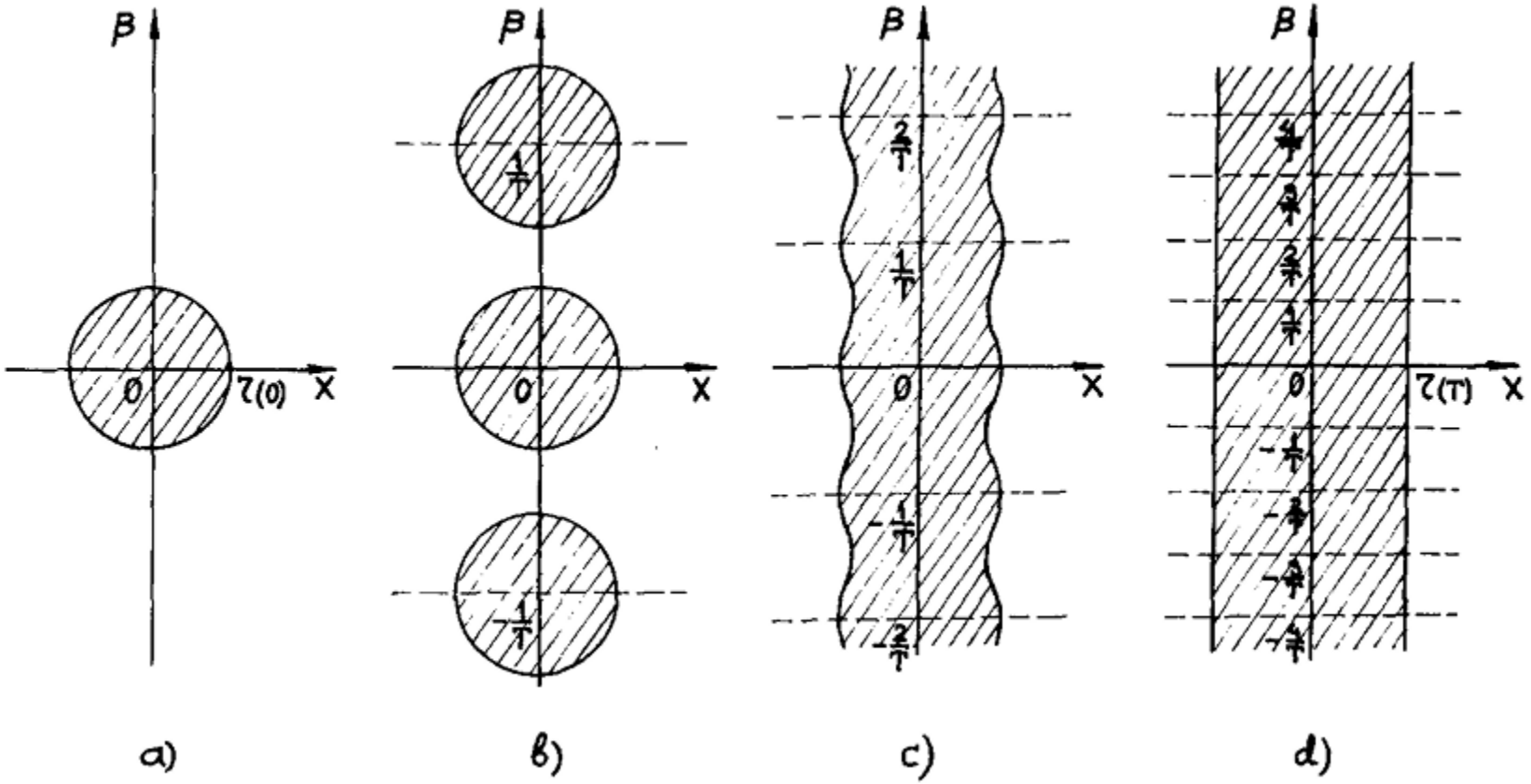


Ultra
Slow-Roll

[LH, Moursy, Wacquez '22]

$$W = \left(1 - \frac{S}{\sqrt{3}}\right)^3 f(Z)$$

$$K = K_1\left(Z, \overline{Z}\right) - 3\log\left[1 - \frac{|S|^2}{3} + \frac{|S|^4}{\Lambda^2}\right]$$


β
0
τ(0)
x
a)
β
1/T
0
x
-1/T
b)
β
2/T
1/T
0
x
-1/T
-2/T
c)
β
4/T
3/T
2/T
1/T
0
τ(T)
x
-1/T
-2/T
-3/T
-4/T
d)

Perturbation Horizon Crossing

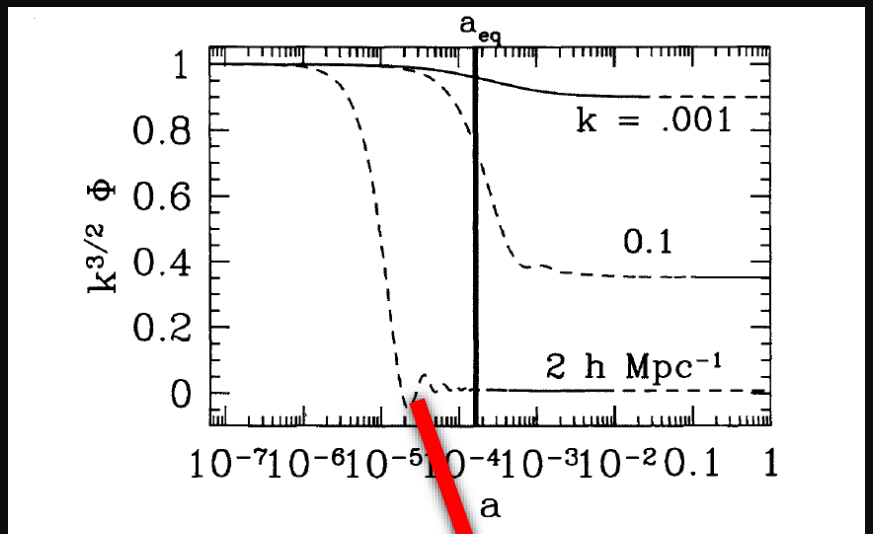


Figure 7.2. The linear evolution of the gravitional potential Φ. Dashed line denotes that the mode has entered the horizon. Evolution through the shaded region is described by the transfer function. The potential is unnormalized, but the relative normalization of the three modes is as it would be for scale-invariant perturbations. Here baryons have been neglected, $\Omega_m = 1$, and $h = 0.5$.

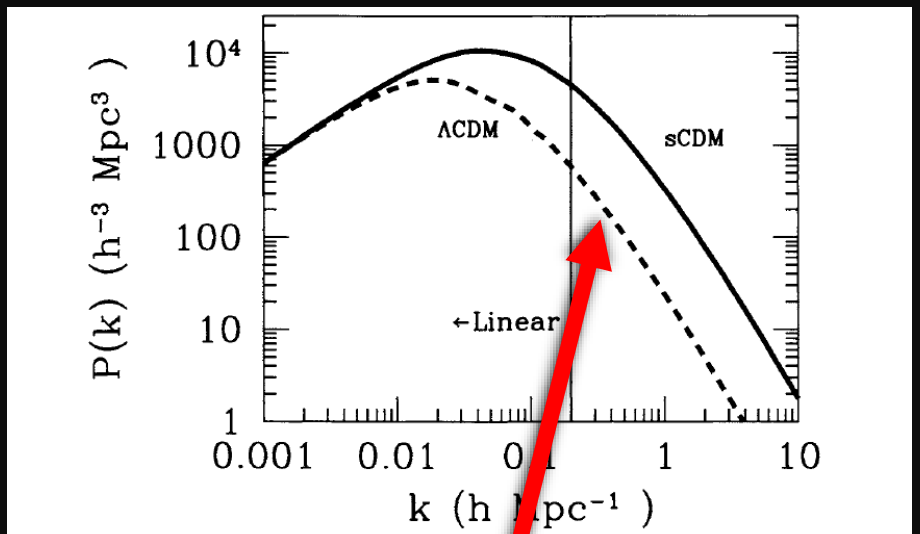
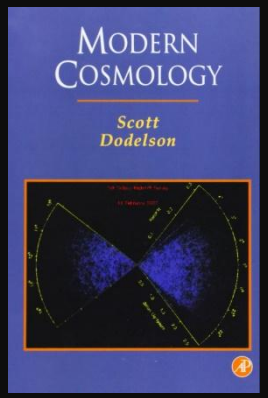


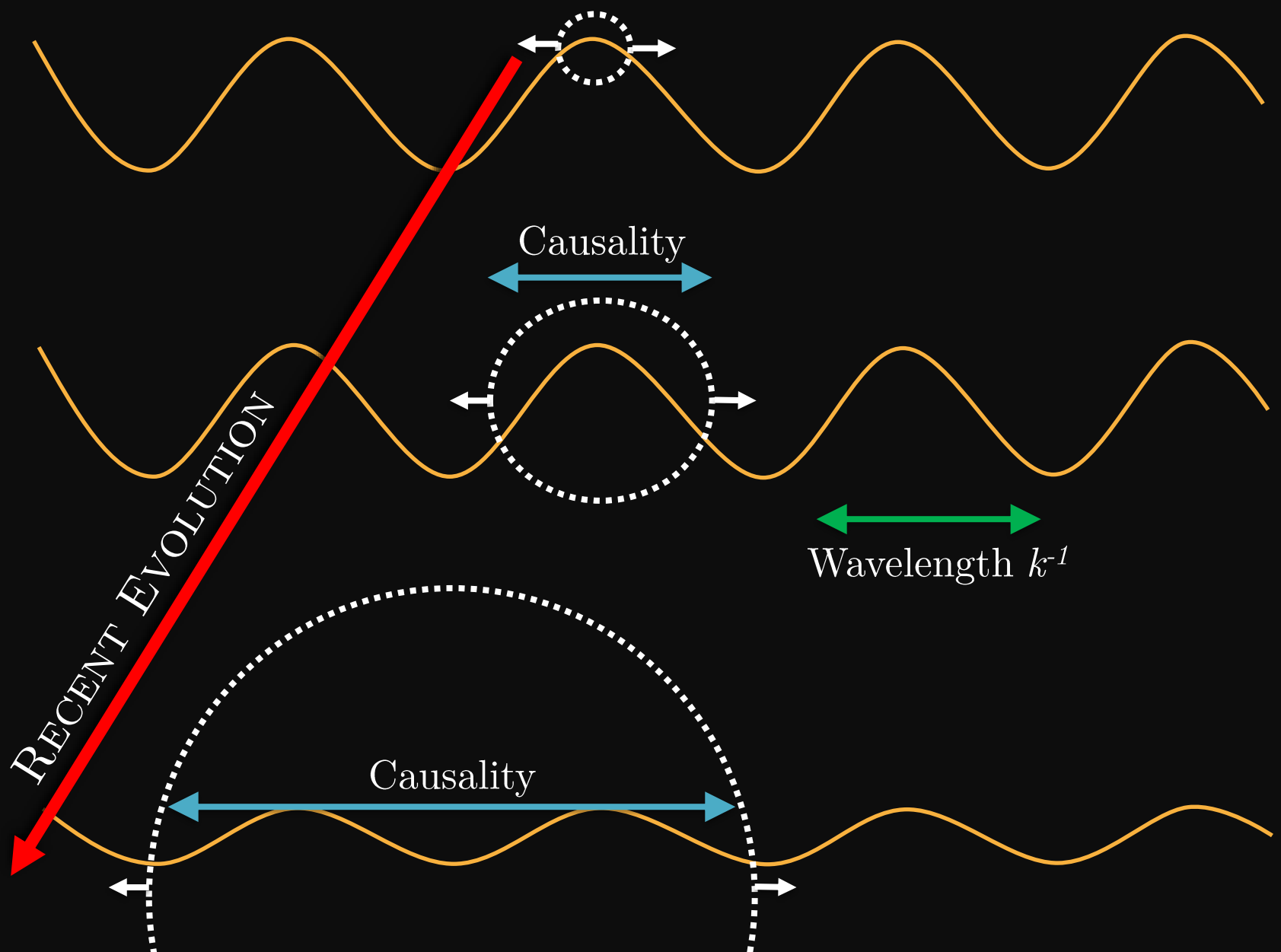
Figure 7.4. The power spectrum in two Cold Dark Matter models, with (ΛCDM) and without (sCDM) a cosmological constant. The spectra have been normalized to agree on large scales. The spectrum in the cosmological constant model turns over on larger scales because of a later $a_{\rm eq}$. Scales to the left of the vertical line are still evolving linearly.



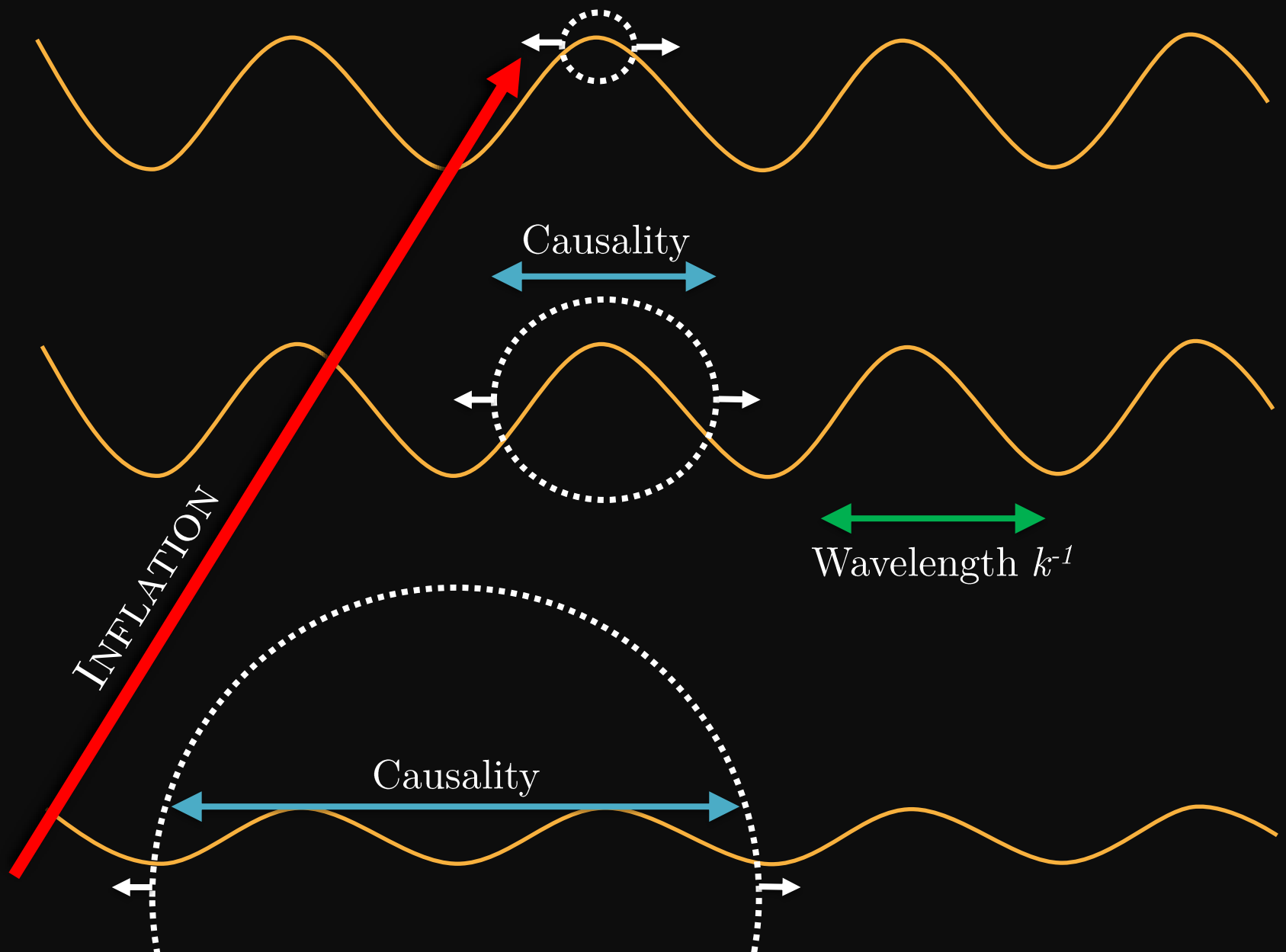
Modes entering the horizon BEFORE matter-radiation equality DECAY...

Causality erases small-scale structures

Primordial Perturbation Evolution



Primordial Perturbation Evolution



Kerr PBHs and Dark Radiation

In the Standard Model

$$\rho_{\rm R}^{\rm SM} = \rho_\gamma \left[1 + \frac{7}{8} \left(\frac{T_\nu}{T_\gamma} \right) N_{\rm eff}^{\rm SM} \right] ,$$

$$T_\nu = (4/11)^{1/3} T_\gamma$$

In the presence of Dark Radiation

$$\rho_{\rm R} \equiv \rho_\gamma \left[1 + \frac{7}{8} \left(\frac{T_\nu}{T_\gamma} \right) (N_{\rm eff}^{\rm SM} + \Delta N_{\rm eff}) \right]$$

$$\Delta N_{\rm eff} = \left\{ \frac{8}{7} \left(\frac{4}{11} \right)^{-\frac{4}{3}} + N_{\rm eff}^{\rm SM} \right\} \frac{\rho_{\rm DR}(T_{\rm ev})}{\rho_{\rm R}^{\rm SM}(T_{\rm ev})} \left(\frac{g_*(T_{\rm ev})}{g_*(T_{\rm eq})} \right) \left(\frac{g_{*S}(T_{\rm eq})}{g_{*S}(T_{\rm ev})} \right)^{\frac{4}{3}}$$

The quantity to evaluate

Kerr PBHs and Dark Radiation

Why ?

$$\frac{\mathrm{d}\mathcal{N}_{\mathrm{DM}}}{\mathrm{d}p} = \int_0^{\tau} \mathrm{d}t' \frac{a(\tau)}{a(t')} \times \frac{\mathrm{d}^2\mathcal{N}_{\mathrm{DM}}}{\mathrm{d}p'\mathrm{d}t'} \left(p\frac{a(\tau)}{a(t')}, t' \right)$$

some redshift is good

[Cheek, LH, Perez-Gonzalez and Turner '22] **Code Publicly available: FRISBHEE**

Kerr PBHs and Dark Radiation

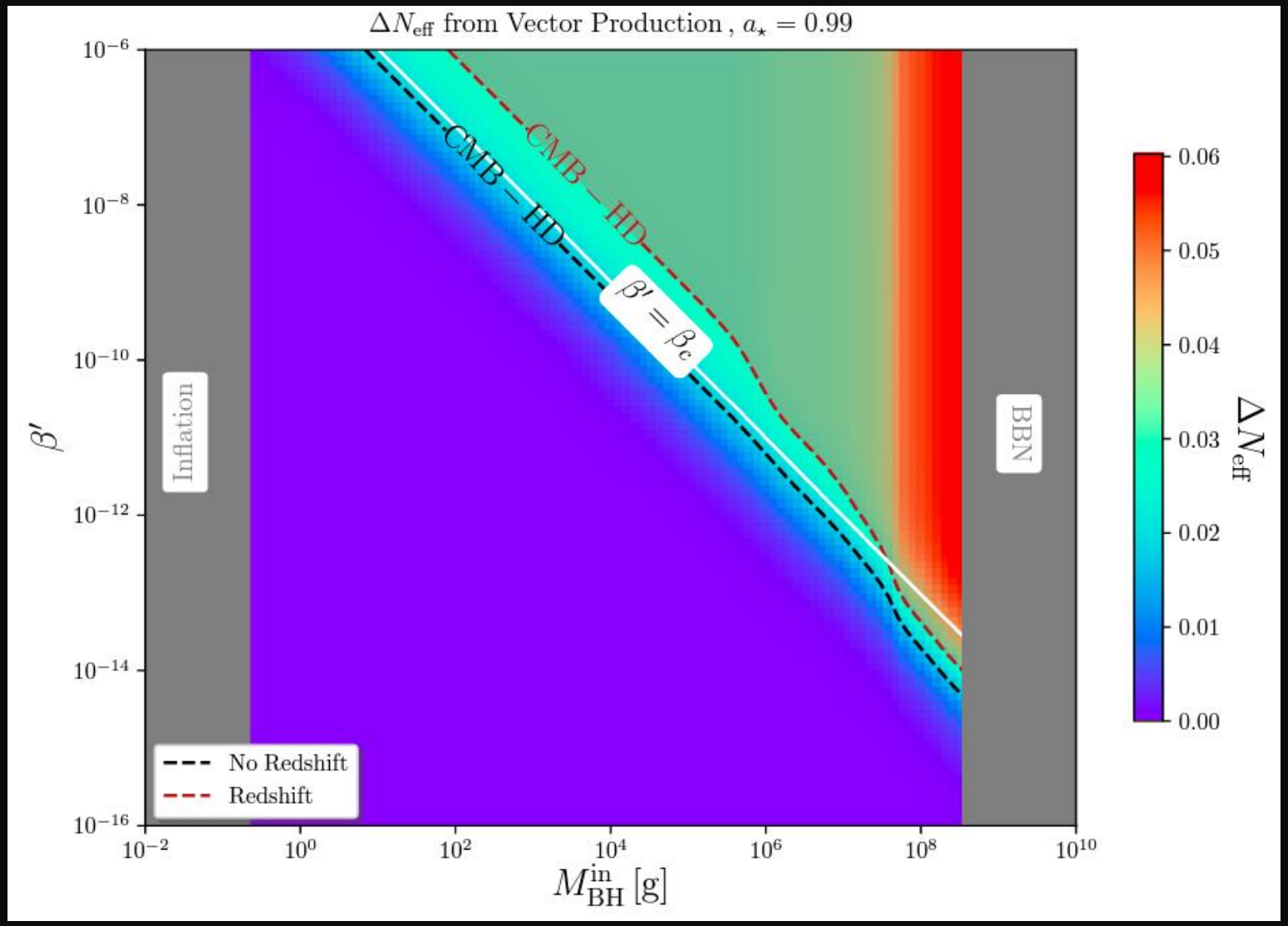
Why ?

$$\frac{\mathrm{d}\mathcal{N}_{\mathrm{DM}}}{\mathrm{d}p} = \int_0^{\tau} \mathrm{d}t' \frac{a(\tau)}{a(t')} \times \frac{\mathrm{d}^2\mathcal{N}_{\mathrm{DM}}}{\mathrm{d}p'\mathrm{d}t'} \left(p\frac{a(\tau)}{a(t')}, t' \right)$$

The correct one is better!

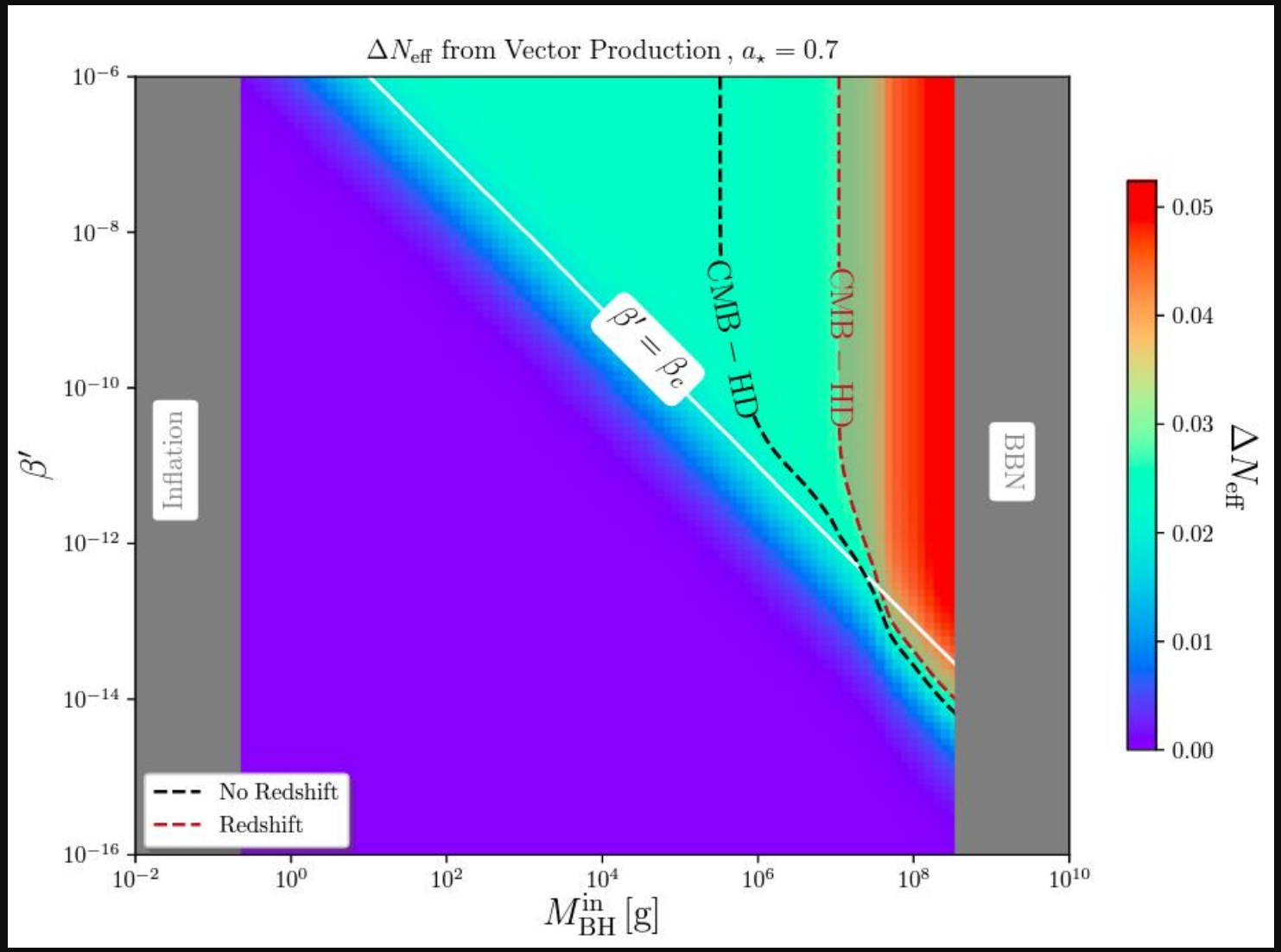
[Cheek, LH, Perez-Gonzalez and Turner '22] **Code Publicly available: FRISBHEE**

Kerr PBHs and Dark Radiation



[Cheek, LH, Perez-Gonzalez and Turner '22] **Code Publicly available: FRISBHEE**

Kerr PBHs and Dark Radiation



[Cheek, LH, Perez-Gonzalez and Turner '22] **Code Publicly available: FRISBHEE**

III. Evaporation of Extended Distributions

[D. N. Page, Phys. Rev. D 14, 3260 (1976)]

$$\frac{dM_{\rm BH}}{dt} = -\epsilon(M_{\rm BH}, a_\star)\frac{M_p^4}{M_{\rm BH}^2}\,,$$

$$\frac{da_\star}{dt} = -a_\star[\gamma(M_{\rm BH}, a_\star) - 2\epsilon(M_{\rm BH}, a_\star)]\frac{M_p^4}{M_{\rm BH}^3}\,,$$

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \frac{a}{M^3}\left[2f(a) - g(a)\right]\,,$$

$$\frac{\mathrm{d}M}{\mathrm{d}t} = -\frac{f(a)}{M^2}\,.$$

$$y \equiv -\ln(a)$$

$$z \equiv -\ln\left(\frac{M}{M_1}\right)$$

$$\tau \equiv M_1^{-3}t$$

Generic solution(z, τ) for any M_1

$$\frac{\mathrm{d}z}{\mathrm{d}y} = \frac{f(a)}{g(a) - 2f(a)}\,,$$

$$\frac{\mathrm{d}\tau}{\mathrm{d}y} = \left(\frac{M}{M_1}\right)^3\frac{1}{g(a) - 2f(a)}\,,$$

$$M = M_i e^{z_i - z}\,,$$

$$(t - t_i) = M_i^3 e^{3z_i}(\tau - \tau_i)\,,$$

III. Evaporation of Extended Distributions

Examples:

$$\frac{\mathrm{d}n}{\mathrm{d}M} \propto \frac{1}{M^2} \exp\left[-\frac{(\log M - \log M_c)^2}{2\,\sigma^2}\right]$$

Evaporation smeared around $\tau(M_c)$

$$\frac{\mathrm{d}n}{\mathrm{d}M} \propto M^{-\alpha} \quad \text{with} \quad \alpha = \frac{2\,(1+2\,w)}{1+w}$$

Regime of 'Cosmological Stasis'

Results

Freeze-Out [Cheek, LH, Perez-Gonzalez and Turner '22]

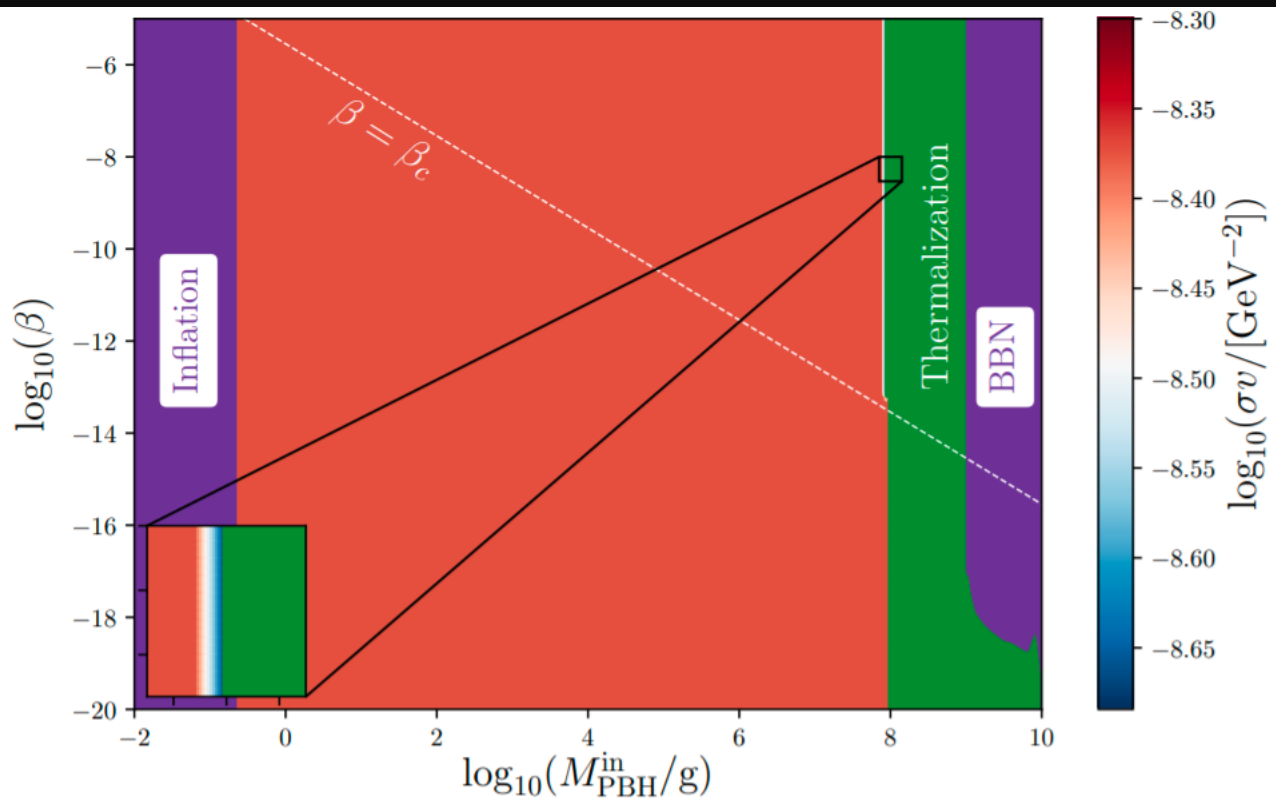


Fig. 7. Two-dimensional scan over the PBH fraction β and mass $M_{\rm BH}$ for a mediator mass $m_X = 10\,{\rm GeV}$ and a dark matter mass $m_{\rm DM} = 1\,{\rm GeV}$, and ${\rm Br}(X \to {\rm DM}) = 0.5$. The color map indicates the value of the non-relativistic cross-section of DM annihilation leading to the correct relic abundance in the Freeze-Out case. See the main text for a description of the different constraints.

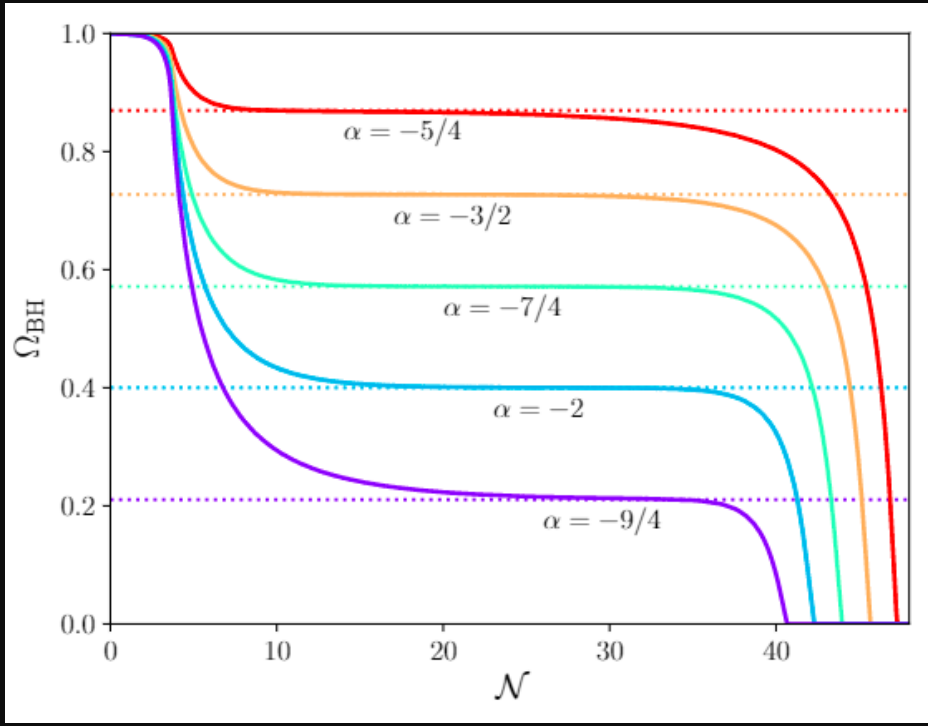
III. Evaporation of Extended Distributions

$$f_{\rm BH}(M) = \begin{cases} CM^{\alpha-1}, & \text{for } M_{\rm min} \leq M \leq M_{\rm max}; \\ 0, & \text{else.} \end{cases}$$

$$\alpha \equiv \frac{(-3w_{\rm form.} - 1)}{(w_{\rm form.} + 1)}$$

$$\frac{{\rm d}H}{{\rm d}t} = -\frac{1}{2}H^2(4 - \Omega_{\rm BH}),$$

$$\frac{{\rm d}\Omega_{\rm BH}}{{\rm d}t} = \Omega_{\rm BH}\left[\frac{\int_0^\infty f_{\rm BH}(M,t)\frac{{\rm d}M}{{\rm d}t}{\rm d}M}{\int_0^\infty f_{\rm BH}(M,t)M{\rm d}M}\right] + H\Omega_{\rm BH}(1 - \Omega_{\rm BH}).$$



$$= \frac{1+\alpha}{3(t - t_i)}$$

[Dienes, LH, Huang, Kim, Tait, Thomas '22]

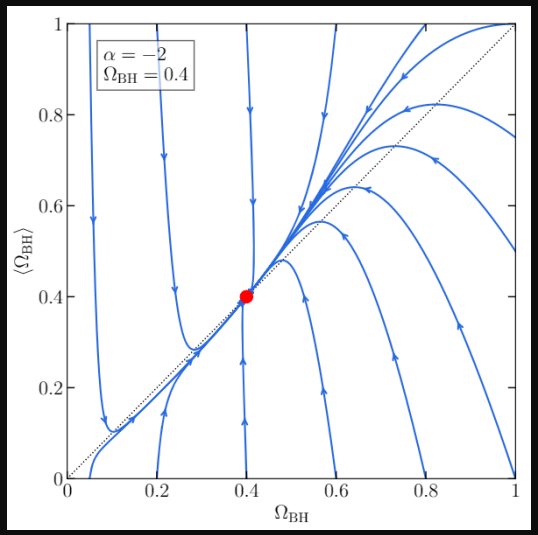
III. Evaporation of Extended Distributions

$$\begin{cases} \dfrac{\mathrm{d}\Omega_{\mathrm{BH}}}{\mathrm{d}t} &= \dfrac{1}{t-t_i}\, f(\Omega_{\mathrm{BH}}, \langle\Omega_{\mathrm{BH}}\rangle) \\ \dfrac{\mathrm{d}\langle\Omega_{\mathrm{BH}}\rangle}{\mathrm{d}t} &= \dfrac{1}{t-t_i}\, g(\Omega_{\mathrm{BH}}, \langle\Omega_{\mathrm{BH}}\rangle)\ , \end{cases}$$

$$f(\Omega_{\mathrm{BH}}, \langle\Omega_{\mathrm{BH}}\rangle) \equiv \Omega_{\mathrm{BH}}\left[\frac{1+\alpha}{3} + \frac{2(1-\Omega_{\mathrm{BH}})}{4-\langle\Omega_{\mathrm{BH}}\rangle}\right]$$

$$g(\Omega_{\mathrm{BH}}, \langle\Omega_{\mathrm{BH}}\rangle) \equiv \Omega_{\mathrm{BH}} - \langle\Omega_{\mathrm{BH}}\rangle\,,$$

$$\langle\Omega_{\mathrm{BH}}\rangle \equiv \frac{1}{t-t_i}\int_{t_i}^{t} dt'\,\Omega_{\mathrm{BH}}(t')\ .$$



$$w_{\mathrm{eff}} = \frac{-\alpha-1}{\alpha+7}$$

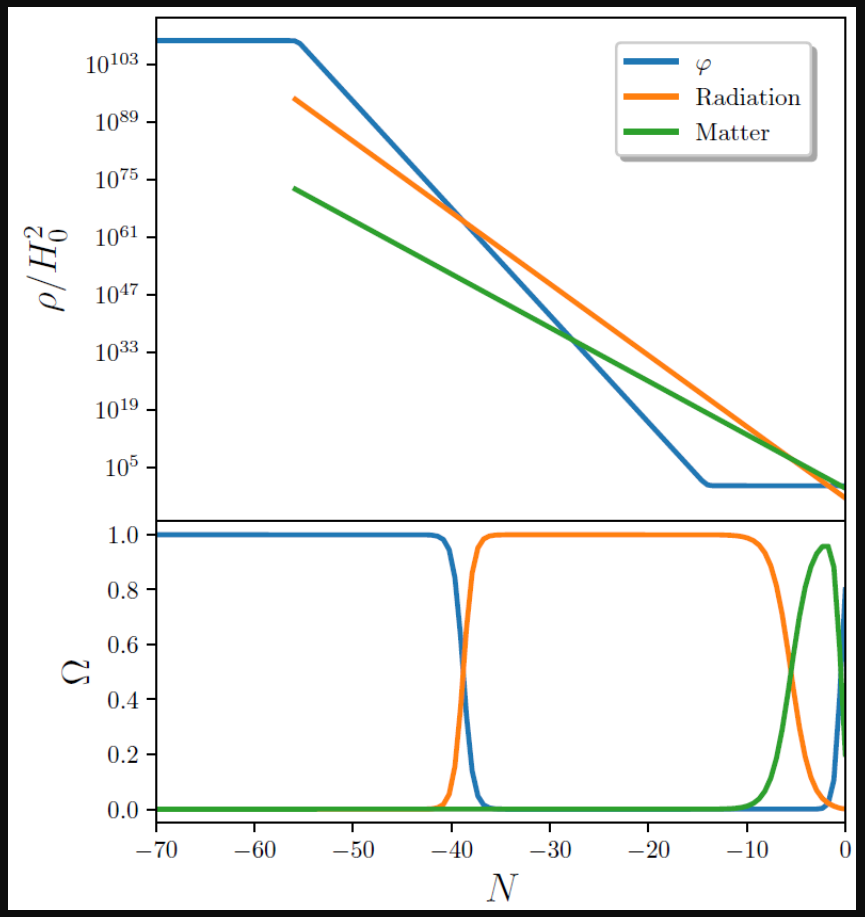
$$\Omega_{\mathrm{BH}} = \langle\Omega_{\mathrm{BH}}\rangle = \frac{4\alpha+10}{\alpha+7} \equiv \overline{\Omega}_{\mathrm{BH}}\,.$$

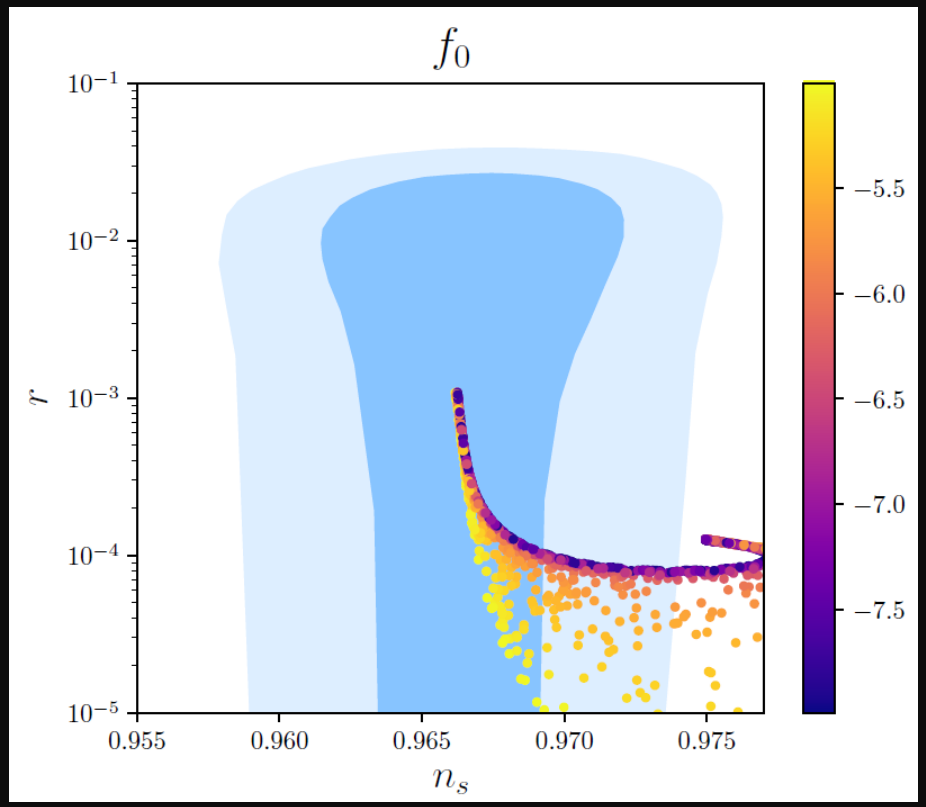
$$\mathcal{J} \equiv \frac{1}{t-t_i}\begin{pmatrix} \partial_{\Omega_{\mathrm{BH}}} f & \partial_{\langle\Omega_{\mathrm{BH}}\rangle} f \\ \partial_{\Omega_{\mathrm{BH}}} g & \partial_{\langle\Omega_{\mathrm{BH}}\rangle} g \end{pmatrix}$$

$$\lambda_{\pm} = \frac{1}{18}\left(-4\alpha \pm \sqrt{-19-4\alpha(2\alpha+19)-59}\right)$$

[Dienes, LH, Huang, Kim, Tait, Thomas '22] [Copeland, Liddle, Barrow '91]

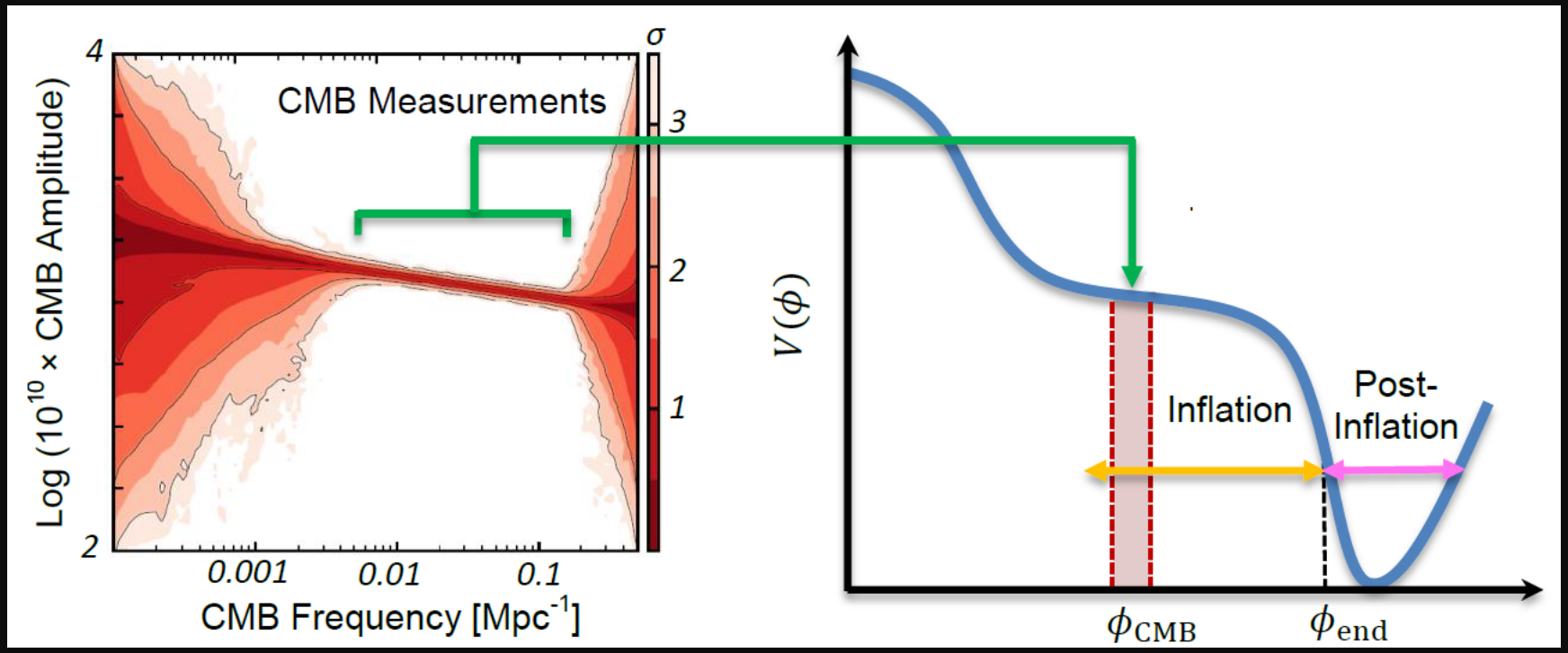
PBH Formation





[LH, Moursy, Wacquez '22]

Primordial Perturbation Evolution



Perturbation Horizon Crossing

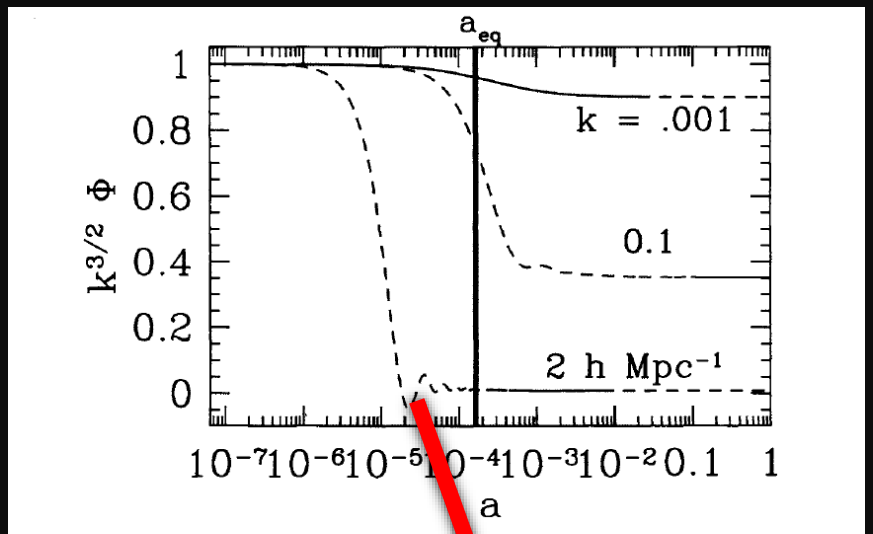


Figure 7.2. The linear evolution of the gravitional potential Φ. Dashed line denotes that the mode has entered the horizon. Evolution through the shaded region is described by the transfer function. The potential is unnormalized, but the relative normalization of the three modes is as it would be for scale-invariant perturbations. Here baryons have been neglected, $\Omega_m = 1$, and $h = 0.5$.

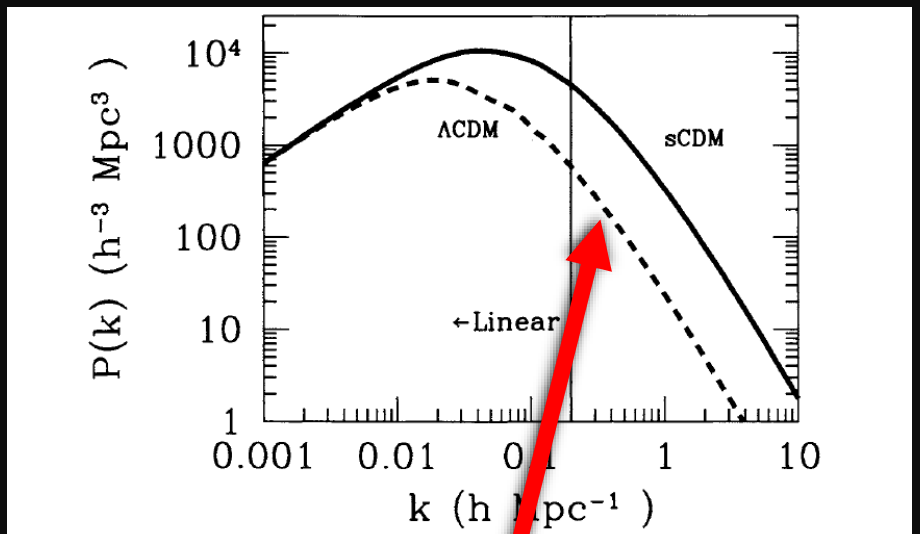
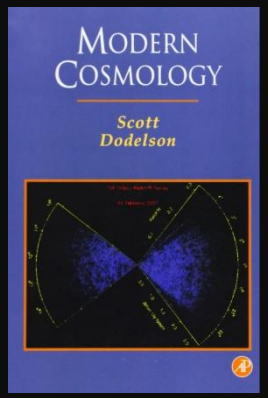


Figure 7.4. The power spectrum in two Cold Dark Matter models, with (ΛCDM) and without (sCDM) a cosmological constant. The spectra have been normalized to agree on large scales. The spectrum in the cosmological constant model turns over on larger scales because of a later $a_{\rm eq}$. Scales to the left of the vertical line are still evolving linearly.



Modes entering the horizon BEFORE matter-radiation equality DECAY...

Causality erases small-scale structures

An example: The EW vacuum

Our Universe may be metastable (at ~2σ)

$$P_{\rm FVD} \equiv 1 - e^{-\Gamma_{\rm FVD}\Delta t}$$

Using $T_{plateau}(M)$

$$\Delta t \sim \Gamma_{\rm ev}^{-1}$$

$$P_{\rm FVD}(M) = 1 - e^{-\Gamma_{\rm FVD}(T_{\rm plateau})/\Gamma_{\rm ev}}$$

Using T_{max}

$$P_{\rm FVD} \approx 1 \text{ as long as } \Delta t \lesssim 10^{-6} \times \Gamma_{\rm ev}^{-1}$$

Constraint:

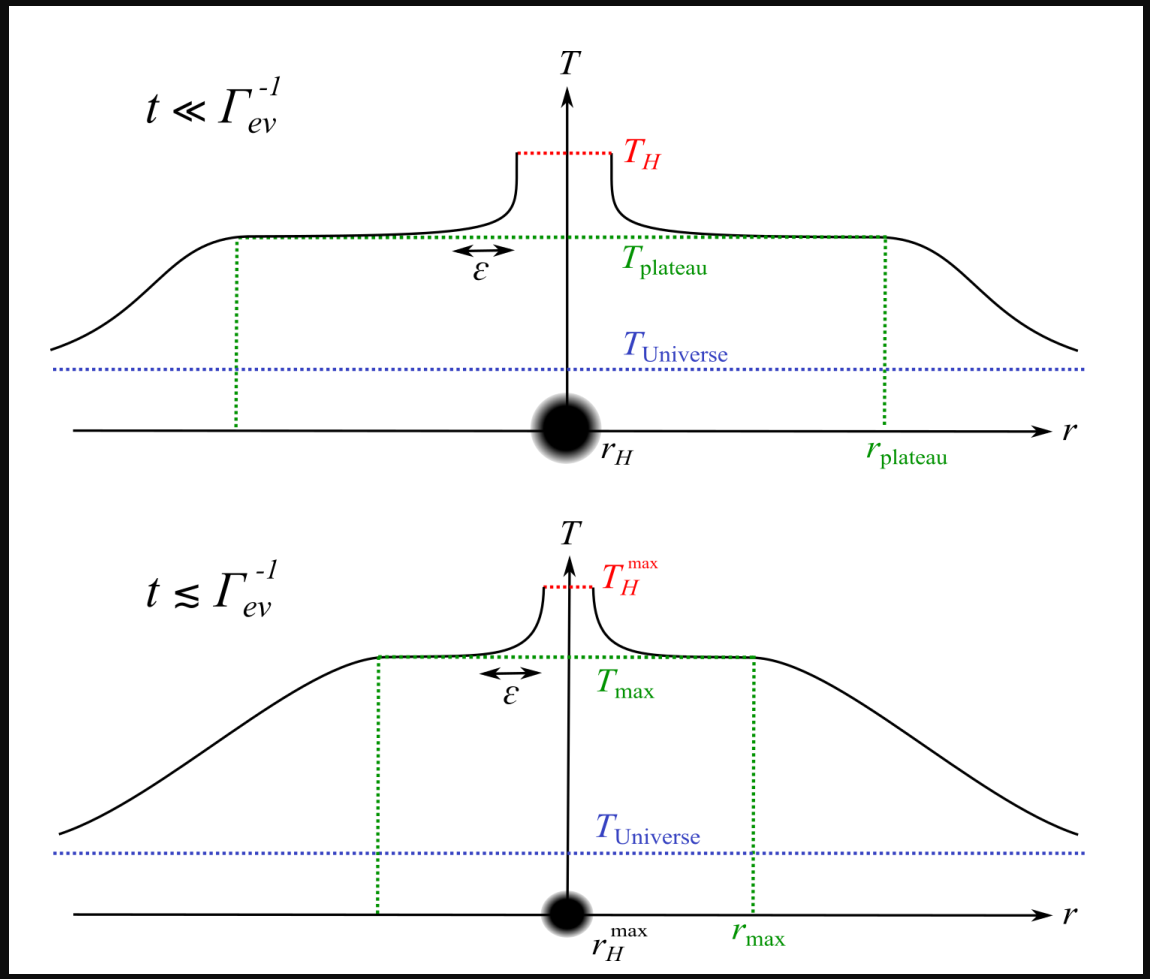
$$\beta_{\rm PBH} = \frac{4}{3}\frac{M N_{\rm PBH} H_0^3}{s_0 T_f} \approx 2 \times 10^{-80} N_{\rm PBH} \left(\frac{M}{M_\star}\right)^{3/2}$$

$$N_{\rm PBH}\, P_d < 2.7$$

[Hamaide, Heurtier, Hu, Cheek, to appear]

In Reality

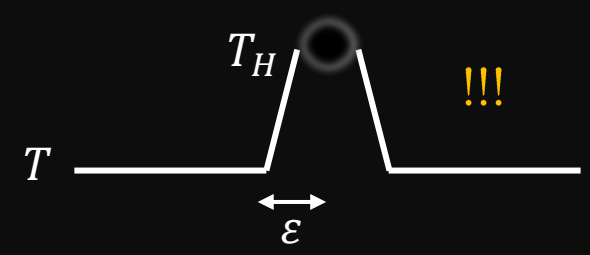
Hawking Radiation heats the ambient plasma locally
Our best guess:



$$\Gamma(T) \sim \alpha^2 T \sqrt{\frac{T}{T_H}} \, .$$

$$dP \sim \Gamma(T) e^{-(r-r_H)\Gamma(T)} dr$$

Physical realisation of



[Hamaide, Heurtier, Hu, Cheek, to appear]