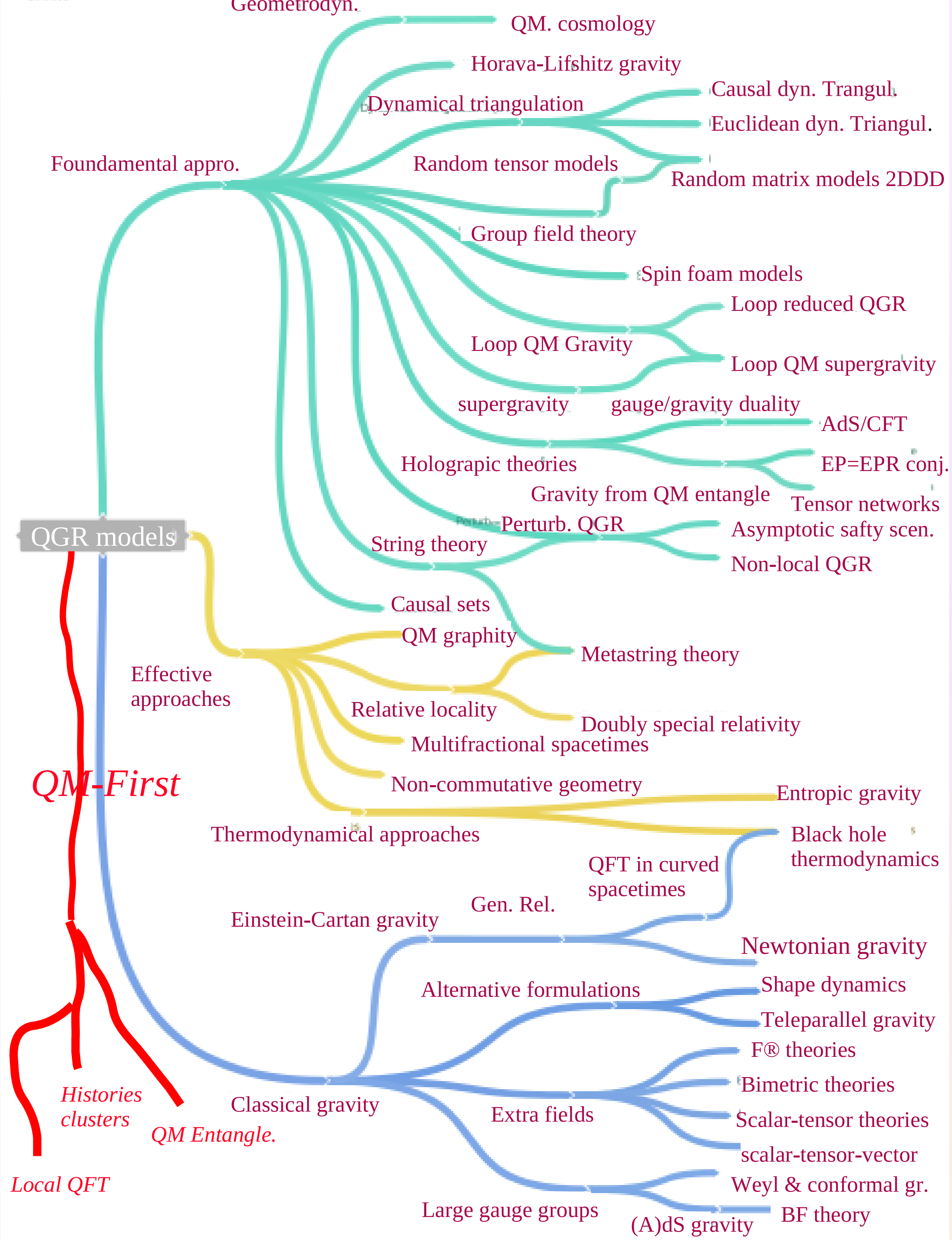
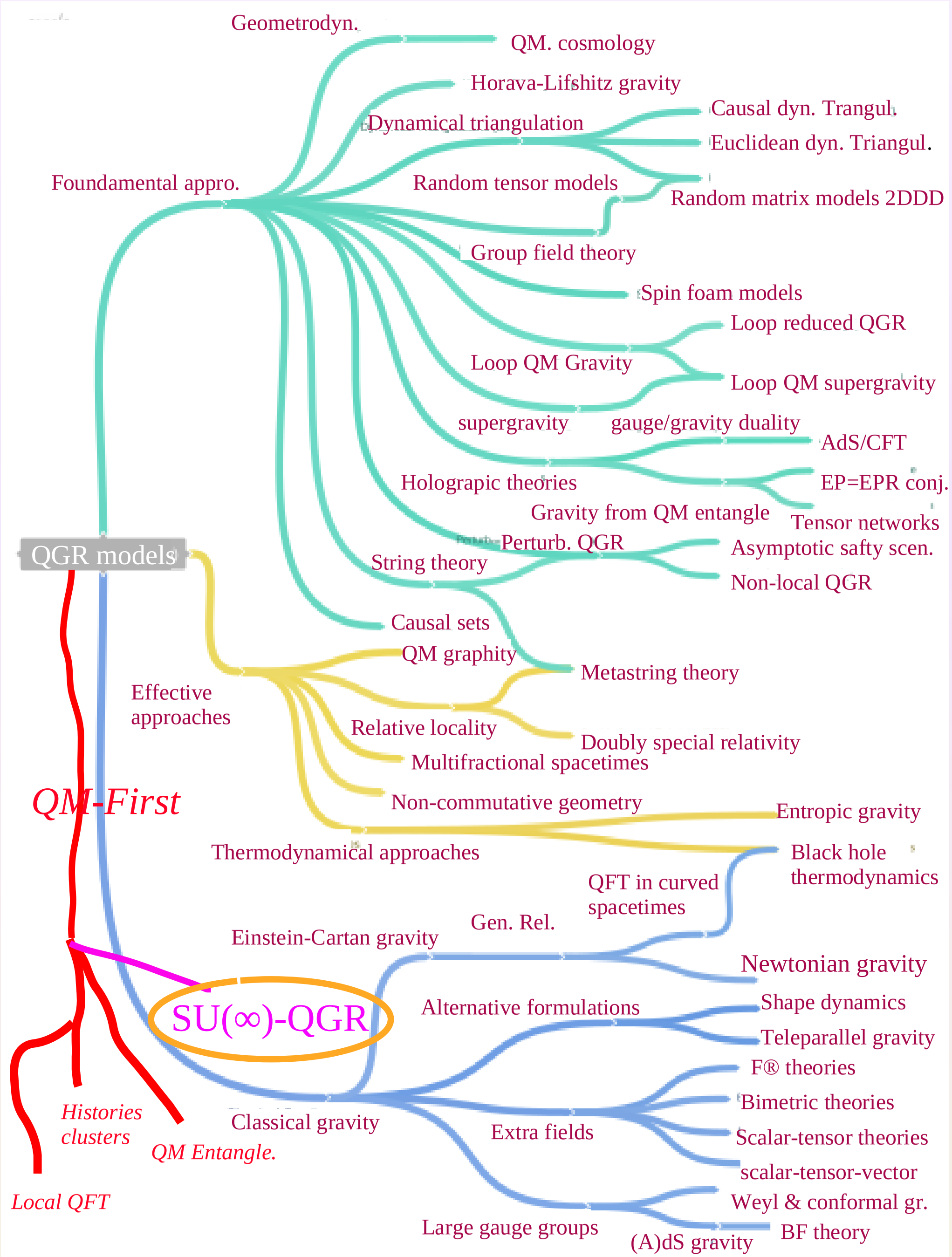


[Mielczarek & Trześniewski 18]

Prospects for strong interactions and PBH formation in $SU(\infty)$-QGR



Based on:

arXiv:2009.03428

arXiv:2304.02761

arXiv:2402.18237

arXiv:2409.08932 (review)

Houri Ziaeepour **NEHOP25, Brussels, 19-22 May 2025**

Contents

- ⋆ Review of $SU(\infty)$-QGR

- ⋆ Prospects for emergence of non-Gaussian, non-thermal, and non-local features

Axioms of $SU(\infty)$-QGR

- ⋆ Canonical and field theoretical approaches to QGR have failed.
- ⋆ Give up the spacetime and go full Quantum !
- ⋆ **Construct a quantum Universe based on 3 well motivated assumptions:**

1. **Quantum Mechanics (QM) is valid at all scales and applies to everything;**
2. **Any quantum system is exclusively specified by a set of symmetries and its Hilbert space represents them;**
3. **The Universe has infinite number of independent - commuting - observables.**

Axioms of $SU(\infty)$-QGR

- ⋆ Canonical and field theoretical approaches to QGR have failed.
- ⋆ Give up the spacetime and go full Quantum !
- ⋆ **Construct a quantum Universe based on 3 well motivated assumptions:**

1. **Quantum Mechanics (QM) is valid at all scales and applies to everything;**
2. **Any quantum system is exclusively specified by a set of symmetries and its Hilbert space represents them;**
3. **The Universe has infinite number of independent - commuting - observables.**

- ⋆ Axiom 1 establishes the theoretical framework of the model.
- ⋆ Axiom 2 is not, strictly speaking, necessary, because postulates of QM can be reformulated with symmetry as a foundational concept. [HZ 13].
- ⋆ **Axiom 3 is the only thing we need for constructing a quantum Universe !**

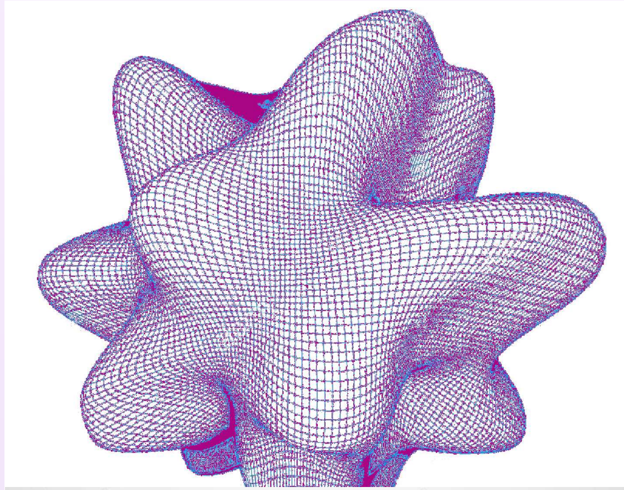
Hilbert space of the Universe and its algebra

- Number of independent and commuting observables = Dimension of Cartan subspace. Thus, Axiom 3 imposes:
  - Hilbert space of the Universe $\mathcal{H}_U$ is infinite dimensional and represents $SU(\infty)$ group.
  - Space of (bounded) linear operators $\mathcal{B}[\mathcal{H}_u] \cong SU(\infty)$ [Hoppe, *et al.*82, 89, 90; Floratos & Iliopoulos 89a; etc]
  - Its algebra can be normalized such that: $[\hat{L}_a, \hat{L}_b] = \frac{i\hbar}{M_P} f^c_{ab} \hat{L}_c$
  - If $M_P \to \infty$ or $\hbar \to 0 \Longrightarrow [\hat{L}_a, \hat{L}_b] = 0,\ \mathcal{B}[\mathcal{H}_u] \cong \bigotimes_{N\to\infty} U(1)$.

Hilbert space of the Universe and its algebra

- ⋆ Number of independent and commuting observables = Dimension of Cartan subspace. Thus, Axiom 3 imposes:
  - ⋆ Hilbert space of the Universe $\mathcal{H}_U$ is infinite dimensional and represents $SU(\infty)$ group.
  - ⋆ Space of (bounded) linear operators $\mathcal{B}[\mathcal{H}_u] \cong SU(\infty)$ [Hoppe, *et al.*82, 89, 90; Floratos & Iliopoulos 89a; etc]
  - ⋆ Its algebra can be normalized such that: $[\hat{L}_a, \hat{L}_b] = \frac{i\hbar}{M_P} f^c_{ab} \hat{L}_c$
  - ⋆ If $M_P \to \infty$ or $\hbar \to 0 \Longrightarrow [\hat{L}_a, \hat{L}_b] = 0,\ \mathcal{B}[\mathcal{H}_u] \cong \bigotimes_{N\to\infty} U(1)$.
- ⋆ **Quantization:**
  - ⋆ The noncommutative algebra of observables can replace quantization. [A. Connes 96]
  - ⋆ Only observables with unbounded spectrum satisfy Heisenberg uncertainty relation. [B.C. Hall, 13]
  - ⋆ In $SU(\infty)$-QGR it is possible to find $\hat{J}_b \in \mathcal{B}[\mathcal{H}^*_U] \cong SU(\infty)$ such that: $[\hat{L}_a, \hat{J}_b] = -i\delta_{ab}\hbar$

Representations of $SU(\infty)$



⋆ $SU(\infty)$ generators depend on 2 continuous parameters and vectors of the Hilbert space are complex-valued functions $v(\eta, \zeta) \in \mathcal{H}_U$.

⋆ $SU(\infty) \cong$ Area-preserving diffeomorphism of 2D Riemann surfaces, called diffeo-surfaces. [Hoppe, *et al.*82, 89, 90; Zunger 01]

⋆ $\mathcal{SU}(\infty)$ is homomorphic to Poisson bracket algebra:

$$\hat{L}_f = \frac{\partial f}{\partial \eta}\frac{\partial}{\partial \zeta} - \frac{\partial f}{\partial \zeta}\frac{\partial}{\partial \eta} \quad , \quad \hat{L}_f \, g = \{f, g\}$$

⋆ **Spherical basis:** $f \equiv Y_{lm}$, $\hat{L}_f = \hat{L}_{lm}$;

⋆ **Torus basis:** $f \equiv \exp[i(m_1 \eta + m_2 \zeta)]$, $\hat{L}_f = \hat{L}_{m \equiv (m_1, m_2)}$;

⋆ **Tensor prod.:** $SU(\infty) \cong \otimes^{\infty} SU(K)$, $K < \infty$, $SU(\infty)^n \cong SU(\infty)$ [Zunger 01]

Universe as a whole is static !

- ⋆ Consistent definition of dynamics in QM is relative to another QM system - **A QM clock.** [Page & Wootters 83]

- ⋆ There is nothing outside the Universe $\Longrightarrow$ **The Universe is globally static !**

- ⋆ A $SU(\infty)$ and reparametrization invariant functional has the form of **2D Yang-Mills** gauge theory:

$$\mathcal{L}_U = \int d^2\Omega \left[\frac{\kappa}{2} \mathrm{tr}(F^{\mu\nu} F_{\mu\nu}) + \frac{1}{2} \mathrm{tr}(\not{D}\rho) \right], \qquad \mu, \nu \in \cos\theta, \phi$$

$$F_{\mu\nu} \equiv F^a_{\mu\nu} \hat{L}^a \equiv [D_\mu, D_\nu], \quad D_\mu = (\partial_\mu - \Gamma_\mu)\mathbb{1} - \sum_a A^a_\mu \hat{L}^a,, \quad F^a_{\mu\nu} F^{\mu\nu}_a = L^*_a L^a$$

- ⋆ Variation of L^a's or $F_{\mu\nu}$ can be compensated by diffeomorphism of the diffeo-surface, which is equivalent to a $SU(\infty)$ gauge transformation, up to an unobservable scaling.

- ⋆ **Field amplitudes $F^a_{\mu\nu}$ are arbitrary, but cannot be all zero.**

Topology matters globally, but locality dominates

- ⋆ The Lagrangian $\mathcal{L}_U$ is topological and proportional to the Euler characteristic:

$$\int d^2\Omega \, \mathrm{tr}(F^{\mu\nu} F_{\mu\nu}) \propto \int d^2\Omega \, \mathcal{R}^{(2)} = 4\pi\chi(\mathcal{M})$$

- ⋆ Topological nature of the Lagrangian $\mathcal{L}_U$ is not surprising, because a single indivisible quantum system is trivial.

- ⋆ The equilibrium solution obtained from Lagrangian $\mathcal{L}_U$ is a trivial vacuum, but unstable under fluctuations.

- ⋆ Quantum fluctuations reduce coherence of an initially fully coherent state $|\psi_U\rangle = |\psi_{cc}\rangle$ and state becomes more clustered:

$$\hat{L}_{l_1 m_1}(\theta,\phi)|\psi_{cc}\rangle \equiv \hat{L}_{l_1 m_1}(\theta,\phi)[C \int d^2\Omega' \sum_{\substack{l \geqslant 0, \\ -l \leqslant m \leqslant l}} |\mathcal{Y}_{lm}(\theta',\phi')\rangle] = |\mathbf{g}_{l_1 m_1}(\theta,\phi)\rangle \neq |\psi_{cc}\rangle$$

Topology matters globally, but locality dominates

⋆ The Lagrangian $\mathcal{L}_U$ is topological and proportional to the Euler characteristic:

$$\int d^2\Omega\, \mathrm{tr}(F^{\mu\nu} F_{\mu\nu}) \propto \int d^2\Omega\, \mathcal{R}^{(2)} = 4\pi\chi(\mathcal{M})$$

⋆ Topological nature of the Lagrangian $\mathcal{L}_U$ is not surprising, because a single indivisible quantum system is trivial.

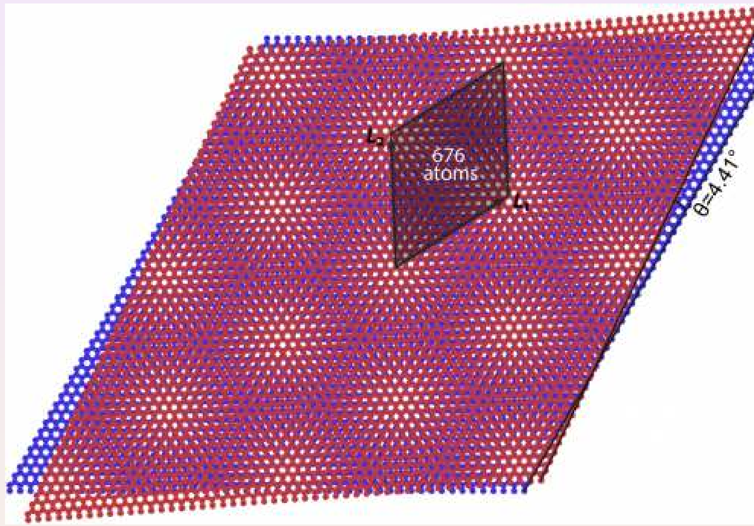
⋆ The equilibrium solution obtained from Lagrangian $\mathcal{L}_U$ is a trivial vacuum, but unstable under fluctuations.

⋆ The Hilbert space can be divided to: $SU(\infty)^2 \cong SU(\infty)$

⋆ Application of an operator to one $SU(\infty)$ is analogous to twisted bilayer graphene.

⋆ **Local finite-rank symmetries arise.**

⋆ One of the Hilbert space factors can be considered as reference and **QM clock.**



Subsystems

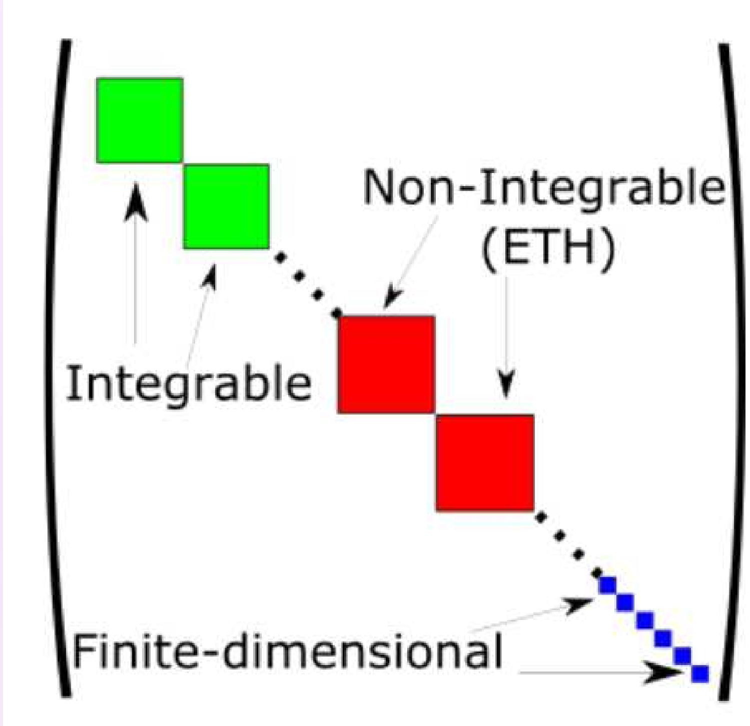
⋆ Clustering of state and Hilbert space fragmentation is observed in closed many body QM systems. [Moudgalaya, *et al.*22] (review)

⋆ Conditions for identifying QM subsystems [Zanardi *et al.*04]:

⋆ Closed subalgebras: $\{A_i\} \subset \mathcal{B}[\mathcal{H}] \Big| \forall \hat{a} \in \{A_i\}, \hat{b} \in \{A_j\}, i \neq j, \ [\hat{a}, \hat{b}] = 0$;

⋆ Complementarity: $\otimes_i \{A_i\} \cong \text{End}(\mathcal{B}[\mathcal{H}])$;

⋆ Locality $\Longrightarrow$ Finite-rank closed subalgebras associated to generic finite-rank group G;



[Moudgalaya, *et al.*22]

⋆ In $SU(\infty)$-QGR due to the global $SU(\infty)$ symmetry these conditions are only approximately satisfied $\Longrightarrow$ No isolated subsystem in the Universe

⋆ **Every subsystem is entangled to the rest of the Universe.** We call this property **the global entanglement.**

Symmetry of subsystems

- **Local reduced density matrix:** Isolate a G factor from $SU(\infty)$ and trace out the rest:

$$\hat{\varrho}_G \equiv \mathrm{tr}_\infty \hat{\varrho}_U = \sum_{\substack{\{k_G, k'_G\},\\ \{y\}}} A_G(k_G; y) A^*_G(k'_G, y)\, \hat{\rho}_G(k_G, k'_G), \quad y \equiv (\eta, \zeta, \cdots)$$

- Due to the global $SU(\infty)$ amplitudes $A_G(k_G; y)$are not factorizable and $\hat{\varrho}_G$ is mixed.

- To a local observer continuous parameters $y \equiv (\eta, \zeta, \cdots)$ look external and **classical**.

- **Purified local state:**

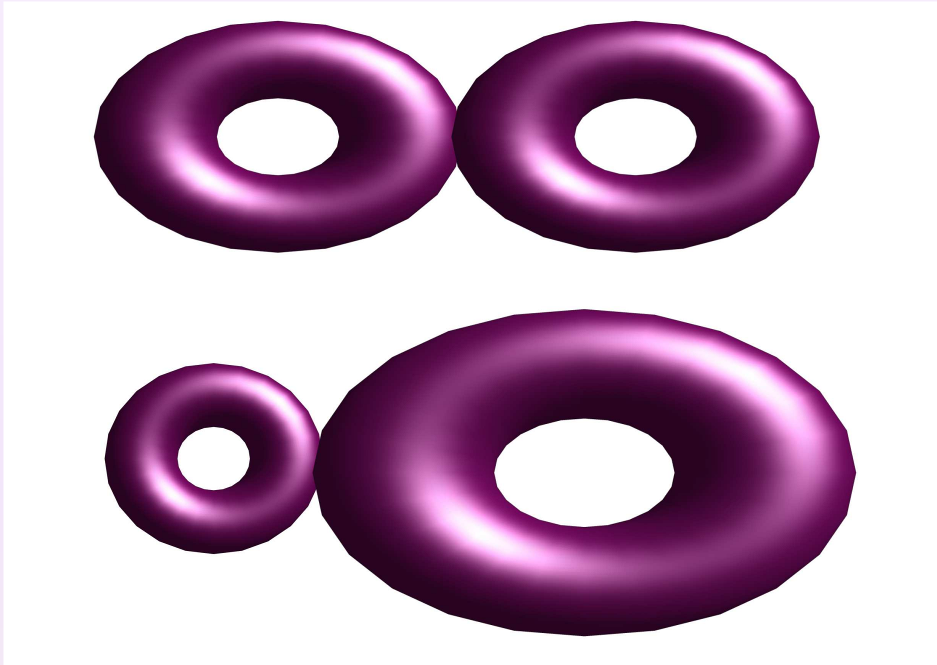
$$|\Psi_{G_\infty}\rangle \equiv \sum_{\{k_G\};(\eta,\zeta,\cdots)\}} A_{G_\infty}(k_G; \eta, \zeta, \cdots)\, |\psi_G(k_G)\rangle \times |\psi_\infty(\eta, \zeta, \cdots)\rangle$$

- **Due to the global entanglement Hilbert space of each subsystem represent $SU(\infty) \times G$.**

- **The common $SU(\infty)$ symmetry is associated to gravity.**

Parameter Space of Subsystems

⋆ Quantum states of emergent subsystems depend on two additional continuous parameters (observables):



⋆ **Area or size of diffeo-surfaces r with respect to an arbitrary subsystem;**

⋆ **A time parameter characterizing state of an arbitrary system chosen as quantum clock.**

⋆ Only area of diffeo-surface of the global $SU(\infty)$ is arbitrary.

⋆ **Full set of continuous parameters:** $y \equiv (t, r, \eta, \zeta)$

⋆ States of subsystems are in general in superposition of these parameters.

Evolution

⋆ At lowest order in number of traces the action has the form of a 4D Yang-Mills action for both $SU(\infty)$ and G symmetries, and is invariant under redefinition parameters: [first studied by Floratos & Illiopoulos 89b]

$$\mathcal{L}_{U_s} = \int d^4x \sqrt{|\eta^{(4)}|}\left[\frac{1}{4}\text{tr}(F^{\mu\nu}F_{\mu\nu}) + \frac{1}{4}\text{tr}(G^{\mu\nu}G_{\mu\nu}) + \frac{\mathcal{M}}{2}\text{tr}(\not{D}\rho_s)\right], \quad \mu,\nu \in 0,1,2,3$$

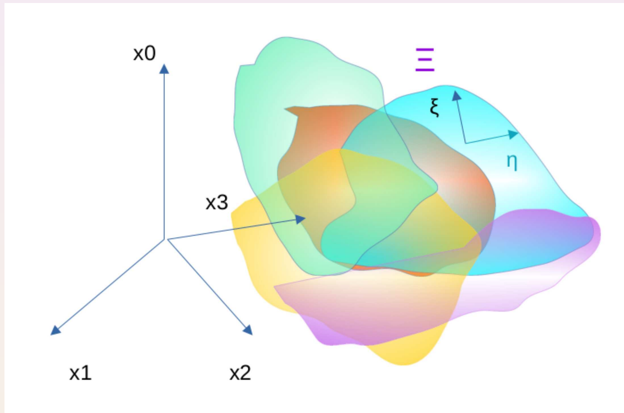
$$F_{\mu\nu} \equiv F^{lm}_{\mu\nu}\hat{L}^{lm} \equiv [D_\mu, D_\nu], \quad D_\mu = \partial_\mu - \Gamma_\mu - i\lambda \sum_{lm} A^{lm}_\mu \hat{L}^{lm}$$

$$G_{\mu\nu} \equiv G^a_{\mu\nu}\hat{T}^a \equiv [D'_\mu, D'_\nu], \quad D'_\mu = \partial_\mu - \Gamma_\mu - i\lambda_G \sum_a B^a_\mu \hat{T}^a$$

Properties of the action $\mathcal{L}_{U_s}$:

⋆ Diffeo-surface of $SU(\infty)$ is embedded in the 4D parameter space.

⋆ This surface was called by *internal space* by Floratos & Illiopoulos 89b. But, in $SU(\infty)$-QGR it is not an independent space.



Evolution

- ⋆ At lowest order in number of traces the action has the form of a 4D Yang-Mills action for both $SU(\infty)$ and G symmetries, and is invariant under redefinition parameters: [first studied by Floratos & Illiopoulos 89b]

$$\mathcal{L}_{U_s} = \int d^4x \sqrt{|\eta^{(4)}|}\left[\frac{1}{4}\mathrm{tr}(F^{\mu\nu}F_{\mu\nu}) + \frac{1}{4}\mathrm{tr}(G^{\mu\nu}G_{\mu\nu}) + \frac{\mathcal{M}}{2}\mathrm{tr}(\not{D}\rho_s)\right], \quad \mu,\nu \in 0,1,2,3$$

$$F_{\mu\nu} \equiv F^{lm}_{\mu\nu}\hat{L}^{lm} \equiv [D_\mu, D_\nu], \quad D_\mu = \partial_\mu - \Gamma_\mu - i\lambda \sum_{lm} A^{lm}_\mu \hat{L}^{lm}$$

$$G_{\mu\nu} \equiv G^a_{\mu\nu}\hat{T}^a \equiv [D'_\mu, D'_\nu], \quad D'_\mu = \partial_\mu - \Gamma_\mu - i\lambda_G \sum_a B^a_\mu \hat{T}^a$$

Properties of the action $\mathcal{L}_{U_s}$:

- ⋆ Diffeo-surface of $SU(\infty)$ is embedded in the 4D parameter space.
- ⋆ **Metric of the parameter space is not observable and can be gauged out.**
- ⋆ Stress-energy tensor is defined with respect to the parameter space metric for both matter and gravity and imposes constraints on amplitudes [HZ (in prep.)].

Evolution

- ⋆ At lowest order in number of traces the action has the form of a 4D Yang-Mills action for both $SU(\infty)$ and G symmetries, and is invariant under redefinition parameters: [first studied by Floratos & Illiopoulos 89b]

$$\mathcal{L}_{U_s} = \int d^4x \sqrt{|\eta^{(4)}|} \left[\frac{1}{4} \text{tr}(F^{\mu\nu} F_{\mu\nu}) + \frac{1}{4} \text{tr}(G^{\mu\nu} G_{\mu\nu}) + \frac{\mathcal{M}}{2} \text{tr}(\not{D} \rho_s) \right], \quad \mu, \nu \in 0, 1, 2, 3$$

$$F_{\mu\nu} \equiv F^{lm}_{\mu\nu} \hat{L}^{lm} \equiv [D_\mu, D_\nu], \quad D_\mu = \partial_\mu - \Gamma_\mu - i\lambda \sum_{lm} A^{lm}_\mu \hat{L}^{lm}$$

$$G_{\mu\nu} \equiv G^a_{\mu\nu} \hat{T}^a \equiv [D'_\mu, D'_\nu], \quad D'_\mu = \partial_\mu - \Gamma_\mu - i\lambda_G \sum_a B^a_\mu \hat{T}^a$$

Properties of the action $\mathcal{L}_{U_s}$:

- ⋆ **A Historical Reminder: For decades the mediator of strong nuclear force was considered to be mesons of various spins and Hideki Yukawa received the Nobel Prize in physics in 1949 for this model.**

- ⋆ **Prediction of a spin-1 field for quantum mediator of gravity is testable in future experiments seeking QGR signature.**

Classical limit

- $SU(\infty)$-QGR becomes trivial for $\hbar \to 0$.

- Classical limit is an observational situation in which quantum nature of the spin-1 $SU(\infty)$ field $F_{\mu\nu}$ is not detectable.

- In this case, the pure Yang-Mills term in the action is perceived as a scalar function.

- **Theorem: Every scalar function of a $D > 2$ manifold is scalar curvature for a (pseudo)-Riemannian metric.** [A.L. Besse 87]

$$\int d^4x \sqrt{|\eta^{(4)}|}\,\mathrm{tr}(F^{\mu\nu}F_{\mu\nu}) \xrightarrow[\text{limit}]{\text{classical}} \propto \int d^4X \sqrt{|g^{(4)}|}\,R^{(4)}(X)$$

- The metric $g_{\mu\nu}$ and **the Einstein-Hilbert action** of classical gravity are effective manifestation underlying quantum interactions.

- **$SU(\infty)$-QGR explains the origin of spacetime dimension.**

Classical spacetime as average path in the parameter space

- **Interpretation of average path in the parameter space as classical spacetime is independent of dynamics.**

- **Quantum Speed Limit (QSL):** A consequence of quantum uncertainty relations [Mandelstam & Tamm, 1945].

- Definition of QSL, using geometry of the Hilbert space [Deffner & Campbell 17 (review)]:

$$\Delta T \geqslant \frac{\mathcal{D}(\hat{\varrho}(T_0), \hat{\varrho}(T))}{\langle\langle \sqrt{g_{tt}} \rangle\rangle}$$

- **Examples:**
  - Fubini-Study(FS) distance and metric for unitarily evolving pure states [Gibbons, 92].
  - Bures distance [Bures, 69] and Winger-Yanase skew information [Winger & Yanase, 63] as metric for mixed states [Uhlmann, 92].

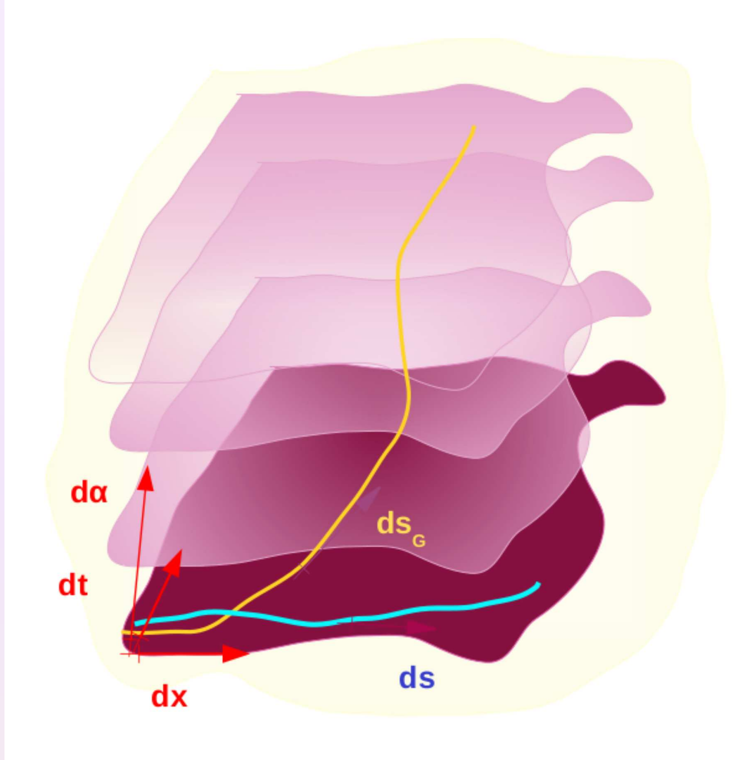
QSL imposes Lorentzian effective geometry

- ⋆ Consider: $\hat{\varrho}_\infty \rightarrow \hat{\varrho}_\infty + d\hat{\varrho}_\infty, \ \hat{\varrho}_\infty \equiv \mathrm{tr}_G \hat{\varrho}_U$
- ⋆ Consider a choice of clock such that:

$$\langle\langle \sqrt{g_{tt}} \rangle\rangle^2 dT^2 = \mathcal{D}^2[\hat{\varrho}_\infty, \hat{\varrho}_\infty + d\hat{\varrho}_\infty] \equiv ds^2$$

⋆ In general, for another choice of parameters the equality is not fulfilled and an additional term is necessary:

$$\langle\langle \sqrt{g'_{tt}} \rangle\rangle^2 dT''^2 - d\mathcal{F}^2 = ds^2 = \mathcal{S}[\mathrm{tr}(f_1(\hat{\varrho}_\infty) f_2(d\hat{\varrho})_\infty]$$



- ⋆ $\mathcal{S}$ includes integration on parameters. For FS metric: $\mathcal{S} = (\mathrm{tr}(\hat{\varrho}_\infty d\hat{\varrho}_\infty))^2$
- ⋆ $ds^2 = \mathcal{S}$ should be a function of effective or average values of parameters X^μ.
- ⋆ Parameterization can be chosen such that:

$$ds^2 = \mathcal{S} = g_{00}(X) dT'^2 - g_{ij}(X) dX^i dX^j, \ \ g_{00}(X) = \langle\langle \sqrt{g'_{tt}} \rangle\rangle^2 > 0$$

- ⋆ **Effective metric is Lorentzian and perceived classical spacetime is (3+1)D.**

Topics to be investigated

- ⋆ Renormalization group flow and running coupling in gravity sector;
- ⋆ Dark energy [HZ (in prep.];
- ⋆ Particle physics - origin of the Standard Model;
- ⋆ Black hole physics;
- ⋆ Early Universe: inflation, reheating, PBH, tensions, etc.;
- ⋆ Predictions for QGR laboratory tests.
- ⋆ etc. **(Suggestions welcome)**

How can $SU(\infty)$-QGRbe relevant for (primordial) black holes ?

- **Asymptotic safety:** In $SU(N)$ Yang-Mills models at 1-loop order:

$$\beta(\mu) \equiv \frac{\mu^2 \partial \alpha_\infty}{\partial \mu^2} = -\alpha_\infty^2 \left(\frac{11N - 2N_f}{12\pi} + \mathcal{O}(\alpha_\infty) \right), \quad \alpha_\infty \equiv \frac{\lambda^2}{4\pi}$$

- The Hilbert space of subsystems is infinite-dimensional $\Longrightarrow N_f \to \infty$.
- As $N, N_f \to \infty$ and have the same cardinal order, the coupling at high energies is expected to approach zero or finite $\Longrightarrow$ **No collapse to BH at high energies.**
- **Many micro-states for the same classical metric** $\Longrightarrow$ **There may be many types of black holes !**
- As QFT's represent type III von Neumann algebras and Lagrangian of $SU(\infty)$-QGR is defined on the parameter space of QM states, **classical horizon is not a barrier for quantum information.** But this claim needs more investigation.

Quantum Many-Body(QMB) phenomena and formation of PBH

- As $SU(\infty) \cong \bigotimes^\infty SU(2)$ QGR interactions are analogous to spin systems: [Hoppe, *et al.*82]

$$\hat{L}_{lm}(\theta,\phi) = \mathcal{R} \sum_{i_\alpha=1,\,2,\,3,\alpha=1,\cdots,l} a^{(m)}_{i_1,\cdots i_l}(\theta,\phi)\sigma_{i_1}\cdots\sigma_{i_l}, \quad \{\sigma_i,\sigma_j\} = 0,\ i \neq j$$

- Their Hamiltonian can be formulated as **Sachdev-Ye-Kitaev(SYK) model**:

$$H_{SYK} = \mathcal{N} \sum_{i_1,i_2,\cdots} = J_{i_1,i_2,\cdot}\psi_{i_1}\psi_{i_2}\ldots, \quad \{\psi_i,\psi_j\} = 0,\ i \neq j$$

- Many QMB models can be considered as variants or combination SYK models.
- SYK is used as an approximate description of holographic (AdS/CFT) models of 2D QGR [Ooguri & Vafa 16, Rosenhaus 18 (review)].
- It is also used for phenomenologically describing state of quantum black holes as a Fermi liquid with long range interactions [Solfanelli, *et al.*24, Sachdev 24 (review)].
- **In $SU(\infty)$-QGR the relation with SYK is exact, but more complicated than phenomenological models.**

Many-body non-local effects: Scars and fragmentation

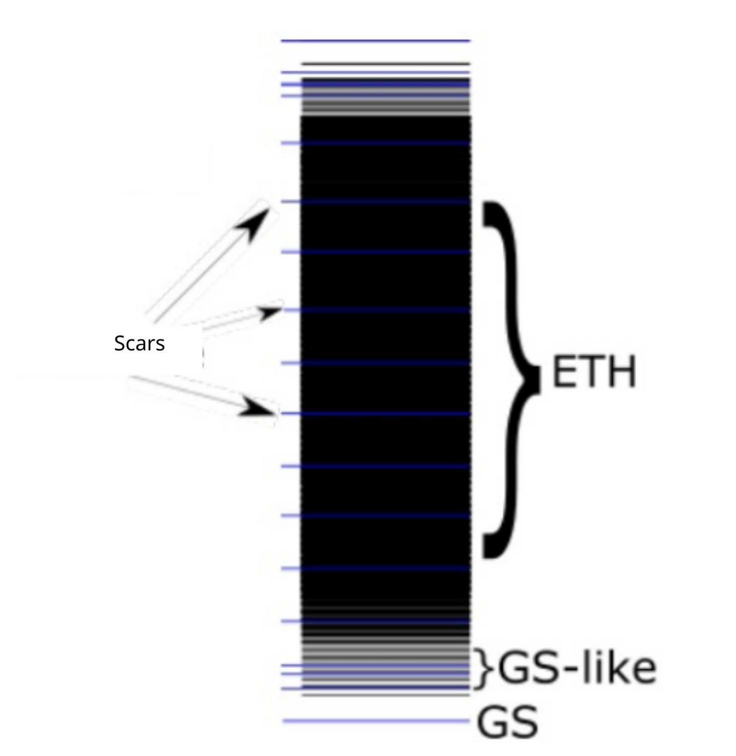
⋆ In QMB non-local interactions can violate Eigen state Thermalization Hypothesis (ETH) $\Longrightarrow$ **QMB scars and Hilbert space fragmentation to isolated Krylov subspaces.**

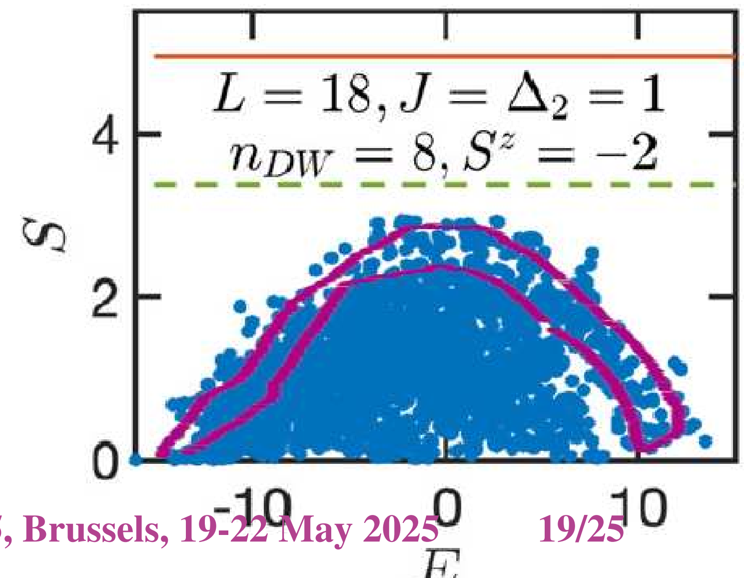
⋆ Scars can be the sought for spikes in the spectrum when QGR effects are important [Moudgalaya, *et al.*22]

⋆ Inflation and reheating can be interpreted as consequences ETH -evolution toward ergodicity and filling the parameter space.

⋆ **They may provide without fine-tuning large but rare fluctuations necessary for the formation of PBH** [Yang, *et al.*16]

⋆ There are not yet a generally applicable understanding of conditions under which non-local features arise.





Outlines

- ⋆ $SU(\infty)$-QGRis a Quantum First model based on the well motivated and simple assumptions
- ⋆ It describes the origin of the observed 3+1 dimension of the spacetime and its Lorentzian geometry.
- ⋆ It relates the classical metric of the spacetime to the quantum state of the Universe content.
- ⋆ Like other fundamental interactions gravity is mediated by a spin-1 boson satisfying a Yang-Mills gauge theory.
- ⋆ As a QFT $SU(\infty)$-QGR is renormalizable, and most probably asymptotic safe.
- ⋆ The homomorphism of $SU(\infty)$ with spin systems can be used to study cosmological phenomena in the early Universe such as inflation and formation conditions for PBH.
- ⋆ $SU(\infty)$-QGR provides new visions and possibilities for cosmology. They need in depth investigation and exploration.