Detection prospects for the GW background of Galactic (sub)solar mass PBHs

FRANS VAN DIE

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WITH IVAN RAPOPORT, YONADAV BARRY GINAT AND VINCENT DESJACQUES

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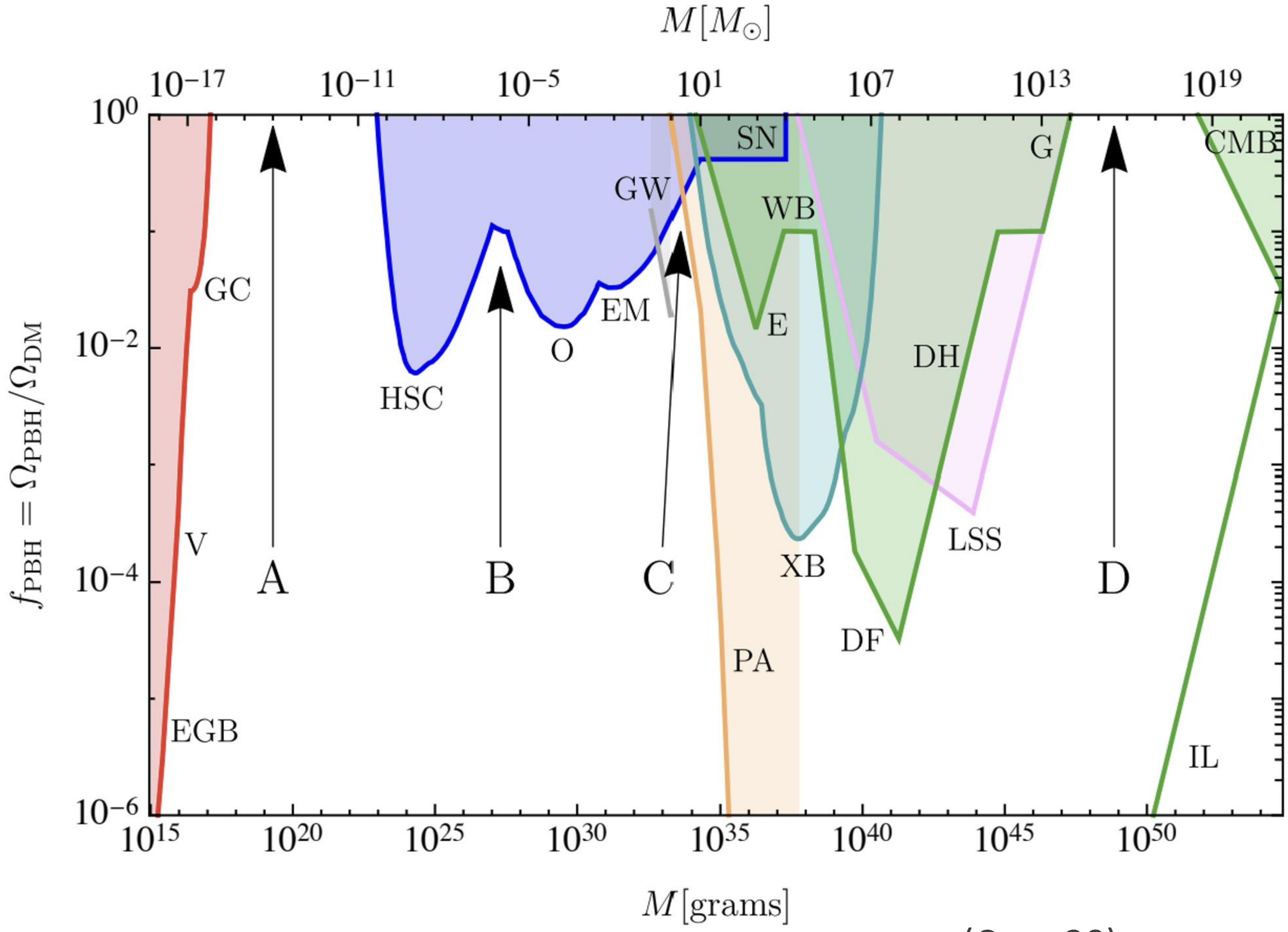
TECHNION
Israel Institute of Technology

Motivation



(Carr+20)

Early-Type PBH Binaries in MW DM halo

- m : Mass number $m \in [\,0.01\,, 0.1\,, 1\,, 30\,, 1000\,]$
- f : Mass fraction $f \in [\,10^{-4}\,, 10^{-3}\,, 10^{-2}\,]$
- η_0 : Binarity

$$N_0 = \frac{M_{\rm DM}}{{\rm M}_\odot}\frac{f_{\rm pbh}\eta_0}{2m} \simeq 1.18\times 10^{10}\, m^{-1}\left(\frac{f}{0.01}\right)\eta_0$$

Steps

- Initialize PBH Binary Population
- Evolve to present-day epoch
- Predict GW Background signal, especially for LISA

PBH Initialization

- Poissonian Spatial Distribution: Nearest Neighbor PBH Pairs

- Angular Momentum: $P(j_i) = P(j_{pbh}) * P(j_\delta)$

(Ali-Haïmoud+17)

Binary Formation

- Newtonian Approximation (Ali-Haïmoud+17, Raidal+19)

$$\ddot{r} - (\dot{H} + H^2)r + \frac{2GM_{\rm pbh}}{r^2}\frac{r}{|r|}(1 - j^2) = 0$$

  - NB: $j = j(t)$

- Binary forms at decoupling from Hubble flow: $\dot{r} = 0$

GW Evolution

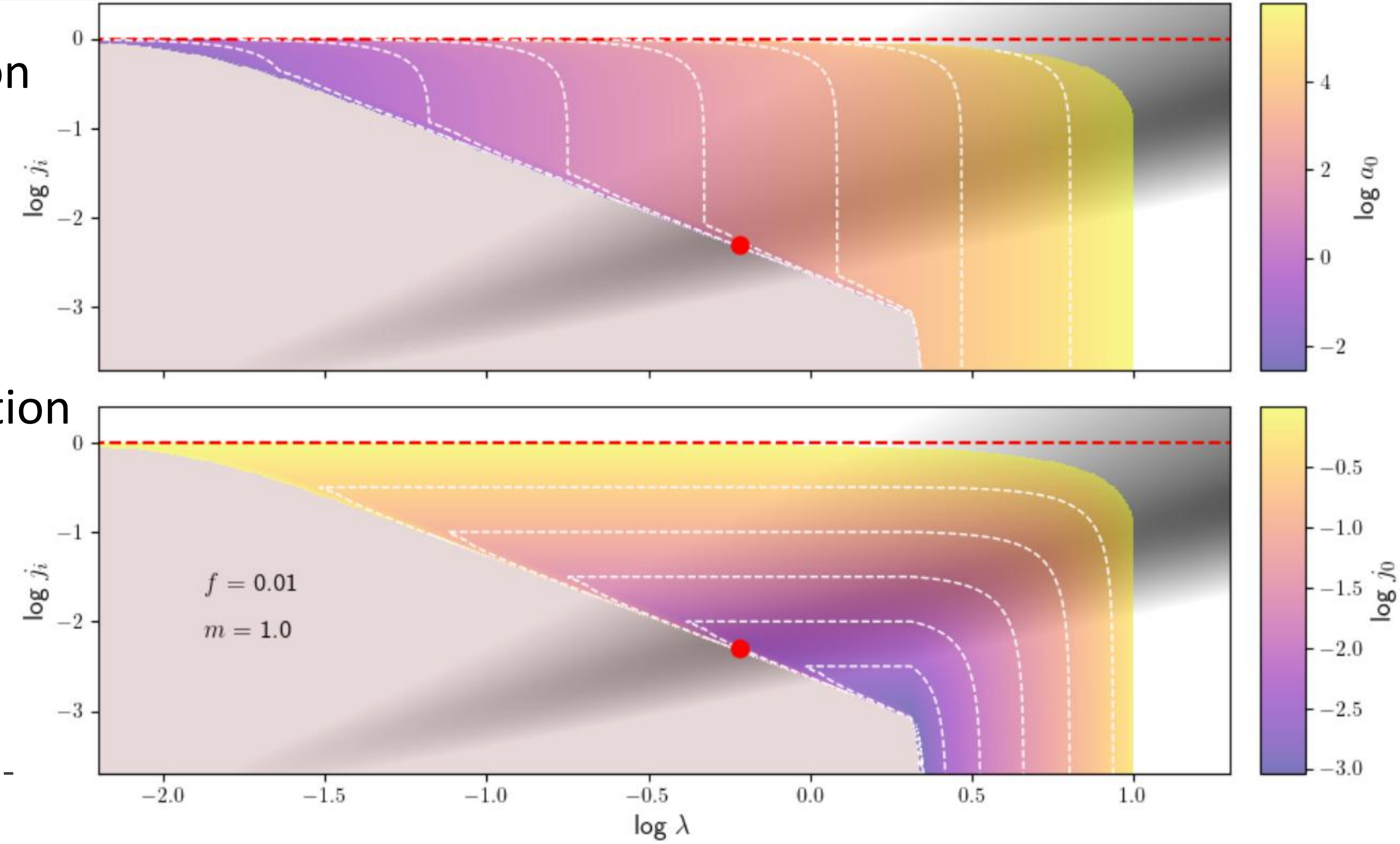
Binary formation for $s < s_{max} \sim 30$

(Peters+64) GW Evolution

$$\lambda \equiv \frac{4\pi \rho_{\rm eq} x_i^3}{3M_{\rm pbh}}$$

η_{merge} : unmerged fraction

$\eta_{merge} \sim 0.9$



Merger Parameters confirm (Ali-Haïmoud+17, Raidal+19)

MW Stellar Disruption

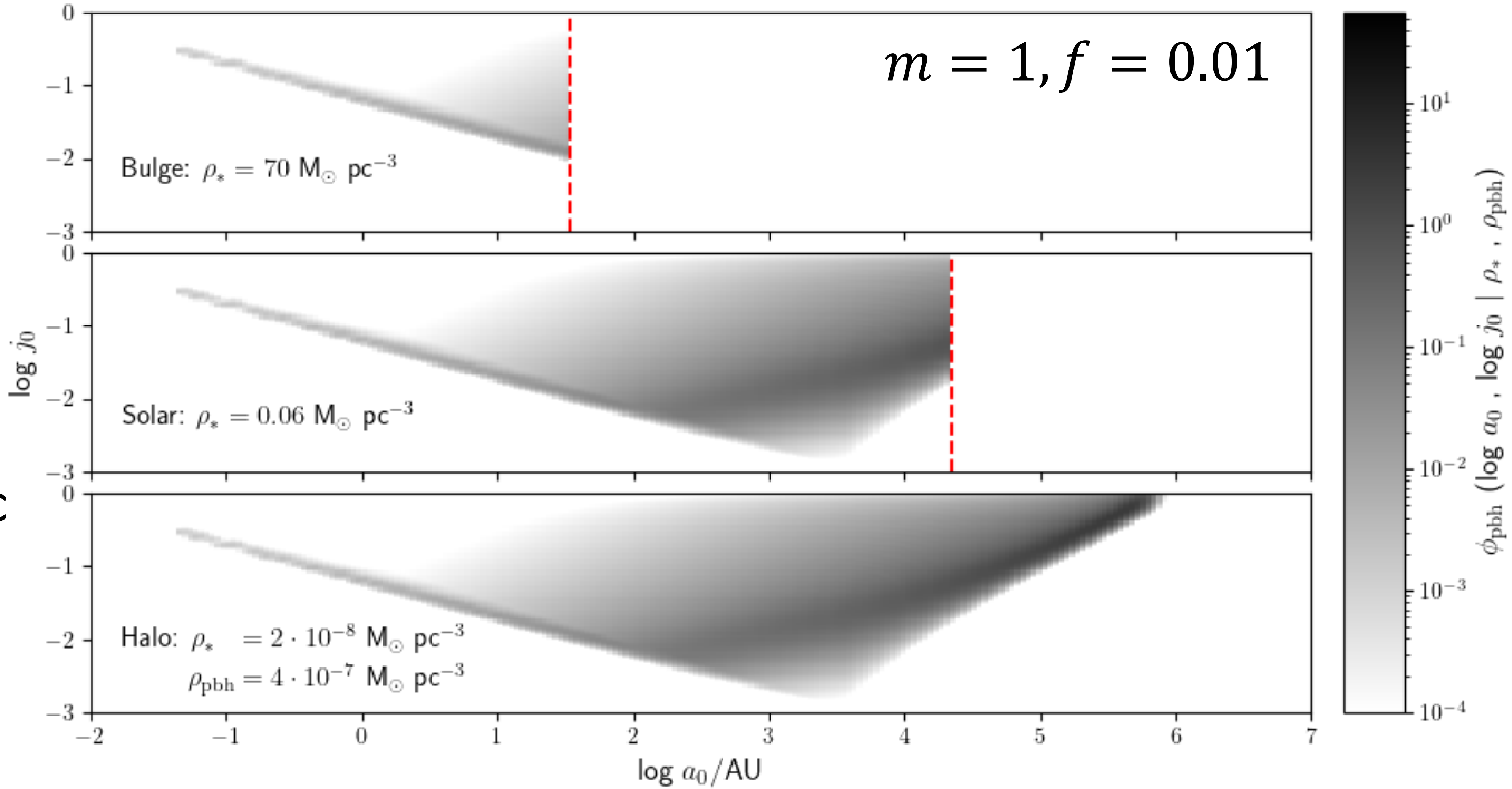
- We model stellar disruption deterministically with the present-day distributions. Disruption can occur via:
  - Ionization rate R_I
  - Evaporation rate R_E
- Soft PBH binaries are disrupted when $R_T = R_I + R_E > 1/t_H$
- NB: Disruption is a location dependent process in the MW, $R_T \propto \rho_*$

MW Stellar Disruption

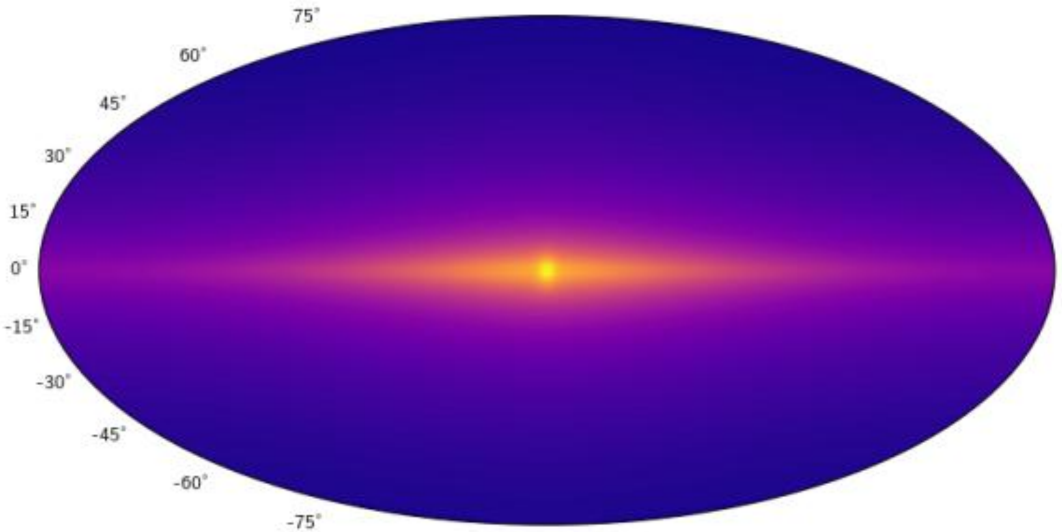
η_d : undisruptedfraction

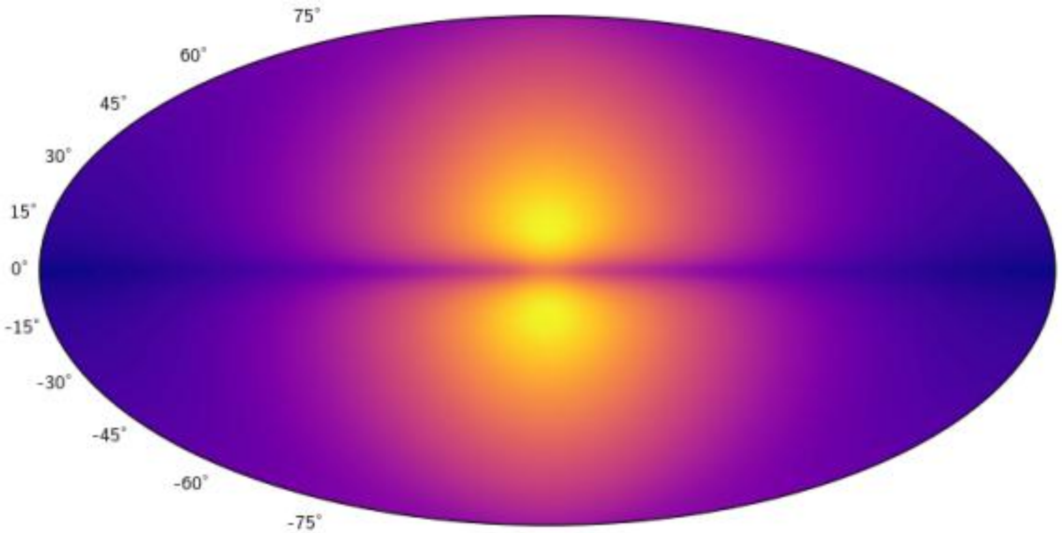
$\eta_d \sim 0.98$

- Higly eccentric
- Merger tail



Galactic Spatial Distributions





Stellar MW Distribution

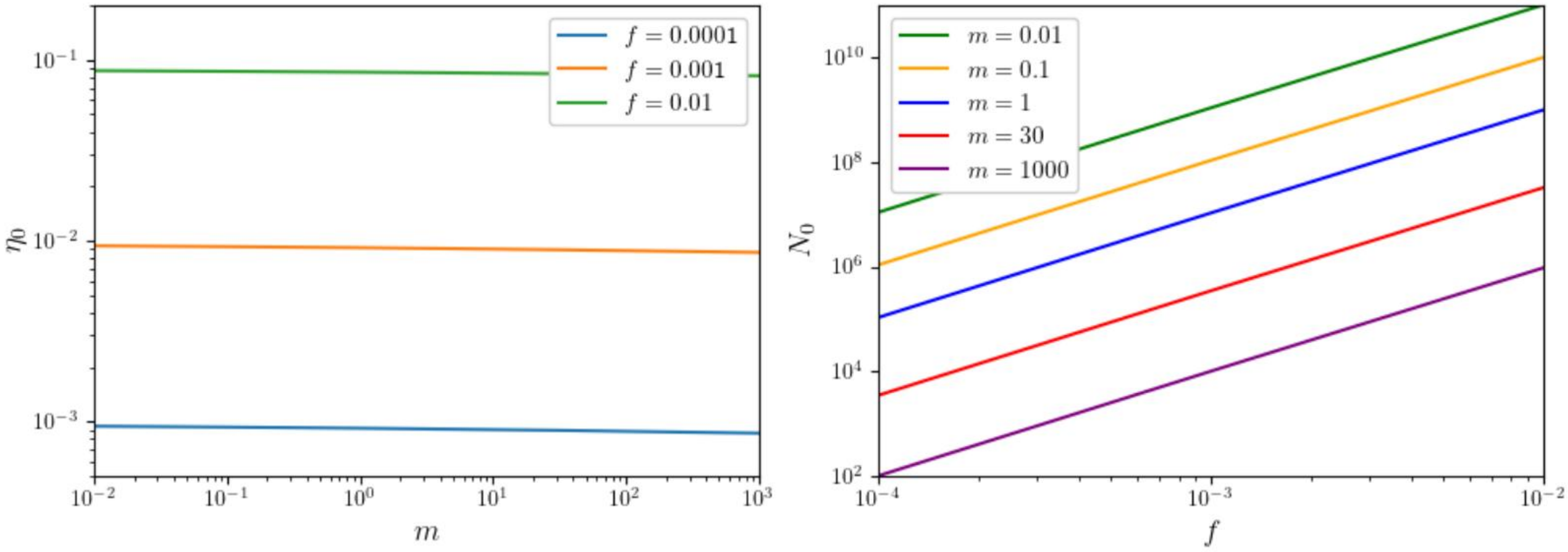
PBH Binary Distribution:

NFW Profile + Disruption

Number of Galactic PBH Binaries

$$N_0 = \frac{M_{\rm DM}}{M_\odot}\frac{f_{\rm pbh}\eta_0}{2m} \simeq 1.18\times 10^{10}\, m^{-1}\left(\frac{f}{0.01}\right)\eta_0$$

$$\eta_0 = \eta_* \cdot \eta_{\rm merge} \cdot \eta_{\rm d}$$



Galactic GWB Construction

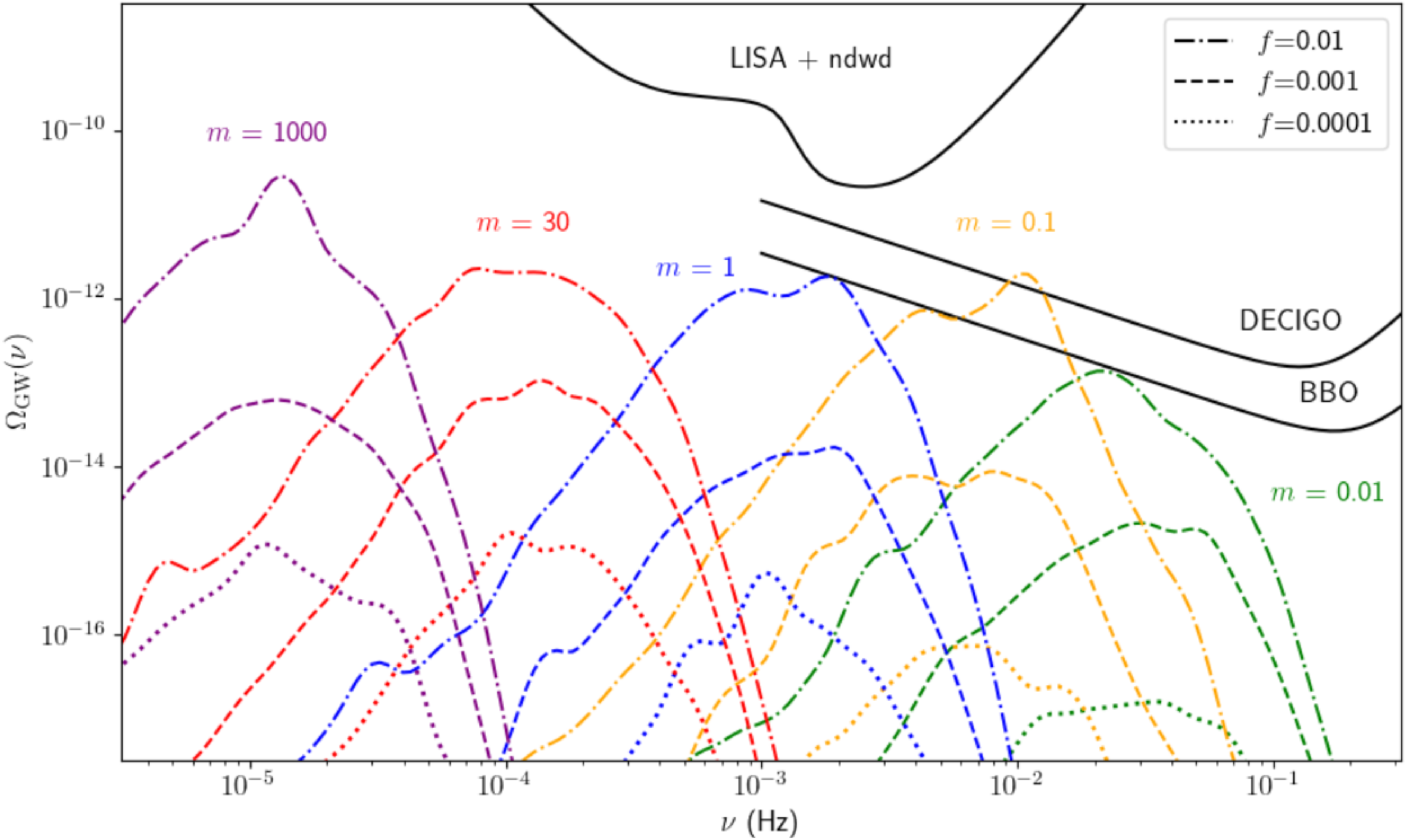
Non-circular sources: $\nu_n \equiv n\nu_0 \ , \ n \geq 2$

$$\Omega_{\rm gw}(\nu) \equiv \frac{1}{\rho_c} \frac{{\rm d}\rho_{\rm gw}}{{\rm d}\ln\nu}$$

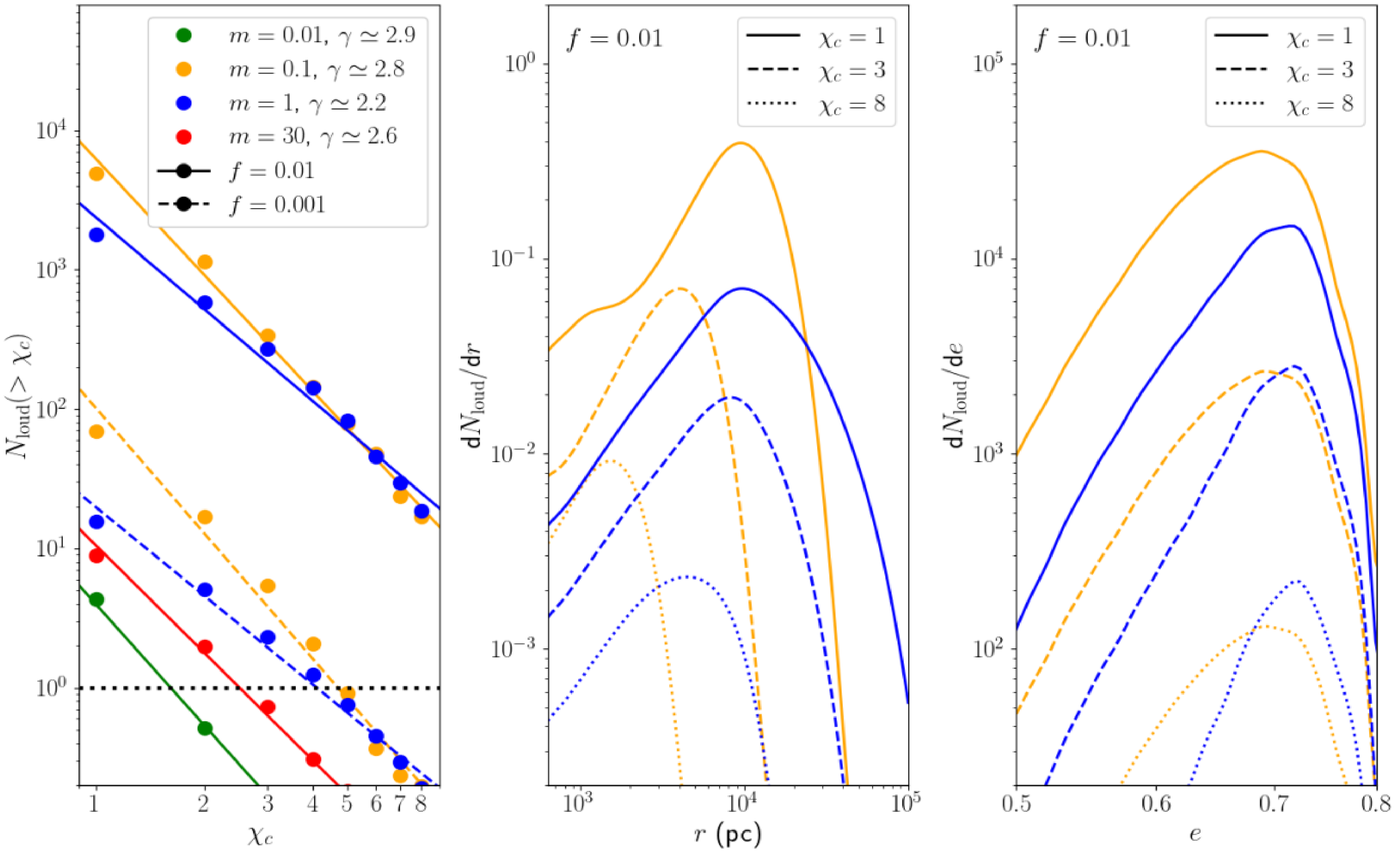
$$h_{0,n}(\nu_n, \boldsymbol{r}, \boldsymbol{\xi}) \equiv h_0(\nu_n, \boldsymbol{r}, \boldsymbol{\xi}) \sqrt{\gamma_n(e)} \quad \Rightarrow \quad \gamma_n(e) \equiv \left(\frac{2}{n}\right)^{2/3} \frac{g(n,e)}{f(e)}$$

$$\Omega_{\rm gw}(\nu) = \frac{16\pi^2 N \nu^3}{15 {H_0}^2} \sum_{n=1}^{\infty} \int {\rm d}\boldsymbol{r}\, {\rm d}\boldsymbol{\xi}\, \phi(\boldsymbol{r}, \boldsymbol{\xi}) \left|\frac{{\rm d}\nu}{{\rm d}t}\right| \delta(\nu - \nu_n)\, h_{0,n}^2(\nu_n, r, \boldsymbol{\xi})$$

Galactic GWB of PBH Binaries



Loud Galactic PBH Binaries



- Above LISA+ndwd
- $t_{coal} \sim 10^3 - 10^4$ yr
- $a < 1$ AU

Summary

- PBHs in the MW halo are expected to form a highly eccentric binary population
- The stochastic GWB of (sub)solar PBH binaries in the Milky Way halo remains undetectable with LISA. For $m \sim 0.01 - 0.1$, it exceeds the DECIGO/BBO thresholds for $f \sim 0.01$
- After 5 years of LISA observation, we expect $O(100)$ (resp. $O(1)$) loud PBH sources ($\mathrm{SNR} \geq 5$) for $f \sim 0.01$ (resp. $f \sim 0.001$) in the mass range $m \sim 0.1 - 1$
- Spatial distribution is key in distinguishing from astrophysical sources

Appendix

$$X'' + \frac{2+\lambda\mathcal{S}}{2+2\lambda\mathcal{S}}\frac{X - X'\mathcal{S}}{\mathcal{S}^2} + \frac{\mathcal{S}^2}{1+\lambda\mathcal{S}}\frac{1}{\epsilon^2 + X^2}\frac{X}{|X|}(1 - j^2) = 0$$

Motivation – Primordial Black Holes

- Hypothesized to have formed in the Early Universe
- Introduced by (Hawking+71, Carr+75)
- Renewed attention since (Bird+16)
- Subsolar mass BHs are a strong indicator of a primordial origin

PBH Initialization

Poissonian Spatial Distribution: Nearest Neighbor PBH Pairs

$$P(x_i) = 4\pi n x_i^2 \exp\left(-\frac{4\pi n x_i^3}{3}\right)$$

$$n = \frac{f \rho_{\rm eq}}{M_{\rm pbh}}$$

$$\lambda \equiv \frac{4\pi \rho_{\rm eq} x_i^3}{3 M_{\rm pbh}}$$

PBH Initialization

Angular Momentum Barrier: $j \equiv \frac{L}{M_{\rm pbh}\sqrt{2GM_{\rm pbh}r}}$

$$P(j_i) = P(j_{pbh}) * P(j_\delta)$$

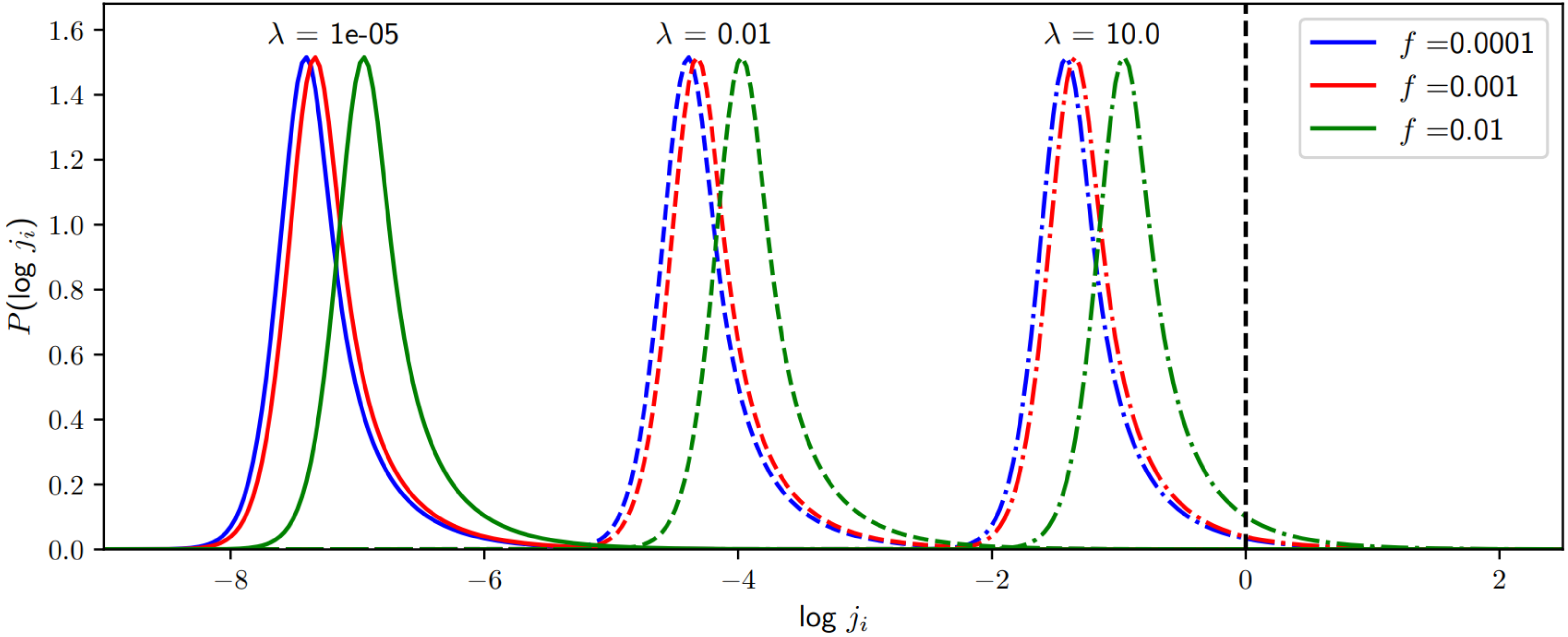
$$P(j_{pbh}) = \frac{j_{pbh}}{{j_f}^2[1 + (j_{pbh}/j_f)^2]^{3/2}}$$ ⟹ $j_f \equiv \frac{1}{2}\lambda f$

$$P(j_\delta) = \frac{j_\delta}{{\sigma_\delta}^2} e^{-{j_\delta}^2/(2{\sigma_\delta}^2)}$$ ⟹ ${\sigma_\delta}^2 = \frac{3}{10}\sigma_{\rm eq}^2\lambda^2$

(Ali-Haïmoud+17)

$\sigma_{\rm eq} = 0.005$

Initial Angular Momentum Distribution



Binary Formation

$$\ddot{r} - (\dot{H} + H^2)r + \frac{2GM_{\rm pbh}}{r^2}\frac{r}{|r|}(1 - j^2) = 0$$

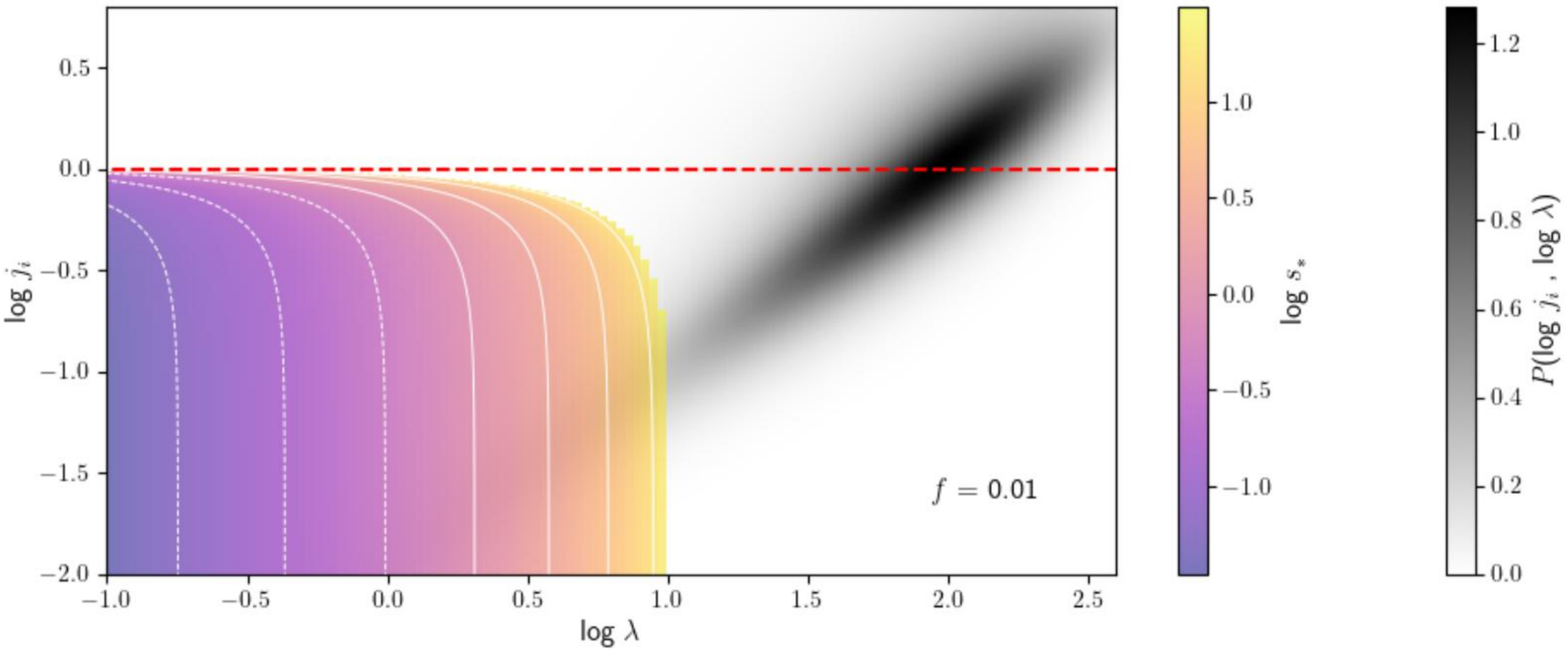
We use a Mean-Field approach, $\langle j_\delta \rangle \propto \sigma_m$, $\sigma_m \simeq s\sigma_{\rm eq}$ to model the time-dependence of the angular momentum barrier in MD as

$$j(t) = j(s) = j_i + \sqrt{\frac{3}{10}}\lambda\sigma_{\rm eq}\,\Xi(s)$$

$$\Xi(s) \equiv \begin{cases} 0 & s < 1 \\ s - 1 & s \geq 1 \end{cases}$$

Binary Formation

- Binary formation for $s < s_{max} \sim 30$
- MD is important
- Linear analysis suffices
- Asymptotic value η_*



GW Evolution

The (Peters+64) equations can be integrated to give:

$$t(a_*, e_*, e) = \frac{15}{304} \frac{c^5 {a_*}^4}{G^3 \mu M^2} \frac{I(e_*) - I(e)}{\mathcal{G}^4(e_*)}$$

$$a(e) = a_* \frac{\mathcal{G}(e)}{\mathcal{G}(e_*)}$$

$$\mathcal{G}(e) \equiv \frac{e^{12/19}}{1 - e^2} \left(1 + \frac{121}{304} e^2\right)^{870/2299}$$

GW Evolution

The (Peters+64) equations also include:

$$g(n,e) = \frac{n^4}{32}\left(\left[J_{n-2}(ne) - 2eJ_{n-1}(ne) + \frac{2}{n}J_n(ne) + 2eJ_{n+1}(ne) - J_{n+2}(ne)\right]^2 + (1-e^2)\left[J_{n-2}(ne) - 2J_n(ne) + J_{n+2}(ne)\right]^2 + \frac{4}{3n^2}\left[J_n(ne)\right]^2\right)$$

$$f(e) \equiv \frac{1}{(1-e^2)^{7/2}}\left(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4\right)$$

$$\sum_{n=1}^{\infty} g(n,e) = f(e)$$

$$I(e) \equiv \frac{e^{10/19}}{3648}\Big(I_0(e) - 3648 A_1(e) - 893 e^2 A_2(e)\Big)$$

$$A_1(e) \equiv F_1\left(\frac{5}{19}, \frac{1}{2}, \frac{1118}{2299}, \frac{24}{19}, e^2, -\frac{121}{304}e^2\right)$$

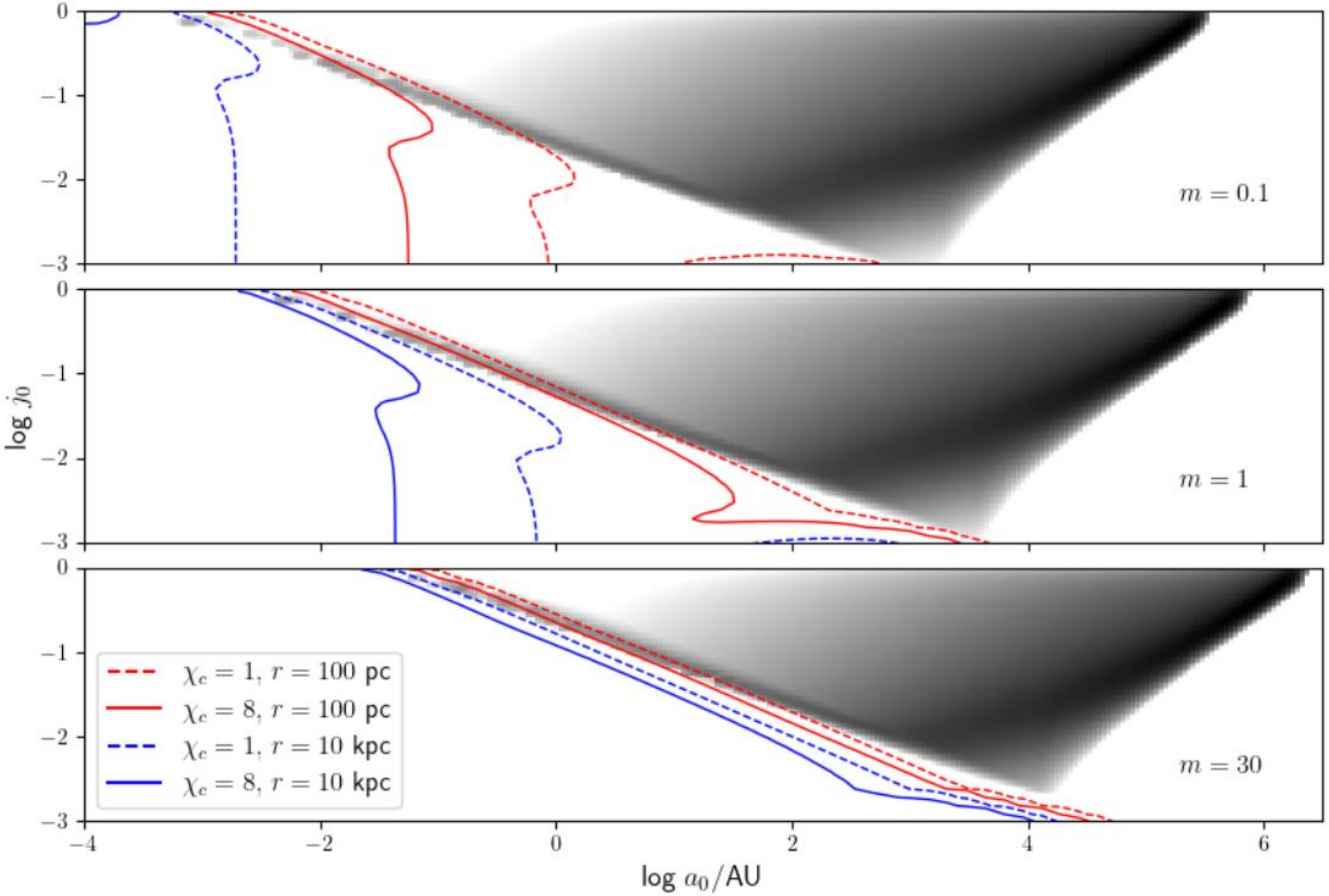
$$A_2(e) \equiv F_1\left(\frac{24}{19}, \frac{1}{2}, \frac{1118}{2299}, \frac{43}{19}, e^2, -\frac{121}{304}e^2\right)$$

$$I_0(e) \equiv \frac{24 \times 2^{2173/2299} \times 19^{1118/2299}}{\sqrt{1-e^2}}\left(304 + 121e^2\right)^{1181/2299}$$

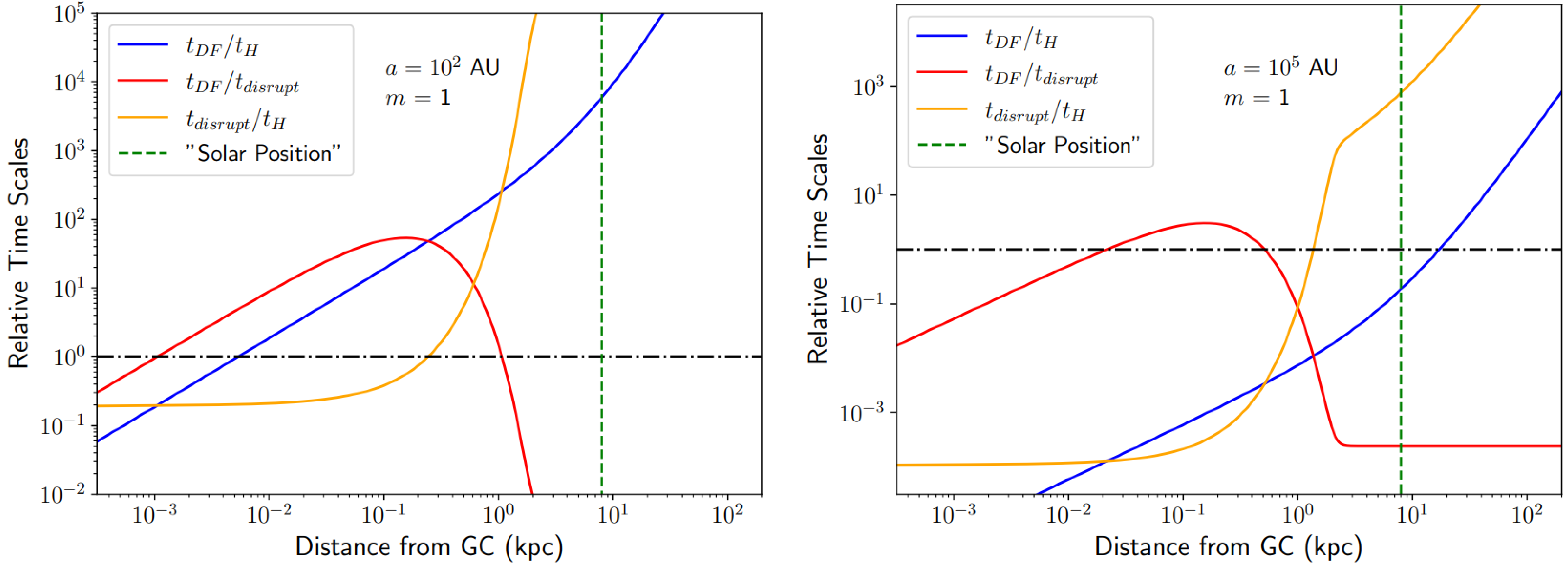
$$F_1(\alpha, \beta_1, \beta_2, \gamma, x, y) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{(\alpha)_{m+n}(\beta_1)_m(\beta_2)_n}{m!n!(\gamma)_{m+n}} x^m y^n$$

$$(z)_p = \prod_{k=0}^{p-1}[z-k]$$

Loud PBH Binaries



Dynamical Friction



Galactic GWB Construction

Loud Source Substraction

$$\chi^2 = \frac{16}{5} T \int \mathrm{d}\nu \left|\frac{\mathrm{d}\nu}{\mathrm{d}t}\right| \delta\big(\nu - \nu(\boldsymbol{r}, \boldsymbol{\xi})\big) \frac{{h_0}^2(\nu, r, \boldsymbol{\xi})}{S_\mathrm{n}(\nu)}$$

$$= \frac{16}{5} T \left(\left|\frac{\mathrm{d}\nu}{\mathrm{d}t}\right| \frac{{h_0}^2(\nu, r, \boldsymbol{\xi})}{S_\mathrm{n}(\nu)} \right)_{\nu=\nu(\boldsymbol{r},\boldsymbol{\xi})} .$$

Galactic GWB Construction

Circular Binaries : DWD GWB in LISA   (Phinney+01, Ginat+20)

$$\Omega_{\mathrm{gw}}(\nu) \equiv \frac{1}{\rho_c}\frac{\mathrm{d}\rho_{\mathrm{gw}}}{\mathrm{d}\ln\nu} = \frac{1}{\rho_c}\int \mathrm{d}\boldsymbol{r}\,\mathrm{d}\boldsymbol{\xi}\, n(\boldsymbol{r},\boldsymbol{\xi})\,\frac{1}{4\pi r^2}\,\frac{\mathrm{d}E_{\mathrm{gw}}}{\mathrm{d}\ln\nu}$$

$$= \frac{16\pi^2 N\nu^3}{15{H_0}^2}\int \mathrm{d}\boldsymbol{r}\,\mathrm{d}\boldsymbol{\xi}\,\phi(\boldsymbol{r},\boldsymbol{\xi})\left|\frac{\mathrm{d}\nu}{\mathrm{d}t}\right|\delta(\nu-\nu_2)\,{h_0}^2(\nu_2,r,\boldsymbol{\xi})$$

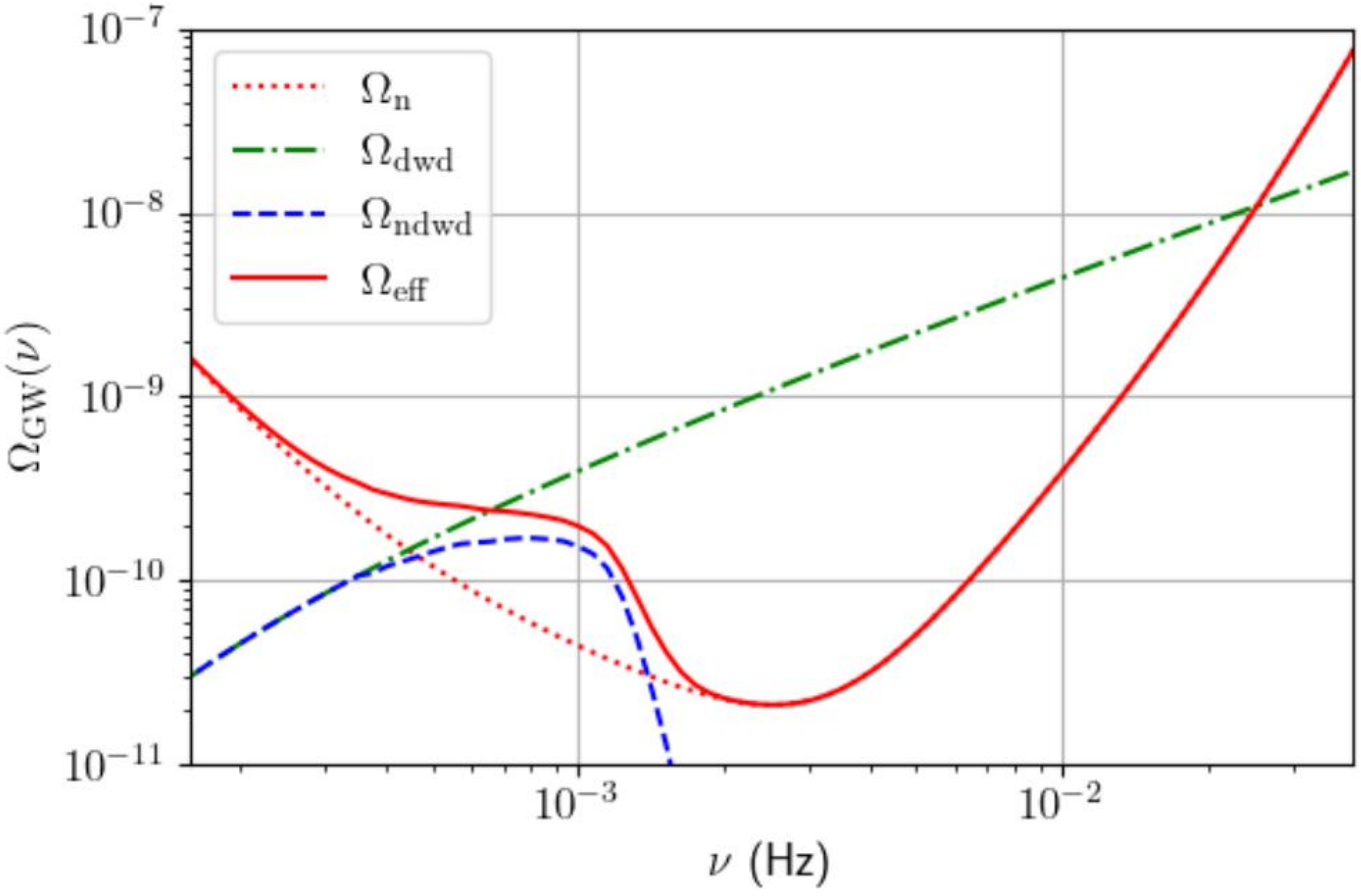
$$h_0(\nu_2,r,\boldsymbol{\xi}) = \frac{1}{\pi^{2/3}}\sqrt{\frac{5}{24}\frac{c}{r}}\left(\frac{GM_c}{c^3}\right)^{5/6}{\nu_2}^{-7/6}$$

$$\nu_n \equiv n\nu_0$$

Galactic GWB Construction

$\chi_c = 8$

Confirms (Korol+22, Boileau+21 and others)



Loud Galactic PBH Binaries

Extract Loud Sources in LISA:

$$\chi_{\mathrm{pbh}}{}^2 = \frac{16}{5} T \sum_{n=0}^{\infty} \left(\left| \frac{\mathrm{d}\nu}{\mathrm{d}t} \right| \frac{h_{0,n}{}^2(\nu, \boldsymbol{r}, \boldsymbol{\xi})}{S_{\mathrm{eff}}(\nu)} \right)_{\nu=\nu_n(\boldsymbol{r},\boldsymbol{\xi})}$$

Here we take $T = 5$ yr