Primordial black hole formation during slow-reheating

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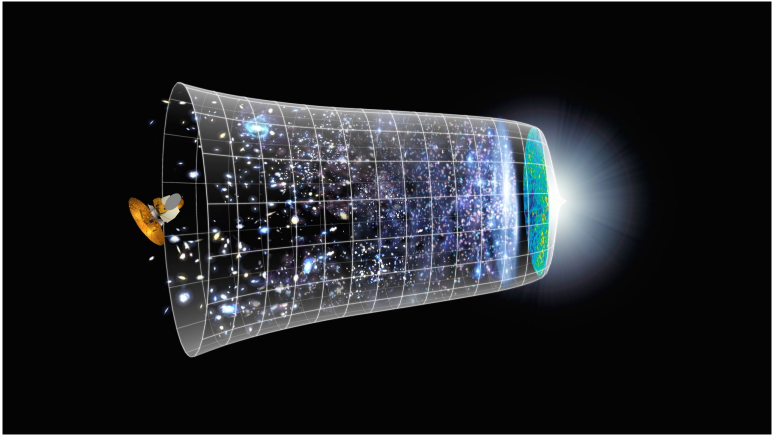
Content

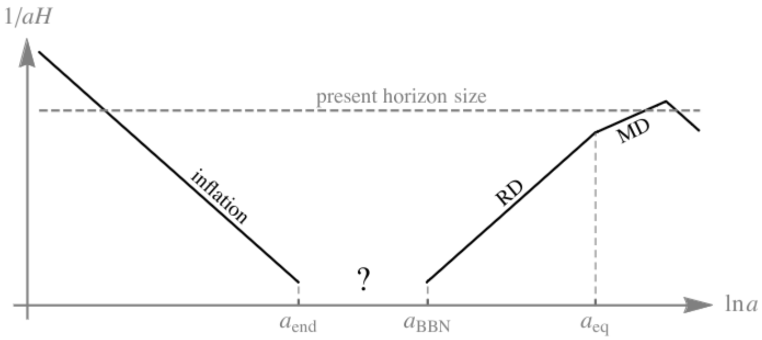
The standard and the early universe

The standard evolution universe



Typically, we assume an early inflationary epoch $\Rightarrow$ Instant reheating process $\Rightarrow$ RD epoch $\Rightarrow$ MD $\Rightarrow$ dark energy domination.

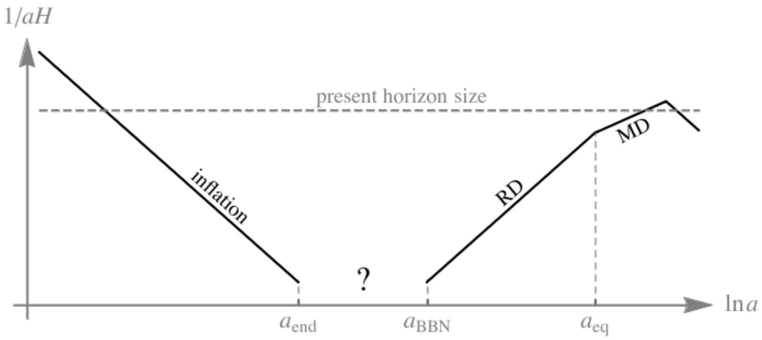
The early universe



From arXiv:2006.16182.

The physics from the end of inflation to the BBN is not completely well understood and in fact we only have evidence of a RD universe at the BBN epoch.

The early universe



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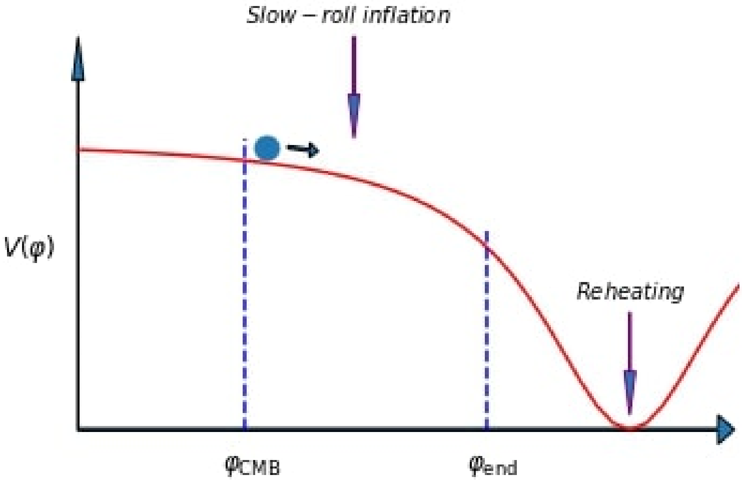
Is there something else?

A simple possibility is a slow-reheating scenario: During the fast-oscillating regime of the inflaton field!



- At the end of inflation, it is necessary to transfer the energy content of the inflaton field to the rest of the standard model particles.
- The reheating of the universe is typically assumed to be instantaneous, but this is not mandatory and, in general, it depends on the inflaton couplings with other fields.

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- We can have couplings between the inflaton field and other fields, which can cause an explosion of particle production $\Rightarrow$ Preheating.
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We shall concentrate in this talk in this last possibility!

The phase of slow-reheating and primordial structure formation

Slow-reheating and structure formation

During slow-reheating, we assume a potential of the form:

$$V(\varphi) = \frac{m^2}{2}\varphi^2. \tag{1}$$

At the background level and neglecting couplings with other fields, the time evolution of the inflaton field is:

$$\varphi_b(t) = \sqrt{\frac{8}{3}}\frac{M_{\rm Pl}}{m}\frac{1}{t}\sin(mt), \quad \Rightarrow \quad \rho_b(a) \simeq \rho_0 \left(\frac{a_0}{a}\right)^3. \tag{2}$$

At linear order in perturbations theory, perturbations grows as standard matter:

$$\delta(a;k) = \delta_{HC}(k)\frac{a}{a_{\rm HC}}, \quad \text{for } k_H < k < k_Q \equiv (16\pi G \rho_0 m^2 a^4)^{1/4}. \tag{3}$$

In the slow-reheating era, we can simplify things and consider gravity in the weak-field approximation in which the Einstein-Hilbert action for subhorizon scales is reduced to:

$$S_{\mathrm{EH}} = \int dx^4 a^3 \left[-\frac{(\partial_i \Psi)^2}{8\pi G a^2} + \left(\frac{1}{2}(1-4\Psi)\dot{\varphi}^2 - \frac{1}{2a^2}(\partial_i \varphi)^2 - (1-2\Psi)\frac{m^2}{2\hbar^2}\varphi^2 \right) \right] \quad (4)$$

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The inflaton during the slow-reheating phase is highly oscillating. So, we can introduce the complex field ψ and factorize the oscillations:

$$\varphi = \frac{\hbar}{\sqrt{2ma^3}}(\psi e^{-imt/\hbar} + \psi^* e^{imt/\hbar}). \quad (5)$$

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After doing some manipulations, we arrive at the Schrodinger-Poisson system of equations:

$$
i\hbar \partial_t \psi = -\frac{\hbar^2}{2ma^2}\nabla^2 \psi + m\Psi\psi, \qquad (6a)
$$

$$
\nabla^2 \Psi = \frac{4\pi G}{a}(\rho - \langle \rho \rangle). \qquad (6b)
$$

Some interesting properties of the SP equations:

- We can consider a Madelung transformation of the form:

$$\psi = \sqrt{\frac{\rho}{m}} e^{im\theta/\hbar} = \sqrt{n} e^{im\theta/\hbar}. \tag{7}$$

Defining the bulk velocity as $\mathbf{v}=\nabla\theta$, the SP system reduces to:

$$\partial_t \rho + \frac{1}{a^2}\nabla(\rho\mathbf{v}) = 0, \qquad \partial_t \mathbf{v} + \frac{1}{a^2}(\mathbf{v}\nabla)\mathbf{v} + \nabla\Psi + \nabla Q = 0, \tag{8a}$$

where Q is given by

$$Q = -\frac{\hbar^2}{2m^2a^2}\left(\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}\right) \tag{8b}$$

i.e. we arrive to a system of equations that are standard in non-relativistic fluid dynamics but this time we have an extra term, a quantum potential term.

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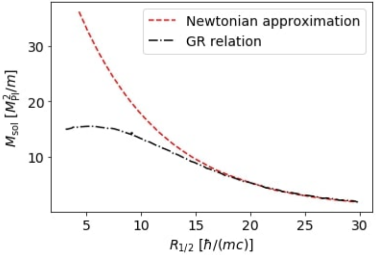
- There exist a maximum stable mass for soliton configurations:

$$M_{\mathrm{sol}}^{(\mathrm{crit})} \simeq 0.633 \frac{m_{\mathrm{Pl}}^2}{m}. \tag{10}$$



- There exist also a Schrodinger-Vlasov correspondence. If we smooth ψ using a Gaussian window and perform its Fourier transformation:

$$\begin{aligned} \tilde{\Psi}(\mathbf{x},\mathbf{p},t) &= \frac{1}{(2\pi\hbar)^{3/2}}\frac{1}{(\eta\sqrt{\pi})^{3/2}} \\ &\times \int e^{-\frac{(\mathbf{x}-\mathbf{y})^2}{2\eta^2}}\psi(\mathbf{y},t)e^{-i\frac{\mathbf{p}\cdot(\mathbf{y}-\mathbf{x}/2)}{\hbar}}d^3\mathbf{y}, \end{aligned} \tag{11}$$

we can obtain the collisionless Boltzmann equation (or Vlasov equation) once smoothing over scales much larger than the deBroglie wavelength:

$$\frac{d\mathcal{F}}{dt} = \frac{\partial\mathcal{F}}{\partial t} + \frac{p_i}{m}\frac{\partial\mathcal{F}}{\partial x_i} - \frac{\partial\Psi}{\partial x_i}\frac{\partial\mathcal{F}}{\partial p_i} = 0, \quad \text{where } \mathcal{F}(\mathbf{x},\mathbf{p},t) = |\tilde{\Psi}(\mathbf{x},\mathbf{p},t)|^2. \tag{12}$$

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From the above equation, we can take momentum moments of the system and obtain:

$$\frac{\partial\rho}{\partial t} + \frac{\partial(\rho v_i)}{\partial x_j} = 0, \qquad \frac{\partial v_i}{\partial t} + v_j\frac{\partial v_i}{\partial x_j} + \frac{1}{\rho}\frac{\partial P_{ij}}{\partial x_j} + \frac{1}{m}\frac{\partial\Psi}{\partial x_i} = 0. \tag{13a}$$

The quantity P_{ij} is defined as $P_{ij} \equiv \rho\sigma_{ij}^2$, where σ_{ij}^2 represents the phase space velocity dispersion. This P_{ij} serves as an effective "pressure" term, analogous to the quantum pressure Q found in the exact QHD equations.

So what has been seen in structure formation simulations with scalar fields?

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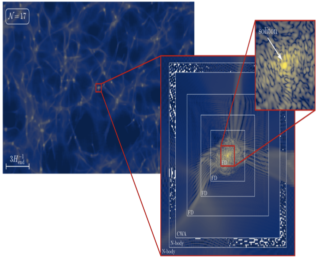
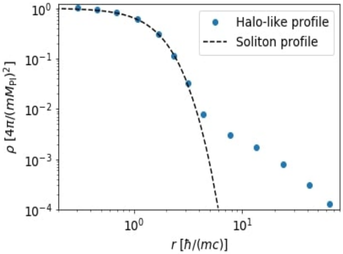
- In non-cosmological simulations with asymptotically empty boundary conditions, the scalar fields relax through a gravitational cooling mechanism to form a soliton configuration.
- Starting with initial conditions in the kinetic regime, the scalar field also forms localized soliton profiles.





From arXiv: 2110.15109v2.

- In cosmological, 3D simulations, it is found a central soliton (inflaton star) that, in mean, can be approximated with a standard soliton solution.
- At scales much larger than the deBroglie wavelength, it is found a NFW-like envelope (inflaton halo) formed by the balance between gravity and velocity dispersion.
- The soliton follows a core-halo mass relation:

$$\left(\frac{M_{\rm sol}(k)}{2.4\times 10^{-5}\ {\rm g}}\right) \simeq \frac{\rho_{11}^{1/6}(a_{\rm NL})}{m_5}\left(\frac{M_{\rm halo}(k)}{7.1\times 10^{-2}\ {\rm g}}\right)^{1/3}, \tag{14}$$

PBHs from slow-reheating

Different criteria

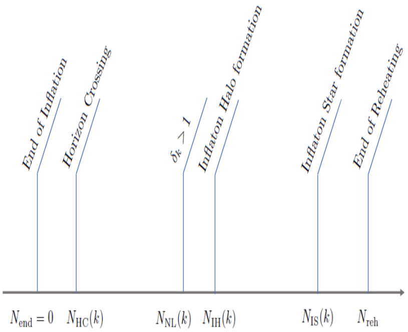
There have been different criteria for PBH formation during slow-reheating, most of them addopted from the conditions for PBH formation in a matter-dominated universe:

- Taking pioneer work in which $\delta_c \sim \omega \quad \Rightarrow$ PBHs form only if the slow-reheating lasted long enough for the perturbations to grow and collapse!
- PBHs form only if the initial perturbations are spherical enough $\Rightarrow$ $\beta \simeq 0.05556\mathcal{P}^{5/2}(k)$.
- Angular momentum can also play an important role $\Rightarrow$ $\beta \sim 1.9 \times 10^{-7}\mathcal{P}(k)\exp\left[-\frac{0.15}{\mathcal{P}^{1/3}(k)}\right]$ (for $\mathcal{P}^{1/2}(k) < 0.005$).
- Inhomogeneities and anisotropies can also play an important role $\Rightarrow$ $\beta \simeq 0.2055\mathcal{P}^{13}(k)$ (for $0.005 \lesssim \mathcal{P}^{1/2}(k) \lesssim 0.2$).
- PBHs form if the 'virial radius' of structures are smaller than its Schwarzschild radius:

$$\delta_{\rm th}^{\rm (halo)} \equiv 0.238, \qquad \delta_{\rm th}^{\rm (soliton)} \equiv 0.019. \tag{15}$$

Important: Even after considering all these effect, the formation of PBHs during slow-reheating is more likely to occur than in the radiation-dominated scenario.

Summary of the PBH formation process during slow-reheating



From arXiv: 2208.09462v2.

- After inflation, the perturbations must reenter the cosmological horizon.
- Reheating must last long enough for the perturbations to grow and become non-linear.
- A process of structure formation begins at this point. Various effects can prevent gravitational collapse into a PBH. If the perturbation does not form a PBH, it is expected to virialize and form an inflaton halo.
- At a later time, an inflaton star is expected to form in the central region of inflaton halos. A new mechanism for PBH formation could also occur through the collapse of these central regions.

Conclusions and discussions

- We have seen that during slow-reheating, the post-inflationary universe can be seen as a matter-dominated universe.
- At the level of structure formation, it is important the quantum force. We can obtain the formation of inflaton stars (produced by the balance of gravity and the quantum force) and NFW-like inflaton halos, where the effective velocity dispersion plays an important role.
- Perturbations can collapse to form PBHs, but there are different effects that needs to be consider to have a good estimation for PBH abundances.
- Most of the results are based in semianalytical calculations or Newtonian simulation results. A full GR numerical simulation needs to be done to have a better characterization of this scenario.
- The scenario of assuming a simple quadratic term is not completely realistic. We could also consider the effects of self-interactions in PBH formation.

Bibliography

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