Primordial correlators from multi-point propagators

Andrea Costantini
NEHOP'25 - Brussels
Based on 2503.15194
In collaboration with Laura Iacconi and David Mulryne

Cosmological inflation

- Period of accelerated expansion in the early universe, solves hot big bang issues and produces seed for the large-scale-structure

$$ds^2 = -dt^2 + a^2\left[(1-2\zeta)\delta_{ij} + \gamma_{ij}\right]dx^i dx^j$$

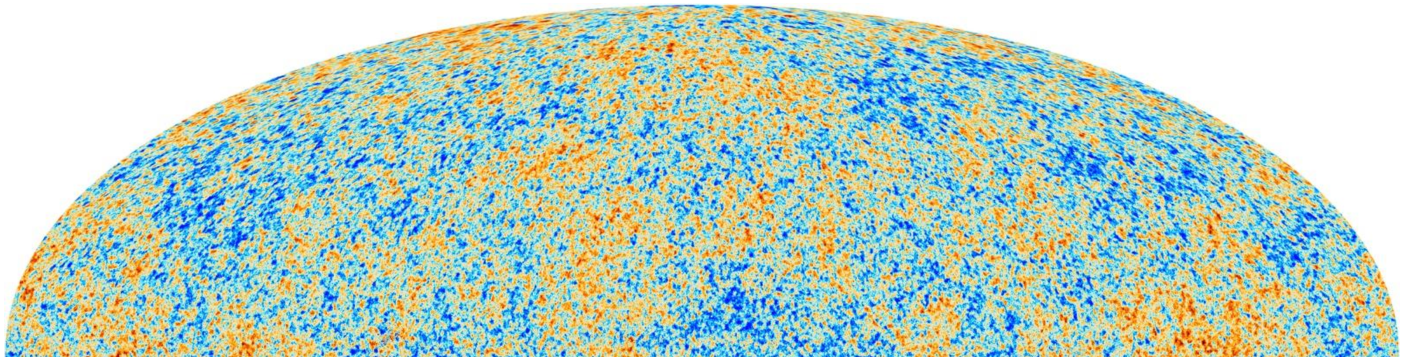
Curvature perturbation

Tensor perturbation

- 2- and 3-point correlators of the curvature perturbation ζ

$$\langle \zeta(\boldsymbol{k}_1)\zeta(\boldsymbol{k}_2)\rangle = (2\pi)^3\delta(\boldsymbol{k}_1 + \boldsymbol{k}_2)P_\zeta(k)$$

$$\langle \zeta(\boldsymbol{k}_1)\zeta(\boldsymbol{k}_2)\zeta(\boldsymbol{k}_3)\rangle \equiv (2\pi)^3\delta(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3)B_\zeta(k_1, k_2, k_3)$$



1807.06209 - Planck collaboration

Phase-space correlators

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left(R - \left(\partial_\mu\phi\right)^2 - \left(\partial_\mu\chi\right)^2 - \frac{1}{2}m_\phi^2\phi^2 - \frac{1}{2}m_\chi^2\chi^2\right)^1$$

1) PhysRevD.35.419 – Silk, Turner

Phase-space correlators

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left(R - (\partial_\mu\phi)^2 - (\partial_\mu\chi)^2 - \frac{1}{2}m_\phi^2\phi^2 - \frac{1}{2}m_\chi^2\chi^2\right)^{1}$$

$$\begin{cases} \phi = \phi_0 + \delta\phi \\ \pi_\phi = \dfrac{\delta S}{\delta[\partial_\mu\phi]} = \pi_{\phi,0} + \delta\pi_\phi \end{cases}$$

$$\begin{cases} \chi = \chi_0 + \delta\chi \\ \pi_\chi = \dfrac{\delta S}{\delta[\partial_\mu\chi]} = \pi_{\chi,0} + \delta\pi_\chi \end{cases}$$

---

1) PhysRevD.35.419 – Silk, Turner

Phase-space correlators

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left(R - (\partial_\mu\phi)^2 - (\partial_\mu\chi)^2 - \frac{1}{2}m_\phi^2\phi^2 - \frac{1}{2}m_\chi^2\chi^2\right)^{1}$$

$$\begin{cases} \phi = \phi_0 + \delta\phi \\ \pi_\phi = \dfrac{\delta S}{\delta[\partial_\mu\phi]} = \pi_{\phi,0} + \delta\pi_\phi \end{cases}$$

$$\begin{cases} \chi = \chi_0 + \delta\chi \\ \pi_\chi = \dfrac{\delta S}{\delta[\partial_\mu\chi]} = \pi_{\chi,0} + \delta\pi_\chi \end{cases}$$

$$\delta X^a \equiv (\delta\phi, \partial\chi, \delta\pi_\phi, \delta\pi_\chi)$$

1) PhysRevD.35.419 – Silk, Turner

Phase-space correlators

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left(R - \left(\partial_\mu\phi\right)^2 - \left(\partial_\mu\chi\right)^2 - \frac{1}{2}m_\phi^2\phi^2 - \frac{1}{2}m_\chi^2\chi^2\right)$$ [1]

$$\begin{cases} \phi = \phi_0 + \delta\phi \\ \pi_\phi = \dfrac{\delta S}{\delta[\partial_\mu\phi]} = \pi_{\phi,0} + \delta\pi_\phi \end{cases}$$

$$\begin{cases} \chi = \chi_0 + \delta\chi \\ \pi_\chi = \dfrac{\delta S}{\delta[\partial_\mu\chi]} = \pi_{\chi,0} + \delta\pi_\chi \end{cases}$$

$$\delta X^a \equiv (\delta\phi, \partial\chi, \delta\pi_\phi, \delta\pi_\chi) \Rightarrow \boxed{\zeta(k) = N_a(k)\delta X^a + \frac{1}{2}N_{ab}(k)\delta X^a\delta X^b + \ldots}$$

---

1) PhysRevD.35.419 – Silk, Turner

Phase-space correlators

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left(R - (\partial_\mu\phi)^2 - (\partial_\mu\chi)^2 - \frac{1}{2}m_\phi^2\phi^2 - \frac{1}{2}m_\chi^2\chi^2\right)$$ [1]

$$\begin{cases} \phi = \phi_0 + \delta\phi \\ \pi_\phi = \dfrac{\delta S}{\delta[\partial_\mu\phi]} = \pi_{\phi,0} + \delta\pi_\phi \end{cases} \qquad \begin{cases} \chi = \chi_0 + \delta\chi \\ \pi_\chi = \dfrac{\delta S}{\delta[\partial_\mu\chi]} = \pi_{\chi,0} + \delta\pi_\chi \end{cases}$$

$$\delta X^a \equiv (\delta\phi, \partial\chi, \delta\pi_\phi, \delta\pi_\chi) \Rightarrow \boxed{\zeta(\mathrm{k}) = \mathrm{N_a}(\mathrm{k})\delta\mathrm{X^a} + \frac{1}{2}\mathrm{N_{ab}}(\mathrm{k})\delta\mathrm{X^a}\delta\mathrm{X^b} + \ldots}$$

$$\boxed{\langle\delta X^a\delta X^b\rangle \equiv (2\pi)^3\delta(\boldsymbol{k}_a + \boldsymbol{k}_b)\Sigma^{ab}(k)} \qquad \boxed{\langle\delta X^a\delta X^b\delta X^c\rangle \equiv (2\pi)^3\delta(\boldsymbol{k}_a + \boldsymbol{k}_b + \boldsymbol{k}_c)\mathrm{B}^{abc}(k_a, k_b, k_c)}$$

---

1) PhysRevD.35.419 – Silk, Turner

How to compute phase-space correlators

In-In formalism[2]

$$\Sigma^{ab}(k) \equiv \langle 0 | \delta x^a \delta x^b | 0 \rangle,$$

$$\mathrm{B}^{abc}(k_a, k_b, k_c) \equiv$$

$$\left\langle 0 \left| \frac{i}{3!} \int_{-\infty}^{t} dt' H_{def} \left[\delta x^d \delta x^e \delta x^f, \delta x^a \delta x^b \delta x^c \right] \right| 0 \right\rangle$$

Transport formalism[3]

$$\frac{\mathrm{d}}{\mathrm{d}N} \Sigma^{ab}(k) = u^a_c \Sigma^{cb}(k) + u^b_c \Sigma^{ac}(k)$$

$$\frac{\mathrm{d}}{\mathrm{d}N} \mathrm{B}^{abc}(k_a, k_b, k_c) = u^a_d(k_a) \mathrm{B}^{dbc}(k_a, k_b, k_c) +$$

$$+ u^a_{de}(k_a, k_b, k_c) \Sigma^{db}(k_b) \Sigma^{ec}(k_c) + 2 \text{ perms}$$

2) 0506236 - Weinberg

3) 1609.00379 – Dias et al.

Multi-point propagators

A different formulation of the Transport formalism can be implemented using the multi-point propagators (**MPPs**)[4]

$$\delta X^{a}(N,\boldsymbol{k}) = \Gamma_{i}^{a\,(N_0,N)}(k)\,\delta X^{i}(N_0,\boldsymbol{k}) + \frac{1}{2!}\int \frac{d^3k_1}{(2\pi)^3}\,\Gamma_{ij}^{a\,(N_0,N)}(k,k_1,|\boldsymbol{k}-\boldsymbol{k_1}|)\,\delta X^{i}(N_0,\boldsymbol{k_1})\,\delta X^{j}(N_0,\boldsymbol{k}-\boldsymbol{k_1}) + \dots$$

4) 1203.2635 – Seery at al.

Multi-point propagators

A different formulation of the Transport formalism can be implemented using the multi-point propagators (**MPPs**)[4]

$$\delta X^a(N,\boldsymbol{k}) = \Gamma_i^{a\,(N_0,N)}(k)\,\delta X^i(N_0,\boldsymbol{k}) + \frac{1}{2!}\int \frac{d^3k_1}{(2\pi)^3}\,\Gamma_{ij}^{a\,(N_0,N)}(k,k_1,|\boldsymbol{k}-\boldsymbol{k_1}|)\,\delta X^i(N_0,\boldsymbol{k_1})\,\delta X^j(N_0,\boldsymbol{k}-\boldsymbol{k_1}) + \ldots$$

$$\frac{\mathrm{d}\Gamma_i^{a\,(N,N_0)}(k)}{\mathrm{d}N} = u_b^{a\,(N)}(k)\Gamma_i^{b\,(N,N_0)}(k)$$

$$\frac{\mathrm{d}\Gamma_{ij}^{a\,(N,N_0)}(k_1,k_2,k_3)}{\mathrm{d}N} = u_b^{a\,(N)}(k_1)\Gamma_{ij}^{b\,(N,N_0)}(k_1,k_2,k_3) + u_{bc}^{a\,(N)}(k_1,k_2,k_3)\Gamma_i^{b\,(N,N_0)}(k_2)\Gamma_j^{c\,(N,N_0)}(k_3).$$

---

4) 1203.2635 – Seery at al.

Correlators from multi-point propagators

Then, we can use the Γ's to propagate the two- and three-point correlators

$$\Sigma^{ab\,(N)}(k) = \Gamma_i^{a\,(N,N_0)}(k)\Gamma_j^{b\,(N,N_0)}(k)\Sigma_{ij}^{(N_0)}(k),$$

$$\begin{aligned}\mathrm{B}^{abc\,(N)}(k_1,k_2,k_3) = {} & \Gamma_i^{a\,(N,N_0)}(k_1)\Gamma_j^{b\,(N,N_0)}(k_2)\Gamma_k^{c\,(N,N_0)}(k_3)\mathrm{B}_{ijk}^{(N_0)}(k_1,k_2,k_3) \\ & +\Gamma_{ij}^{a\,(N,N_0)}(k_1,k_2,k_3)\Gamma_k^{b\,(N,N_0)}(k_2)\Gamma_l^{c\,(N,N_0)}(k_3)\Sigma_{ik}^{(N_0)}(k_2)\Sigma_{jl}^{(N_0)}(k_3) \\ & +\ 2\ \text{perms}\end{aligned}$$

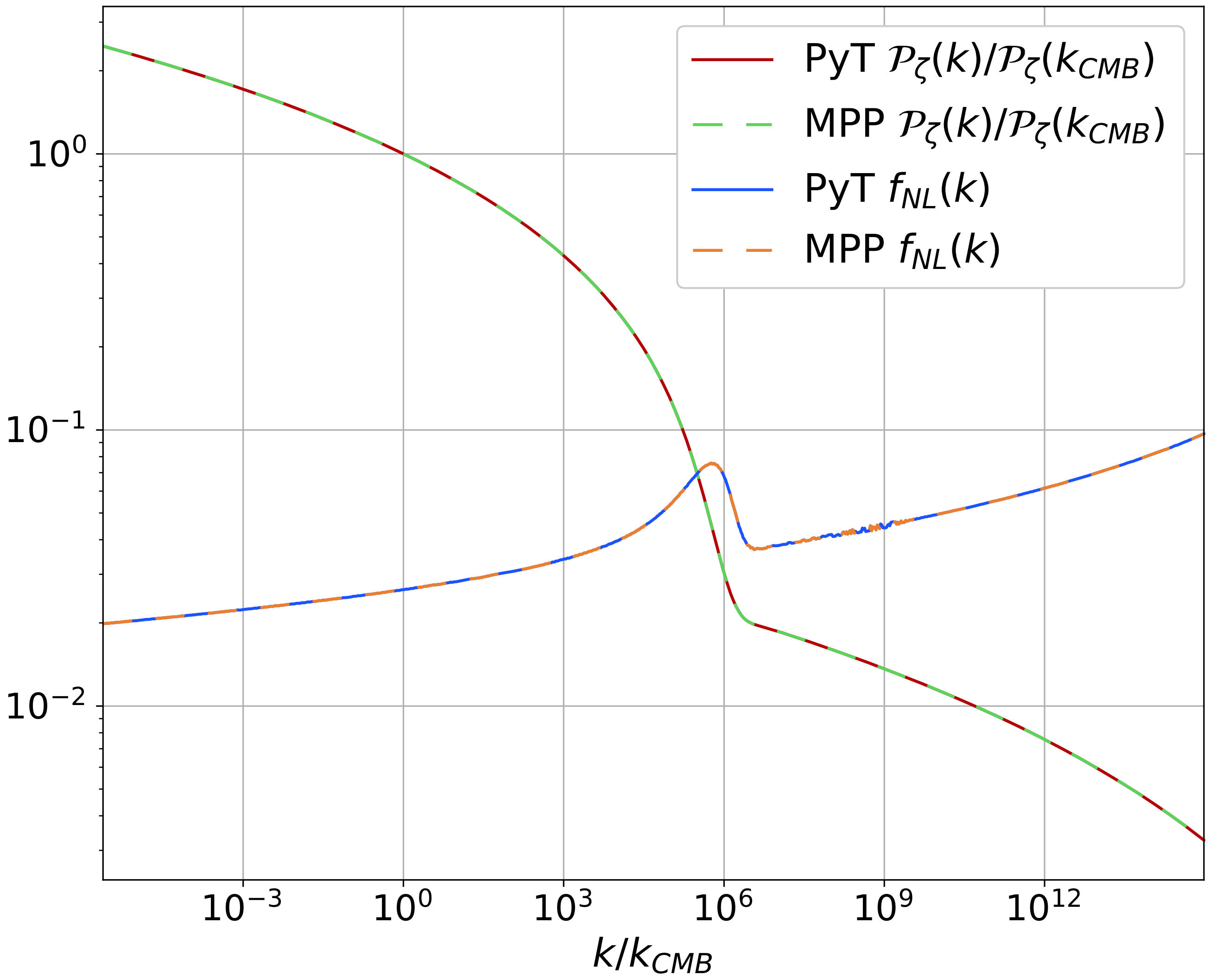
PyTransport 3.0

- Is a package that computes the evolution of the 2- and 3-point correlation functions at tree-level, for Multi-field inflationary models with curved field-space.

- In the last update we included functions to compute the evolution of the correlation functions using the MPPs.

Tests: Double Inflation

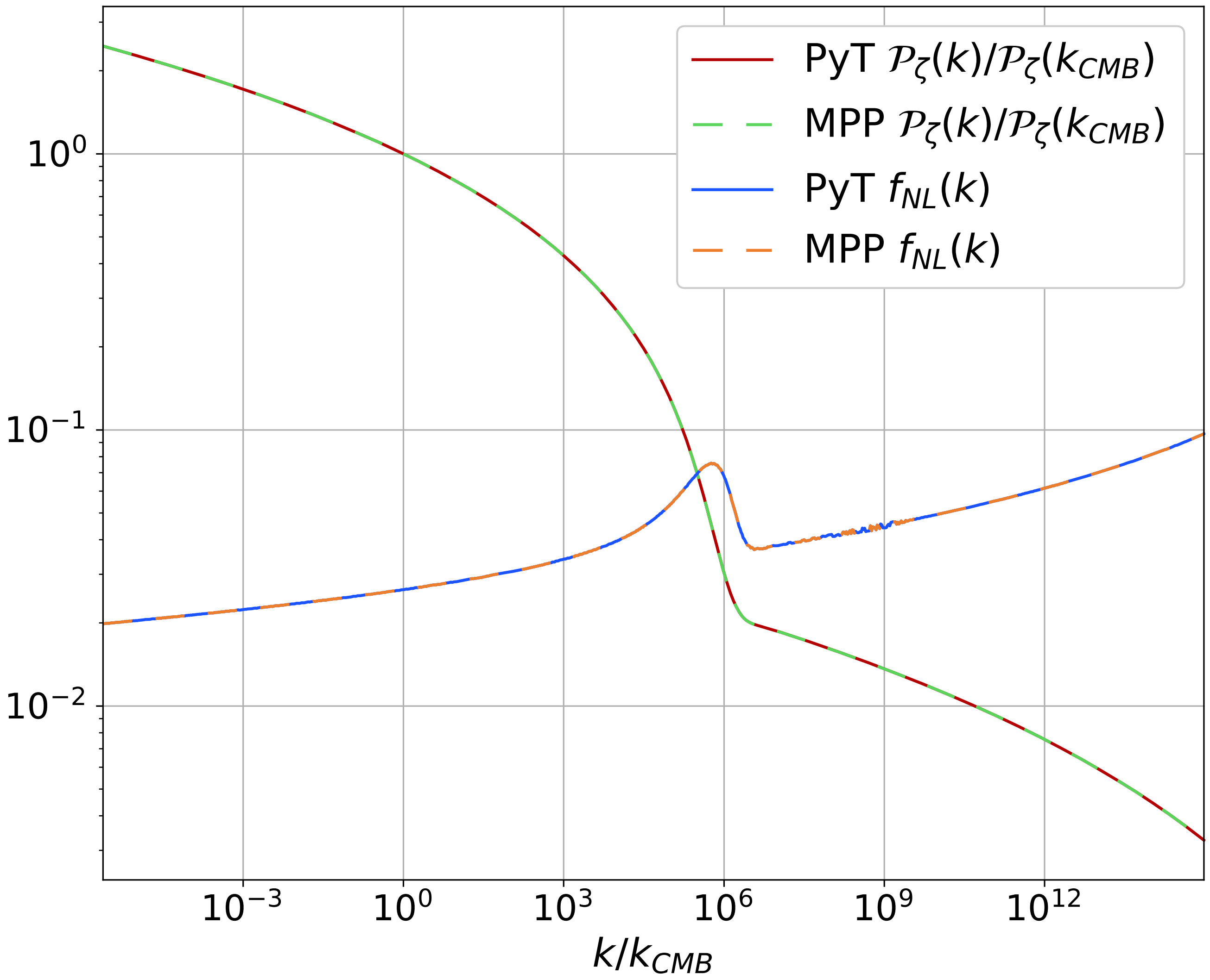
$$V = \frac{1}{2}m_\chi^2\chi^2 + \frac{1}{2}m_\phi^2\phi^2$$



Tests: Double Inflation

$$V = \frac{1}{2} m_\chi^2 \chi^2 + \frac{1}{2} m_\phi^2 \phi^2$$

$$\mathcal{P}_\zeta(k) \equiv \frac{k^3}{2\pi^2} P_\zeta(k)$$

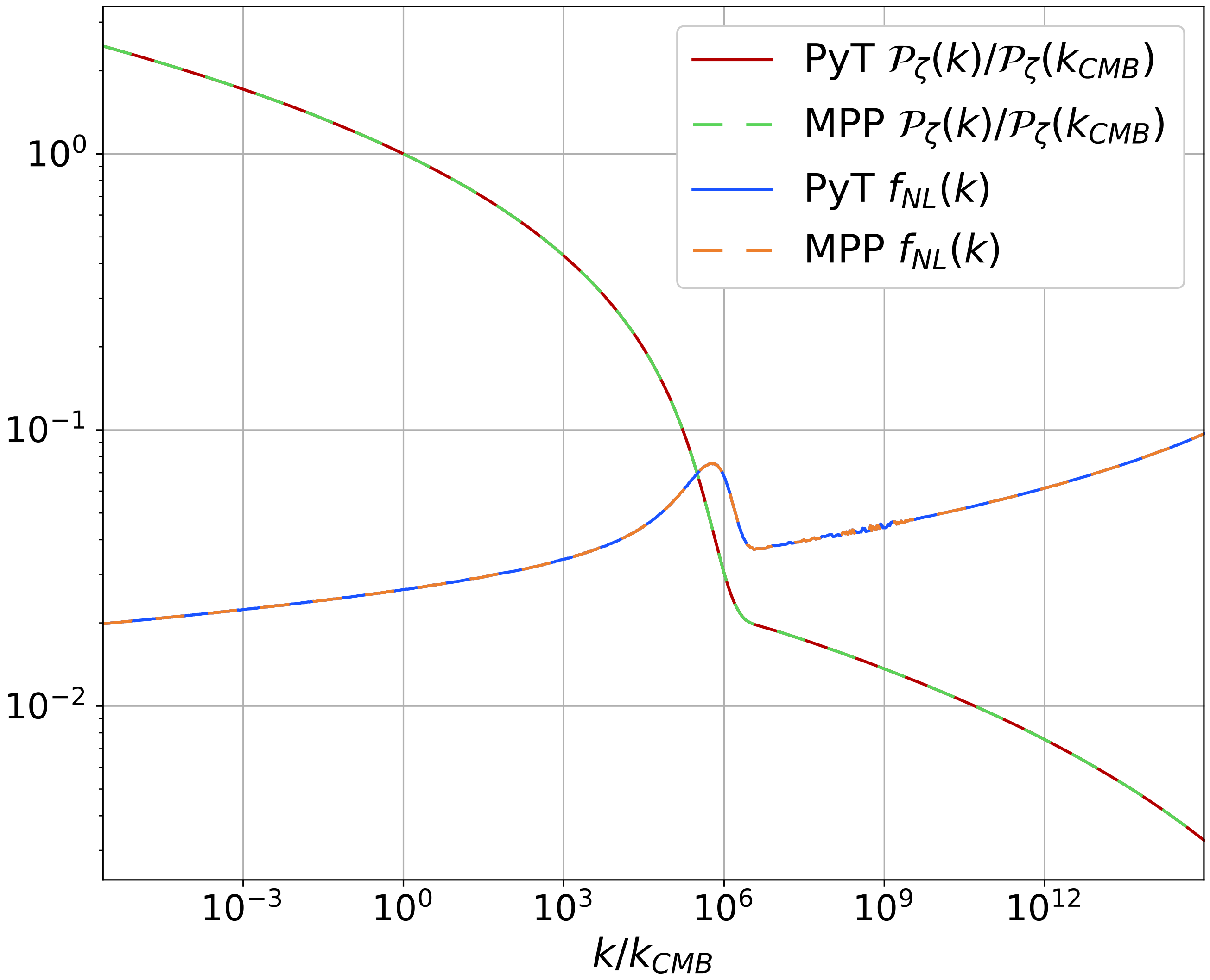


Tests: Double Inflation

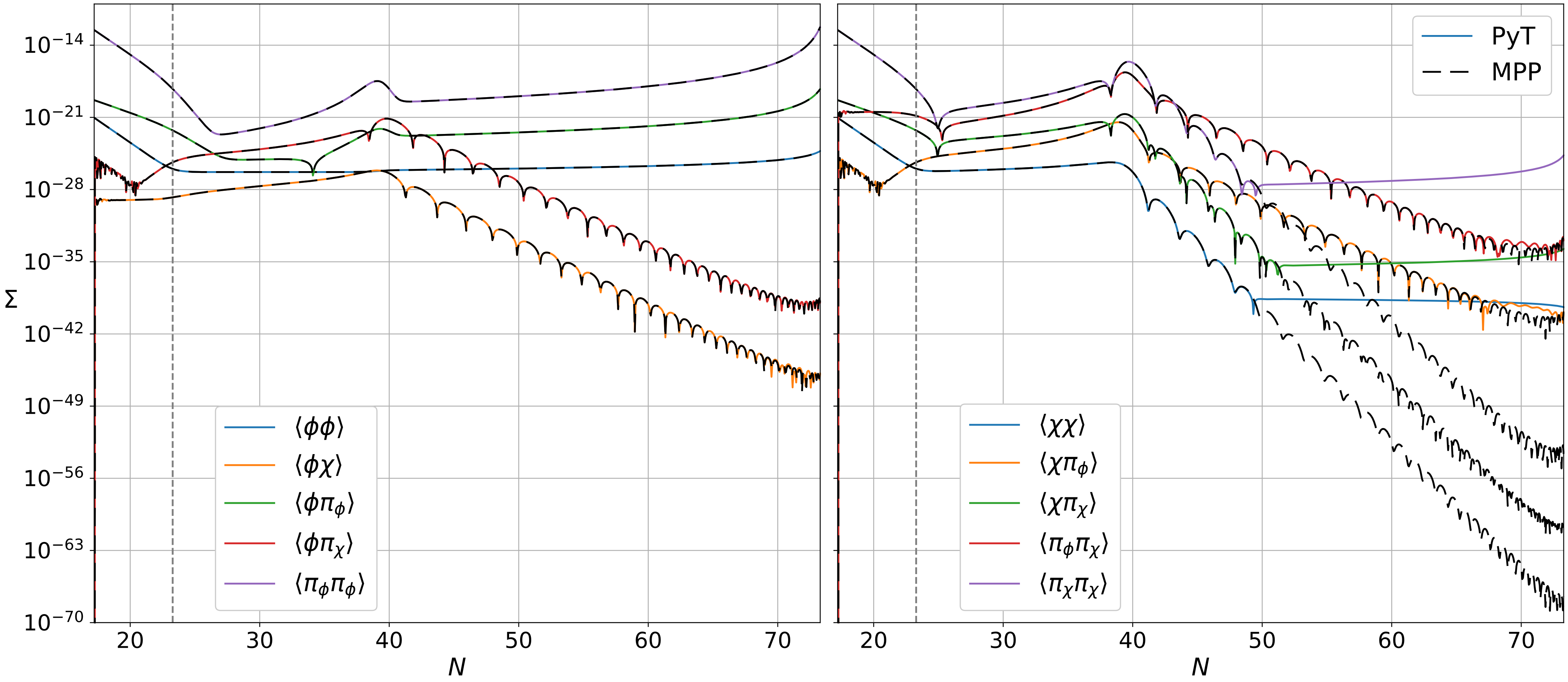
$$V = \frac{1}{2} m_\chi^2 \chi^2 + \frac{1}{2} m_\phi^2 \phi^2$$

$$\mathcal{P}_\zeta(k) \equiv \frac{k^3}{2\pi^2}\, P_\zeta(k)$$

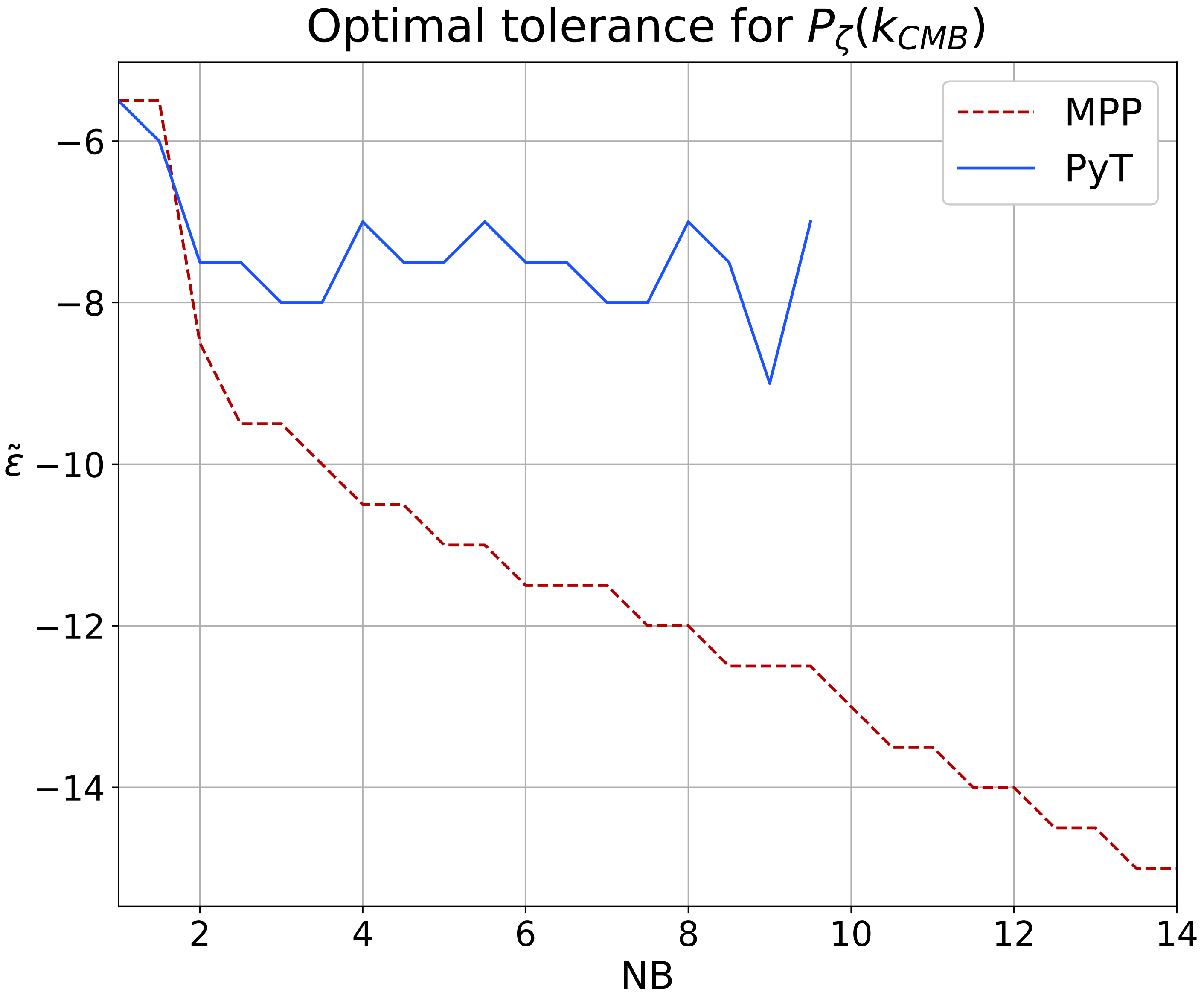
$$f_{\mathrm{NL}} \equiv \frac{5}{6} \frac{B_\zeta(k_1,k_2,k_3)}{P_\zeta(k_1)P_\zeta(k_2)+P_\zeta(k_2)P_\zeta(k_3)+P_\zeta(k_1)P_\zeta(k_3)}$$



Tests: 2pt evolution

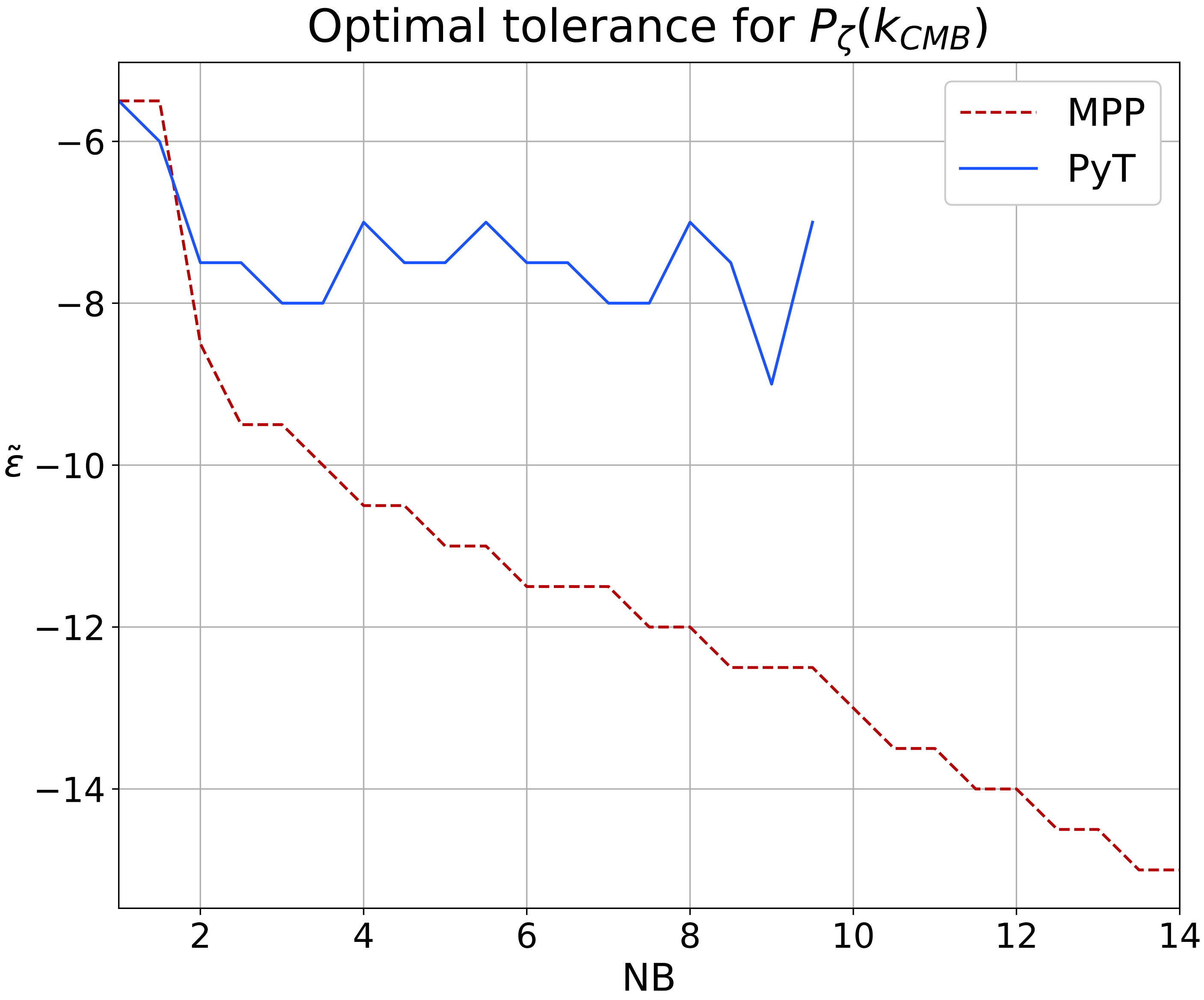


Tests: precision



Tests: precision

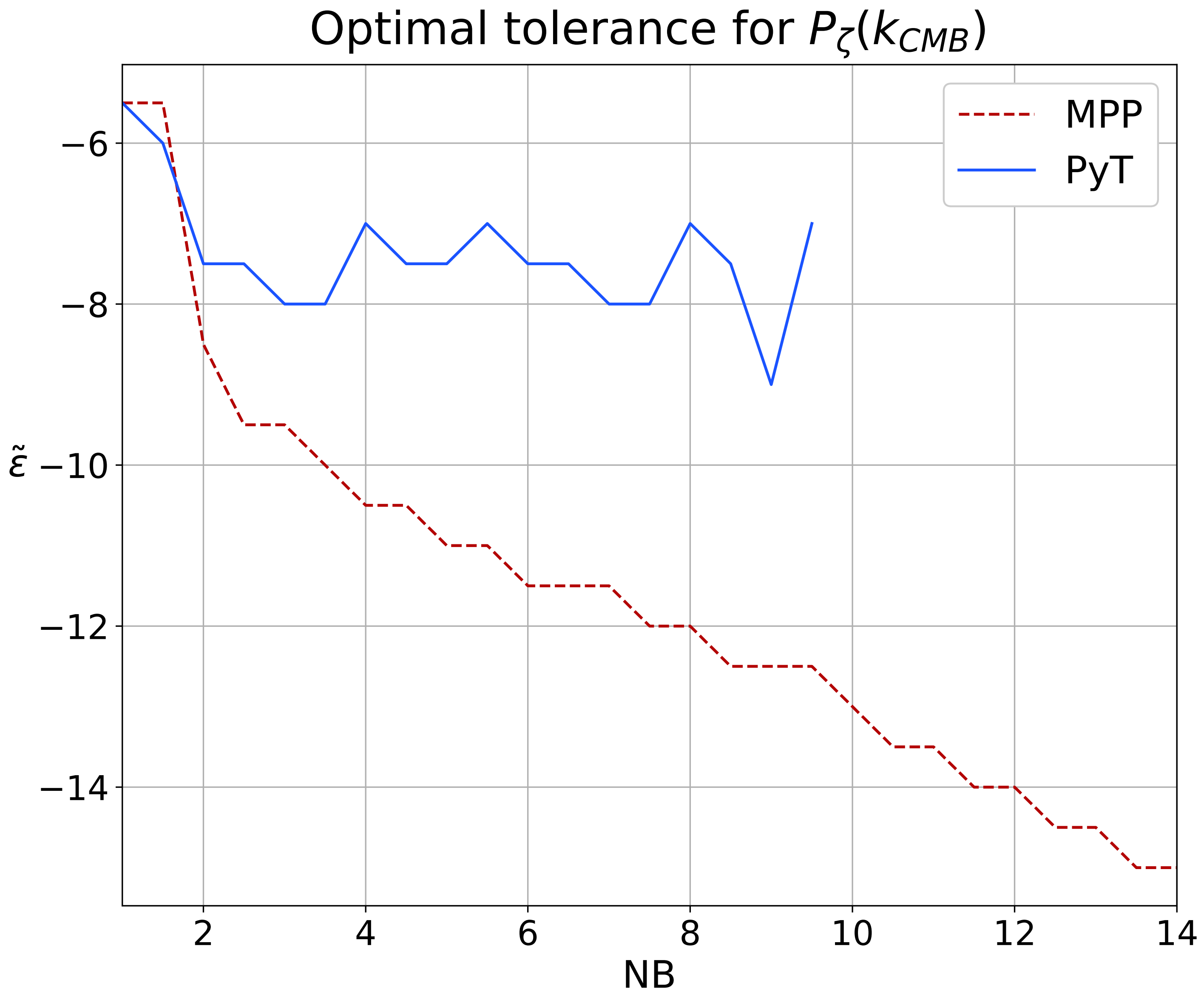
NB ≡ e-folds of sub-horizon evolution



Tests: precision

NB ≡ e-folds of sub-horizon evolution

$$\epsilon \;=\; \log_{10}(tol)$$

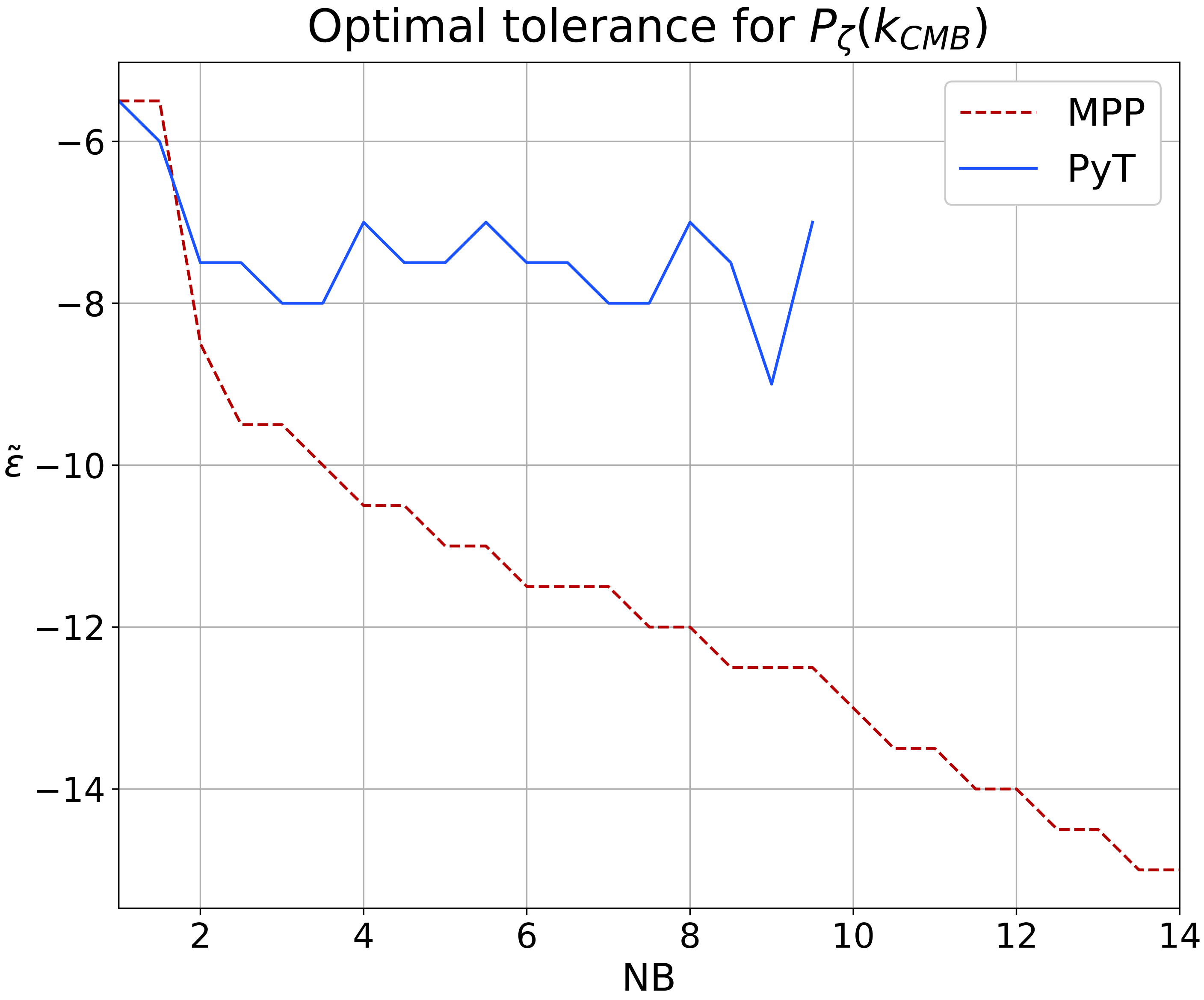


Tests: precision

NB ≡ e-folds of sub-horizon evolution

$$\epsilon = \log_{10}(tol)$$

$$\Delta\mathcal{P}_\zeta = \left|\frac{{\mathcal{P}_\zeta}^{(n-1)} - {\mathcal{P}_\zeta}^{(n)}}{{\mathcal{P}_\zeta}^{(n)}}\right|$$



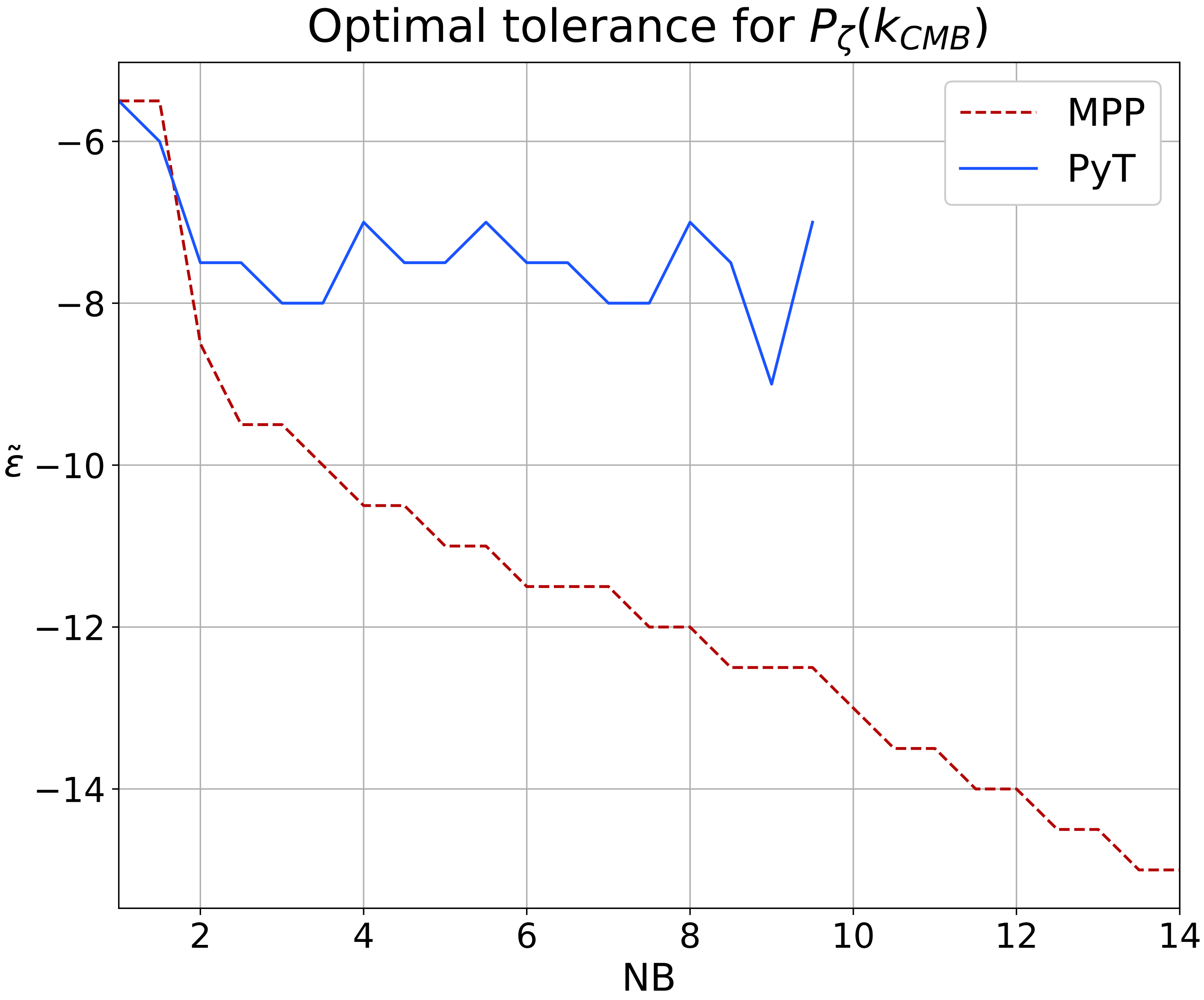
Tests: precision

NB ≡ e-folds of sub-horizon evolution

$$\epsilon \;=\; \log_{10}(tol)$$

$$\Delta\mathcal{P}_\zeta = \left|\frac{\mathcal{P}_\zeta{}^{(n-1)} - \mathcal{P}_\zeta{}^{(n)}}{\mathcal{P}_\zeta{}^{(n)}}\right|$$

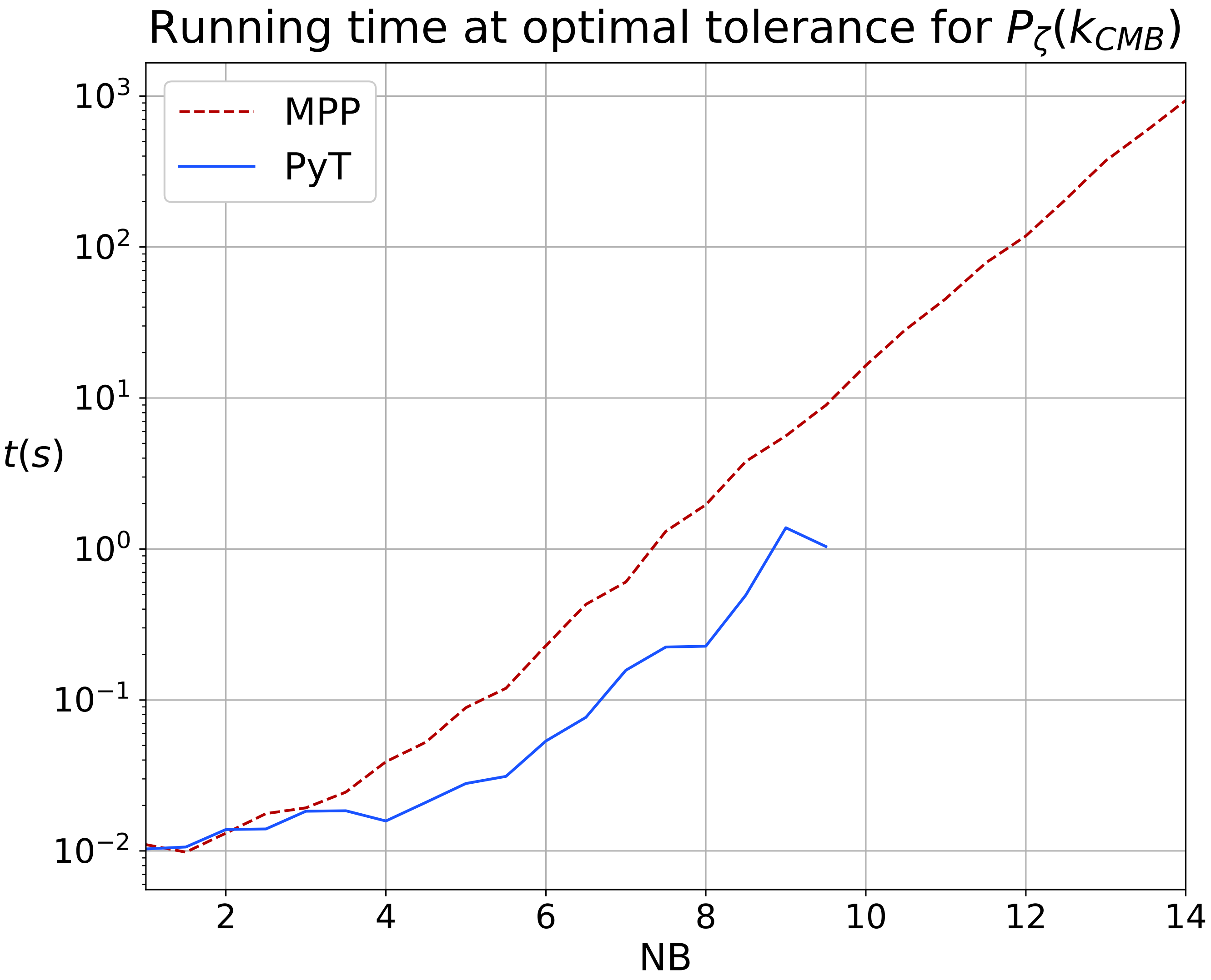
$$\tilde{\epsilon} \to \Delta\mathcal{P}_\zeta(\epsilon) < 10^{-8}$$



Tests: running time

$$\mathcal{P}_\zeta(k) \equiv \frac{k^3}{2\pi^2} P_\zeta(k)$$

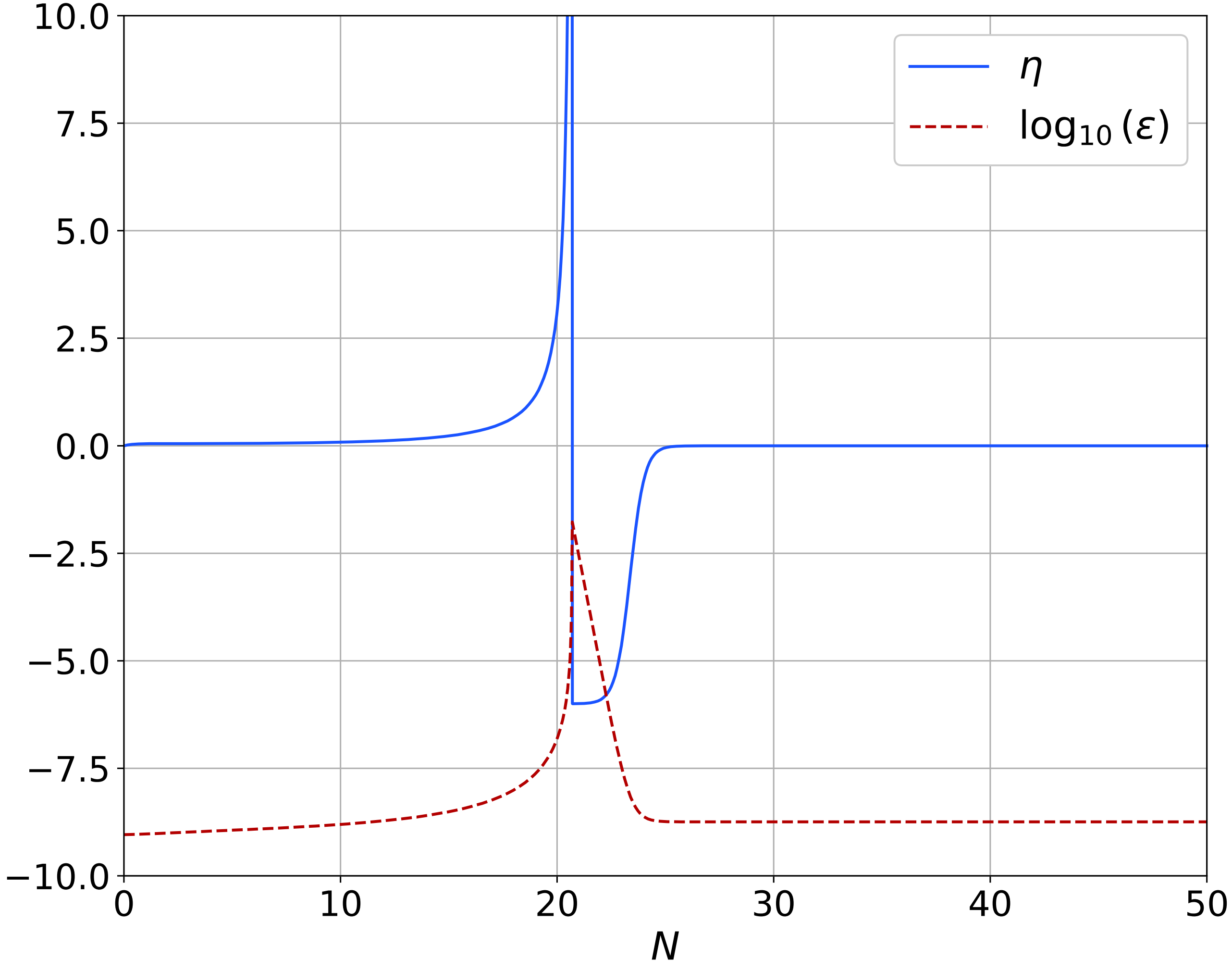
$$\tilde{\epsilon} \to \Delta\mathcal{P}_\zeta(\epsilon) < 10^{-8}$$



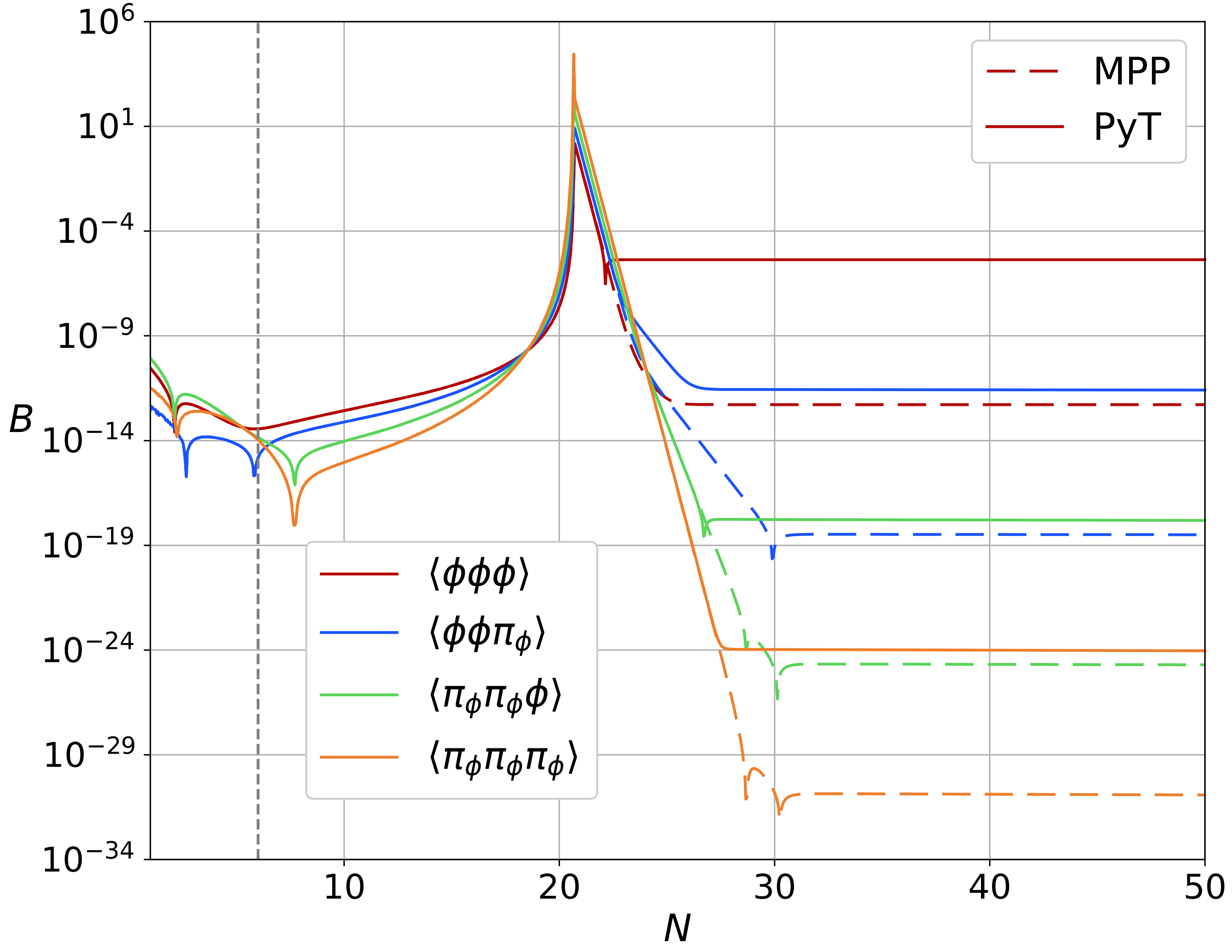
Ultra-slow-roll model

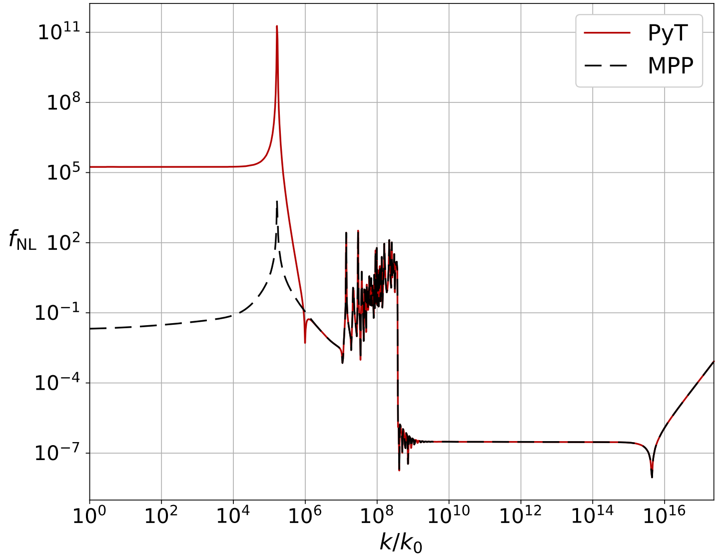
USR phase → $\epsilon \ll 1,\ \eta \sim -6$

$$\epsilon = -\frac{\dot{H}}{H^2} \qquad \eta = \frac{\dot{\epsilon}}{\epsilon H}$$

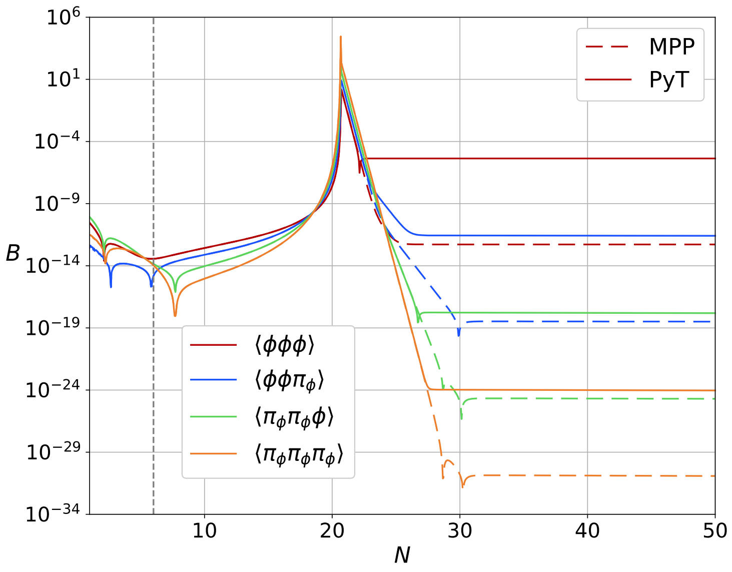


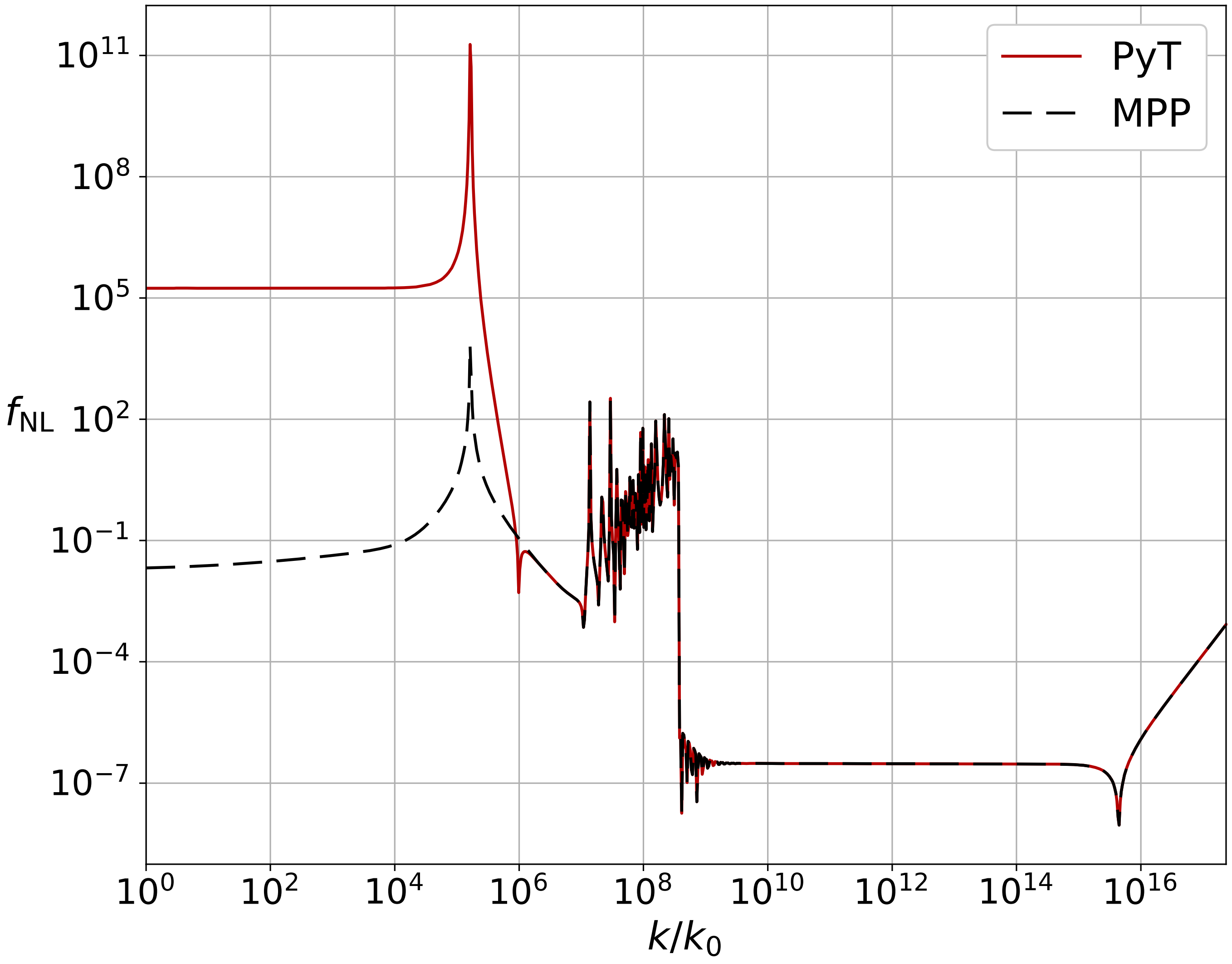
USR model: 3-point correlators



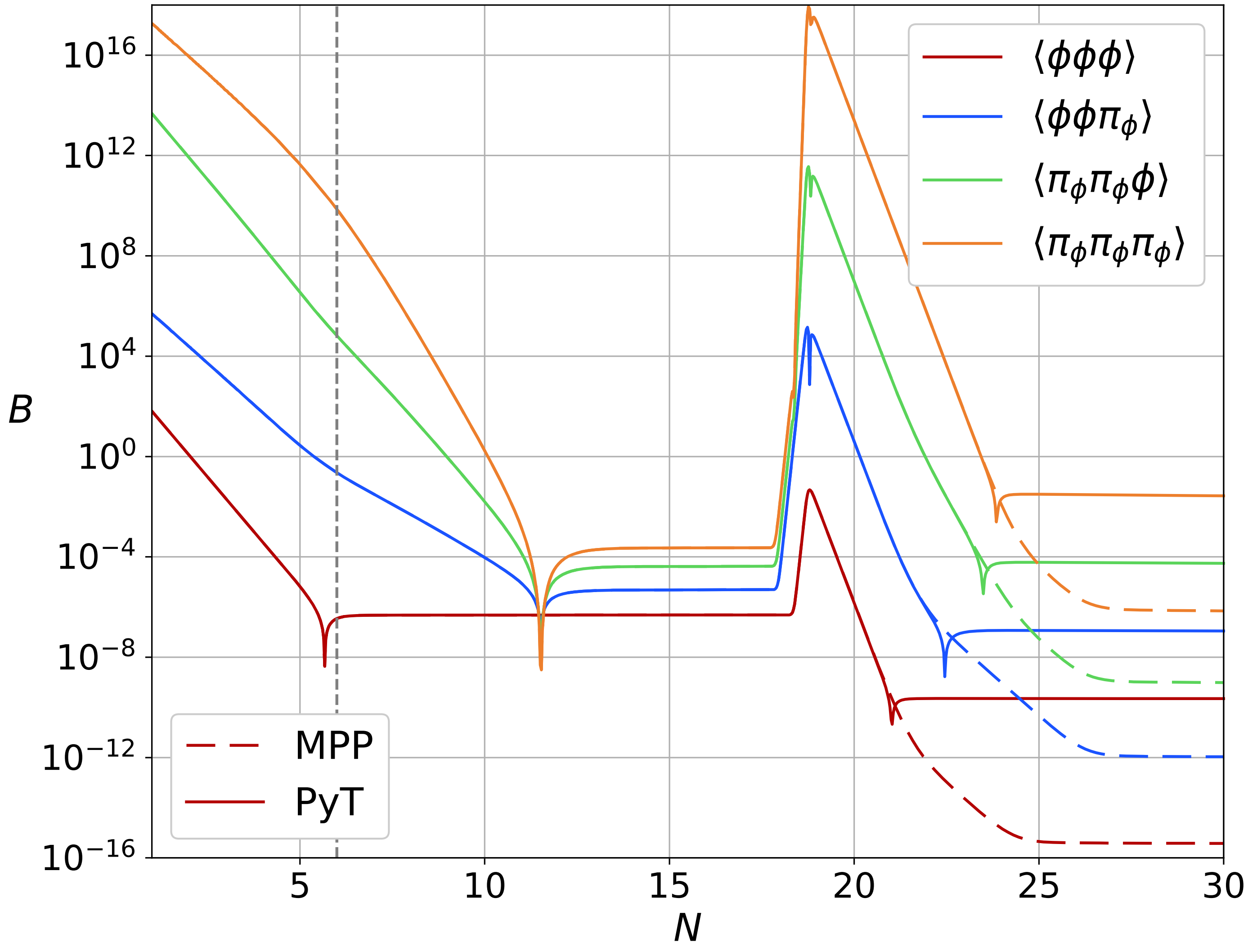


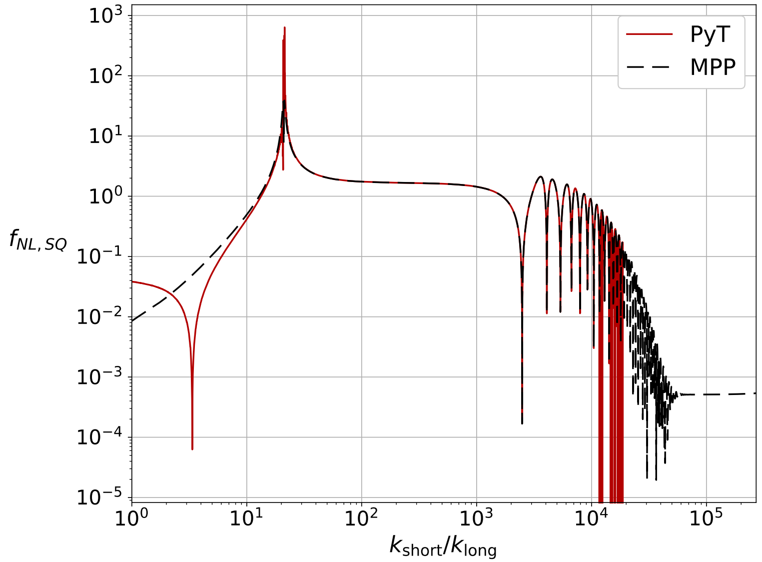
USR model : 3-point correlators



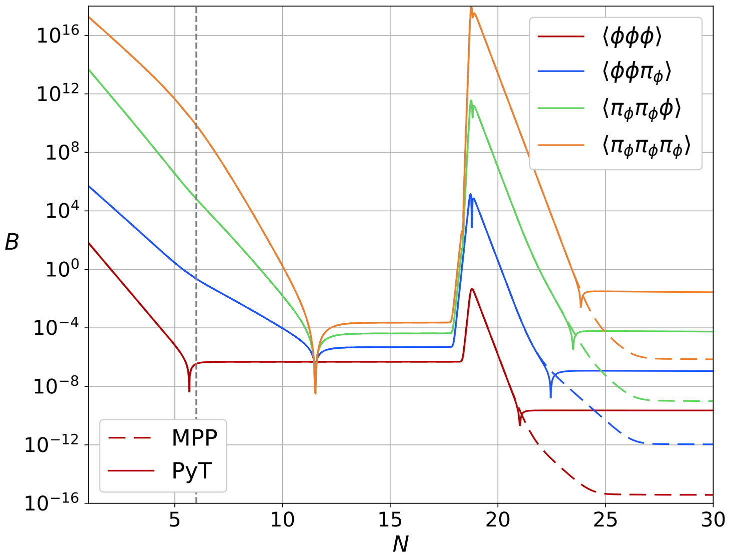


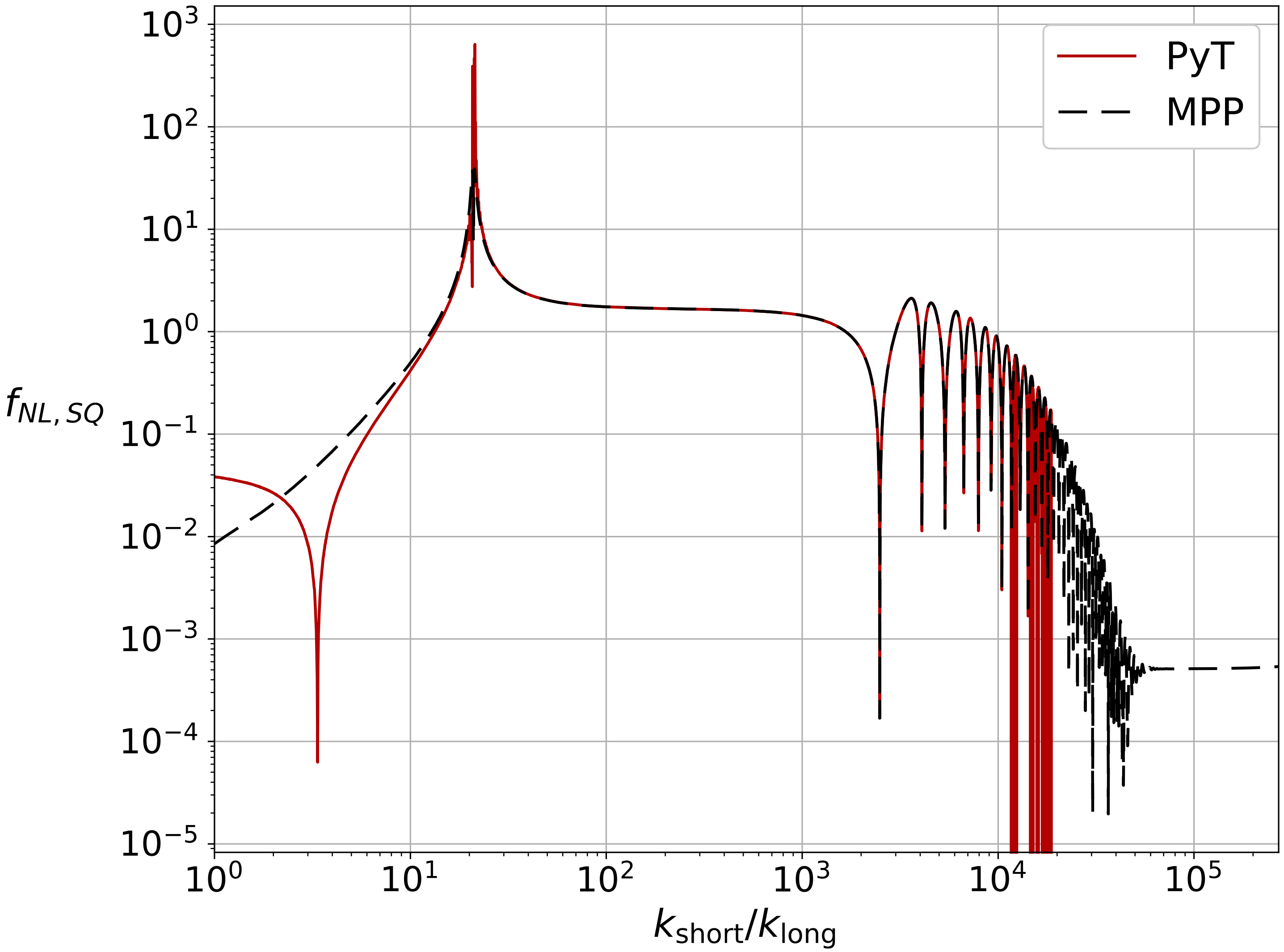
USR model: squeezed bispectrum





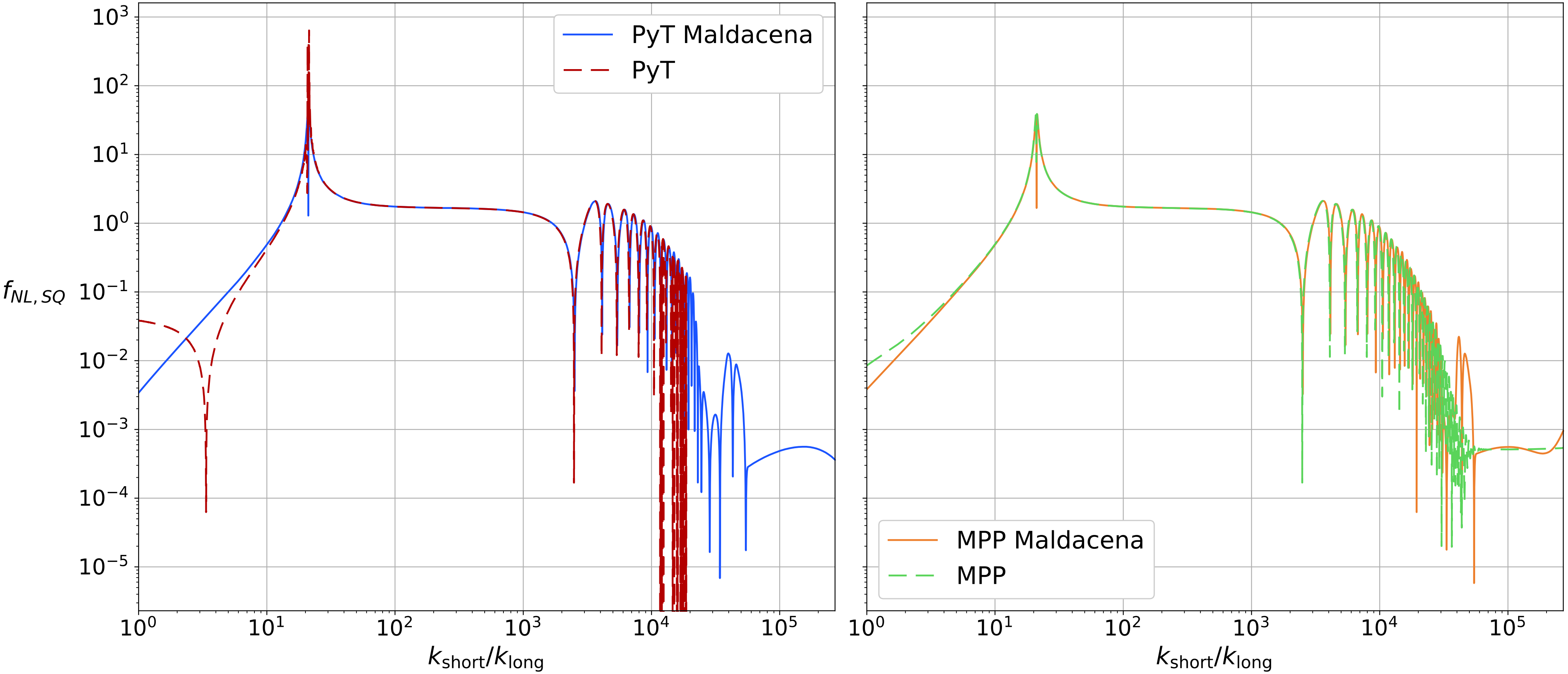
USR model: squeezed bispectrum





USR squeezed bispectrum: Maldacena consistency relation

$$f_{\mathrm{NL,sq}}\left(k_{\mathrm{long}}, k_{\mathrm{short}}\right) = \frac{5}{12}\left[1 - n_s\left(k_{\mathrm{short}}\right)\right] \quad \Rightarrow \quad \boxed{n_s = \frac{\mathrm{d}\log \mathcal{P}_\zeta(k)}{\mathrm{d}\log k}}$$



Conclusion

- We implemented into PyTransport the MPP formalism
- Tested the accuracy and the performance of the MPPs vs PyTransport
- Verified that the MPP approach can track better the dacay of correlators
- Verified that the MPP can correctly compute the bispectrum in cases where PyTransport fails

Future perspectives

- Use the MPPs to compute the power-spectrum and the bispectrum at **1-loop**
- Use the MPPs to compute the trispectrum at tree-level

$$
\begin{aligned}
\delta X^a(N,\boldsymbol{k}) = {} & \Gamma_i^{a\,(N_0,N)}(k)\,\delta X^i(N_0,\boldsymbol{k}) + \frac{1}{2!}\int \frac{\mathrm{d}^3 k_1}{(2\pi)^3}\,\Gamma_{ij}^{a\,(N_0,N)}(k,k_1,|\boldsymbol{k}-\boldsymbol{k_1}|)\,\delta X^i(N_0,\boldsymbol{k_1})\,\delta X^j(N_0,\boldsymbol{k}-\boldsymbol{k_1}) \\
& + \frac{1}{3!}\int \frac{\mathrm{d}^3 k_1}{(2\pi)^3}\int \frac{\mathrm{d}^3 k_2}{(2\pi)^3}\,\Gamma_{ijk}^{a\,(N_0,N)}(k,k_1,k_2,|\boldsymbol{k}-\boldsymbol{k_1}-\boldsymbol{k_2}|)\,\delta X^i(N_0,\boldsymbol{k_1})\,\delta X^j(N_0,\boldsymbol{k_2})\delta X^k(N_0,\boldsymbol{k}-\boldsymbol{k_1}-\boldsymbol{k_2}) + \cdots
\end{aligned}
$$