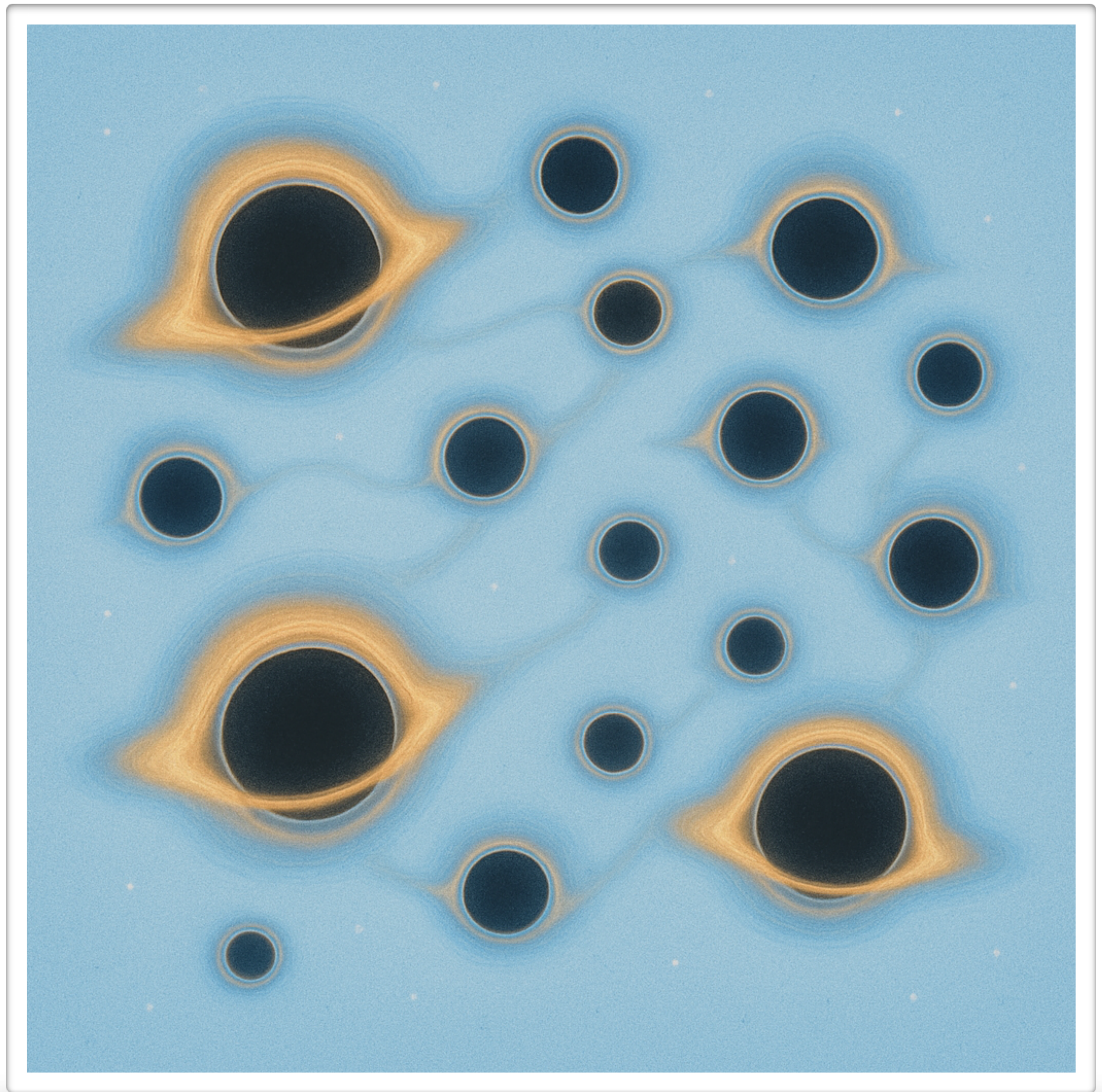
The Irrelevance of Primordial Black Hole Clustering in the LVK mass range



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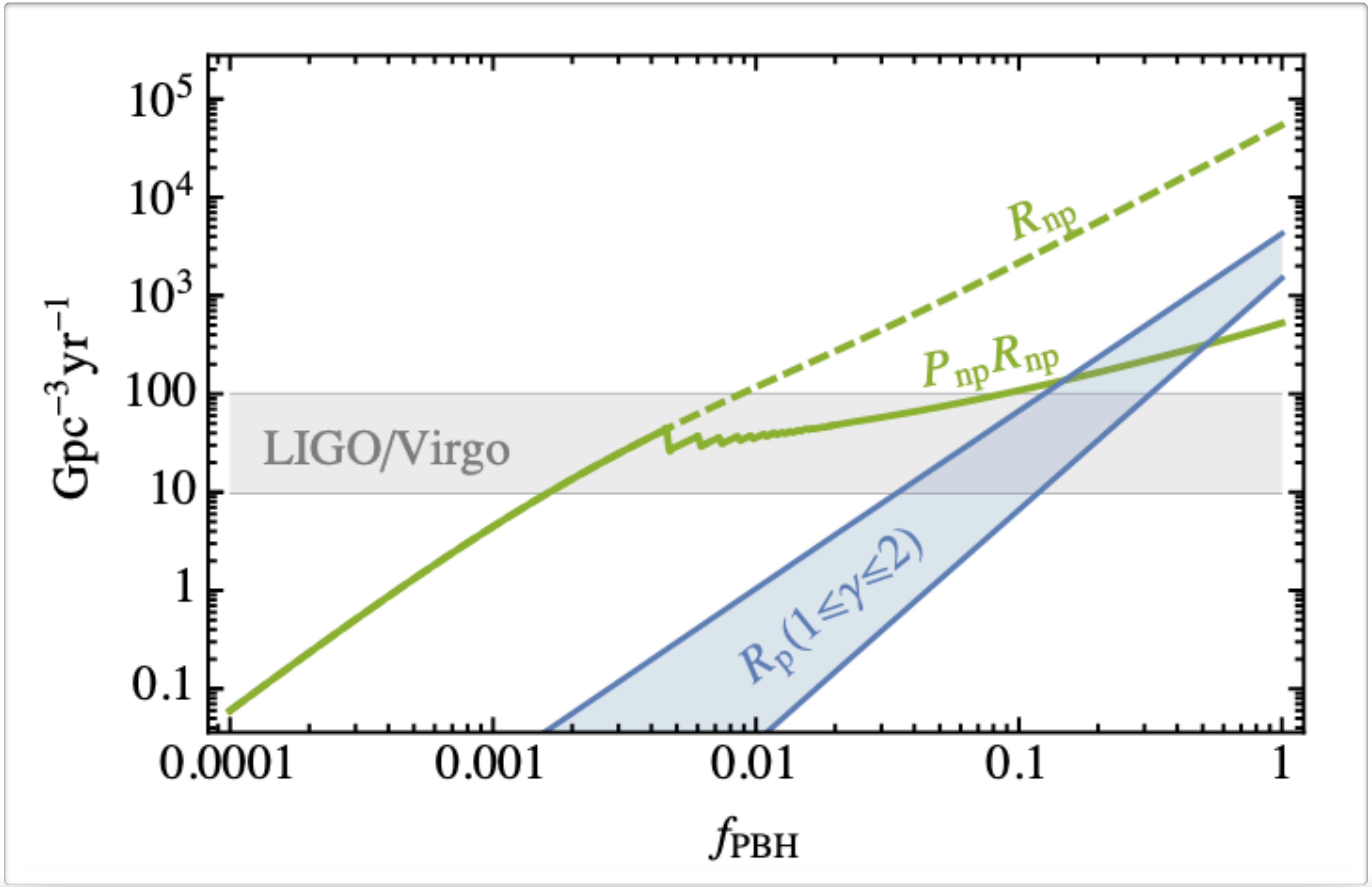
Does the initial clustering change the merger rate ?



R_{np} := Two Body merger rate

R_p := Three Body merger rate

P_{np} := Probability of non-perturbation of the Binary

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Outline

- The PBH clustering
- Typical Non-Gaussianity and clustering
- Merger rate scales and clustering

The PBH Clustering

- The PBH overdensity definition

$$\delta_{\rm PBH}(\vec{x}) = \frac{1}{\bar{n}_{\rm PBH}} \sum_i \delta_D(\vec{x} - \vec{x}_i) - 1$$

- The two-point correlation function

$$\left\langle \delta_{\rm PBH}(\vec{x})\, \delta_{\rm PBH}(0) \right\rangle = \frac{1}{\bar{n}_{\rm PBH}} \delta_D(\vec{x}) + \xi_{\rm PBH}(x)$$

- The linear bias definition

$$\xi_{\rm PBH}(x) \approx b_1^2\, \xi_r(x)$$

The PBH Clustering

- The maximum Clustering: the Broad curvature power spectrum

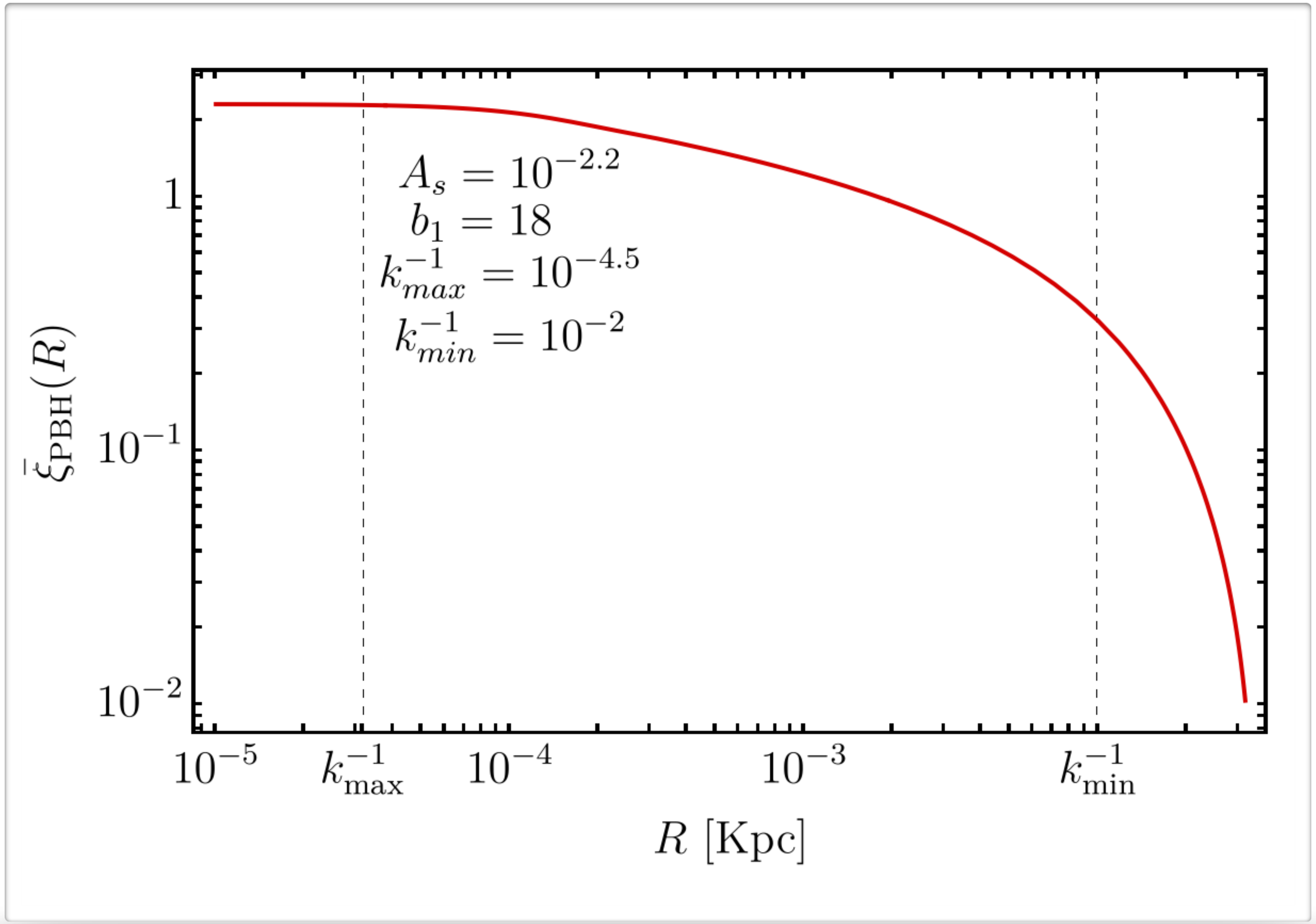
$$P_\zeta(k) = A_s \frac{2\pi^2}{k^3}\,\theta(k_{\rm max} - k)\,\theta(k - k_{\rm min})$$

- The average Number density of PBH

$$\langle N(R) \rangle = \bar{n}_{\rm PBH} V(R) \left[1 + \bar{\xi}_{\rm PBH}(R)\right]$$

- Clustering beats Poisson when

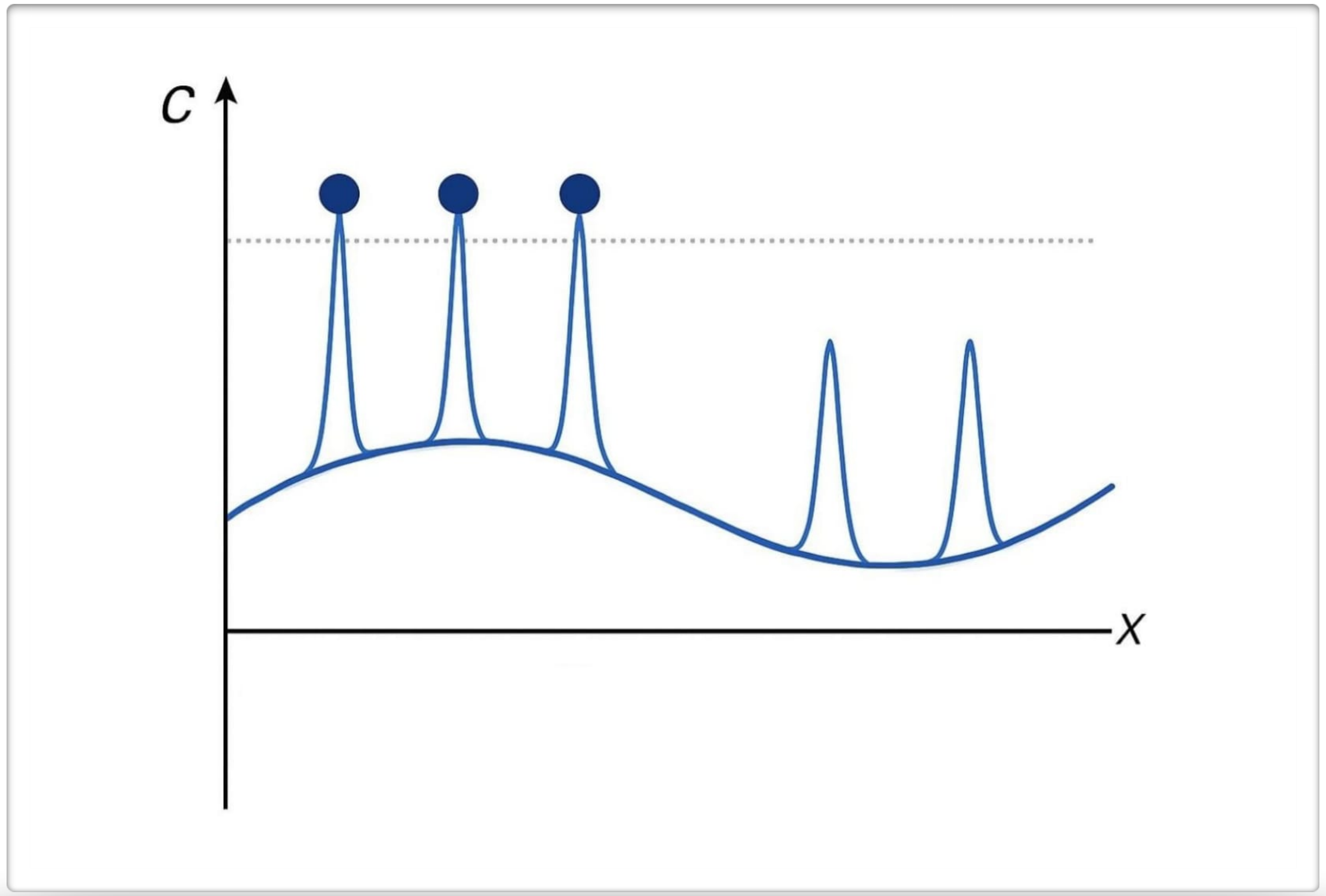
$$\bar{\xi}_{\rm PBH}(R) \gg 1$$



The local Bias

- Compaction function and NG curvature perturbation

$$\zeta = F(\zeta_g) \quad \cdots\blacktriangleright \quad C(r) = C_g(r)F' - \frac{3}{8}C_g^2(r)(F')^2$$



- Peak Background Split Picture

$$\boxed{b_1}$$

$$\delta_{\rm PBH}(\vec{x}) = \frac{P\left(C > C_c \mid \zeta_g^l(\vec{x})\right)}{P(C > C_c)} - 1 \simeq \frac{\partial \ln P\left(C > C_c \mid \zeta_g^l(\vec{x})\right)}{\partial \zeta_g^l(\vec{x})}\Bigg|_{\zeta_g^l=0} \zeta_g^l(\vec{x})$$

The local Bias

- The long modes modify the statistics of the short one

$$C_g^s(r_m) \to \left(1 + \frac{F_s''}{F_s'}\zeta_g^l(\vec{x})\right) C_g^s(r_m)$$

- The bias calculation

$$b_1 = \left\langle \left(\frac{C_g^2\,\sigma_r^2 - C_g\,\zeta_g\,\sigma_{\rm cr}^2}{\sigma_c^2\,\sigma_r^2 - \sigma_{\rm cr}^4} - 1 \right) \left(\frac{F_s''}{F_s'} \right) \right\rangle$$

Some Explicit Scenarios

- Broad Curvature Power spectrum

$$k_{\max} = k_{\min}\Delta$$

- Two cases of NG for ζ :

- Quadratic Ansatz

$$\zeta = \zeta_g + \frac{3}{5} f_{NL} \zeta_g^2$$

- Curvaton Scenario

$$\zeta = \ln[X(r_{dec}, \zeta_g)]$$

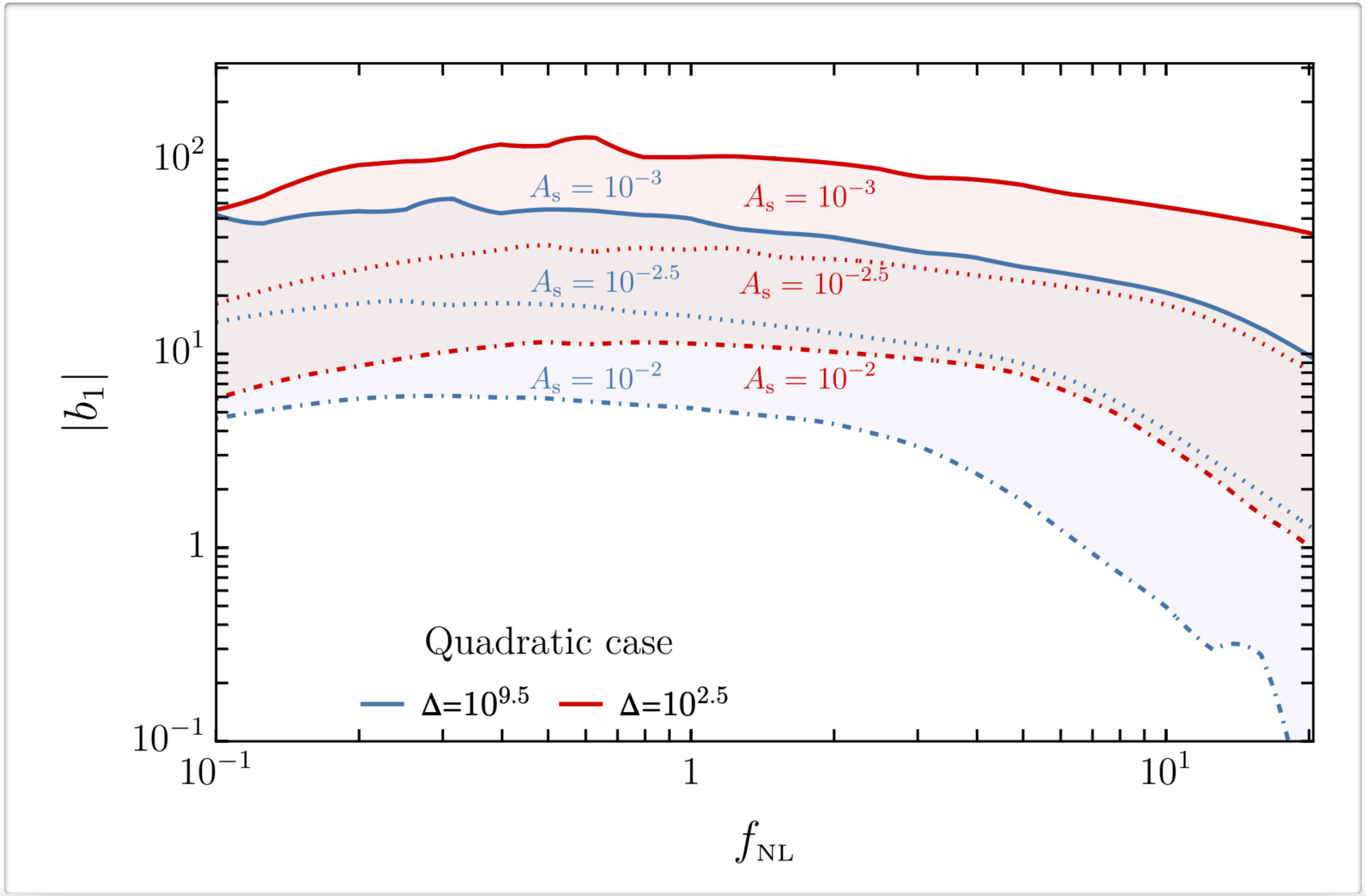
Δ	A_s	NG	$f_{\rm PBH}$	b_1
$10^{2.5}$	$\simeq 10^{-2.2}$	Quadratic $f_{\rm NL} = 0.42$	$\simeq 5 \cdot 10^{-4}$	18
$10^{2.5}$	$\simeq 10^{-3.2}$	Quadratic $f_{\rm NL} = 10.75$	$\simeq 3 \cdot 10^{-3}$	80
$10^{9.5}$	$\simeq 10^{-2.4}$	Quadratic $f_{\rm NL} = 0.42$	$\simeq 1$	15
$10^{9.5}$	$\simeq 10^{-3.5}$	Quadratic $f_{\rm NL} = 10.75$	$\simeq 1$	63
$10^{2.5}$	$\simeq 10^{-2.0}$	Curvaton $r_{\rm dec} = 0.5$	$\simeq 3 \cdot 10^{-4}$	8
$10^{2.5}$	$\simeq 10^{-2.5}$	Curvaton $r_{\rm dec} = 0.1$	$\simeq 3 \cdot 10^{-4}$	5
$10^{9.5}$	$\simeq 10^{-2.0}$	Curvaton $r_{\rm dec} = 0.5$	$\simeq 1$	1.8
$10^{9.5}$	$\simeq 10^{-2.6}$	Curvaton $r_{\rm dec} = 0.1$	$\simeq 1$	1.5

Some Explicit Scenarios

The local Bias in Quadratic NG

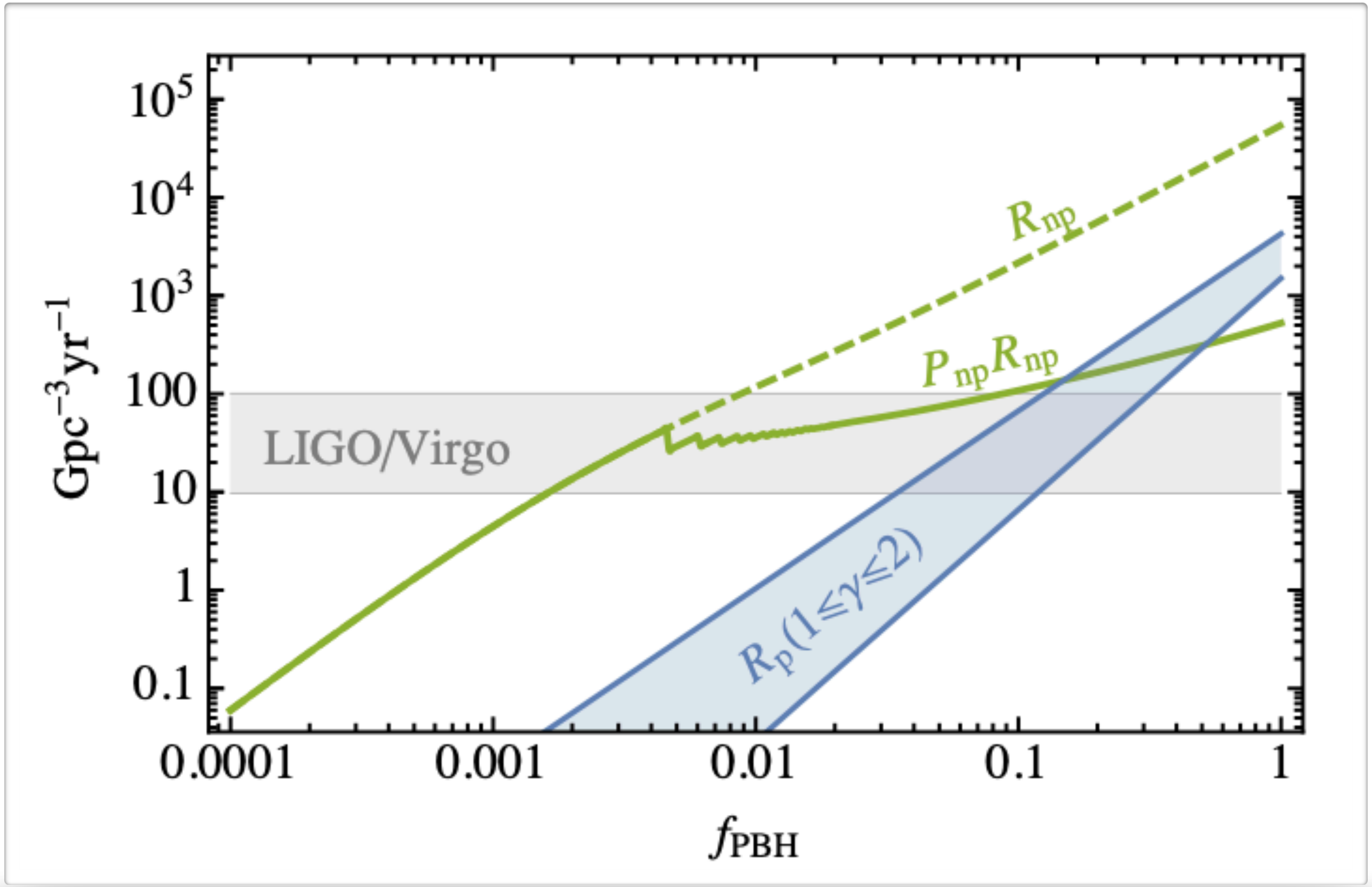
- The Local Bias

$$b_1 \sim \left\langle f(\zeta_g, C_g) \frac{f_{\rm NL}}{1 + \frac{6}{5} f_{\rm NL} \zeta_g} \right\rangle$$



Clustering and Merger length scales

- The 2-body mechanism VS 3-body mechanism



R_{np} := Two Body merger rate

R_p := Three Body merger rate

P_{np} := Probability of non-perturbation of the Binary

$$\bar{\xi}_{PBH}(R) \gg 1$$

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Clustering and Merger length scales

- Scales involved :

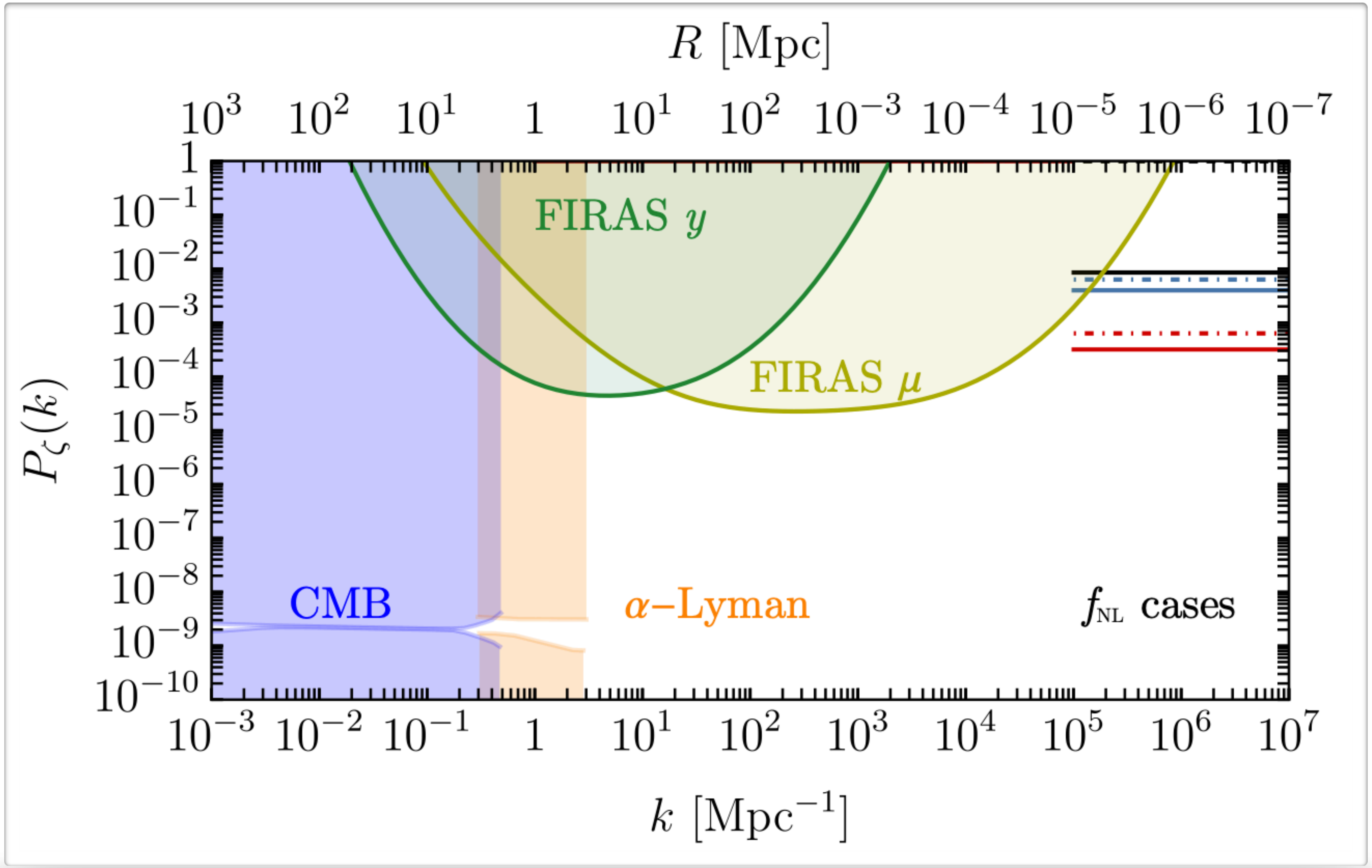
$$R_{\rm max} \lesssim \left(\frac{f_{\rm PBH}}{\bar{n}_{\rm PBH}}\right)^{1/3} \simeq 0.31 \left(\frac{M}{M_\odot}\right)^{1/3} \rm kpc$$

$$R_{\rm min} \sim 9.5 \cdot 10^{-3} \left(\frac{M}{M_\odot}\right)^{7/16} \rm kpc$$

$$\bar{\xi}_{\rm PBH}(R) \gg 1 \quad , \quad R_{\rm min} \lesssim R \lesssim R_{\rm max}$$

Clustering and Merger length scales

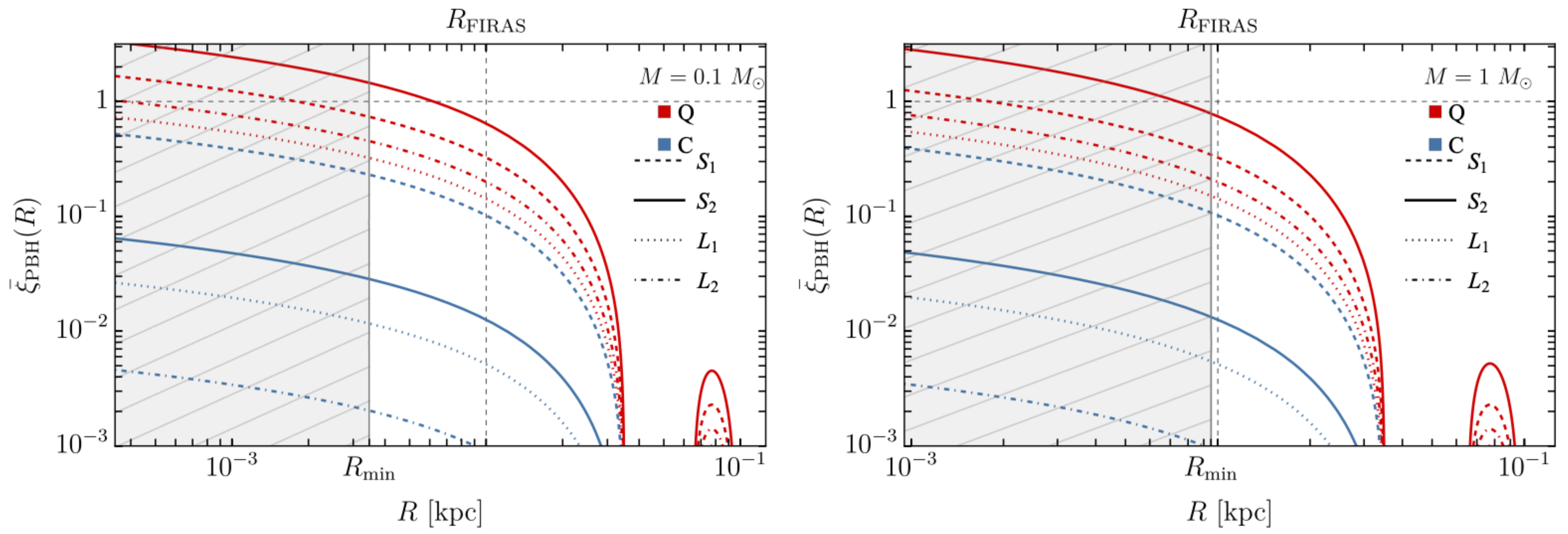
- The CMB spectral distortion and the choice of k_{min}



$$k_{\rm min}^{-1} \lesssim R_{\rm FIRAS} \simeq 10^{-2}\,{\rm kpc}$$

The Irrelevance of Clustering in the LVK regime

$$\bar{\xi}_{\rm PBH}(R) \gg 1 \quad , \quad R_{\rm min} \lesssim R \lesssim R_{\rm FIRAS}$$



Conclusion

- Due to the CMB distortion the initial clustering in the LVK mass range does not affect the standard merger rate estimate

- Future Work: going to smaller mass , the cluster could modify the merger rate channel

The PBH abundance

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- Definition of Compaction function

$$C(r,t) = \frac{2\left[M(r,t) - M_b(r,t)\right]}{R(r,t)} = \frac{2}{R(r,t)} \int_{V_R} d^3\vec{x}\, \rho_b(t)\, \delta(\vec{x},t)$$

- Density contrast on Superhorizon scale , using gradient expansion in spherical symmetry

$$\delta(r,t) = -\frac{4}{9}\left(\frac{1}{aH}\right)^2 e^{-2\zeta(r)}\left[\zeta''(r) + \frac{2}{r}\zeta'(r) + \frac{1}{2}\left(\zeta'(r)\right)^2\right]$$

- Compaction function

$$C(r) = -\frac{4}{3} r\,\zeta'(r)\left[1 + \frac{r}{2}\zeta'(r)\right] = C_1(r) - \frac{3}{8}C_1^2(r), \quad C_1(r) = -\frac{4}{3} r\,\zeta'(r)$$

The PBH abundance

Backup Slide

- From the NG definition in the curvature perturbation the compaction now has the form

$$\zeta = F(\zeta_g) \quad \longrightarrow \quad C(r) = C_g(r)F' - \frac{3}{8}C_g^2(r)(F')^2$$

- Since both the Compaction and the curvature perturbation has the same gaussian statistics , the joint Gaussian Bivariate is

$$P_g(C_g, \zeta_g) = \frac{1}{(2\pi)\sigma_c\sigma_r\sqrt{1-\gamma_{\rm cr}^2}} \exp\left(-\frac{\zeta_g^2}{2\sigma_r^2}\right) \exp\left[-\frac{1}{2(1-\gamma_{\rm cr}^2)}\left(\frac{C_g}{\sigma_c} - \frac{\gamma_{\rm cr}\zeta_g}{\sigma_r}\right)^2\right]$$

The PBH abundance

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- The variance are here defined

$$\sigma_c^2 = \langle C_g C_g \rangle = \frac{16}{81} \int_0^\infty \frac{\mathrm{d}k}{k} (kr_m)^4 W^2(k, r_m) P_\zeta^T(k)$$

$$\sigma_{\mathrm{cr}}^2 = \langle C_g \zeta_g \rangle = \frac{4}{9} \int_0^\infty \frac{\mathrm{d}k}{k} (kr_m)^2 W(k, r_m) W_s(k, r_m) P_\zeta^T(k)$$

$$\sigma_r^2 = \langle \zeta_g \zeta_g \rangle = \int_0^\infty \frac{\mathrm{d}k}{k} W_s^2(k, r_m) P_\zeta^T(k)$$

The PBH abundance
Backup Slide

- The PBH formation: the Compaction function

$$C(r) = C_1(r) - \frac{3}{8}C_1^2(r), \quad C_1(r) = -\frac{4}{3}r\,\zeta'(r)$$

- The curvature perturbation is NG

$$\zeta = F(\zeta_g) \quad \longrightarrow \quad C(r) = C_g(r)F' - \frac{3}{8}C_g^2(r)(F')^2$$

- The collapse Probability

$$P(C > C_c) = \int_D \mathrm{P_g}(C_\mathrm{g}, \zeta_\mathrm{g})\,\mathrm{d}C_\mathrm{g}\,\mathrm{d}\zeta_\mathrm{g} D = \left\{ C_\mathrm{g}, \zeta_\mathrm{g} \in \mathbb{R} \,:\, C(C_\mathrm{g}, \zeta_\mathrm{g}) > C_c \wedge C_1(C_\mathrm{g}, \zeta_\mathrm{g}) < \frac{4}{3} \right\}$$

The PBH abundance

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- The mass fraction

$$\beta(M_{\rm PBH}, M_H) = \int_{\mathscr{D}} \frac{M_{\rm PBH}}{M_H} \delta\left(\ln \frac{M_{\rm PBH}}{M_{\rm PBH}(C)}\right) P_g(C_g, \zeta_g)\, \mathrm{d}C_g\, \mathrm{d}\zeta_g$$

- The PBH abundance

$$f_{\rm PBH}(M_{\rm PBH}) \equiv \frac{1}{\Omega_{\rm DM}} \frac{\mathrm{d}\Omega_{\rm PBH}}{\mathrm{d}\ln M_{\rm PBH}} = \frac{1}{\Omega_{\rm DM}} \int \mathrm{d}\ln M_H \left(\frac{M_H}{M_\odot}\right)^{-1/2} \left(\frac{g_{*s}^4/g_*^3}{106.75}\right)^{-1/4} \left(\frac{\beta(M_{\rm PBH}, M_H)}{7.9 \times 10^{-10}}\right)$$

Some explicit Scenarios

The Mass function

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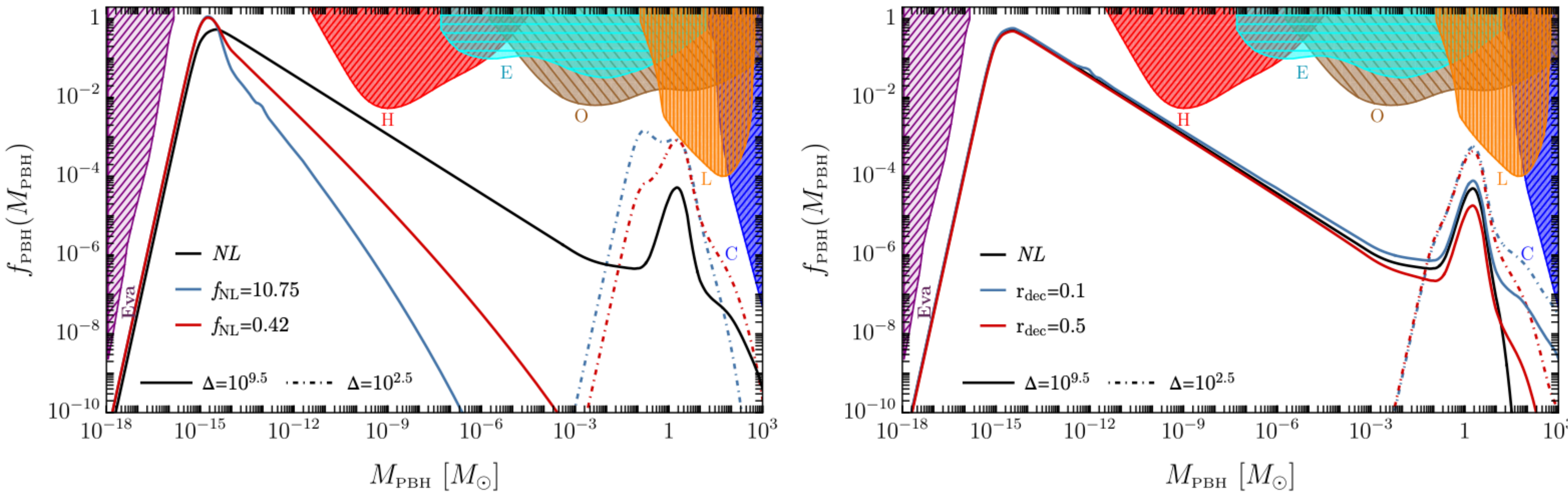

G.Ferrante et al. 2023

The PBH Clustering

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- The two-point function in Fourier space

$$\xi_{\rm PBH}(x) = \frac{1}{2\pi^2}\int_0^\infty {\rm d}k\, k^2 P_{\rm PBH}(k) j_0(kx)$$

- From the bias definition

$$P_{\rm PBH}(k) = \left(\frac{4}{9}\right)^2 b_1^2 P_\zeta(k)$$

- The maximum Clustering: the Broad curvature power spectrum

$$P_\zeta(k) = A_s \frac{2\pi^2}{k^3}\,\theta(k_{\rm max} - k)\,\theta(k - k_{\rm min}) \qquad k_{\rm max} = k_{\rm min}\Delta$$

The PBH clustering

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- Two point correlation function

$$\xi_{\rm PBH}(x) = \frac{1}{2\pi^2}\int_0^\infty {\rm d}k\, k^2 P_{\rm PBH}(k) j_0(kx)$$

$$\xi_{\rm PBH}(x) = \left(\frac{4}{9}\right)^2 A_s b_1^2 \left[{\rm Ci}(k_{\rm max}x) - {\rm Ci}(k_{\rm min}x) - \frac{\sin(k_{\rm max}x)}{k_{\rm max}x} + \frac{\sin(k_{\rm min}x)}{k_{\rm min}x}\right]$$

$$\approx \left(\frac{4}{9}\right)^2 A_s b_1^2 \begin{cases} \ln\Delta, & x \ll k_{\rm max}^{-1} \\ \ln\left(\frac{1.53}{xk_{\rm min}}\right), & k_{\rm max}^{-1} \ll x \ll k_{\rm min}^{-1} \\ \mathcal{O}((xk_{\rm min})^{-2}), & x \gg k_{\rm min}^{-1} \end{cases}$$

The PBH clustering

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- The average Number density of PBH into a Volume

$$\langle N(R) \rangle = \bar{n}_{\text{PBH}} V(R) + \bar{n}_{\text{PBH}} \int_0^R \mathrm{d}^3x \, \xi_{\text{PBH}}(x) = \bar{n}_{\text{PBH}} V(R) \left[1 + \bar{\xi}_{\text{PBH}}(R)\right]$$

$$\bar{\xi}_{\text{PBH}}(R) = \left(\frac{4}{9}\right)^2 A_s b_1^2 \left[\text{Ci}(k_{\text{max}}R) - \text{Ci}(k_{\text{min}}R) + \frac{\cos(k_{\text{max}}R)}{k_{\text{max}}^2 R^2} - \frac{\cos(k_{\text{min}}R)}{k_{\text{min}}^2 R^2} - \frac{\sin(k_{\text{max}}R)}{k_{\text{max}}^3 R^3}(1 + k_{\text{max}}^2 R^2) + \frac{\sin(k_{\text{min}}R)}{k_{\text{min}}^3 R^3}(1 + k_{\text{min}}^2 R^2)\right]$$

$$\approx \left(\frac{4}{9}\right)^2 A_s b_1^2 \begin{cases} \ln \Delta, & R \ll k_{\text{max}}^{-1} \\ \ln\left(\frac{2.12}{Rk_{\text{min}}}\right), & k_{\text{max}}^{-1} \ll R \ll k_{\text{min}}^{-1} \\ \mathcal{O}((Rk_{\text{min}})^{-3}), & R \gg k_{\text{min}}^{-1} \end{cases}$$

The local Bias

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- The Peak Background Split Picture

$$\delta_{\rm PBH}(\vec{x}) = \frac{P\left(C > C_c \mid \zeta_g^l(\vec{x})\right)}{P(C > C_c)} - 1 \simeq \frac{\partial \ln P\left(C > C_c \mid \zeta_g^l(\vec{x})\right)}{\partial \zeta_g^l(\vec{x})}\Bigg|_{\zeta_g^l = 0} \zeta_g^l(\vec{x})$$

- Expanding the Compaction function in term of short and long mode

$$C_1(\vec{x}, r_m) = -\frac{4}{3} r_m \zeta_g^{s'}(r_m) F_s' - \frac{4}{3} r_m \zeta_g^{s'}(r_m) F_s'' \zeta_g^l(\vec{x}) = C_1^s(r_m)\left(1 + \frac{F_s''}{F_s'}\zeta_g^l(\vec{x})\right),$$

with $\quad C_1^s(r_m) = F_s' C_g^s(r_m) \quad$ and $\quad C_g^s(r_m) = -\frac{4}{3} r_m \zeta_g^{s'}(r_m)$

The local Bias

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- The new form of the Compaction

$$C(\vec{x}, r_m) = C_1(\vec{x}, r_m) - \frac{3}{8}C_1^2(\vec{x}, r_m) \simeq C_1^s(r_m)\left(1 + \frac{F_s''}{F_s'}\zeta_g^l(\vec{x})\right) - \frac{3}{8}\left(C_1^s(r_m)\right)^2\left(1 + 2\frac{F_s''}{F_s'}\zeta_g^l(\vec{x})\right)$$

- The short modes can be redefine

$$C_g^s(r_m) \to \left(1 + \frac{F_s''}{F_s'}\zeta_g^l(\vec{x})\right)C_g^s(r_m)$$

- And so the Variances

$$\sigma_c^2 \to \left(1 + 2\frac{F_s''}{F_s'}\zeta_g^l(\vec{x})\right)\sigma_c^2,\ \sigma_{\rm cr}^2 \to \left(1 + \frac{F_s''}{F_s'}\zeta_g^l(\vec{x})\right)\sigma_{\rm cr}^2, \sigma_r^2 \to \sigma_r^2\,.$$

The local Bias

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- As a results, the PBH overdensity becomes

$$\delta_{\rm PBH}(\vec{x}) \simeq \left(2\frac{\partial \ln P(C > C_c)}{\partial \ln \sigma_c^2} + \frac{\partial \ln P(C > C_c)}{\partial \ln \sigma_{\rm cr}^2}\right)\frac{F_s''}{F_s'}\zeta_g^l(\vec{x})$$

- And so the linear bias

$$b_1 = \frac{1}{P(C > C_c)}\int_{\mathscr{D}} \frac{\left(C_g^2\sigma_r^2 - C_g\zeta_g\sigma_{\rm cr}^2 - \sigma_c^2\sigma_r^2 + \sigma_{\rm cr}^4\right)\exp\left(\frac{C_g^2\sigma_r^2 - 2C_g\zeta_g\sigma_{\rm cr}^2 + \zeta_g^2\sigma_c^2}{2\sigma_{\rm cr}^4 - 2\sigma_c^2\sigma_r^2}\right)}{2\pi\sigma_c\sigma_r\sqrt{1 - \frac{\sigma_{\rm cr}^4}{\sigma_c^2\sigma_r^2}}\left(\sigma_c^2\sigma_r^2 - \sigma_{\rm cr}^4\right)} \cdot \frac{F_s''}{F_s'}\,{\rm d}C_g\,{\rm d}\zeta_g$$

Clustering and the merger length scale

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- Constraint at small scales: at some redshifts , the energy injections into the primordial plasma cause spectral distortion in the CMB
- These distortion are divided into the chemical potential μ-type distortions created ad higher redshifts, and the Compton y-type at lower redshifts.
- The Spectral distortion, given a specific Power spectrum is

$$X = \int_{k_\mathrm{m}}^{\infty} \frac{\mathrm{d}k}{k} \mathscr{P}_\zeta(k)\, W_X(k)$$

Clustering and the merger length scale

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- The definition for the Window function

$$W_\mu(k) \simeq 2.2 \left[\exp\left(-\frac{(\hat{k}/1360)^2}{1 + (\hat{k}/260)^{0.6} + \hat{k}/340} \right) - \exp\left(-(\hat{k}/32)^2 \right) \right], \qquad W_y(k) \simeq 0.4 \, \exp\left(-(\hat{k}/32)^2 \right)$$

- The COBE FIRAS measurements constrain the μ distortion to be $\mu < 4.7 \times 10^{-5}$, and $y < 1.5 \times 10^{-5}$ at the 95% confidence level.