Bayesian reconstruction of curvature perturbations from SIGW

Ameek Malhotra

Swansea University
Prifysgol Abertawe

NEHOP
Brussels, 22 May 2025

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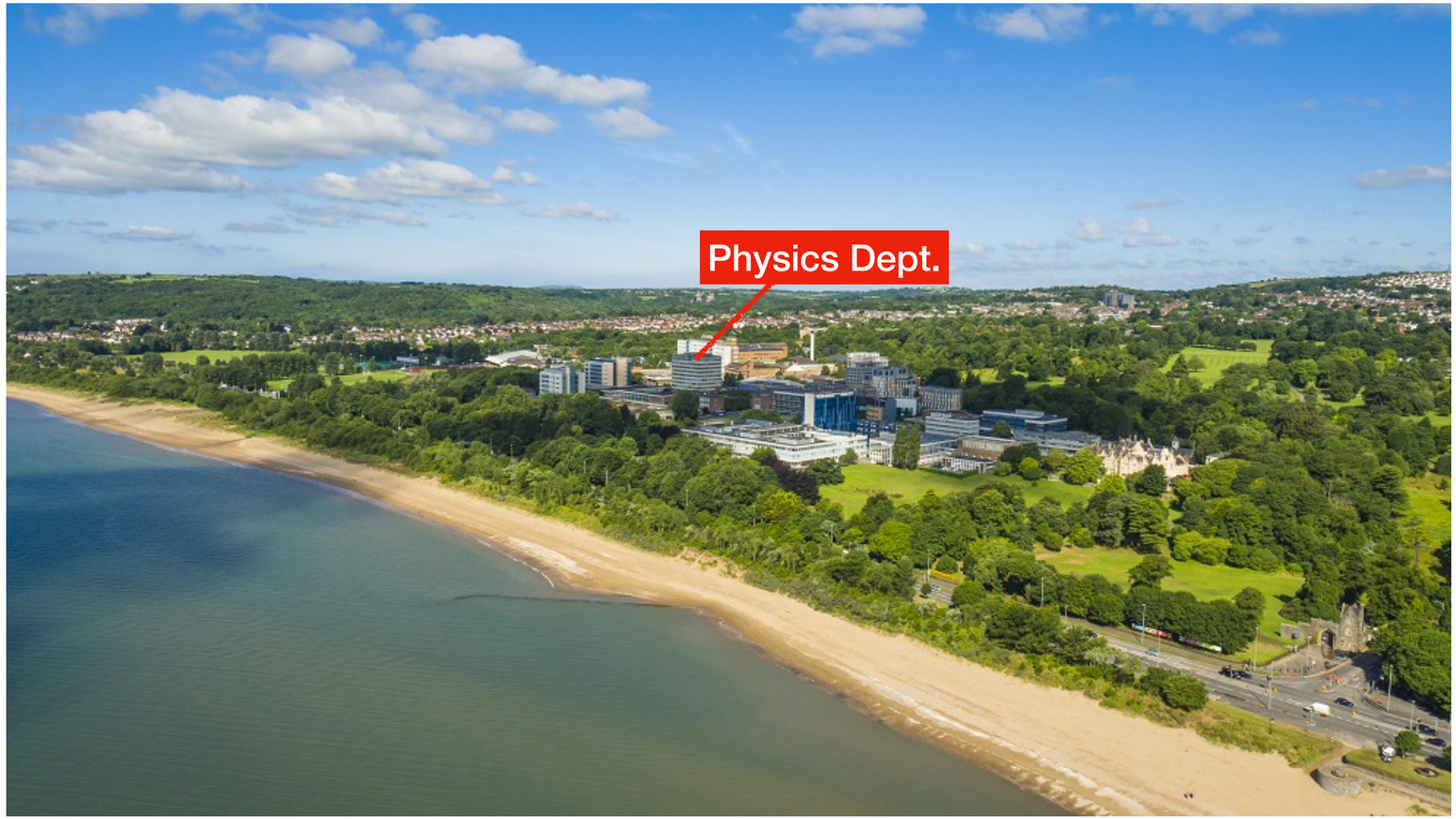

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Outline

How well can we constrain the primordial curvature power spectrum and potentially the equation of state from scalar induced GW observations?

Based on work in progress with Aya Ghaleb, Gianmassimo Tasinato and Ivonne Zavala (Swansea Cosmology group)

Outline

How well can we constrain the primordial curvature power spectrum and potentially the equation of state from scalar induced GW observations?

(assuming Gaussian P_ζ)

Based on work in progress with Aya Ghaleb, Gianmassimo Tasinato and Ivonne Zavala (Swansea Cosmology group)

Scalar induced GW

Theory

$$h''_{ij}(\eta, \mathbf{x}) + 2\,\mathcal{H}\,h'_{ij}(\eta, \mathbf{x}) - \nabla^2 h_{ij}(\eta, \mathbf{x}) \;=\; -4\,T_{ij}{}^{lm}\,S_{lm}(\eta, \mathbf{x})\,.$$

Gravitational waves at second order in the scalar perturbations, typically sourced around horizon crossing of the scalar modes

$$S_{lm} \propto \Phi\partial_l\partial_m\Phi,\; \partial_l\Phi\partial_m\Phi\ldots$$

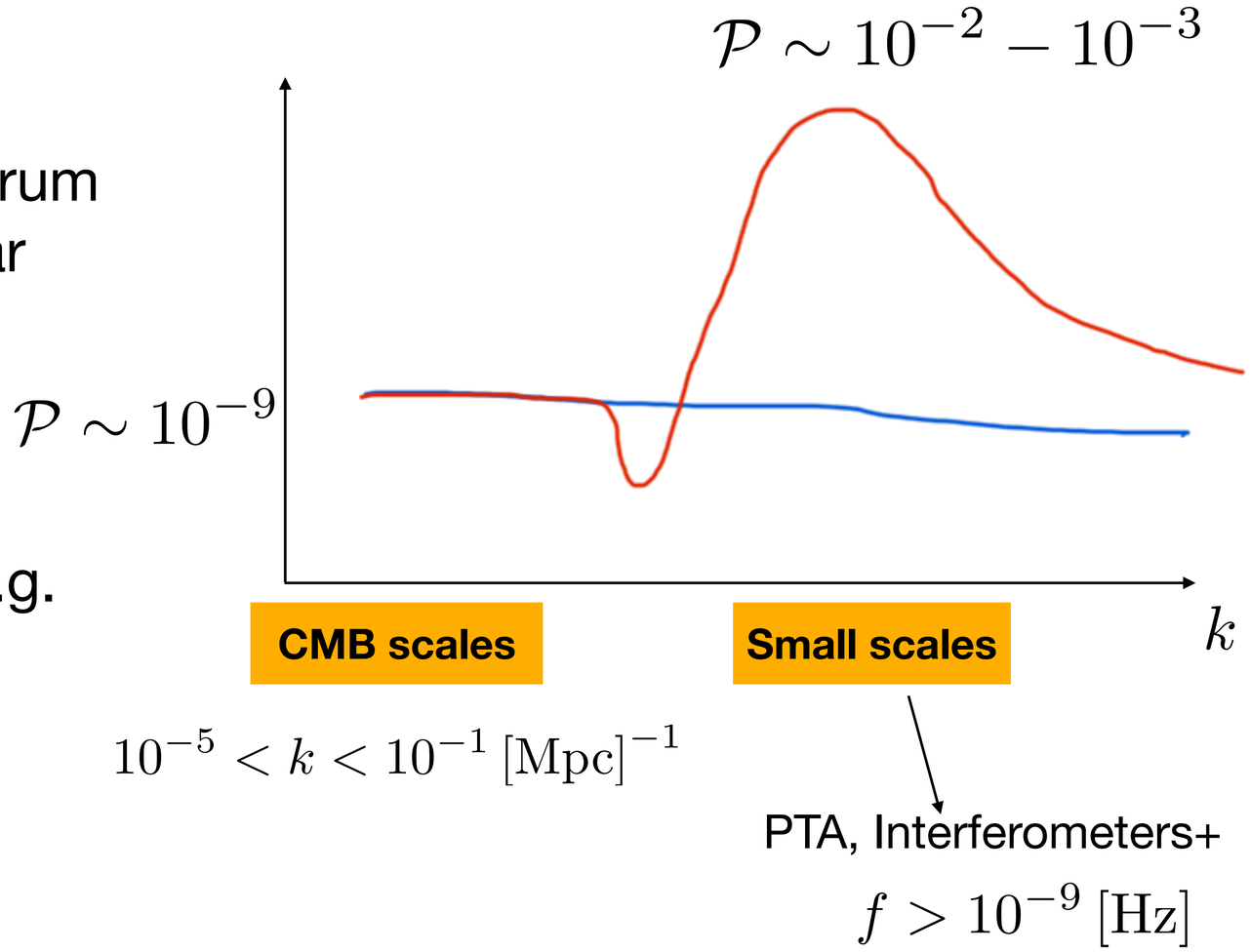
[Tomita (1967), Mataresse et al (1993+), Ananda et al. (2006), Baumann et al. (2007)]

Scalar induced GW

Theory

Enhancement of the scalar power spectrum on small scales can lead to PBH + scalar induced GW

Can be realised in single field inflation e.g. ultra-slow roll or multi-field scenarios



Scalar induced GW

Theory

$$\Omega_{\mathrm{GW}}(k,\eta) \propto \int_0^\infty dt \int_0^1 ds\, I^2(t,s,k,\eta) P_\zeta\left(k\frac{t+s+1}{2}\right) P_\zeta\left(k\frac{t-s+1}{2}\right)$$

Primordial curvature power spectrum

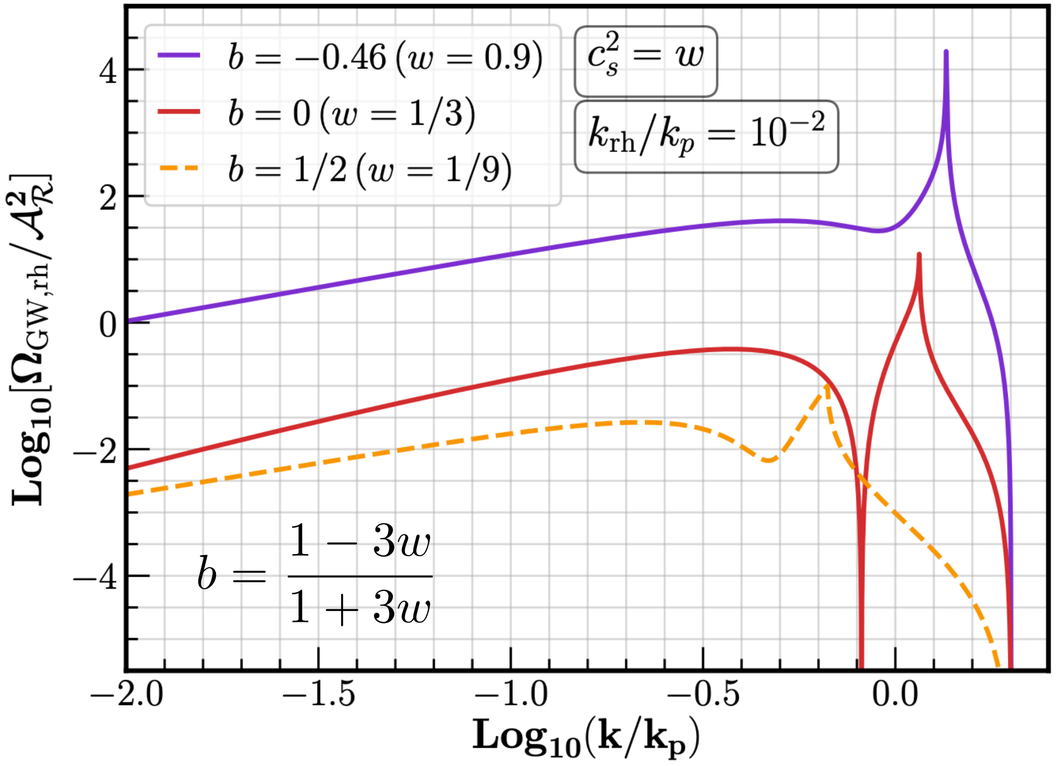
Kernel, depends on background cosmology

The induced GW spectrum contains information about the background cosmology and the primordial curvature power spectrum.

Scalar induced GW

Theory

Resonant amplification at $k_{\rm GW} \approx 2c_s k_p$
(for Dirac-Delta spectrum)



See review by Domenech (2021)

Scalar induced GW

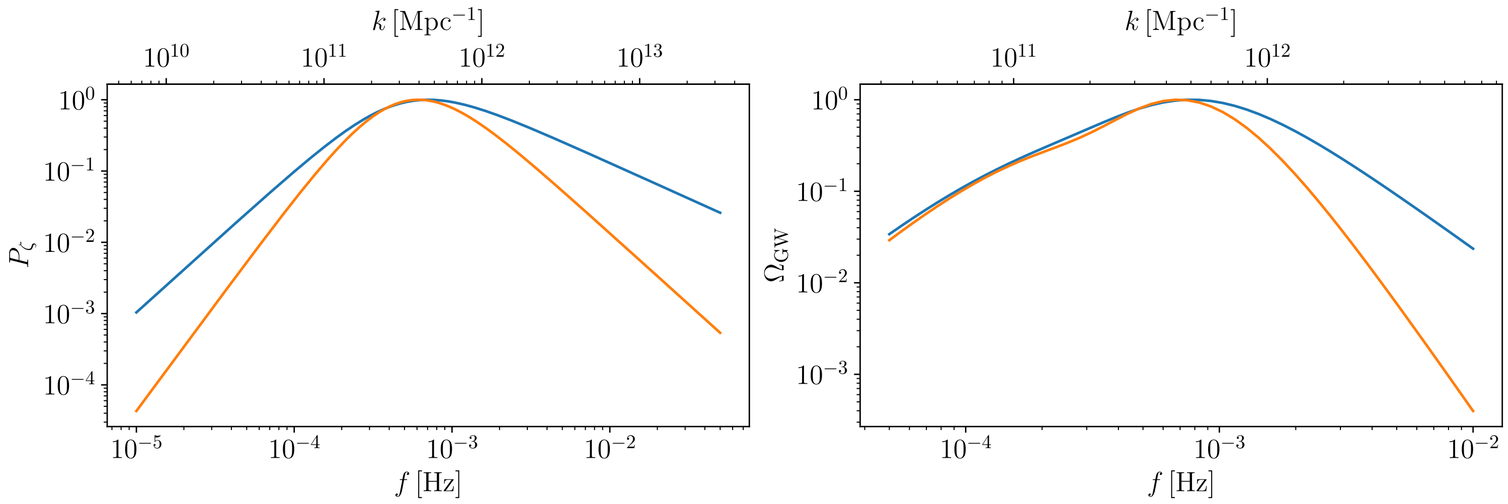
Theory

Analytic approx. for broad peaked spectra (RD) [See review by Domenech (2021)]

$$\Omega_{\rm GW}(k) \propto \begin{cases} \left(\frac{k}{k_{\rm p}}\right)^{3}, & k \ll k_{\rm p}, \\ \left(\frac{k}{k_{\rm p}}\right)^{-\Delta}, & k \gg k_{\rm p}, \end{cases} \quad \text{with} \quad \Delta = \begin{cases} 2\,n_{\rm UV}, & 0 < n_{\rm UV} < 4, \\ 4 + n_{\rm UV}, & n_{\rm UV} > 4, \end{cases}$$

Universal IR behaviour
[Cai, Pi, Sasaki (2020)]

Dependence on P_ζ (RD)



Scalar induced GW

Theory

Dependence on w

$$\Omega_{\rm GW}(k) \propto \begin{cases} \left(\frac{k}{k_{\rm p}}\right)^{3}, & k \ll k_{\rm rh}, \\ \left(\frac{k}{k_{\rm p}}\right)^{3-2|b|}, & k_{\rm rh} \ll k \ll k_{\rm p}, \\ \left(\frac{k}{k_{\rm p}}\right)^{-\Delta-2|b|}, & k \gg k_{\rm p}, \end{cases} \qquad \text{with} \quad \Delta = \begin{cases} 2\,n_{\rm UV}, & 0 < n_{\rm UV} + b < 4, \\ 4 + n_{\rm UV}, & n_{\rm UV} + b > 4, \end{cases}$$

$$b = \frac{1-3w}{1+3w}$$

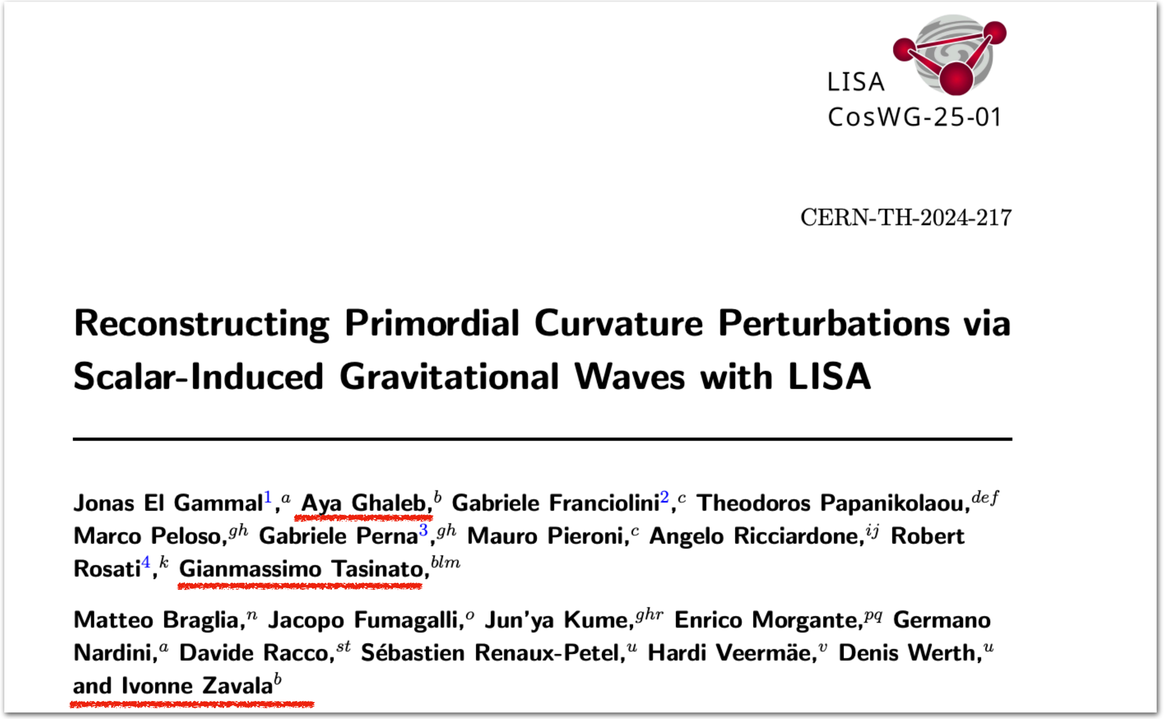
[Domenech (2021)]

Reconstructing curvature power spectrum

Previous work

Development of fast code (SIGWAY) to compute $\Omega_{\rm GW}$ from P_ζ (and V_ϕ) for radiation and early matter-radiation transition

Reconstruction based on templates (power spectrum+potential), binned amplitudes, NG + more



LISA CosWG-25-01

CERN-TH-2024-217

Reconstructing Primordial Curvature Perturbations via Scalar-Induced Gravitational Waves with LISA

Jonas El Gammal[1],[a] Aya Ghaleb,[b] Gabriele Franciolini[2],[c] Theodoros Papanikolaou,[def] Marco Peloso,[gh] Gabriele Perna[3],[gh] Mauro Pieroni,[c] Angelo Ricciardone,[ij] Robert Rosati[4],[k] Gianmassimo Tasinato,[blm]

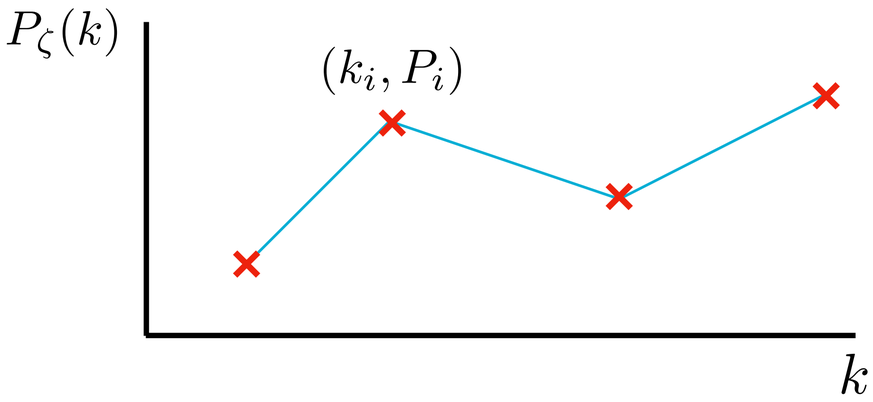
Matteo Braglia,[n] Jacopo Fumagalli,[o] Jun'ya Kume,[ghr] Enrico Morgante,[pq] Germano Nardini,[a] Davide Racco,[st] Sébastien Renaux-Petel,[u] Hardi Veermäe,[v] Denis Werth,[u] and Ivonne Zavala[b]

2501.11320

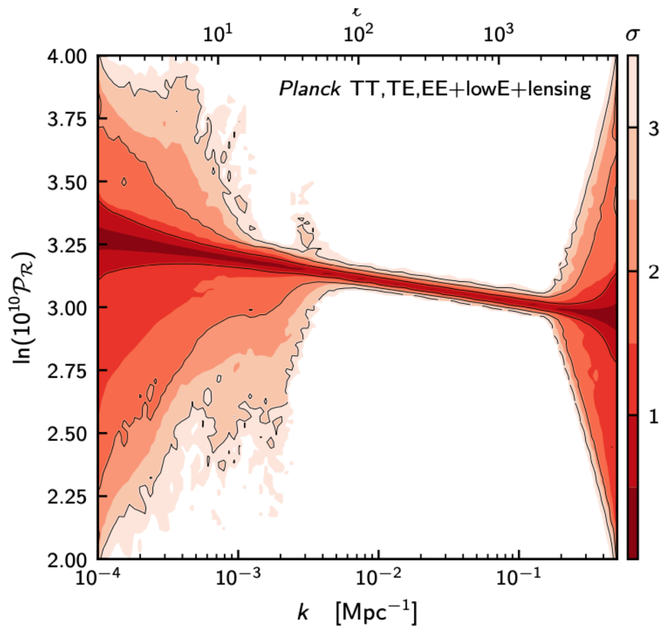
Reconstructing curvature power spectrum

Our method

Reconstruct P_ζ using linear interpolating splines (other possibilities as well) — no strong assumptions regarding functional form of spectrum



Similar methods have been previously used for the CMB



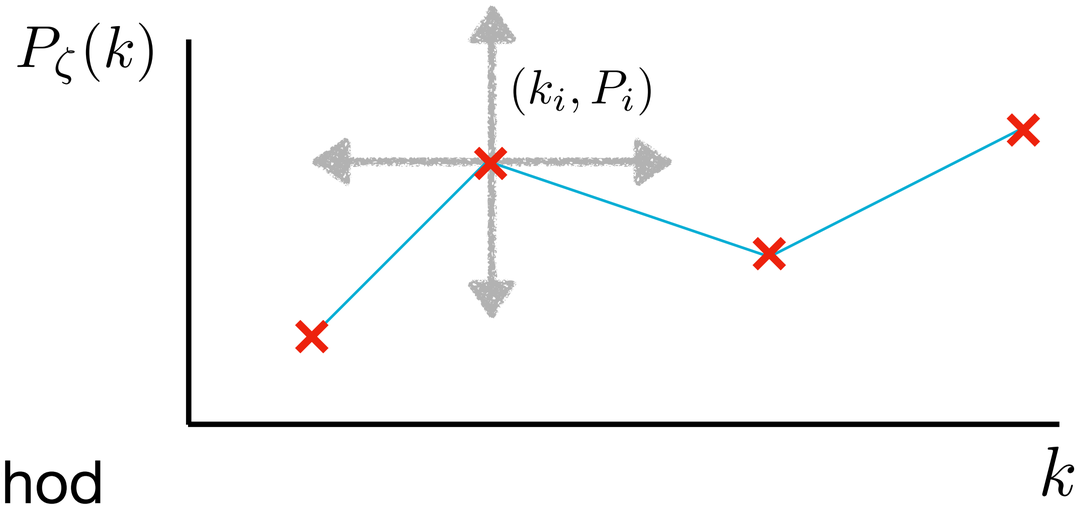
Planck 2018: Constraints on Inflation

Reconstructing curvature power spectrum

Our method

We sample the likelihood using a nested sampler,
$\theta \rightarrow$ spline node positions and amplitudes

$$-2\ln\mathcal{L}(\theta) = \sum_f \left(\frac{\Omega_{\mathrm{GW,model}}(\theta) - \Omega_{\mathrm{GW,obs}}}{\Delta\Omega_{\mathrm{GW}}}\right)^2 .$$



We assume $\Delta\Omega_{\mathrm{GW}}$ given but not necessary, method can be integrated into SGWB estimation pipeline e.g. `SGWBinner` for LISA

Reconstructing curvature power spectrum

Our method

Let the data determine node positions and amplitudes. Use Bayesian evidence for relative probabilities of spline models with different number of nodes.

At the end, reconstruct distribution of $P_\zeta(k)$ and $\Omega_{\rm GW}(k)$ from the samples after marginalising over the reconstructions with different number of nodes

$$\text{e.g.} \quad \mathbb{E}[P_{\zeta,k}] = \frac{\sum_n \mathbb{E}[P_{\zeta,k}|\mathcal{M}_n]\mathcal{Z}_n}{\sum_n \mathcal{Z}_n}.$$

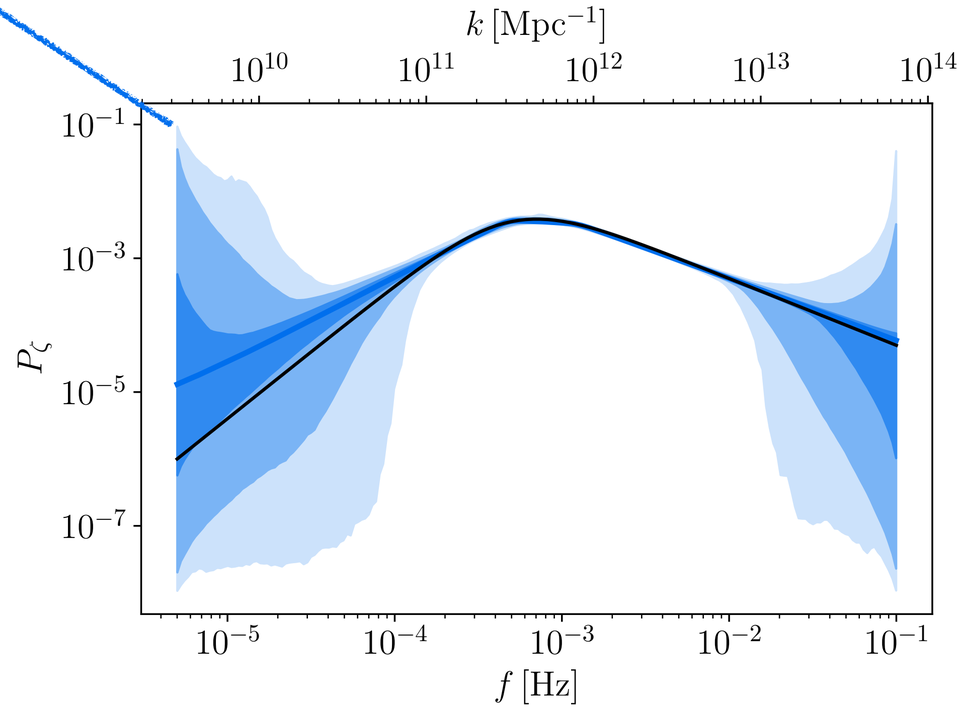
Relative probability/weight of model with n nodes

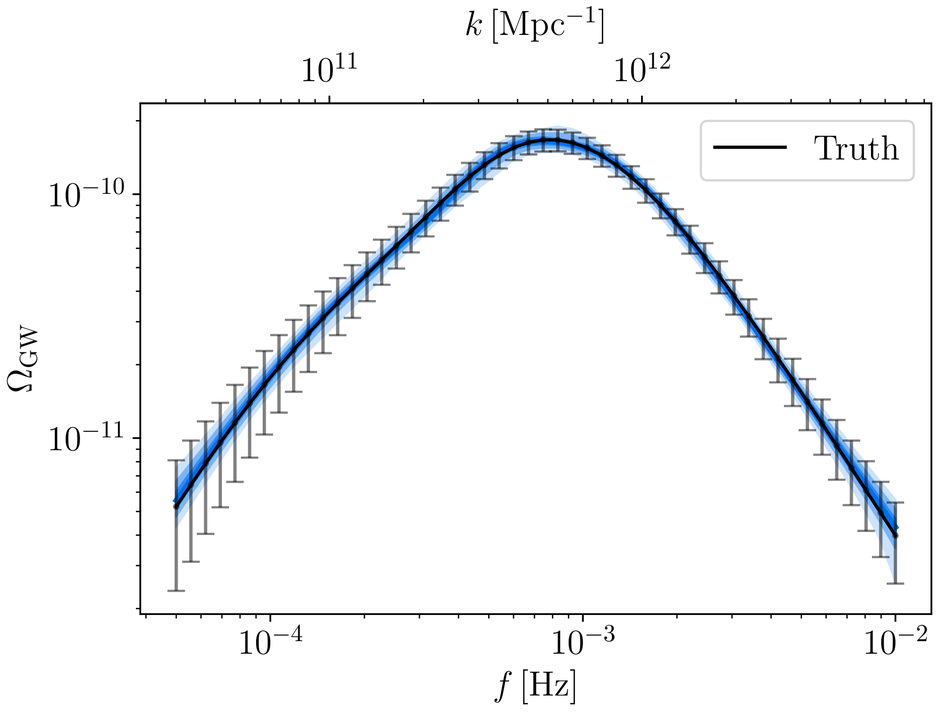
Reconstructing curvature power spectrum

Example: Radiation domination

Reconstruction is great if peak of $\Omega_{\rm GW}$ is observed

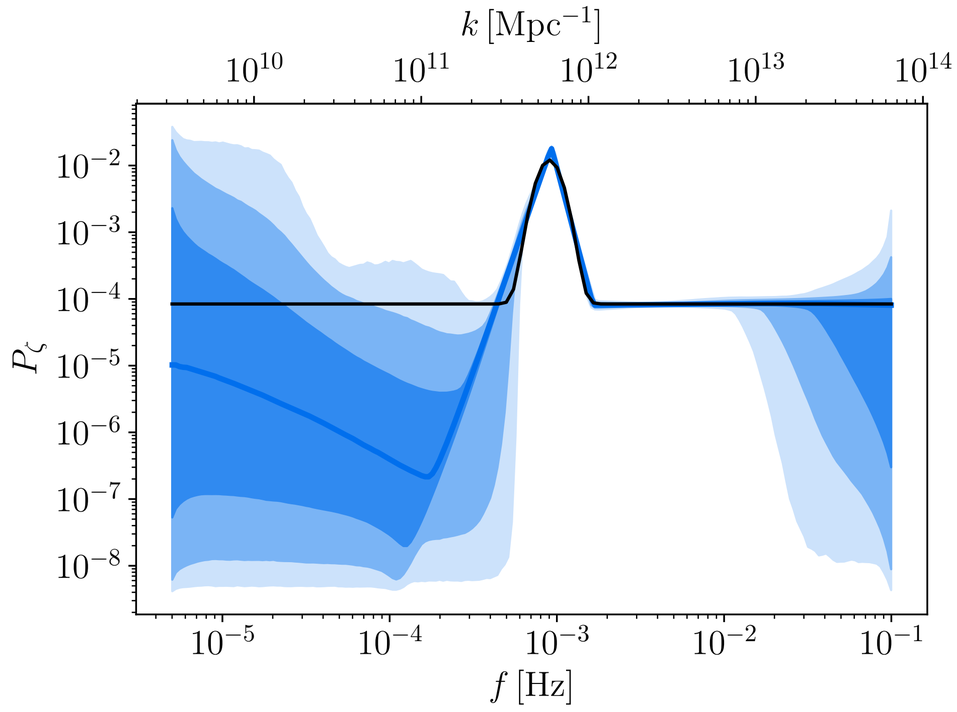
68%,
95%,
99.7%
credible
intervals

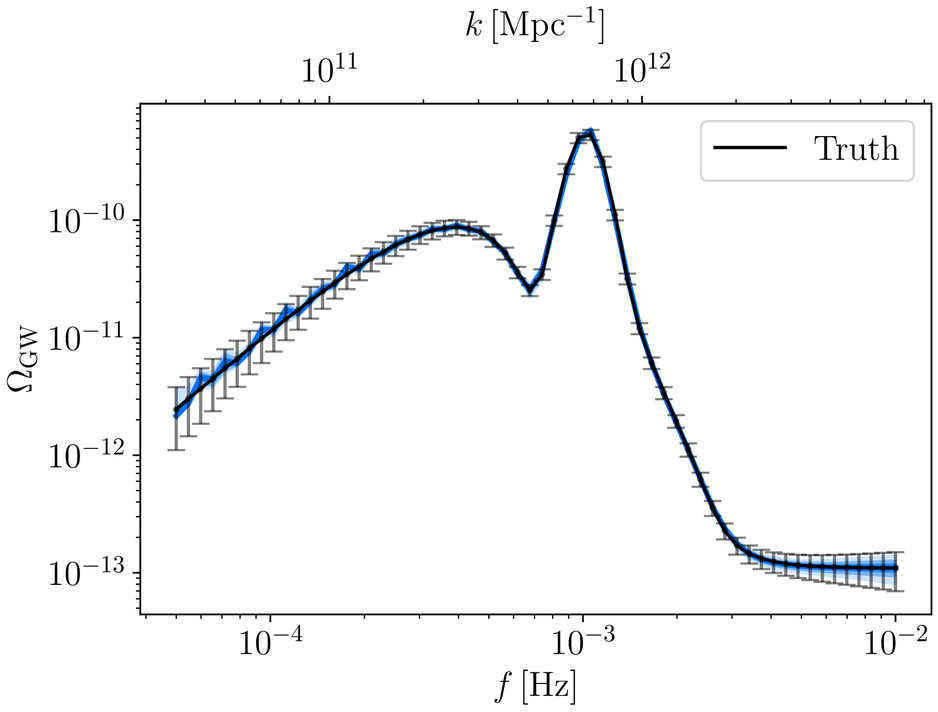




Reconstructing curvature power spectrum

Example: Radiation domination

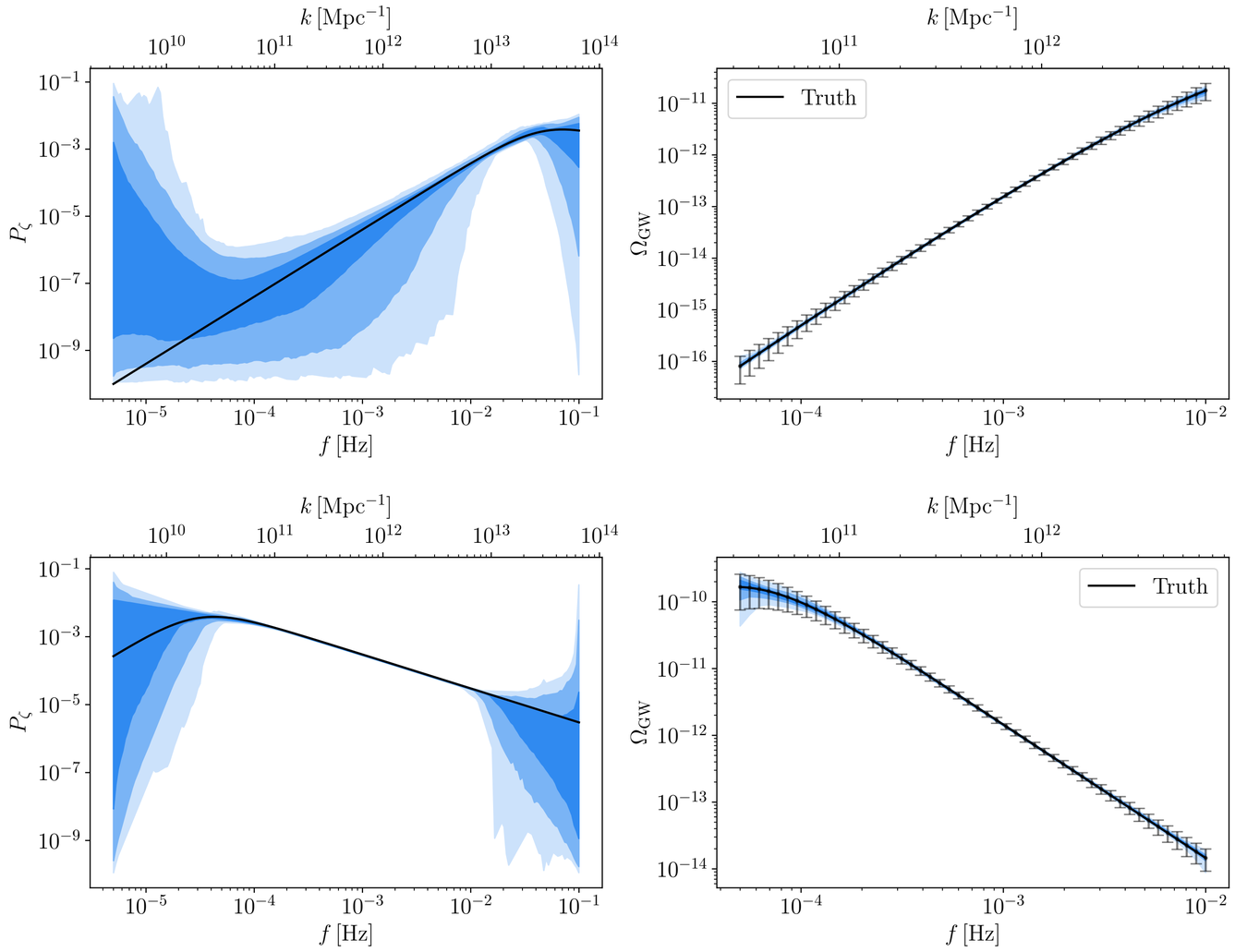




Reconstructing curvature power spectrum

Example: Radiation domination

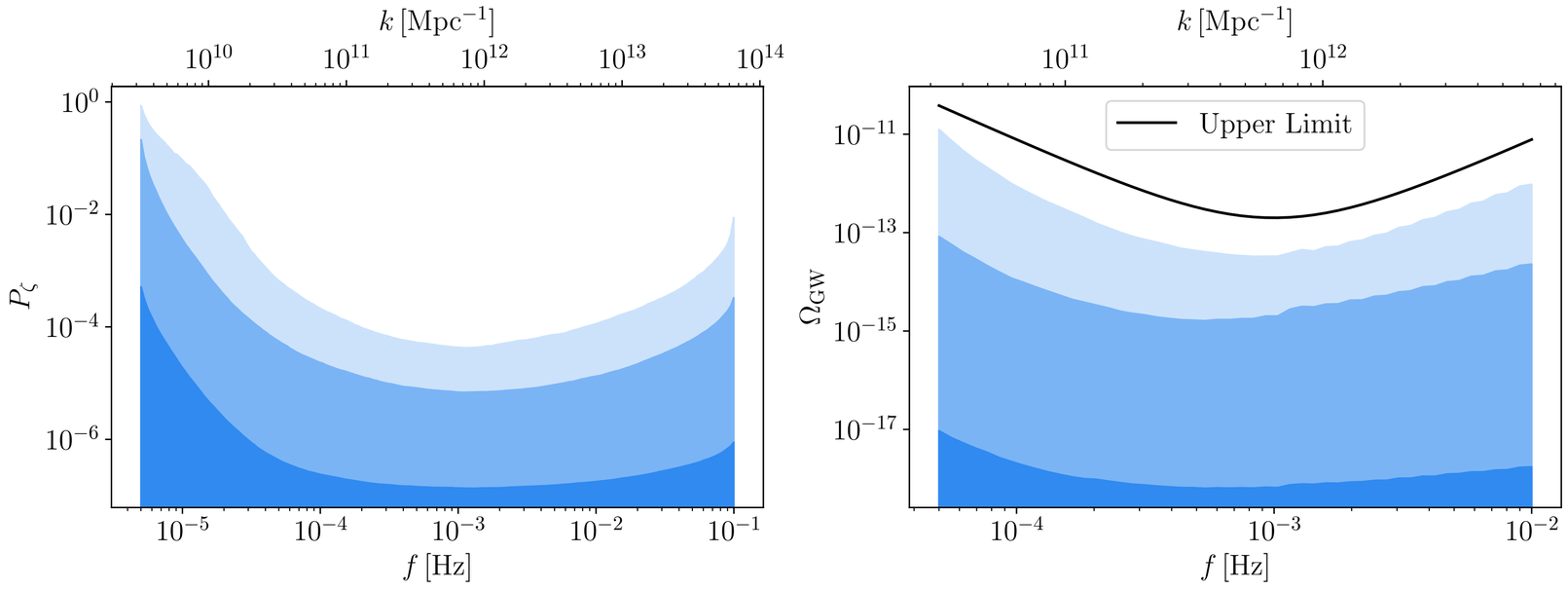
What if peak is not observed?



Reconstructing curvature power spectrum

Example: Radiation domination

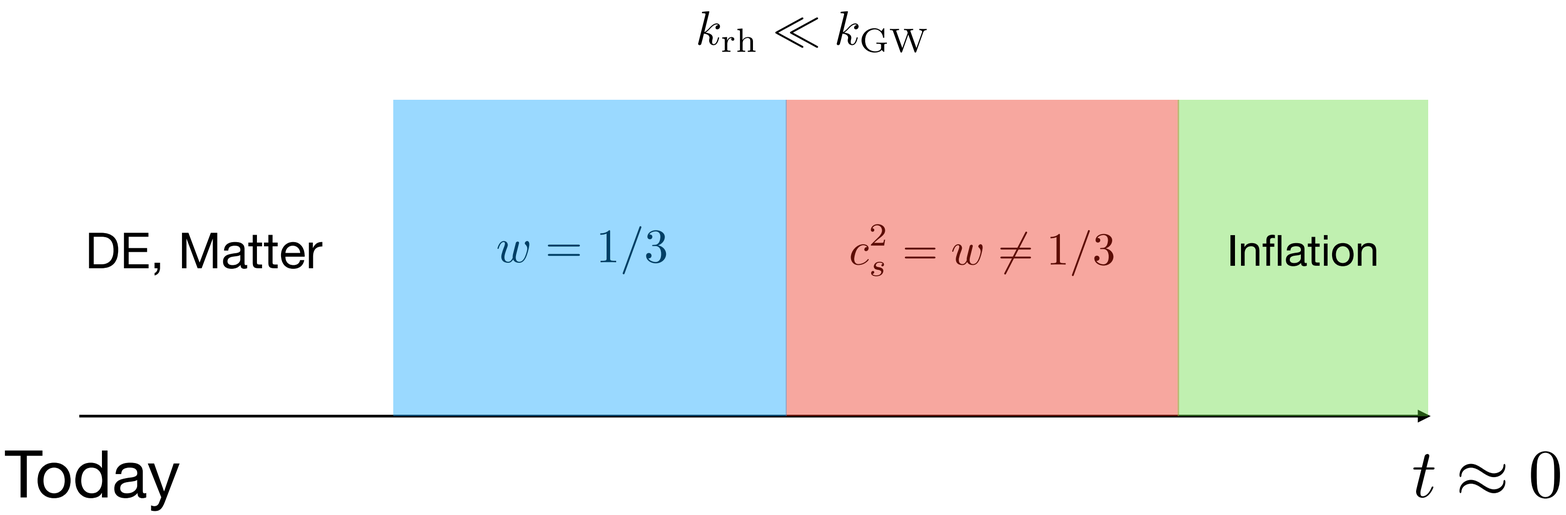
Case of non-detection: upper limits on P_ζ



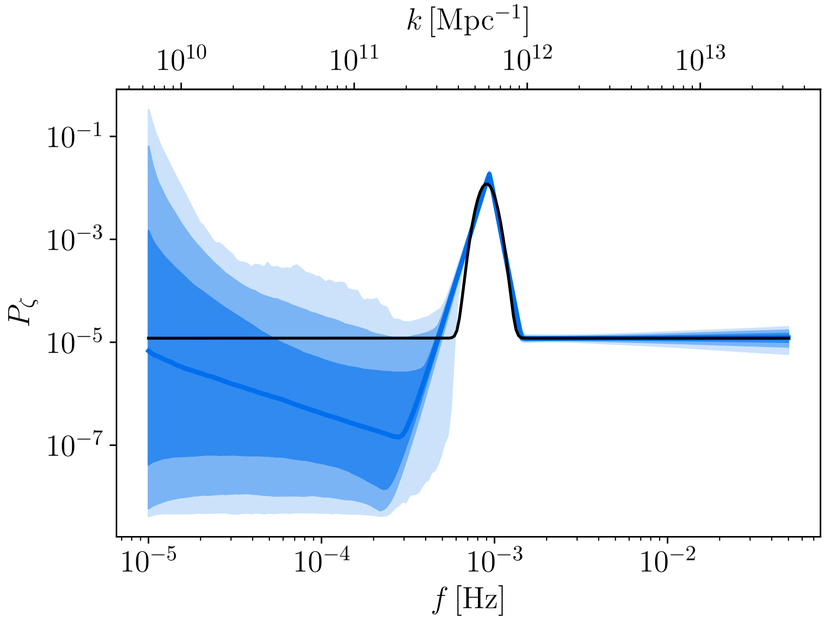
Reconstructing the equation of state

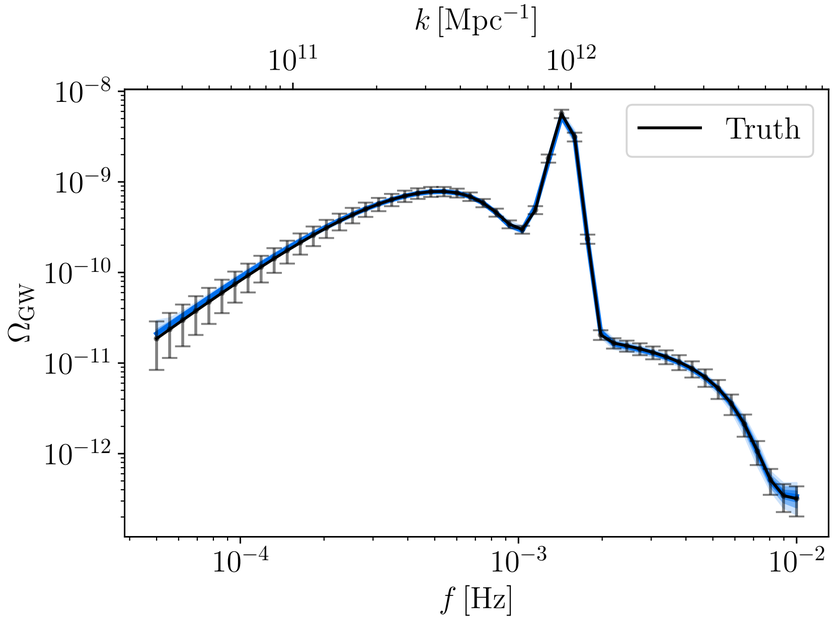
If SIGW are observed, how well can we reconstruct the primordial curvature power spectrum **and the equation of state**?

We consider the following scenario, (using modified version of `SIGWFast` [Witkowski 2022])

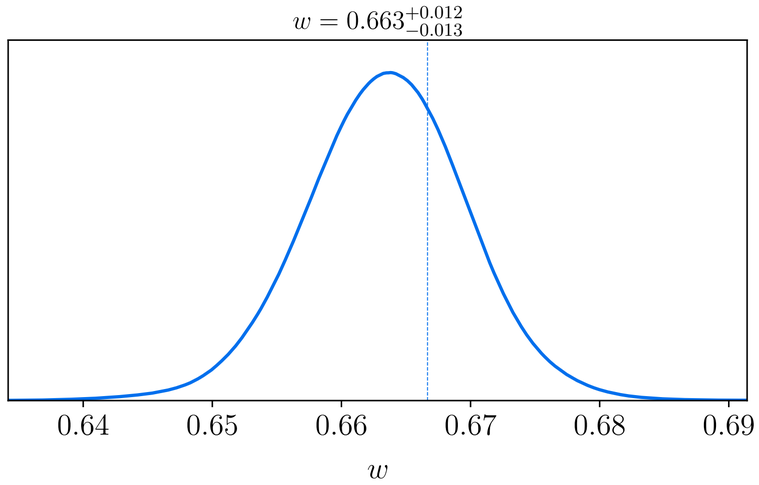


Equation of state

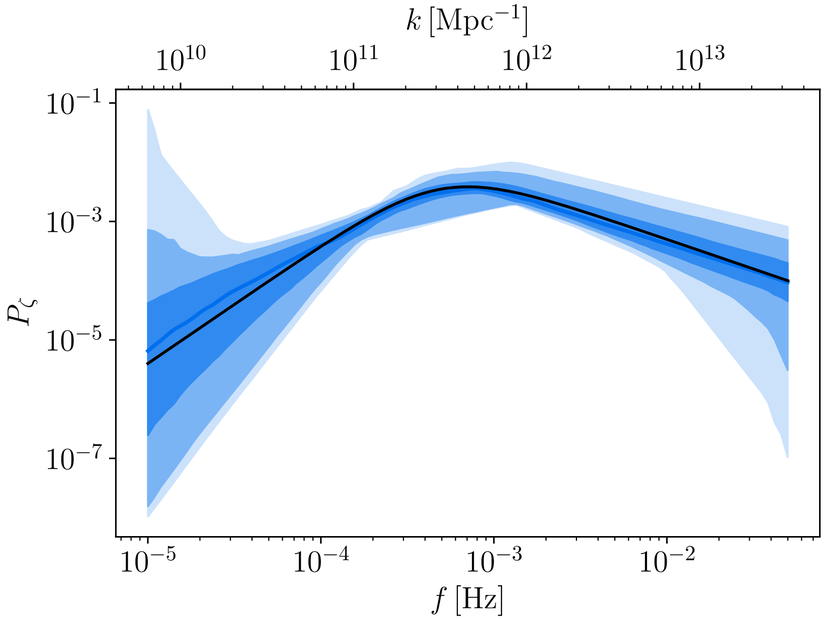


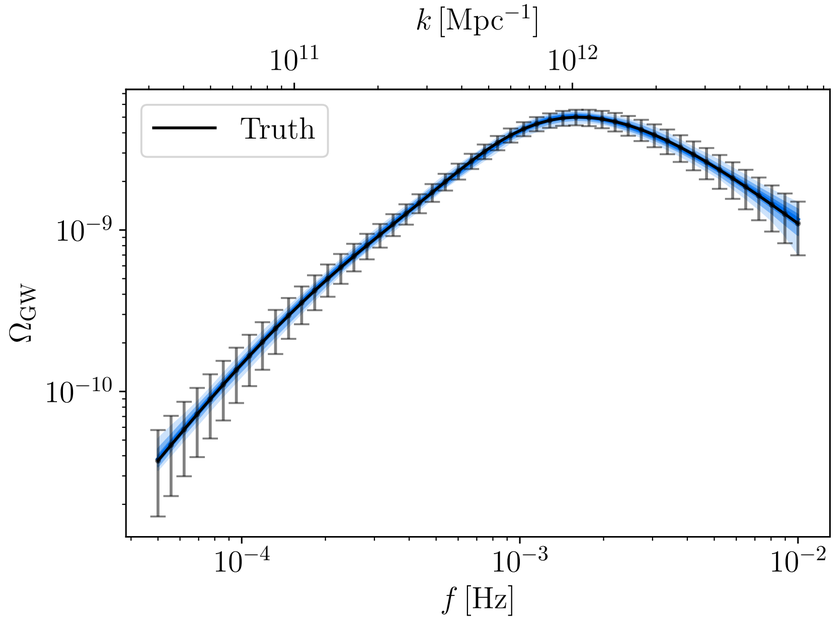


Background equation of state can be inferred from the data since peak shape depends on w

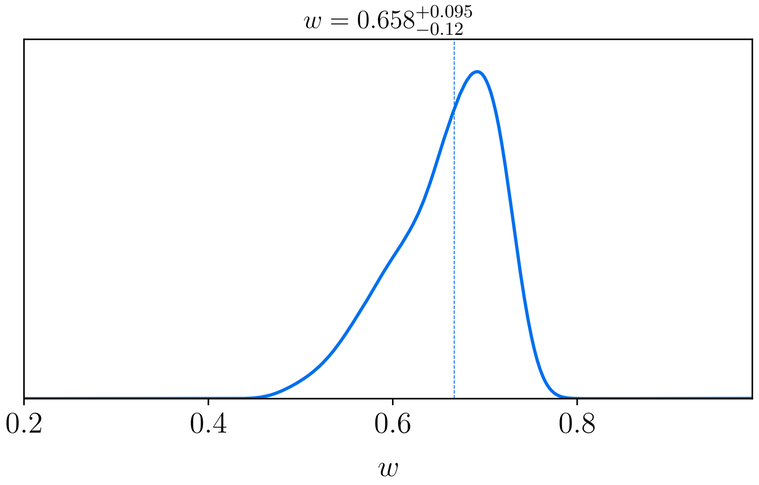


Equation of state





Peak location and UV behaviour depend on w, but there are can be degeneracies between w and P_ζ...



SIGW reconstruction

Summary

SIGW observations would allow for constraints on the small scale curvature power spectrum and equation of state (more examples in paper: oscillatory features, eMD-RD transition, $c_s = 1$ case)

SIGW reconstruction

Summary

SIGW observations would allow for constraints on the small scale curvature power spectrum and equation of state (more examples in paper: oscillatory features, eMD-RD transition, $c_s = 1$ case)

But, this is just a starting point…

Can further incorporate non-Gaussianity, gradual transitions to radiation domination, PBH constraints, modifications of GR

Reconstruction for power spectrum could also be extended to directly reconstruct the inflationary potential

Thank you!

Contact: ameek.malhotra@swansea.ac.uk