The unexpected shape of the primordial black hole mass function

Jacopo Fumagalli

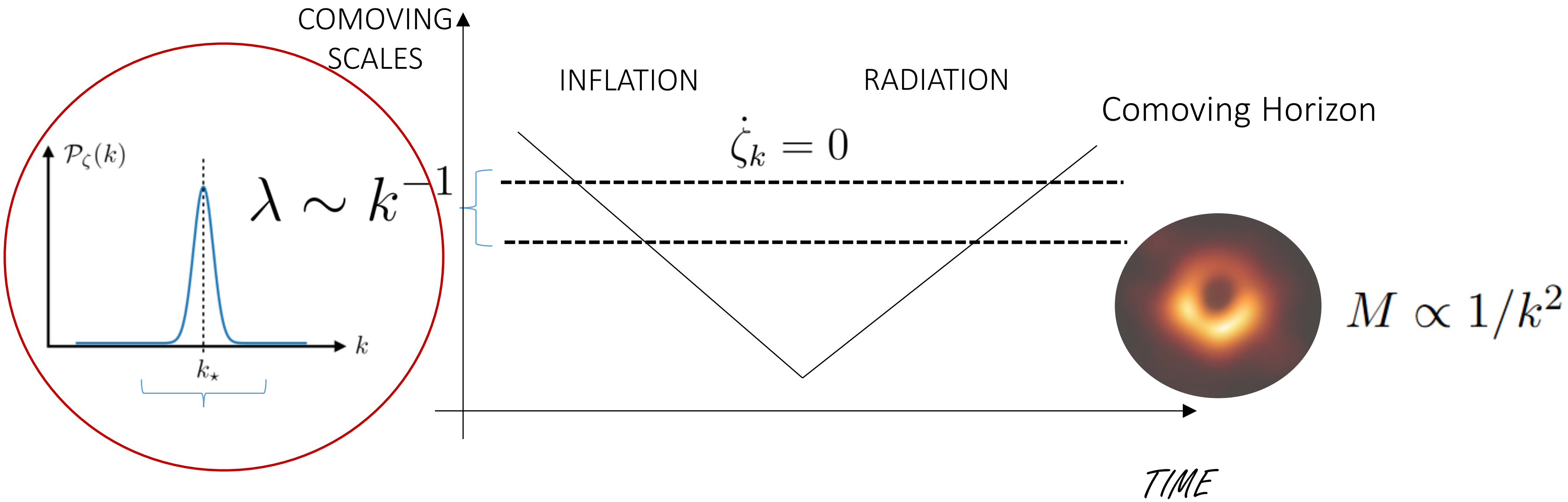
NEHOP Brussels – 25/05/2025

Based on 2412.07709

J. F., J. Garriga, C. Germani and R.K. Shet

ICCUB | Institut de Ciències del Cosmos
UNIVERSITAT DE BARCELONA

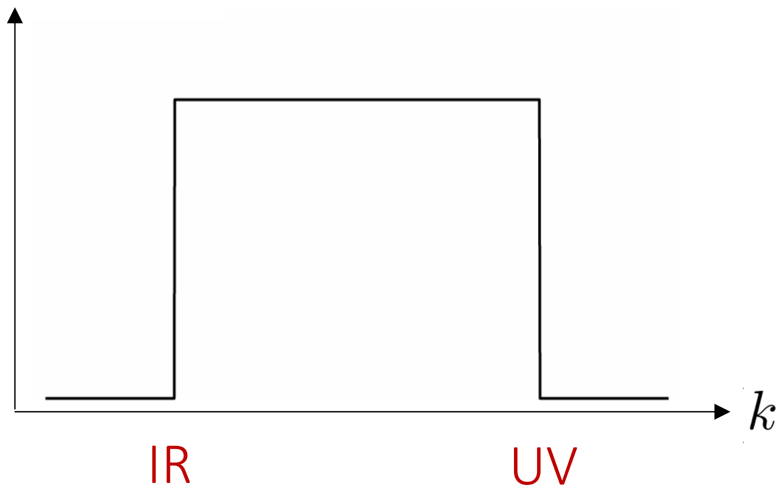
Primordial black hole from inflation



Common sense vs unexpected

Consider a "Broad" spectrum between an IR and an UV scale

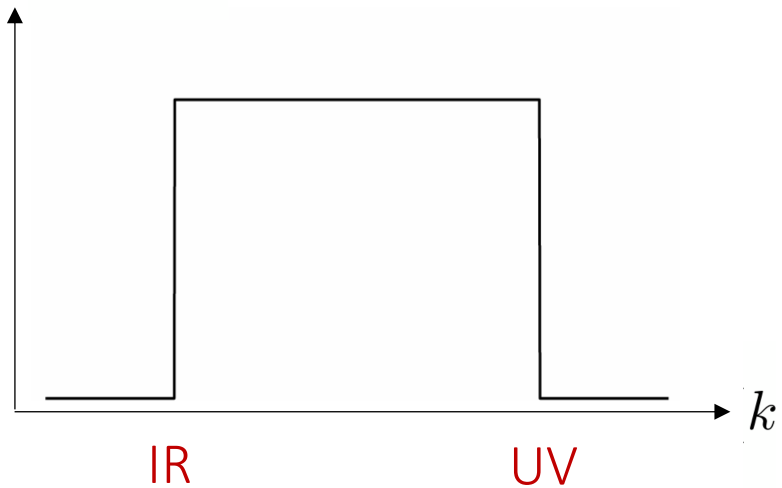
$\mathcal{P}_\zeta(k)$: Power spectrum



Common sense vs unexpected

Consider a "Broad" spectrum between an IR and an UV scale

$\mathcal{P}_\zeta(k)$: Power spectrum



Same probability at formation → Number density grows

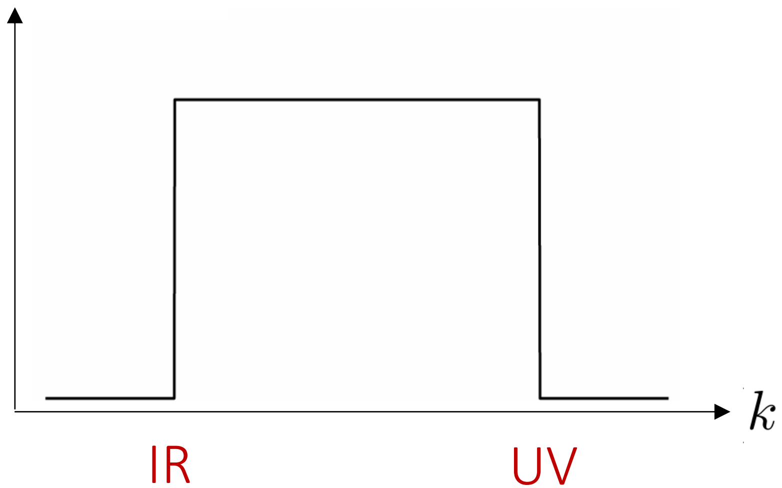
$$\beta_p(M) = \text{const} \qquad \beta(M) \propto \rho_{\rm PBH}/\rho_{\rm rad} \propto a$$

$$\rho_{\rm PBH} \to 1/a^3 \qquad \rho_{\rm rad} \to 1/a^4$$

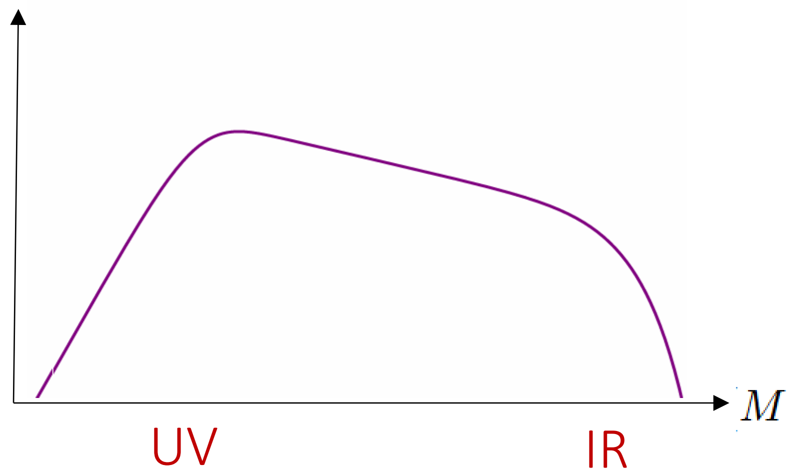
Common sense vs unexpected

Consider a "Broad" spectrum between an IR and an UV scale

$\mathcal{P}_\zeta(k)$: Power spectrum



$f_{\rm PBH}(M)$: Mass function



Same probability at formation → Number density grows → Light PBH dominate

$$\beta_p(M) = \text{const}$$

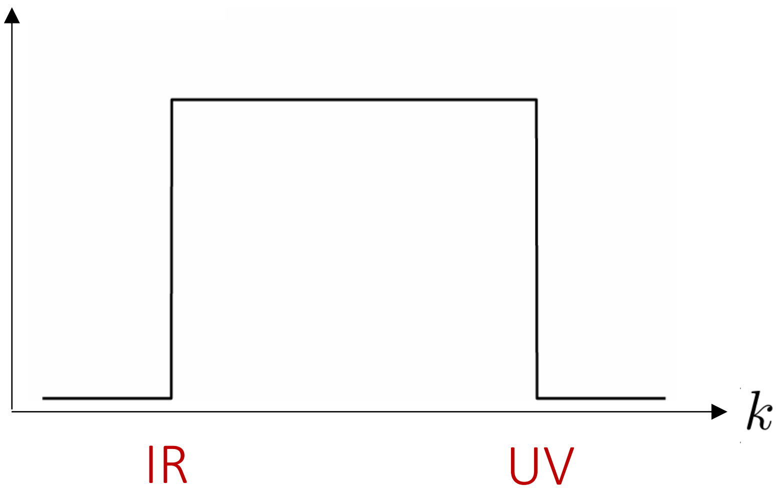
$$\beta(M) \propto \rho_{\rm PBH}/\rho_{\rm rad} \propto a$$

$\rho_{\rm PBH} \to 1/a^3$, $\rho_{\rm rad} \to 1/a^4$

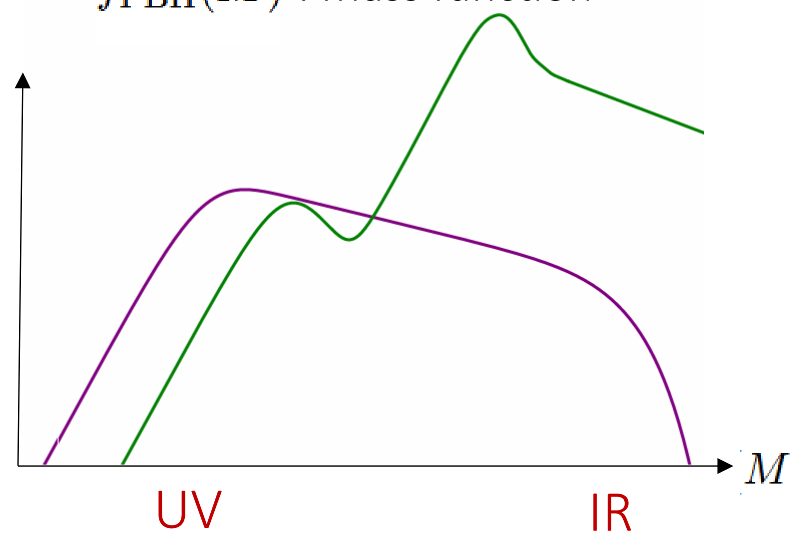
Common sense vs unexpected

Consider a "Broad" spectrum between an IR and an UV scale

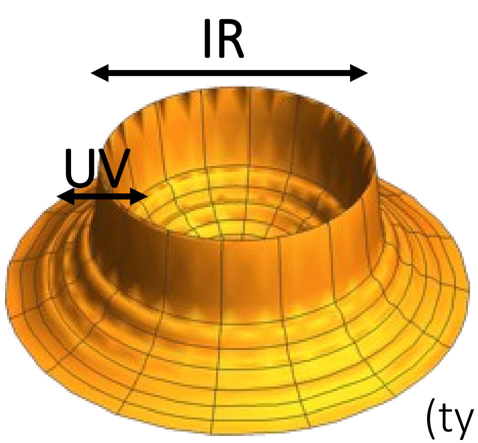
$\mathcal{P}_\zeta(k)$: Power spectrum



$f_{\rm PBH}(M)$: Mass function



J.F., J. Garriga, C. Germani and R.K. Shet 2412.07709



→ Unexpected IR contribution

(typical shape of the density contrast)

Compaction function statistics

Excess mass over the area radius:

$$C = 2\frac{M - M_b}{R}$$ when $C > 1$ gravitational collapse into a PBH is triggered

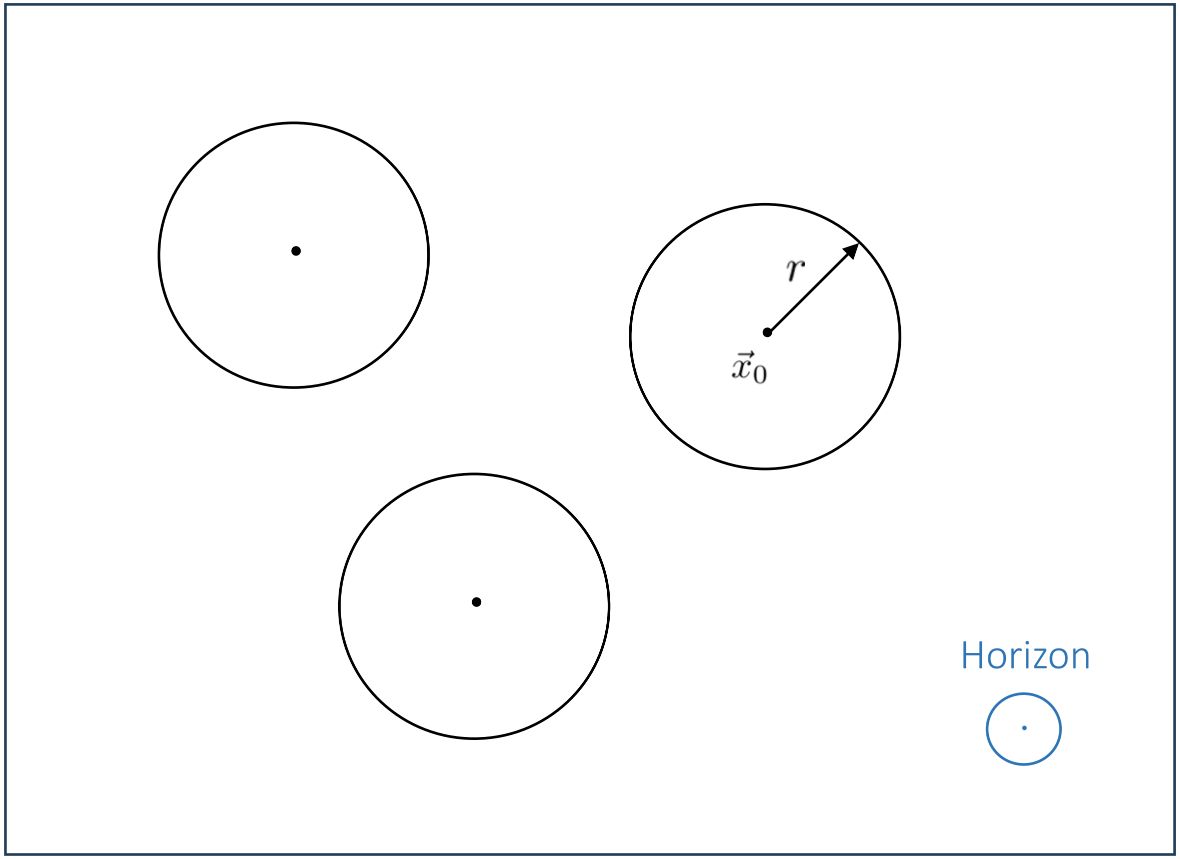
Shibata-Sasaki '99

Compaction function statistics

Compaction function on super-horizon scale (statistics on the initial conditions!):

$$C = 2\frac{M - M_b}{R}$$

Take a comoving volume and the random field ζ

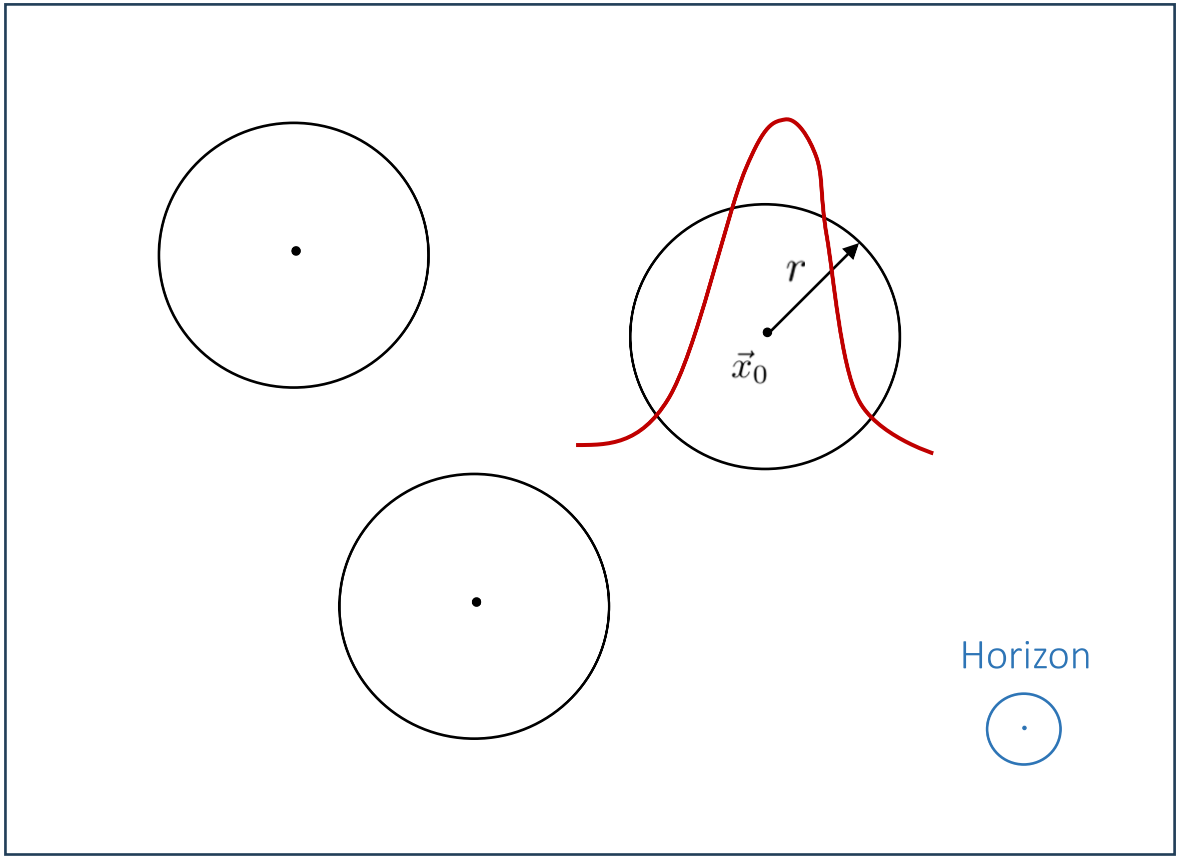


Compaction function statistics

Compaction function on super-horizon scale (statistics on the initial conditions!):

$$C = 2\frac{M - M_b}{R}$$

Take a comoving volume and the random field ζ



Look for:

1. Peaks of the over-density
2. Max. of C
3. Max. of C over threshold

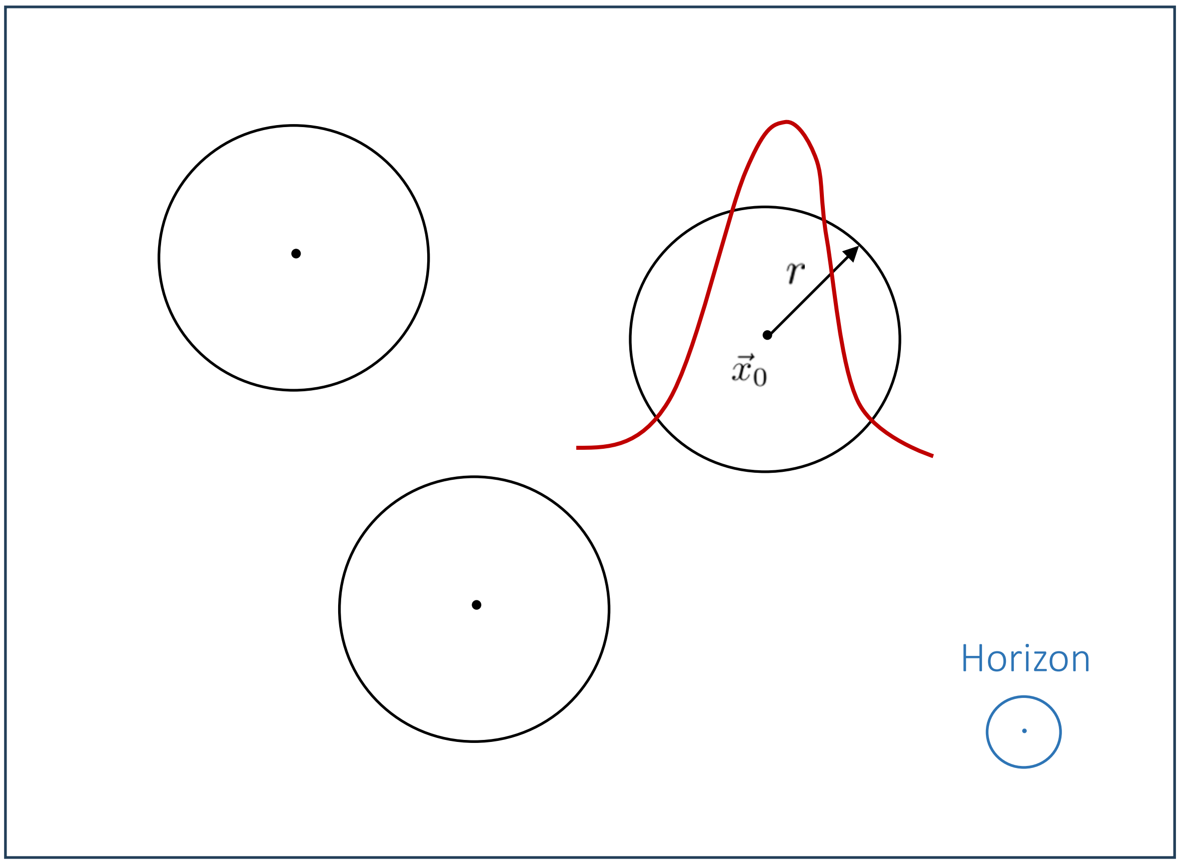
BBKS 86
Germani, Sheth 2020, 2023

Compaction function statistics

Compaction function on super-horizon scale (statistics on the initial conditions!):

$$C = g(r;\vec{x}_0)\left(1 - \frac{3}{8}g(r;\vec{x}_0)\right) \qquad g = -(4/3)\, r\partial_r\zeta$$

Take a comoving volume and the random field ζ



Look for:

1. Peaks of the over-density

   $g(r;\vec{x}_0)$ peak as a function of $\vec{x}_0$

2. Max. of C

   $$v(r_m) \equiv r_m\partial_r g(r_m;\vec{x}_0) = 0$$

   $$w \equiv -r_m^2\partial_r^2 g(r_m;\vec{x}_0) \geq 0$$

3. Max. of C over threshold

   $$g(r_m;\vec{x}_0) \geq g_c(w)$$

BBKS 86
Germani, Sheth 2020, 2023

Abundance

$$\Omega_{\rm PBH} \propto$$

1. Peaks of the over-density

   $g(r;\vec{x}_0)$ peak as a function of $\vec{x}_0$

2. Max. of C

   $$v(r_m) \equiv r_m \partial_r g(r_m;\vec{x}_0) = 0$$

   $$w \equiv -r_m^2 \partial_r^2 g(r_m;\vec{x}_0) \geq 0$$

3. Max. of C over threshold

   $$g(r_m;\vec{x}_0) \geq g_c(w)$$

Abundance

$$\Omega_{\rm PBH} \propto \int \frac{d\ln r}{r} \int_0^\infty dw w \int_{g_c(w)}^{4/3} dg \, \frac{M}{M_H} \frac{f\left(\frac{2g+w}{\sigma_2}\right)}{(2\pi)^{3/2}(\sqrt{3}\sigma_1/\sigma_2)} \cdot P(g, w, v=0)$$

with

$$\sigma_j^2 \equiv \frac{16}{81} \int \frac{dk}{k} (kr)^{4+2j} W^2(kr) \mathcal{P}_\zeta(k)$$

$$\langle gg \rangle \equiv \sigma_g^2 = \sigma_0^2$$

Red box: Number density of peak per comoving volume BBKS '86

Green box: Multivariate Gaussian on the conditions 2. 3.

$$P(g, w, v=0) = p(v=0)p(g,w)$$

$$= \frac{1}{\sqrt{2\pi\sigma_v^2}} \frac{1}{2\pi\sqrt{\det\Sigma}} \exp\left(-\frac{1}{2}\vec{X}^T \Sigma^{-1} \vec{X}\right)$$

$$\vec{X}^T = (w \;\; g), \quad \Sigma = \begin{pmatrix} \tilde{\sigma}_w^2 & \tilde{\sigma}_{wg}^2 \\ \tilde{\sigma}_{gw}^2 & \tilde{\sigma}_g^2, \end{pmatrix}$$

1. Peaks of the over-density

   $g(r; \vec{x}_0)$ peak as a function of $\vec{x}_0$

2. Max. of C

   $$v(r_m) \equiv r_m \partial_r g(r_m; \vec{x}_0) = 0$$

   $$w \equiv -r_m^2 \partial_r^2 g(r_m; \vec{x}_0) \geq 0$$

3. Max. of C over threshold

   $$g(r_m; \vec{x}_0) \geq g_c(w)$$

Abundance

$$\Omega_{\rm PBH} \propto \int \frac{d\ln r}{r} \int_0^\infty dw w \int_{g_c(w)}^{4/3} dg \, \frac{M}{M_H} \frac{f\left(\frac{2g+w}{\sigma_2}\right)}{(2\pi)^{3/2}(\sqrt{3}\sigma_1/\sigma_2)} \cdot P(g,w,v=0)$$

with

$$\sigma_j^2 \equiv \frac{16}{81} \int \frac{dk}{k} (kr)^{4+2j} W^2(kr) \mathcal{P}_\zeta(k)$$

$$\langle gg \rangle \equiv \sigma_g^2 = \sigma_0^2$$

→ Critical scaling

$$\frac{M}{M_H} = \mathcal{K}(r) \left[C(g) - C(g_c(w))\right]^{\gamma_{\rm cr}}$$

1. Peaks of the over-density

   $g(r;\vec{x}_0)$ peak as a function of $\vec{x}_0$

2. Max. of C

   $$v(r_m) \equiv r_m \partial_r g(r_m;\vec{x}_0) = 0$$

   $$w \equiv -r_m^2 \partial_r^2 g(r_m;\vec{x}_0) \geq 0$$

3. Max. of C over threshold

   $g(r_m;\vec{x}_0) \geq g_c(w)$

Abundance

$$\Omega_{\rm PBH} \propto \int \frac{d\ln r}{r} \int_0^\infty dw w \int_{g_c(w)}^{4/3} dg \, \frac{M}{M_H} \frac{f\left(\frac{2g+w}{\sigma_2}\right)}{(2\pi)^{3/2}(\sqrt{3}\sigma_1/\sigma_2)} \cdot P(g,w,v=0)$$

with

$$\sigma_j^2 \equiv \frac{16}{81} \int \frac{dk}{k} (kr)^{4+2j} W^2(kr) \mathcal{P}_\zeta(k)$$

$$\langle gg \rangle \equiv \sigma_g^2 = \sigma_0^2$$

Threshold

We use an analytic fit to numerical simulations with Type I initial conditions

$$g_c(w \ll 1) \approx \frac{1}{2} \quad \text{and} \quad g_c(w \gg 1) \approx \frac{4}{3} - \frac{32}{9w}.$$

1. Peaks of the over-density

   $g(r;\vec{x}_0)$ peak as a function of $\vec{x}_0$

2. Max. of C

   $$v(r_m) \equiv r_m \partial_r g(r_m;\vec{x}_0) = 0$$
   $$w \equiv -r_m^2 \partial_r^2 g(r_m;\vec{x}_0) \geq 0$$

3. Max. of C over threshold

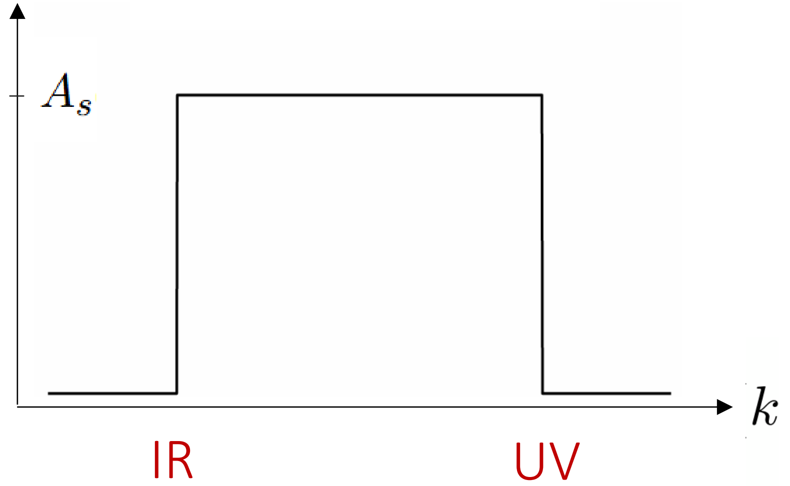
   $g(r_m;\vec{x}_0) \geq g_c(w)$

Escrivà,Germani, Sheth, 2020

Broad spectrum

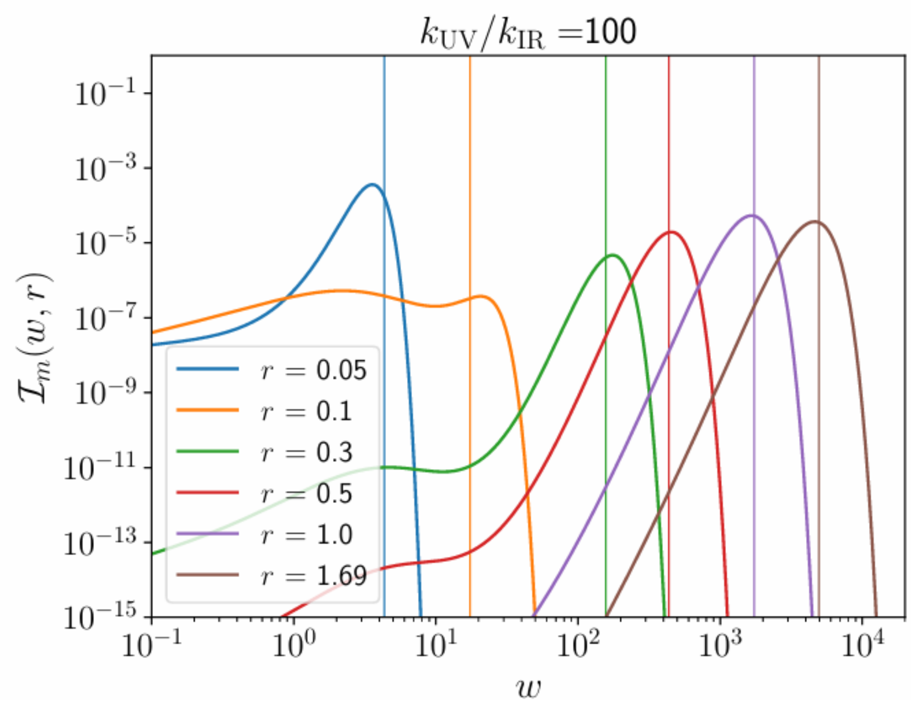
$$\mathcal{P}_\zeta(k) = A_s \theta(k - k_{\rm IR})\theta(k_{\rm UV} - k),$$

$$\alpha \equiv k_{\rm UV}/k_{\rm IR} \gg 1$$



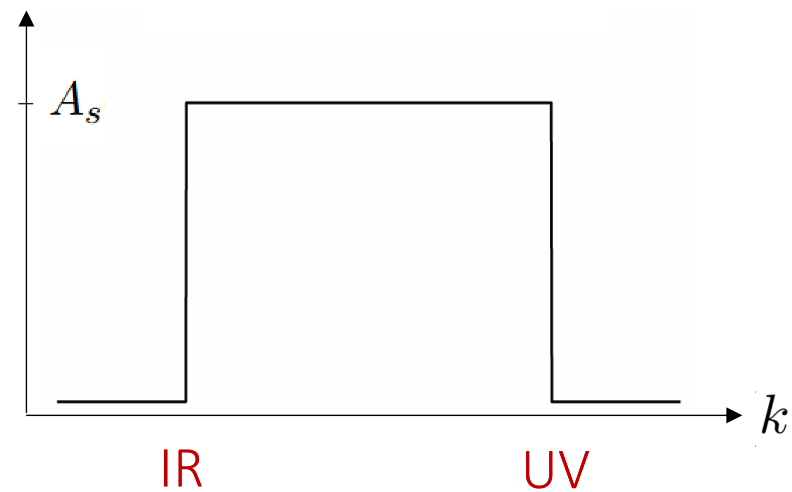
Broad spectrum

1. For IR scales, PDF favors peaked profiles of the compaction function whenever $A_s(\ln\alpha)^2 \gtrsim 0.05$



$$\mathcal{P}_\zeta(k) = A_s\theta(k - k_{\rm IR})\theta(k_{\rm UV} - k),$$

$$\alpha \equiv k_{\rm UV}/k_{\rm IR} \gg 1$$



J.F., J. Garriga, C. Germani and R.K. Shet 2412.07709

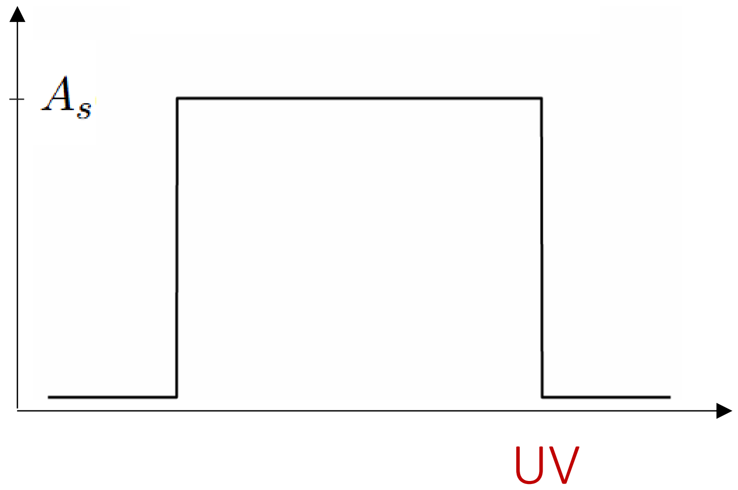
Broad spectrum

2. σ_g^2 has maximum around the IR scale independent on α !

$$\sigma_g^2 \simeq \frac{8}{9} A_s \left(\ln\alpha + \mathrm{CosIntegral}(2r) + \frac{\sin^2 r - r\sin(2r)}{r^2} \right)$$

$$\mathcal{P}_\zeta(k) = A_s \theta(k - k_{\rm IR}) \theta(k_{\rm UV} - k),$$

$$\alpha \equiv k_{\rm UV}/k_{\rm IR} \gg 1$$



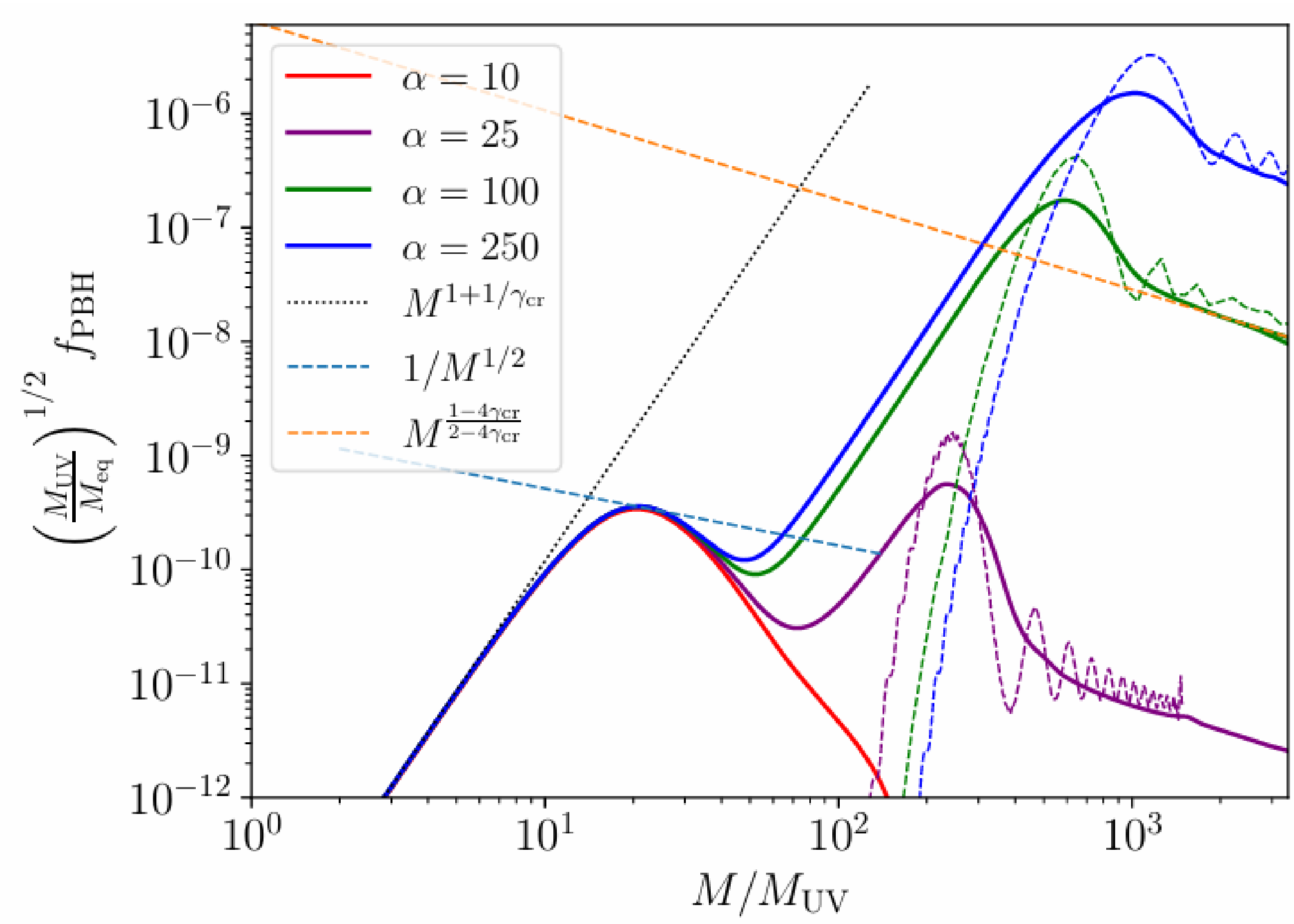
J.F., J. Garriga, C. Germani and R.K. Shet 2412.07709

All that leads to an unexpected bimodal mass function!

$$\Omega_{\rm PBH} = \int d\log M \, \underline{f_{\rm PBH}(M)}$$

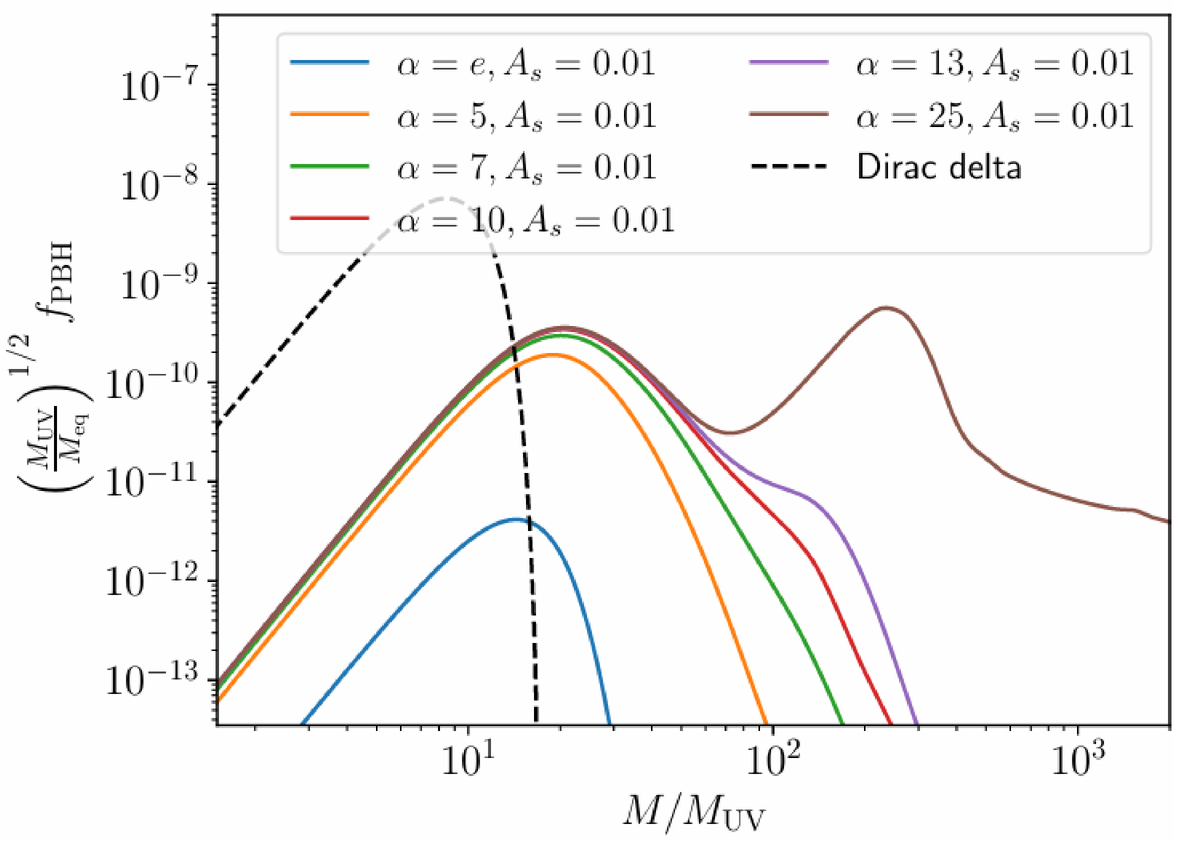
Formula and analytical approximation in

J.F., J. Garriga, C. Germani and R.K. Shet 2412.07709



From Narrow to Broad

$$\Omega_{\rm PBH} = \int d\log M \, \underline{f_{\rm PBH}(M)}$$



J.F., J. Garriga, C. Germani and R.K. Shet
2412.07709

Conclusions and outlook

A broad enhancement in the primordial perturbations leads to a significant contribution to the PBH mass function from heavy black holes associated with the IR scale.

Conclusions and outlook

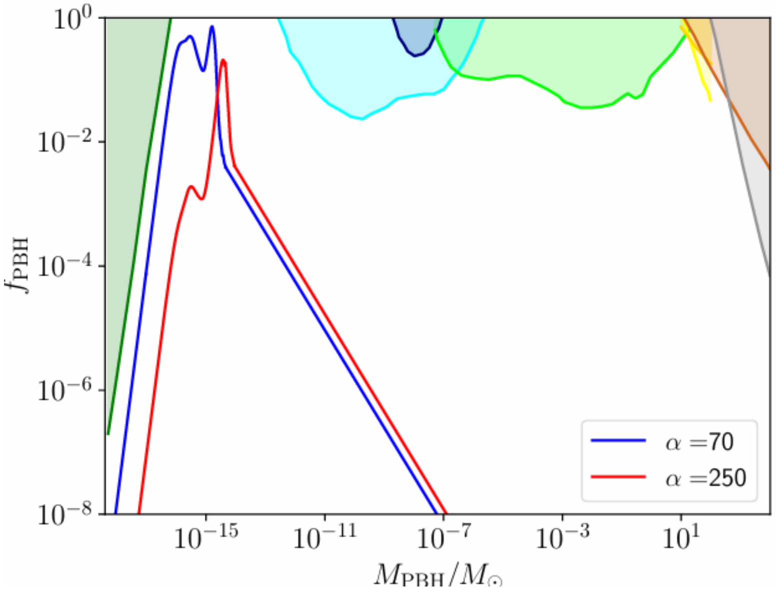
A broad enhancement in the primordial perturbations leads to a significant contribution to the PBH mass function from heavy black holes associated with the IR scale.

- This may significantly impact on overproduction constraints for GWs

Conclusions and outlook

A broad enhancement in the primordial perturbations leads to a significant contribution to the PBH mass function from heavy black holes associated with the IR scale.

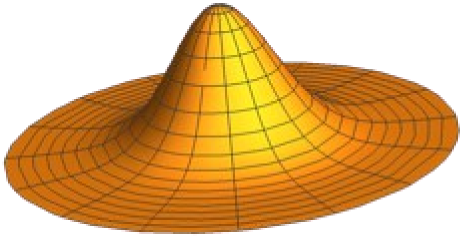
- This may significantly impact on overproduction constraints for GWs
- It can reduce the effective size of the asteroid mass window.



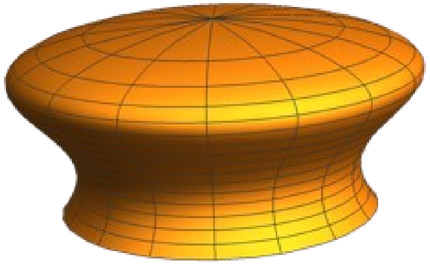
Conclusions and outlook

A broad enhancement in the primordial perturbations leads to a significant contribution to the PBH mass function from heavy black holes associated with the IR scale.

- This may significantly impact on overproduction constraints for GWs
- It can reduce the effective size of the asteroid mass window.
- The statistics is dominated by fluctuations which are at the boundary between Type I and Type II.
  Dedicated studies needed to clarify the contribution of the Type II perturbations.



Type I fluctuation $g < 4/3$



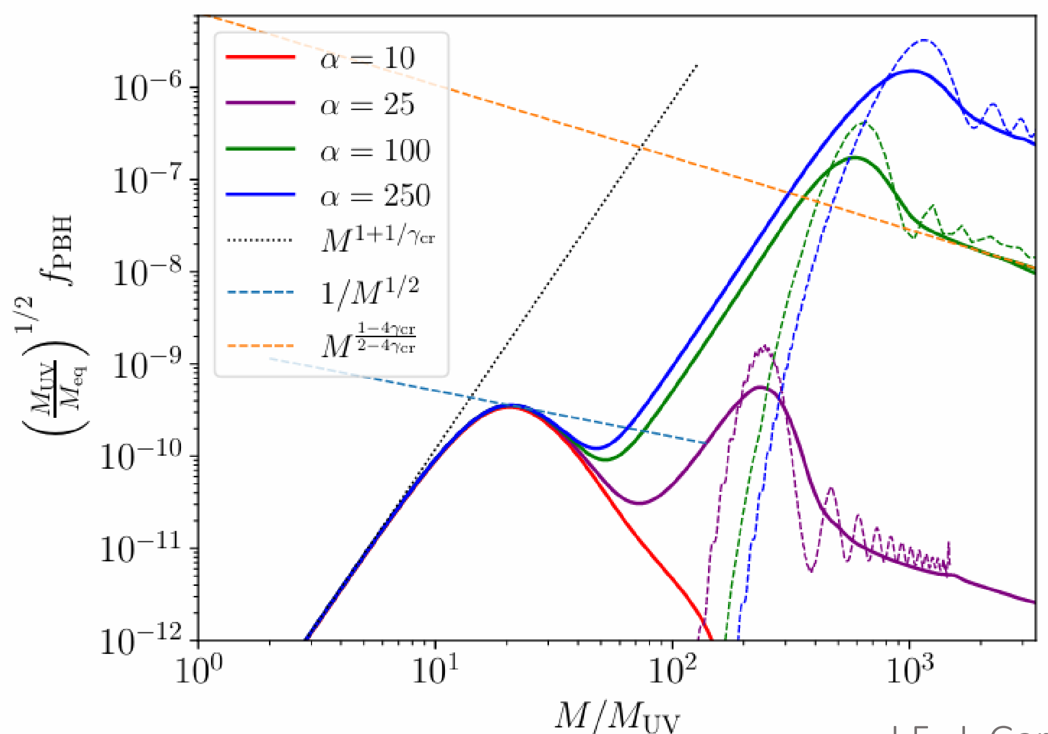
Type II fluctuation $g > 4/3$

Back up slides

Explicit formula mass function

$$\Omega_{\rm PBH} = \int d\log M\, f_{\rm PBH}(M)$$

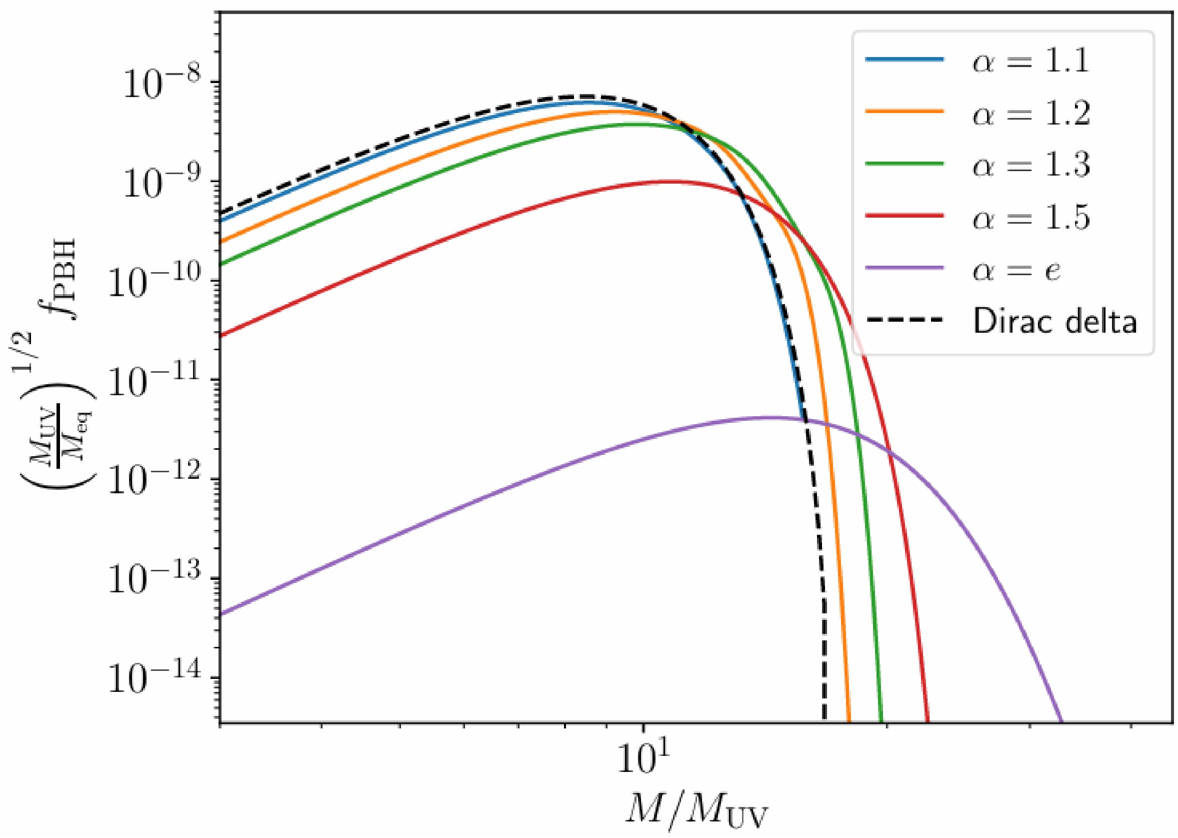
$$f_{\rm PBH}(M) = \left(\frac{M}{\mathcal{K}M_{\rm eq}}\right)^{\frac{1}{\gamma_{\rm cr}}+1} \int \frac{dM_H}{M_H}\left(\frac{M_{\rm eq}}{M_H}\right)^{\frac{1}{\gamma_{\rm cr}}+1} \times \frac{2\pi\mathcal{K}}{3}\int_0^\infty dw \frac{w}{\gamma_{\rm cr}(1-\frac{3}{4}g_\bullet)}\left(\frac{M_{\rm eq}}{M_H}\right)^{1/2}$$
$$\times\, p(g_\bullet, w, v=0)\frac{f\left(\frac{2g_\bullet+w}{\sigma_\chi}\right)}{(2\pi)^{3/2}(\sqrt{3}\sigma_1/\sigma_2)^3}\,,$$



J.F., J. Garriga, C. Germani and R.K. Shet
2412.07709

Narrow spectrum

$$\Omega_{\rm PBH} = \int d\log M \, f_{\rm PBH}(M)$$



J.F., J. Garriga, C. Germani and R.K. Shet
2412.07709