CALCULATION OF PRIMORDIAL nG FROM USR INFLATION

Alejandro Pérez Rodríguez
IFT UAM-CSIC

NEHOP. Brussels, May 2025

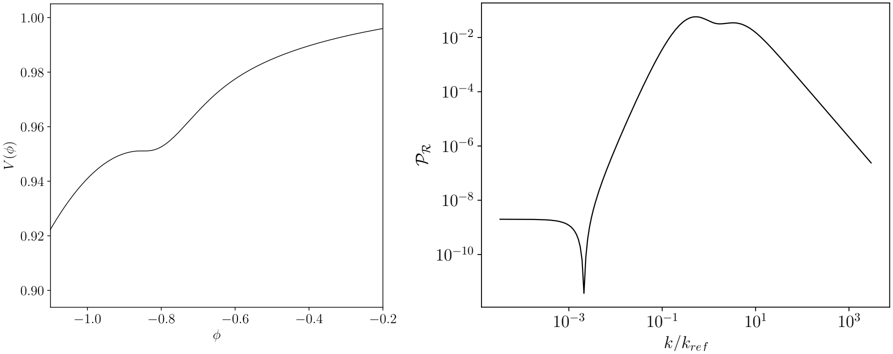
OUTLINE

1. Setup: USR inflation
2. The δN formalism: non-Gaussianity from non-linearity
3. Going beyond: non-Gaussian $\delta\phi$

Based on:

- 2406.02417 (G. Ballesteros, T. Konstandin, **APR**, M. Pierre, J. Rey)
- 2412.14106 (G. Ballesteros, J. Gambín Egea, T. Konstandin, **APR**, M. Pierre, J. Rey)

1. The USR setup | Inflection point



Inflection point in $V(\phi)$ → Violation of slow-roll →

Peak in $\mathcal{P}_{\mathcal{R}}$ and non-Gaussianity

2. THE δN FORMALISM

Fully non-linear *classical* relation between ζ and $\delta\phi$, $\delta\pi = \delta\phi'$ *to first order in gradients*

$$\boxed{\zeta = \delta N \equiv N\left[\{\bar{\phi}(N_i) + \delta\phi, \bar{\pi}(N_i) + \delta\pi\} \to \bar{\phi}(N_f)\right] - N\left[\{\bar{\phi}(N_i), \bar{\pi}(N_i)\} \to \bar{\phi}(N_f)\right]\ ,}$$

- No dependence on final time (ζ conserved) [Lyth, Malik, Sasaki '05]
- Number of efolds N computed solving $\underline{\text{background}}$ EOMs (no $\boldsymbol{x}$ dependence)
  ↔ *Separate universe approach* [Sugiyama, Komatsu, Futamase '13]
- Reference N_i chosen once relevant Fourier modes $\mathcal{R}_k$ $\underline{\text{have frozen}}$
  → $\delta\pi = (\pi'/\pi)|_N\, \delta\phi$, cf. [Jackson et al. '24] (gradient contamination)
- N.B. Gradient neglecting enters at two points in the formula

2. THE δN FORMALISM | Gaussian input

Main idea

Assuming Gaussian $\delta\phi$, non-linear relation with ζ yields non-Gaussian ζ:

$$P[\zeta] = P[\delta\phi] \left| \frac{d\delta\phi}{d\zeta} \right|$$

- Assumption of $\delta\phi$ gaussianity motivated by smallness of self-interactions [Biagetti et al. '21], [Hooshangi, Namjoo, Noorbala '23]
- Gaussian PDF fully determined by its variance $\sigma^2_{\delta\phi} = \int_{k_{IR}}^{k(N_i)} d\log k \, \mathcal{P}_{\delta\phi}(k)$
- This approach provides only *local non-Gaussianity* of ζ

2. THE δN FORMALISM | CR attractor

Constant-roll $\leftrightarrow \epsilon_2 = \frac{2\phi''}{\phi'} = \text{cst} \leftrightarrow$ linear EOM for $\phi(N) \leftrightarrow \phi(N) \propto e^{\frac{\epsilon_2}{2}N}$.

$$\boxed{\delta\phi = \bar{\phi}'(N_i)\left(e^{-\frac{\epsilon_2}{2}\delta N} - 1\right)} \qquad \text{Recall: } \zeta = \delta N$$

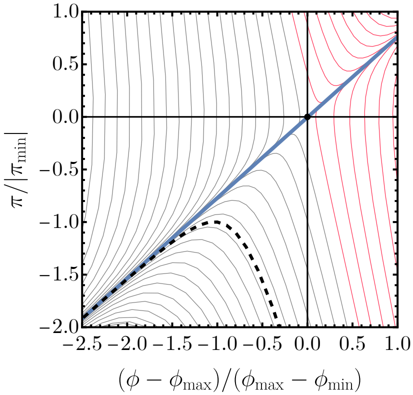
Change of variables: $P(\delta N) = P(\delta\phi)\left|\frac{d\delta\phi}{d\delta N}\right|$ ($\delta\phi$: Gaussian with $\sigma^2_{\delta\phi}$)

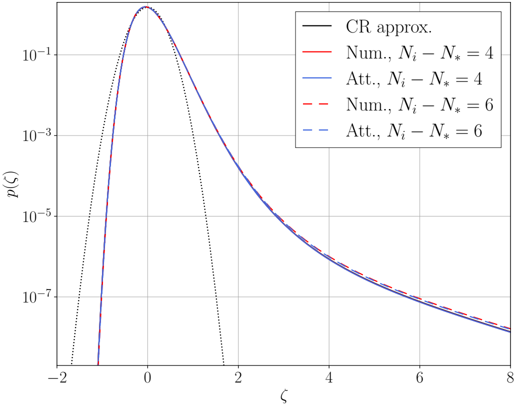
$$P(\zeta) = \frac{1}{\sqrt{2\pi\,\sigma_\zeta^2}}\exp\left[-\frac{1}{2\sigma_\zeta^2}\left(\frac{2}{\epsilon_2}\right)^2\left(1 - e^{-\frac{\epsilon_2}{2}\zeta}\right)^2 - \frac{\epsilon_2}{2}\zeta\right].$$

- Small $\epsilon_2\delta N$: $P(\zeta) \sim e^{-\frac{\zeta^2}{2\sigma_{\mathcal{R}}}}$. Gaussian with variance $\sigma^2_{\mathcal{R}}$ (agrees with LPT)
- Large $\epsilon_2\delta N$: $\boxed{P(\zeta) \sim e^{-\frac{\epsilon_2}{2}\zeta}}$. For $\epsilon_2 > 0$, exponential tail

CR attractor

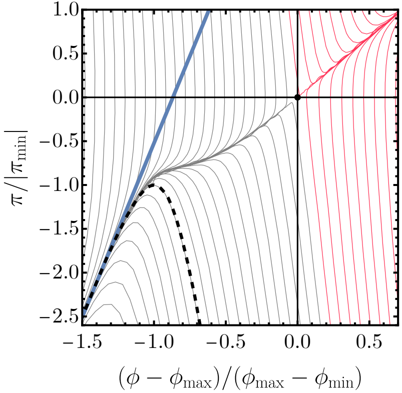
N.B.: $\epsilon_2 \propto d\pi/d\phi$. Straight line in phase space ↔ CR trajectory ☞ [2406.02417]

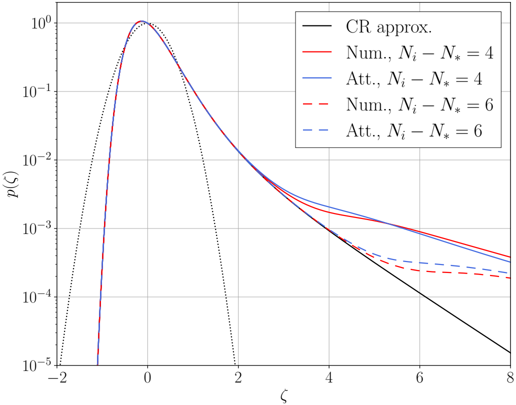




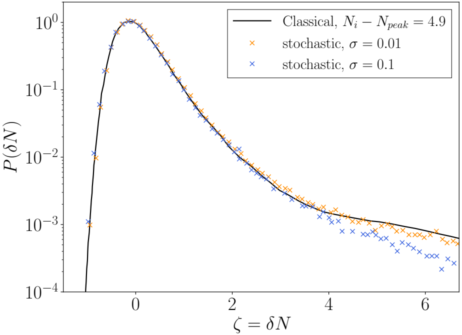
2. THE δN FORMALISM | Non-CR attractor

N.B.: $\epsilon_2 \propto d\pi/d\phi$. Straight line in phase space ↔ CR trajectory ☞ [2406.02417]





2. THE δN FORMALISM | A comment on stochastic approach



Models studied: Same result as classical δN provided $k_{\text{peak}} \simeq \sigma\, a(N_i)H(N_i)$ [2406.02417] (irrelevant for CR attractor [Tomberg '23])
$N_i - N_{\text{peak}} \approx -\log(0.01) \approx 4.6 \neq -\log(0.1) \approx 2.3$

N.B. Stochastic and classical δN do not predict same result for attractor with $\epsilon_2 = 0$ (Exponential vs. Gaussian) [Jackson et al. '24], cf. *first-passage time analysis* [Pattison, Vennin, Assadullahi, Wands '17], [Ezquiaga, García-Bellido, Vennin '20]

3. NON-GAUSSIAN $\delta\phi$ | Main idea

Recent update: 2412.14106

[Ballesteros, Gambín Egea, Konstadin, APR, Pierre, Rey '24 *(arXiv preprint)*]

Question:

- δN formalism: assumes *as input* Gaussian $\delta\phi$
- Computes local NG due to *non-linear* relation between $\delta\phi$ and ζ
- Is $\delta\phi$ at N_i really Gaussian?

Approach to answer:

- Use in-in formalism to compute higher-order (≥ 3) correlators of $\delta\phi$
- Construct non-Gaussian PDF of $\delta\phi$ (*intrinsic* non-Gaussianity)
- Use non-linear δN transformation, check how intrinsic NG affects the PDF of ζ

3. NON-GAUSSIAN $\delta\phi$ | A first estimate

Neglecting gradients,

$$\zeta = \varphi + \frac{\epsilon_2}{4}\varphi^2 + \mathcal{O}(\varphi^3), \quad \varphi = -\delta\phi/\phi'$$

Truncation of δN formula to first non-linear order.

Two first contributions to the bispectrum of ζ:

$$\langle \zeta_{\boldsymbol{p}}\zeta_{\boldsymbol{k}}\zeta_{\boldsymbol{q}}\rangle = \langle \varphi_{\boldsymbol{p}}\varphi_{\boldsymbol{k}}\varphi_{\boldsymbol{q}}\rangle + \frac{\epsilon_2}{4}\langle (\varphi^2)_{\boldsymbol{p}}\varphi_{\boldsymbol{k}}\varphi_{\boldsymbol{q}}\rangle + \mathcal{O}(\varphi^5) \equiv [\mathcal{I}(p,q,k) + \mathcal{N}(p,q,k)](2\pi)^3\delta(\boldsymbol{p}+\boldsymbol{k}+\boldsymbol{q})$$

3. NON-GAUSSIAN $\delta\phi$ | A first estimate

$$\langle \zeta_{\boldsymbol{p}} \zeta_{\boldsymbol{k}} \zeta_{\boldsymbol{q}} \rangle = \langle \varphi_{\boldsymbol{p}} \varphi_{\boldsymbol{k}} \varphi_{\boldsymbol{q}} \rangle + \frac{\epsilon_2}{4} \langle (\varphi^2)_{\boldsymbol{p}} \varphi_{\boldsymbol{k}} \varphi_{\boldsymbol{q}} \rangle + \mathcal{O}(\varphi^5) \equiv [\mathcal{I}(p,q,k) + \mathcal{N}(p,q,k)](2\pi)^3 \delta(\boldsymbol{p}+\boldsymbol{k}+\boldsymbol{q})$$

where

$$\langle \varphi_{\boldsymbol{p}} \varphi_{\boldsymbol{k}} \varphi_{\boldsymbol{q}} \rangle = 2\mathrm{Im}\left[\int dt' \, \langle 0 | \varphi_{\boldsymbol{p}} \varphi_{\boldsymbol{k}} \varphi_{\boldsymbol{q}} H_I(t') | 0 \rangle \right] \tag{1}$$

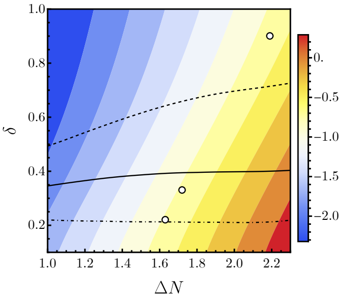
$$\frac{\epsilon_2}{4} \langle (\varphi^2)_{\boldsymbol{p}} \varphi_{\boldsymbol{k}} \varphi_{\boldsymbol{q}} \rangle = \frac{\epsilon_2}{2} \left(|\varphi_p|^2 |\varphi_q|^2 + |\varphi_p|^2 |\varphi_k|^2 + |\varphi_k|^2 |\varphi_q|^2 \right) \tag{2}$$

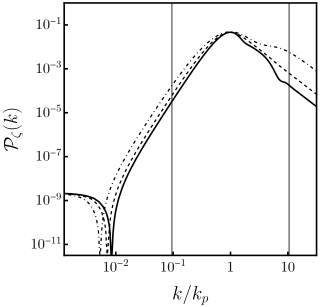
(1) First correction to Gaussianity of $\delta\phi$. H_I: cubic interaction Hamiltonian in the $\delta\phi$-gauge. *Instrinsic non-Gaussianity*

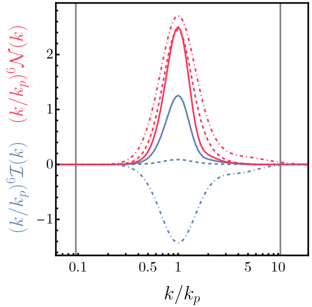
(2) Local non-Gaussinity from change of gauge. First term of the non-linear result obtained through δN formalism. *Non-linear non-Gaussianity*

3. Non-Gaussian $\delta\phi$ | A first estimate

Relative size of $\mathcal{I}$ and $\mathcal{N}$ depending on smoothness of SR-USR transition







See details in [2406.02417]

3. NON-GAUSSIAN $\delta\phi$ | PDF of ζ

Next step: impact of first order correction to Gaussianity of $\delta\phi$ in all orders of ζ

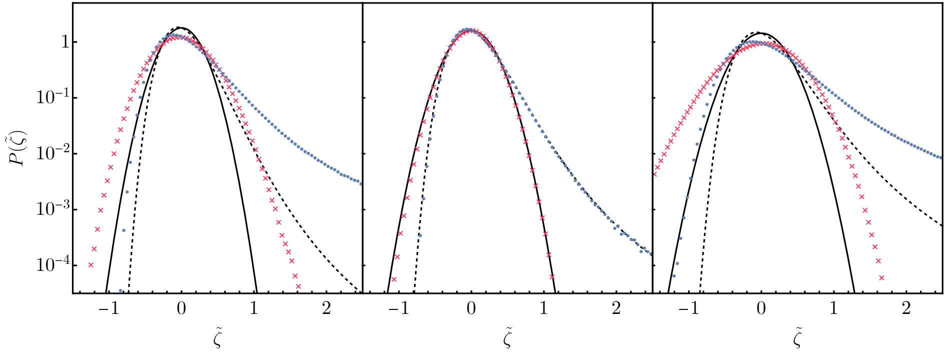
Procedure: (details ☞ 2412.14106)

1. Compute $\mathcal{P}_{\delta\phi}(k)$, $\mathcal{I}(p,k,q)$ (spectrum and bispectrum of $\delta\phi$)
2. Generate a (random) lattice of non-Gaussian $\delta\phi(\boldsymbol{x})$ with the right spectrum and bispectrum:
   - 2.1 Fourier transform Gaussian $\delta\phi(\boldsymbol{x}) \to \delta\phi_{\boldsymbol{k}}$
   - 2.2 Multiply by $\mathcal{P}_{\delta\phi}(k)$, convolve with $\mathcal{I}(p,k,q)$
   - 2.3 Inverse-Fourier transform non-Gaussian $\delta\phi_{\boldsymbol{k}} \to \delta\phi(\boldsymbol{x})$
3. From the lattice, sample the PDF of $\delta\phi$
4. Use δN formula to compute non-linear ζ from non-Gaussian $\delta\phi$

Remark: Convergence? Relation to perturbativity breakdown in the in-in formalism (large loop corrections)? See 2412.14106

3. NON-GAUSSIAN $\delta\phi$ | PDF of ζ

See details in [2406.02417]



— Gaussian $\delta\phi$ - - - - non-linear ζ from Gaussian $\delta\phi$

× non-Gaussian $\delta\phi$ · non-linear ζ from non-Gaussian $\delta\phi$

CONCLUSIONS

- δN formula: non-linear relation between inflaton and curvature perturbations. Gaussian $\delta\phi \rightarrow$ non-Gaussian ζ
- USR phase: Exponential tail in PDF of ζ due to non-linear change of variables along attractor in phase space
- Refinement: account for inflaton non-Gaussianity
  - Two sources of non-Gaussianity: *intrinsic non-Gaussianity* and non-linearities
  - A first approach: sample non-Gaussian $\delta\phi$ (using QFT input), apply non-linear relation to ζ.