From the inflaton potential to the PBH abundance

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Dark matter without dark matter particles

- Primordial black holes (PBH) are a candidate for dark matter. (Hawking 1971)

- PBHs could also seed supermassive black holes in centres of galaxies.

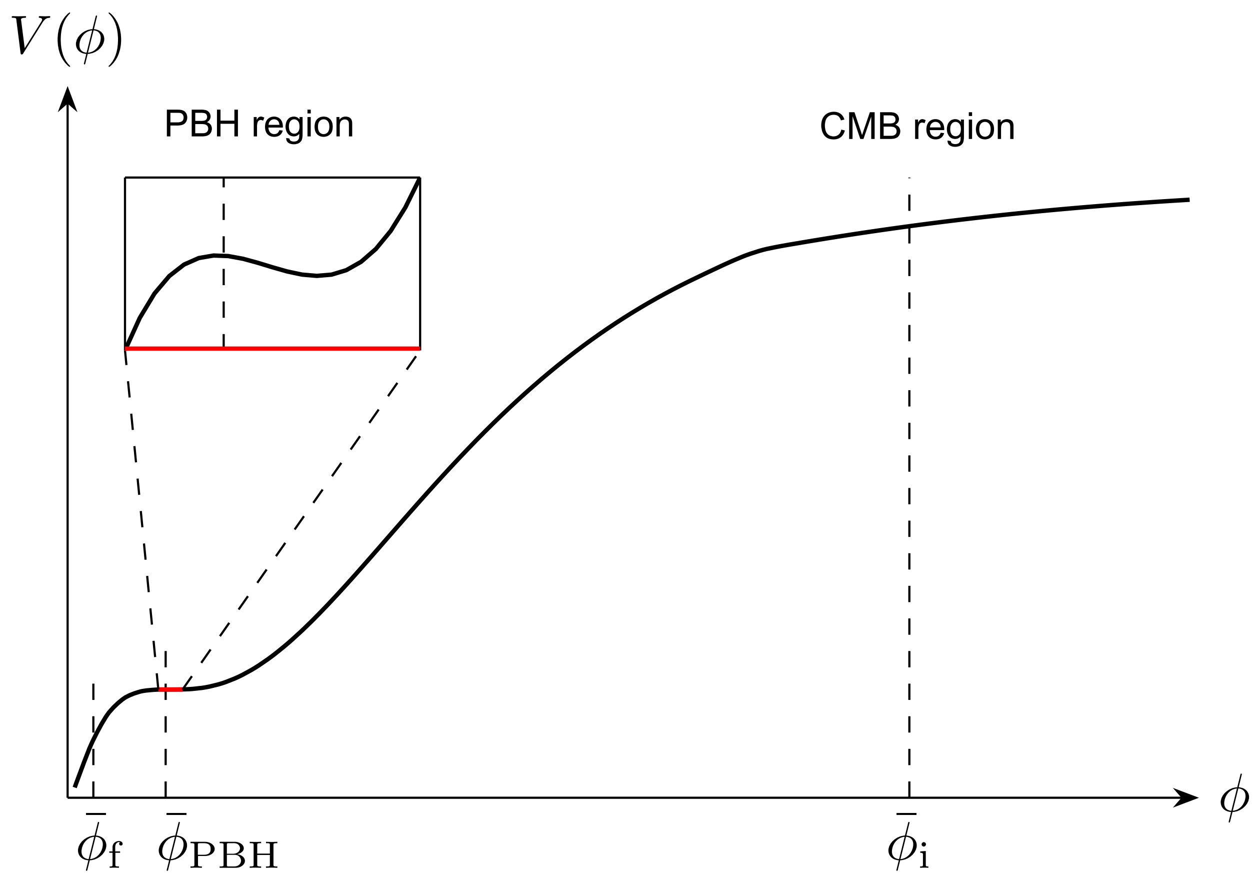
Slow means large

- PBHs require large (~1) fluctuations on small scales.
- They could be generated by the same process as the small (~10^{-5}) fluctuations on large scales.
- The simplest and most successful scenario is inflation.
  (Dolgov and Silk 1993, Ivanov, Naselsky and Novikov 1994)
- The curvature perturbation (in slow-roll) is
  $\zeta = -H\delta\phi/\dot{\phi} \sim H^2/\dot{\phi}$.
- The slower the field, the larger the perturbations.

The road to black holes

- During inflation, $\vec{k}$-modes stretch to super-Hubble scales and freeze.
- After inflation, they cross back inside the Hubble radius and start evolving.
- If a Hubble patch is overmassive enough, it collapses into a PBH.

Potential for asteroid-mass PBHs



SR and Tomberg: 1810.12608

All inflation is stochastic inflation

- Inflaton evolution is stochastic when the coupling of small- and long-wavelength modes is taken into account. (Starobinsky 1986)

- As modes become super-Hubble and classicalise, they change the background in which shorter modes evolve.

- Amplitude of every $\vec{k}$-mode is independently drawn from a Gaussian distribution, so the background is subject to Gaussian white noise: we get a Langevin equation.

Large and rare

- Stochastic effects are important for PBHs.
  - Perturbations are large.
  - PBHs are rare, and the tail of the distribution is sensitive to small corrections.

Compact enough

- According to simulations, PBH formation is determined by the maximum of the compaction function $\mathcal{C}(r)=2G_N\Delta M/R$. (Shibata and Sasaki: gr-qc/9905064)
  - Here ΔM is mass excess over the background and R is areal radius.
- PBH forms when $\mathcal{C}>\mathcal{C}_{\rm th}$, where $\mathcal{C}_{\rm th}$ = 0.4…2/3.
- Need to know the radial profile.
  - Earlier studies used ad hoc profiles or the Gaussian mean profile.

From the inflaton potential to the compaction function

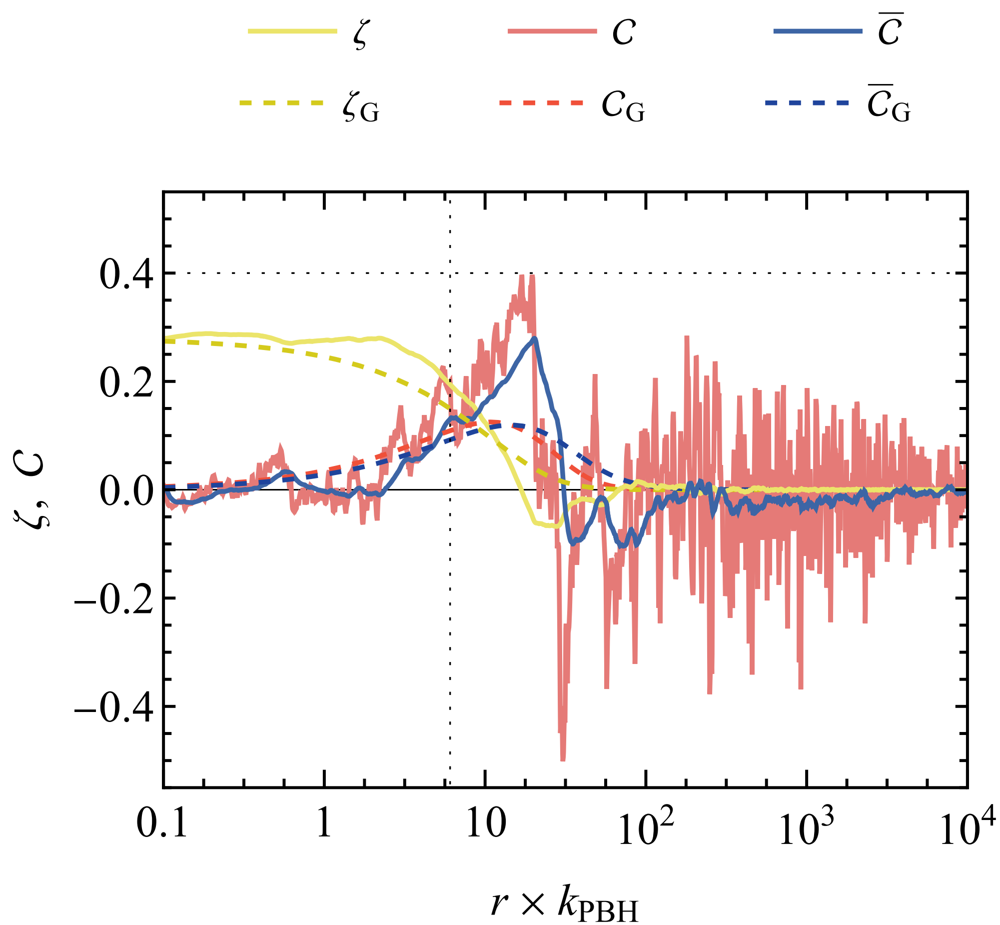
- We use an analytical approximation to solve $\Delta N_\mathbf{k}=\zeta_\mathbf{k}$ from the Langevin equation. (Tomberg: 2210.17441, 2304.10903)
  - Consider 10^4 k values in one patch.
  - Calculate 10^8 patches to get statistics.

- We construct the spatial profile directly via inverse Fourier transform:

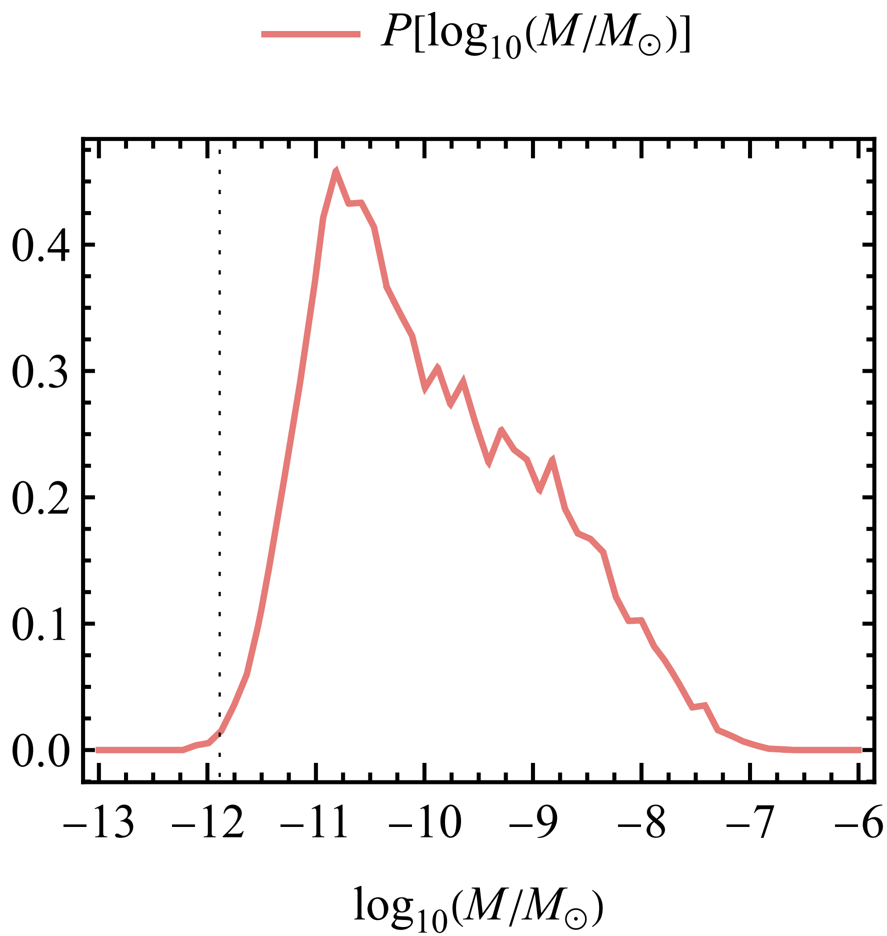
$$\zeta(r) = \frac{1}{(2\pi)^{3/2}} \int \mathrm{d}^3 k \zeta_{\vec{k}} e^{i\vec{k}\cdot\vec{x}} = \sqrt{\frac{2}{\pi}} \int_0^\infty \mathrm{d}k k^2 \zeta_k \frac{\sin(kr)}{kr}$$

- Calculate $\mathcal{C}(r)$ in the gradient approximation.

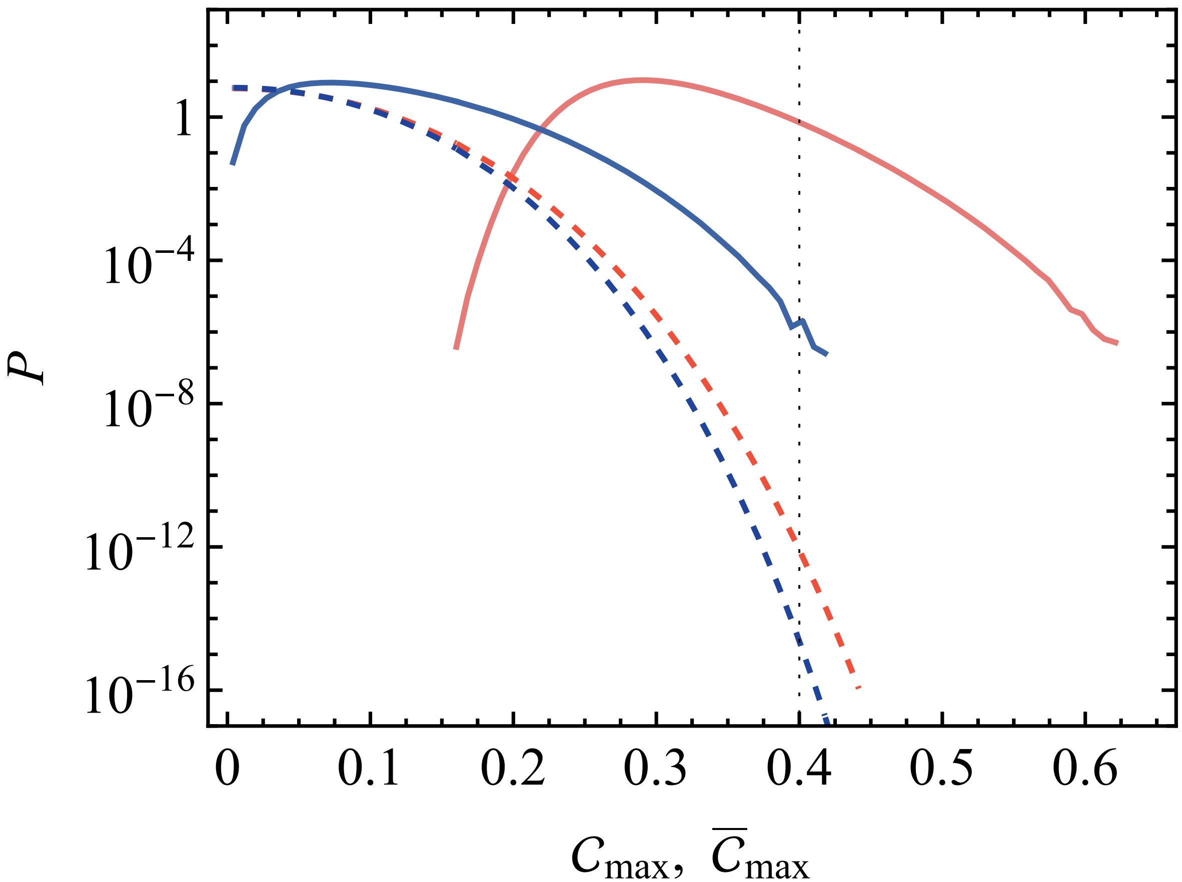
One realisation



PBH mass is stochastic



Distribution of max($\mathcal{C}$)



Stochastic spikes

- Stochastic kicks generate an exponential tail that enhances PBH abundance by 10^5 compared to the Gaussian case.

- Stochasticity of the individual patches enhances PBH abundance by an extra factor of 10^9.
  - Very different from the case of smooth ad hoc profiles or the Gaussian mean profile.

Merits and flaws

- No window function for PBH mass: no arbitrary choice of scales.

- No cloud-in-a-cloud problem: we resolve the peaks on different scales.

- Open issues:
  - Once inside Hubble radius, spikes lead to large pressure gradients that smoothen the profile.
  - Validity of spherical symmetry unclear, not expanded around a maximum.

Spiking the conclusions

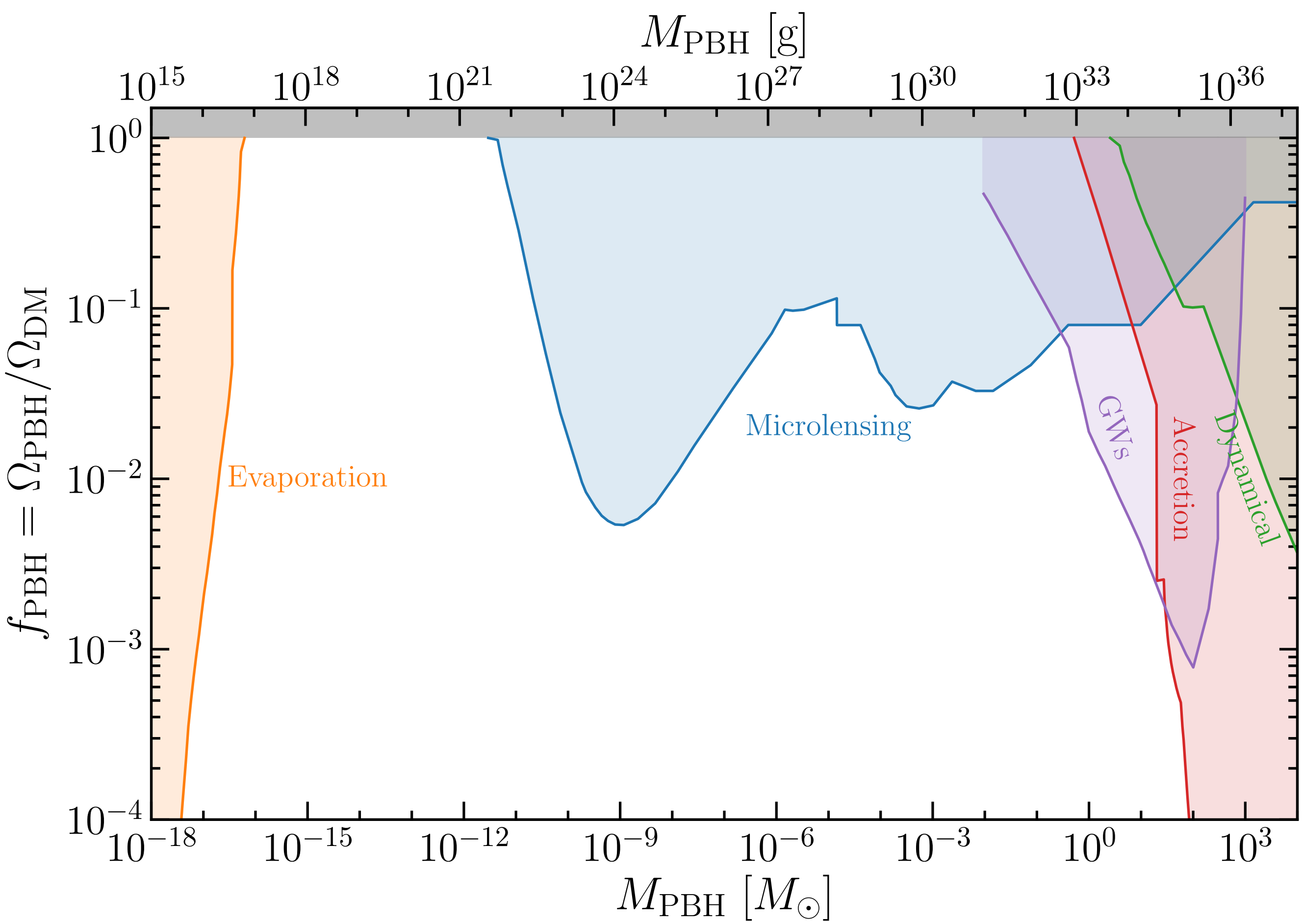
- Producing PBH seeds from inflation requires large perturbations, so stochastic kicks are important.

- We calculate $\zeta(r)$ and $\mathcal{C}(r)$ starting from the inflaton potential, realisation by realisation.
  - Patches are spiky, far from the Gaussian mean profile.
  - Spikes enhance PBH abundance by orders of magnitude.

- Need to redo collapse simulations with spiky profiles, check spherical symmetry, and use new techniques to treat small values of the power spectrum.

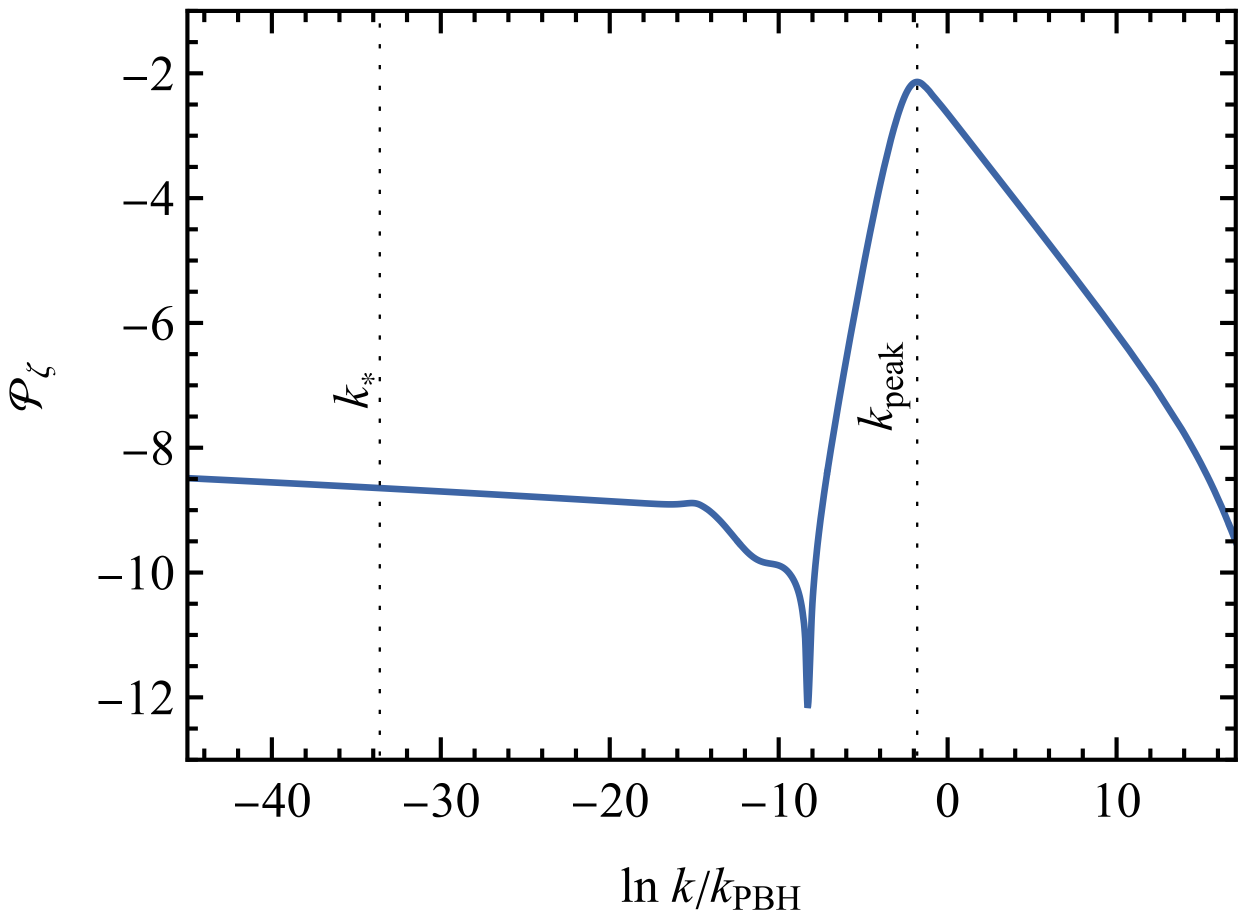
Backup slides

Observational PBH mass window

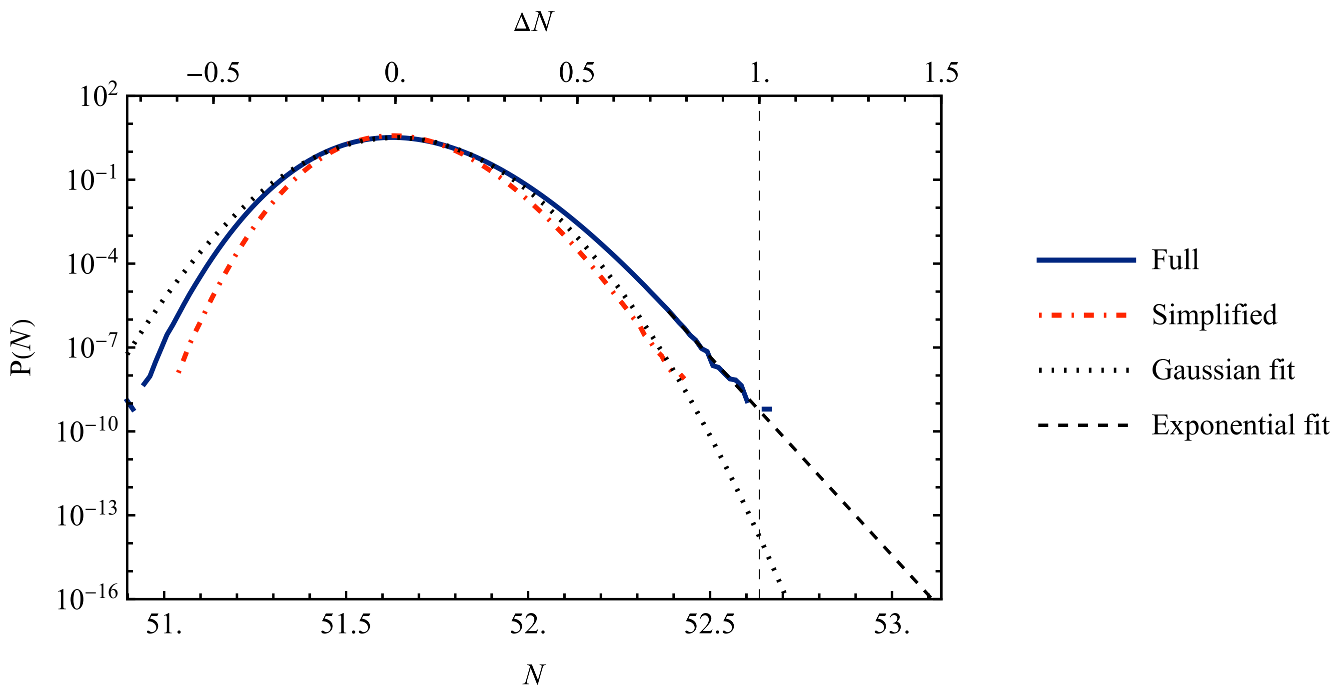


Green and Kavanagh: 2007.10722

Curvature power spectrum

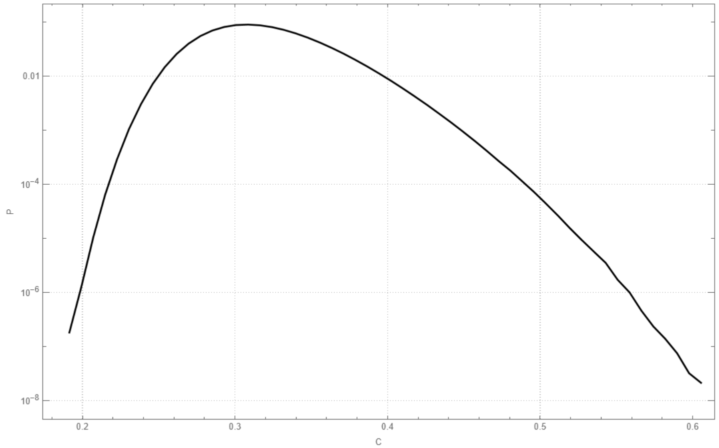


Stochastic tail

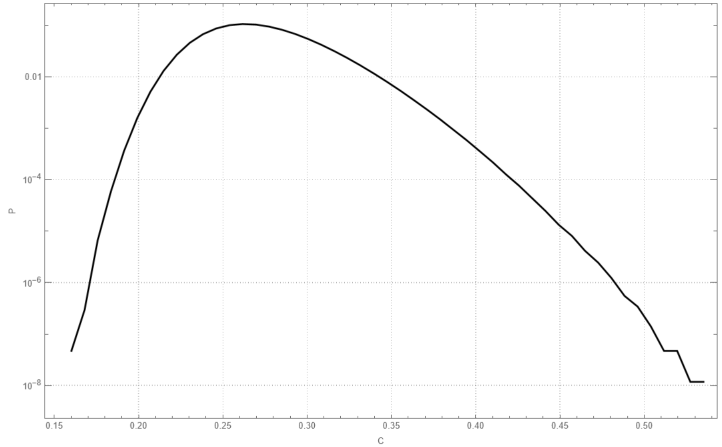


- Stochastic kicks generate an exponential tail.
  (Pattison, Vennin, Assadullahi, Wands: 1707.00537; Ezquiaga, Garcia-Bellido, Vennin: 1912.05399)
- This enhances PBH abundance by $\sim 10^5$.

Effect of the transfer function

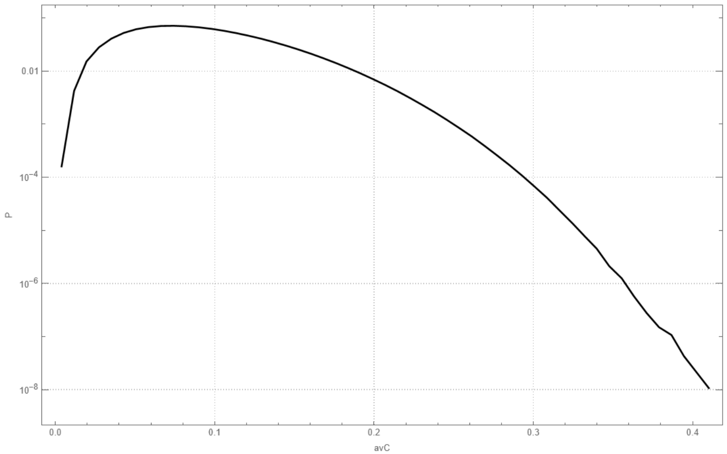


Without transfer function

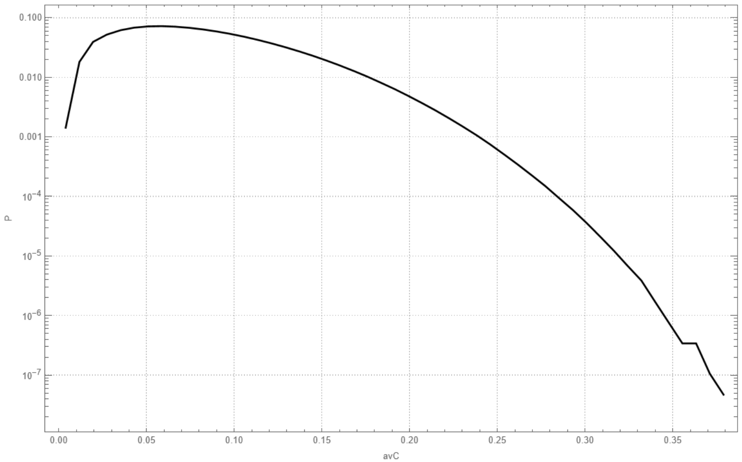


With transfer function

Average of $\mathcal{C}(r)$



Without transfer function



With transfer function