Metastable bound states and dark matter freeze-out

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Frontiers in particle dark matter searches

(very simplistic summary)



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Heavy (m_{DM} ≥ TeV) dark matter

How does the phenomenology of dark matter look like? (in popular scenarios, e.g. thermal-relic DM)

New type of dynamics emerges:

Long-range interactions

$$egin{aligned} \lambda_B &\sim rac{1}{\mu v_{ ext{rel}}}, \, rac{1}{\mu lpha} &\lesssim rac{1}{m_{ ext{mediator}}} &\sim ext{interaction range} \ &\mu: ext{ reduced mass } (m_{ ext{dm}}/2) \end{aligned}$$







Sommerfeld

Bound

states

Distortion of scattering-state wavefunctions ⇒ affects all cross-sections e.g. annihilation, elastic scattering

- Production in early universe, e.g. freeze-out
 ⇒ changes correlation of parameters (mass couplings)
- Indirect detection signals
- Elastic scattering

Unstable bound states (positronium-like) ⇒ extra annihilation channel

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- Novel low-energy indirect detection signals
- Colliders

Stable bound states

- Elastic scattering (usually screening)
- Novel low-energy indirect detection signals
- Inelastic scattering in direct detection experiments (?)

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Bound states

Sommerfeld

Dark matter production via thermal freeze-out



Dark U(1) model: Dirac DM X, \overline{X} coupled to γ_{D}



Thermal freeze-out with bound states Boltzmann equations

$$\text{free particles:} \quad \frac{dn}{dt} + 3Hn = -\left<\sigma^{\text{ann}} v_{\text{rel}}\right> \left(n^2 - n^{\text{eq} \ 2}\right) - \sum_{_{\mathcal{B}}} \left(\left<\sigma^{_{\mathcal{B}}\text{SF}} v_{\text{rel}}\right> n^2 - \Gamma^{_{\text{ion}}}_{_{\mathcal{B}}} n_{_{\mathcal{B}}}\right)$$

bound states:

$$\text{ates:} \quad \frac{dn_{\scriptscriptstyle \mathcal{B}}}{dt} + 3Hn_{\scriptscriptstyle \mathcal{B}} = + \left(\left\langle \sigma_{\scriptscriptstyle \mathcal{B}}^{\scriptscriptstyle \mathrm{BSF}} v_{\rm rel} \right\rangle n^2 - \Gamma_{\scriptscriptstyle \mathcal{B}}^{\rm ion} n_{\scriptscriptstyle \mathcal{B}} \right) - \Gamma_{\scriptscriptstyle \mathcal{B}}^{\rm dec} \left(n_{\scriptscriptstyle \mathcal{B}} - n_{\scriptscriptstyle \mathcal{B}}^{\rm eq} \right) - \sum_{\scriptscriptstyle \mathcal{B}' \neq \scriptscriptstyle \mathcal{B}} \left(\Gamma_{\scriptscriptstyle \mathcal{B} \rightarrow \scriptscriptstyle \mathcal{B}'}^{\rm trans} n_{\scriptscriptstyle \mathcal{B}} - \Gamma_{\scriptscriptstyle \mathcal{B}' \rightarrow \scriptscriptstyle \mathcal{B}}^{\rm trans} n_{\scriptscriptstyle \mathcal{B}'} \right)$$

Processes			Detailed balance
Bound state formation (BSF) Ionisation (ion)	$X+ar{X}$ $\mathcal{B}(Xar{X})+\gamma_{\scriptscriptstyle D}$	$egin{array}{lll} ightarrow \mathcal{B}(Xar{X})+\gamma_{\scriptscriptstyle D} \ ightarrow X+ar{X} \end{array}$	$\langle \sigma^{\scriptscriptstyle \mathrm{BSF}}_{oldsymbol{eta}} v_{ m rel} angle (n^{ m eq})^2 = \Gamma^{ m ion}_{oldsymbol{eta}} n^{ m eq}_{oldsymbol{eta}}$
Decay (dec)	$\mathcal{B}(Xar{X})$	$ ightarrow 2\gamma_{\scriptscriptstyle D} ~{ m or}~ 3\gamma_{\scriptscriptstyle D}$	
Transitions (trans)	${\cal B}(Xar X) \ {\cal B}(Xar X) + \gamma_{\scriptscriptstyle D}$	$egin{array}{lll} ightarrow \mathcal{B}'(Xar{X})+\gamma_{\scriptscriptstyle D} \ ightarrow \mathcal{B}'(Xar{X}) \end{array}$	$\Gamma^{ ext{trans}}_{{m eta} o {m eta}'} n^{ ext{eq}}_{{m eta}} = \Gamma^{ ext{trans}}_{{m eta}' o {m eta}} n^{ ext{eq}}_{{m eta}'}$

Thermal freeze-out with bound states Boltzmann equations



Complete treatement: Binder, Filimonova, Petraki, White 2112.00042

Thermal freeze-out with bound states Boltzmann equations and effective cross-section



Thermal freeze-out with bound states Effective cross-section

$$\frac{dn}{dt} + 3Hn = -\langle \sigma^{\text{eff}} v_{\text{rel}} \rangle \left(n^2 - n^{\text{eq} \ 2}\right)$$
where, neglecting bound-to-bound transitions,
 $\langle \sigma^{\text{eff}} v_{\text{rel}} \rangle \equiv \langle \sigma^{\text{ann}} v_{\text{rel}} \rangle + \sum_{g} \langle \sigma_{g}^{\text{BSF}} v_{\text{rel}} \rangle \times \frac{\Gamma_{g}^{\text{dec}}}{\Gamma_{g}^{\text{dec}} + \Gamma_{g}^{\text{ion}}}$
At $T \gg \text{ Binding Energy} \Rightarrow \Gamma_{g}^{\text{ion}} \gg \Gamma_{g}^{\text{dec}}$,
 $\sigma_{g}^{\text{BSF}} v_{\text{rel}} \rangle \frac{\Gamma_{g}^{\text{dec}}}{\Gamma^{\text{dec}} + \Gamma^{\text{ion}}} \simeq \langle \sigma_{g}^{\text{BSF}} v_{\text{rel}} \rangle \frac{\Gamma_{g}^{\text{dec}}}{\Gamma_{g}^{\text{ion}}} = \frac{n_{g}^{\text{eq}}}{(n^{\text{eq}})^2} \Gamma_{g}^{\text{dec}}$
At $T \lesssim \text{ Binding Energy} \Rightarrow \Gamma_{g}^{\text{ion}} \ll \Gamma_{g}^{\text{dec}}$,

$$\langle \sigma^{\scriptscriptstyle \mathrm{BSF}}_{\scriptscriptstyle \mathcal{B}} v_{\operatorname{rel}}
angle \; rac{\Gamma^{\operatorname{dec}}_{\scriptscriptstyle \mathcal{B}}}{\Gamma^{\operatorname{dec}}_{\scriptscriptstyle \mathcal{B}} + \Gamma^{\operatorname{ion}}_{\scriptscriptstyle \mathcal{B}}} \simeq \langle \sigma^{\scriptscriptstyle \mathrm{BSF}}_{\scriptscriptstyle \mathcal{B}} v_{\operatorname{rel}}
angle.$$

Typically, most of DM destruction due to BSF occurs in this regime.

Independent of actual BSF cross-section!

 $\simeq rac{g_{\scriptscriptstyle {\cal B}}}{g_{\scriptscriptstyle X}^2} \left(rac{4\pi}{m_{\scriptscriptstyle X}T}
ight)^{3/2} imes e^{|E_{\cal B}|/T} \ \Gamma_{\scriptscriptstyle {\cal B}}^{
m dec}$

 $\Gamma_{\scriptscriptstyle B}^{
m dec} \propto (\sigma^{
m ann} v_{
m rel}) \rightarrow {
m modest} \ {
m increase} \ {
m over the direct annihilation,} \ {
m but increases} \ {
m exponentially} \ {
m as} \ T \ {
m drops}.$

Effective cross-section in dark U(1) model

Cross-sections

Thermally averaged cross-sections



Thermal freeze-out with long-range interactions Dark U(1) model: Dirac DM X, \overline{X} coupled to γ_{D}



Thermal freeze-out with long-range interactions Dark U(1) model: Dirac DM X, \overline{X} coupled to γ_{D}



Thermal freeze-out with bound states Boltzmann equations and effective cross-section



A corollary

Saha equilibrium for metastable bound states

$$egin{aligned} rac{n_{\mathcal{B}}}{n_{\mathcal{B}}^{ ext{eq}}} = \left(rac{n_{ ext{free}}}{n_{ ext{free}}^{ ext{eq}}}
ight)^2 - \left[\left(rac{n_{ ext{free}}}{n_{ ext{free}}^{ ext{eq}}}
ight)^2 - 1
ight]r_{\mathcal{B}} \end{aligned}$$

Binder, Filimonova, Petraki, White 2112.00042



Standard Saha equilibrium

Particles with decay rate > Hubble

Neutralino-squark co-annihilation scenarios

Squark-neutralino co-annihilation scenarios

- Degenerate spectrum \rightarrow soft jets \rightarrow evade LHC constraints
- Large stop-Higgs coupling reproduces measured Higgs mass and brings the lightest stop close in mass with the LSP

⇒ DM density determined by "effective" Boltzmann equation $n_{\text{tot}} = n_{\text{LSP}} + n_{\text{NLSP}}$ $\sigma_{\text{ann}}^{\text{eff}} = [n_{\text{LSP}}^2 \sigma_{\text{ann}}^{\text{LSP}} + n_{\text{NLSP}}^2 \sigma_{\text{ann}}^{\text{NLSP}} + n_{\text{LSP}} n_{\text{NLSP}} \sigma_{\text{ann}}^{\text{LSP-NLSP}}]/n_{\text{tot}}^2$ Scenario probed in colliders. Important to compute DM density accurately! → QCD corrections

$$egin{aligned} \mathcal{L} &\supset \; rac{1}{2} \overline{\chi^c} \, i \partial \!\!\!/ \chi - rac{1}{2} m_\chi \, \overline{\chi^c} \chi \ &+ \; \left[(\partial_\mu + i g_s G^a_\mu T^a) X
ight]^\dagger \left[(\partial^\mu + i g_s G^{a,\mu} T^a) X
ight] - m_X^2 |X|^2 \ &+ \; (\chi \leftrightarrow X, X^\dagger) ext{ interactions in chemical equilibrium during freeze-out} \end{aligned}$$

Bound-state formation and decay





Bound-state formation vs Annihilation



Harz, KP: 1805.01200



Squark-neutralino co-annihilation scenarios

- Degenerate spectrum \rightarrow soft jets \rightarrow evade LHC constraints
- Large stop-Higgs coupling reproduces measured Higgs mass and brings the lightest stop close in mass with the LSP
 - ⇒ DM density determined by "effective" Boltzmann equation

$$\sigma_{ann}^{eff} = [n_{LSP}^{2} \sigma_{ann}^{LSP} + n_{NLSP}^{2} \sigma_{ann}^{NLSP} + n_{LSP} n_{NLSP} \sigma_{ann}^{LSP-NLSP}]/n_{tot}^{2}$$
Scenario probed in colliders.
Important to compute DM density accurately!
$$\rightarrow \text{ QCD corrections}$$

The Higgs as a light mediator

- Sommerfeld enhancement of direct annihilation
- Binding of bound states

Harz, KP: 1711.03552

Harz, KP: 1901.10030

DM coannihilation with scalar colour triplet MSSM-inspired toy model The effect of the Higgs-mediated potential



The Higgs as a light mediator

- Sommerfeld enhancement of direct annihilation
- Harz, KP: 1711.03552

Binding of bound states

Harz, KP: 1901.10030

Formation of bound states via Higgs (doublet) emission ?

Capture via emission of neutral scalar suppressed, due to selection rules: quadruple transitions

March-Russel, West 0812.0559 KP, Postma, Wiechers: 1505.00109 An, Wise, Zhang: 1606.02305 KP, Postma, de Vries: 1611.01394

Capture via emission of charged scalar [or its Goldstone mode] very very rapid: monopole transitions ! Ko,Matsui,Tang: 1910:04311 Oncala, KP: 1911.02605

Ko,Matsui,Tang: 1910:04312 Oncala, KP: 1911.02605 Oncala, KP: 2101.08666 Oncala, KP: 2101.08667

Sudden change in effective Hamiltonian precipitates transitions. Akin to atomic transitions precipitated by β decay of nucleus.

Renormalisable Higgs-portal WIMP models

Singlet-Doublet coupled to the Higgs: $L \supset -y \overline{D} H S$

 $m_D \simeq m_S \rightarrow D$ and S co-annihilate. Freeze-out begins before the EWPT if $m_{DM} > 5$ TeV



Oncala, KP: 2101.08666/7

Renormalisable Higgs-portal WIMP models

Singlet-Doublet coupled to the Higgs: $L \supset -y \overline{D} H S$ $m_D \simeq m_S \rightarrow D$ and S co-annihilate.

Freeze-out begins before the EWPT if $m_{DM} > 5TeV$



Oncala, KP: 2101.08666/7

Is it a coincidence that non-perturbative effects arise in all these models at the multi-TeV regime?

Or is there a model-independent way to understand and *predict* it?

If so, what else can we learn from it?

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Or is there a model-independent way to understand and *predict* it?

If so, what else can we learn from it?



Partial-wave unitarity limit

$$\sigma_{
m inel}^{(\ell)} \ \leqslant \ rac{\pi(2\ell+1)}{k_{
m cm}^2} \quad \stackrel{
m non-rel}{
ightarrow} \ rac{\pi(2\ell+1)}{\mu^2 v_{
m rel}^2} \quad \stackrel{\mu=M_{
m DM}/2}{
ightarrow} \ rac{4\pi(2\ell+1)}{M_{
m DM}^2 v_{
m rel}^2}$$

[Griest, Kamionkowski (1990); Hui (2001)]

Physical meaning: saturation of probability for inelastic scattering

Partial-wave unitarity limit in non-relativistic regime

$$\sigma_{
m inel}^{(\ell)} v_{
m rel} ~\leqslant~ \sigma_{
m uni}^{(\ell)} v_{
m rel} ~=~ rac{4\pi(2\ell+1)}{M_{
m _DM}^2 v_{
m rel}}$$

Implies upper bound on the mass of thermal-relic DM

Griest, Kamionkowski (1990)

$$egin{aligned} &\sigma_{
m ann} v_{
m rel} &\simeq 2.2 imes 10^{-26} \ {
m cm}^3/{
m s} &\leqslant rac{4\pi}{M_{
m DM}^2 v_{
m rel}} \ &\langle v_{
m rel}^2
angle^{1/2} &= (6T/M_{
m DM})^{1/2} \quad {
m freeze-out} M_{
m DM}/T pprox 25 \ &0.49 \ &M_{
m DM}/T pprox 25 \ &0.49 \ &M_{
m uni} &\simeq egin{cases} 117 \ {
m TeV}, & {
m self-conjugate DM} \ {
m 83 \ TeV}, & {
m non-self-conjugate DM} \ & {
m non-self-conjugate DM} \ &M_{
m uni} & {
m DM} \ &M_{
m uni} & {
m cm} \ &M_{
m uni}$$

- Assumes contact-type interactions, $\sigma v_{rel} = constant$
- Considers only s-wave annihilation



- Parametric dependence on mass and velocity implies that
- σ_{uni} can be approached or attained only by long-range interactions

Long-range interactions imply **bound states**, which may form by **higher partial waves** of the scattering state that contribute at the same order.

- Thermal relic DM can be much heavier than anticipated.
 - In viable thermal scenarios, expect long-range behavior at m_{DM} ≥ few TeV (important for exps)
 - No model-independent unitarity limit on mass of thermal relic DM!

Baldes, KP: 1703.00478

Conclusions

 Bound states impel complete reconsideration of thermal decoupling at / above the TeV scale: emergence of a new type of inelasticity

Unitarity limit can be approached / attained only by long-range interactions → bound states play very important role! Baldes, KP: 1703.00478

There is no unitarity limit on the mass of thermal relic DM!

- Experimental implications:
 - DM heavier than anticipated: multi-TeV probes very important

⇒ build the 100 TeV collider :)

- Indirect detection:

Enhanced rates due to BSF Novel signals: low-energy radiation emitted in BSF Indirect detection of asymmetric DM

- Colliders: improved detection prospects due increased mass gap in coannihilation scenarios
- Effects not limited freeze-out scenario: freeze-in, asymmetric DM, self-interacting DM, stable bound states