Dark Matter in the form of Compact Objects

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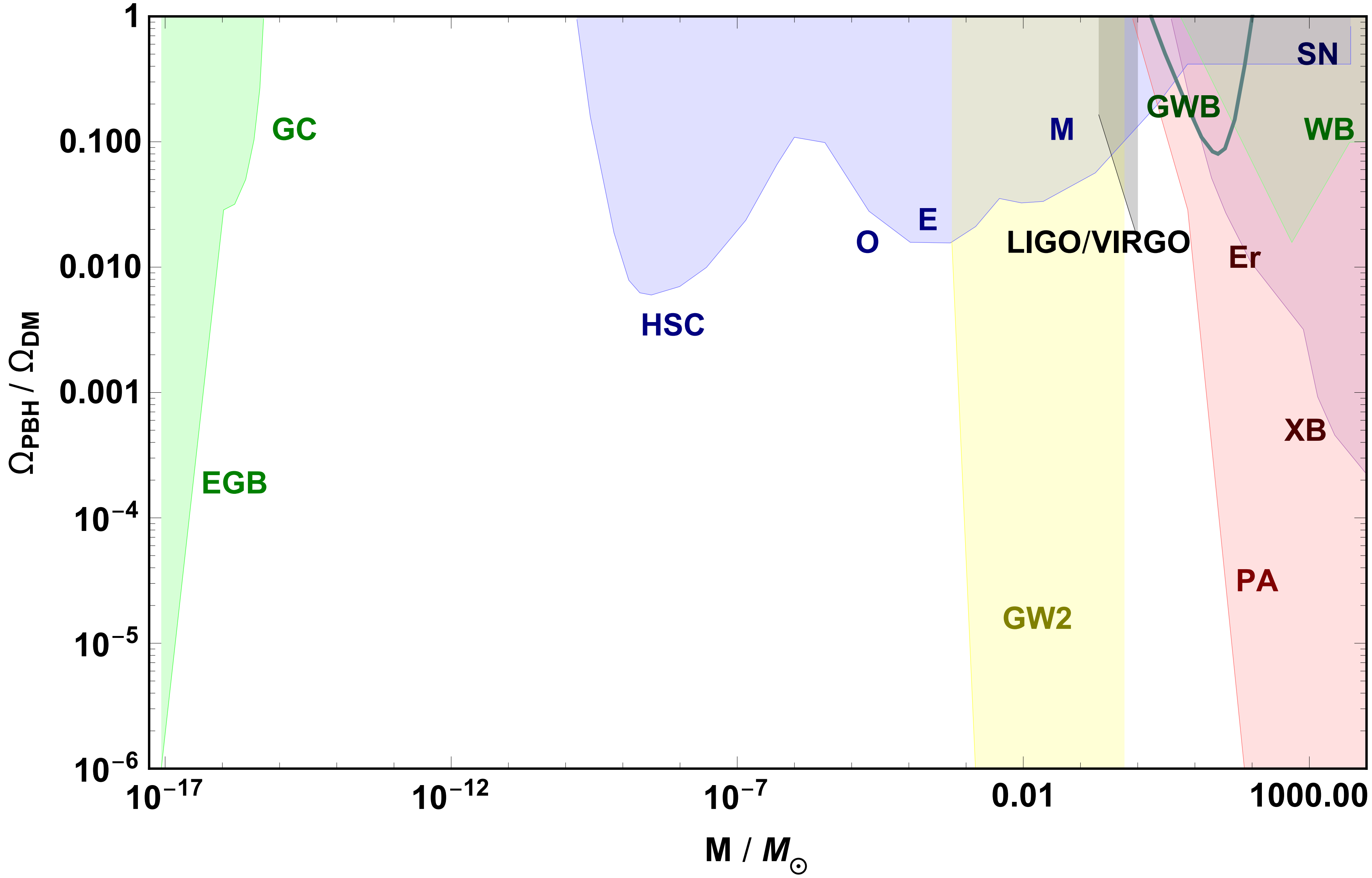
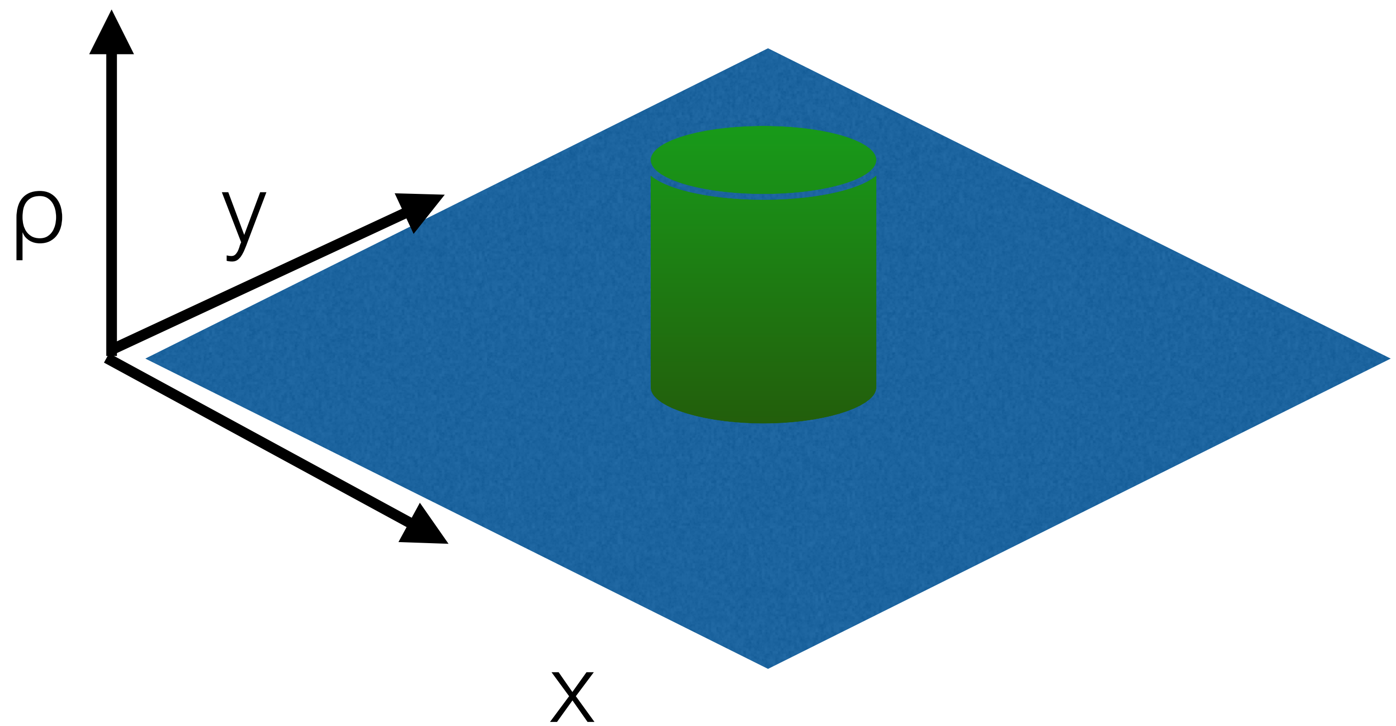
Dark Matters Bruxelles 30/11/2022

PBH Constraints



Top Hat Spherical Collapse



- The Friedmann Equation gives

$$\left(\frac{dr}{dt}\right)^2 = \frac{2GM_i}{r} - \mathcal{K} \qquad \mathcal{K} = 8\pi G\rho_{\rm H}(t_i) r_i^2\, \delta\, /3$$

$$r(\theta) = \frac{GM_i}{\mathcal{K}}(1 - \cos\theta)$$

$$t(\theta) = \frac{GM_i}{\mathcal{K}^{3/2}}(\theta - \sin\theta)$$

- θ=π maximum expansion
- θ=2π collapse

$$\delta_{\rm lin} = 1.062$$

Formation of PBH

- Actual perturbations are not completely spherical

- To form a PBH, the hoop conjecture should be satisfied

$$\mathcal{C} \lesssim 2\pi r_s$$

- In a RD Universe, the pressure plays a double role:

1. It makes the collapsing perturbation more spherical, so it easier the satisfy the hoop conjecture
2. The pressure impedes the collapse, so large perturbations are needed in order for M to be larger than the Jeans mass.

- In a eMD Universe, there is no pressure. The lack of pressure from one hand facilitates the collapse but the same time small deviations from sphericity can grow larger thus making harder to satisfy the hoop conjecture.

GW Production

- To first order in perturbation theory scalar, vector and tensor perturbations are decoupled.

This means that no GW can be produced from scalar perturbations in that order.
One needs to go to 2nd order

$$\mathrm{d}s^2 = a^2(\eta)\left[-\left(1+2\Phi^{(1)}+2\Phi^{(2)}\right)\mathrm{d}\eta^2 + 2V_i^{(2)}\mathrm{d}\eta\mathrm{d}x^i + \left\{\left(1-2\Psi^{(1)}-2\Psi^{(2)}\right)\delta_{ij} + \frac{1}{2}h_{ij}\right\}\mathrm{d}x^i\mathrm{d}x^j\right]$$

$$\hat{T}_{ij}{}^{lm}G^{(2)}_{lm} = \kappa^2\hat{T}_{ij}{}^{lm}T^{(2)}_{lm}$$

Acquaviva Bartolo Matarrese Riotto '02
Baumann Steinhardt Takahashi Ichiki '07

$$h''_{\mathbf{k}}(\eta) + 2\mathcal{H}h'_{\mathbf{k}}(\eta) + k^2 h_{\mathbf{k}}(\eta) = 4S_{\mathbf{k}}(\eta)$$

$$S_{\mathbf{k}} = \int \frac{\mathrm{d}^3q}{(2\pi)^{3/2}} e_{ij}(\mathbf{k})q_i q_j \left(2\Phi_{\mathbf{q}}\Phi_{\mathbf{k-q}} + \frac{4}{3(1+w)}\left(\mathcal{H}^{-1}\Phi'_{\mathbf{q}} + \Phi_{\mathbf{q}}\right)\left(\mathcal{H}^{-1}\Phi'_{\mathbf{k-q}} + \Phi_{\mathbf{k-q}}\right)\right)$$

Kohri Terada '18

$$\Phi''_{\mathbf{k}}(\eta) + \frac{6(1+w)}{(1+3w)\eta}\Phi'_{\mathbf{k}}(\eta) + wk^2\Phi_{\mathbf{k}}(\eta) = 0 \qquad \Phi_{\mathbf{k}} = \Phi(k\eta)\phi_{\mathbf{k}}$$

$$\langle \phi_{\mathbf{k}}\phi_{\mathbf{k}'}\rangle = \delta(\mathbf{k}+\mathbf{k}')\frac{2\pi^2}{k^3}\left(\frac{3+3w}{5+3w}\right)^2\mathcal{P}_\zeta(k)$$

GW Production

- Perturbation theory is not valid through the whole collapse process.

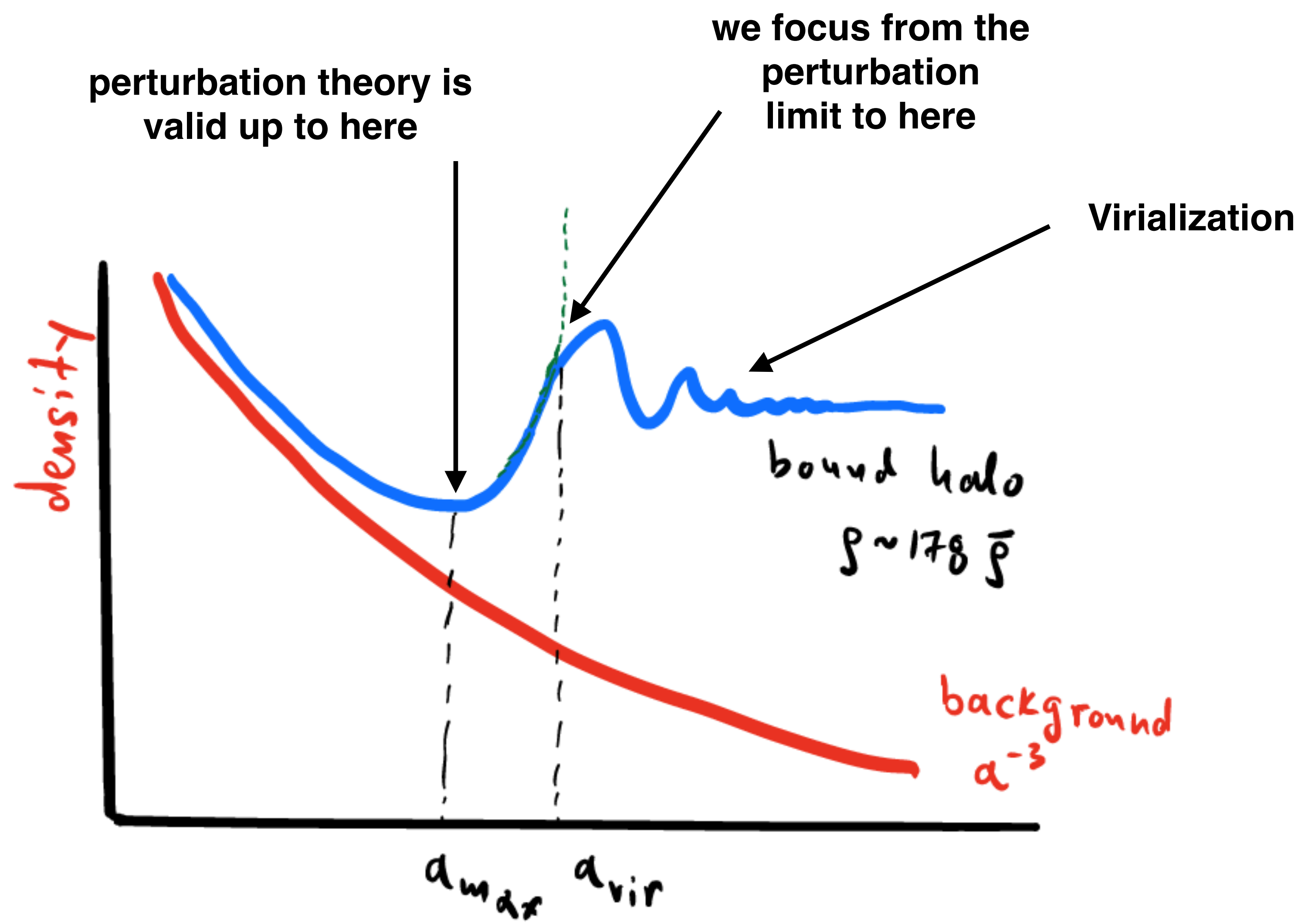
- The turnaround in linear theory happens at $\delta_{lin}=1.062$ and perturbations theory is not trustworthy soon after the turnaround since $\delta>>1$

- The part of the collapse from maximum expansion to black hole or halo formation is not covered by perturbation theory. This part could potentially give a strong signal especially in a virialization process where shell crossing and oscillations can induce large quadrupole moments

- We implement a nonlinear approach, hopefully capturing more accurately the form of the produced GW.

Collapse in early Matter Domination

- The absence of pressure magnifies deviations from sphericity leading to the formation of the so called Zel'dovich pancakes. Different shells start oscillating and crossing each other, forming eventually a virialized bound halo.



- If the collapsing pancake satisfies at some point the hoop conjecture, a PBH forms and there is no further virialization stage.

Zel'dovich Pancakes

$$r_i = a(t)q_i + b(t)p_i(q_i)$$

scaling factor from Hubble → $a(t)$; growing gravitational Instability → $b(t)$; initial deviation → $p_i(q_i)$

$$D_{ik} = \frac{\partial r_i}{\partial q_k} = a(t)\delta_{ik} + b(t)\frac{\partial p_i}{\partial q_k} = \mathrm{diag}(a - \alpha b, a - \beta b, a - \gamma b)$$

perturbation entering the horizon $\quad a(t_q)q = H^{-1}(t_q)$

$$M = \int \rho a^3 d^3 r = \bar{\rho} a^3 \int d^3 q$$

mass conservation $\quad \rho(a - \alpha b)(a - \beta b)(a - \gamma b) = \bar{\rho} a^3$

$$\delta_L \equiv \left(\frac{\rho - \bar{\rho}}{\bar{\rho}}\right)_L = (\alpha + \beta + \gamma)\frac{b}{a}$$

δ ~ a in MD $\quad b \propto a^2$

at turnaround $\quad \dot{r}_1(t_{\max}) = 0 \quad r_1(t_{\max}) = a(t_{\max})q_1 - \alpha b(t_{\max})q_1 = \frac{1}{2}a(t_{\max})q_1 \quad \frac{b(t_{\max})}{a(t_{\max})} = \frac{1}{2\alpha}$

GW Production

Doroshkevich probability density for deviations from sphericity ‘70

$$\mathcal{F}_{\mathrm{D}}(\alpha,\beta,\gamma)d\alpha d\beta d\gamma = -\frac{27}{8\sqrt{5}\pi\sigma_3^6}\times\exp\left[-\frac{3}{5\sigma_3^2}\left((\alpha^2+\beta^2+\gamma^2)-\frac{1}{2}(\alpha\beta+\beta\gamma+\gamma\alpha)\right)\right]$$
$$\times(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)d\alpha d\beta d\gamma,$$

Moment of Inertia
$$I_{ij} = \frac{1}{5}M\begin{pmatrix} r_2^2+r_3^2 & 0 & 0 \\ 0 & r_1^2+r_3^2 & 0 \\ 0 & 0 & r_1^2+r_2^2 \end{pmatrix}$$

Quadrupole
$$Q_{ij} = -I_{ij}(t) + \frac{1}{3}\delta_{ij}\mathrm{Tr}I(t)$$

GW quadrupole radiation
$$\frac{dE_e}{dt} = \frac{G}{5c^5}\sum_{ij}\dddot{Q}_{ij}(t)\dddot{Q}_{ji}(t)$$

GW Production

We focus on the time interval between t_{max} and t_{col} or t_{BH} whichever happens first

We break the interval in N subintervals [t_i , t_i +δt]

$$dE_{\rm GW}(\alpha,\beta,\gamma) = \sum_N \frac{1}{1+z_N}\frac{4\pi G}{5c^5}\omega^7 \sum_{ij} |\tilde{Q}^N_{ij}(\omega)|^2 \frac{V_{\rm com}(t_0)}{\frac{4\pi}{3}q^3}\mathcal{F}_{\rm D}(\alpha,\beta,\gamma)\,d\alpha d\beta d\gamma\, d\ln\omega$$

Dalianis CK '21

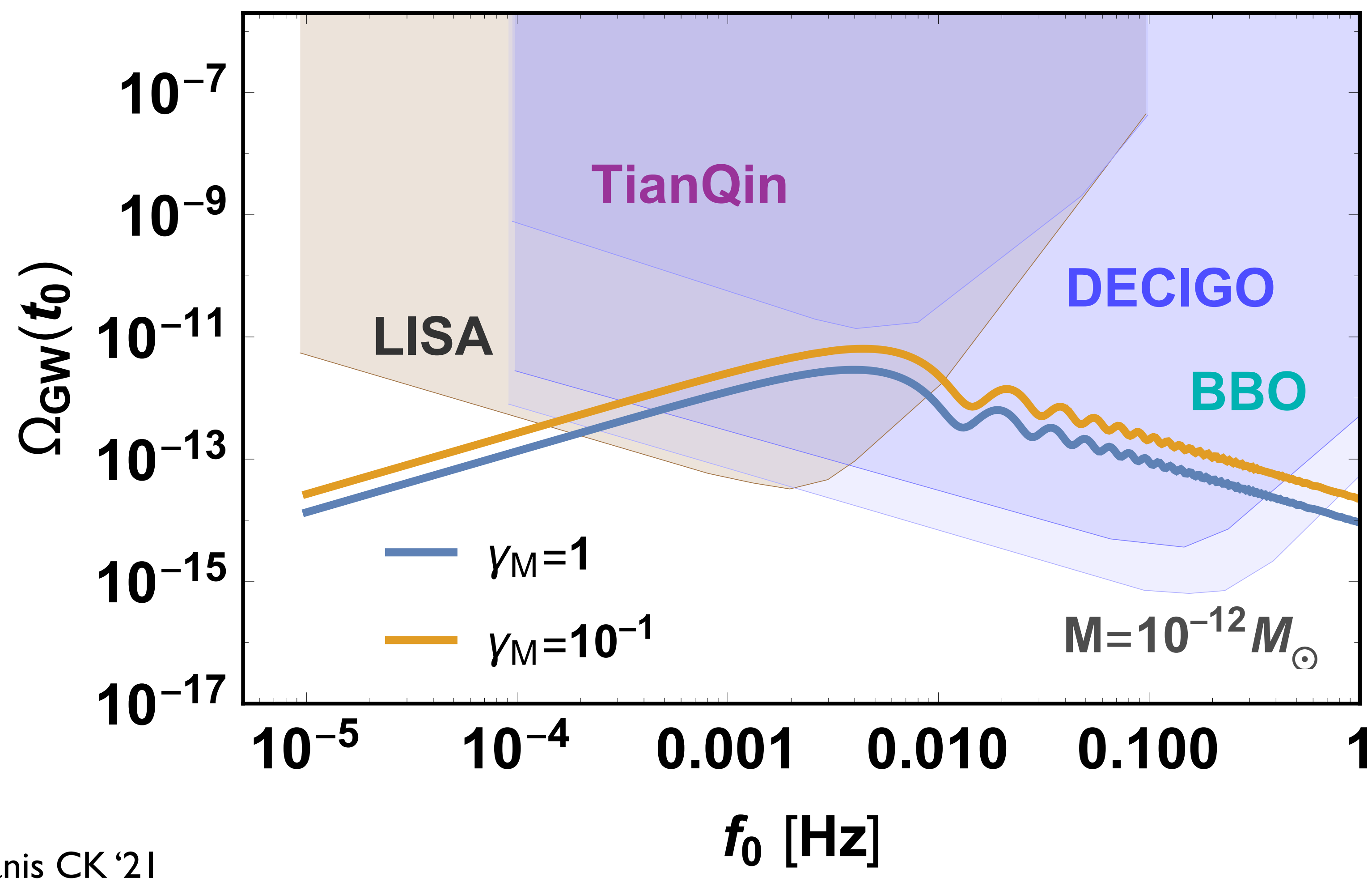
To save computational time we take N=1. This introduces a horizontal error ~2

We have to integrate over α, β, γ

We need to insert a step function so the reheating takes place after the collapse

If we want to form PBH, the hoop conjecture should be satisfied

GW Signal for 100% DM contribution

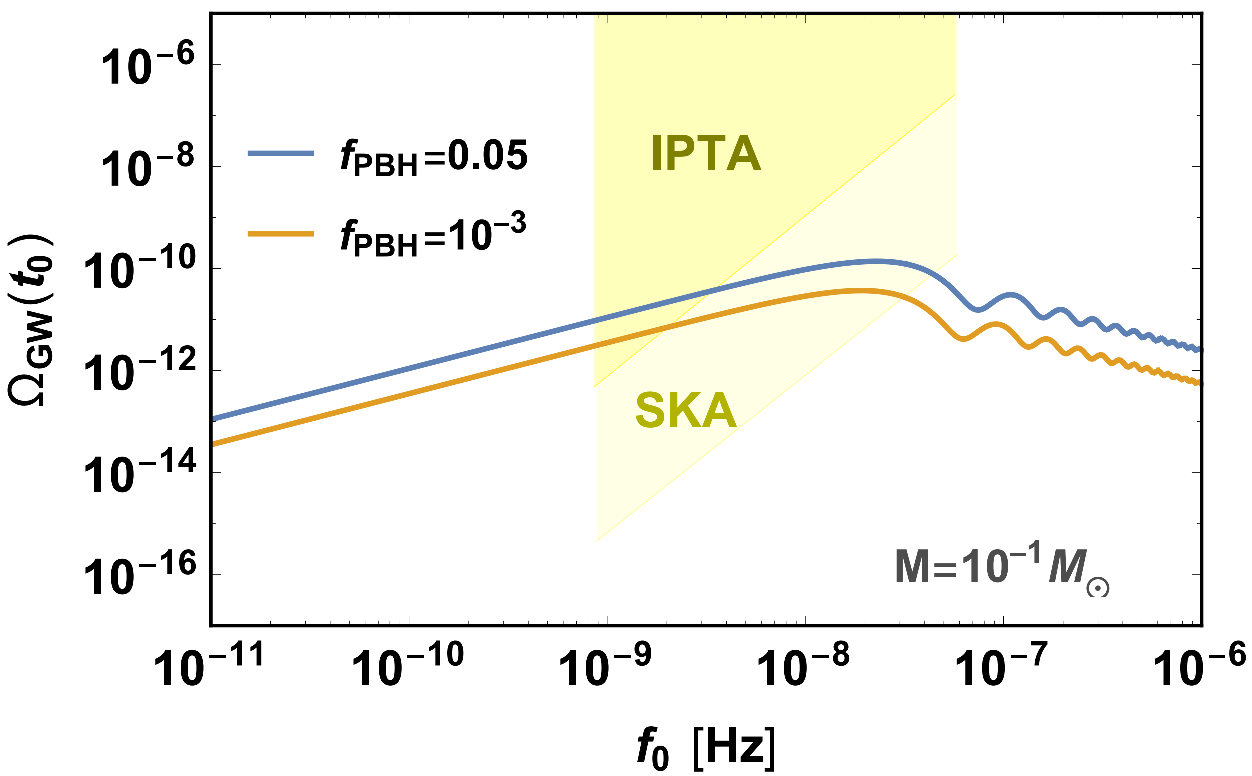


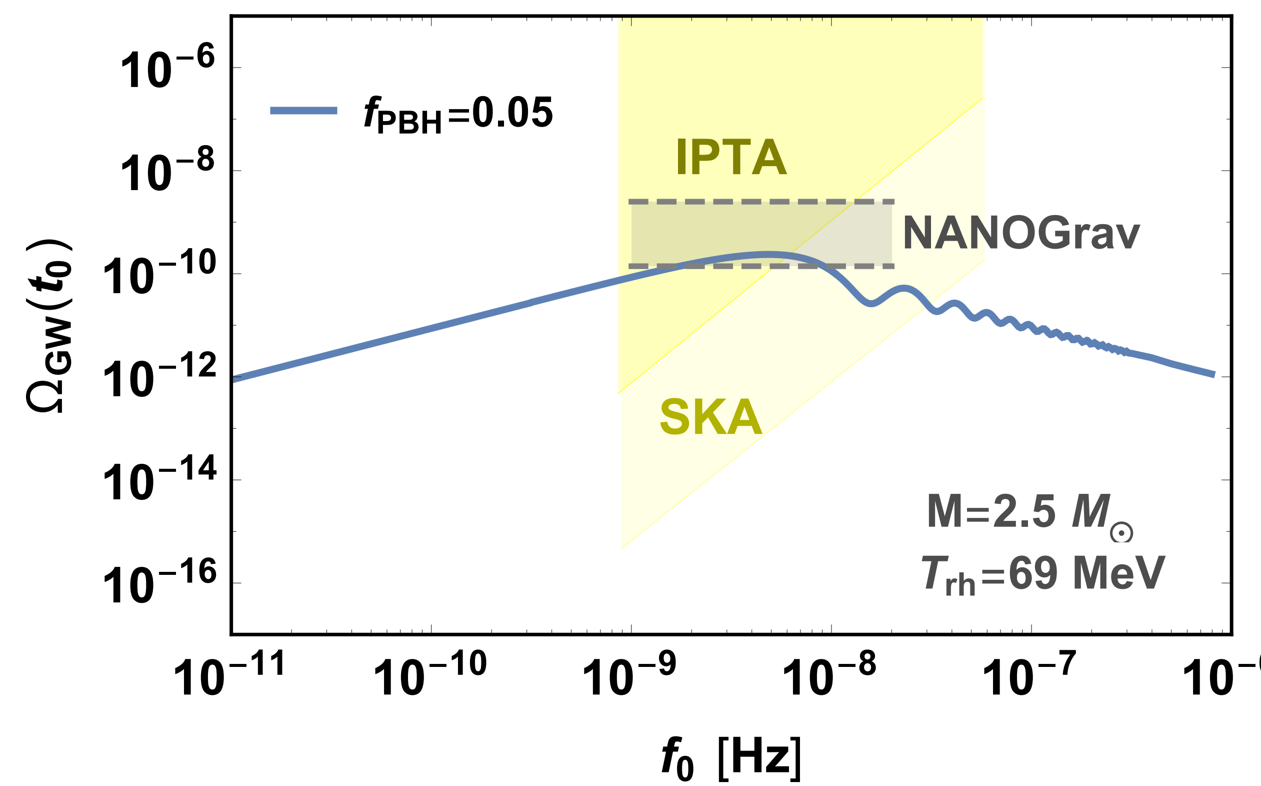
Dalianis CK '21

This is the region where PBH could consist 100% of the dark matter abundance.

Smaller γ_M create larger signal

PTA Detection





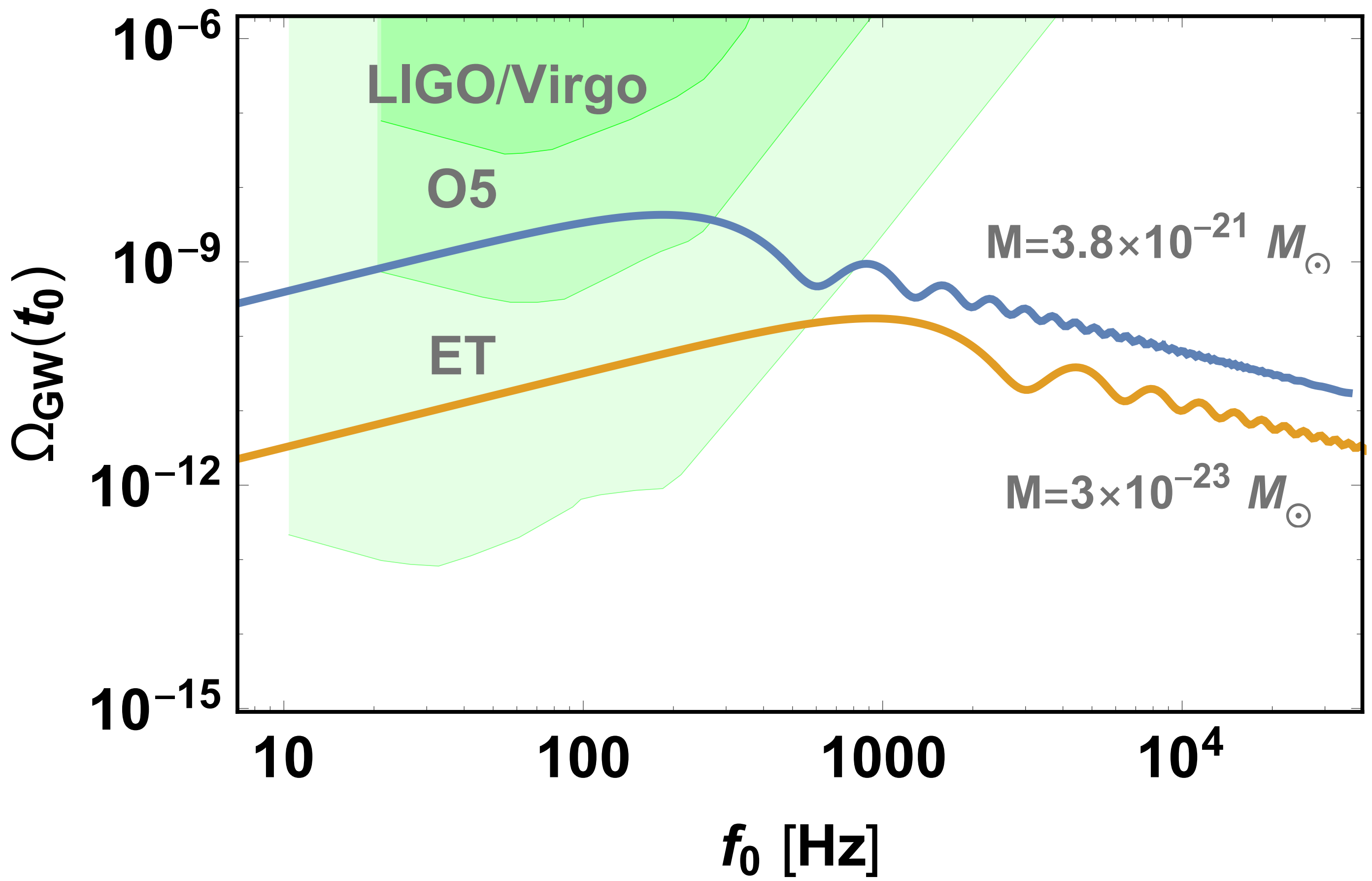
PBH formed in eMD avoid exclusion from PTA unlike the corresponding scenarios in RD.

in RD $\beta \approx \mathrm{Erfc}\left[\frac{\delta_c}{\sqrt{2}\,\sigma}\right]$ there is a threshold in σ to produce sufficient number of PBH

in eMD $\beta_0(\sigma) = 0.056\,\sigma^5$ We can build easier a PBH population. Therefore for the same number of PBH, RD produces stronger GW signal than eMD.

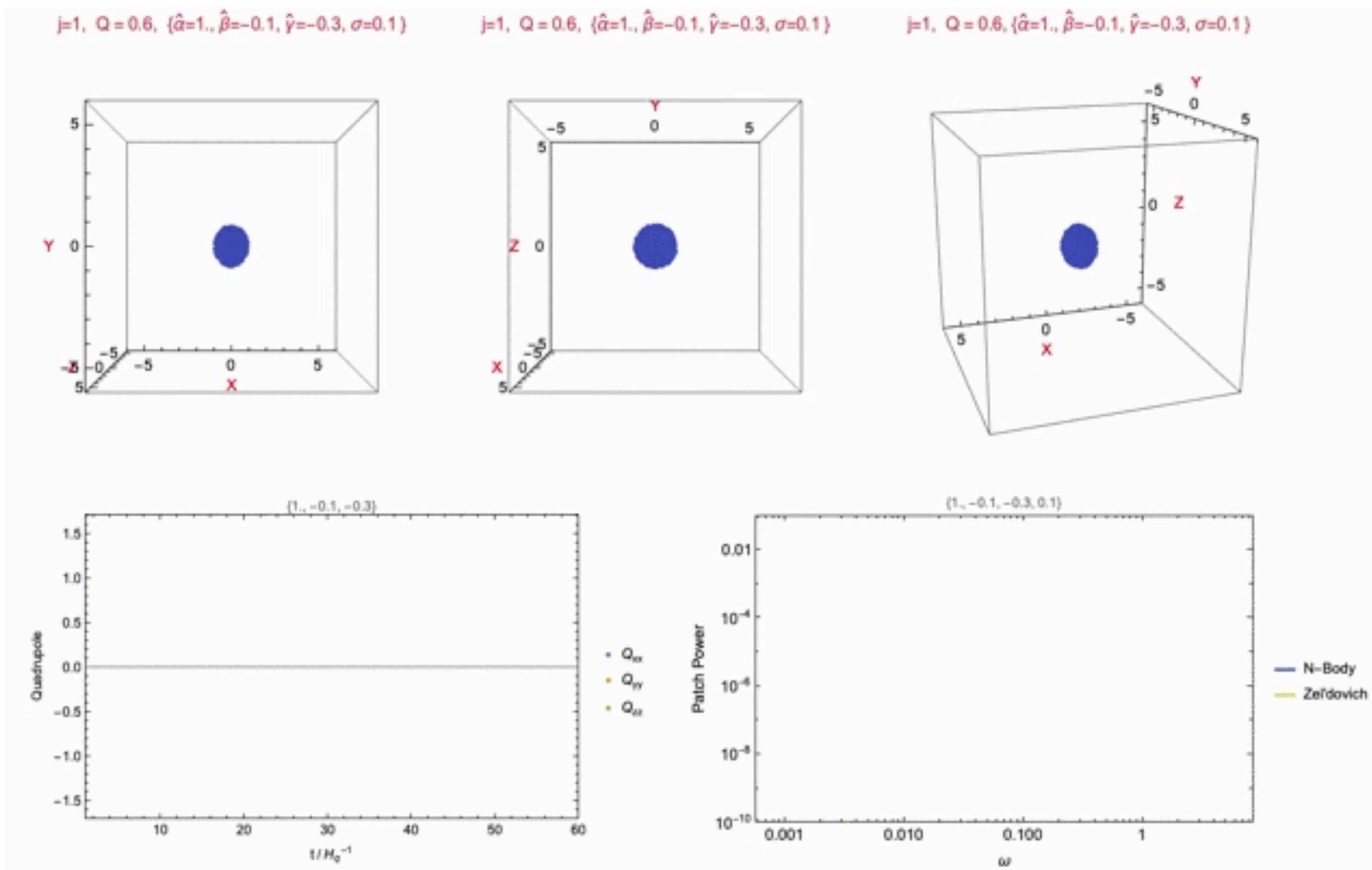
Harada Yoo Kohri Nakao Jhingan '16

Evaporating PBH at LIGO and ET



- Evaporating PBH can be probed in LIGO/Virgo & ET
- The peak can be at a different place compared to PBH formed in RD because γ_M can be much smaller
- If the evaporation leaves a Planck remnant, these PBH could explain 100% of DM relic abundance

Simulating Violent Relaxation



Dalianis CK in prep

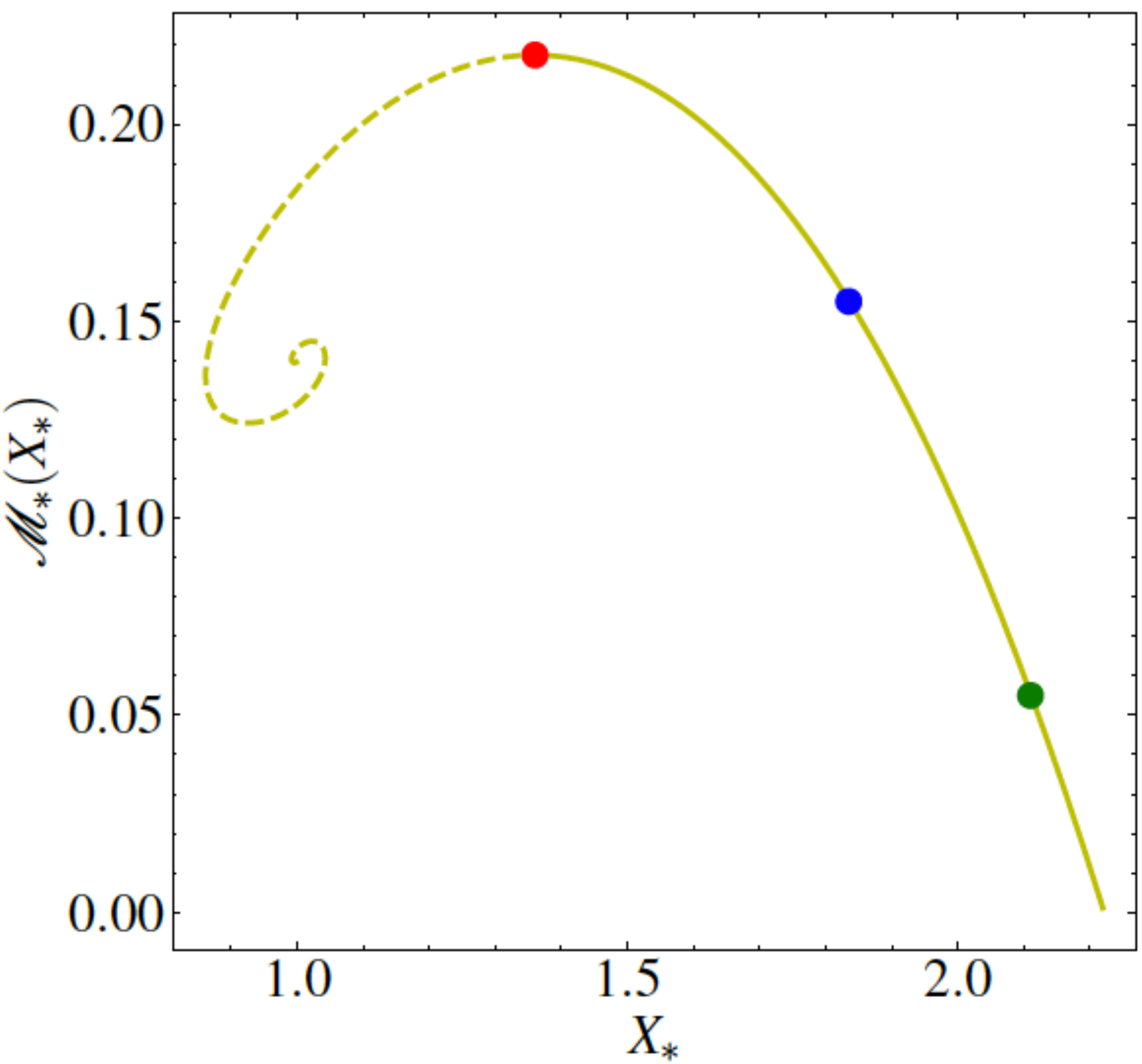
Why Dark Matter Self-Interactions?

Problems with Collisionless Cold Dark Matter

- Core-cusp profile in dwarf galaxies
- “Too big to fail”
- Diversity Problem
- Supermassive Black Holes

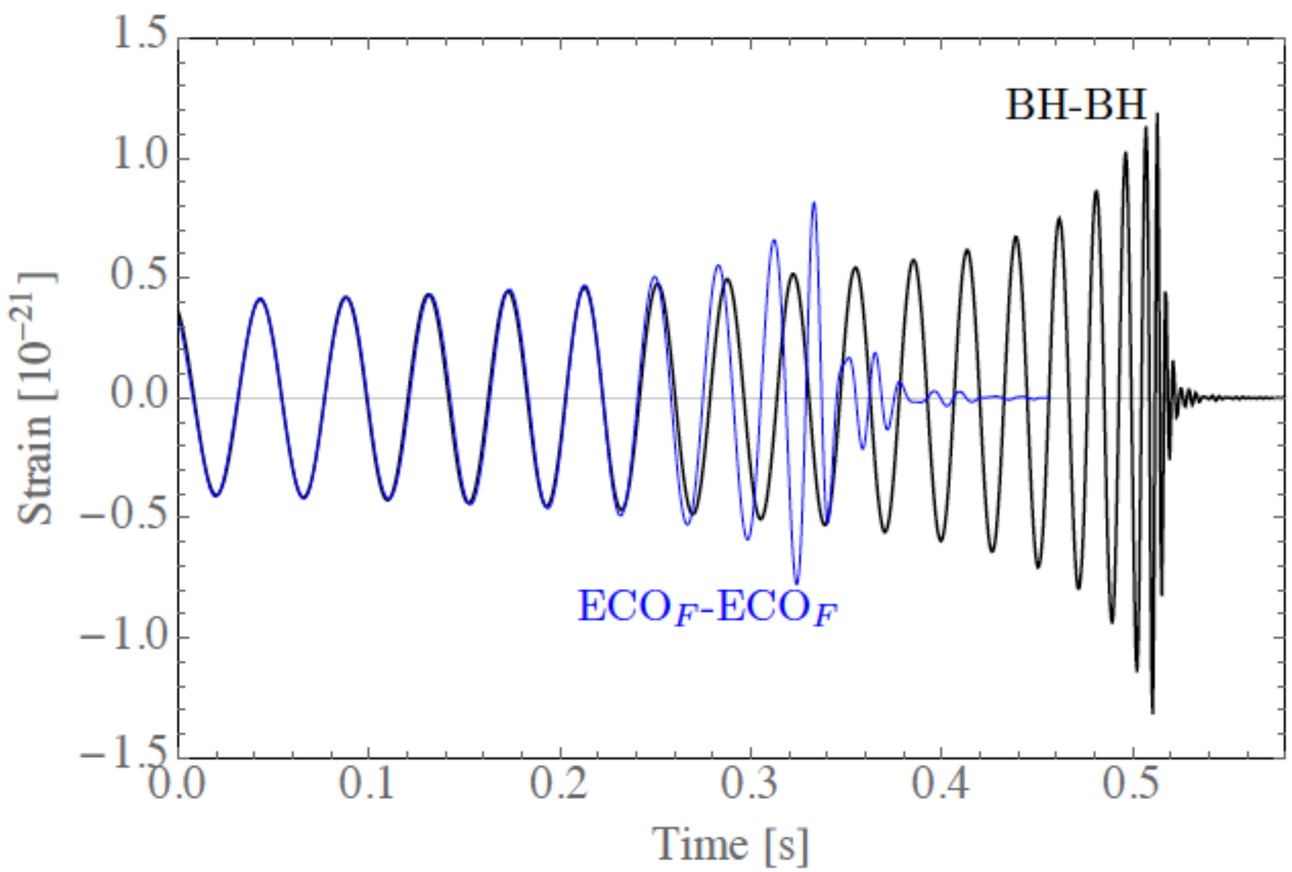
Can asymmetric dark matter with self-interactions form its own compact objects?

Asymmetric Bosonic Dark Stars



Eby Nielsen CK Wijewardhana '16

Gravitational Waves from Dark Stars



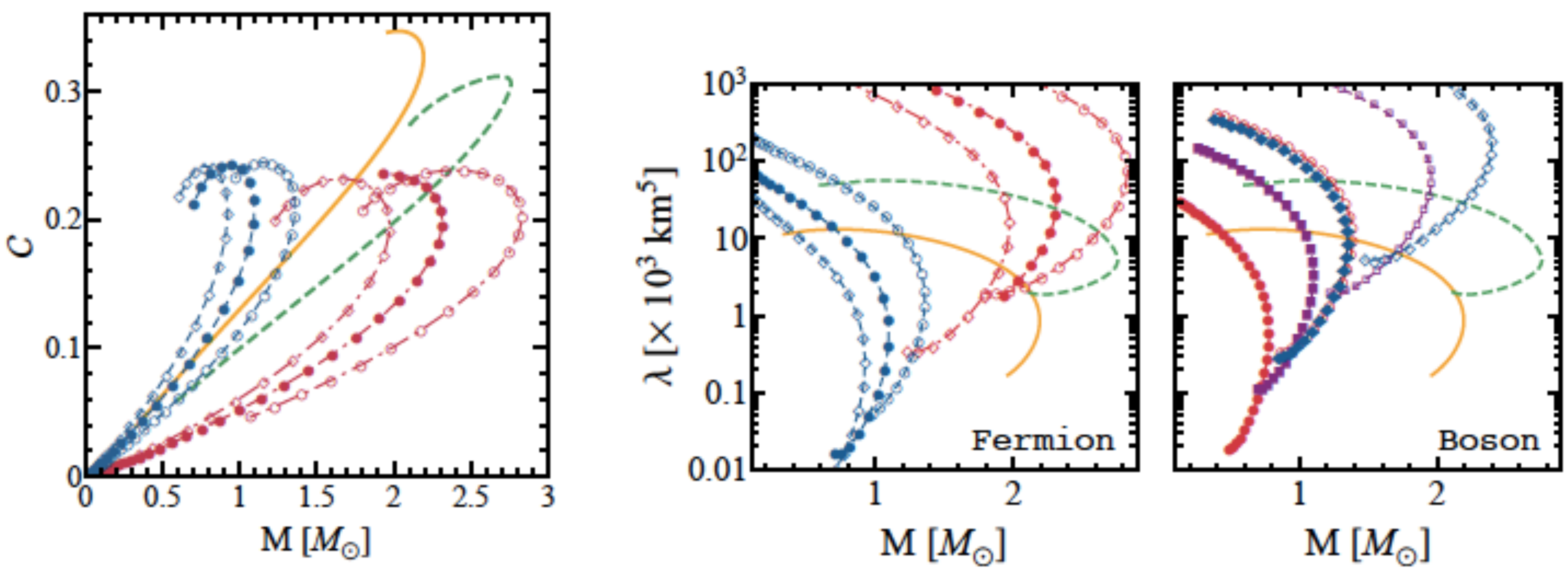
Giudice, McCullough,
Urbano '16

Tidal Deformations of Dark Stars

How stars deform in the presence of an external gravitational field?

$V=-(1/2)\varepsilon_{ij}x^i x^j$ $Q_{ij}=-\lambda\varepsilon_{ij}$ $\lambda=\frac{2}{3}k_2R^5$

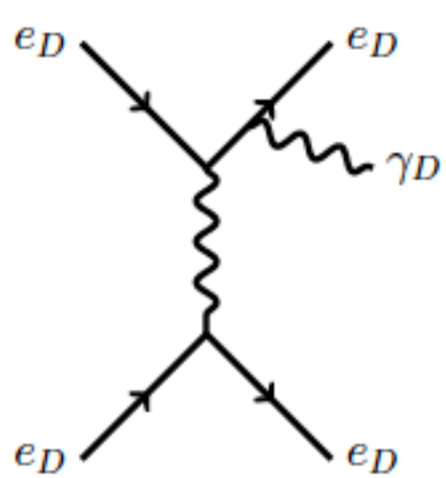
Love number



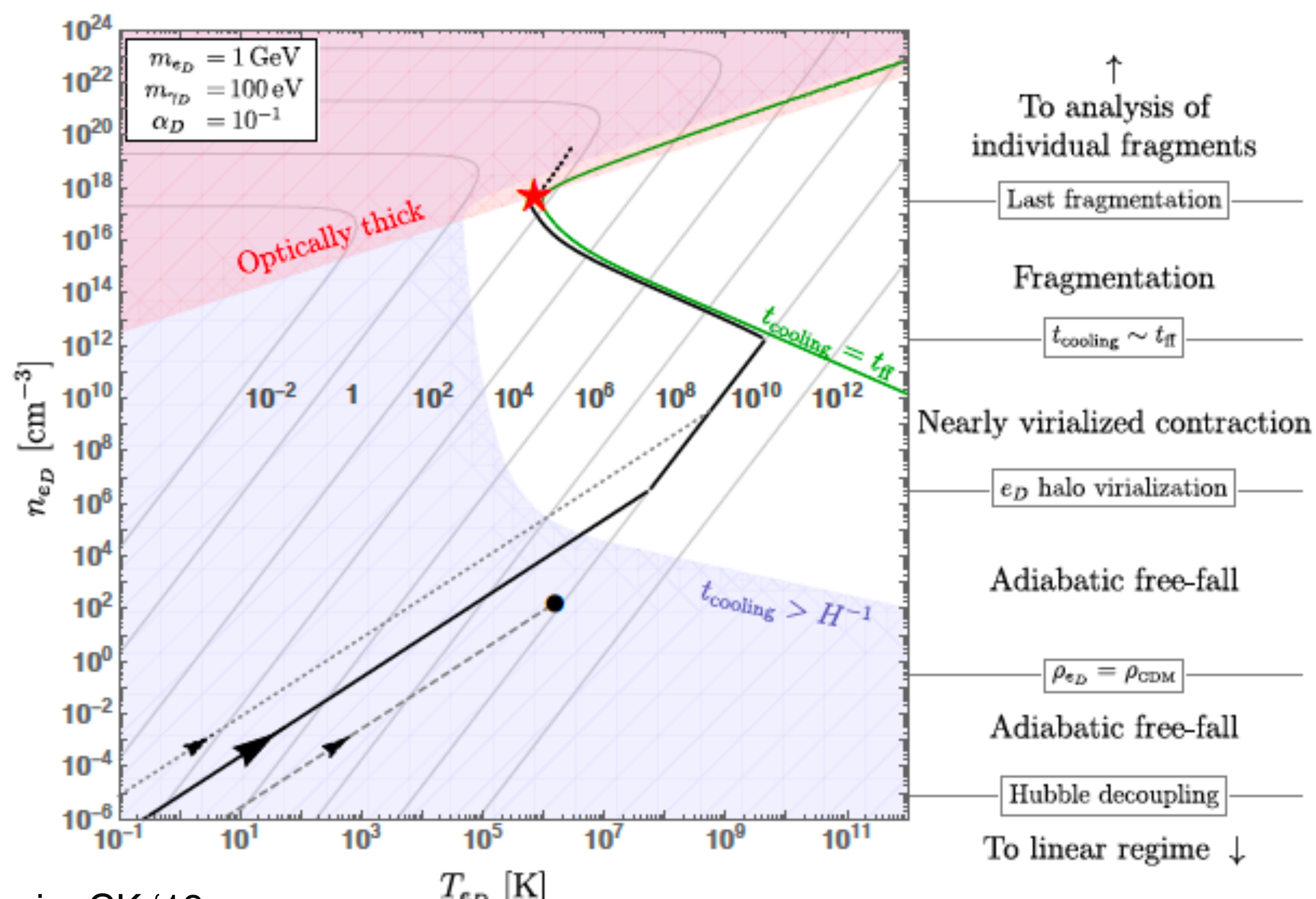
Maselli, Pnigouras,Nielsen,
CK, Kokkotas, 17

Formation of Asymmetric Dark Stars

Collapse can proceed via dark photon Bremsstrahlung Cooling



$$\frac{3}{2m_{e_D}}\frac{dT_{e_D}}{dt} = -\frac{P_{e_D}}{M}\frac{dV}{dt} - \Lambda$$



Chang, Egana-Ugrinovic, Essig, CK '19

Relativistic Proton Capture rate

Dark stars can accrete protons and electrons

$$ds^2 = -B(r)dt^2 + A(r)dt^2 + r^2 d\Omega^2$$

$$\frac{dt}{d\sigma} = \frac{1}{B(r)}, \quad \theta = \frac{\pi}{2}, \quad r^2 \frac{d\varphi}{d\sigma} = J = \text{const.}$$

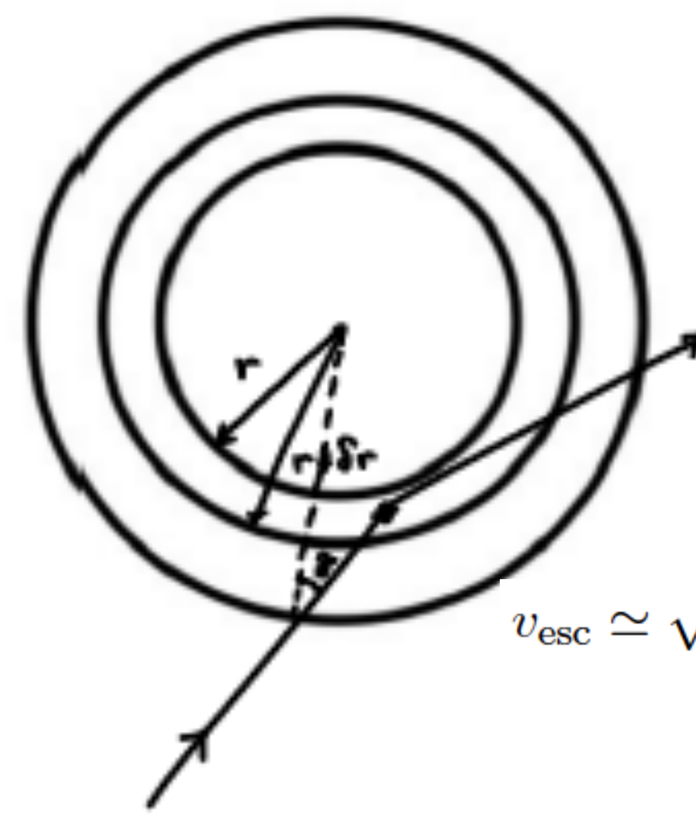
$$A(r)\left(\frac{dr}{d\sigma}\right)^2 + \frac{J^2}{r^2} - \frac{1}{B(r)} = -E = \text{const.}$$

$$J_{\rm max} = \sqrt{\frac{1 - B(r)}{B(r)} + u^2}$$

$$dF = n f(u) u \cos\theta \frac{1}{2} d\cos\theta du$$

$$dF_{\rm tot} = \pi n \frac{f(u)}{u} du dJ^2$$

$$dC = dF_{\rm tot} \frac{dl}{l_{\rm mfp}}$$



$$v_{\rm esc} \simeq \sqrt{1 - B(r)}$$

$$C = \frac{n\sigma}{m} \int_0^\infty 4\pi \frac{f(u)}{u} du \int_0^{r_*} \frac{1 - B(r)}{B(r)} r^2 \rho(r) dr$$

Betancourt, Brenner, Ibarra, CK ’22

$$C = n_0 \left(\frac{3}{2\pi \bar{v}^2}\right)^{3/2} 4\pi^2 (2GMR) \frac{1}{1 - 2GM/R} \frac{1}{3} \bar{v}^2$$

Goldman Nussinov ’89, CK ‘07

Dark Star Outbursts

after capture there is a thermalisation stage where protons settle in a thermal radius

$$r_{\rm th} \approx \sqrt{\frac{15 k_B T}{4\pi G \rho_{\rm core} m_{\rm p}}}.$$

Thermal Evolution of star

$$\frac{dT}{dt} = -\frac{L_\gamma + L_{\gamma'}}{C_{\rm v}}$$

BEC critical temperature

$$T_{\rm c} = \frac{2\pi}{m k_{\rm B}} \left(\frac{n}{\xi(\frac{3}{2})} \right)^{2/3}$$

Heat Capacity

$$C_{\rm v} = \frac{15}{4} k_{\rm B} N \frac{\xi(\frac{5}{2})}{\xi(\frac{3}{2})} \left(\frac{T}{T_{\rm c}} \right)^{\frac{3}{2}}$$

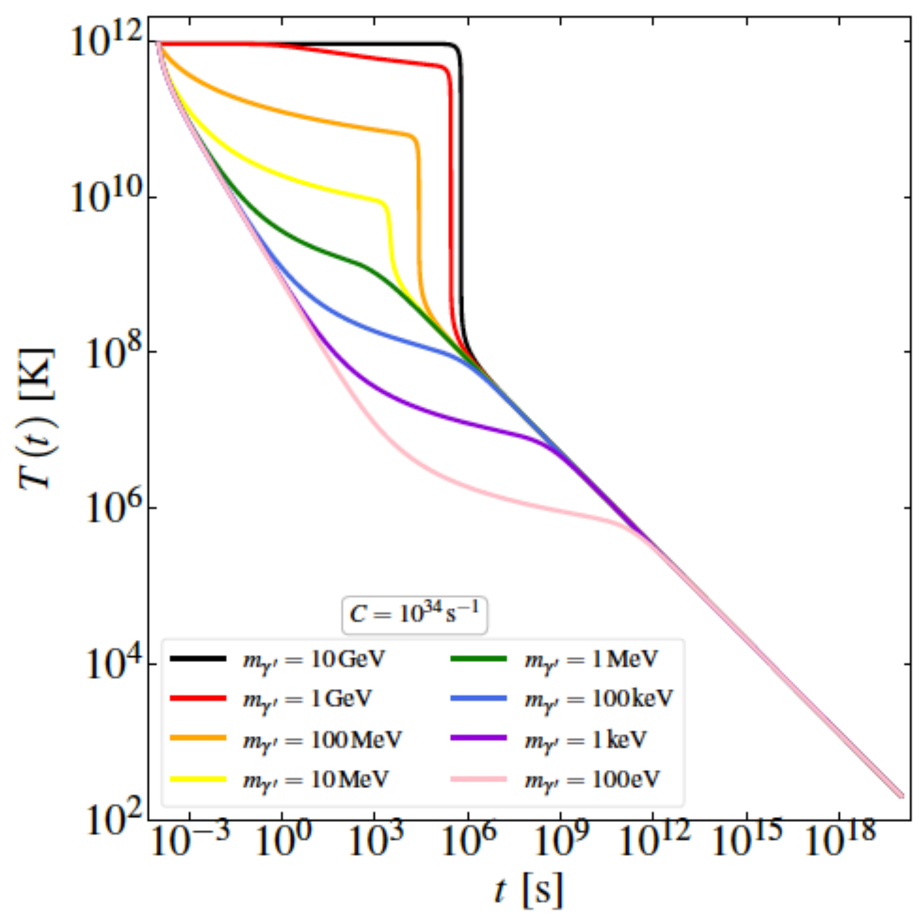
$$L_\gamma = (4\pi r_{\rm th}^2) \int_0^\infty I(\nu) d\nu \qquad I(\nu) = \frac{2h}{c^2} \frac{\nu^3}{{\rm e}^{\frac{h\nu}{k_B T}} - 1} \left(1 - {\rm e}^{-\tau(\nu)} \right)$$

$$\tau(\nu) \equiv r_{\rm th} \alpha_\nu \qquad \alpha_\nu = \frac{4}{3} \left(\frac{2\pi}{3} \right)^{\frac{1}{2}} g_{\bar{f}f} \frac{e^6}{h m_e^2 c^2} n_e n_p \left(\frac{m_e c^2}{k_B T} \right)^{1/2} \frac{1 - {\rm e}^{-\frac{h\nu}{k_B T}}}{\nu^3}$$

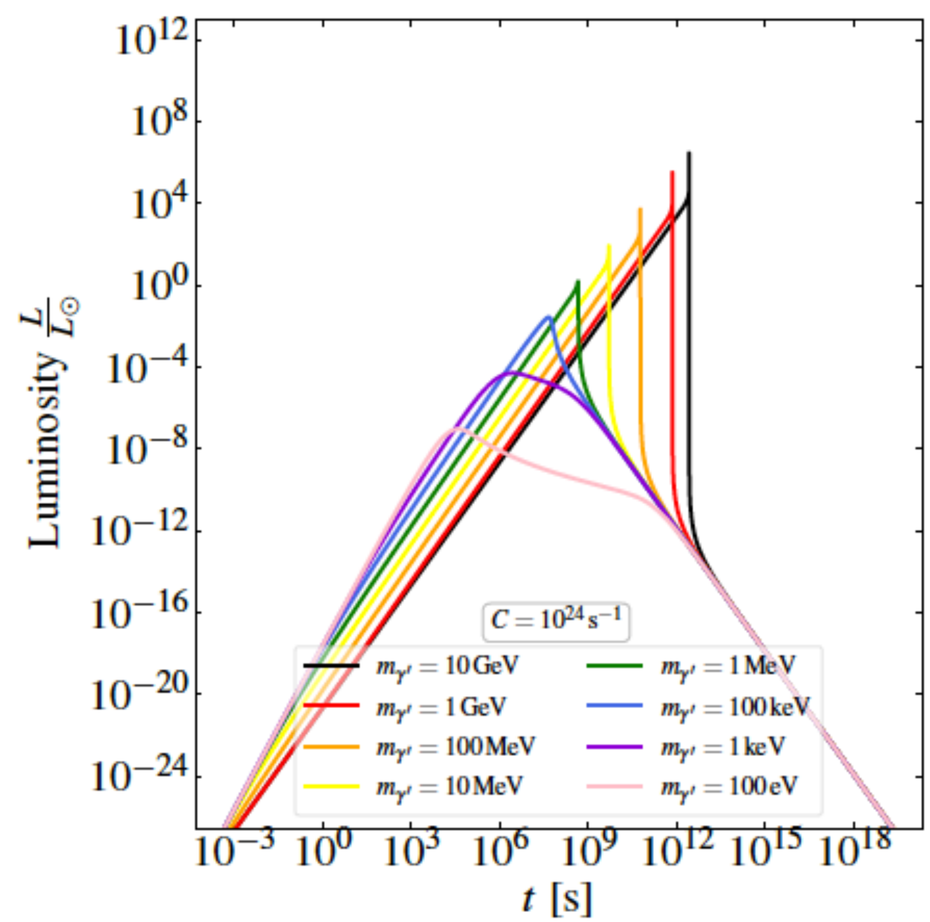
with $\tau \ll 1$ (optically thin limit)-> Bremsstrahlung
when $\tau \gg 1$ blackbody radiation

Dark Star Outbursts

Temperature



Luminosity



Outbursts can last from days to months

At first the photon luminosity scales as $n_p^2 T^2 \sim t^2/T$

As temperature reduces, the luminosity and the energy loss increase dramatically until the thermal radius becomes opaque for the photons.

At this point the spectrum becomes the blackbody one with luminosity~ T^5

there is one extra power of T due to the thermal radius dependence on T.

Conclusions

Gravitational Wave Production in a eMD era

- Perturbation theory fails after maximum expansion
- Zel'dovich method is valid until violent relaxation
- eMD formed PBH can avoid PTA constraints because it is easier to make BH
- Can be distinguished from RD formation
- Can be tested in current and future interferometers

Dark Stars

- Could form by a strongly self-interacting component of dark matter
- Dark stars can be distinguishable from black holes or neutron stars merger events
- They could have significant luminosity outbursts once they accrete sufficient baryonic and leptonic matter