Primordial Black Holes: formation and cosmological signature

Ilia Musco

(**INFN Fellini Fellow** - *SAPIENZA* University of Rome)

Dark Matters 2022

30 November 2022 - Brussels

G. Franciolini, IM, P.Pani, A. Urbano - arXiv:2209.05959
IM, K. Jedamzik, S. Young - in preparation

Introduction: a very brief overview

- **Primordial Black Holes (PBHs)** [**Zeldovich & Novikov** (1967), **Hawking** (1971)] could form from the collapse of cosmological perturbation during the radiation dominated era.

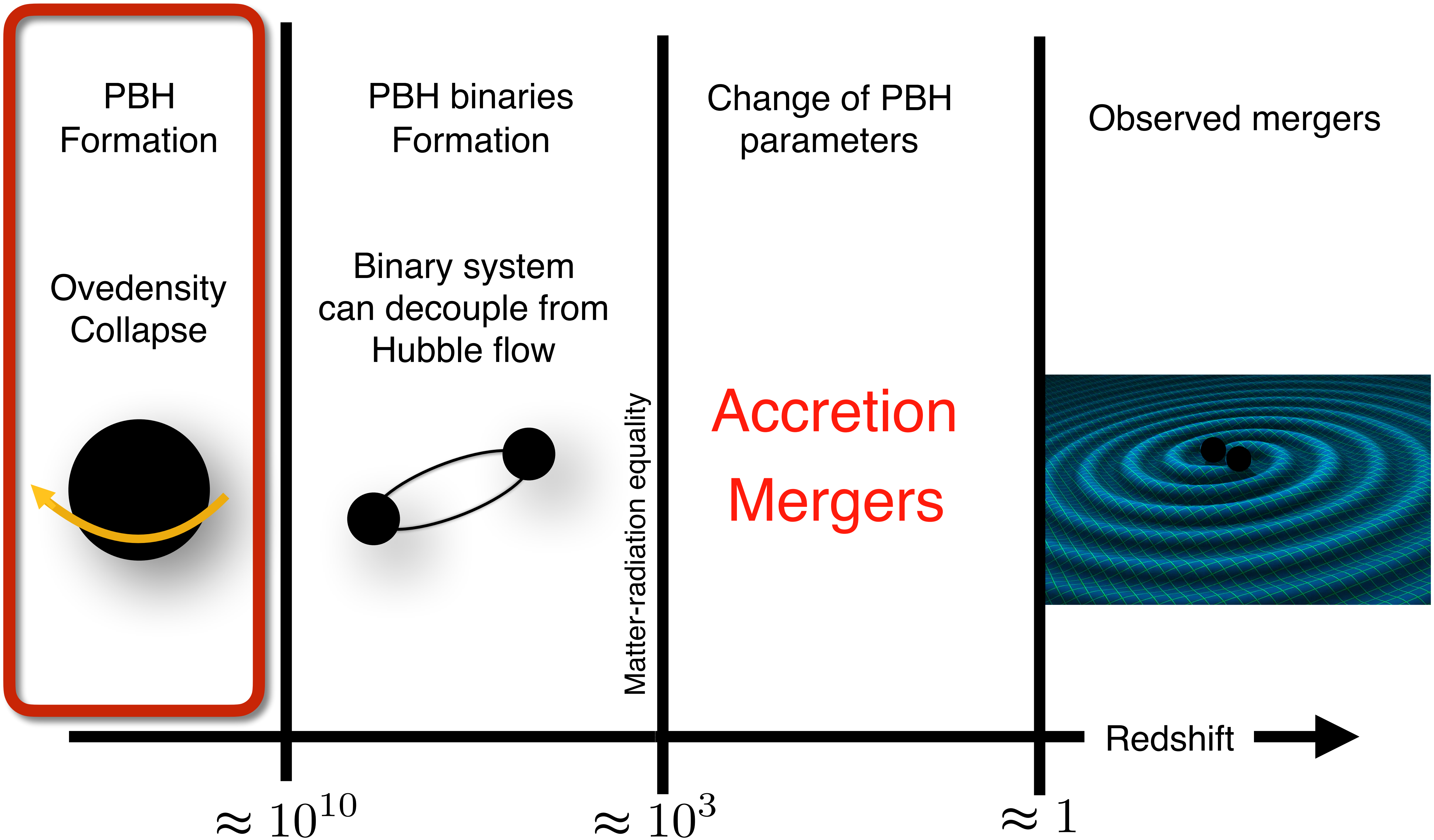
$$p = \frac{\rho}{3}$$

- **PBHs could span a large wide range of masses and if not evaporated** [BH evaporation **Hawking** (1974)]: PBHs with $M > 10^{15}\,g$ are interesting candidates for dark matter, intermediate mass black holes and the seeds of supermassive black holes.

- **Numerical hydrodynamical simulations in spherical symmetry** of a cosmological perturbation, characterized by an amplitude δ, have shown:

    - $\delta > \delta_c \;\Rightarrow$ PBH formation
    - $\delta < \delta_c \;\Rightarrow$ perturbation bounce
    - $\delta_c \sim c_s^2 \equiv \frac{\partial p}{\partial \rho}$ (**Carr 1975**)

PBH evolution

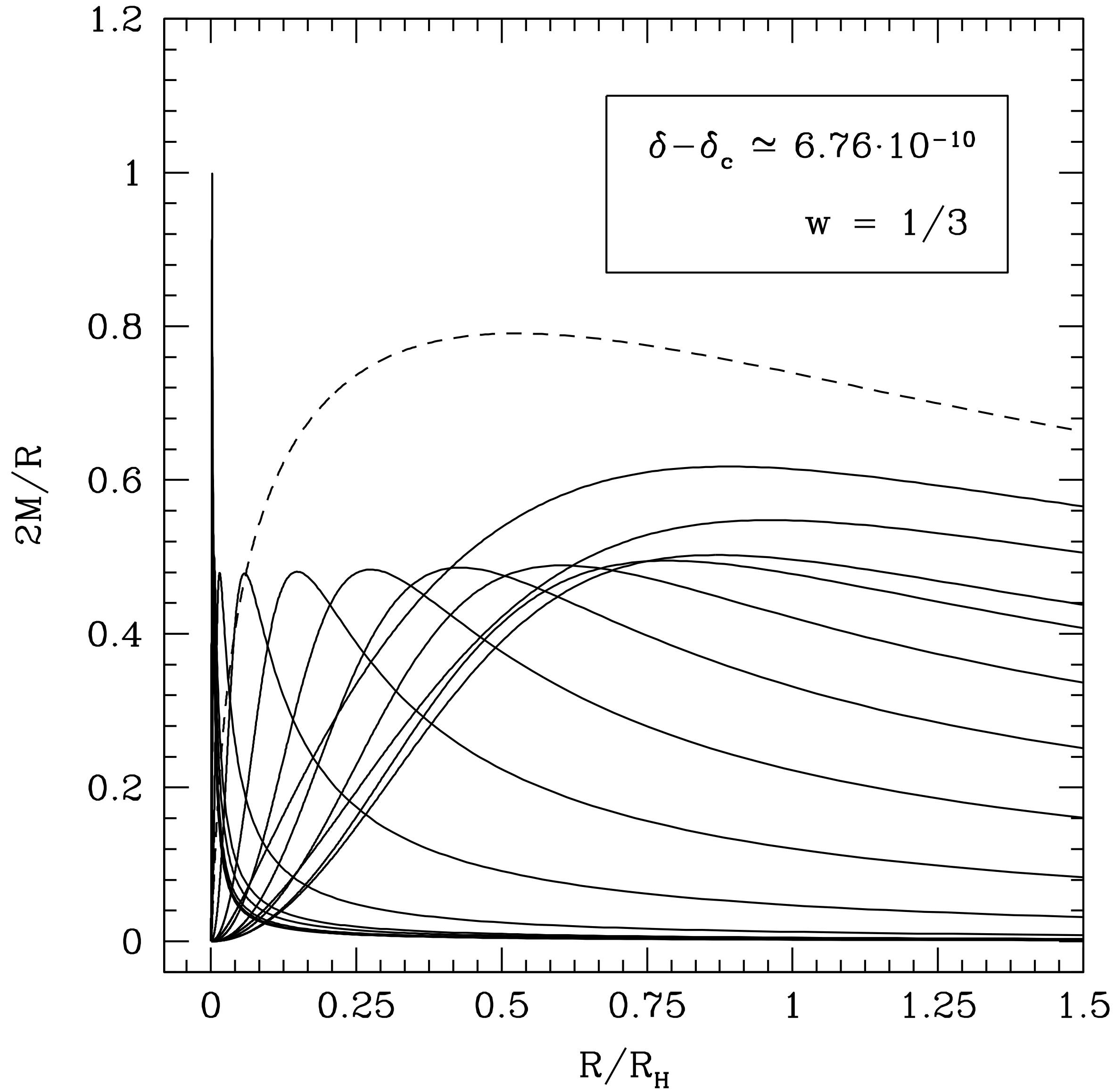


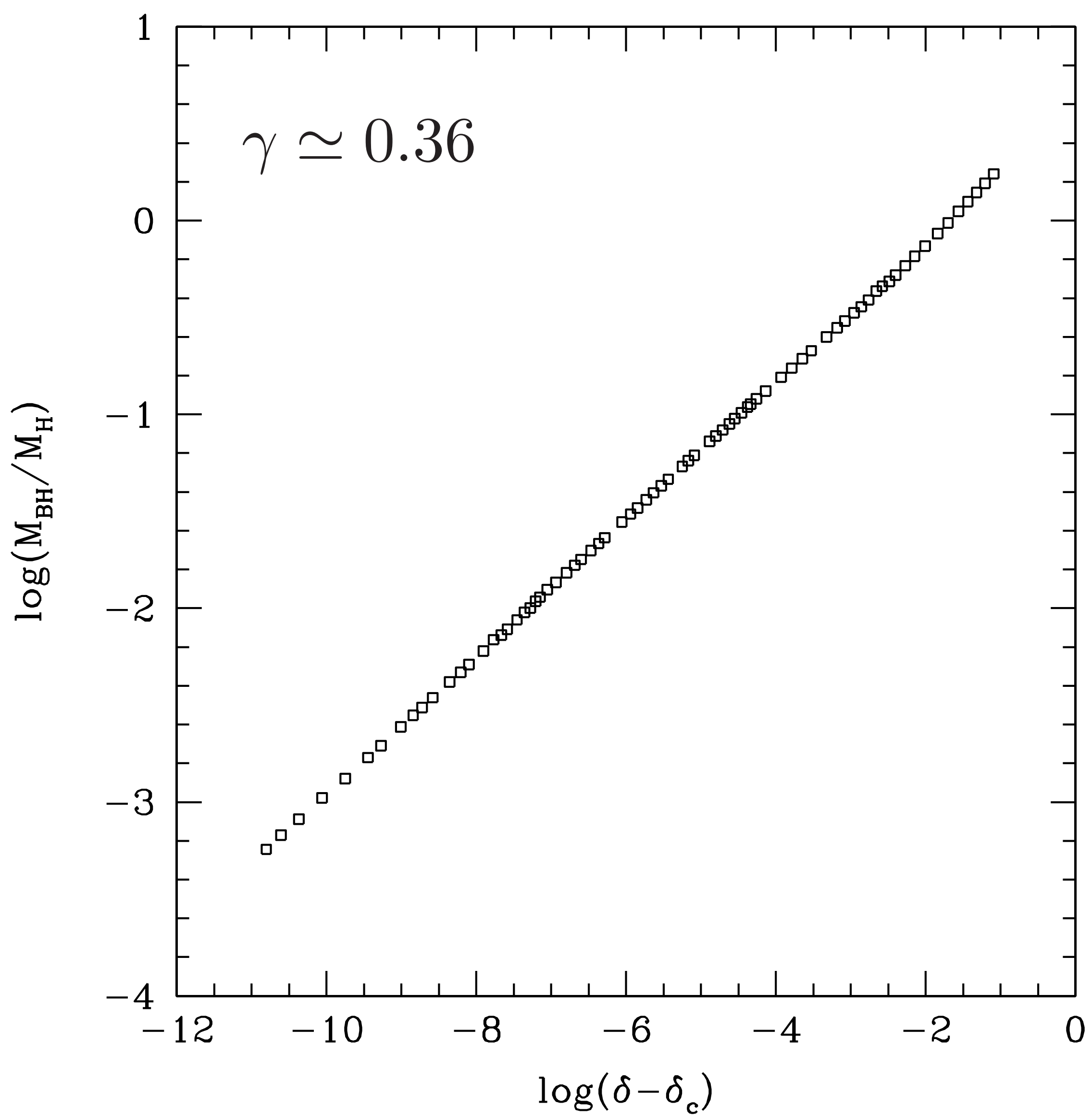
Courtesy of *Antonio Riotto*

Numerical Results: PBH formation / mass spectrum

$$R(r,t) = 2M(r,t)$$

$$M_{PBH} = \mathcal{K}(\delta - \delta_c)^{\gamma} M_H$$





$\mathcal{K}, \delta_c$ — shape dependent

IM, J. Miller, Polnarev, Rezzolla - CQG (2005 - 2013)

Initial conditions: curvature profile

- The asymptotic metric ($t \rightarrow 0$), describing super-horizon cosmological perturbations in the comoving synchronous gauge can be written as:

$$ds^2 \simeq -dt^2 + a^2(t)e^{2\zeta(r)}\left[dr^2 + r^2 d\Omega^2\right]$$

- In the "linear regime" of cosmological perturbations, adiabatic perturbations on super horizon scales can be described by a time independent curvature profile using the **quasi-homogeneous / gradient expansion approach**.

$$\frac{\delta\rho}{\rho_b} = -\left(\frac{1}{aH}\right)^2 \frac{4}{9}\left[\nabla^2\zeta(r) + \frac{1}{2}\left(\nabla\zeta(r)\right)^2\right] e^{-2\zeta(r)}$$

- The perturbation amplitude δ is measured by the peak of the compaction function, corresponding to the excess of mass of the over density.

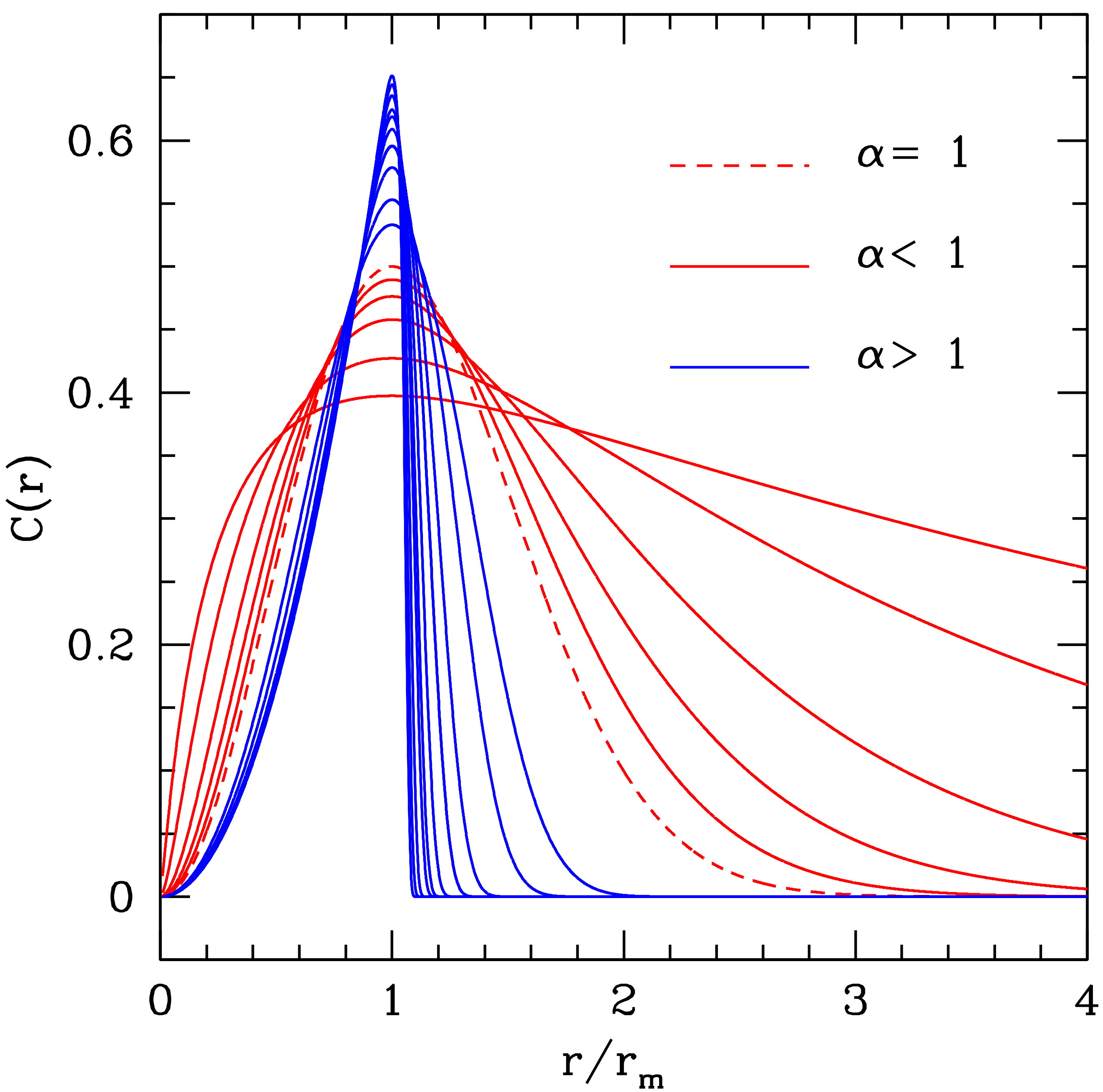
$$\mathcal{C}(r) := \frac{2[M(r,t) - M_b(r,t)]}{R(r,t)} = -\frac{4}{3}\tilde{r}\zeta'(r)\left[1 + \frac{1}{2}\tilde{r}\zeta'(r)\right] \quad \Rightarrow \quad \delta = \delta_G\left[1 - \frac{3}{8}\delta_G\right]$$

Shape parameter

I. Musco - PRD (2019)

$$\mathcal{C}'(r_m) = 0, \quad \Phi_m \equiv -r_m \zeta'(r_m)$$

$$\boxed{\delta(r_m, t_H) = 3\frac{\delta\rho}{\rho_b}(r_m, t_H)}$$



$$\alpha \equiv -\frac{\mathcal{C}''(r_m) r_m^2}{4\mathcal{C}(r_m)} = \frac{\alpha_G}{\left(1 - \frac{1}{2}\Phi_m\right)\left(1 - \Phi_m\right)}$$

$$\boxed{0.4 \leq \delta_c(\alpha) \leq \frac{2}{3}}$$

$$\delta_c \simeq \begin{cases} \alpha^{0.047} - 0.50 & 0.1 \lesssim \alpha \lesssim 7 \\ \alpha^{0.035} - 0.475 & 7 \lesssim \alpha \lesssim 13 \\ \alpha^{0.026} - 0.45 & 13 \lesssim \alpha \lesssim 30 \end{cases}$$

Escrivá, Germani, Sheth - PRD (2020)

PBH Abundance (Peak Theory)

C.Germani, IM - PRL (2019)

- **PDF of δ follows a Gaussian distribution:** $P(\delta) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\delta^2}{2\sigma^2}\right)$

$$\sigma^2 = \langle \delta^2 \rangle = \int_0^\infty \frac{dk}{k} \mathcal{P}_\delta(k, r_m) = \left(\frac{2}{3}\right)^4 \int_0^\infty \frac{dk}{k} (kr_m)^4 T^2(k, r_m) W^2(k, r_m) \mathcal{P}_\zeta(k)$$

$$\beta_f \simeq \sqrt{\frac{2}{\pi}} \mathcal{K} \left(\frac{k_*}{a_m H_m}\right)^3 \sigma^\gamma \nu_c^{1-\gamma} \gamma^{\gamma+1/2} e^{-\frac{\nu_c^2}{2}} \qquad \nu_c \equiv \frac{\delta_c}{\sigma}$$

- **If** $M_{PBH} \sim 10^{16} g$ are Dark Matter $\Rightarrow \beta_f \simeq 10^{-8} \sqrt{\frac{M_{PBH}}{M_\odot}} \simeq 10^{-16}$

- **Narrow peak:** $\frac{k_*}{\sigma} \gg 1 \;\Rightarrow\; \nu_c \simeq 0.22 \sqrt{\frac{k_*}{\sigma \mathcal{P}_0}} \;\Rightarrow\; \mathcal{P}_0 \sim 7 \times 10^{-4} \frac{k_*}{\sigma} \gg 10^{-3}$

- **Broad peak:** $\frac{k_*}{\sigma} \ll 1 \;\Rightarrow\; \nu_c \simeq 0.46 (\mathcal{P}_0)^{-1/2} \;\Rightarrow\; \mathcal{P}_0 \sim 3 \times 10^{-3}$

- **Non linear effects:** $\delta = \delta_G \left[1 - \frac{3}{8}\delta_G\right] \quad \Rightarrow \quad 1.5 \lesssim \frac{\mathcal{P}_{0_{NL}}}{\mathcal{P}_{0_L}} = \frac{16\left(1 - \sqrt{1 - \frac{3}{2}\delta_c}\right)^2}{9\delta_c^2} \lesssim 4$

S.Young, IM, C.Byrnes JCAP (2019)

De Luca, Franciolini, Kehagias, Peloso, Riotto and Unal (2019)

PBH threshold prescription

Curvature power spectrum $\mathcal{P}_\zeta$

$\Downarrow$

Characteristic overdensity scale $k_* \hat{r}_m$

$\Downarrow$

Characteristic shape parameter α

$\Downarrow$

Threshold δ_c

IM, De Luca, Franciolini, Riotto - PRD (2021)

1. **The power spectrum of the curvature perturbation**: take the primordial power spectrum $\mathcal{P}_\zeta$ of the Gaussian curvature perturbation and compute, on superhorizon scales, its convolution with the transfer function $T(k,\eta)$

$$P_\zeta(k,\eta) = \frac{2\pi^2}{k^3}\mathcal{P}_\zeta(k) T^2(k,\eta).$$

2. **The comoving length scale** $\hat{r}_m$ of the perturbation is related to the characteristic scale k_* of the power spectrum P_ζ. Compute the value of $k_* \hat{r}_m$ by solving the following integral equation

$$\int dk k^2 \left[(k^2 \hat{r}_m^2 - 1) \frac{\sin(k\hat{r}_m)}{k\hat{r}_m} + \cos(k\hat{r}_m) \right] P_\zeta(k,\eta) = 0\,.$$

3. **The shape parameter:** compute the corresponding shape parameter α of the collapsing perturbation, including the correction from the non linear effects, by solving the following equation

$$F(\alpha)\left[1+F(\alpha)\right]\alpha = -\frac{1}{2}\left[1 + \hat{r}_m \frac{\int dk k^4 \cos(k\hat{r}_m) P_\zeta(k,\eta)}{\int dk k^3 \sin(k\hat{r}_m) P_\zeta(k,\eta)}\right]$$

$$F(\alpha) = \sqrt{1 - \frac{2}{5} e^{-1/\alpha} \frac{\alpha^{1-5/2\alpha}}{\Gamma\left(\frac{5}{2\alpha}\right) - \Gamma\left(\frac{5}{2\alpha}, \frac{1}{\alpha}\right)}}\,.$$

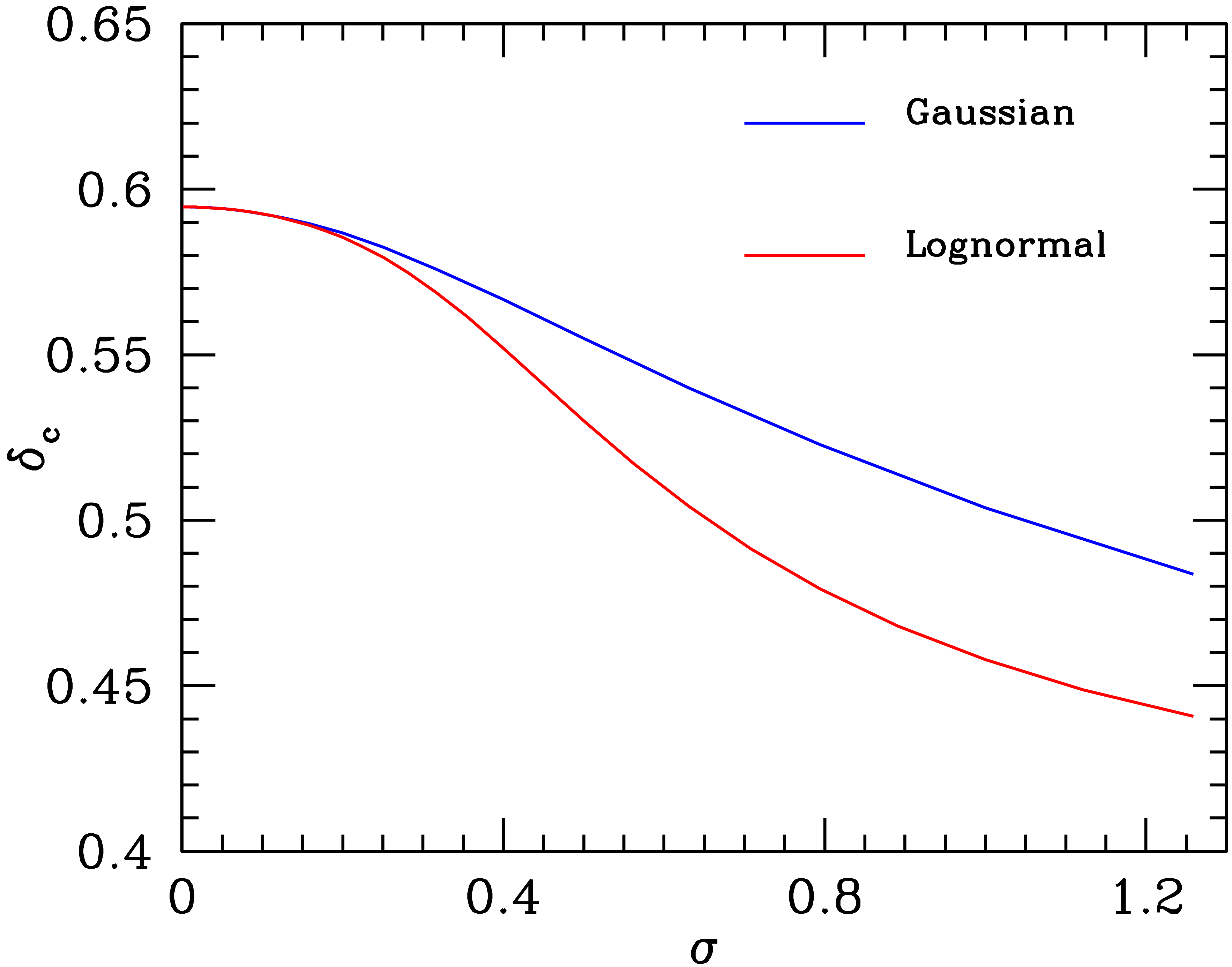
4. **The threshold δ_c:** compute the threshold as function of α, fitting the numerical simulations, at *superhorizon scales*, making a linear extrapolation at horizon crossing ($aHr_m = 1$).

$$\delta_c \simeq \begin{cases} \alpha^{0.047} - 0.50 & 0.1 \lesssim \alpha \lesssim 7 \\ \alpha^{0.035} - 0.475 & 7 \lesssim \alpha \lesssim 13 \\ \alpha^{0.026} - 0.45 & 13 \lesssim \alpha \lesssim 30 \end{cases}$$

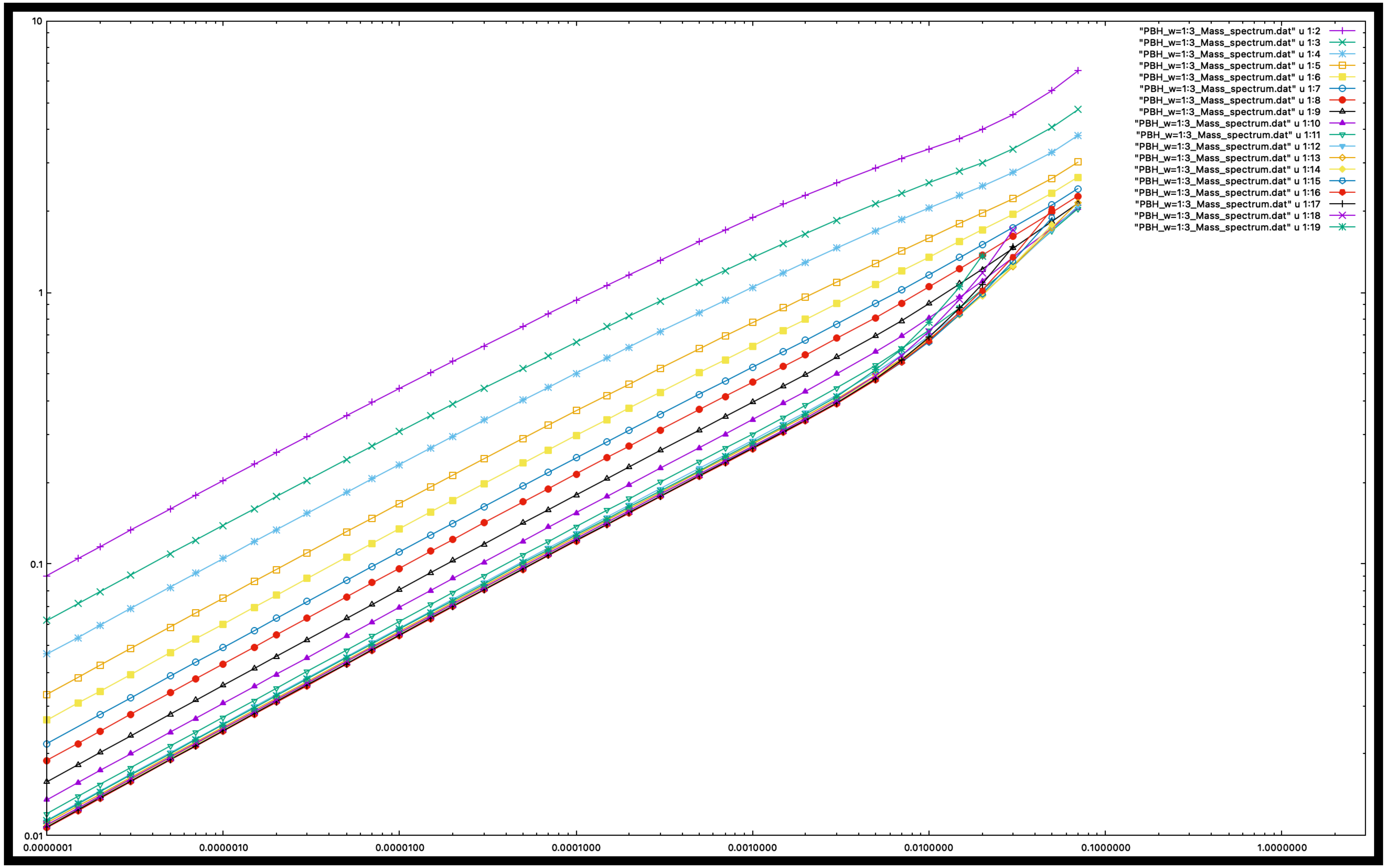
Power Spectrum:

Gaussian: $\mathcal{P}_\zeta(k) = \mathcal{P}_0 \exp\left[-(k-k_*)^2/2\sigma^2\right]$

Lognormal: $\mathcal{P}_\zeta(k) = \mathcal{P}_0 \exp\left[-\ln^2(k/k_*)/2\sigma^2\right]$

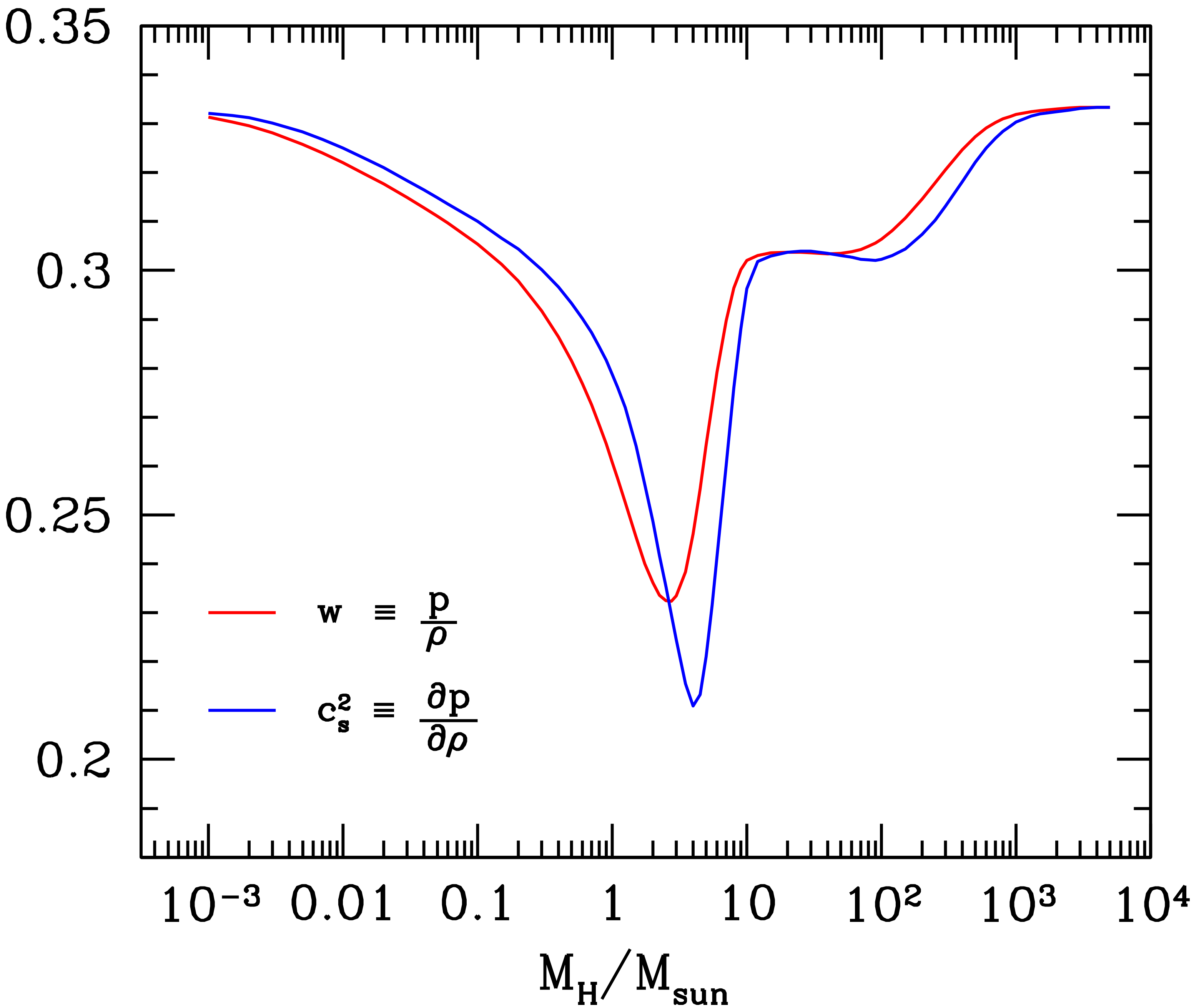


Mass Spectrum:



QCD Phase-transition

- Significant softening of the equation of state (lattice QCD simulations)
- Introducing an intrinsic scale



$$\rho = \frac{\pi^2}{30} g_{\rm eff}(T) T^4$$

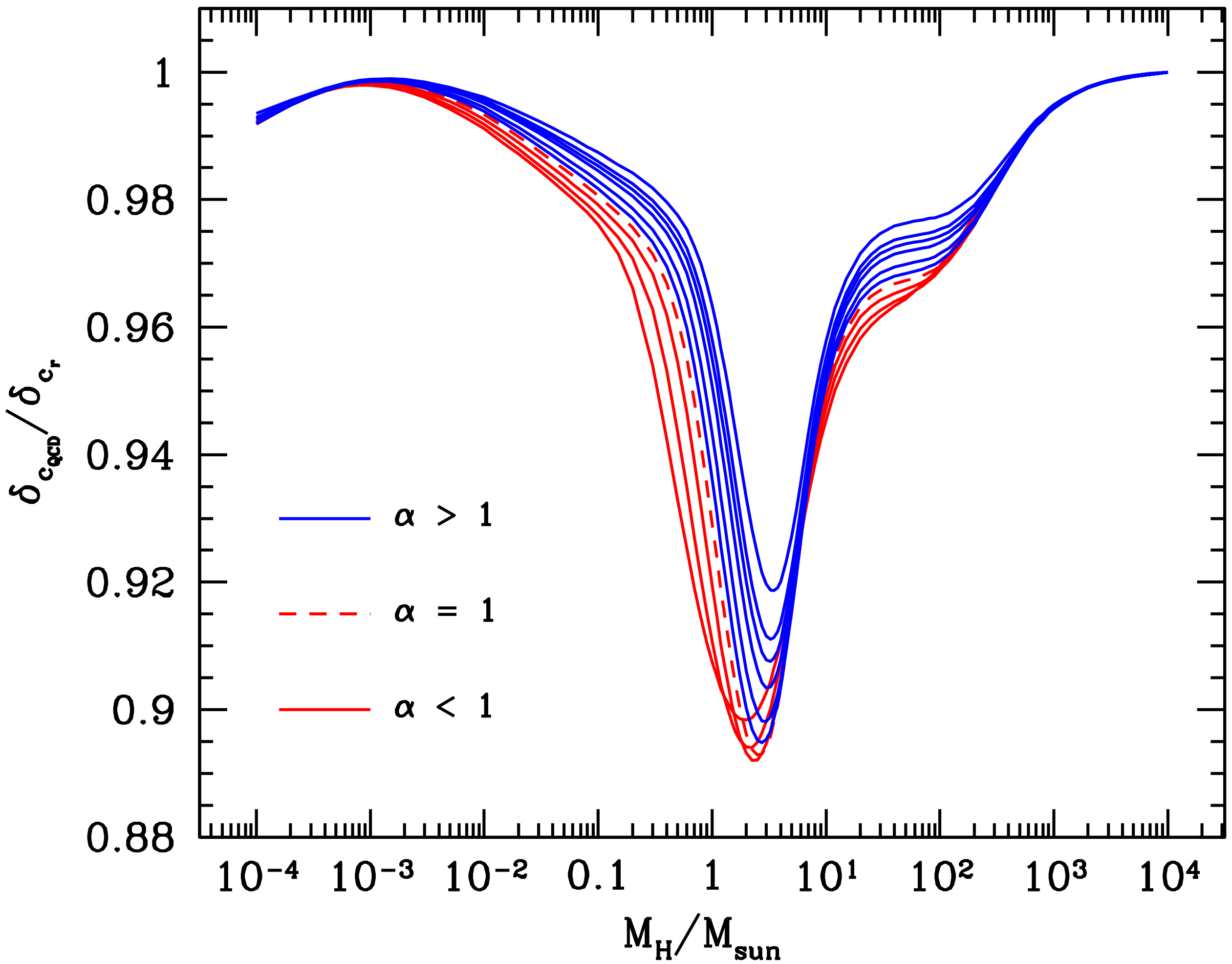
$$s = \frac{2\pi^2}{45} h_{\rm eff}(T) T^3$$

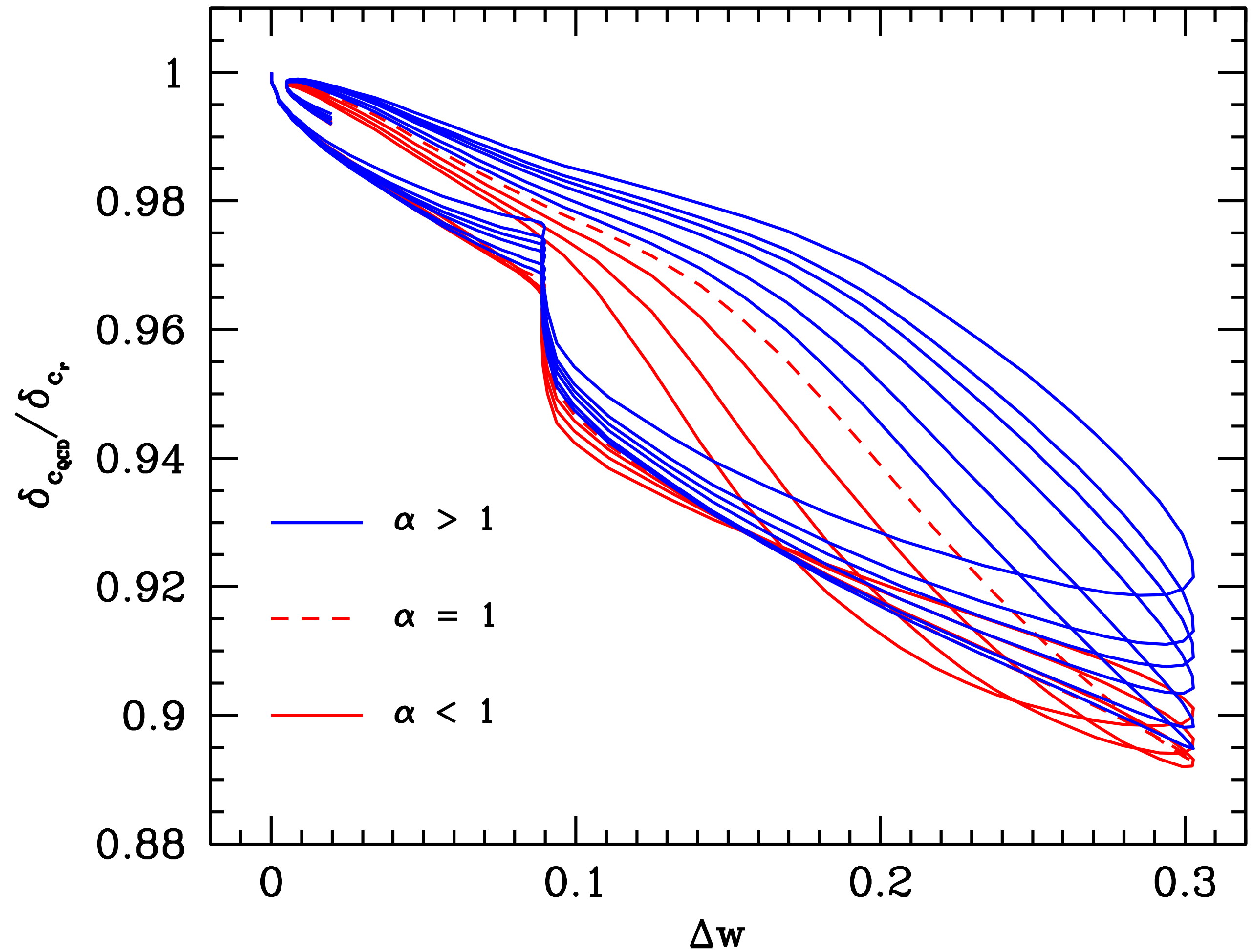
$$p = sT - \rho = w(T)\rho$$

$$w(T) = \frac{4h_{\rm eff}(T)}{3g_{\rm eff}(T)} - 1$$

$$c_s^2(T) = \frac{\partial p}{\partial \rho} = \frac{4(4h_{\rm eff} + Th'_{\rm eff})}{3(4g_{\rm eff} + Tg'_{\rm eff})} - 1$$

PBH Threshold during the QCD





Depending on the shape, the threshold for PBH formation during the QCD phase transition is reduced about 10% around the minimum of ***w(T)*.**

Significant enhancement of PBH formation around the solar mass scale!

$$\Delta w = \frac{1/3 - w}{1/3} = 1 - 3w$$

***IM, K. Jedamzik, Sam Young* - in progress…**

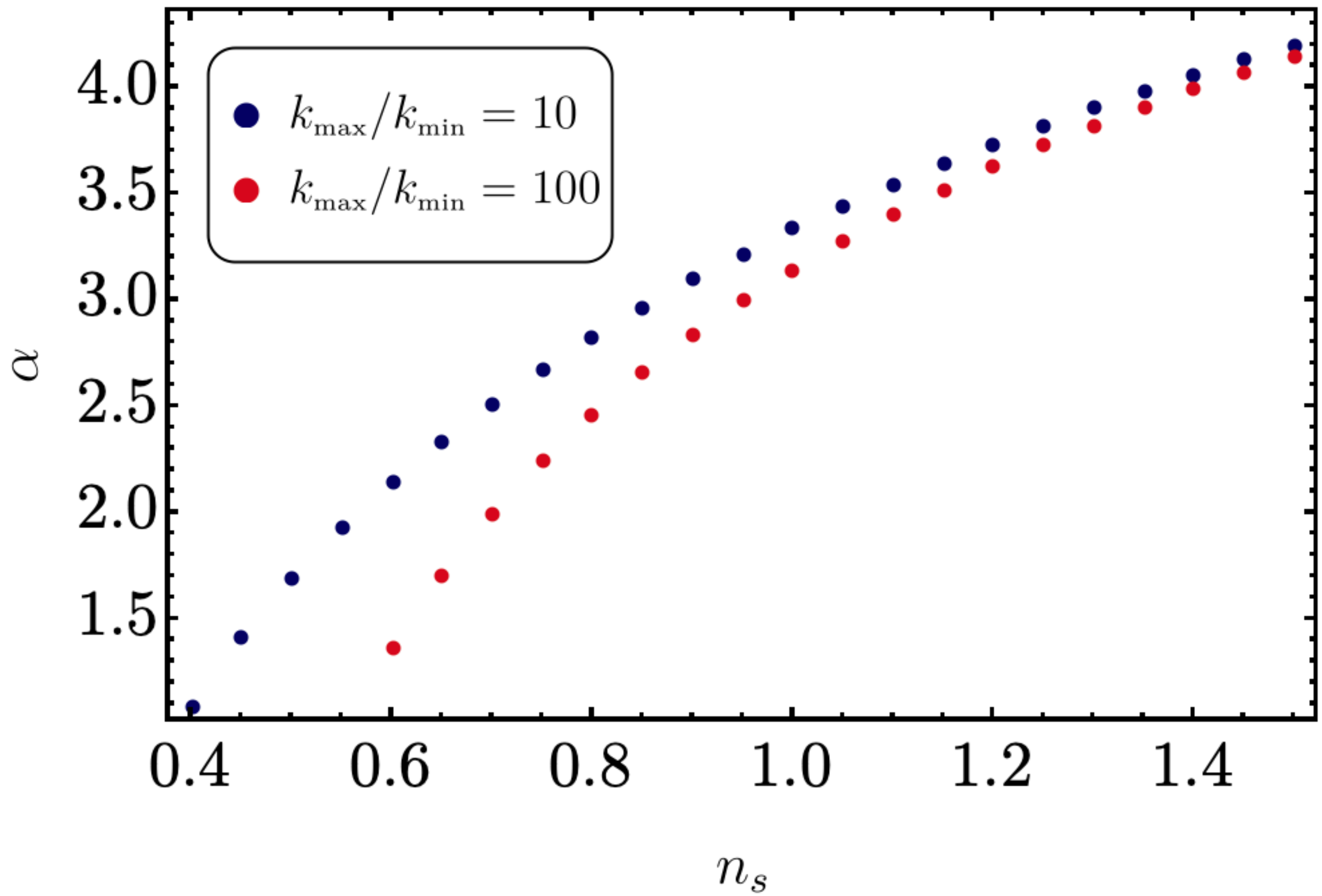
A. Escriva, E. Baguai, S. Clesse - arXiv:2209.06196

From the Power Spectrum to the shape parameter

$$P_\zeta(k) = A\,(k/k_{\rm min})^{n_s-1}\,\Theta(k-k_{\rm min})\Theta(k_{\rm max}-k)$$

n_s — spectrum tilt

$k_{\rm max}/k_{\rm min}$ — cut-off scale

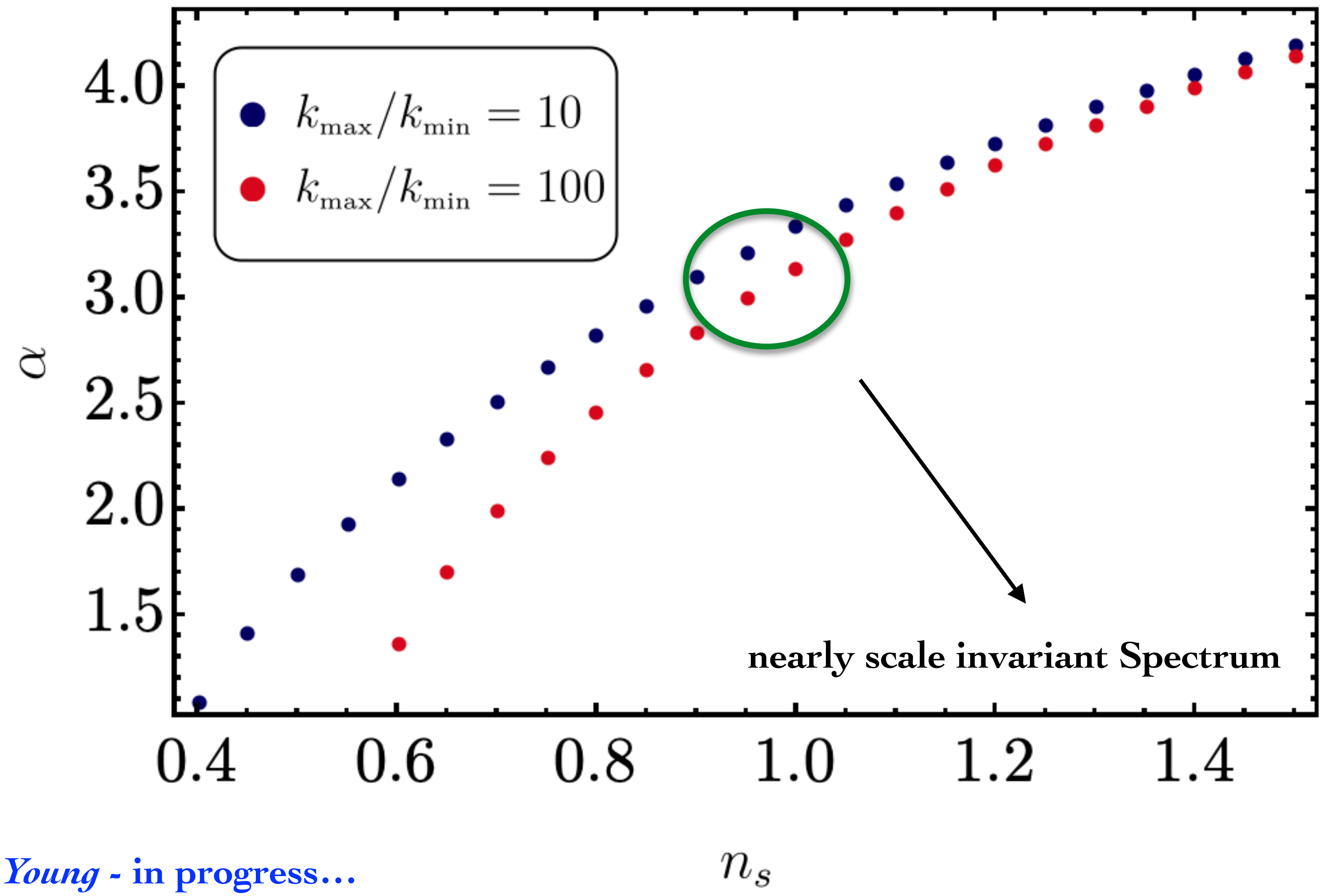


From the Power Spectrum to the shape parameter

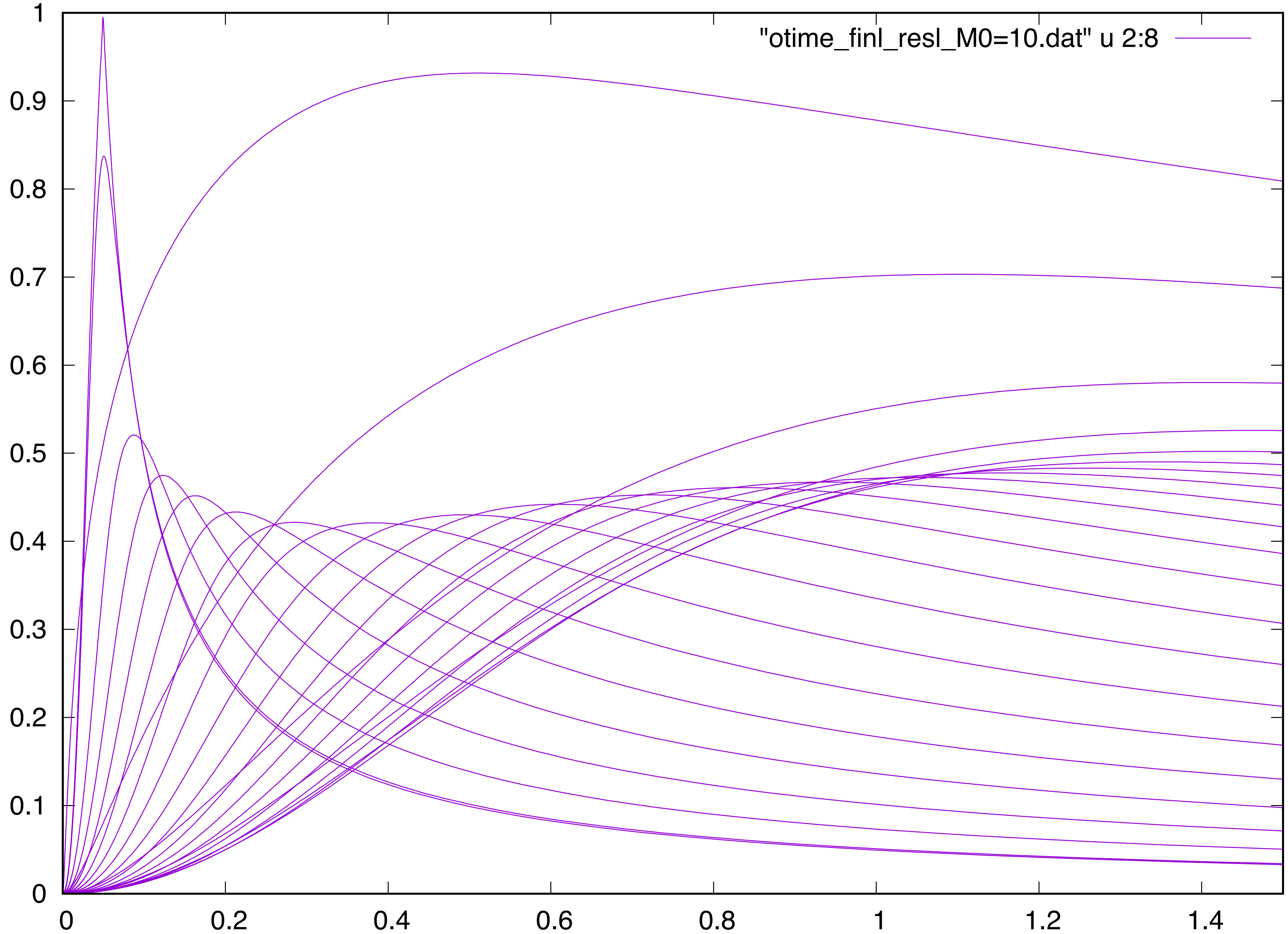
$$P_\zeta(k) = A\,(k/k_{\rm min})^{n_s-1}\,\Theta(k-k_{\rm min})\Theta(k_{\rm max}-k)$$

n_s – spectrum tilt

$k_{\rm max}/k_{\rm min}$ – cut-off scale



IM, K. Jedamzik, Sam Young - in progress…


"otime_finl_resl_M0=10.dat" u 2:8
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1
1.2
1.4

PBH Mass Spectrum - QCD

For a nearly scale invariant Power Spectrum

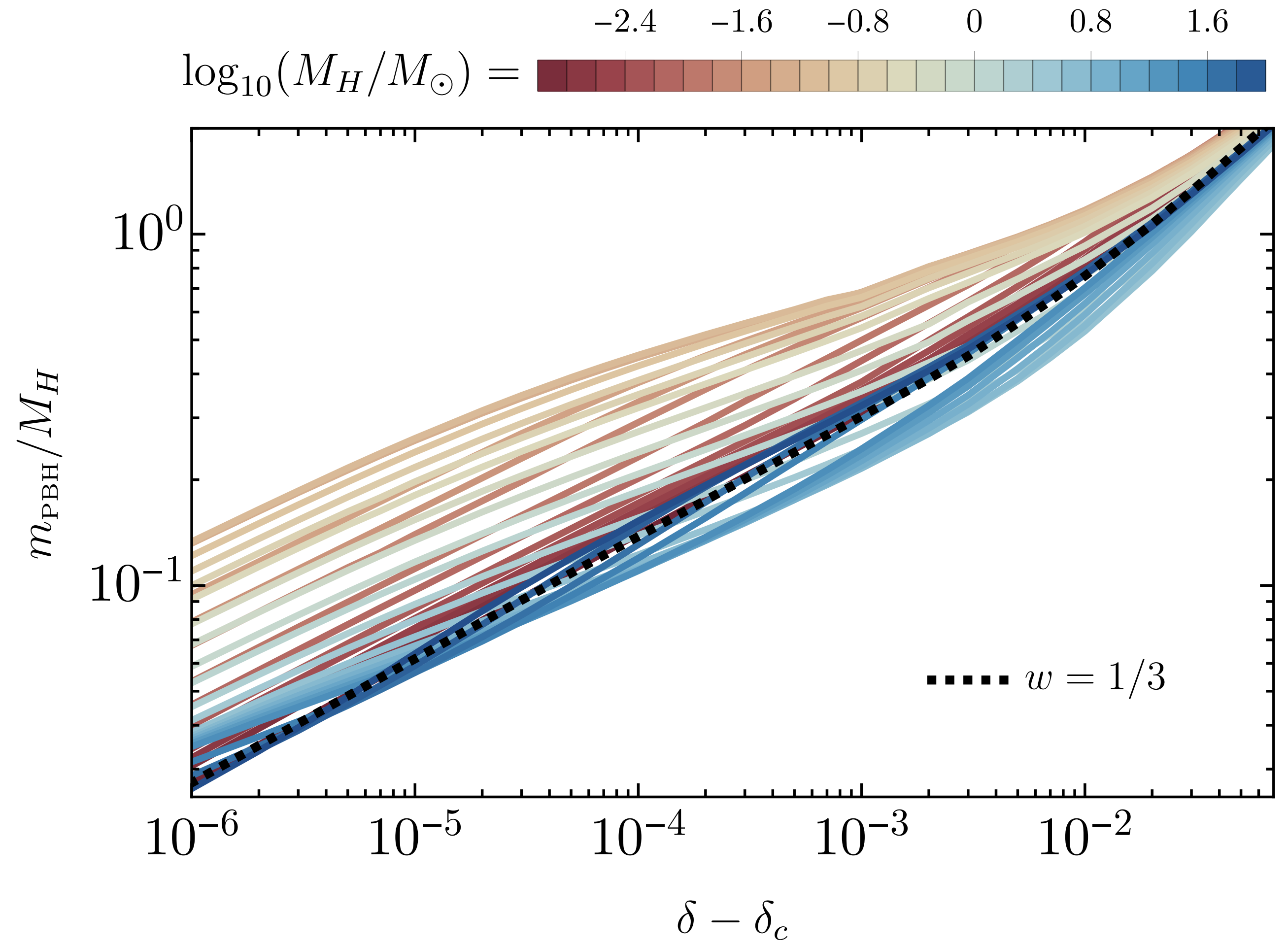
↓

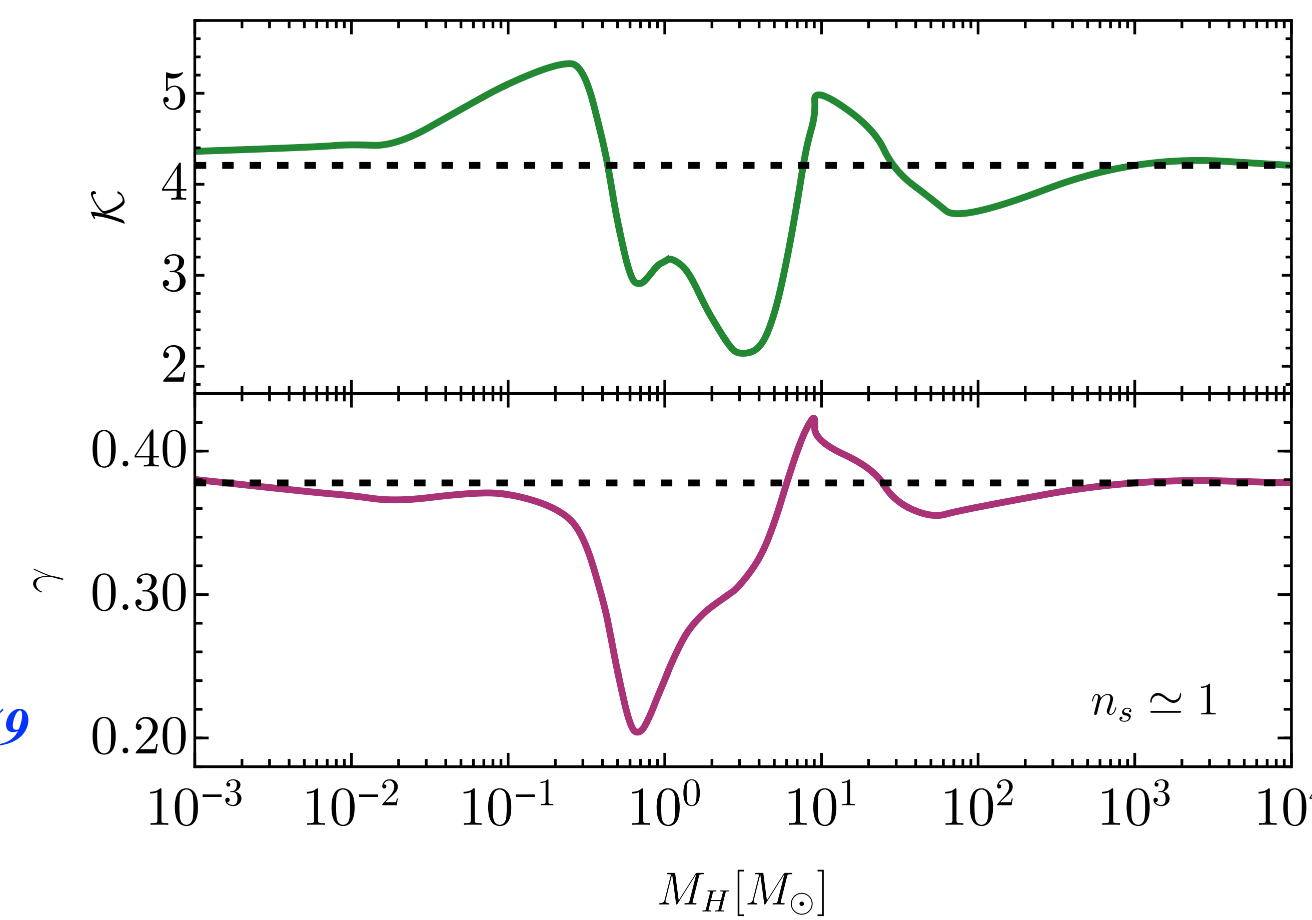
$$\alpha \simeq 3$$

$$M_{PBH} = \mathcal{K}(\delta - \delta_c)^\gamma M_H$$

$$\delta_c(M_H), \gamma(M_H), \mathcal{K}(M_H)$$

G. Franciolini, IM, P.Pani, A urbano - arXiv:2209.05959





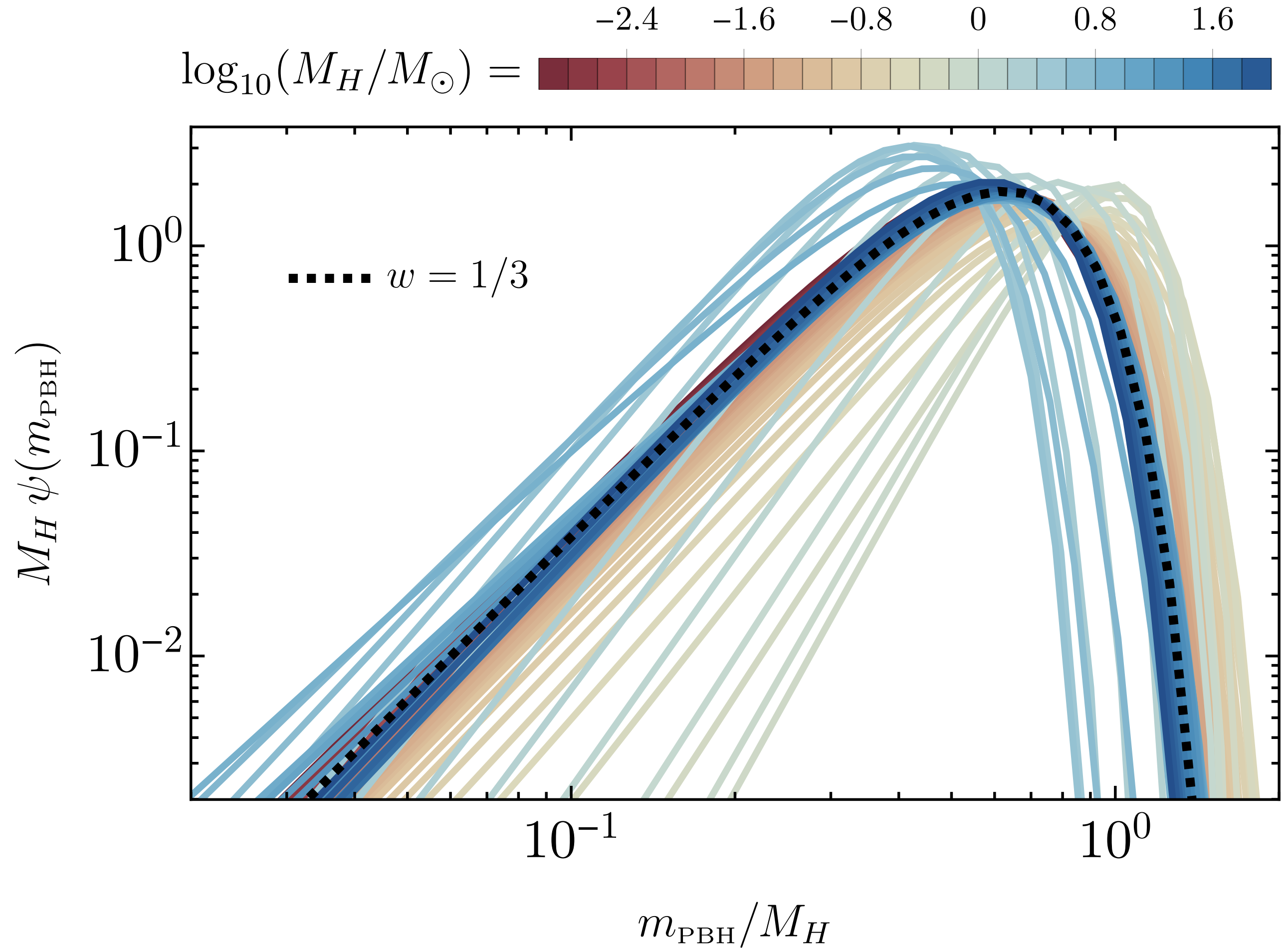
PBH mass distribution - QCD

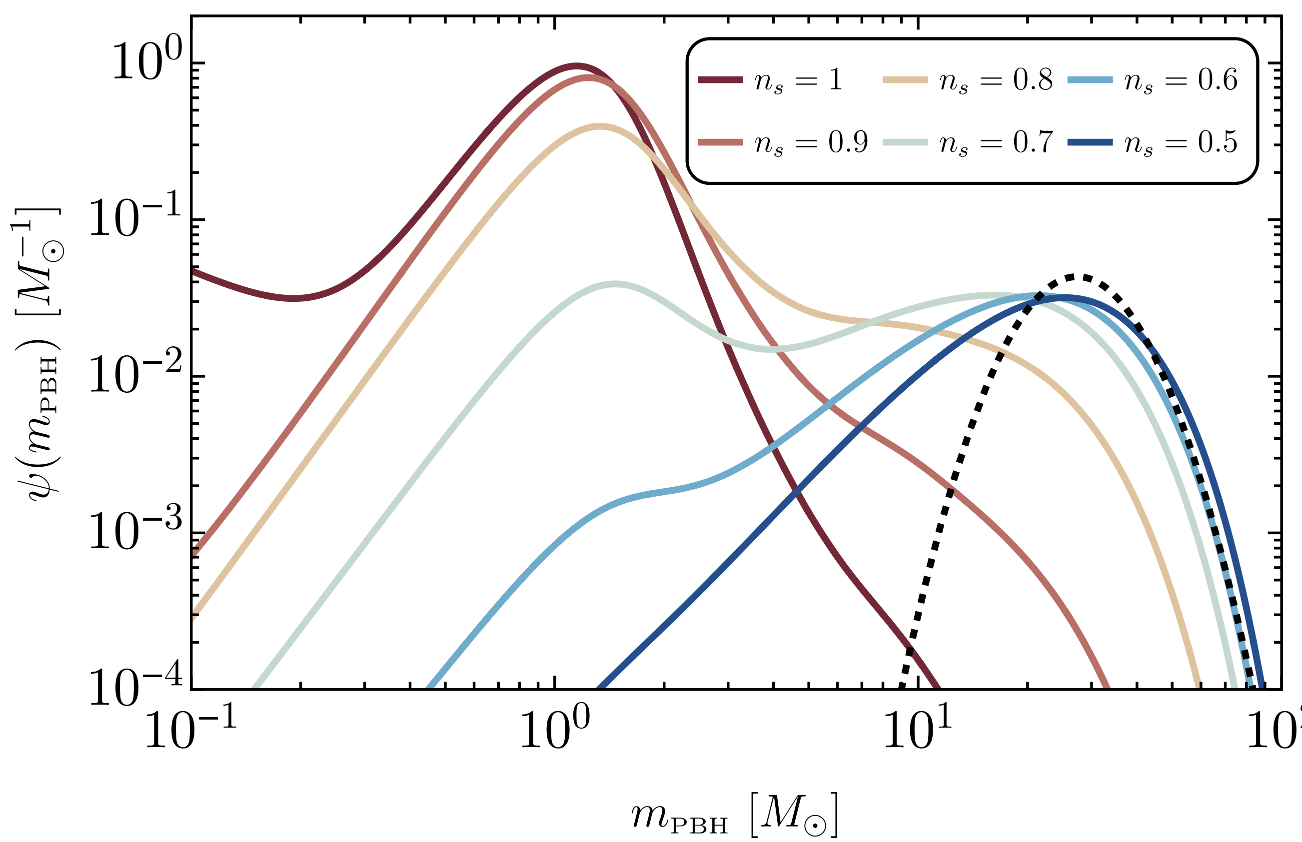
Mass Function $\psi(m_{PBH})$: fraction of PBHs with mass in the infinitesimal interval of m_{PBH}

$$\psi(M_{\rm PBH}) = \frac{1}{\Omega_{\rm PBH}} \frac{d\Omega_{\rm PBH}}{dM_{\rm PBH}}$$

$$\int dm_{\rm PBH}\ \psi(m_{\rm PBH}) = 1$$

G. Franciolini, IM, P.Pani, A urbano - arXiv:2209.05959





Conclusions

- The **non linear threshold for PBH** and **the mass spectrum** could be fully computed from the **shape of the power spectrum of cosmological perturbations**, making relativistic numerical simulations.

- A softening of the equation of state **(QCD) significantly enhances the formation of PBHs**, with a **mass distribution peaked between 1 and 2 solar masses** (the range of heavy NSs and light BHs).

- This could gives **a sub-population of BH mergers compatible with the LVK catalog**, explaining mass gap events as **GW190814**.

- Our analysis predicts a **constraint on the abundance of DM in PBHs formed during the QCD (up to 0.1%)**, compatible with the current observational constraints.

- **A large enough feature of the power spectrum could account for all dark matter in PBHs in the asteroidal mass range (USR inflation models)**.