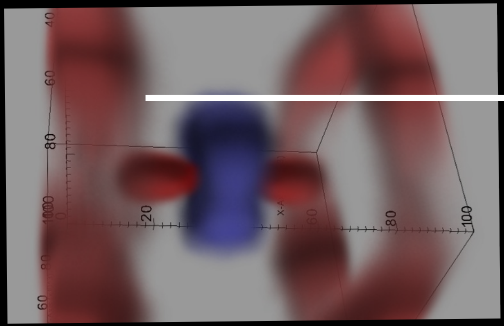
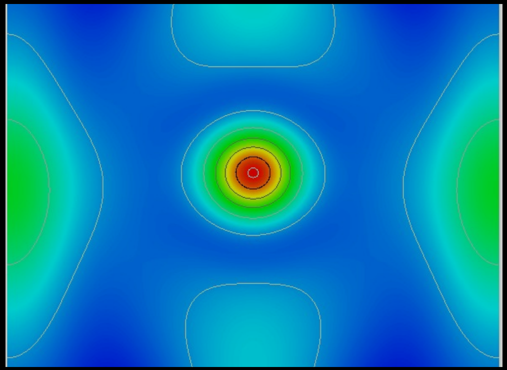


Graviational Waves from the inhomogeneous pre-inflationar Era:

Cristian Joana - UCLouvain (CURL)

with S. Clesse and C. Ringeval

Belgian GWs seminar 27/10/2020



fnrs LA LIBERTÉ DE CHERCHER

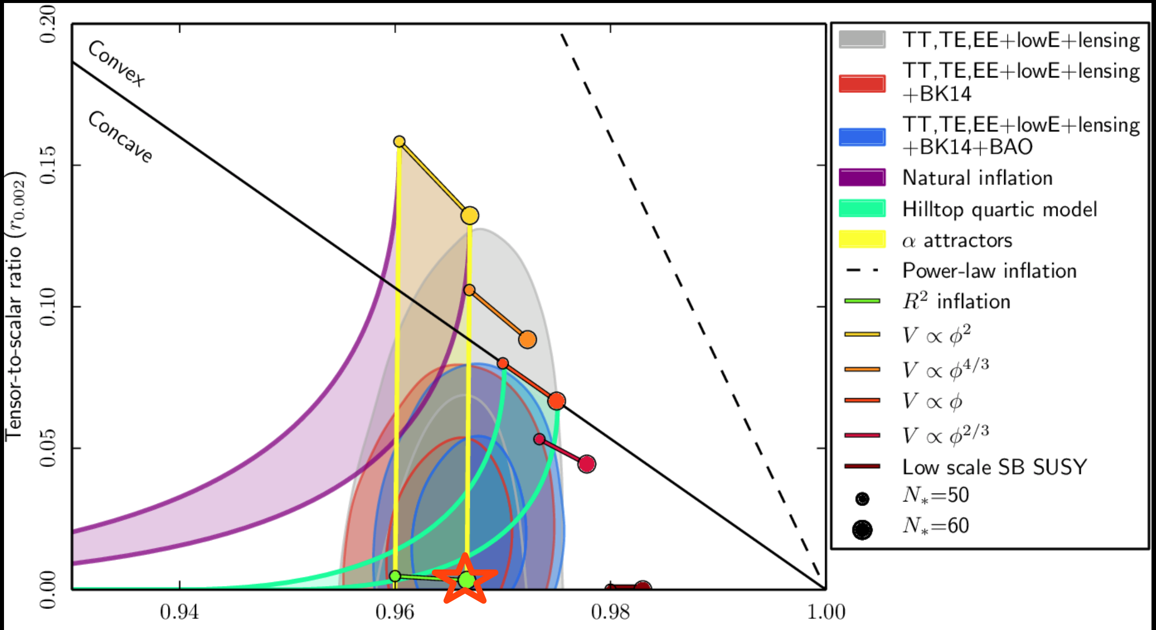
UCLouvain

Our main Goals

- Check the robustness of inflation to its initial conditions.
- Explore the non-linear dynamics from highly inhomogeneous cosmologies. While considering only a single scalar field.
- Learn and derive some conclusions from it.

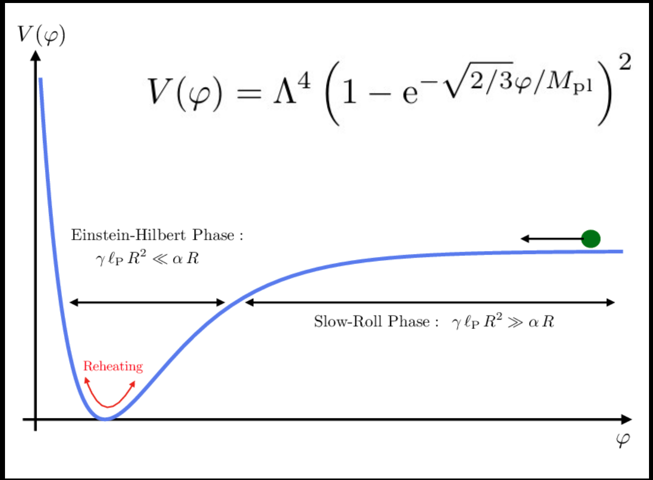
[Disclaimer]

- There are many models of inflation, we didn't test them all.
  [e.g. see Encyclopædia Inflationaris, arXiv: 1303.3787]



Planck '18, arxiv:1807.06211

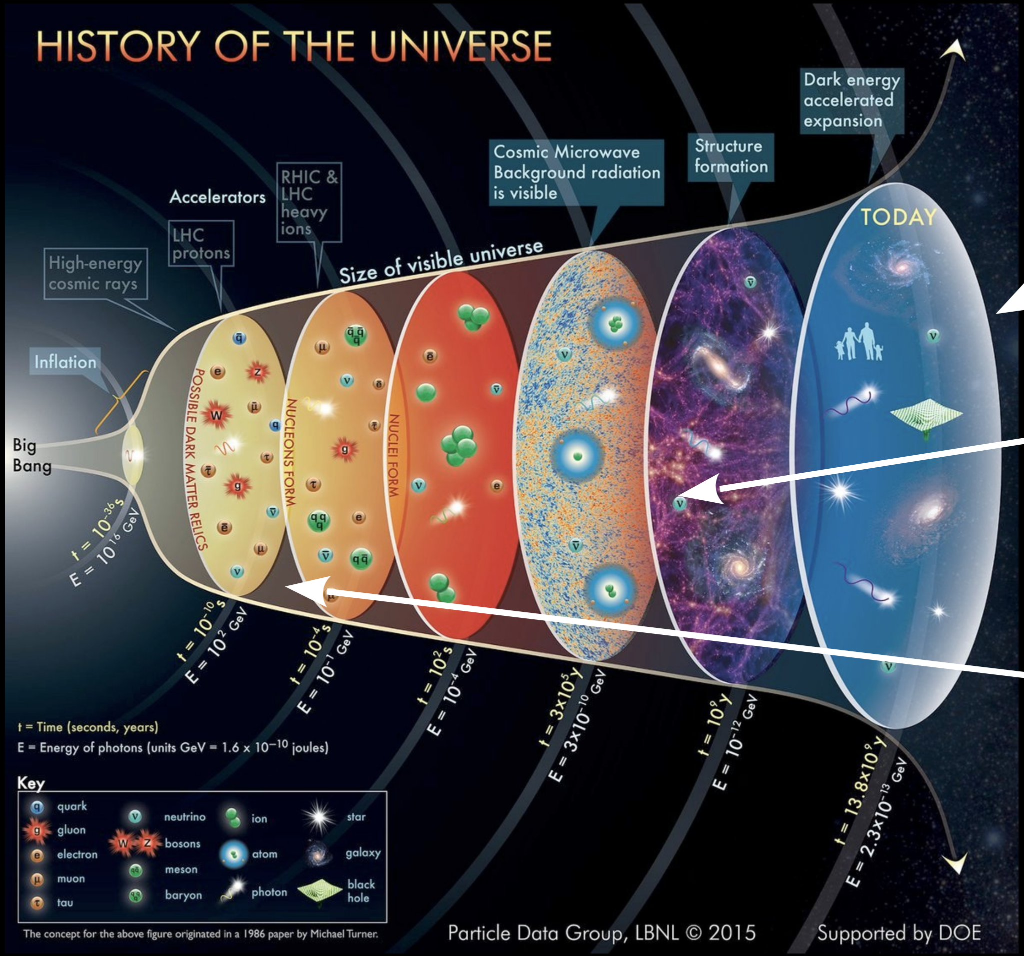
Starobinsky Inflation



Outline

- Brief introduction to inflation
- How we simulate it?
- Results
- Future prospects

I.C. for inflation:

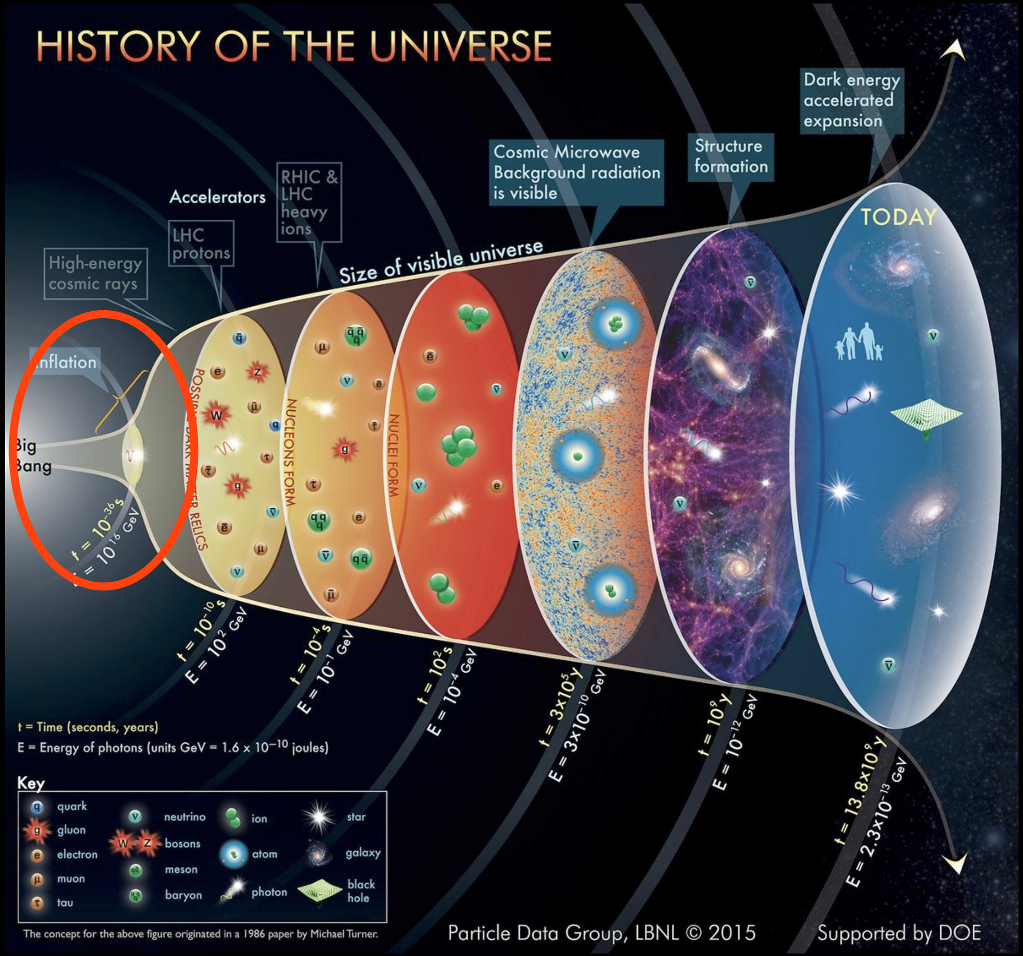


Today: Vacuum (DE) domination

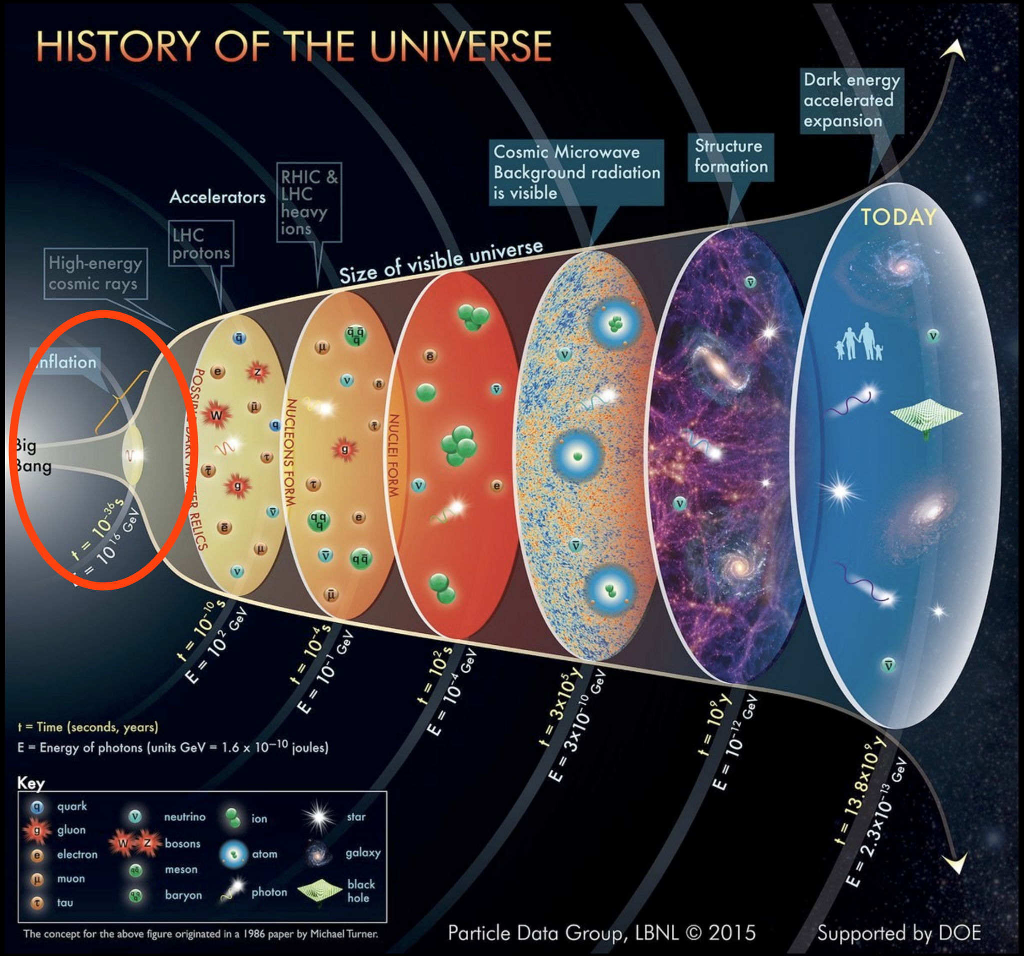
Matter domination

Radiation domination

I.C. for inflation:

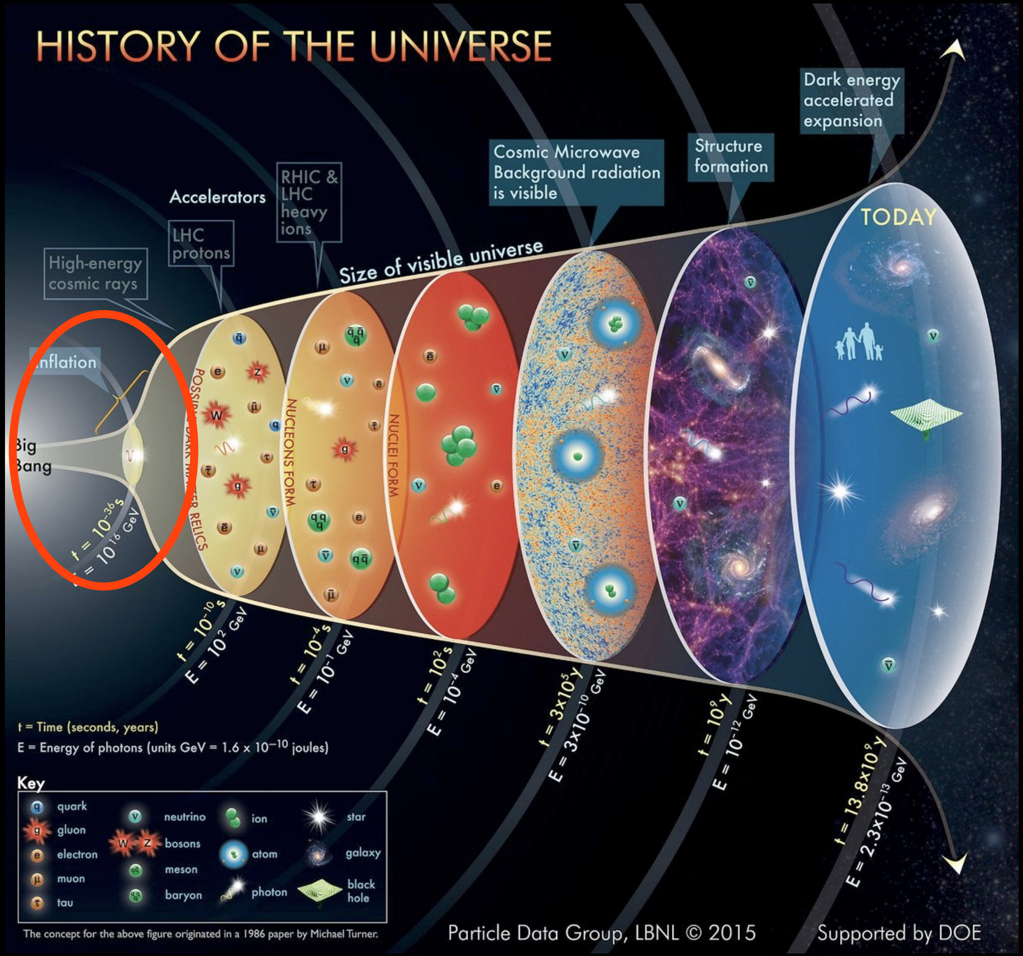


I.C. for inflation:



- Inflation is a hypothetical phase of exponential expansion of the Universe

I.C. for inflation:



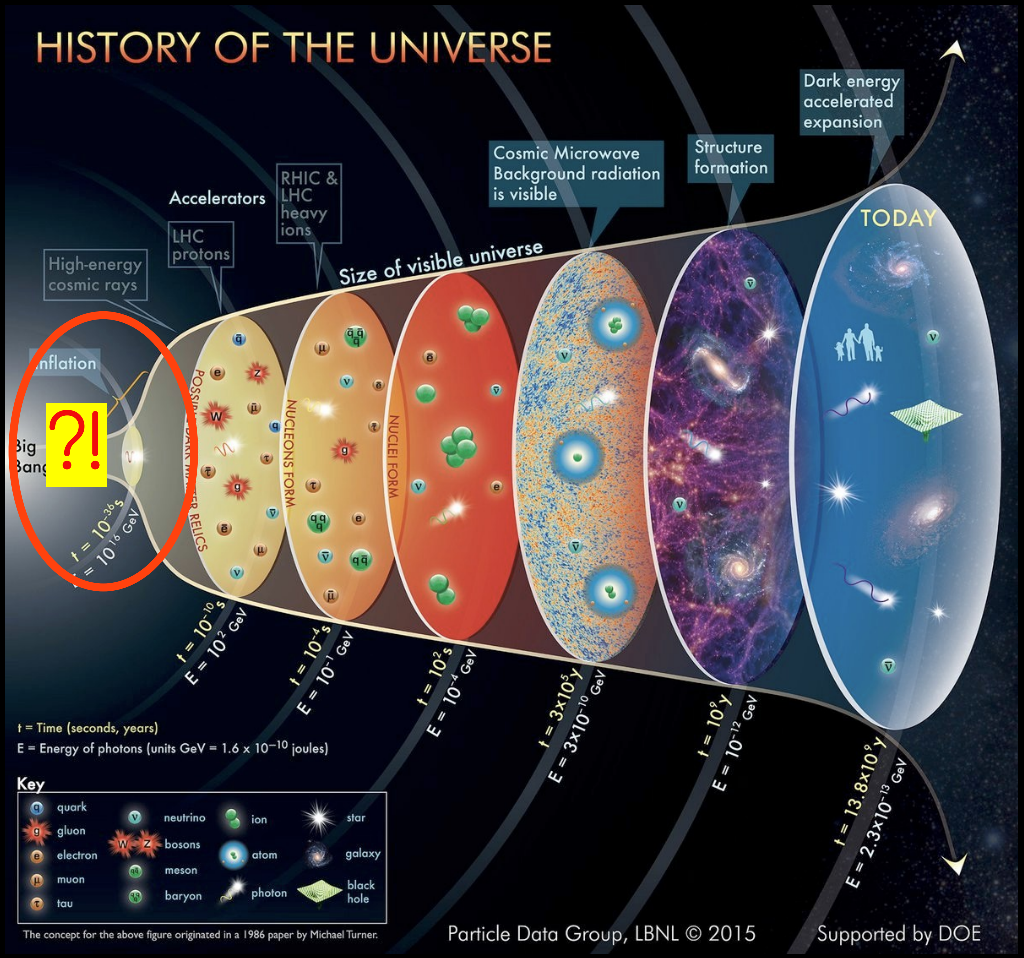
- Inflation is a hypothetical phase of exponential expansion of the Universe

- Naturally solves Big Bang problems:

  → horizon problem
  → flatness problem
  → lack BB relics

  → Provides seeds for LSS

I.C. for inflation:



- Inflation is a hypothetical phase of exponential expansion of the Universe

- Naturally solves Big Bang problems:

  → horizon problem
  → flatness problem
  → lack BB relics

  → Provides seeds for LSS

IF (!) Inflation is natural.

I.C. for inflation:

- The main idea of inflation

FLRW :

$$\frac{\ddot{a}}{a} = -\frac{4}{3}\pi(\rho + 3p)$$

$$\ddot{a} > 0 \rightarrow \omega = \frac{p_\varphi}{\rho_\varphi} < -\frac{1}{3}$$

- Single scalar field inflation:

$$\rho_\varphi = \frac{1}{2}\dot{\varphi}^2 + V(\varphi) + \frac{1}{2}\frac{(\nabla\varphi)^2}{a^2}$$
$$p_\varphi = \frac{1}{2}\dot{\varphi}^2 - V(\varphi) - \frac{1}{6}\frac{(\nabla\varphi)^2}{a^2}$$

I.C. for inflation:

- The main idea of inflation

FLRW :

$$\frac{\ddot{a}}{a} = -\frac{4}{3}\pi(\rho + 3p)$$

$$\ddot{a} > 0 \rightarrow \omega = \frac{p_\varphi}{\rho_\varphi} < -\frac{1}{3}$$

- Single scalar field inflation:

$$\rho_\varphi = \frac{1}{2}\dot{\varphi}^2 + V(\varphi) + \frac{1}{2}\frac{(\nabla\varphi)^2}{a^2}$$

$$p_\varphi = \frac{1}{2}\dot{\varphi}^2 - V(\varphi) - \frac{1}{6}\frac{(\nabla\varphi)^2}{a^2}$$

Ok! ($\omega \rightarrow -1$)

I.C. for inflation:

- The main idea of inflation

FLRW : ?!

$$\frac{\ddot{a}}{a} = -\frac{4}{3}\pi(\rho + 3p)$$ + GR... $$\ddot{a} > 0 \rightarrow \omega = \frac{p_\varphi}{\rho_\varphi} < -\frac{1}{3}$$

- Single scalar field inflation:

$$\rho_\varphi = \frac{1}{2}\dot{\varphi}^2 + V(\varphi) + \frac{1}{2}\frac{(\nabla\varphi)^2}{a^2}$$
$$p_\varphi = \frac{1}{2}\dot{\varphi}^2 - V(\varphi) - \frac{1}{6}\frac{(\nabla\varphi)^2}{a^2}$$

Ok! (ω → -1)

What if not in FLRW?
i.e. gradients?
kinetics?
Unforeseen GR effects?
Fine-tuned?

I.C. for inflation:

- The main idea of inflation

FLRW : ?!

$$\frac{\ddot{a}}{a} = -\frac{4}{3}\pi(\rho + 3p)$$ + GR... $$\ddot{a} > 0 \rightarrow \omega = \frac{p_\varphi}{\rho_\varphi} < -\frac{1}{3}$$

- Single scalar field inflation:

$$\rho_\varphi = \frac{1}{2}\dot{\varphi}^2 + V(\varphi) + \frac{1}{2}\frac{(\nabla\varphi)^2}{a^2}$$
$$p_\varphi = \frac{1}{2}\dot{\varphi}^2 - V(\varphi) - \frac{1}{6}\frac{(\nabla\varphi)^2}{a^2}$$

Does inflation still it work?

I.C. for inflation:

- The main idea of inflation

FLRW : ?!

$$\frac{\ddot{a}}{a} = -\frac{4}{3}\pi(\rho + 3p)$$ + GR… $$\ddot{a} > 0 \rightarrow \omega = \frac{p_\varphi}{\rho_\varphi} < -\frac{1}{3}$$

- Single scalar field inflation:

$$\rho_\varphi = \frac{1}{2}\dot{\varphi}^2 + V(\varphi) + \frac{1}{2}\frac{(\nabla\varphi)^2}{a^2}$$
$$p_\varphi = \frac{1}{2}\dot{\varphi}^2 - V(\varphi) - \frac{1}{6}\frac{(\nabla\varphi)^2}{a^2}$$

Does inflation still it work?

YES!
It's basically unavoidable.

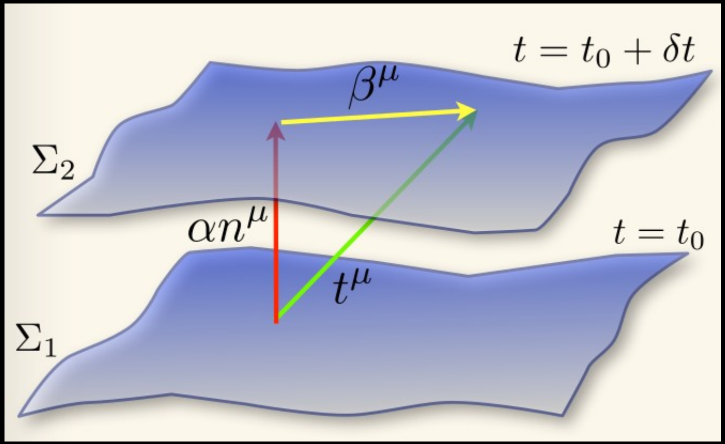
GR simulations with scalar fields :

- What we do them?

GR simulations with scalar fields :

- How we do it?

→ 3+1 decomposition Einstein eq.



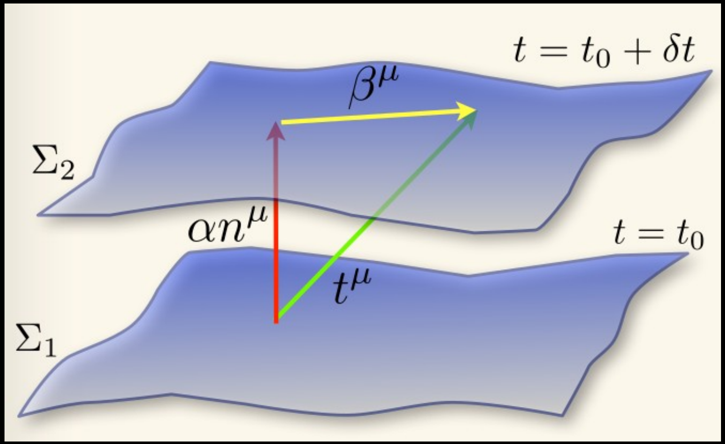
$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi T_{\mu\nu}$$

GR simulations with scalar fields :

- How we do it?

→ 3+1 decomposition Einstein eq.



$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

→ Constrains Equations (ICs)

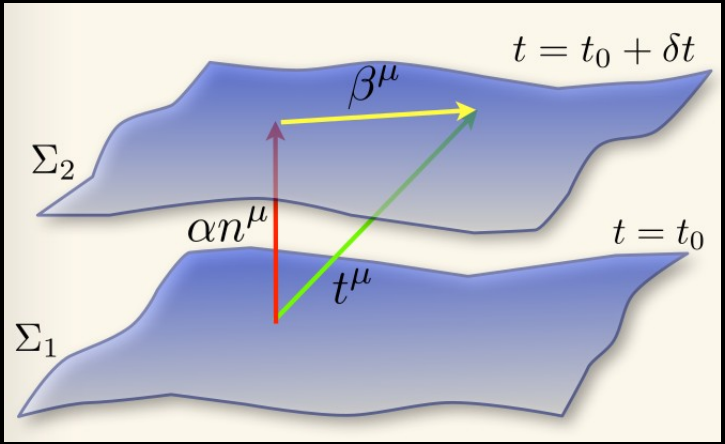
$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi T_{\mu\nu}$$

GR simulations with scalar fields :

- How we do it?

→ 3+1 decomposition Einstein eq.

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$



→ Constrains Equations (ICs)

→ Evolution equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi T_{\mu\nu}$$

$$\partial_t\chi = \frac{2}{3}\,\alpha\,\chi\,K - \frac{2}{3}\,\chi\,\partial_k\beta^k + \beta^k\,\partial_k\chi\ ,$$

$$\partial_t\tilde{\gamma}_{ij} = -2\,\alpha\,\tilde{A}_{ij} + \tilde{\gamma}_{ik}\,\partial_j\beta^k + \tilde{\gamma}_{jk}\,\partial_i\beta^k - \frac{2}{3}\,\tilde{\gamma}_{ij}\,\partial_k\beta^k + \beta^k\,\partial_k\tilde{\gamma}_{ij}\ ,$$

$$\partial_t K = -\gamma^{ij} D_i D_j\alpha + \alpha\left(\tilde{A}_{ij}\tilde{A}^{ij} + \frac{1}{3}K^2\right) + \beta^i\partial_i K + 4\pi\,\alpha(\rho + S)\ ,$$

$$\partial_t\tilde{A}_{ij} = \left[-D_iD_j\alpha + \chi\alpha\left(R_{ij} - 8\pi\ S_{ij}\right)\right]^{\rm TF} + \alpha(K\tilde{A}_{ij} - 2\tilde{A}_{il}\,\tilde{A}^l{}_j) + \tilde{A}_{ik}\,\partial_j\beta^k + \tilde{A}_{jk}\,\partial_i\beta^k - \frac{2}{3}\,\tilde{A}_{ij}\,\partial_k\beta^k + \beta^k\,\partial_k\tilde{A}_{ij}\ ,$$

$$\partial_t\tilde{\Gamma}^i = 2\,\alpha\left(\tilde{\Gamma}^i_{jk}\,\tilde{A}^{jk} - \frac{2}{3}\,\tilde{\gamma}^{ij}\partial_j K - \frac{3}{2}\,\tilde{A}^{ij}\,\frac{\partial_j\chi}{\chi}\right) - 2\,\tilde{A}^{ij}\,\partial_j\alpha + \beta^k\partial_k\tilde{\Gamma}^i + \tilde{\gamma}^{jk}\partial_j\partial_k\beta^i + \frac{1}{3}\,\tilde{\gamma}^{ij}\partial_j\partial_k\beta^k + \frac{2}{3}\,\tilde{\Gamma}^i\,\partial_k\beta^k - \tilde{\Gamma}^k\partial_k\beta^i - 16\pi\,\alpha\,\tilde{\gamma}^{ij}\,S_j\ ,$$

$$\begin{aligned}\partial_t\alpha &= -\eta_\alpha\alpha K + \beta^i\partial_i\alpha\ ,\\ \partial_t\beta^i &= B^i\,,\\ \partial_t B^i &= \frac{3}{4}\,\partial_t\tilde{\Gamma}^i - \eta_B\,B^i\ ,\end{aligned}$$

$$T_{\mu\nu} = \partial_\mu\varphi\,\partial_\nu\varphi - \frac{1}{2}g_{\mu\nu}\,\partial_\lambda\varphi\,\partial^\lambda\varphi - g_{\mu\nu}V(\varphi)$$

$$\begin{aligned}\partial_t\varphi &= \alpha\Pi_{\rm M} + \beta^i\partial_i\varphi\ ,\\ \partial_t\Pi_{\rm M} &= \beta^i\partial_i\Pi_{\rm M} + \alpha\partial_i\partial^i\varphi + \partial_i\varphi\,\partial^i\alpha + \alpha\left(K\Pi_{\rm M} - \gamma^{ij}\Gamma^k_{ij}\partial_k\varphi - V'(\varphi)\right)\ ,\end{aligned}$$

$$\begin{aligned}\mathcal{H} &= R + K^2 - K_{ij}K^{ij} - 16\pi\rho = 0\,,\\ \mathcal{M}_i &= D^j(K_{ij} - \gamma_{ij}K) - 8\pi S_i = 0\,.\end{aligned}$$

GRChombo

$$\partial_t \chi = \frac{2}{3}\,\alpha\,\chi\,K - \frac{2}{3}\,\chi\,\partial_k\beta^k + \beta^k\,\partial_k\chi\ ,$$

$$\partial_t \tilde{\gamma}_{ij} = -2\,\alpha\,\tilde{A}_{ij} + \tilde{\gamma}_{ik}\,\partial_j\beta^k + \tilde{\gamma}_{jk}\,\partial_i\beta^k - \frac{2}{3}\,\tilde{\gamma}_{ij}\,\partial_k\beta^k + \beta^k\,\partial_k\tilde{\gamma}_{ij}\ ,$$

$$\partial_t K = -\gamma^{ij}D_iD_j\alpha + \alpha\left(\tilde{A}_{ij}\tilde{A}^{ij} + \frac{1}{3}K^2\right) + \beta^i\partial_i K + 4\pi\,\alpha(\rho + S)\ ,$$

$$\partial_t\tilde{A}_{ij} = \left[-D_iD_j\alpha + \chi\alpha\left(R_{ij} - 8\pi\ S_{ij}\right)\right]^{\mathrm{TF}} + \alpha(K\tilde{A}_{ij} - 2\tilde{A}_{il}\,\tilde{A}^l{}_j) + \tilde{A}_{ik}\,\partial_j\beta^k + \tilde{A}_{jk}\,\partial_i\beta^k - \frac{2}{3}\,\tilde{A}_{ij}\,\partial_k\beta^k + \beta^k\,\partial_k\tilde{A}_{ij}\ ,$$

$$\partial_t\tilde{\Gamma}^i = 2\,\alpha\left(\tilde{\Gamma}^i_{jk}\,\tilde{A}^{jk} - \frac{2}{3}\,\tilde{\gamma}^{ij}\partial_j K - \frac{3}{2}\,\tilde{A}^{ij}\,\frac{\partial_j\chi}{\chi}\right) - 2\,\tilde{A}^{ij}\,\partial_j\alpha + \beta^k\partial_k\tilde{\Gamma}^i + \tilde{\gamma}^{jk}\partial_j\partial_k\beta^i + \frac{1}{3}\,\tilde{\gamma}^{ij}\partial_j\partial_k\beta^k + \frac{2}{3}\,\tilde{\Gamma}^i\,\partial_k\beta^k - \tilde{\Gamma}^k\partial_k\beta^i - 16\pi\,\alpha\,\tilde{\gamma}^{ij}\,S_j\ ,$$

$$\begin{aligned}\partial_t\alpha &= -\eta_\alpha\alpha K + \beta^i\partial_i\alpha\ ,\\ \partial_t\beta^i &= B^i\,,\\ \partial_t B^i &= \frac{3}{4}\,\partial_t\tilde{\Gamma}^i - \eta_B\,B^i\ ,\end{aligned}$$

$$T_{\mu\nu} = \partial_\mu\varphi\,\partial_\nu\varphi - \frac{1}{2}g_{\mu\nu}\,\partial_\lambda\varphi\,\partial^\lambda\varphi - g_{\mu\nu}V(\varphi)$$

$$\begin{aligned}\partial_t\varphi &= \alpha\Pi_{\mathrm{M}} + \beta^i\partial_i\varphi\ ,\\ \partial_t\Pi_{\mathrm{M}} &= \beta^i\partial_i\Pi_{\mathrm{M}} + \alpha\partial_i\partial^i\varphi + \partial_i\varphi\,\partial^i\alpha + \alpha\left(K\Pi_{\mathrm{M}} - \gamma^{ij}\Gamma^k_{ij}\partial_k\varphi - V'(\varphi)\right)\ ,\end{aligned}$$

$$\begin{aligned}\mathcal{H} &= R + K^2 - K_{ij}K^{ij} - 16\pi\rho = 0\,,\\ \mathcal{M}_i &= D^j(K_{ij} - \gamma_{ij}K) - 8\pi S_i = 0\,.\end{aligned}$$

Full GR code! →

GRChombo

Simulation results :

Simulation results :

- Interesting variables for inflation:

$$\frac{\ddot{a}}{a} = -\frac{1}{3}\alpha^3 \left(\aleph + 4\pi\rho \left(\frac{1}{3} + \omega_\varphi \right) \right) + (\text{gauge terms})$$

→ The necessary conditions for an accelerated expansion of the universe :

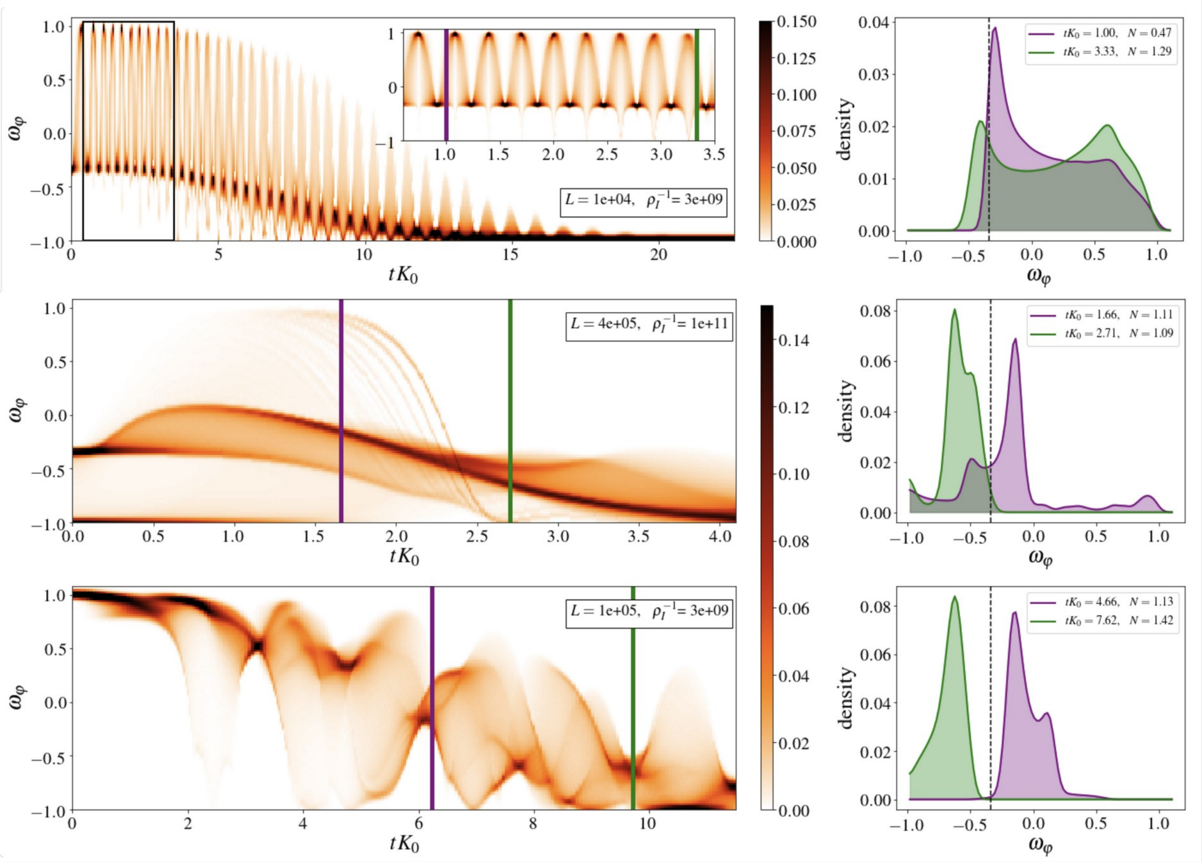
Energy density of the metric

$$\omega_\varphi < -\frac{1}{3} \qquad \aleph < \left| 12\pi\rho \left(\frac{1}{3} + \omega_\varphi \right) \right| \qquad \aleph \propto \dot{h}_{ij}\dot{h}_{ij}$$

Simulation results :

- Dynamical Equation of State :

$$\omega_\varphi(t) \simeq \begin{cases} 1 & \rightarrow \quad \rho \simeq \rho_{\rm kin} \\ -1/3 & \rightarrow \quad \rho \simeq \rho_{\rm grad} \\ -1 & \rightarrow \quad \rho \simeq \rho_{\rm V} \end{cases}$$



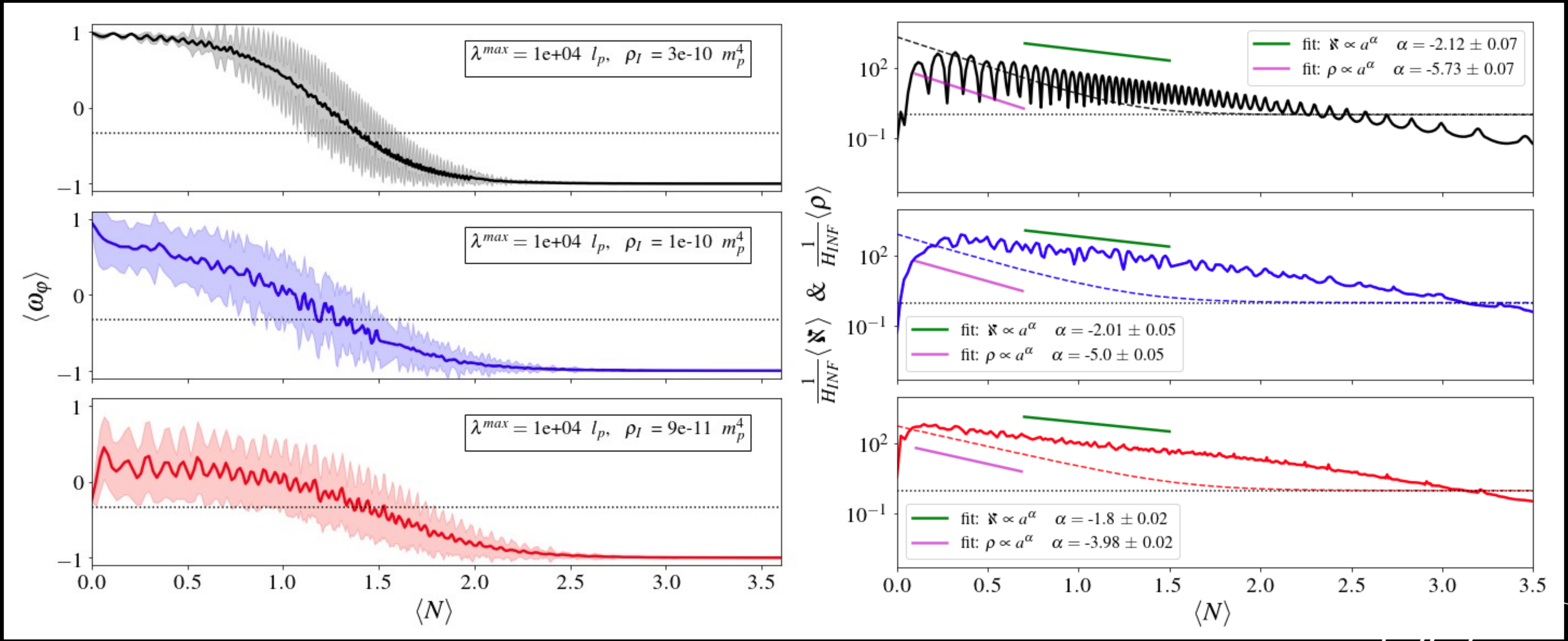
← gradients ICs (sub-Hubble)

← gradients ICs (super-Hubble)

← Kination ICs (super-Hubble)

Simulation results :

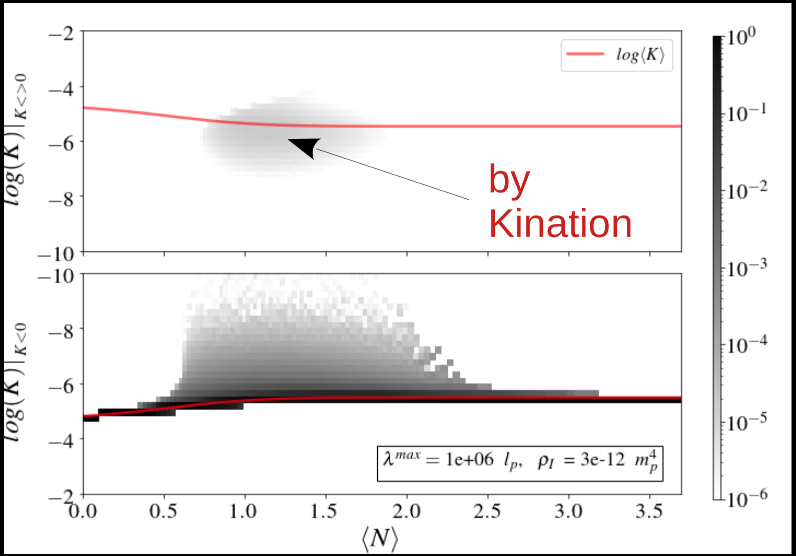
- Evolution of $\omega_\varphi,\ \rho,\ \aleph\ (\mathrm{ECMs})$:



Simulation results :

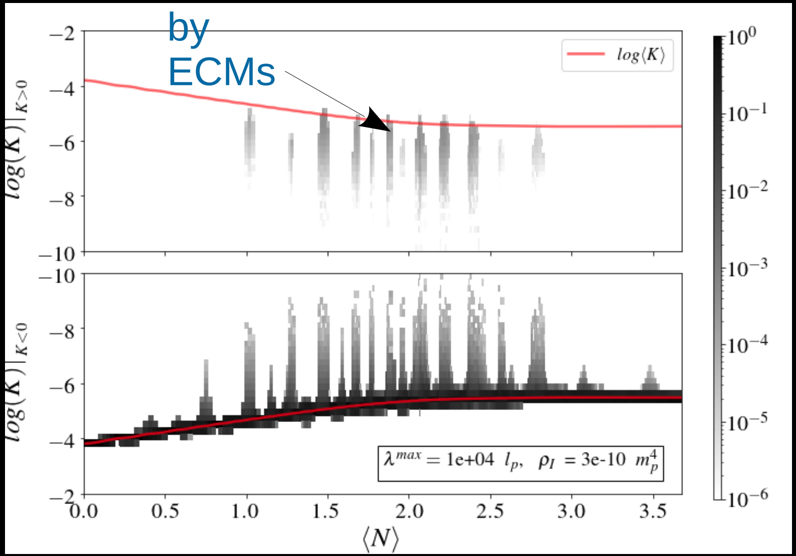
- Pre-Inflation BHs

$$-\frac{1}{3}K \approx \frac{\dot{a}}{a}$$



Super-Hubble
(confirmed by the AH-finder)

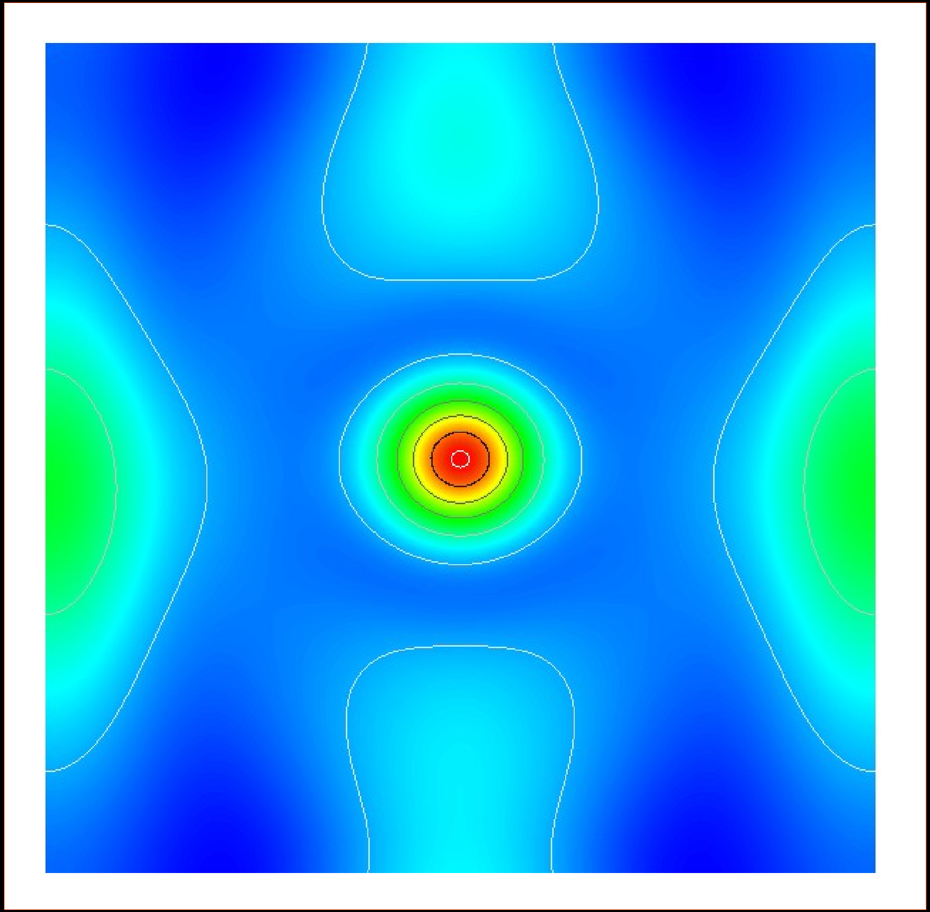
$M_{BH} \sim 10 - 10^5\ M_{Pl}$



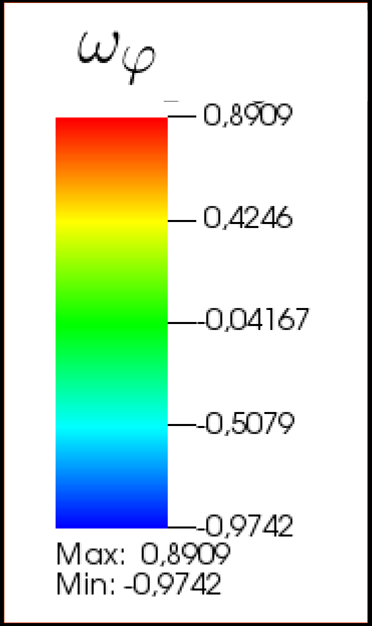
Sub-Hubble
(not found by the AH-finder)

$M_{BH} \overset{(?)}{\sim} 1 - 10^2\ M_{Pl}$

Simulation results :



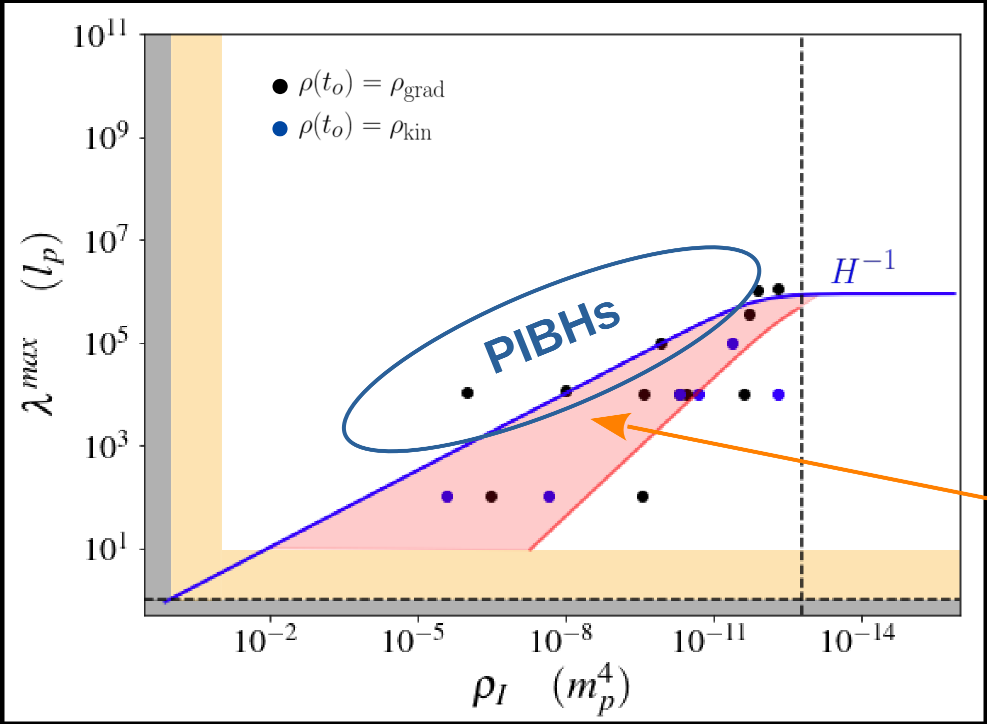
$M_{BH} \approx 2 \cdot 10^4 \; M_P$



Do PIBHs facilitate inflation onset?

Take home message

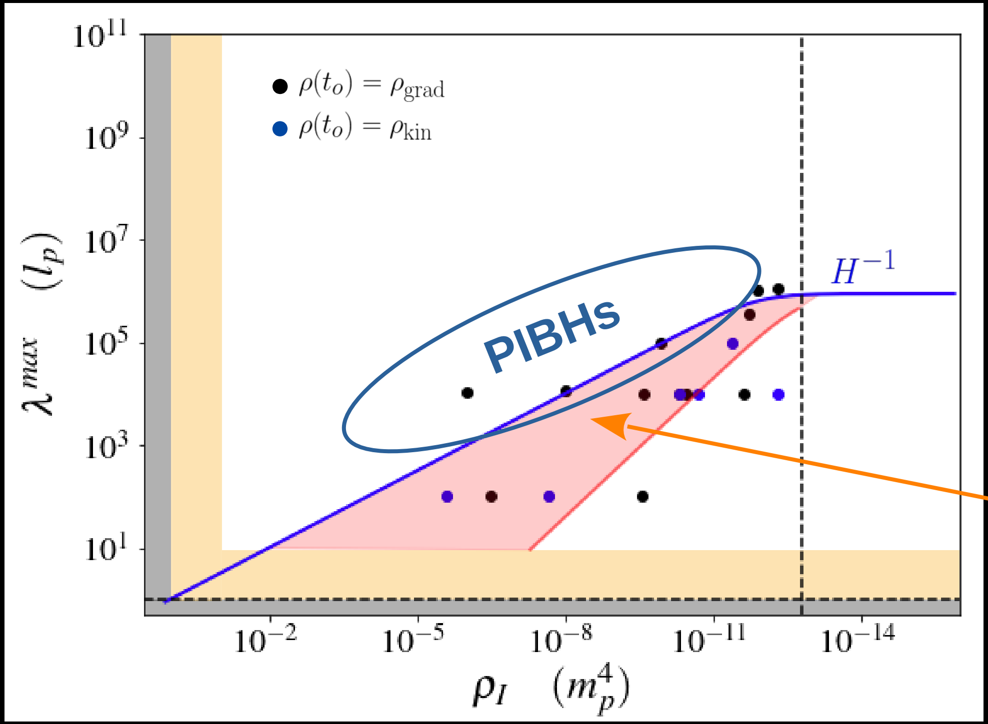
- Simualted many ICs, different scenarios :



Generate ECMs!
(vector&tensor metric modes)

Take home message

- Simualted many ICs, different scenarios :



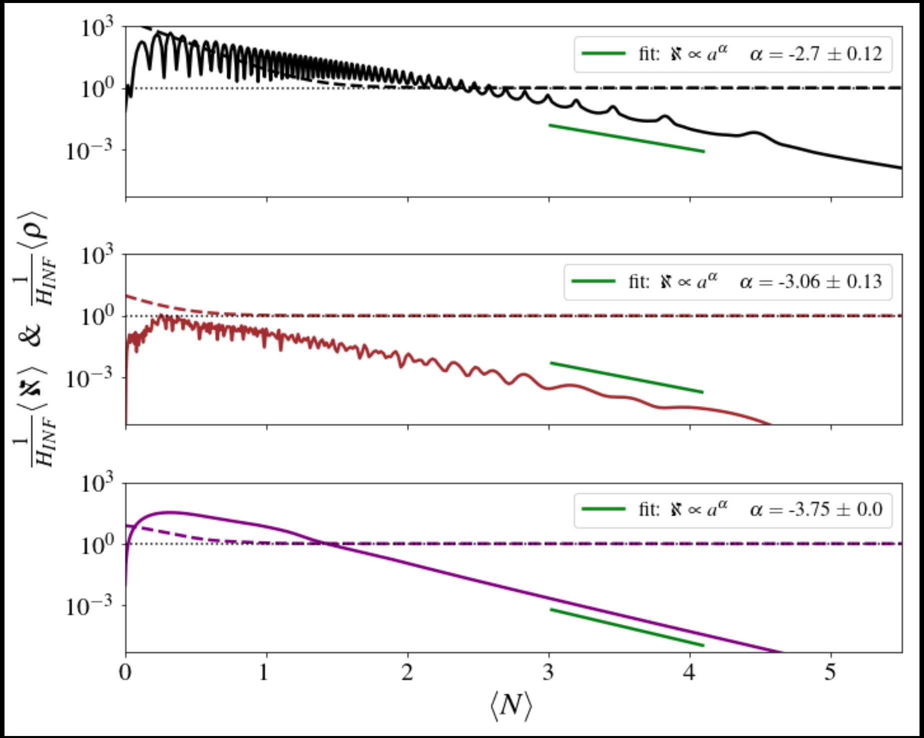
→ Inflation is robust

→ Formation of ECMs and PIBHs

Generate ECMs!
(vector&tensor metric modes)

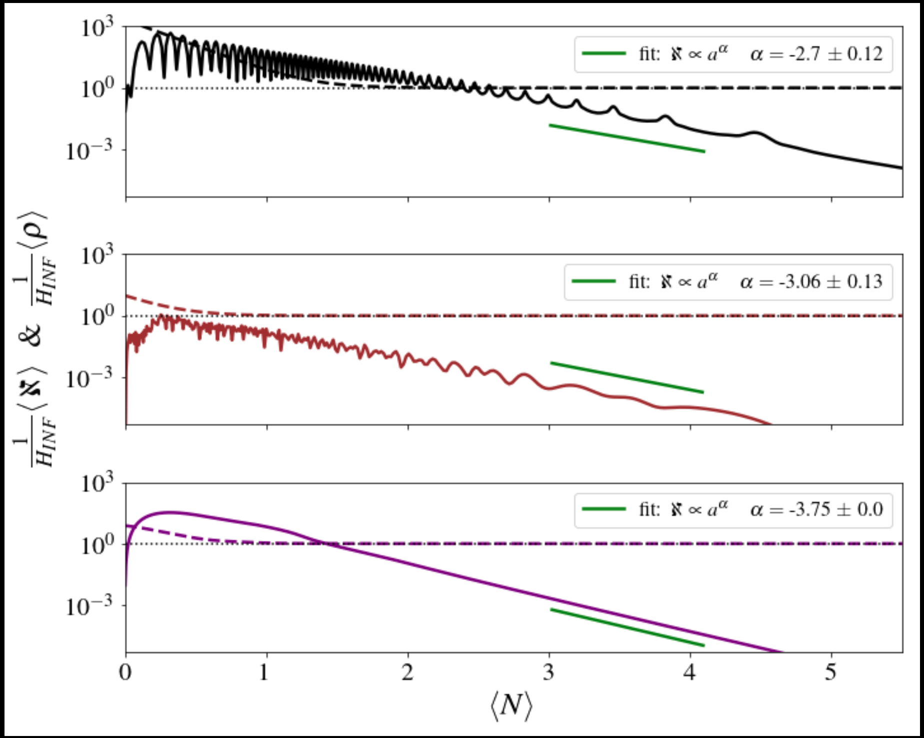
Observables?

- Decay rates .... $\aleph \propto a^{-4}$

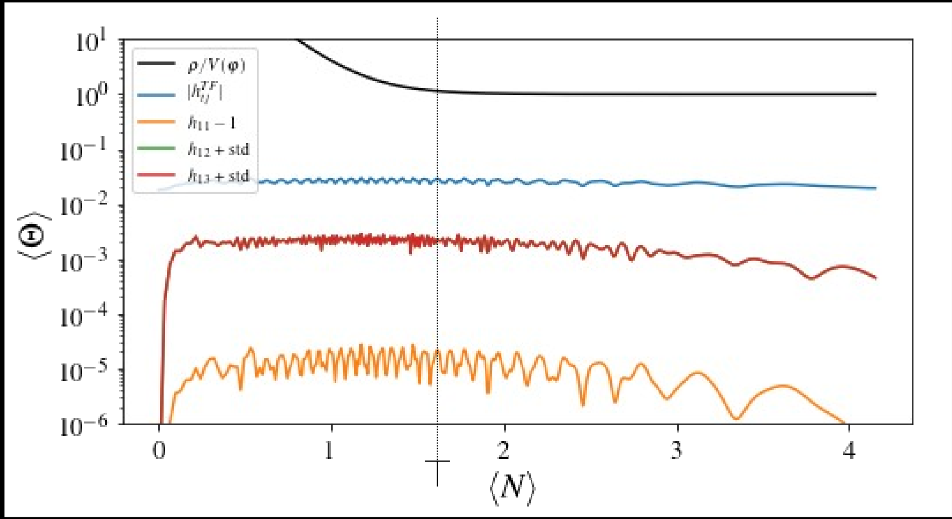


Observables?

- Decay rates …. $\aleph \propto a^{-4}$

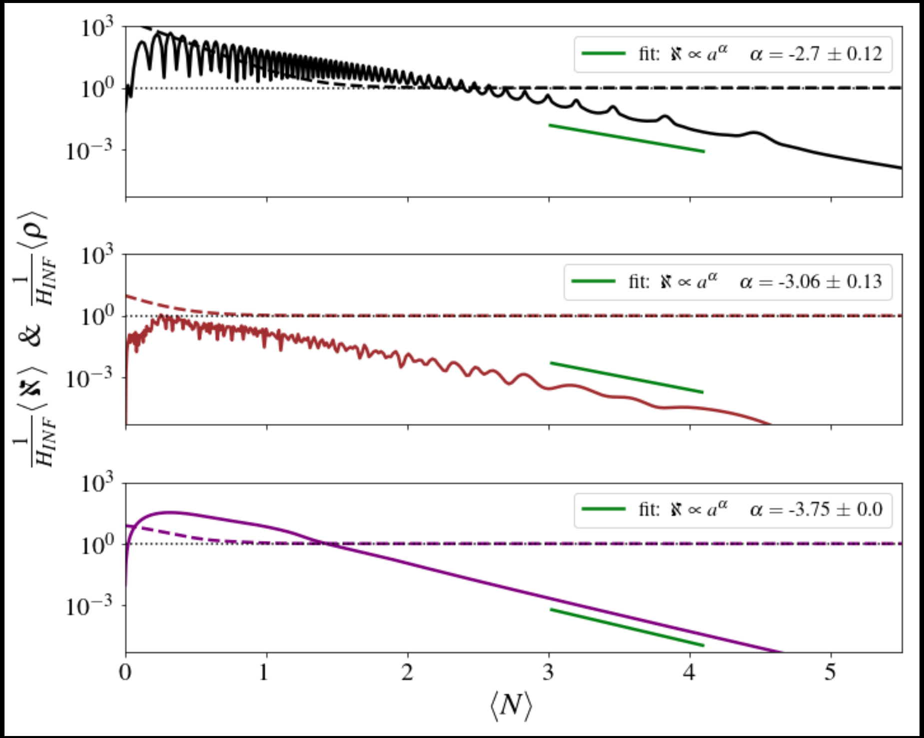


& metric components?

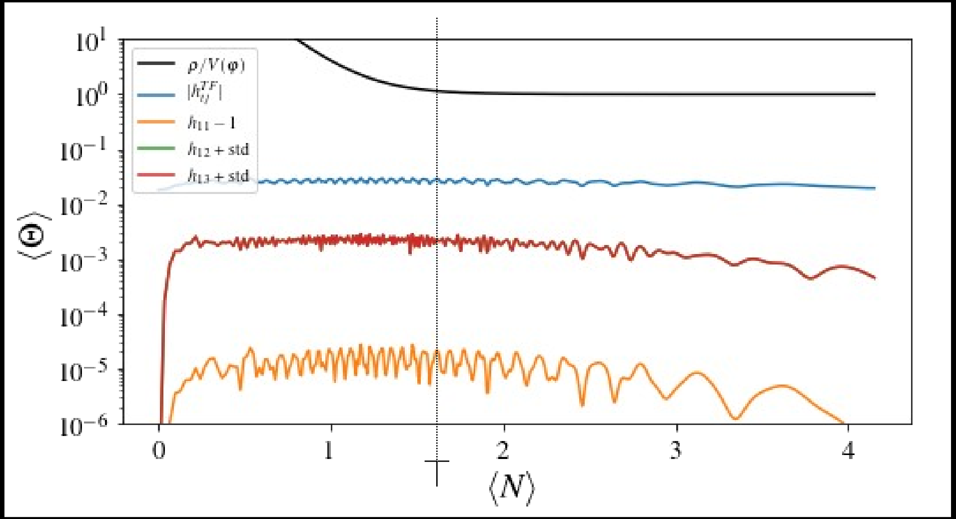


Observables?

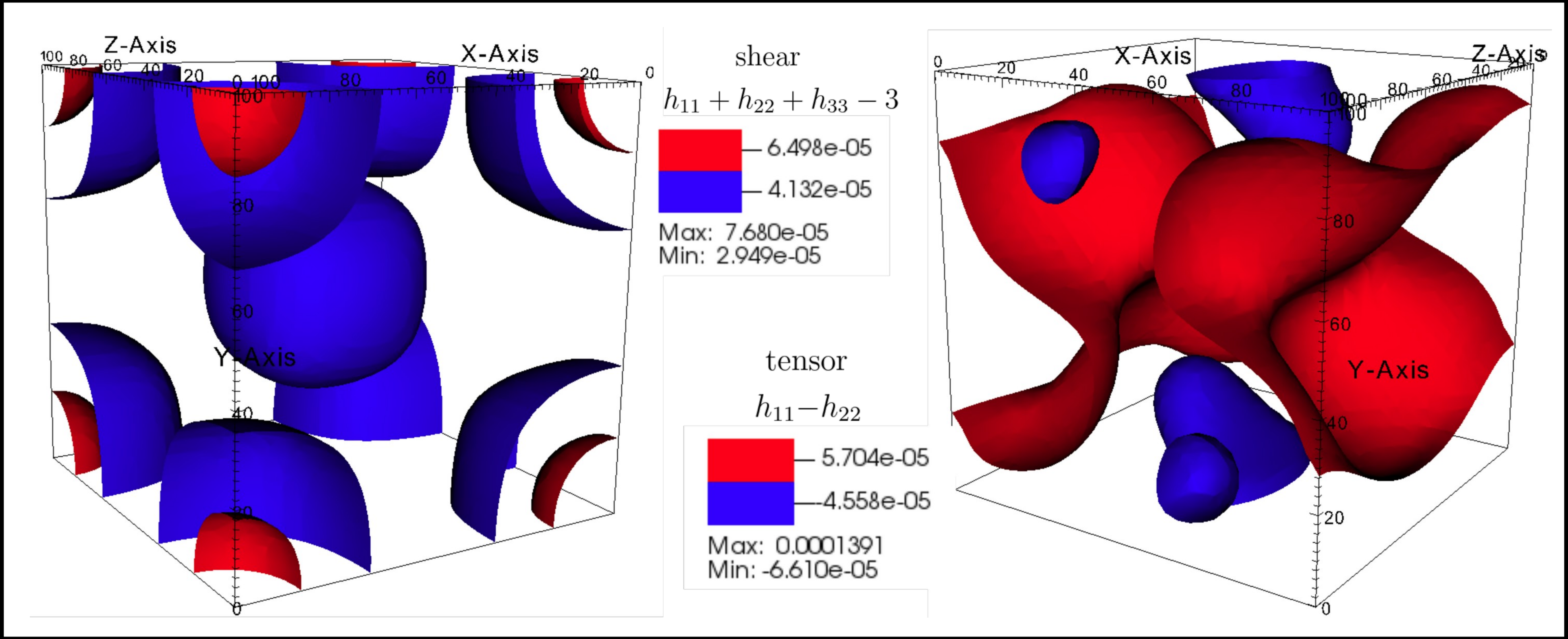
- Decay rates .... $\aleph \propto a^{-4}$



& metric components?



If inflation last just
N ~ 60 efolds! Maybe!


Z-Axis
X-Axis
Y-Axis
shear
$h_{11} + h_{22} + h_{33} - 3$
6.498e-05
4.132e-05
Max: 7.680e-05
Min: 2.949e-05
tensor
$h_{11} - h_{22}$
5.704e-05
-4.558e-05
Max: 0.0001391
Min: -6.610e-05
X-Axis
Z-Axis
Y-Axis

I.C. for inflation: Summary (key points)

- We extended the analysis on the initial conditions for Starobinsky/Higgs inflation (and alike)
- We simulated varius plaussible configurations of the pre-inflationary era. Including **Kination domination.**
- Large perturbation in the scalar field induce pertubations in the metric sector (ECMs) and may produce PIBHs.
- Inflation prevails robust!
  ECMs and PIBHs may give as a more complex picture of the Pre-inflationary era!

A paper is coming very soon!

- Late October/ Beginning of November…

**Inhomogeneous initial conditions for inflation:
A wibbly-wobbly timey-wimey path to Salvation**

Cristian Joana[1,*] and Sébastien Clesse[1,2,3,†]

[1]*Cosmology, Universe and Relativity at Louvain (CURL),
Institut de Recherche en Mathematique et Physique (IRMP),
University of Louvain, 2 Chemin du Cyclotron, 1348 Louvain-la-Neuve, Belgium*
[2]*Namur Institute of Complex Systems (naXys), Departement of Mathematics,
University of Namur, Rempart de la Vierge 8, 5000 Namur, Belgium*
[3]*Service de Physique Théorique, Université Libre de Bruxelles (ULB),
Boulevard du Triomphe, CP225, 1050 Brussels, Belgium.*
(Dated: October 24, 2020)

We use of the 3+1 formalism of numerical relativity to investigate the robustness of Starobinsky and Higgs inflation to inhomogeneous initial conditions, in the form of either field gradient or kinetic energy density. Sub-Hubble and Hubble-sized fluctuations generically lead to inflation after an oscillatory phase between gradient and kinetic energies. Hubble-sized inhomogeneities also produce contracting regions that end up in the formation of primordial black holes, subsequently diluted by inflation. We analyse the dynamics of the pre-inflation era and the generation of vector and tensor fluctuations. Our analysis further supports the robustness of inflation to any size of inhomogeneity, in the field, velocity or equation-of-state. The pre-inflation dynamics only marginally depends on the field potential and it is expected that such a behaviour is universal and applies to any inflaton potential of plateau-type, favored by CMB observations after Planck.

PACS numbers: 98.80.Cq, 98.70.Vc

I. INTRODUCTION

In the inflationary paradigm, the Universe undergoes an

density fluctuations certainly do not prevent the onset of inflation [4, 5]. But dealing with the fully relativistic non-linear dynamics of large inhomogeneities, including

Thank you !

Back up

Inhomogeneous initial conditions for Inflation:

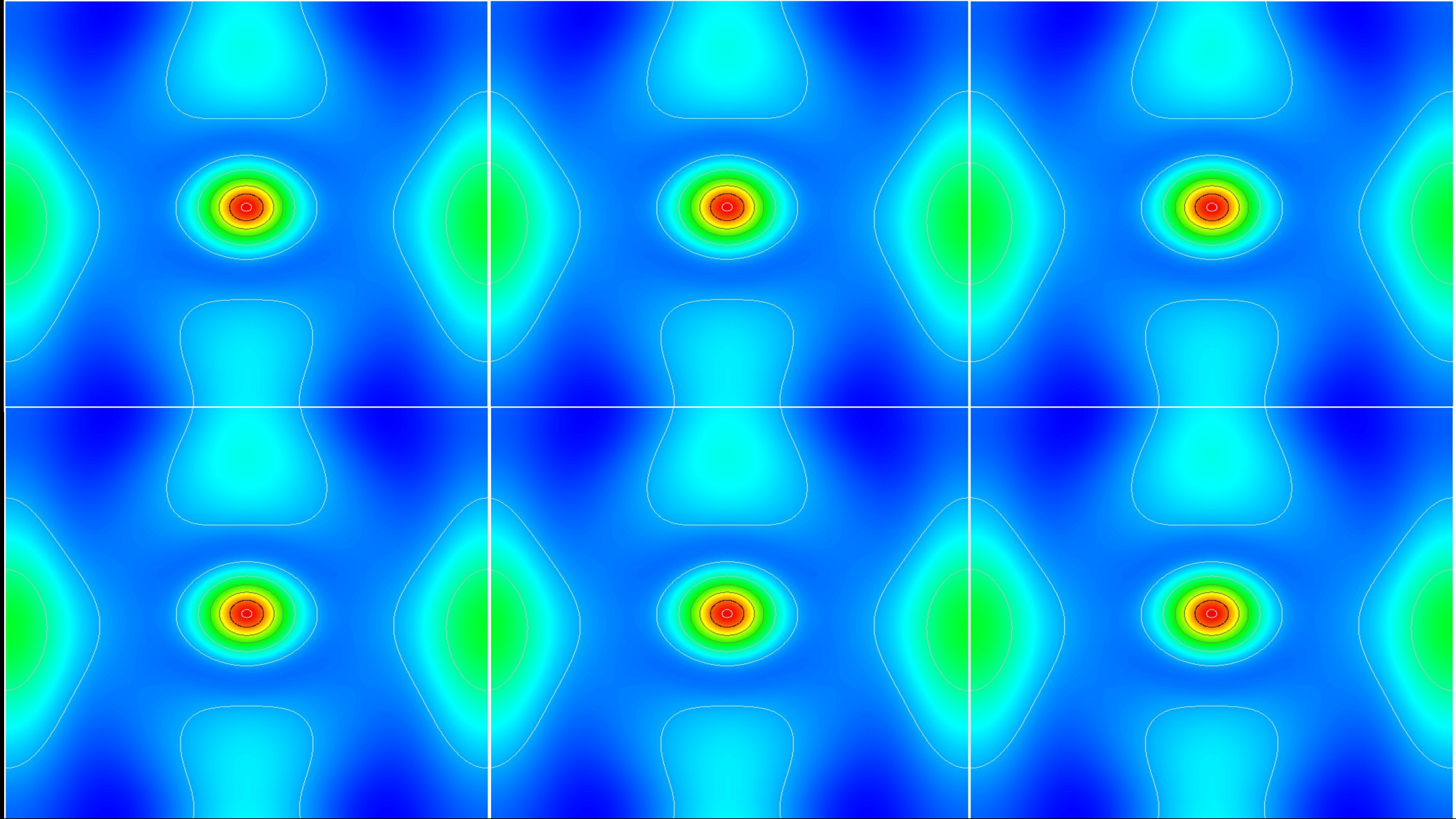
A wibbly-wobbly timey-wimey path to Salvation.

A journey through:
Kination phases, space-time waves & black holes

[Coming soon: arXiv : 2011.XXXX]

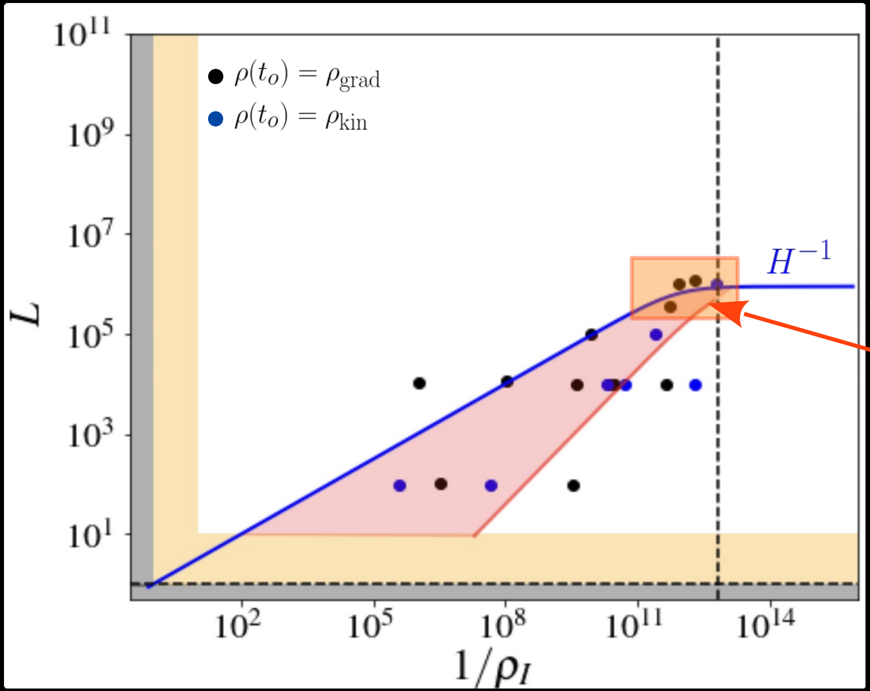
with S. Clesse and C. Ringeval

Cristian Joana (UCLouvain-CURL)



I.C. for inflation: Initial data

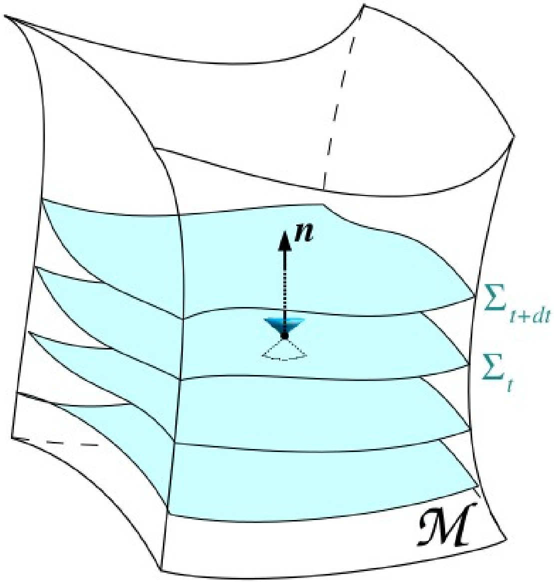
- ICs in terms of energy scale (ρ_I) & perturbation length (L):



$$\rho_i(\partial_i\varphi, \dot{\varphi}, a) = \rho - V(\varphi)$$

$$L \leftarrow \text{grid size}$$

In previous works



$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}\left(d\beta^i dt + dx^i\right)\left(d\beta^j dt + dx^j\right)$$

$$\partial_t \gamma_{ij} - \mathcal{L}_\beta \gamma_{ij} = -2\alpha K_{ij}$$

Conformal decomposition:

$$\gamma_{ij} = e^{4\phi}\,\tilde{\gamma}_{ij} \quad \& \quad K_{ij} = e^{4\phi}\,\tilde{A}_{ij} + \frac{1}{3}\gamma_{ij} K$$

The (3+1) decomposition of the Einstein equation raises four constrain equations, given by

$$0 = \mathcal{H} = R + \frac{1}{8}\tilde{A}_{ij}\tilde{A}^{ij} - \frac{1}{12}K^2 + 2\pi\rho \qquad (3)$$

$$0 = \mathcal{M}_i = e^{-6\phi}\tilde{D}_j\left(e^{6\phi}\tilde{A}^j{}_i\right) - \frac{2}{3}\tilde{D}_iK - 8\pi S_i \qquad (4)$$

(with $R = e^{-5\phi}\tilde{\gamma}^{ij}\tilde{D}_i\tilde{D}_je^{\phi} - \frac{1}{8}e^{-4\phi}\tilde{R}$)

$$\partial_t \chi = \frac{2}{3}\,\alpha\,\chi\,K - \frac{2}{3}\,\chi\,\partial_k\beta^k + \beta^k\,\partial_k\chi\ ,$$

$$\partial_t \tilde\gamma_{ij} = -2\,\alpha\,\tilde A_{ij} + \tilde\gamma_{ik}\,\partial_j\beta^k + \tilde\gamma_{jk}\,\partial_i\beta^k - \frac{2}{3}\,\tilde\gamma_{ij}\,\partial_k\beta^k + \beta^k\,\partial_k\tilde\gamma_{ij}\ ,$$

$$\partial_t K = -\gamma^{ij} D_i D_j\alpha + \alpha\left(\tilde A_{ij}\tilde A^{ij} + \frac{1}{3}K^2\right) + \beta^i\partial_i K + 4\pi\,\alpha(\rho + S)\ ,$$

$$\partial_t \tilde A_{ij} = \left[-D_i D_j\alpha + \chi\alpha\left(R_{ij} - 8\pi\ S_{ij}\right)\right]^{\mathrm{TF}} + \alpha(K\tilde A_{ij} - 2\tilde A_{il}\,\tilde A^l{}_j) + \tilde A_{ik}\,\partial_j\beta^k + \tilde A_{jk}\,\partial_i\beta^k - \frac{2}{3}\,\tilde A_{ij}\,\partial_k\beta^k + \beta^k\,\partial_k\tilde A_{ij}\ ,$$

$$\partial_t\tilde\Gamma^i = 2\,\alpha\left(\tilde\Gamma^i_{jk}\,\tilde A^{jk} - \frac{2}{3}\,\tilde\gamma^{ij}\partial_j K - \frac{3}{2}\,\tilde A^{ij}\,\frac{\partial_j\chi}{\chi}\right) - 2\,\tilde A^{ij}\,\partial_j\alpha + \beta^k\partial_k\tilde\Gamma^i + \tilde\gamma^{jk}\partial_j\partial_k\beta^i + \frac{1}{3}\,\tilde\gamma^{ij}\partial_j\partial_k\beta^k + \frac{2}{3}\,\tilde\Gamma^i\,\partial_k\beta^k - \tilde\Gamma^k\partial_k\beta^i - 16\pi\,\alpha\,\tilde\gamma^{ij}\,S_j\ ,$$

$$\frac{\ddot a}{a} = -\frac{\alpha}{3}\left(\frac{\dot\alpha}{\alpha}K - D^i D_i\alpha + \alpha\left(\aleph + 4\pi T\right)\right)$$

$$T \equiv \rho + S = 3\rho\left(\frac{1}{3} + \omega_\varphi\right) \qquad \aleph \equiv \tilde A_{ij}\tilde A^{ij}$$

$$\omega_\varphi(t) \simeq \begin{cases} 1 & \rightarrow \quad \rho \simeq \rho_{\mathrm{kin}} \\ -1/3 & \rightarrow \quad \rho \simeq \rho_{\mathrm{grad}} \\ -1 & \rightarrow \quad \rho \simeq \rho_{\mathrm{V}} \end{cases}$$

- **Flatness Problem:** The big bang model doesn't explain why the Universe seems so flat. ($|\Omega_{k,0}| < 0.005$)

$$\Omega_k = -\frac{k}{\dot{a}^2}$$

$$\dot{\Omega}_k = H\Omega_k(1+3\omega)$$

$$\Omega_k = 0 \rightarrow \text{unstable}$$

$$\Omega_{k,BBN} \sim 10^{-18}$$

$$\Omega_{k,Planck} \sim 10^{-63}$$