INTRODUCTION TO

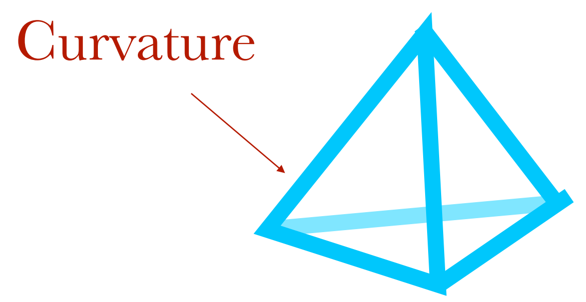
COVARIANT LOOP QUANTUM GRAVITY

FRANCESCA VIDOTTO
Western University, Canada
& IEM-CSIC, Spain

CHRONOLOGY

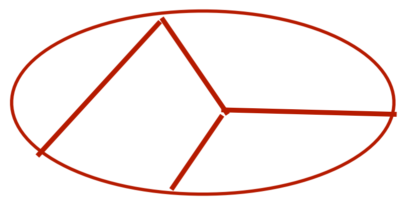
old QG

- 1957 $Z(q) = \int_{\partial g = q} Dg \;\; e^{iS_{EH}[g]}$ [Misner]
- 1961 Regge calculus → truncation of GR [Regge]
- 1967 W-DeW equation [Wheeler, DeWitt]
- 1971 Spin-geometry theorem → spin network [Penrose]



old LQG

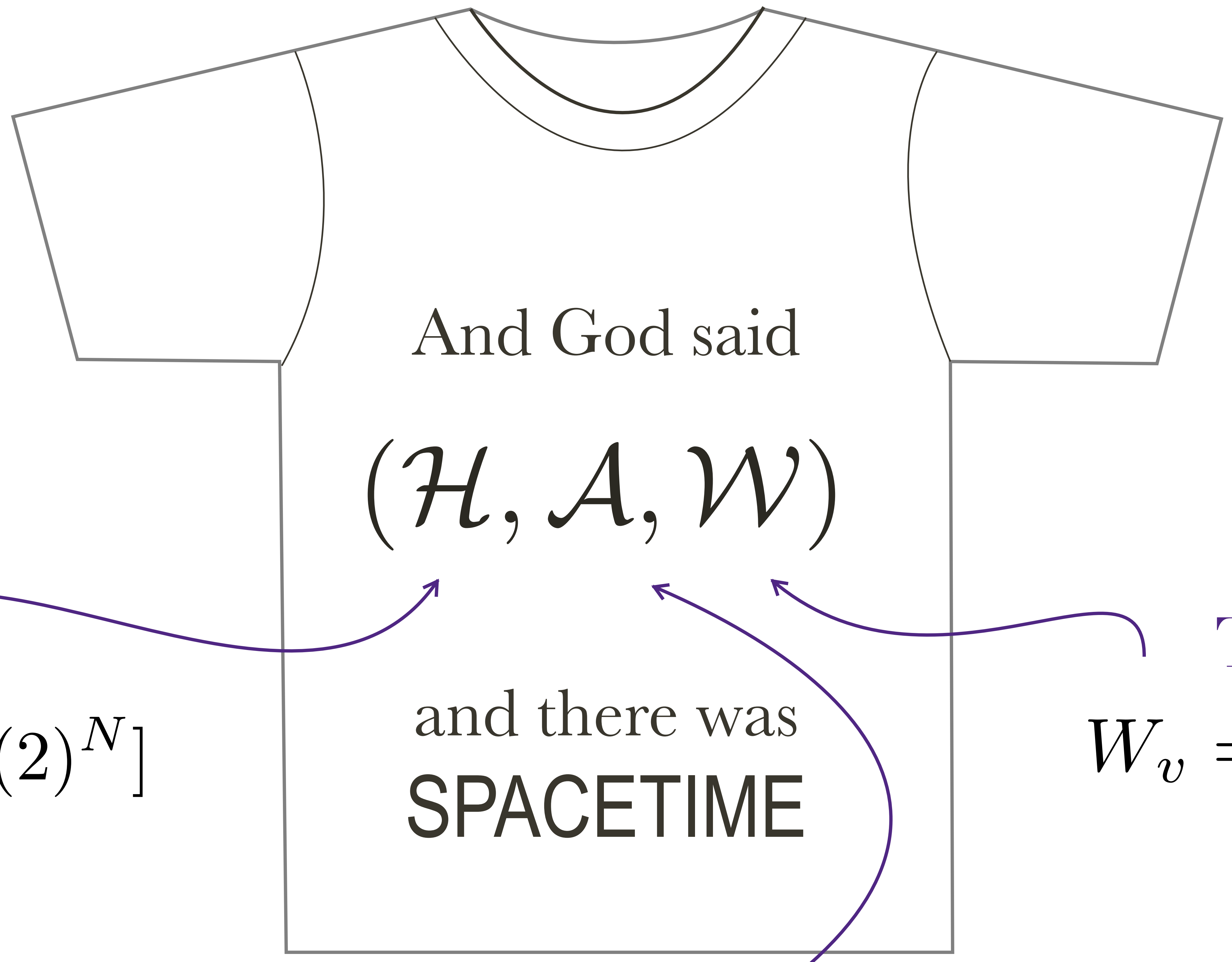
- 1988 Complex variables for GR [Ashtekar]
- 1988 Loop solutions to WdW eq → LQG [Rovelli-Smolin]
- 1994 Spectral problem for geometrical operators → spin networks
- 1996 Covariant dynamics → spinfoams [Reisenberger-Rovelli]



Modern LQG

- 2008 Covariant dynamics of LQG [Engle-Pereira-Livine-Rovelli, Freidel-Krasnov]
- 2010 Asymptotic of the new dynamics → recovery of Regge action [Barrett et al, ...]
- 2011 Cosmological constant → finiteness of the transition amplitudes [Han, ...]

THE THEORY



Hilbert Space:

$$\mathcal{H}_\Gamma = L_2[SU(2)^L / SU(2)^N]$$

Transition Amplitude:

$$W_v = (P_{SL(2,\mathbb{C})} \circ Y_\gamma \, \psi_v)(\mathbb{1})$$

Operator Algebra:

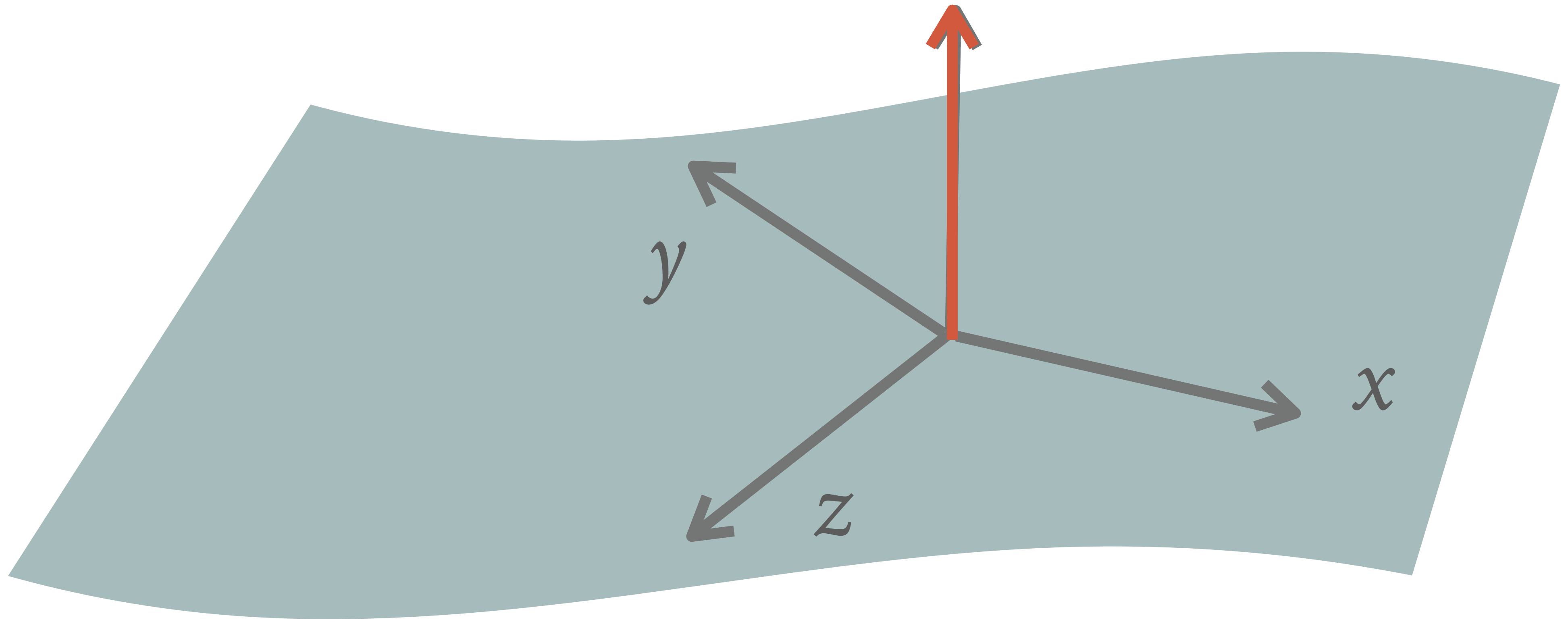
$$[L_a^i, L_b^j] = i\delta_{ab}\ell^2 \epsilon^{ij}_k L_a^k$$

PLAN OF THE LECTURES

1. Quantum Geometry
2. Spinfoam Dynamics: Introduction
3. Spinfoam Dynamics: Full Definition
4. Main Results and Discussion
5. Application: Cosmology


FRANCESCA VIDOTTO'S INTRODUCTION TO
COVARIANT LOOP QUANTUM GRAVITY
LECTURE 1
QUANTUM GEOMETRY
...

GENERAL RELATIVITY



- Reference frames $\longrightarrow$ reference fields (tetrads)
- ADM formalism: select a foliation at a given time
- Hamiltonian formulation: (densitized) triads are conjugate to the Ashtekar connection
- Area units: $8\pi\gamma\hbar G = 1$
- Triads have a rotation symmetry: $so(3) \longrightarrow su(2)$

GRAVITY AS A GAUGE THEORY

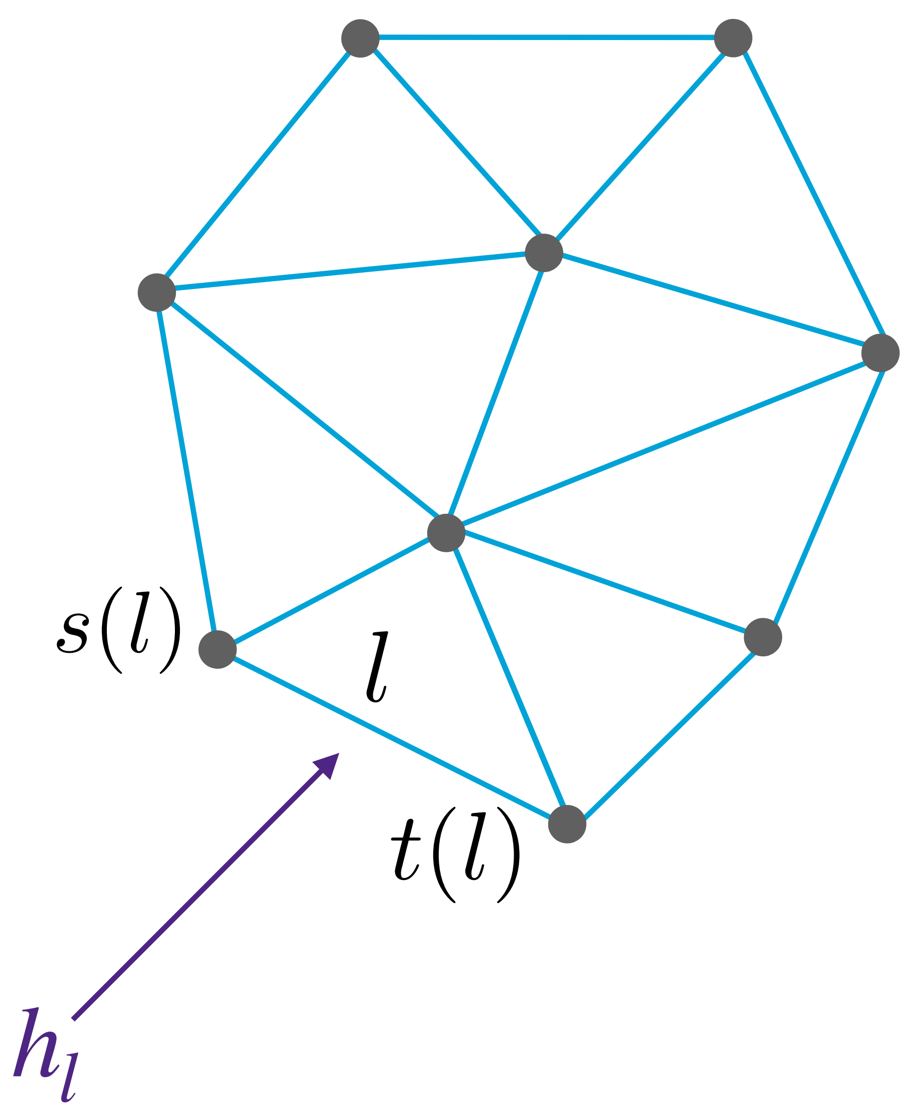
GAUGES IN GENERAL RELATIVITY ⟶ { - diffeos - Lorentz ⟶ { - graph/lattice - group variables ⟶ LOOP QUANTUM GRAVITY

- Abstract graphs: $\Gamma = \{N, L\}$ $\qquad \tilde{\mathcal{H}} = \bigoplus_{\Gamma} \mathcal{H}_\Gamma$

- Group variables: $\begin{cases} h_l \in SU(2) \\ \vec{L}_l \in su(2) \end{cases}$

- Graph Hilbert space: $\mathcal{H}_\Gamma = L_2[SU(2)^L/SU(2)^N]$

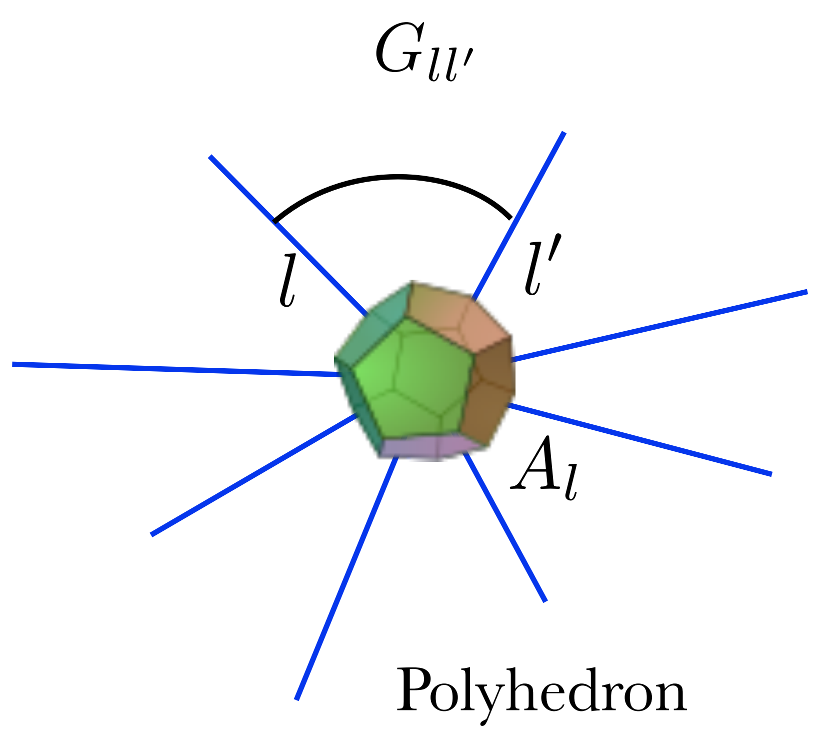
- States: $\psi(h_l) \to \psi(g_{s(l)}\, h_l\, g_{t(l)}^{-1}) \qquad g_n \in SU(2) \quad \forall n$



GRAVITATIONAL FIELD OPERATOR

State space: $\mathcal{H}_\Gamma = L^2[SU(2)^L/SU(2)^N]$

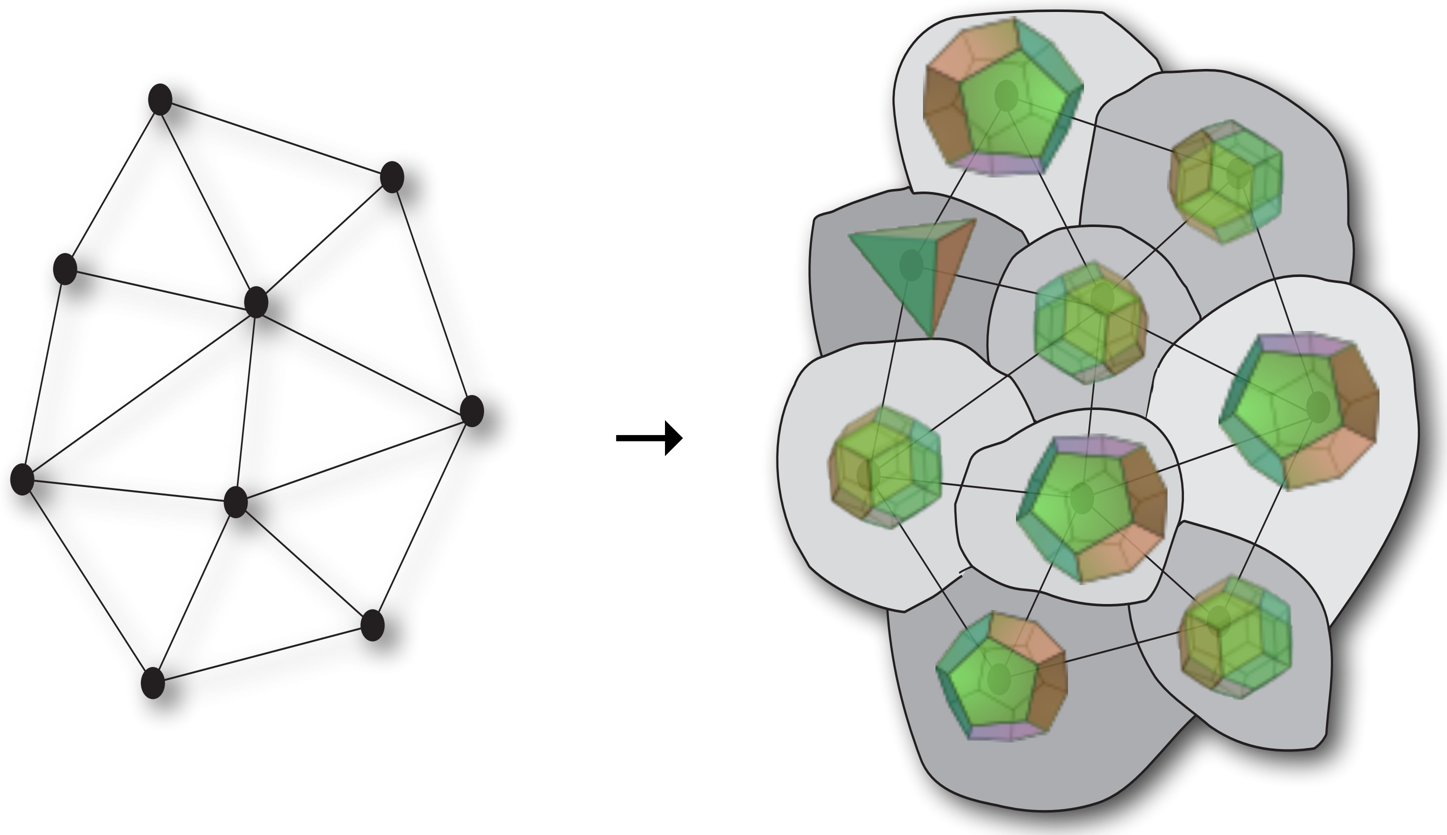
Operator: $\vec{L}_l = \{L_l^i\}, i = 1,2,3$ where $L^i\psi(h) \equiv \frac{d}{dt}\psi(he^{t\tau_i})\Big|_{t=0}$ $\qquad \sum_{l\in n} \vec{L}_l = 0$



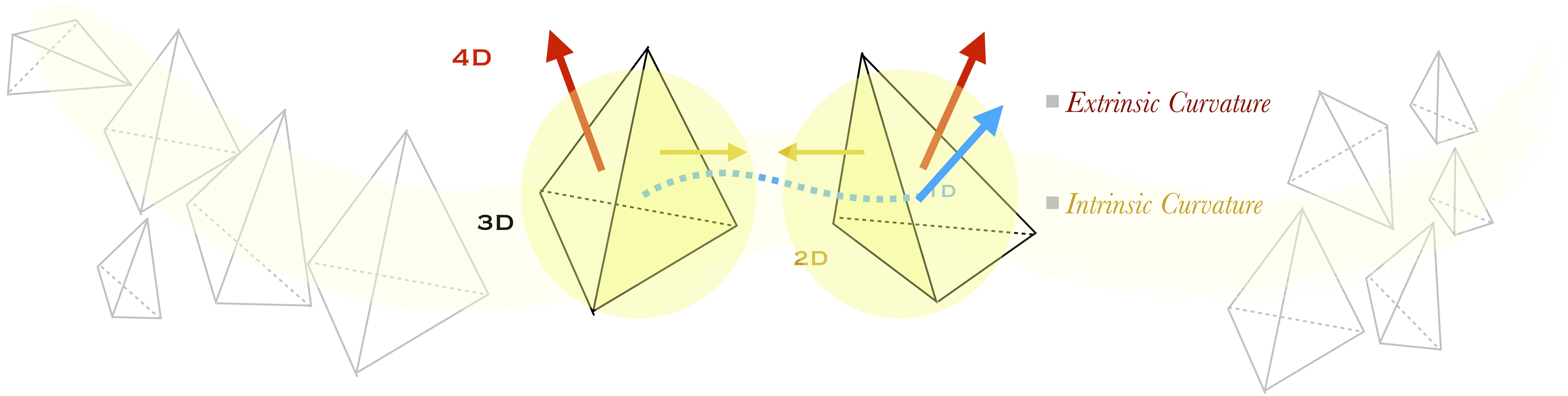
The gauge invariant operator: $G_{ll'} = \vec{L}_l \cdot \vec{L}_{l'}$ satisfies $\qquad \sum_{l\in n} G_{ll'} = 0$

Penrose metric operator on the graph

1971 Penrose spin-geometry theorem
(1897 Minkowski theorem):
semiclassical states have a
geometrical interpretation as polyhedra.



QUANTUM GEOMETRY



- h_l "Holonomy of the Ashtekar-Barbero connection along the link"
- $\vec{L}_l = \{L_l^i\}, i = 1, 2, 3$ SU(2) generators $L^i\psi(h) \equiv \frac{d}{dt}\psi(he^{t\tau_i})\Big|_{t=0}$

  gravitational field operator (tetrad)

  $g_{ab} = e_a^i \; e_b^i \qquad e = e_a dx^a \in \mathbb{R}^{(1,3)}$

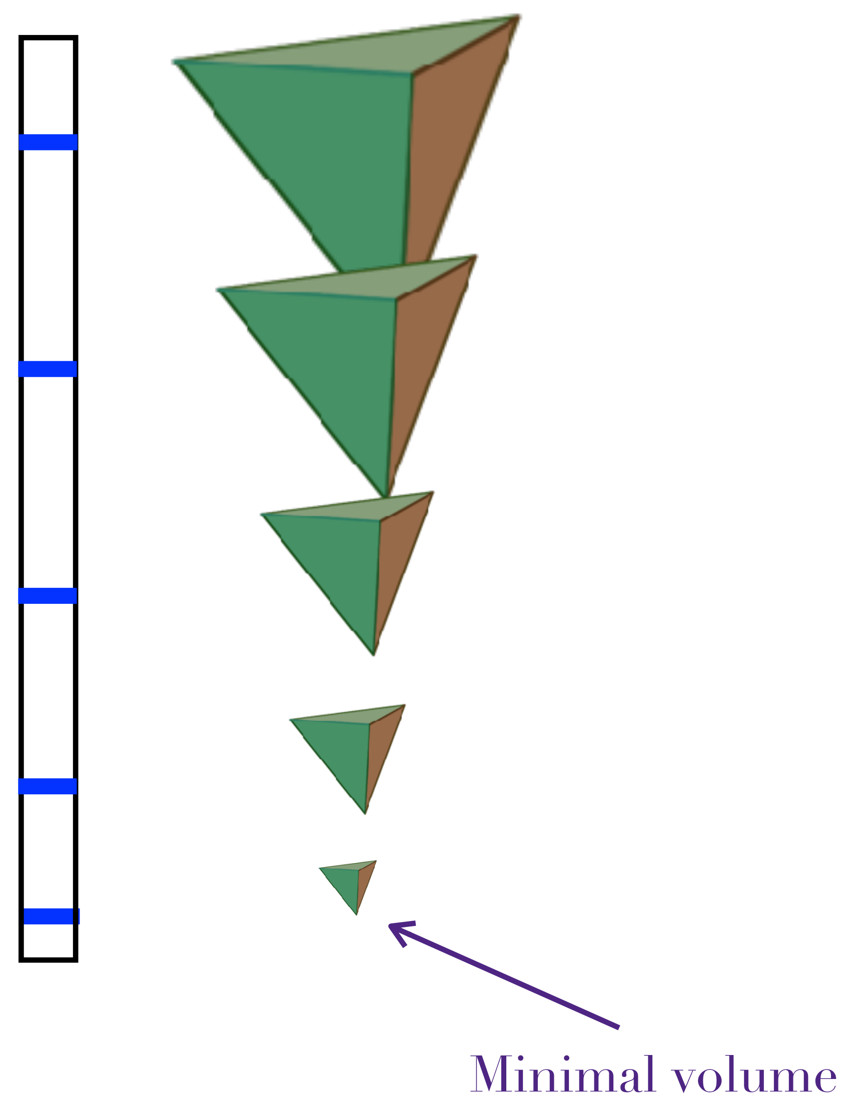
GEOMETRIC OPERATORS

REPRESENTING GEOMETRIES

[Rovelli, Smolin '93]

- Composite operators:
  - Angle: $L_l^i L_{l'}^i$
  - Area: $A_\Sigma = \sum_{l \in \Sigma} \sqrt{L_l^i L_l^i}$ $\qquad A_\Sigma = \sum_{l \in \Sigma} \sqrt{j_l(j_l+1)}$
  - Volume: $V_R = \sum_{n \in R} V_n$ $\qquad V_n^2 = \frac{2}{9} \, |\epsilon_{ijk} L_l^i L_{l'}^j L_{l''}^k|$

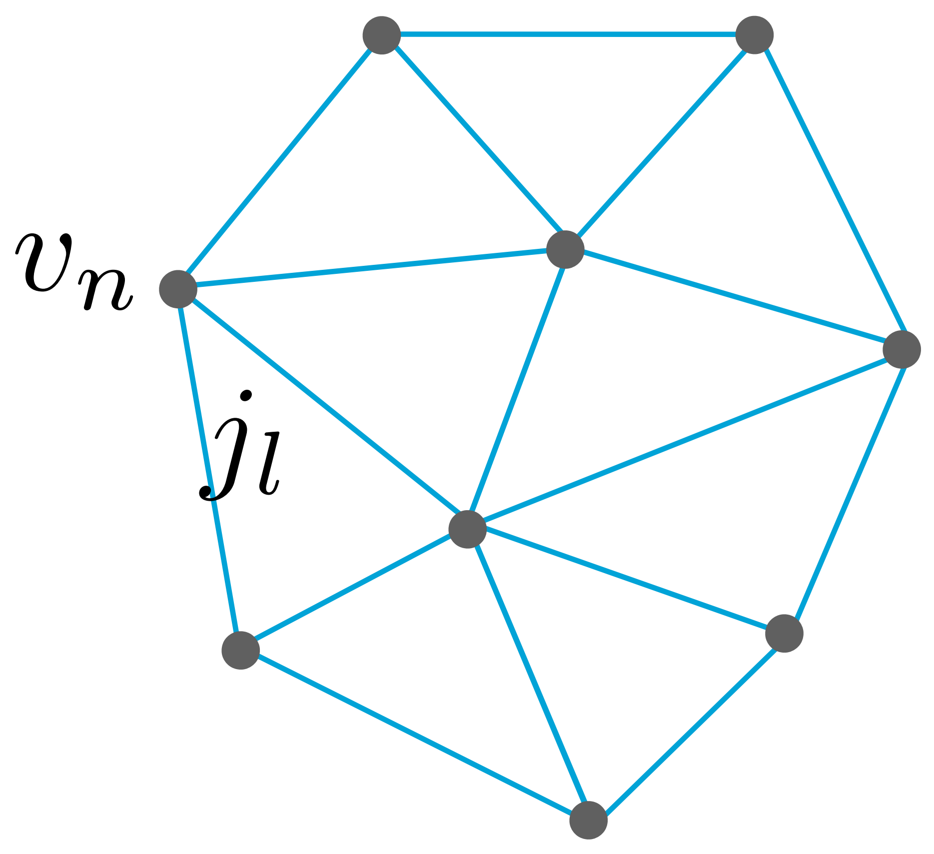
Discrete spectra!



SPINNETWORK
STATES

REPRESENTING GEOMETRIES

- Peter-Weyl Theorem: $\tilde{\mathcal{H}}_\Gamma = L_2[SU(2)^L] = \bigoplus_{j_l} \bigotimes_l (\mathcal{H}^*_{j_l} \otimes \mathcal{H}_{j_l})$
- Intertwiner Space: $\mathcal{H}_n = \mathrm{Inv}_{SU(2)}[\tilde{\mathcal{H}}_n]$
- Basis: $|\Gamma, j_l, v_n\rangle \in \mathcal{H}_\Gamma = \bigoplus_{j_l} \bigotimes_n \mathcal{H}_n$



- eigenvalues are discrete
- the operators do not commute
- quantum superposition

  ↳ *coherent states*

Quantum states of space,
rather than states on space.

REPRESENTING GEOMETRIES

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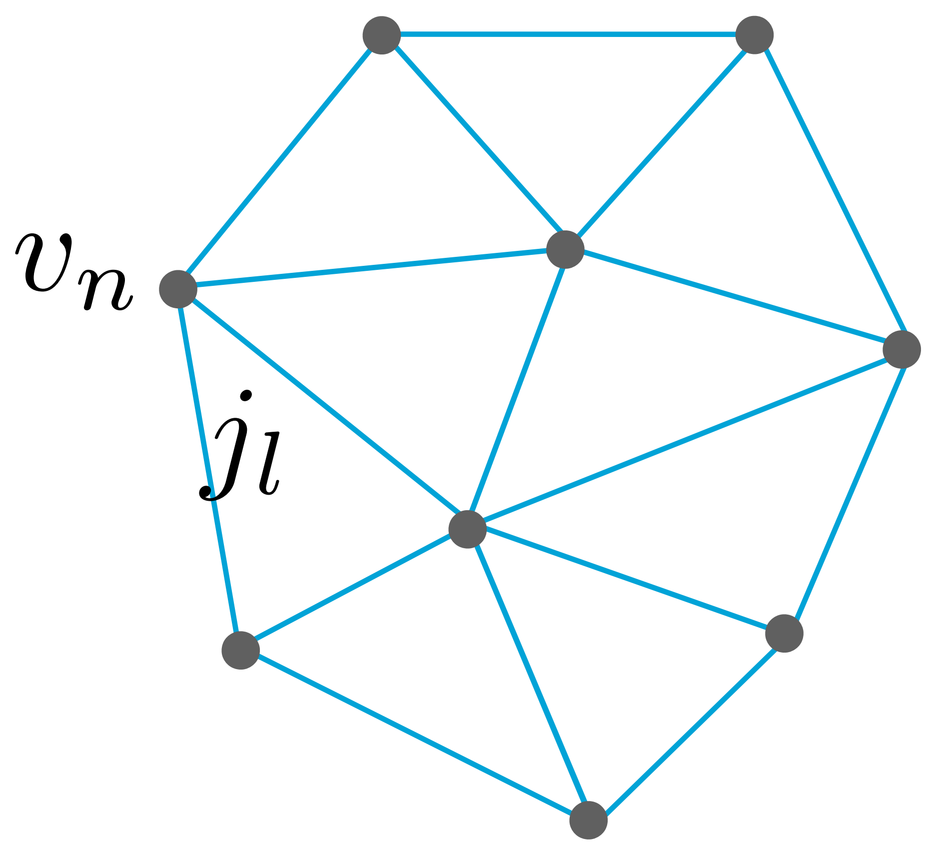

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- the operators do not commute
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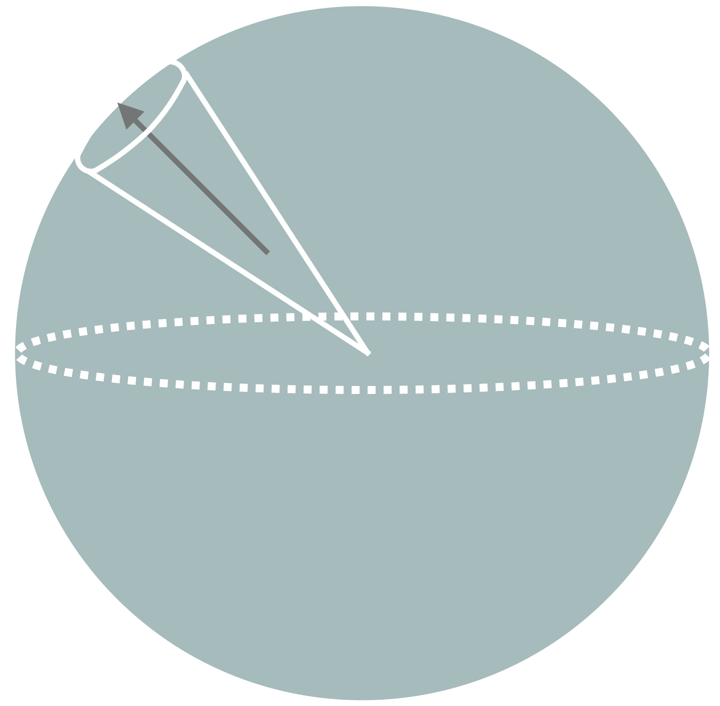
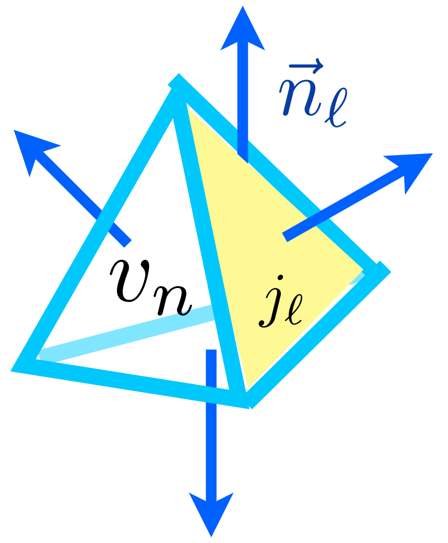
INTRINSIC COHERENT STATES



- Spinnetwork states $|\Gamma, j_l, v_n>$ $\leftrightarrow$ Coherent states
- $|j,j>$ has minimal spread $\Rightarrow$ coherence
- $\forall \vec{n}$ direction: $|j, \vec{n}> = h_{\vec{n}}\, |j,j>$
- $\| j_i, \vec{n}_i > = \int_{SU(2)} dh \bigotimes_i h \triangleright |j_i, \vec{n}_i>$ $\quad \forall i = 1,2,3,4$ faces



- intrinsic coherent states: equally spread on 3d geometry (intrinsic curvature)

(extrinsic coherent states: also spread in j, i.e. area, so that the extrinsic curvature is not spread)

FRANCESCA VIDOTTO'S INTRODUCTION TO

COVARIANT LOOP QUANTUM GRAVITY

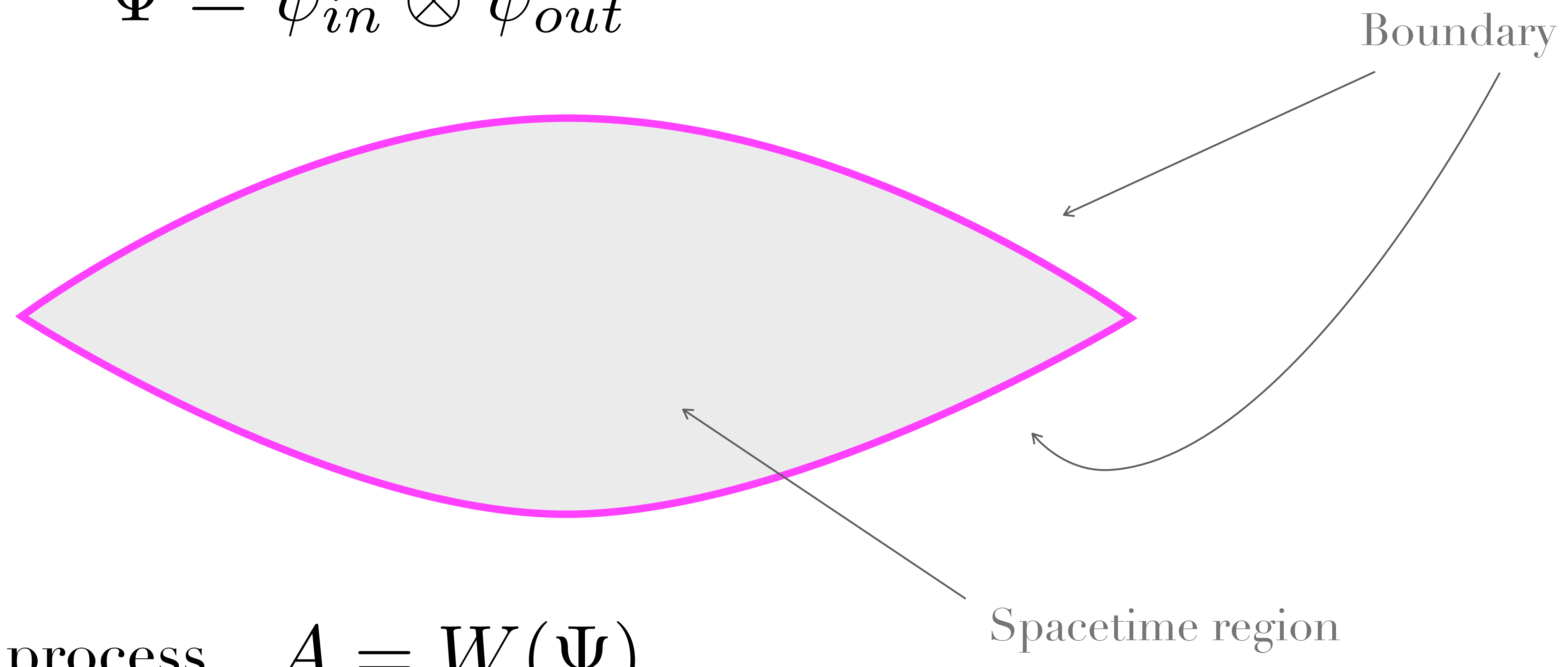
LECTURE 2
SPINFOAM DYNAMICS:
Introduction

SPACETIME IS A PROCESS

QUANTUM MECHANICS		GENERAL RELATIVITY
Process	← Locality →	Spacetime region
State		Boundary, space region

Spacetime is a process, a state is what happens at its boundary.

Boundary state $\Psi = \psi_{in} \otimes \psi_{out}$



Amplitude of the process $A = W(\Psi)$

LORENTZIAN LATTICE GAUGE THEORY

GAUGES IN GENERAL RELATIVITY $\longrightarrow$ $\begin{cases} \text{- diffeos} \\ \text{- Lorentz} \end{cases}$ $\longrightarrow$ $\begin{cases} \text{- graph/lattice} \\ \text{- group variables} \end{cases}$ $\longrightarrow$ LOOP QUANTUM GRAVITY

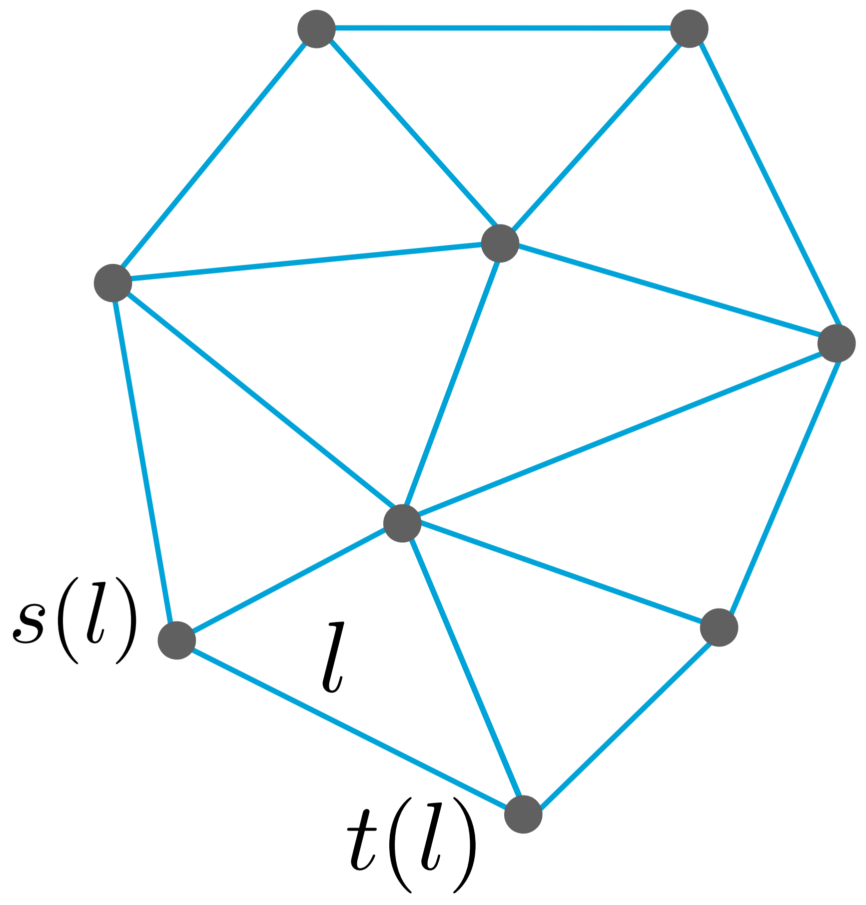
➤ Γ is the two-skeleton of the boundary of the lattice

➤ Graph Hilbert space: $\mathcal{H}_\Gamma^{SL(2,\mathbb{C})} = L_2[SL(2,\mathbb{C})^L/SL(2,\mathbb{C})^N]$

➤ States lives on the boundary of a 4D region

➤ States: $\psi(H_l)$ $\quad H_l \in SL(2,\mathbb{C})$

are wave functions of the holonomies $H_l = \mathcal{P}\exp\int_l \omega$



MATHEMATICAL TOOLKIT FOR GENERAL RELATIVITY

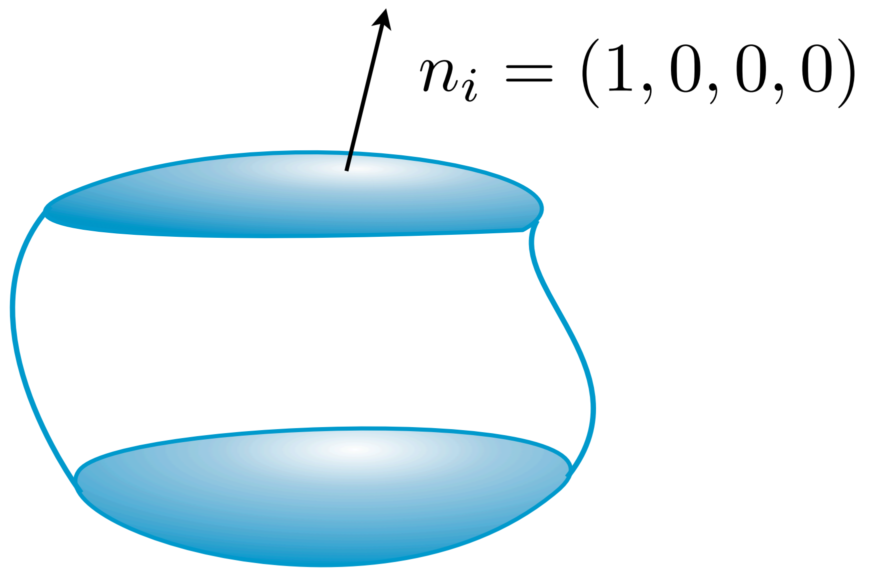
- Differential Geometry $\longrightarrow$ Pseudo-Riemannian Manifold $\longrightarrow$ Einstein-Cartan formalism
- Tetrads $\quad g_{ab} \to e^i_a \qquad g_{ab} = e^i_a \, e^i_b \qquad e^i = e^i_a dx^a \in \mathbb{R}^{(1,3)}$
- Spin connection $\quad \omega = \omega_a dx^a \in sl(2,\mathbb{C}) \qquad \omega(e): \quad de + \omega \wedge e = 0$
- GR action $\quad S[e,\omega] = \int e \wedge e \wedge F^*[\omega] + \frac{1}{\gamma} \int e \wedge e \wedge F[\omega] \qquad$ (*Holst term*)
- Conjugate momentum $\quad J = e \wedge e + \frac{1}{\gamma}(e \wedge e)^*$

SIMPLICITY CONSTRAINT

- Classical theory:

$$J \quad \begin{cases} L^i = \frac{1}{2}\epsilon^i{}_{jk}J^{jk} \\ K^i = J^{0i} \end{cases}$$

$$\vec{K} = -\gamma \vec{L}$$



- Quantum theory: J = generator of $SL(2,\mathbb{C})$

SPINFOAM DYNAMICS

$SU(2)$unitary representations: $2j \in Z$ $\quad |j;m\rangle \in \mathcal{H}_j$

$SL(2,\mathbb{C})$unitary representations: $2k \in N, \quad \nu \in R$ $\quad |k,\nu;j,m\rangle \in \mathcal{H}_{k,\nu} = \bigoplus_{j=k,\infty} \mathcal{H}^j_{k,\nu}$

$SL(2,\mathbb{C})$ Casimir's: $K^2 - L^2 = \nu^2 - k^2 + 1$ $\quad \vec{K} \cdot \vec{L} = \nu k$

γ-simple representations: $\nu = \gamma k$

Define a map Y_γ s.t. on its image: $j = k$ *Langlands classification: Vogan's minimal k-type*

$SU(2) \to SL(2,\mathbb{C})$ map:

$$Y_\gamma: \quad \mathcal{H}_j \longmapsto \mathcal{H}_j \subset \mathcal{H}_{(k=j,\nu=\gamma j)}$$
$$|j,m\rangle \longrightarrow |j,\gamma j;j,m\rangle$$

Main property: $\vec{K} + \gamma \vec{L} = 0$ weakly on the image of Y_γ : $\langle \psi | \vec{K} + \gamma \vec{L} | \phi \rangle = 0$

Boost generator ($\vec{K}$) — Rotation generator ($\vec{L}$)


FRANCESCA VIDOTTO'S INTRODUCTION TO
COVARIANT
LOOP
QUANTUM
GRAVITY
LECTURE 3
SPINFOAM DYNAMICS:
Lorentzian 4D theory

SPINFOAM AMPLITUDES

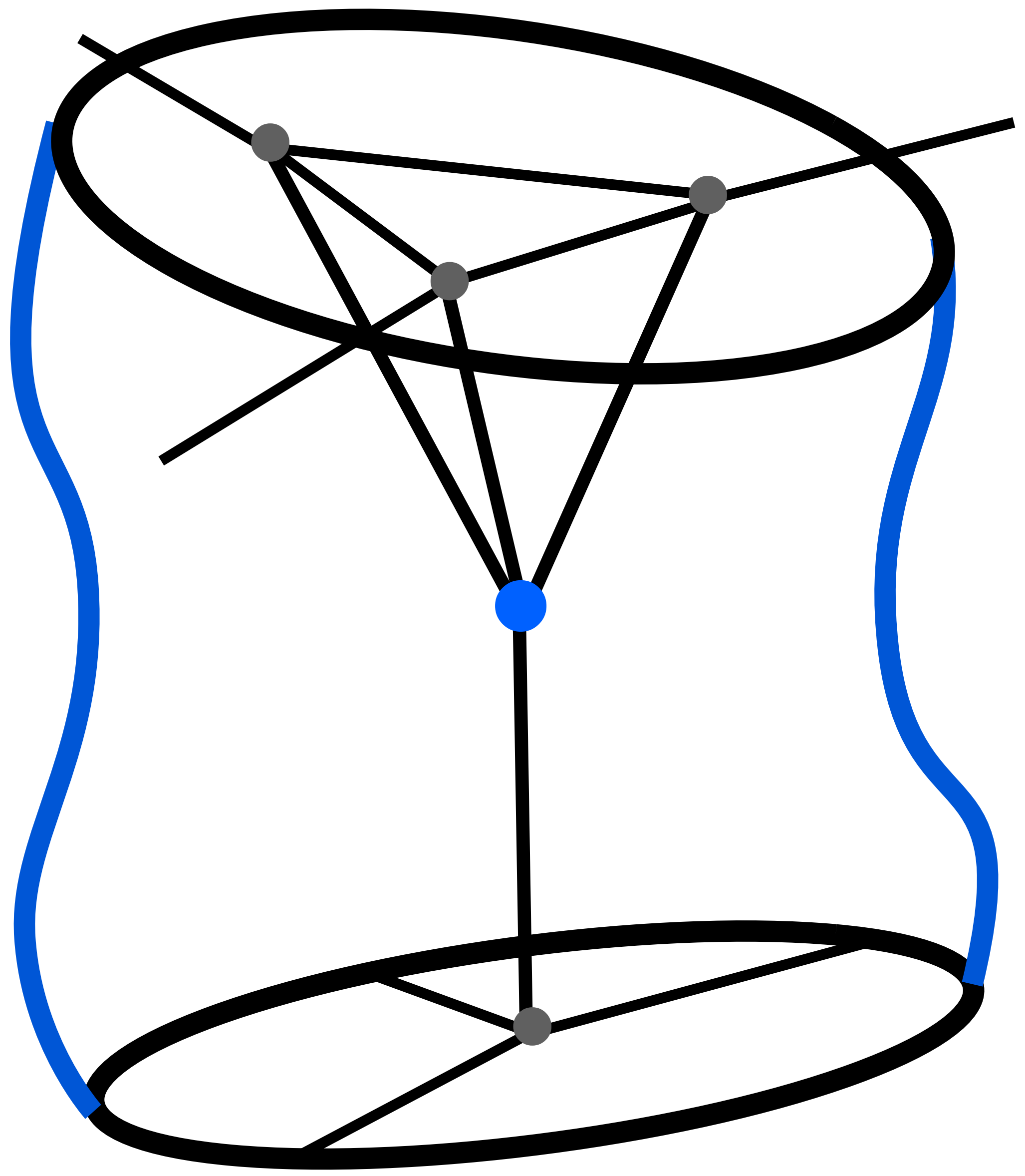
[Engle-Pereira-Livine-Rovelli, Freidel-Krasnov, Kaminski-Kisielowski-Lewandowski '08-'09]

Probability amplitude $P(\psi) = |\langle W|\psi\rangle|^2$

for a state ψ associated to the boundary of a 4d region

$$W(q) \approx \int_{\partial g = q} Dq\; e^{iS[q]}$$

- Superposition $\langle W|\psi\rangle = \sum_\sigma W(\sigma)$
- Local vertex expansion $W(\sigma) \sim \prod_v W_v.$



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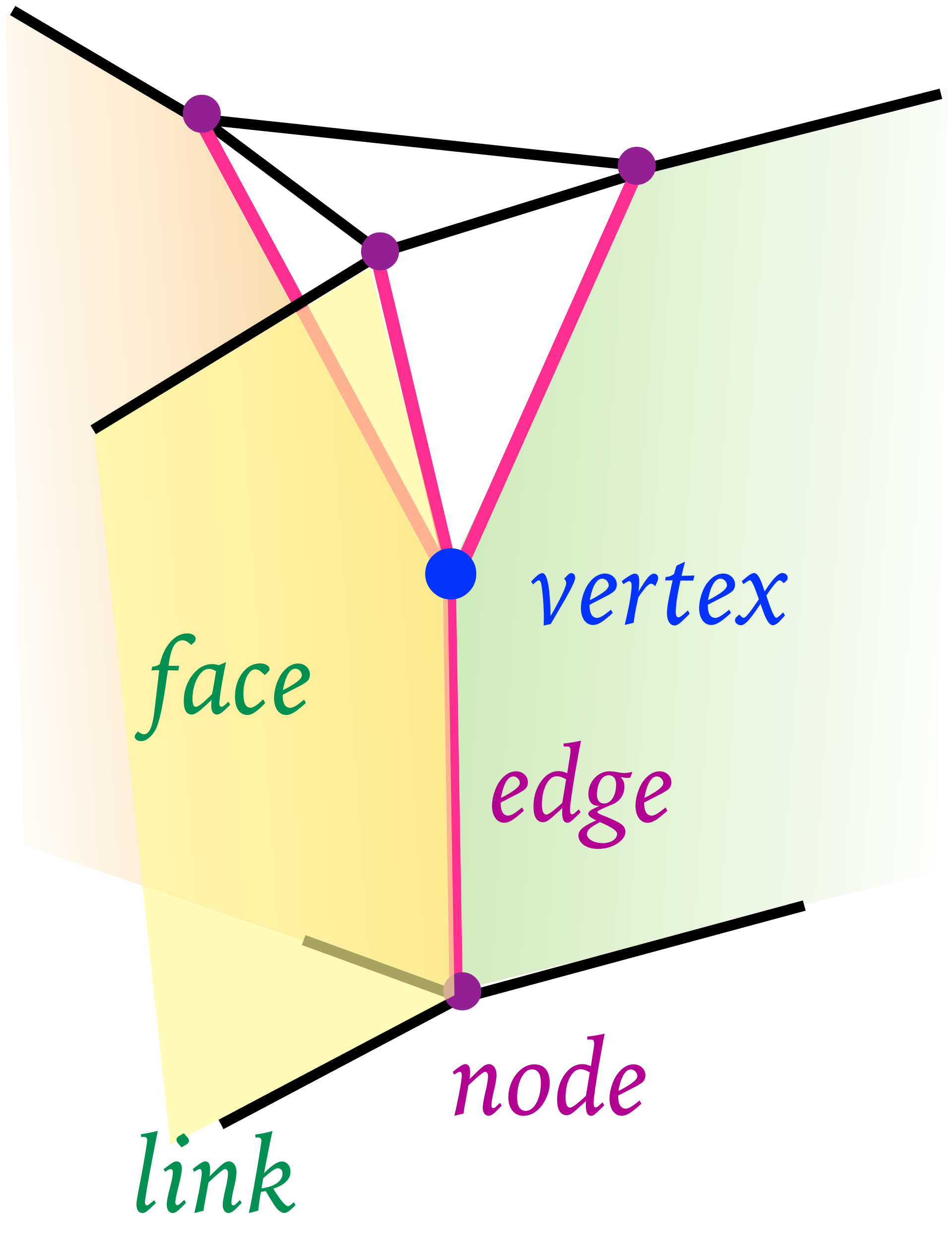

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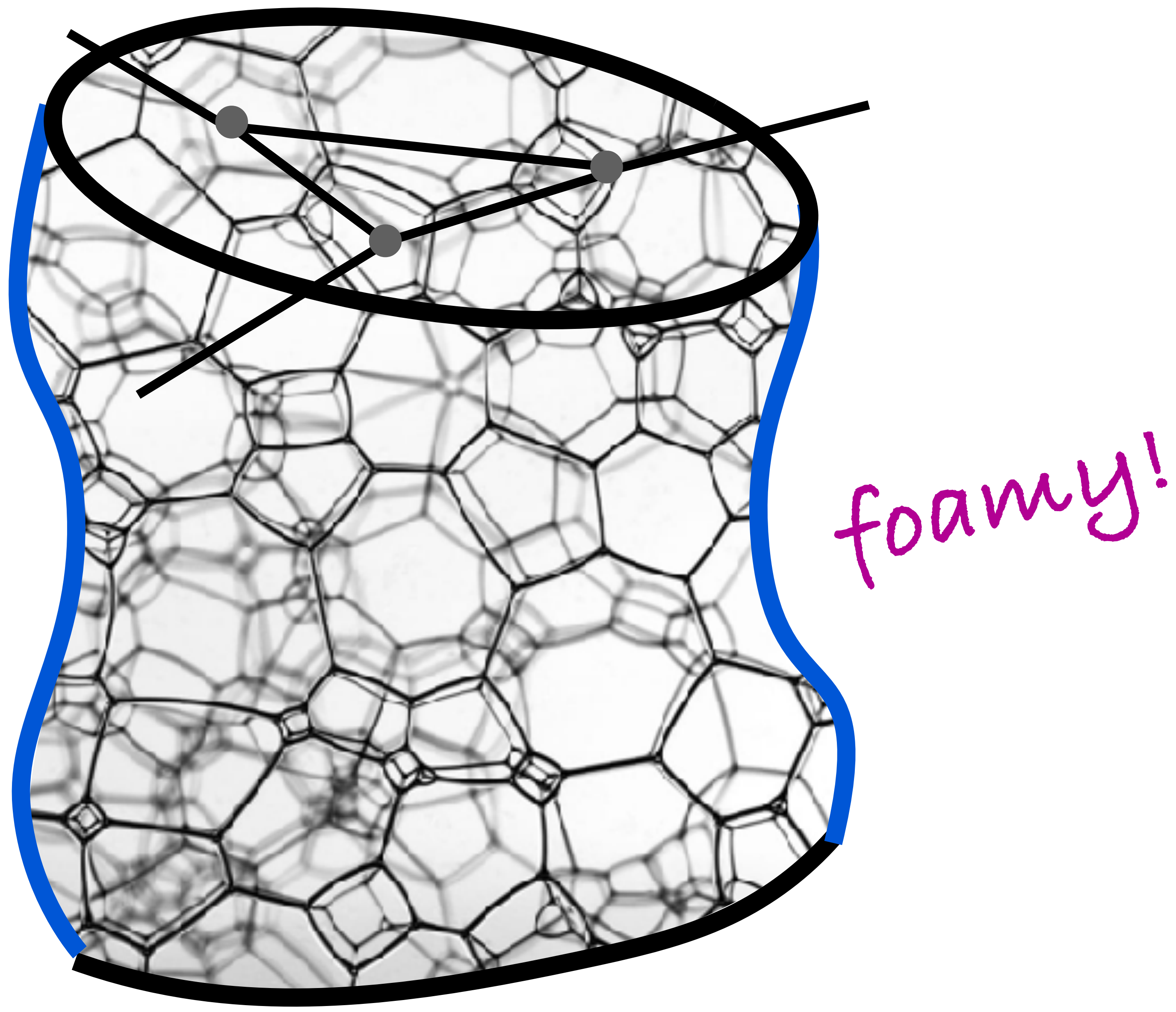
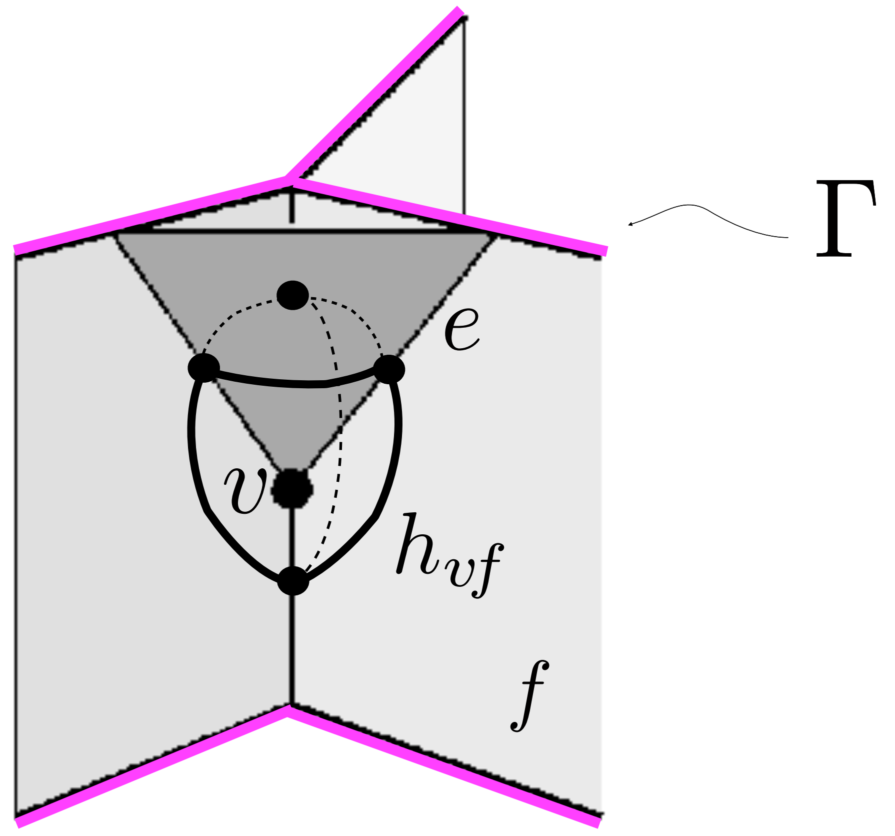


VERTEX AMPLITUDE

[Engle-Pereira-Livine-Rovelli, Freidel-Krasnov, Kaminski-Kisielowski-Lewandowski '08-'09]

2-complex $\mathcal{C}$
(vertices, edges, faces)



$$W_v = (P_{SL(2,\mathbb{C})} \circ Y_\gamma \; \psi_v)(\mathbb{1})$$

CLASSICAL LIMIT

[Barret et al. '09]

$$\langle W_v | \psi_g \rangle \sim e^{\frac{i}{\hbar} S_{Regge}[g]}$$

Coherent state peaked on the boundary geometry *g*

Regge action of a flat 4 simplex with the boundary geometry *g*

The simple vertex expression codes the Einstein equations!

AMPLITUDES

VERTEX AMPLITUDE

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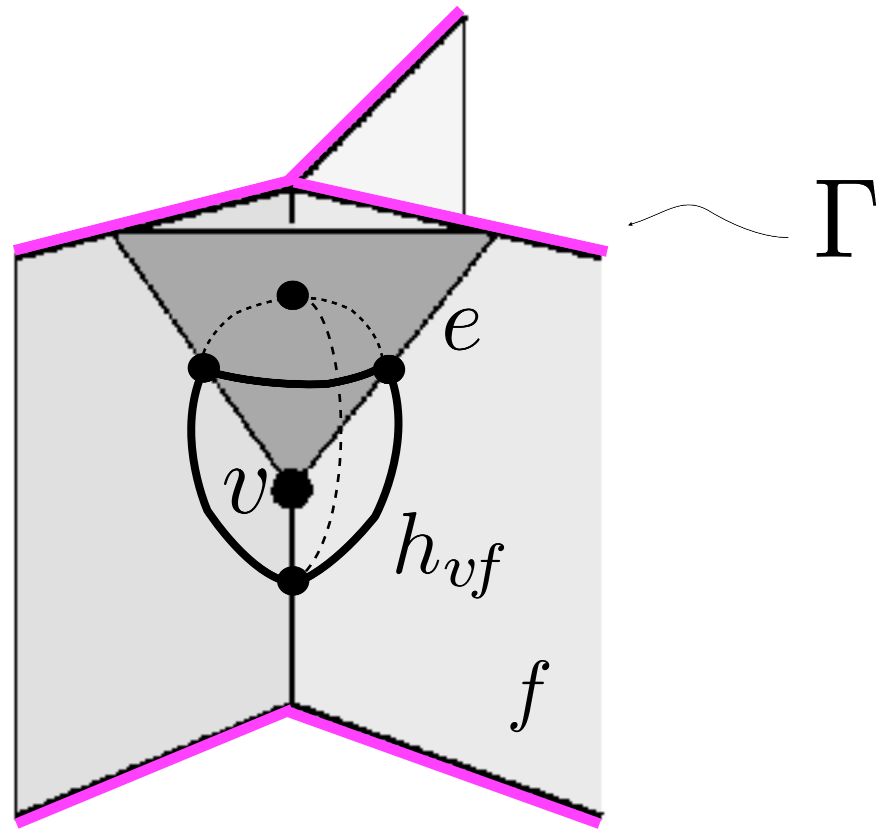


$$W_v = (P_{SL(2,\mathbb{C})} \circ Y_\gamma \ \psi_v)(\mathbb{1})$$

How can we extract a number from this, concretely?

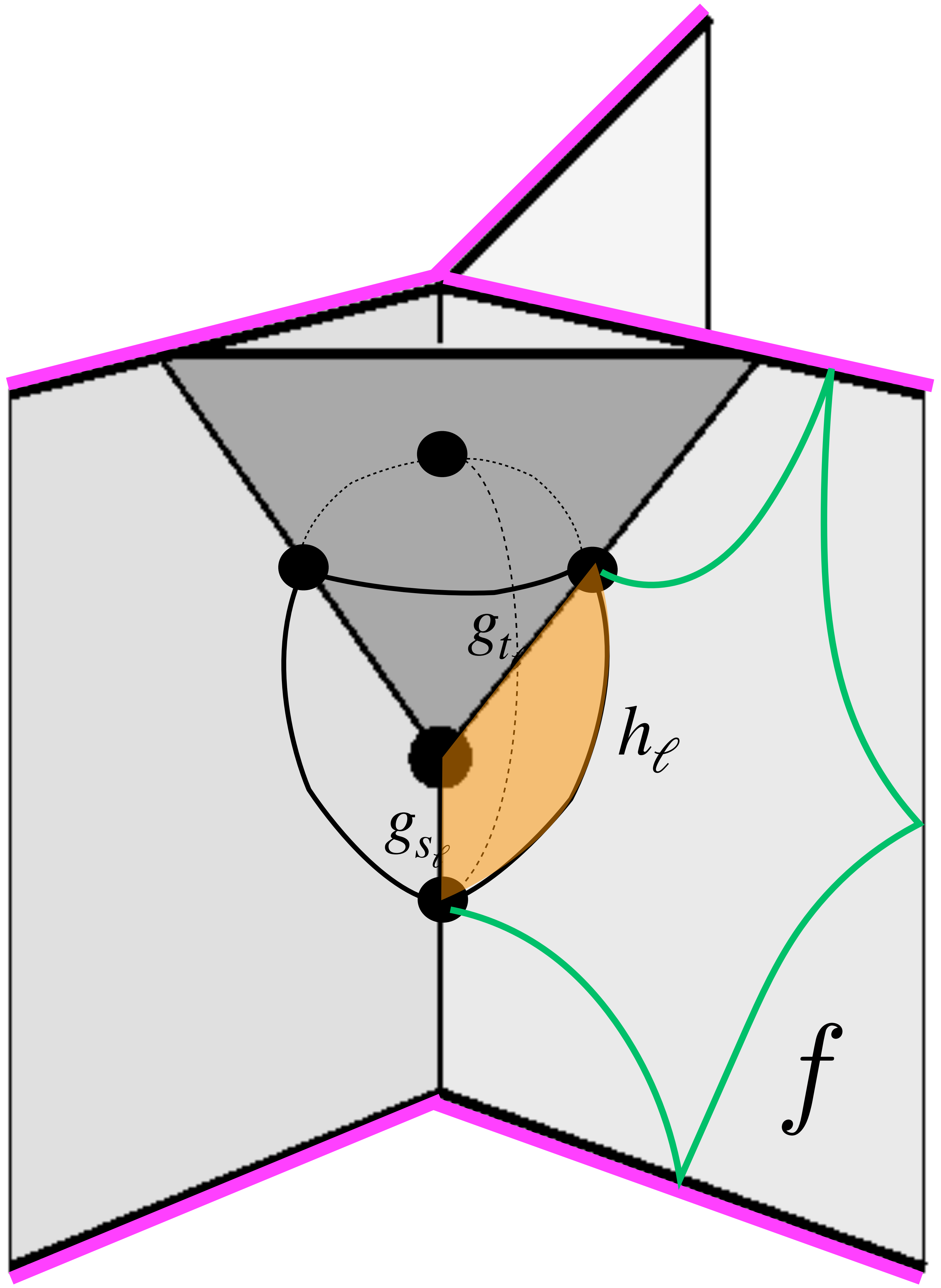
SPINFOAM DYNAMICS: EXPLICIT FORMULAS

- $SU(2)$ Wigner matrices $\quad h \triangleright |j,m\rangle = D^j_{mn}(h)\ |j,n\rangle \qquad D^j_{nm}(h) = \langle j,n\,|\,h\,|\,j,m\rangle$
- $SL(2,\mathbb{C})$ Wigner matrices $\quad g \triangleright |k,\nu;j,m\rangle = D^{k\nu}_{jm,j'n}(g)\ |k,nu;j,n\rangle$
- Change of basis $\quad L_2[SU(2)] = \oplus_j\,(H_j^* \otimes H_j) \qquad \langle j,n,m\,|\,h\rangle = D^j_{mn}(h)$

$$L_2[SL(2,\mathbb{C})] = \oplus_{k,\nu}\,(H_{k\nu}^* \otimes H_{k\nu}) \qquad \langle k,\nu;j,n,j',m\,|\,g\rangle = D^{k,\nu}_{jm,j'n}(g)$$

- Y_γ map $\quad Y_\gamma\,|j,m\rangle = |j,\gamma j;j,m\rangle \qquad \langle g\,|\,Y_\gamma\,|\,h\rangle = \sum_{j,m,n} D^{j,\gamma j}_{jm,jn}(g)\ D^j_{m,n}(h) \equiv P(g,h)$

VERTEX AMPLITUDE



$$\langle W_v | h_\ell \rangle = (P_{SL(2,\mathbb{C})} Y_\gamma | h_\ell \rangle)(\mathbb{1}) = \int_{SL(2,\mathbb{C})} dg_n \prod_l P(g_{s_e} g_{t_e}^{-1}, h_\ell)$$

Wedge amplitude $\qquad W_w(g, g', h) = P(gg', h)$

2-Complex amplitude

$$W_C = \int_{SU2} dh_{vf} \int_{SL2C} dg_{ve} \prod_w P(g_{s_\ell}, g_{t_\ell}, h_{vf}) \prod_f \delta(h_{l_1} \dots h_{l_{N_f}})$$

$$P(g, h) = \sum_{j,m,n} D^j_{m,n}(h)\, D^{j,\gamma j}_{jmjn}(g)$$

NUMERICAL METHODS

- Exploit factorization of the amplitude:

$$W(j_l, i_n) = \sum_{l_f, k_e} \left(\prod_e (2k_e + 1) B(j_l, l_f; i_n, k_e) \right) \{15j\}(l_f, k_e)$$

[Speziale'17]

- New sl2cfoam-next library

[Gozzini'21, Doná, Frisoni '22]

CLASSICAL LIMIT

$$\langle W_v | \psi_g \rangle \sim Re\,[e^{\frac{i}{\hbar} S_{Regge}[g]}]$$

[Barrett et al. '09]

Coherent state peaked on the boundary geometry *g*

Regge action of a flat 4 simplex with the boundary geometry *g*

The simple vertex expression codes the Einstein equations!

COMPUTING THE AMPLITUDE ON A GIVEN 2-COMPLEX

Analytical results

- N-point functions
- Black to White hole transition
- Big bounce transition
- …

Numerical results

- Small spin regime: Monte Carlo [Donà, Gozzini, Frisoni…]
- High spin regime: Complex saddle point methods [Han, Liu, Qu, Huang …]
- …

From properties of the amplitude

- Curvature Bound
- BH entropy
- …

FRANCESCA VIDOTTO'S INTRODUCTION TO

COVARIANT LOOP QUANTUM GRAVITY

LECTURE 4
MAIN RESULTS
& DISCUSSION

RESULTS FROM THE COVARIANT DYNAMICS

- Classical Limit ☑
- Coupling of fermions, scalars, Yang-Mills fields… ☑
- Graviton propagator ☑
- Scattering Amplitudes ☑
- Radiative Corrections ☑
- Black Holes Entropy ☑
- Cosmology ☑
- …. ☑

CLASSICAL LIMIT

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[Batterrt et al. '09]

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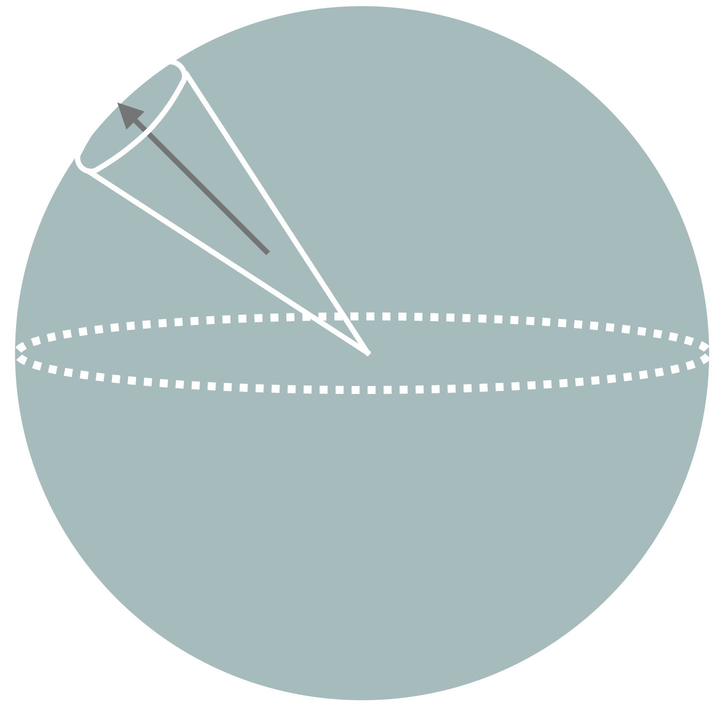
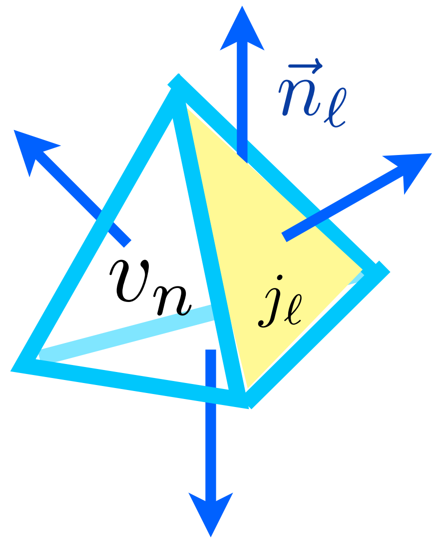
INTRINSIC COHERENT STATES

- Spinnetwork states $|\Gamma, j_l, v_n>$ $\leftrightarrow$ Coherent states



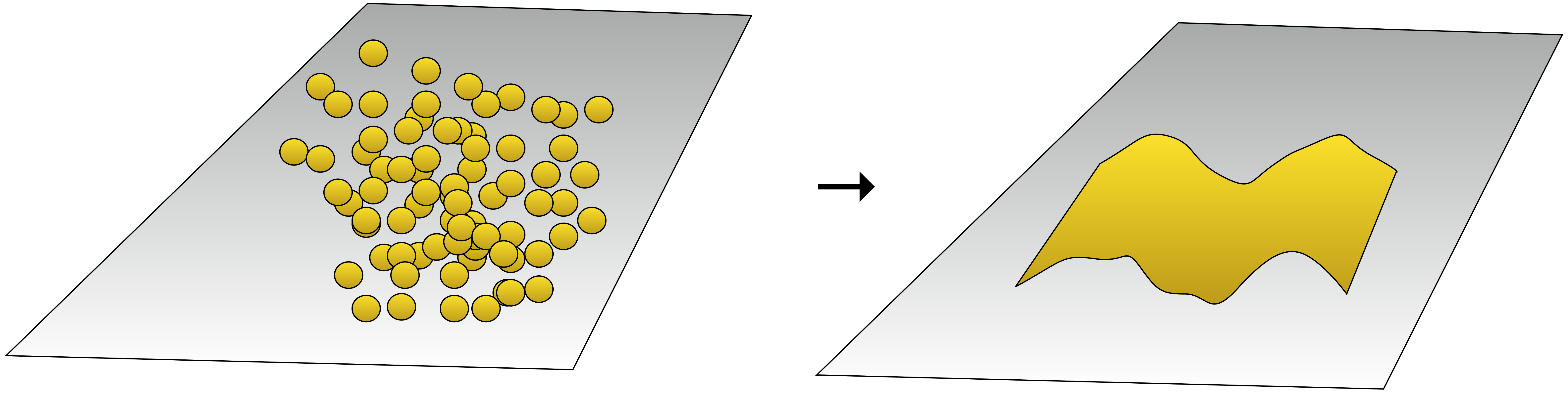
- $|j, j>$ has minimal spread $\Rightarrow$ coherence
- $\forall \vec{n}$ direction: $|j, \vec{n}> = h_{\vec{n}} \, |j, j>$
- $\| j_i, \vec{n}_i > = \int_{SU(2)} dh \bigotimes_i h \triangleright |j_i, \vec{n}_i >$ $\quad \forall i = 1,2,3,4$ faces



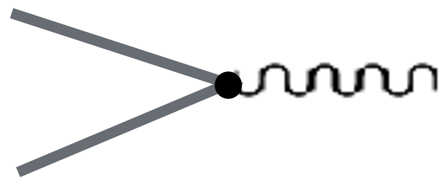
- intrinsic coherent states: equally spread on 3d geometry (intrinsic curvature)

(extrinsic coherent states: also spread in j, i.e. area, so that the extrinsic curvature is not spread)

LIMIT $\hbar \longrightarrow 0$

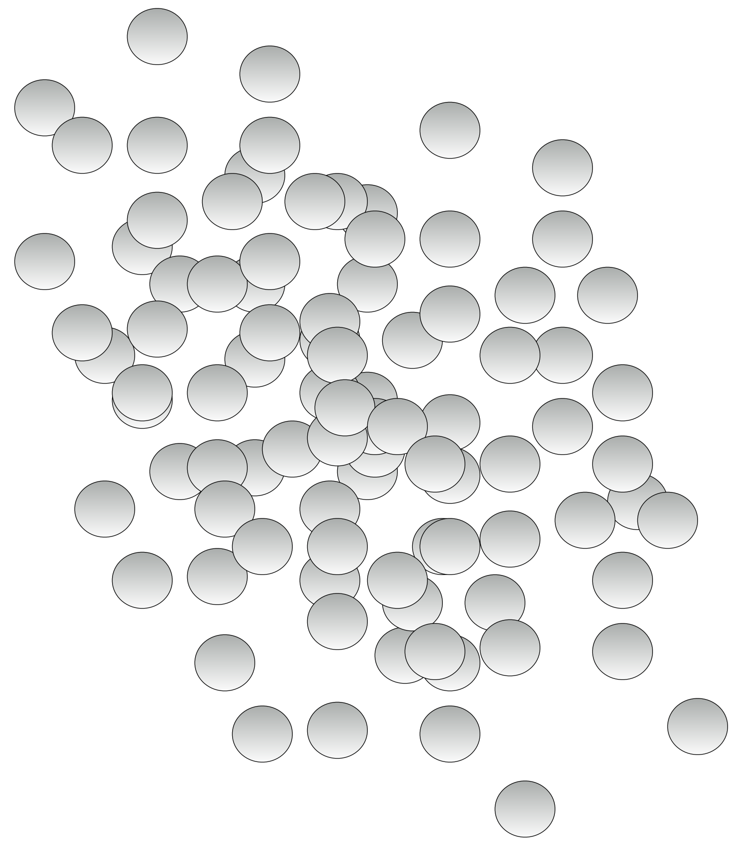


- DISCRETE
- FUZZY
- PROBABILISTIC

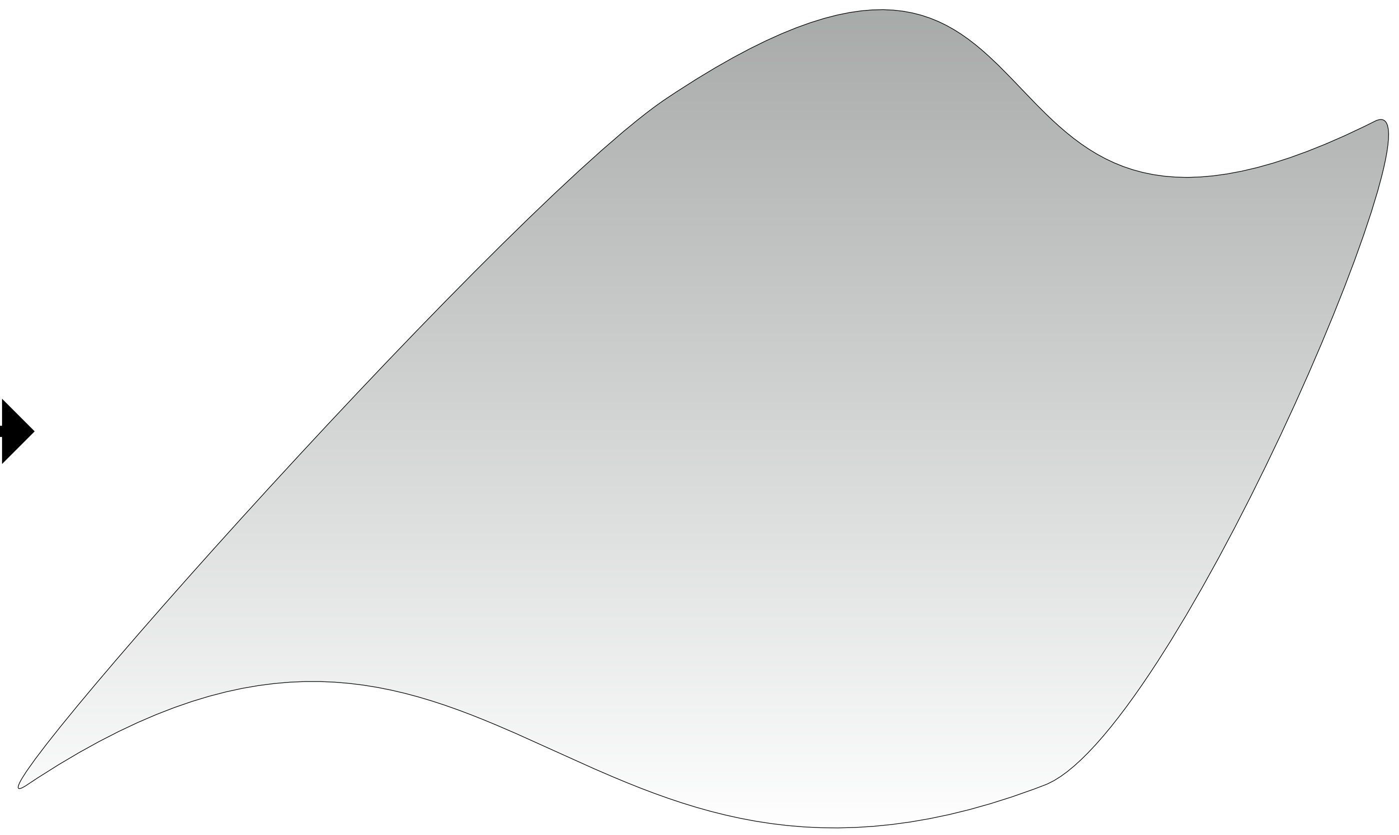


- NO DISCRETENESS
- NO FUZZYNESS
- A CLASSICAL FIELD

LIMIT $\hbar \longrightarrow 0$



$\rightarrow$

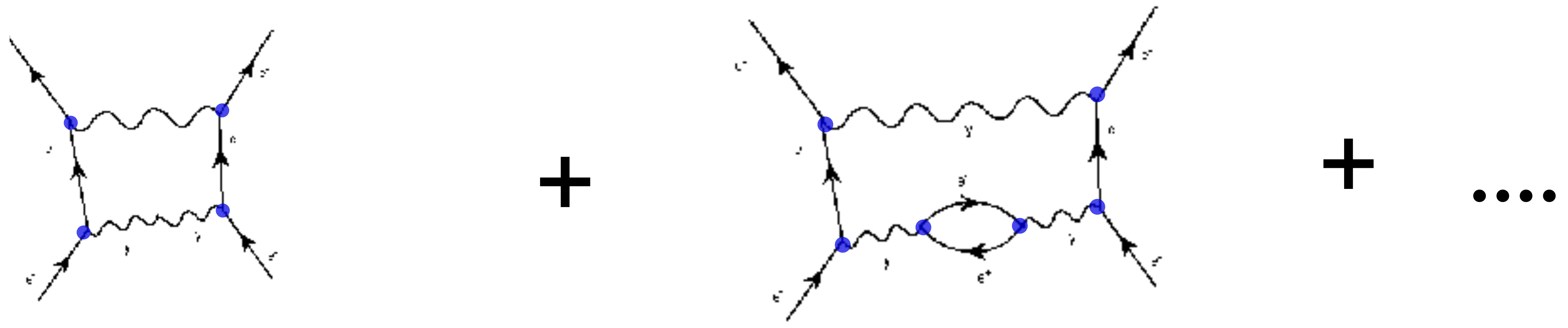


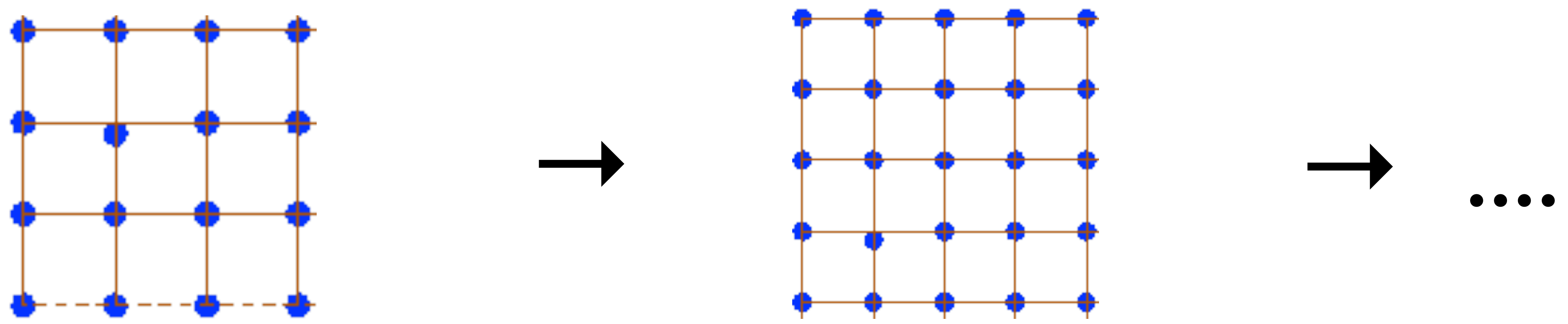
- DISCRETE $\ell_{Pl}^2 = \hbar G$
- FUZZY
- PROBABILISTIC

- NO DISCRETENESS $\ell_{Pl} \rightarrow 0$
- NO FUZZYNESS
- A CLASSICAL FIELD $E_a^i(x) \rightarrow g_{\mu\nu}(x)$

CONVERGENCE BETWEEN QED AND QCD

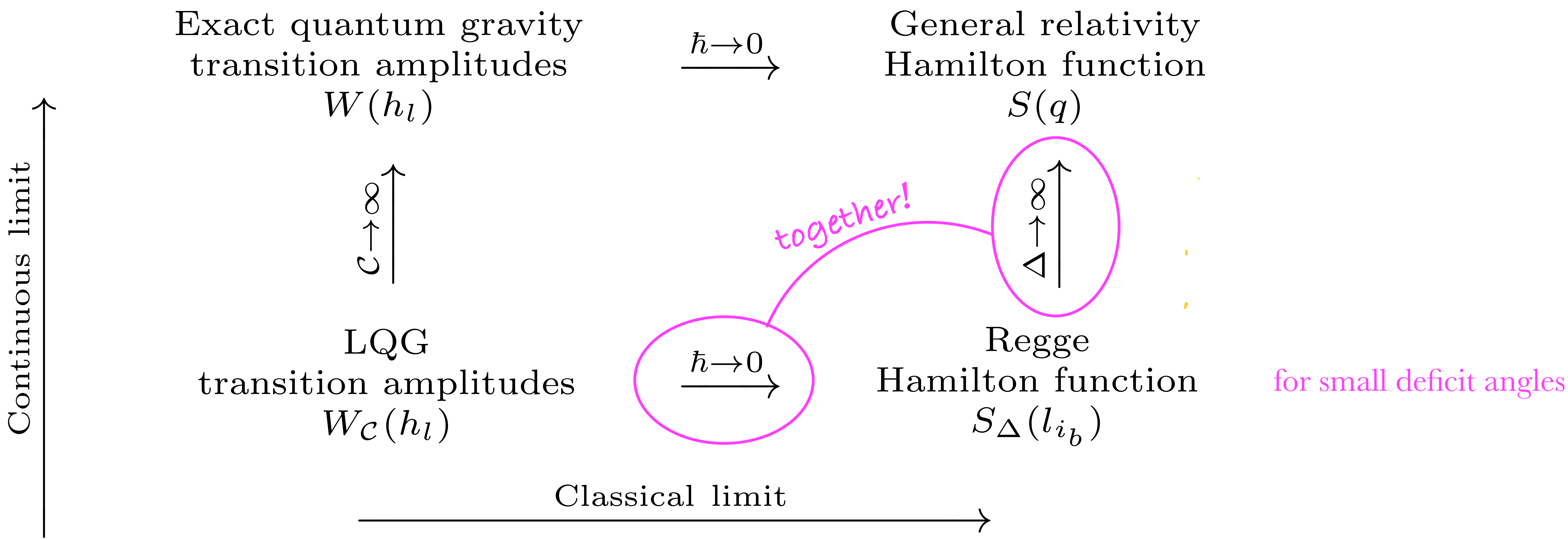
- All physical QFT are constructed via a truncation of the d.o.f.   (QED: particles, QCD: lattice)
- All physical calculation are performed within a truncation.
- The limit in which all d.o.f. is then recovered is pretty different in QED qnd QCD:





- Quantum Gravity: Diff invariance !
- Lattice site = small region of space = excitations of the gravitational field = quanta of space = quanta of the field

STRUCTURE OF THE THEORY



- No critical point
  - QFT : critical phenomenon
  - Quantum Gravity: non-critical phenomenon
- No infinite renormalization
- Physical Scale ℓ_{Pl}

FROM QUANTUM TO CLASSICAL

- FROM QUANTUM TO CLASSICAL
  The *classical* limit is $\hbar \longrightarrow 0$, the limit for ∞ quanta is relevant for the *continuous* limit

  No thermodynamical limit is needed for this.

- EMERGENCE OF SPACETIME IS STANDARD CLASSICAL EMERGENCE
  just as the electromagnetic field emerges from photons

- SPACETIME IN THE QUANTUM REGIME IS MADE OF QUANTA
  - there is no classical spacetime in the quantum regime
  - same as in Q.E.D. where there are photons

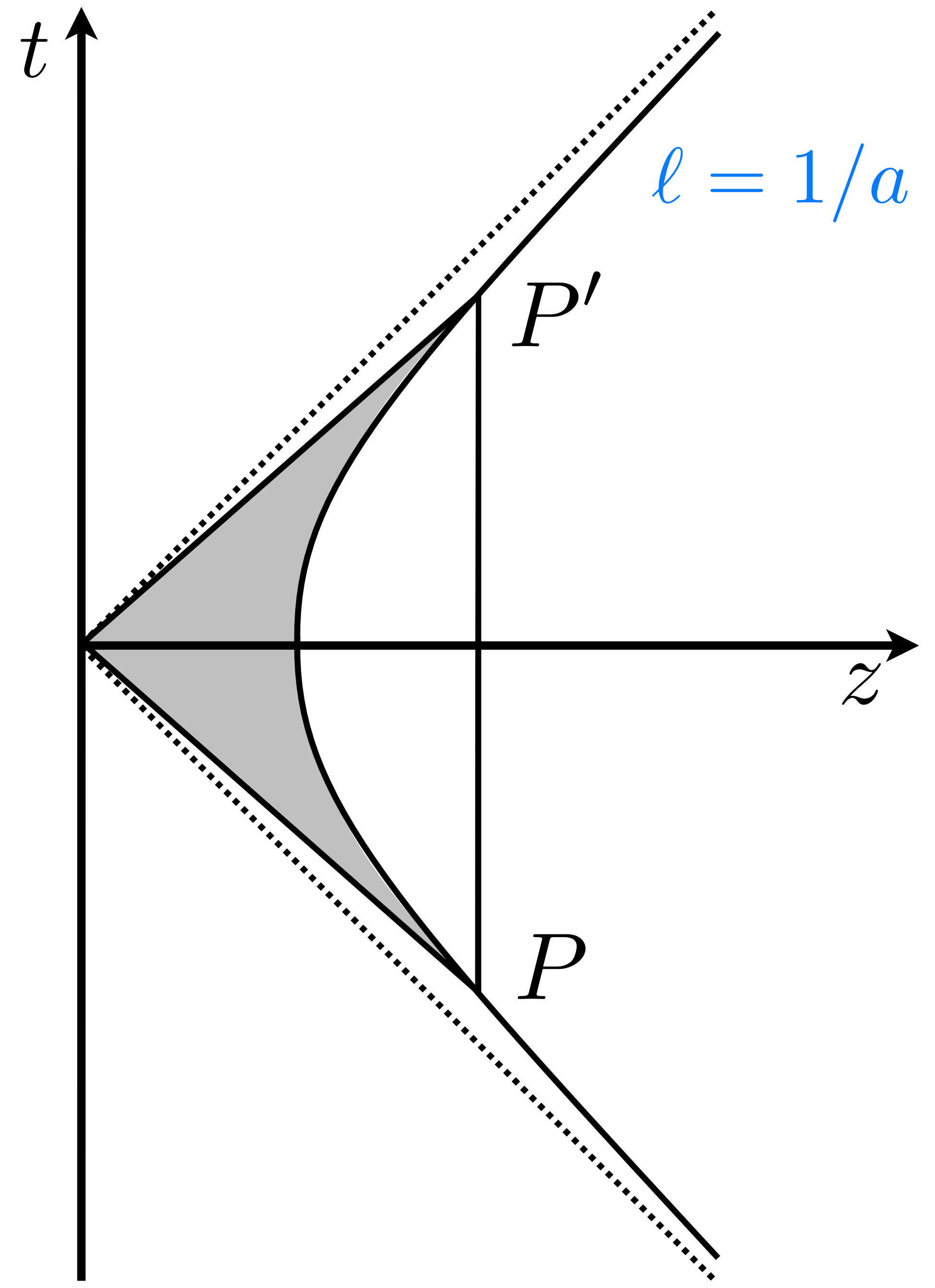
- SPACETIME IN THE QUANTUM REGIME IS A QUANTUM PROCESS
  - states are defined by the continuity relations between quanta
  - a spinfoam is a quantum interaction, but also a spacetime region

- THERE IS NO TIME, THERE IS ONLY CHANGE
  in fact change is everything we measure!

FROM SIMPLICITY TO SINGULARITY RESOLUTION

MAXIMAL ACCELERATION

[Vidotto, Rovelli '13]



$$dA = \frac{\ell^2}{2} d\eta = \frac{1}{2a^2} d\eta$$

η is the boost parameter along the trajectory from P to P′

- Constantly accelerated observer:
  - K generator of boost
  - $E=aK$ generator of proper time evolution

Linear simplicity constraint $\vec{K} = \gamma \vec{L}$

- Lorentzian area: $A = \int_{\mathcal{R}} \frac{1}{\gamma} K^z = \int_{\mathcal{R}} L^z$

$\ell_{min} = \sqrt{8\pi G\hbar}$ $A_{min} = 4\pi G\hbar$ $a_{max} = \sqrt{\frac{1}{8\pi G\hbar}}$

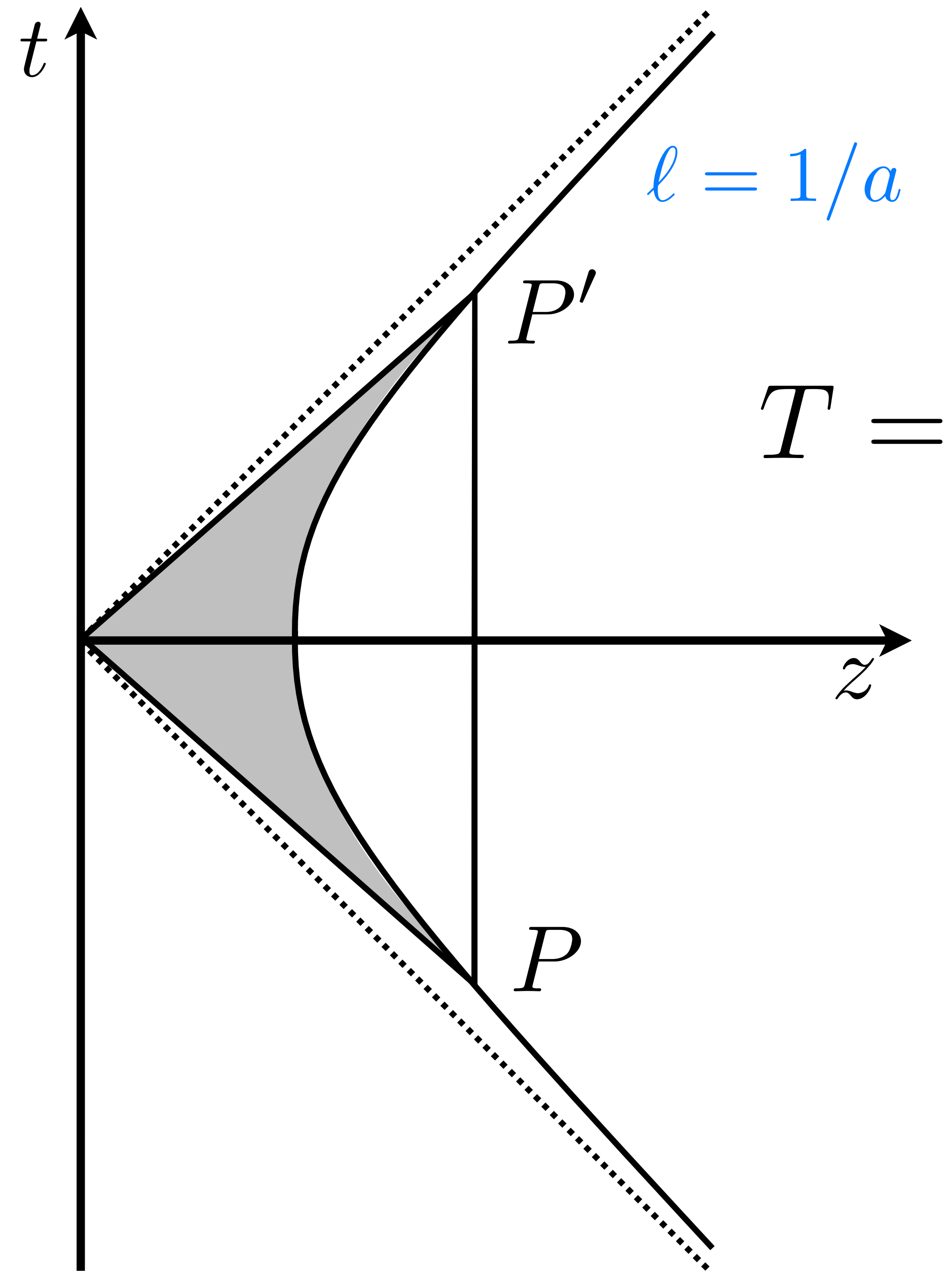
[Cainiello '81]
[Cainiello, Gasperini, Scarpetta '91]
[Bozza, Feoli, Lambiase, Papini, Scarpetta]

HEURISTICS FOR NO CURVATURE SINGULARITIES IN LQG

FROM SIMPLICITY TO BLACK HOLE ENTROPY

THERMODYNAMICS

[Frodden, Gosh, Perez '11]



$$T = \frac{\hbar a}{2\pi}$$

- Constantly accelerated observer:
  - K generator of boost
  - $E=aK$ generator of proper time evolution

$$E = \frac{A}{8\pi G}\, l^{-1} \quad \Rightarrow \quad S_{\rm BH} = \frac{A}{4\,G\hbar}$$

- **Boost Hamiltonian** $\quad H_A = 2\pi \sum_l K_l$

  Define density matrix: $\quad \rho_A = e^{-H_A}$

- **Entanglement Entropy** $\quad S_{\rm EE} = -{\rm Tr}(\rho_A \log \rho_A) = \frac{2\pi}{\hbar}{\rm Tr}(\sum_l K_l\, \rho_A)$

(No unknown degrees of freedom)

$$S_{\rm EE} = \frac{\mathcal{A}_\Sigma}{4G_0}\,.$$

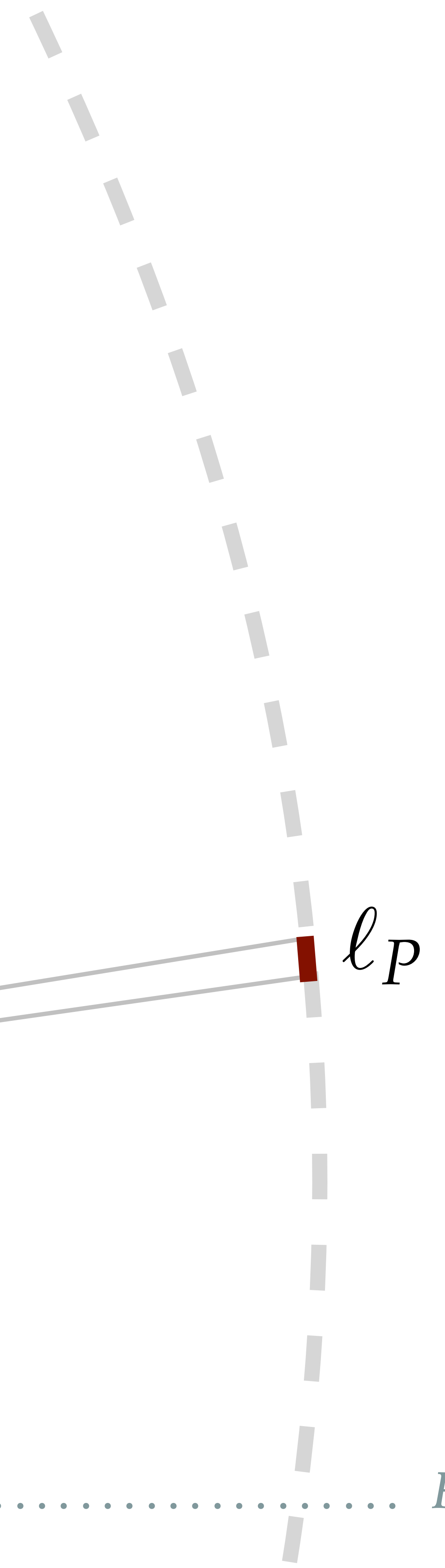
[Bianchi '12]

COSMOLOGICAL CONSTANT

REMOVAL OF IR INFINITIES: FINITENESS OF THE AMPLITUDE

- Early perturbative quantum gravity: **non-renormalizability**
  - Local: *observables at arbitrarily small regions in a continuous manifold*
  - Infinite *renormalization group*
  - Cut-off: *it is a mathematical trick*
- Perturbations methods are some kind of approximation.
- Infinities: we perturb around points that are not really good.
  - Non-perturbative approach: presence of a fundamental scale!
  - Minimal area $a_o = 8\pi G\hbar\gamma\frac{\sqrt{3}}{2}$ → natural UV cut-off
  - Cosmological constant $\Lambda > 0$ → natural IR cut-off

    ↳ *horizon*



$$\phi_{min} = \sqrt{\Lambda}\,\ell_P$$

Han, Fairbairn-Moesburger, 2011
see also Bianchi, Rovelli 2011

REMOVAL OF IR INFINITIES: FINITENESS OF THE AMPLITUDE

- Planck length + horizon = minimal angular resolution j_{max}
- Mathematically a *fuzzy spheres:* spherical harmonics with $SU(2)_q$
- A maximum angular momentum characterizes the representations of $SU(2)_q$ (Majid'88)

$$q = e^{i2\pi/k} \qquad \text{with } k\sim 2$$

- The local rotational symmetry is better described by $SU(2)_q$

  than by SU(2), with $q = e^{i\Lambda l_P^2}$

- Physically: non-commutativity, fuzziness of any angular function, (Connes'94)

  impossibility of resolving small dihedral angles.

- Loop gravity: ϕ is an operator with a discrete spectrum.
- Best angular resolution: $\phi_{min} = \sqrt{2/j_{max}}$ with $j_{max} \sim \frac{1}{l_P^2\,\Lambda}$ (Major'99)

FRANCESCA VIDOTTO'S INTRODUCTION TO

COVARIANT LOOP QUANTUM GRAVITY

LECTURE 5

An application of the covariant formalism: COSMOLOGY

QUANTUM COSMOLOGY

[Bianchi, Rovelli, Vidotto'10]

canonical / covariant
quantization

gravity

$$R_{\mu\nu} - \tfrac{1}{2}\, g_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

quantum gravity

$$W_v = (P_{SL(2,\mathbb{C})} \circ Y_\gamma\ \psi_v)(\mathbb{1})$$

symmetry
reduction

canonical
quantization

cosmology

$$ds^2 = dt^2 - a^2(t)\, d^3\vec{x}$$

+ perturbations

quantum
cosmology

LOOP QUANTUM COSMOLOGY

- (canonical) LQG

Input:

- SU(2) group variables
  - Minimal area gap
- Hamiltonian constraint
  - Holonomy corrections
  - Inverse-volume corrections

Output:

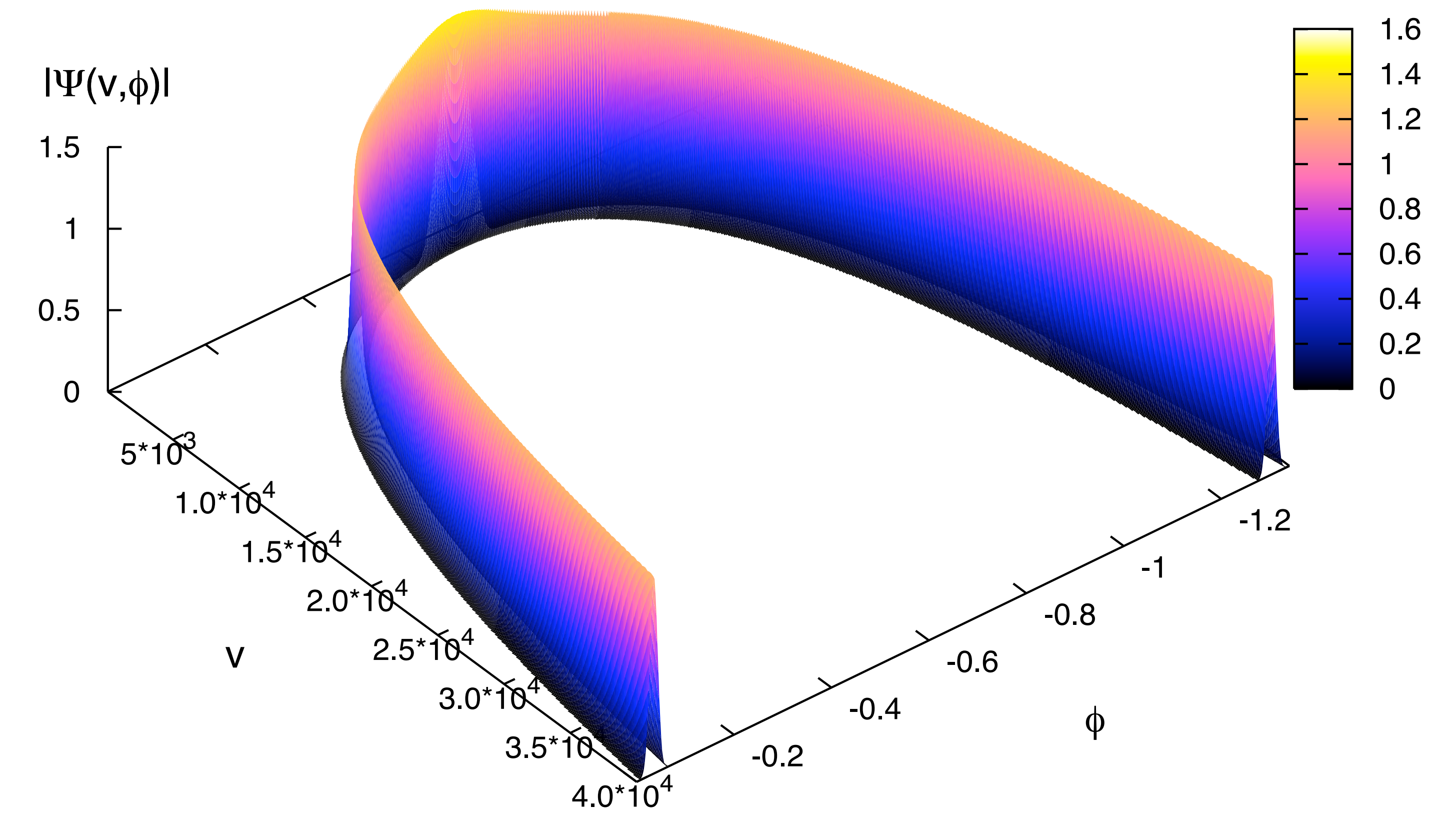
- Singularity resolution
  - No need to violate the SEC
- Modified Friedmann equations
  - Wave-packet non-singular trajectories
- Modified Muhanov-Sasaki equations
  - Predictions for the CMB

- [Bojowald '99]

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right)$$

$$v'' - \left(1 - 2\frac{\rho}{\rho_c}\right)\nabla^2 v - \frac{z''}{z}v = 0$$

- [Ashtekar, Pawlowski, Singh '04]



THE STANDARD SCENARIO $\longrightarrow$ LOOP QUANTUM COSMOLOGY

[Agullo, Wang, Wilson-Ewing 2301.10215]

- SINGULARITY
- BIG BANG: The universe starts hot and dense in a quantum regime $\Rightarrow$ quantum fluctuations
- INFLATION: a non-identified field governs the dynamics of the universe driving the expansion and putting in place the seeds of structure formation
- INITIAL CONDITION: kinetic energy of the inflaton should dominate over the potential $\Rightarrow$ power spectra depends on the choice of vacuum

- NO SINGULARITIES: maximal curvature
- BIG BOUNCE: maximal energy density deep quantum regime $\Rightarrow$ tunnelling
- INFLATION IS GENERIC: no fine-tuned initial conditions are required
- INITIAL CONDITION: the contracting phase makes the inflaton to climb up the potential

QUANTUM COSMOLOGY

[Bianchi, Rovelli,Vidotto'10]

canonical / covariant quantization

gravity

$$R_{\mu\nu} - \tfrac{1}{2}\, g_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

quantum gravity

$$W_v = (P_{SL(2,\mathbb{C})} \circ Y_\gamma \; \psi_v)(\mathbb{1})$$

symmetry reduction

cosmology

$$ds^2 = dt^2 - a^2(t)\, d^3\vec{x}$$

+ perturbations

quantum cosmology

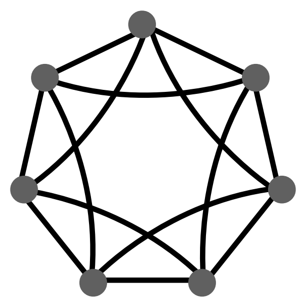
IN THIS LECTURE

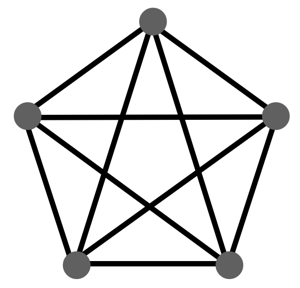
- THEORY: *Covariant* Loop Quantum Gravity (*Spinfoam*)
- STATE: Cosmological Lorentzian Spinfoam State — with Rovelli and Bianchi
- BOUNCE: Semiclassical techniques — with Han, Liu, Qu, and Zhang
- QUANTUM FLUCTUATIONS: Numerical Evaluation — with Gozzini and Frisoni
- FUTURE ROADMAP

GRAPH STATES

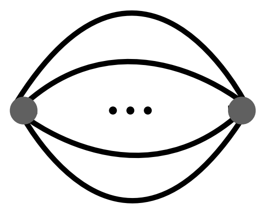
[Borja, Garay, Vidotto 2011]

- Restrict the states to a fixed graph with a finite number N of nodes.
  This defines an approximated kinematics of the universe,
  inhomogeneous but truncated at a finite number of cells.





- The graph captures the large scale d.o.f. obtained averaging the metric
  over the faces of a cellular decomposition formed by N cells.



- The full theory can be regarded as an expansion for growing N.
  For instance FRW cosmology corresponds to the lower order where there is only
  a regular cellular decomposition: the only d.o.f. is given by the volume.

- Different graphs can be useful to model different physical situations.

FEW-NODE THEORY: REGGE CALCULUS

[Collins & Williams '72]

- IDEA — Evolve one or few tetrahedra, triangulating a 3-sphere.
- PROBLEM — Compare the evolution for 5, 16 and 600 tetrahedra.
- RESULT — The qualitative behavior is the same!

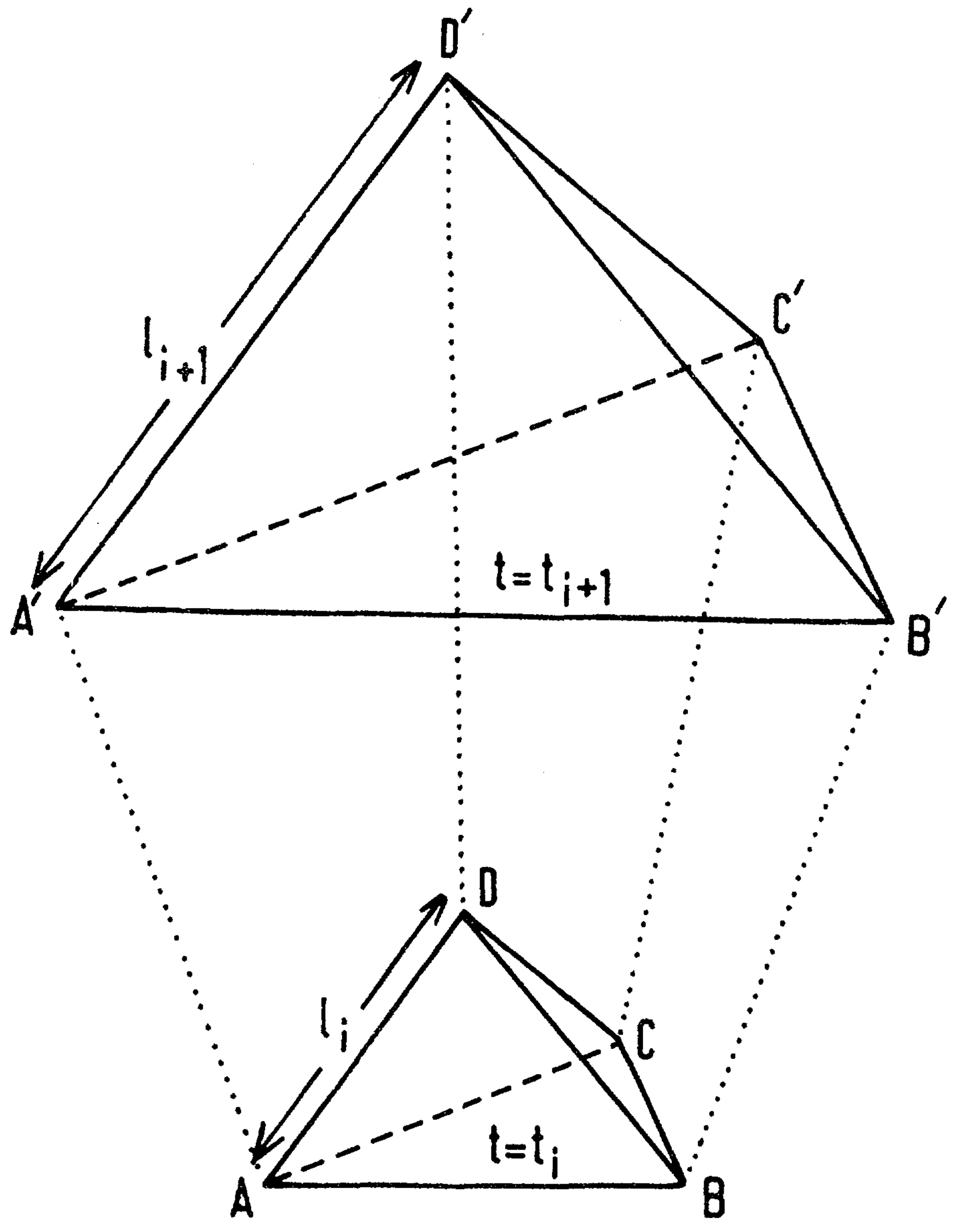


FIG. 1. Diagram illustrating a 4-dimensional block.

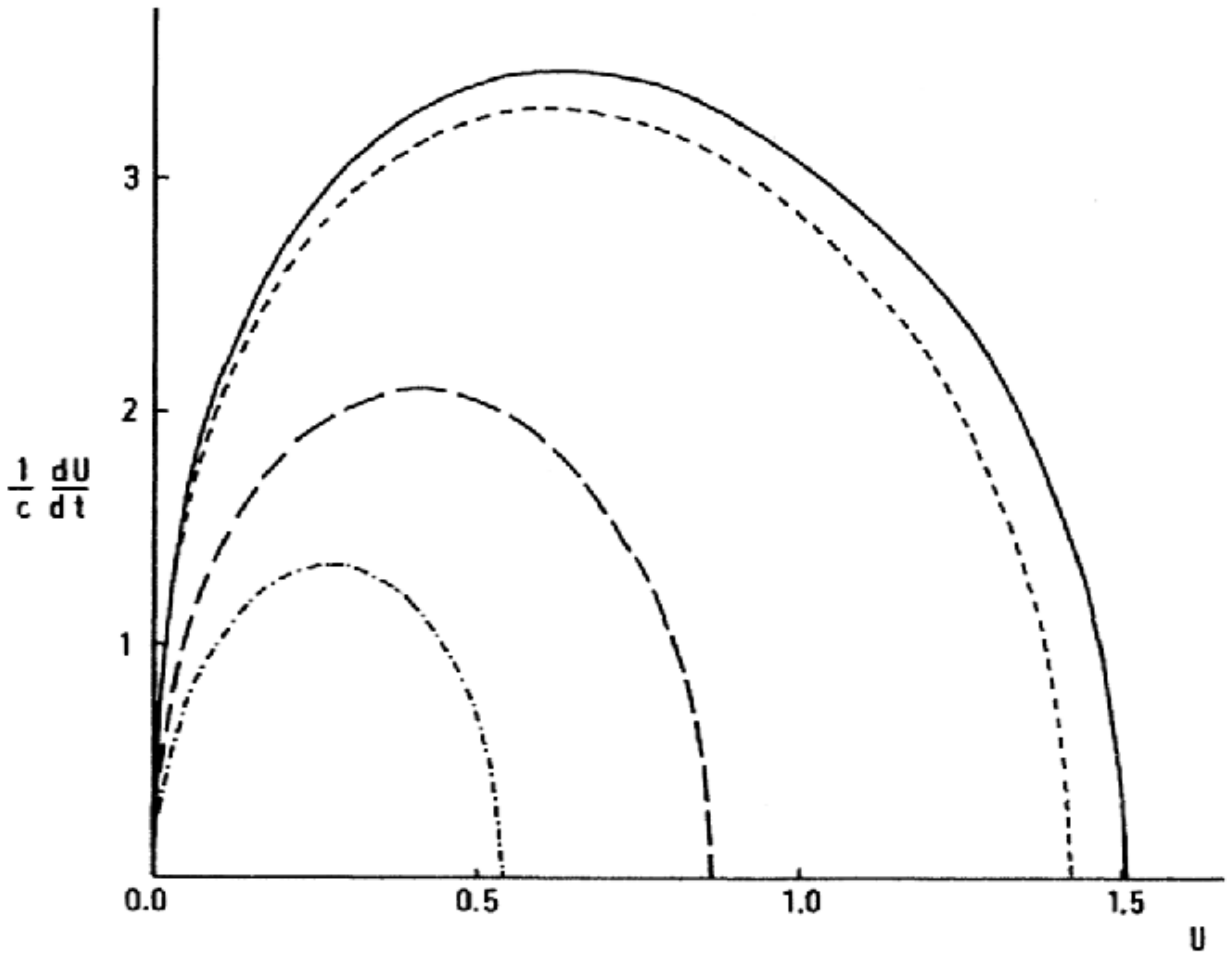
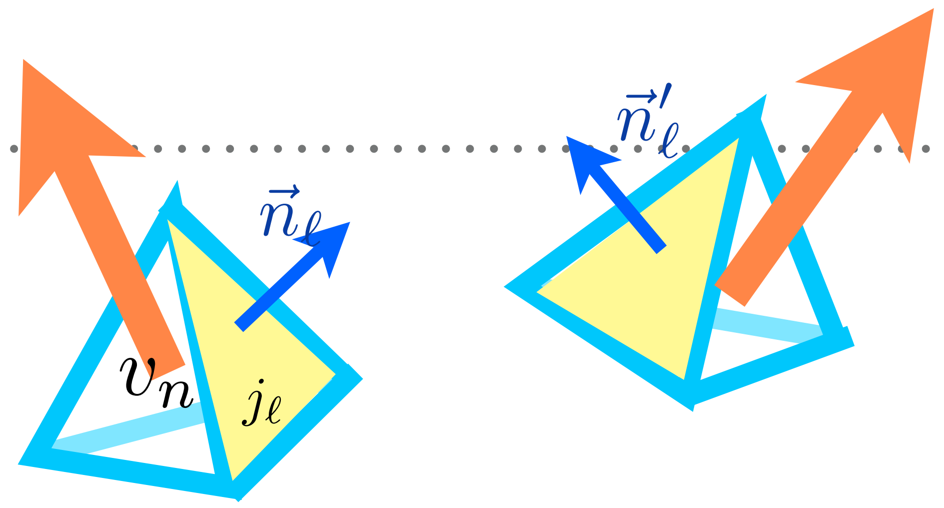


FIG. 2. Rate of change of the volume of the universe plotted against the volume for $MG/c^2 = 1$; analytic solution ———, 600-tetrahedron model - - - - - - -, 16-tetrahedron model — — — —, 5-tetrahedron model —·—·—·.

EXTRINSIC COHERENT STATES



- Spinnetwork states $|\Gamma, j_\ell, v_n\rangle \leftrightarrow$ Coherent states $|\Gamma, z_\ell, \vec{n}_\ell, \vec{n}'_\ell\rangle$

$$\psi_{H_\ell}(h_\ell) = \int_{SU(2)^N} dg_n \prod_{\ell=1}^{L} K_t(\, g_{s(\ell)}\, U_\ell\, g_{t(\ell)}^{-1}\; H_\ell^{-1})$$

$H_\ell \in SL(2,\mathbb{C})$ [Bianchi, Magliaro, Perini '09]

"group average" to get gauge invariant states

The heat kernel K_t peaks each U_ℓ on H_ℓ

- Geometrical interpretation for the labels $(z_\ell, \vec{n}_\ell, \vec{n}'_\ell)$ [Freidel, Speziale '10]

  $\vec{n}_\ell, \vec{n}'_\ell$ are the 3d normals to the faces of the cellular decomposition;

- $Im(z_\ell) \leftrightarrow$ curvature at the faces and $Re(z_\ell) \leftrightarrow$ area of the face

$$Re(z_\ell) = \theta(\gamma K + \Gamma)$$

- Hom&Iso coherent states $|\Gamma, z\rangle$

  $\vec{n}_\ell, \vec{n}'_\ell$ fixed by requiring a regular cellular decomposition [Marcianò, Magliaro, Perini, Rovelli, FV…]

- in terms of the scale factor $Re(z) \sim \dot{a}$ and $\sqrt{Im(z)} \sim a$

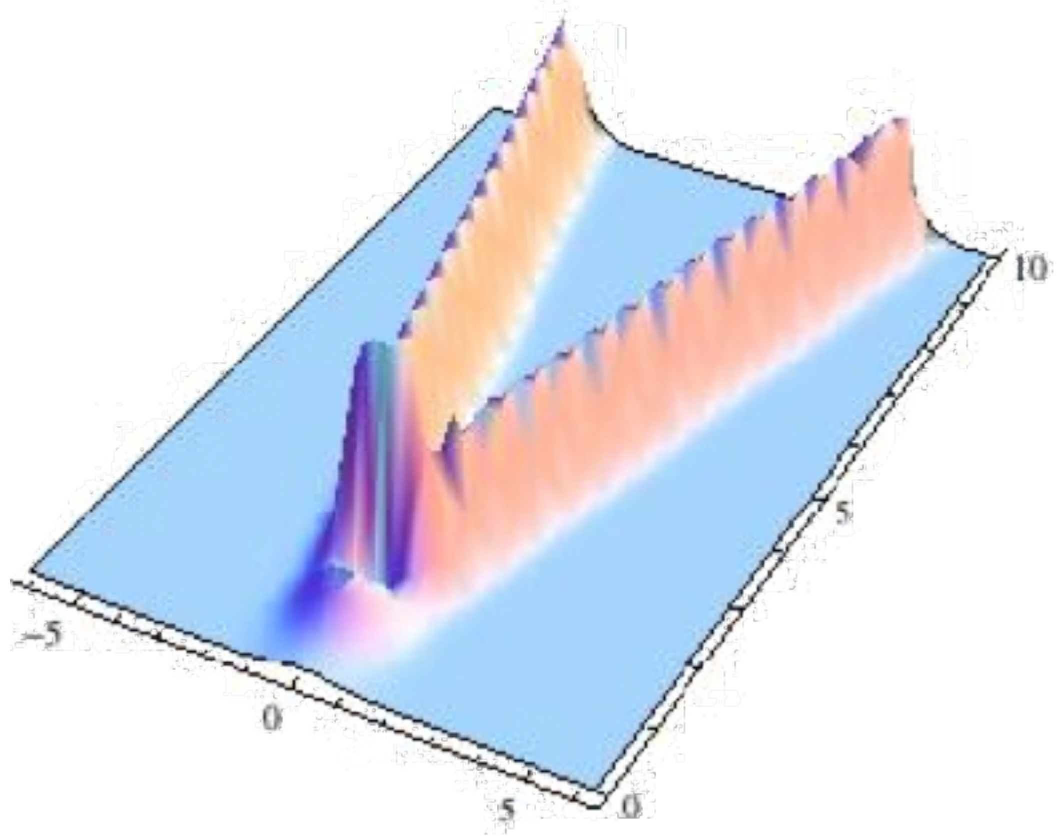
SEMICLASSICAL REGIME

[Bianchi, Krajewski, Rovelli, Vidotto'11]

- LQG coherent states
  peaked on a homogenous and isotropic geometry

- Spinfoam amplitude with an effective Λ:

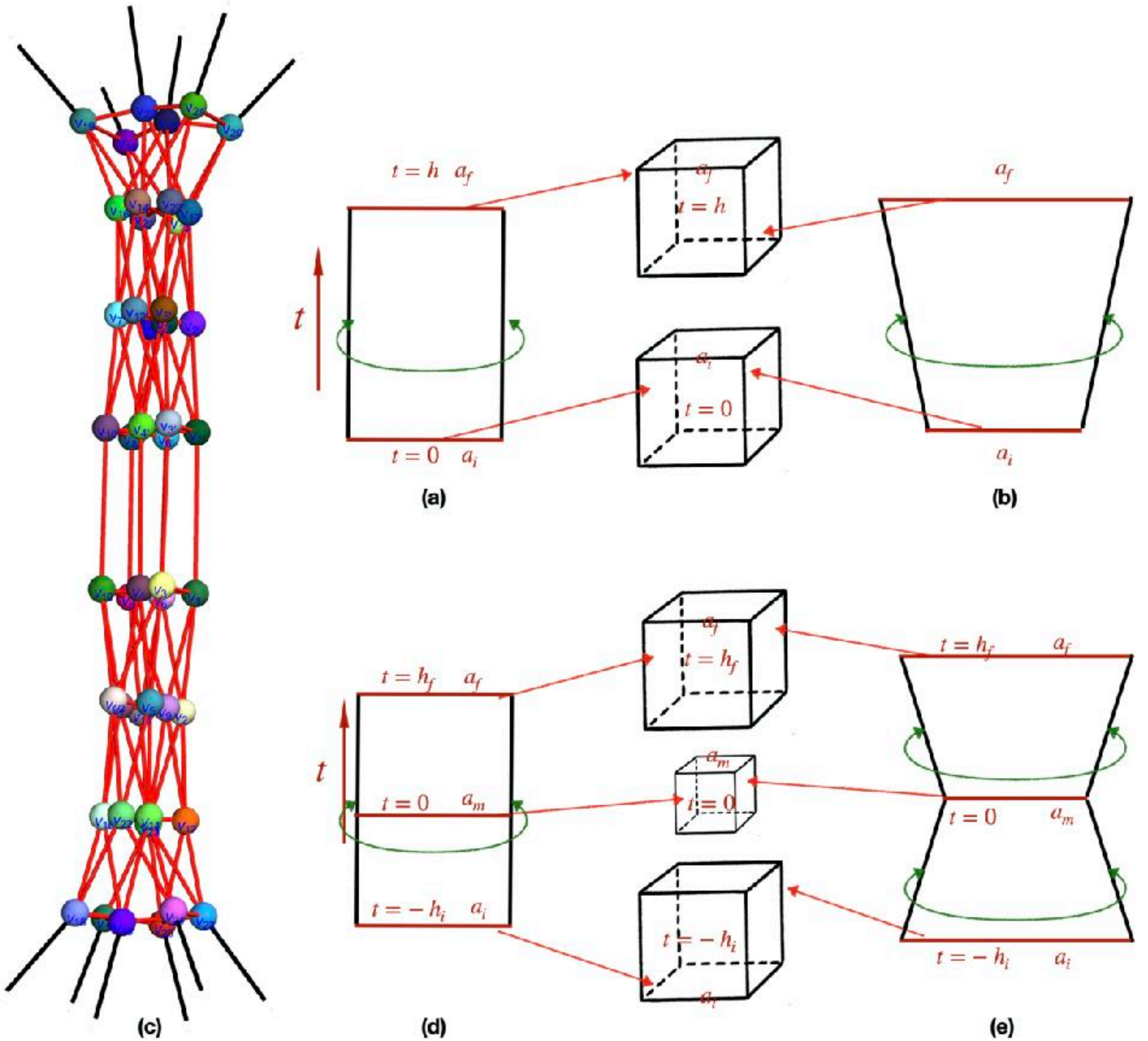
$$Z_{\mathcal{C}} = \sum_{j_f, \mathtt{v}_e} \prod_f (2j+1) \prod_e e^{i\lambda \mathtt{v}_e} \prod_v A_v(j_f, \mathtt{v}_e)$$



BOUNCE FROM SPINFOAM

[Han, Liu, Qu, Vidotto, Zhang '24]

- Hypercube ~ torus
- Coupling with a scalar field
- Same initial and final state
  but for a flip
  in the extrinsic curvature
- Suppressed but non-vanishing amplitude for the process
- In the semi-classical limit we get an action with extra higher-derivative terms



1^{ST}-ORDER FACTORIZATION

[Vidotto '11]

- classical dynamics

$$H = const\,(a\dot{a}^2 - \frac{\Lambda}{3}a^3) = 0 \qquad \dot{a} = \pm\sqrt{\frac{\Lambda}{3}}a$$

1^{ST}-ORDER FACTORIZATION

[Vidotto '11]

- classical dynamics

$$S_H = const \int \mathrm{d}t \, (a\dot{a}^2 + \frac{\Lambda}{3} a^3)\Big|_{\dot{a} = \pm\sqrt{\frac{\Lambda}{3}} a} = const \frac{2}{3} \sqrt{\frac{\Lambda}{3}} (a_f^3 - a_i^3)$$

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- quantum dynamics

$$W(a_f, a_i) = e^{\frac{i}{\hbar}S_H(a_f,a_i)} = W(a_f)\overline{W(a_i)}$$

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- loop dynamics

$$\langle W|\psi_{H_{(z,z')}}\rangle = W(z, z') = W(z)\,\overline{W(z')}$$

1^{ST}-ORDER FACTORIZATION

[Vidotto '11]

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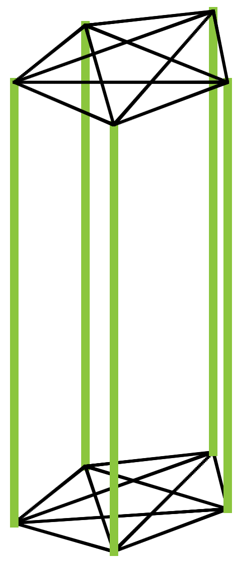
- loop dynamics $$\langle W|\psi_{H_{(z,z')}}\rangle = W(z,z') = W(z)\,\overline{W(z')}$$

order (0)



$$= \; W_0(h_\ell, h_{\ell'}) \;=\; \delta_{\Gamma_\ell}(h_\ell, h_{\ell'})$$

1^{ST}-ORDER FACTORIZATION

[Vidotto '11]

- classical dynamics

$$S_H = const \int \mathrm{d}t \, (a\dot{a}^2 + \frac{\Lambda}{3} a^3)\Big|_{\dot{a} = \pm\sqrt{\frac{\Lambda}{3}}a} = const \frac{2}{3}\sqrt{\frac{\Lambda}{3}}(a_f^3 - a_i^3)$$

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- loop dynamics

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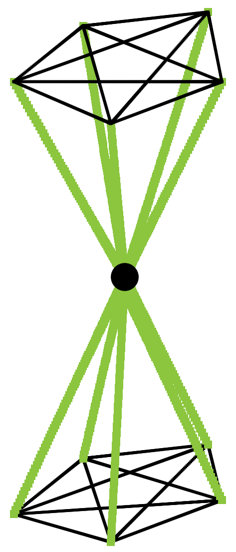
order (1)



$$W_{\mathcal{C}_\infty}(z', z) = \int h_\ell \int h'_\ell \; \overline{\psi_{z'}(h'_\ell)} \; W_1(h'_\ell, h_\ell) \; \psi_z(h'_\ell)$$

$$W_1(h'_\ell, h_\ell) = \int_{SL(2,\mathbb{C})} \prod_{n=1}^{N-1} dG_n \prod_{\ell=1}^{L} P(h_\ell, G_\ell) P(h'_\ell, G'_\ell)$$

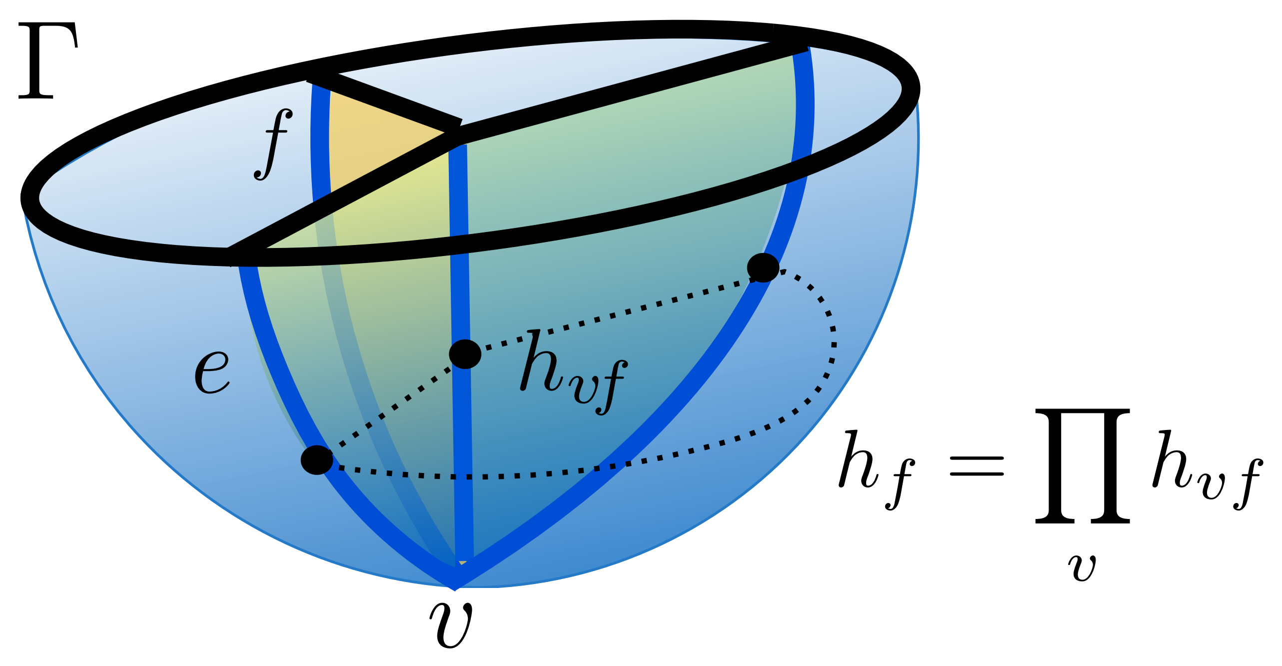
$$G_\ell = G_{n_s} G_{n_t}^{-1}$$

SPINFOAM HARTLE-HAWKING STATES

[Bianchi, Rovelli,Vidotto'10]

- Hartle-Hawking states:

$$\psi_H(q) = \int_{\partial g = q} Dg\, e^{iS[g]}$$



- Spinfoam HH states:

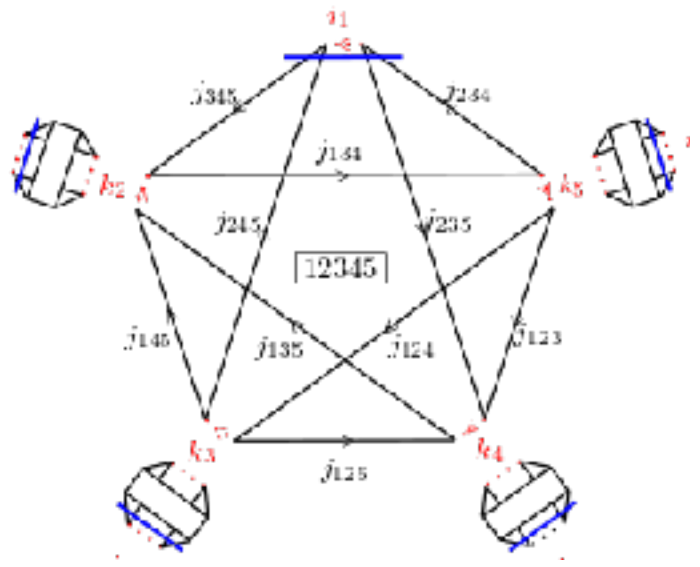
$$W_{\mathcal{C}}(h_l) = \int_{SU(2)} dh_{vf} \prod_f \delta(h_f) \prod_v A(h_{vf})$$

CORRELATIONS

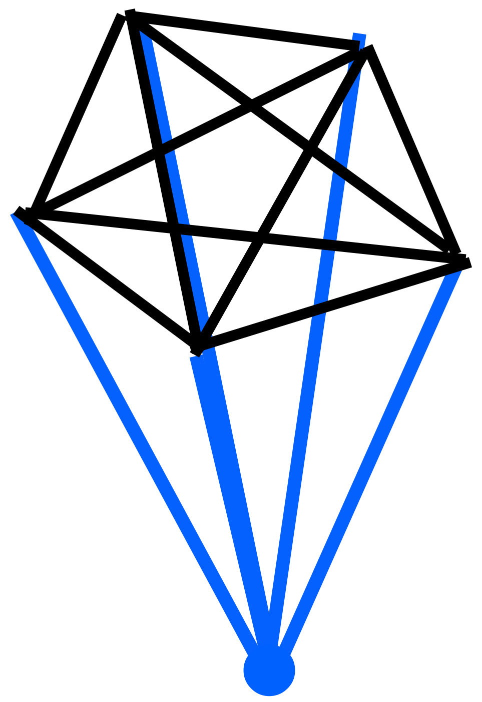
5-CELL PENTACHORDS

[Frisoni, Gozzini, Vidotto '22]

- Simplest regular 4-polytope



- Regular triangulation of S_3



OBSERVABLES

- Area
- Volume

$$\langle O \rangle = \langle \psi_o | O | \psi_o \rangle$$

- Dihedral Angles $\Rightarrow$ Curvature

spread

- **Correlations**

$$C(O_1, O_2) = \frac{\langle \psi_o | O_1 O_2 | \psi_o \rangle - \langle O_1 \rangle \langle O_2 \rangle}{(\Delta O_1)\ (\Delta O_2)}$$

$$\Delta O = \sqrt{\langle \psi_o | O^2 | \psi_o \rangle - \langle O \rangle^2}$$

- **Entanglement**

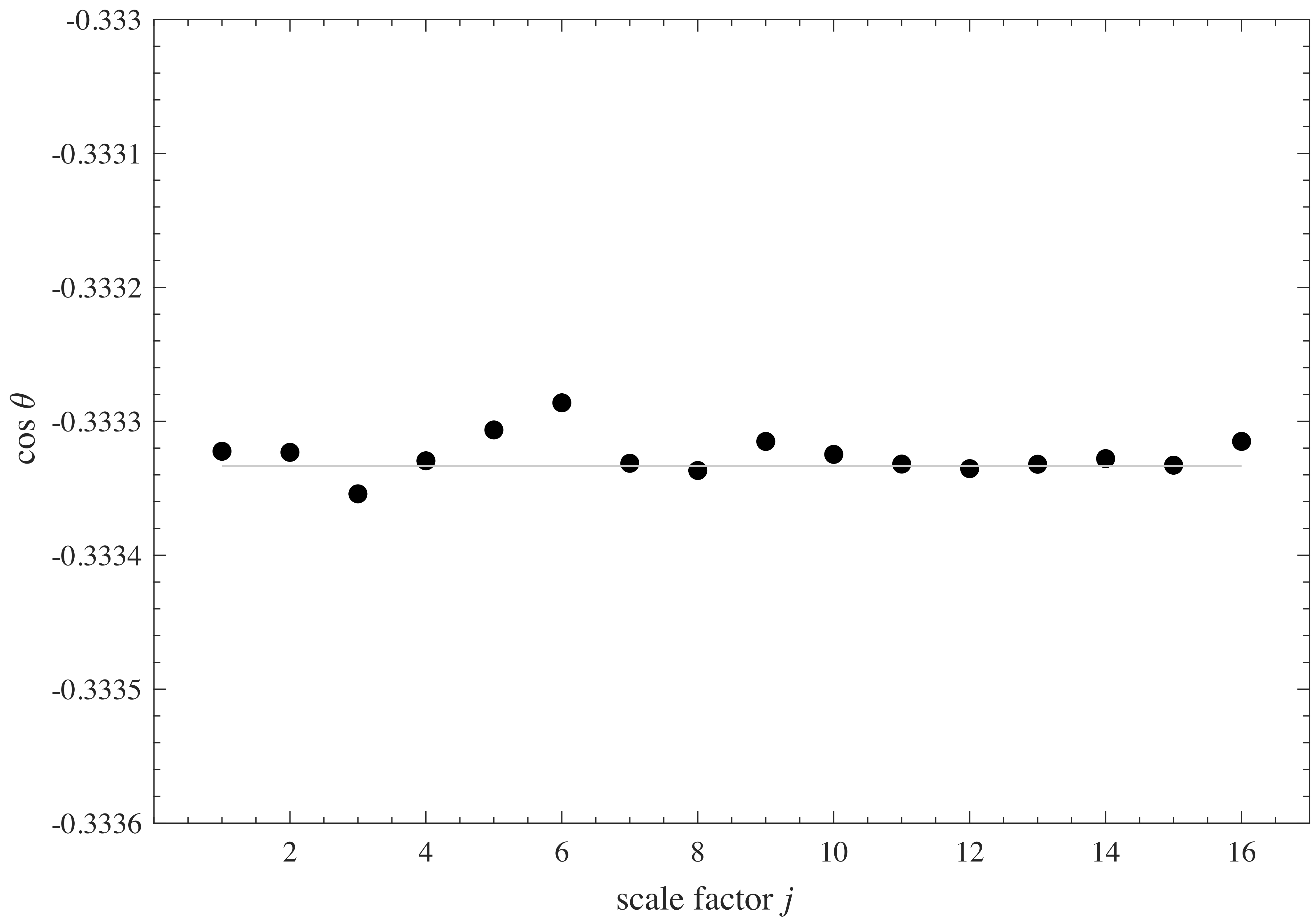
RESULTS

Gozzini, Vidotto 1906.02211

1. 3-sphere as emerging geometry

2. large fluctuations

3. large correlations



RESULTS

Gozzini, Vidotto 1906.02211

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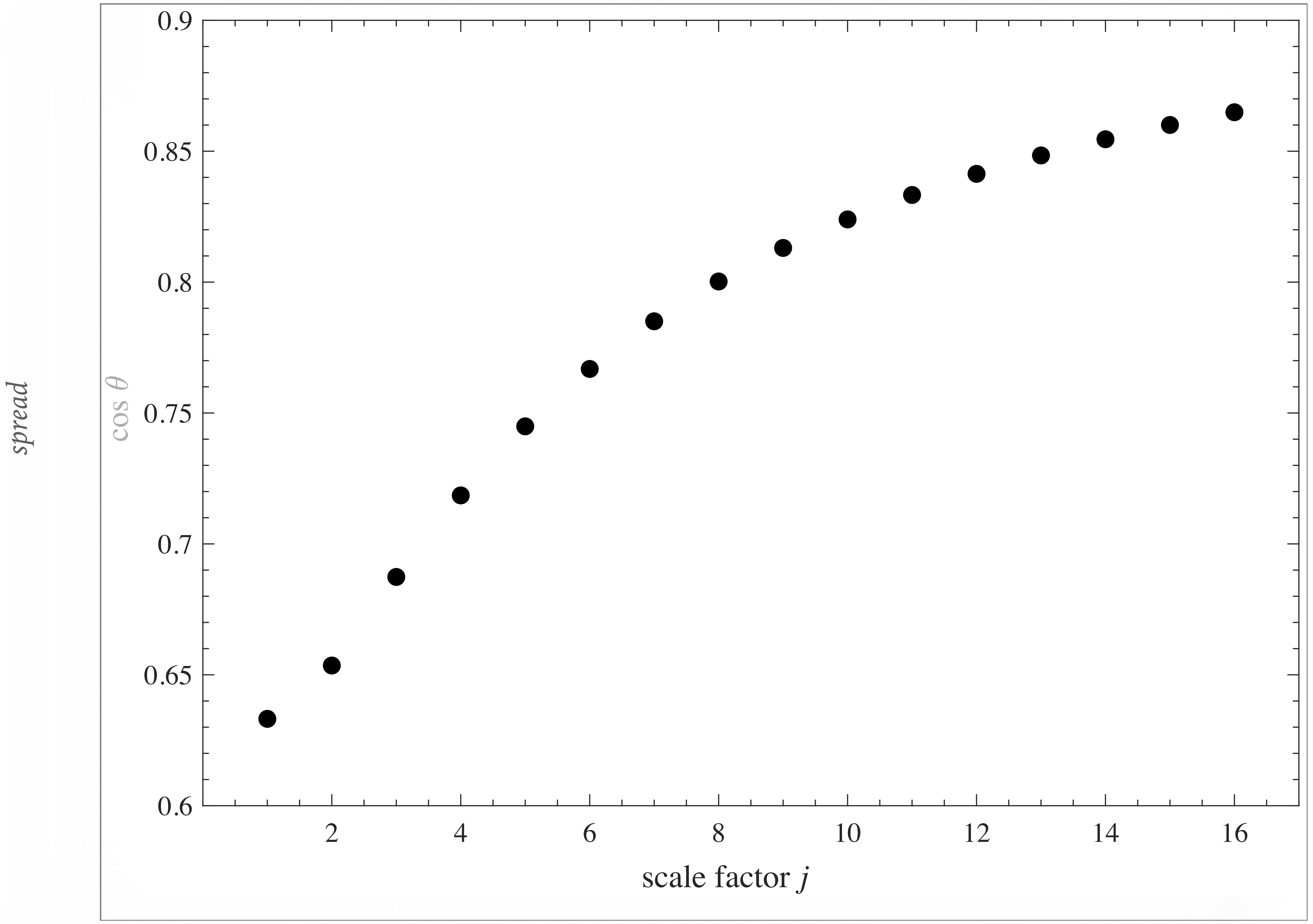

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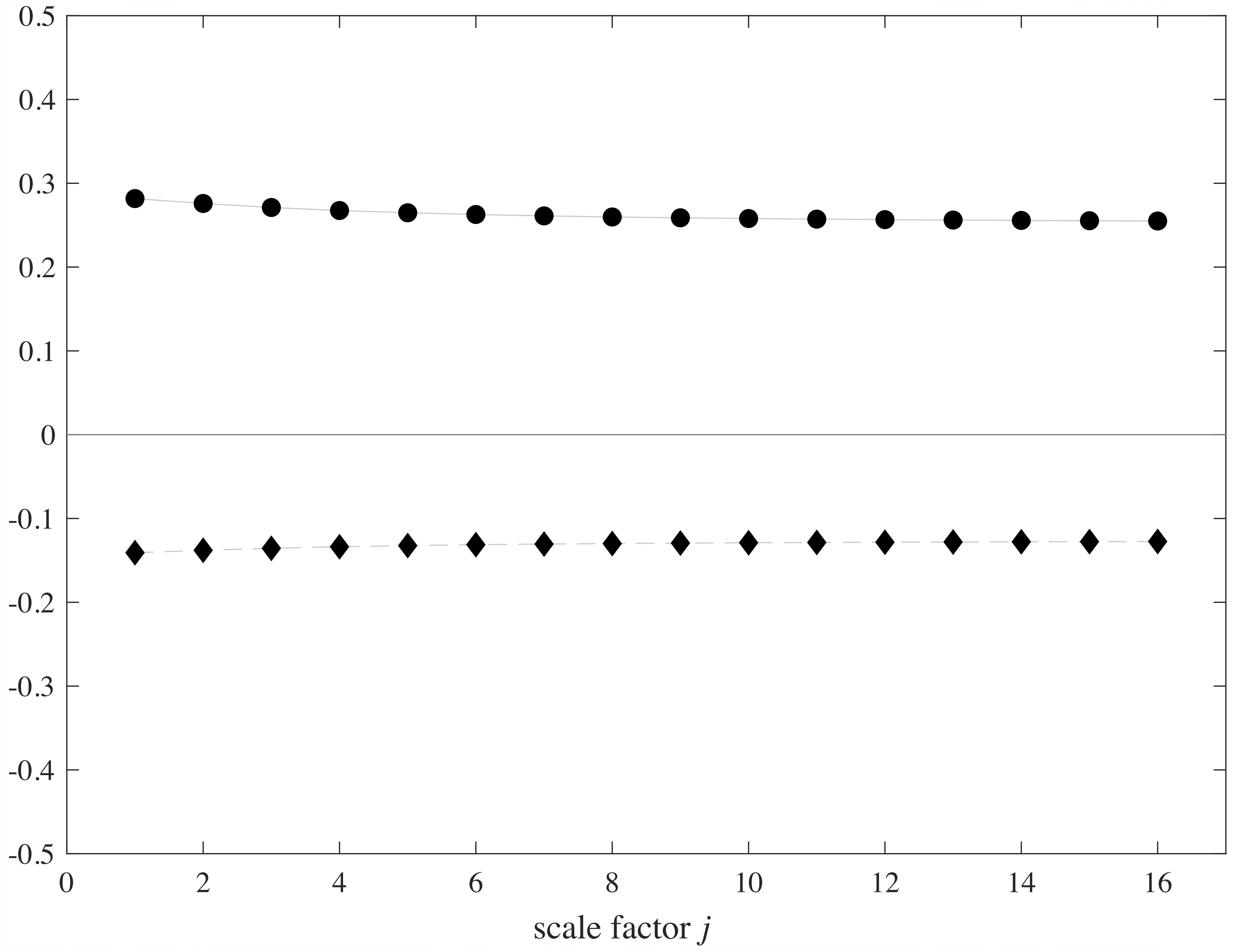

RESULTS

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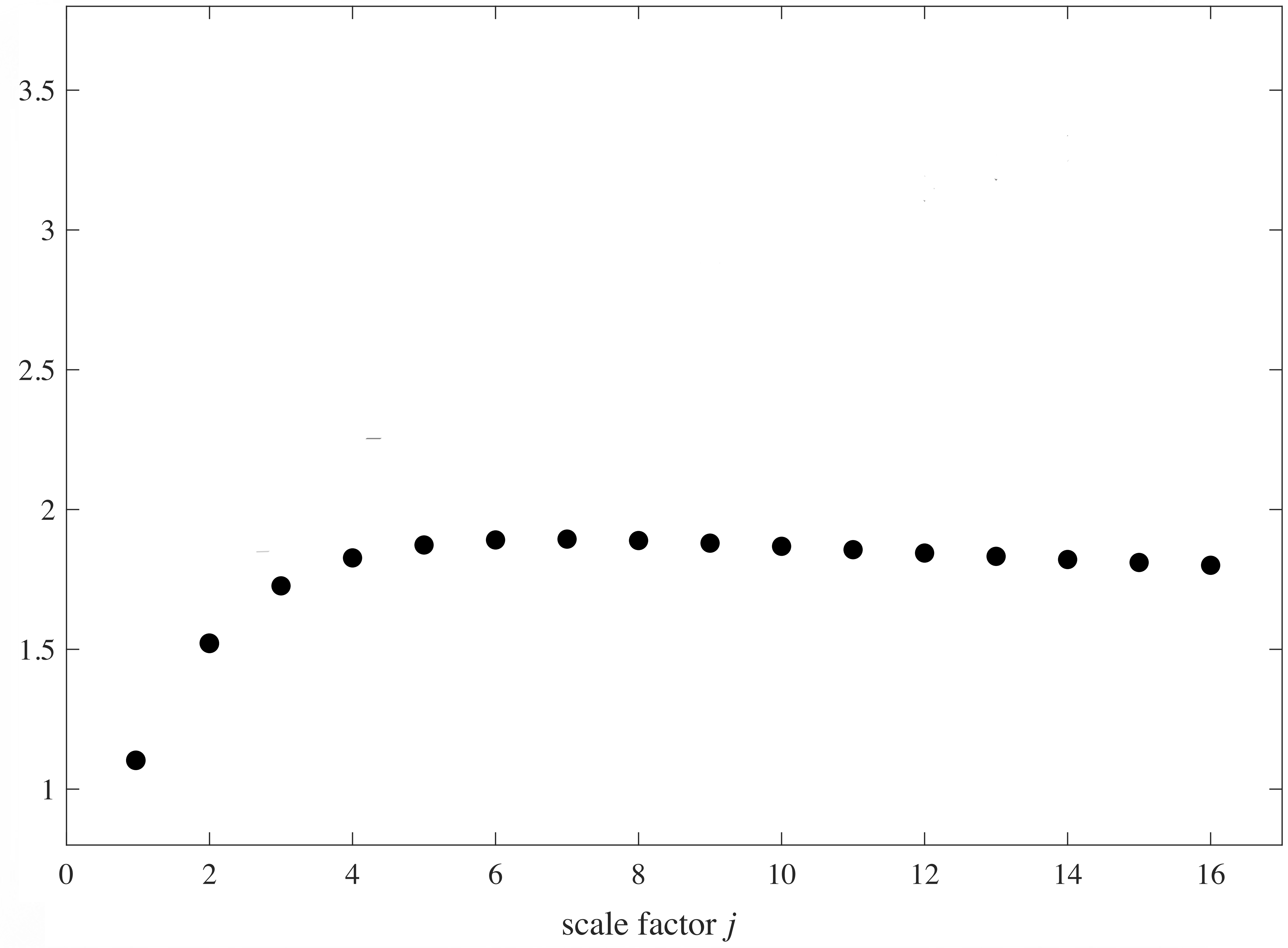
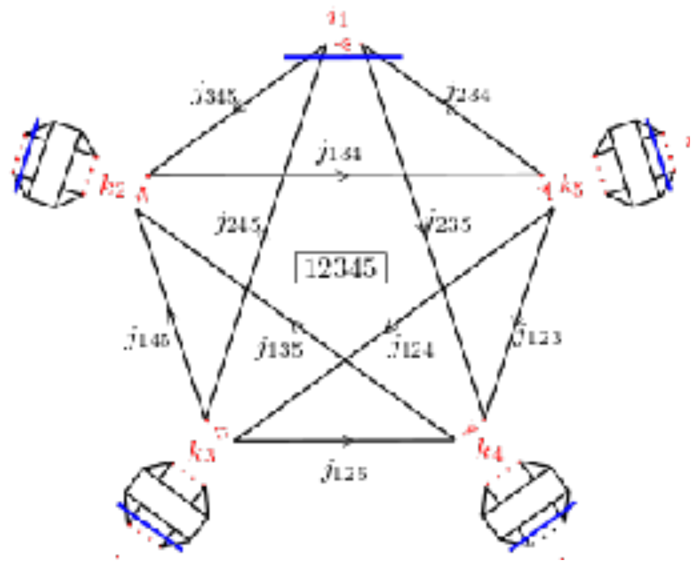
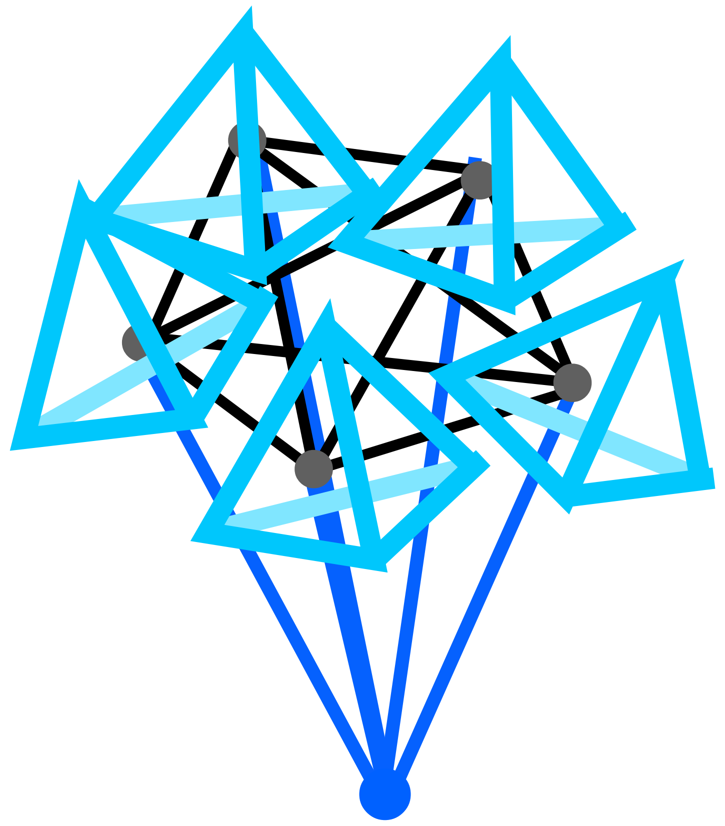


GRAPH REFINEMENT

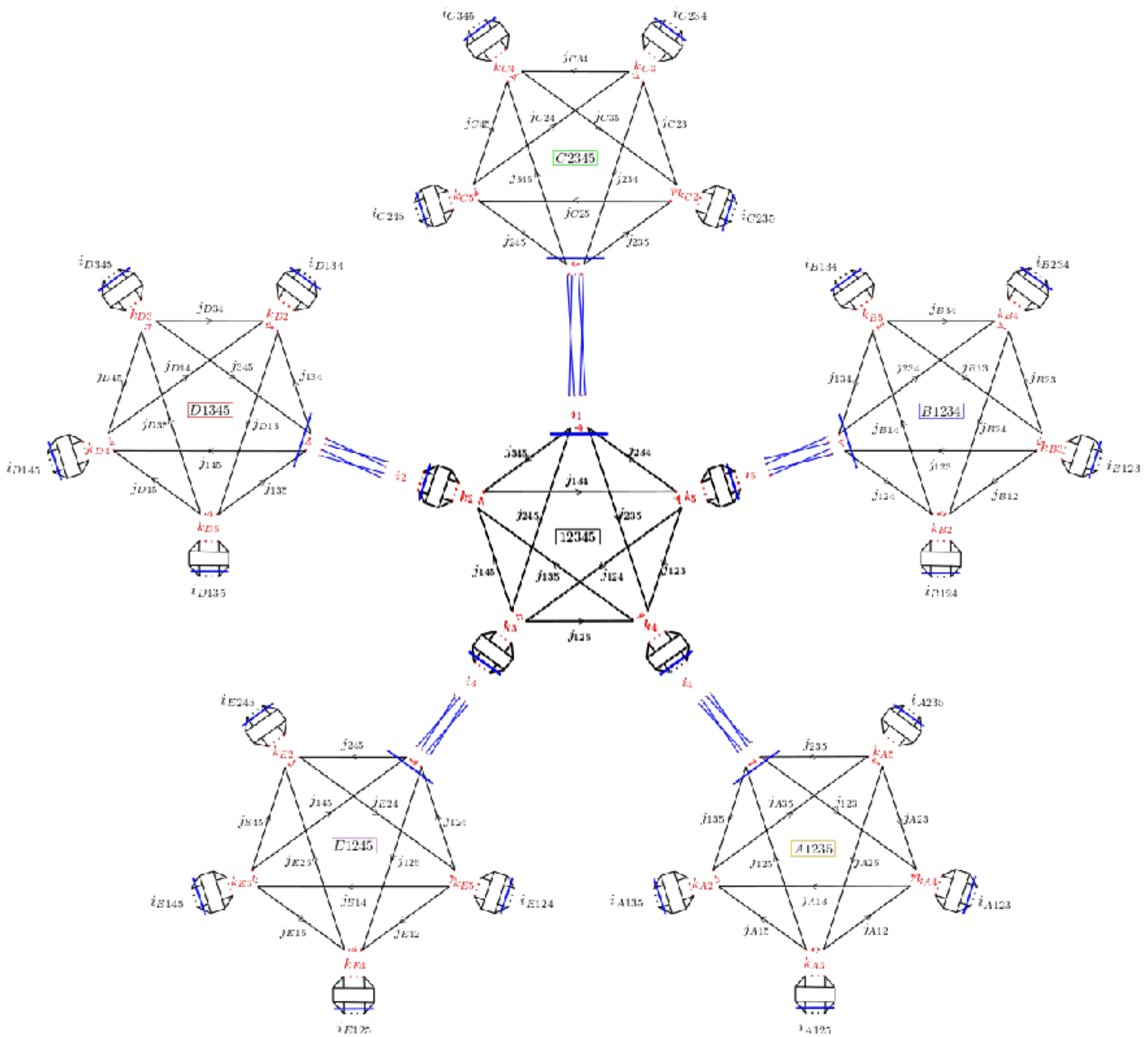
[Frisoni, Gozzini, Vidotto '22]

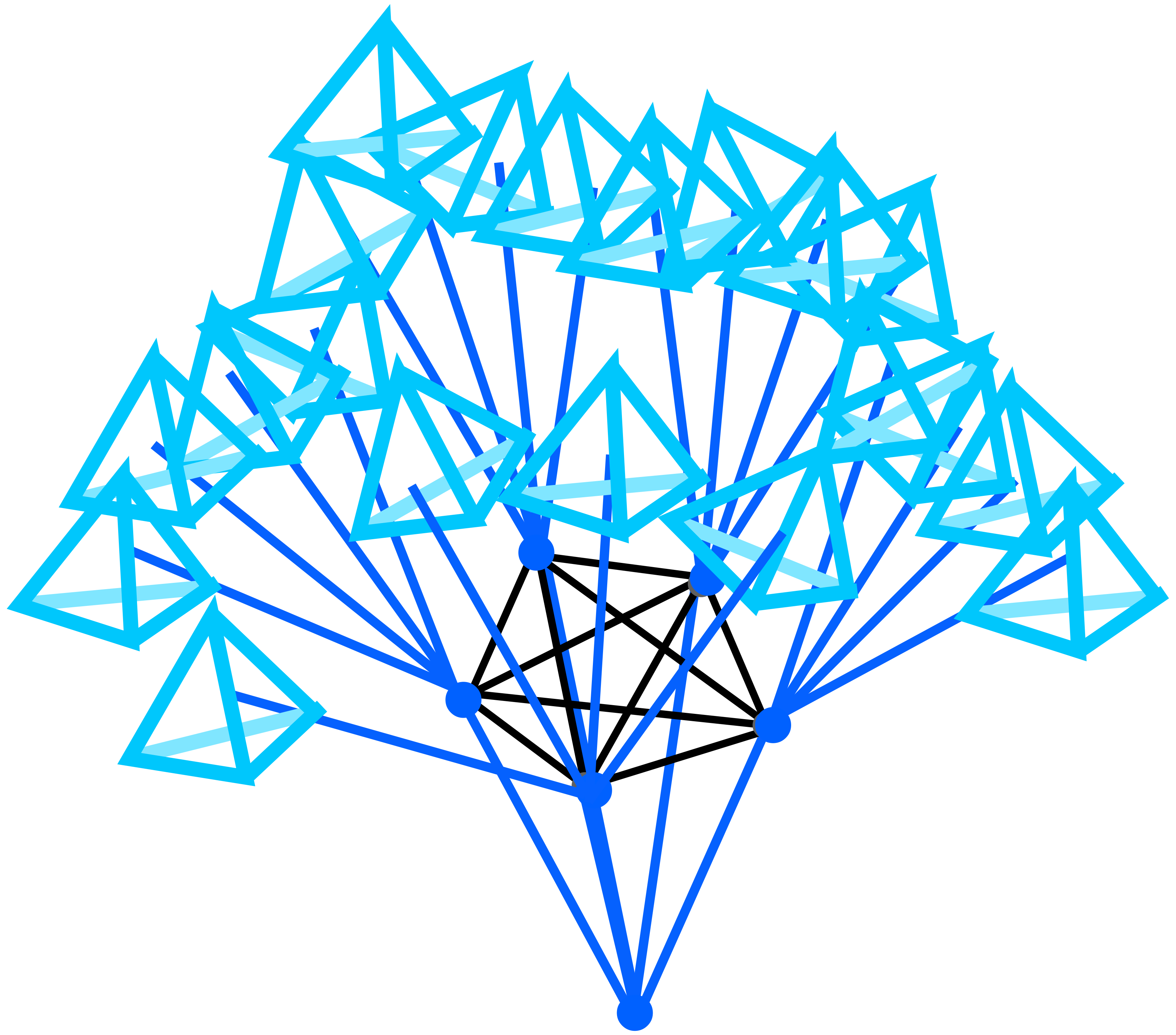




GRAPH REFINEMENT

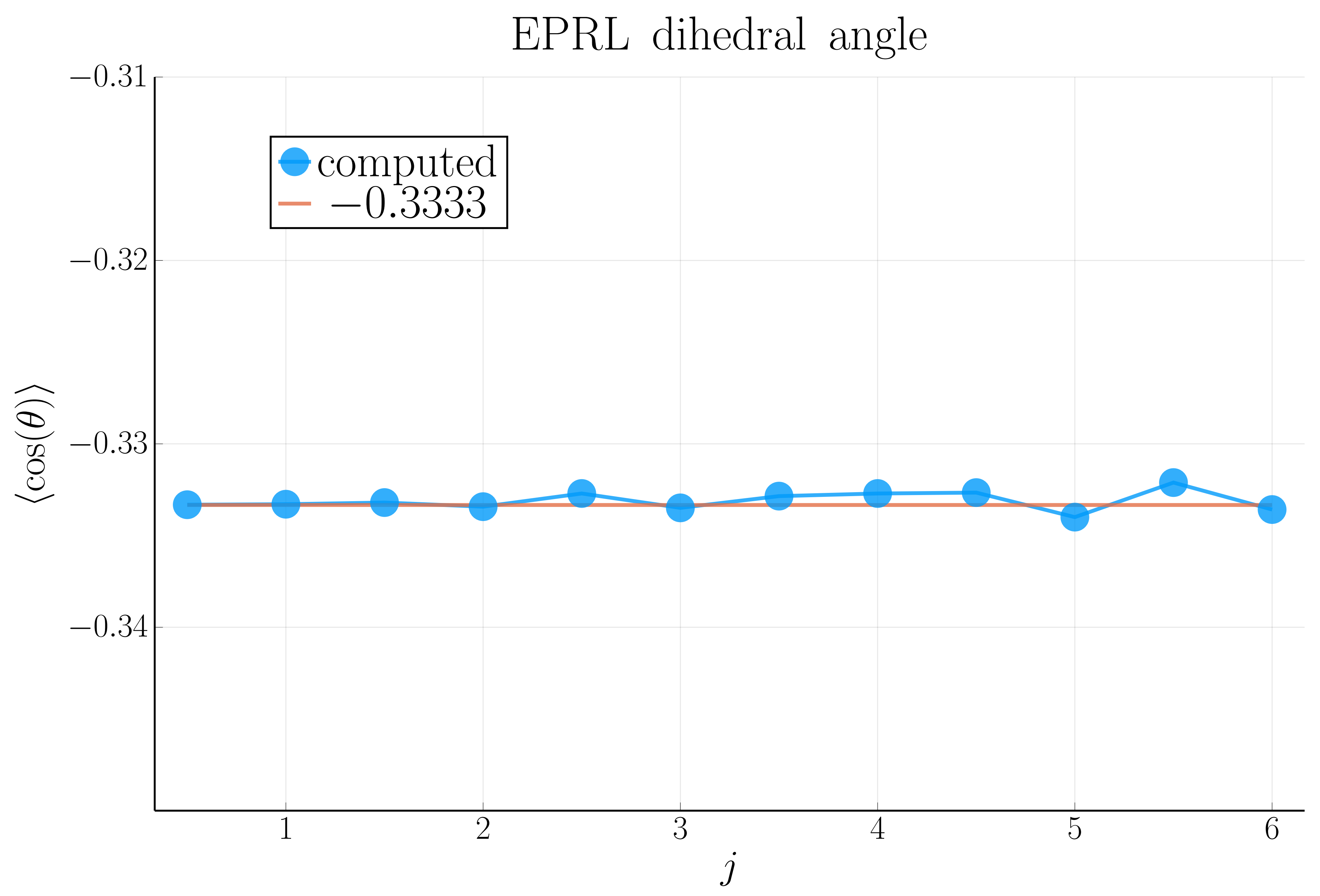
[Frisoni, Gozzini, Vidotto '22]

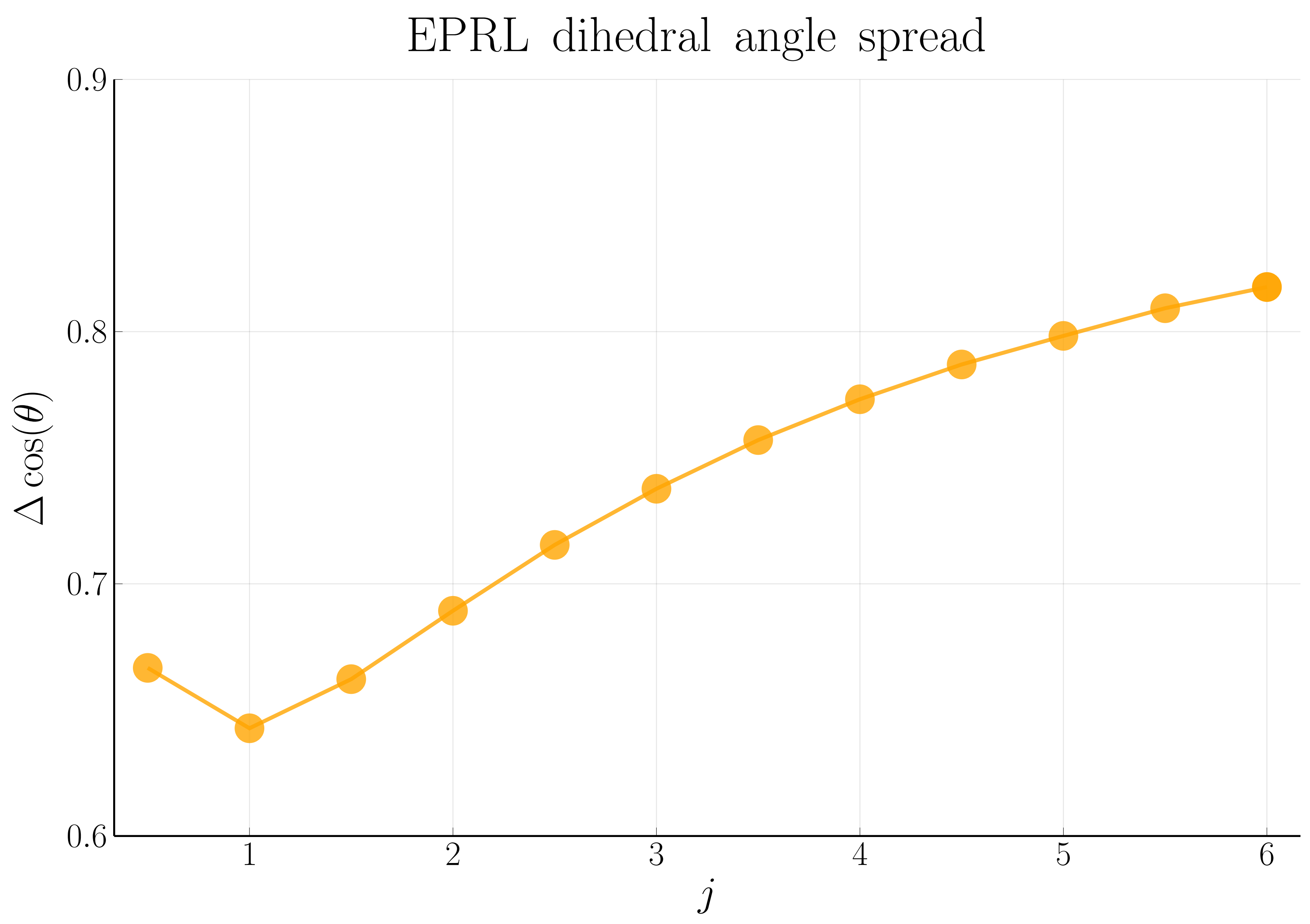




DIHEDRAL ANGLE

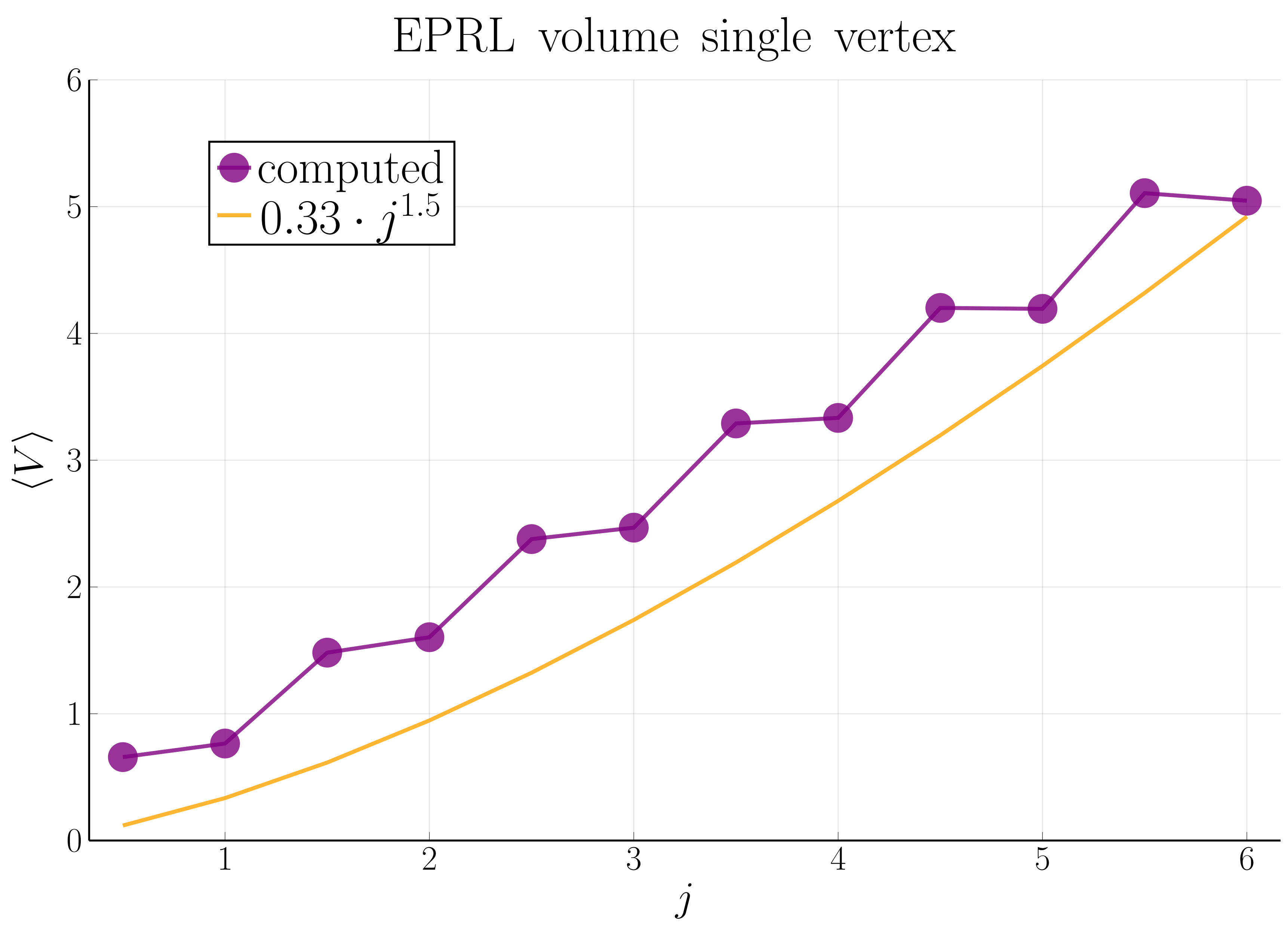
[Frisoni, Gozzini, Vidotto '22]

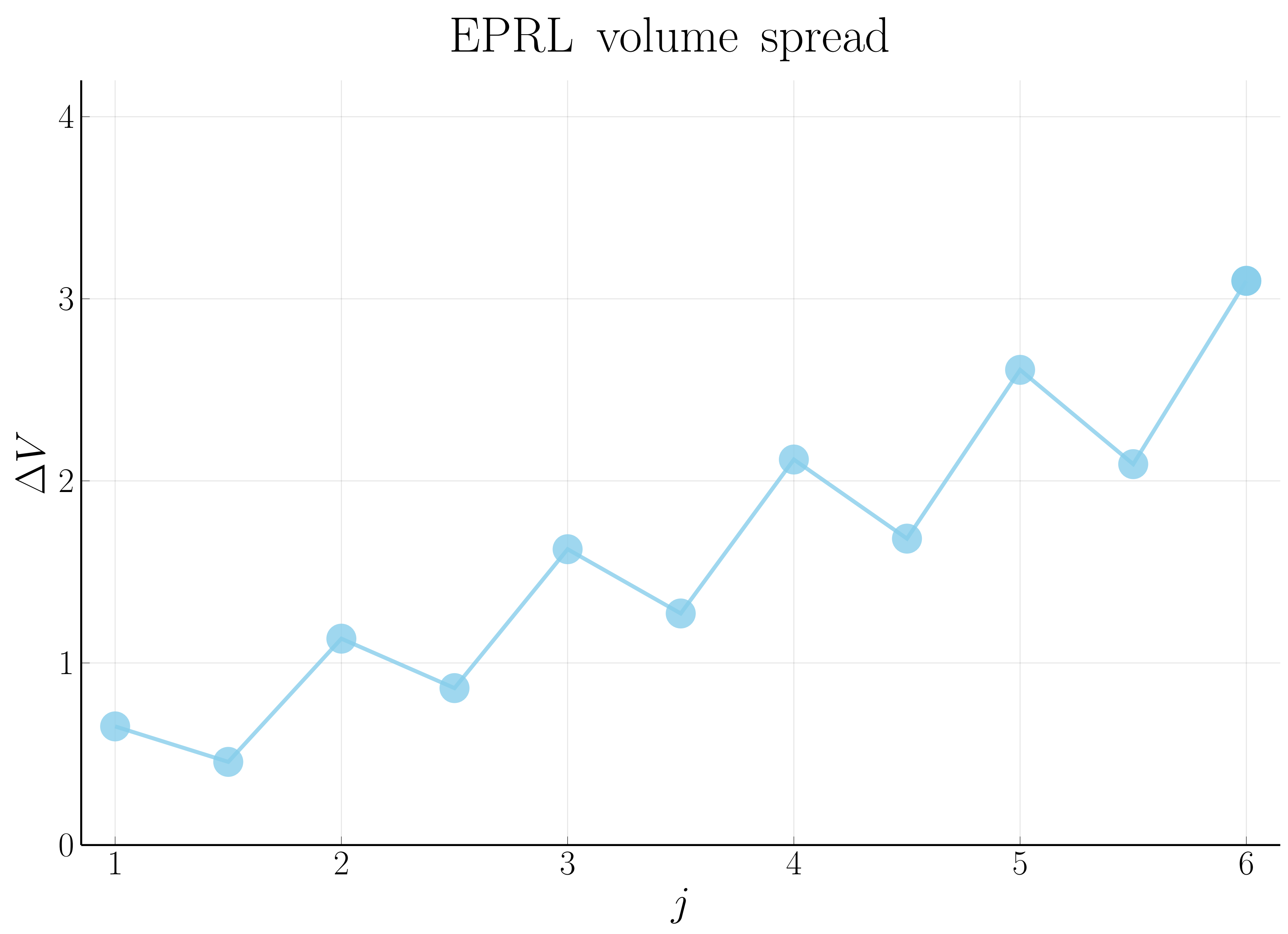




VOLUME

[Frisoni, Gozzini, Vidotto ’22]

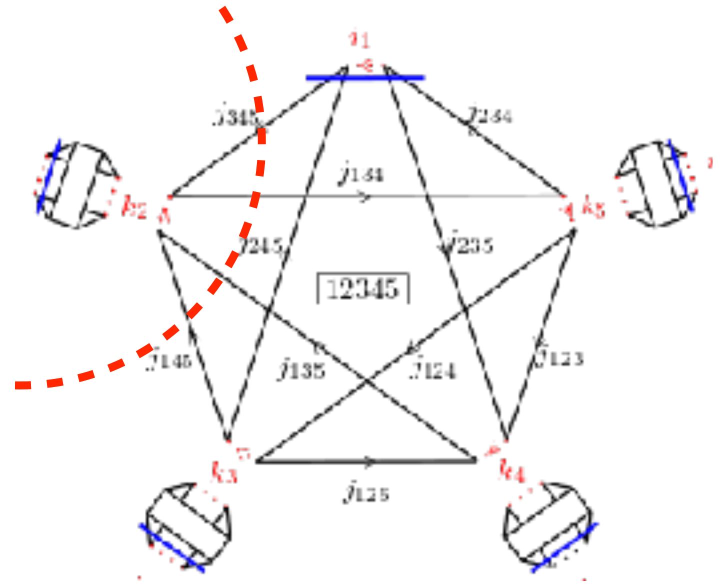




ENTANGLEMENT ENTROPY

[Frisoni, Gozzini, Vidotto ’22]

- Partition: $\mathscr{H} = \mathscr{H}_A \otimes \mathscr{H}_{\bar{A}}$
- Reduced density matrix: $\rho_A = \frac{1}{Z} Tr_{\bar{A}} \, |\psi_0\rangle\langle\psi_0|$
- Entanglement entropy: $S_A = - Tr\left(\rho_A \log \rho_A\right)$



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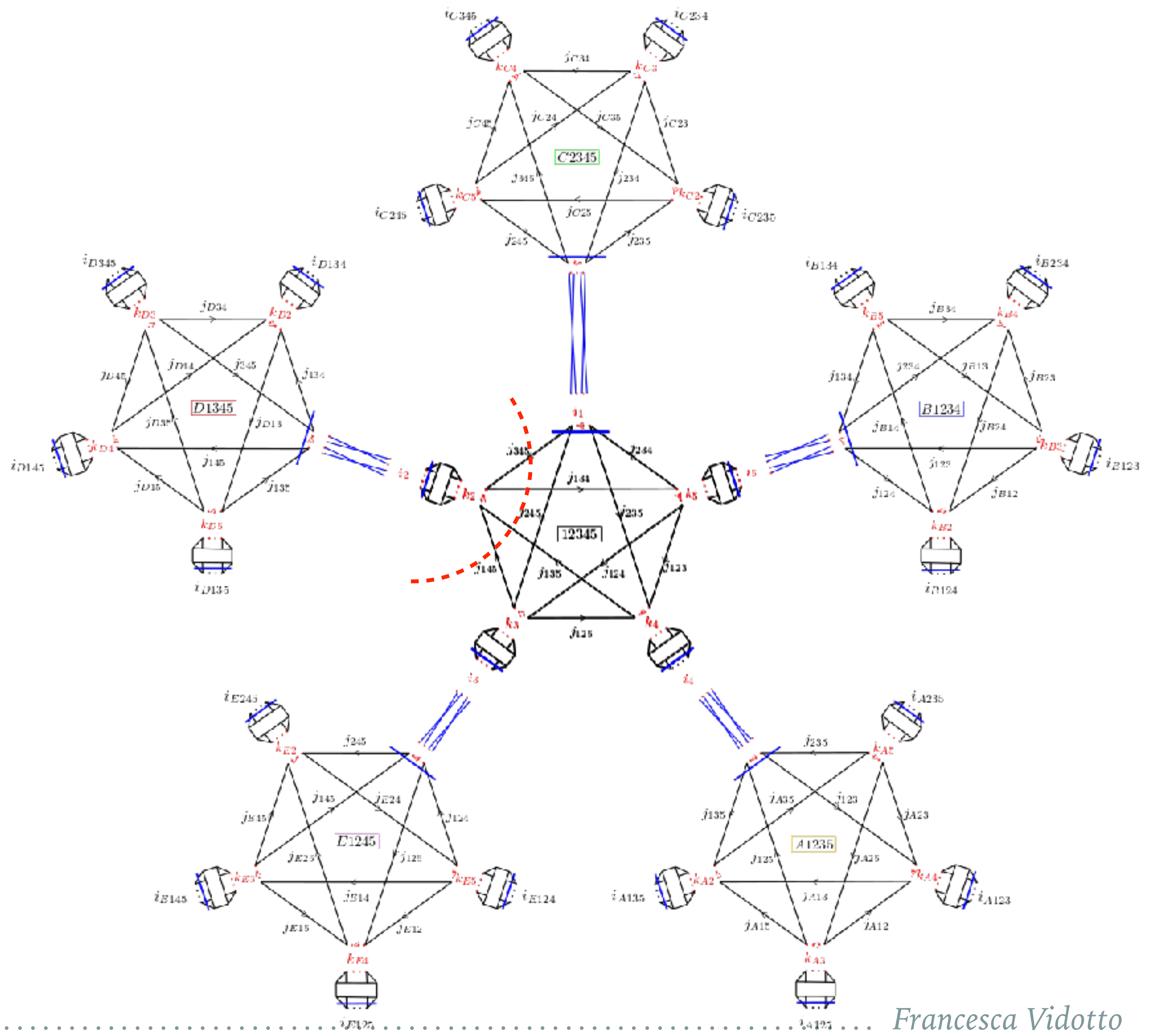
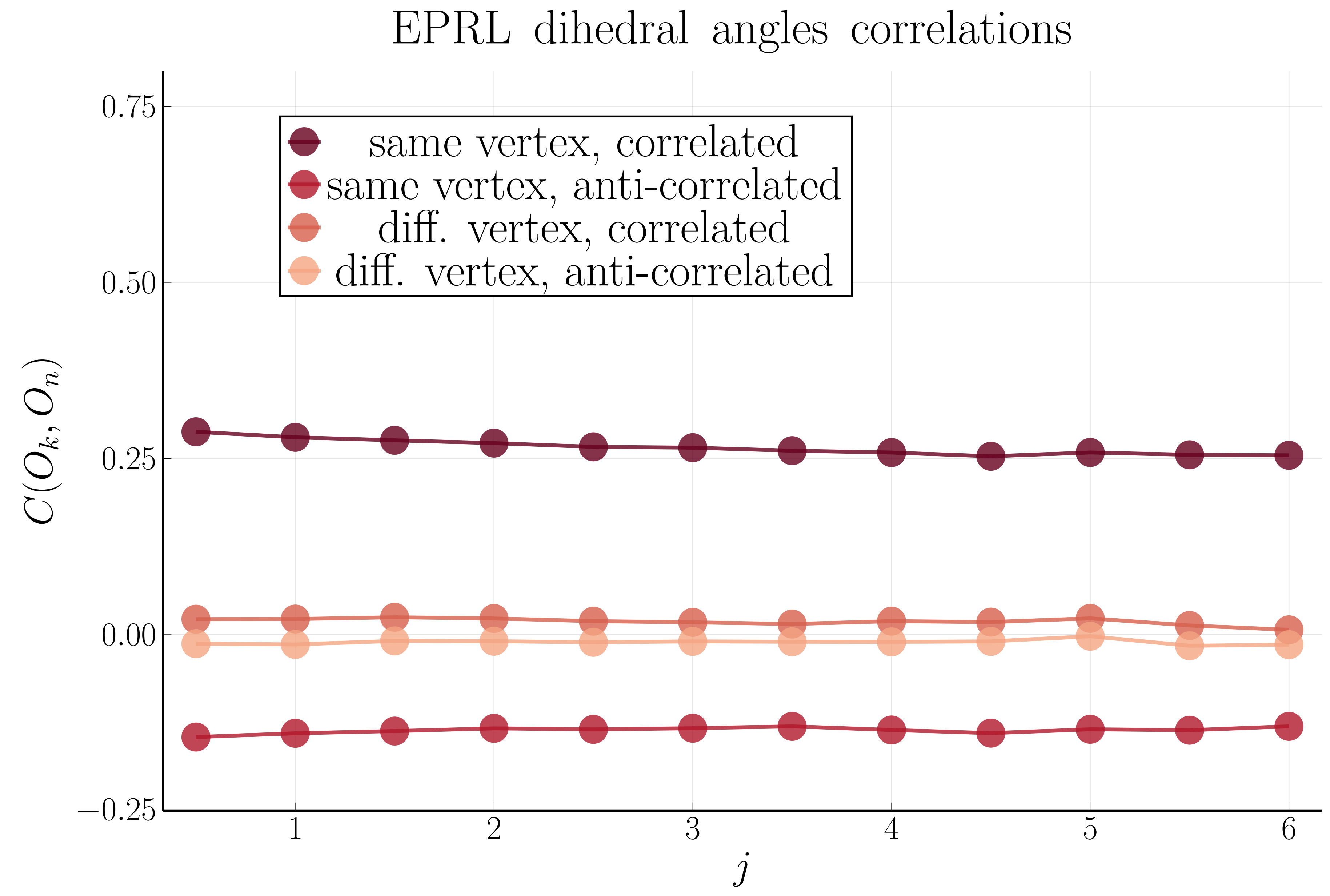
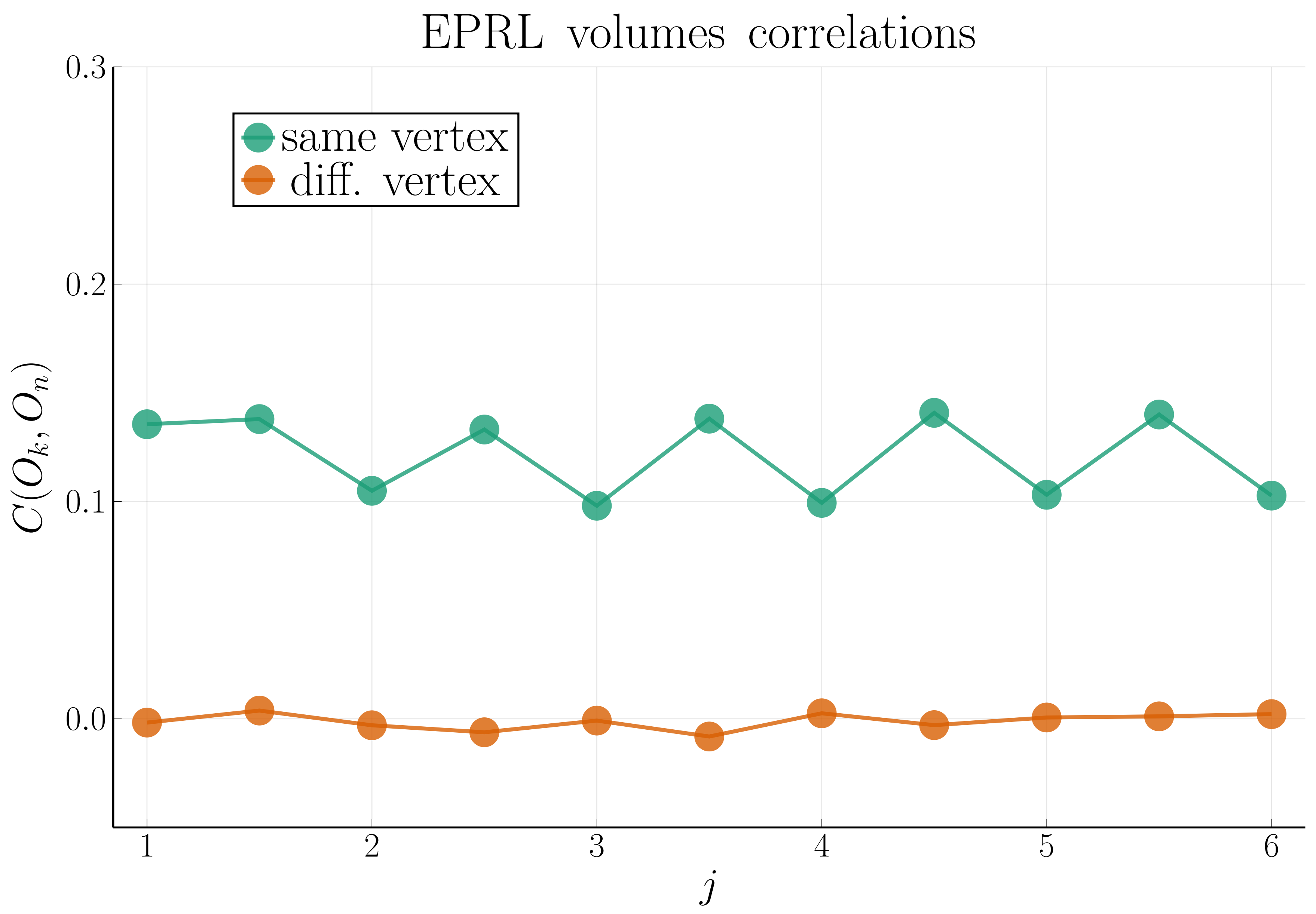


CORRELATIONS

[Frisoni, Gozzini, Vidotto '22]





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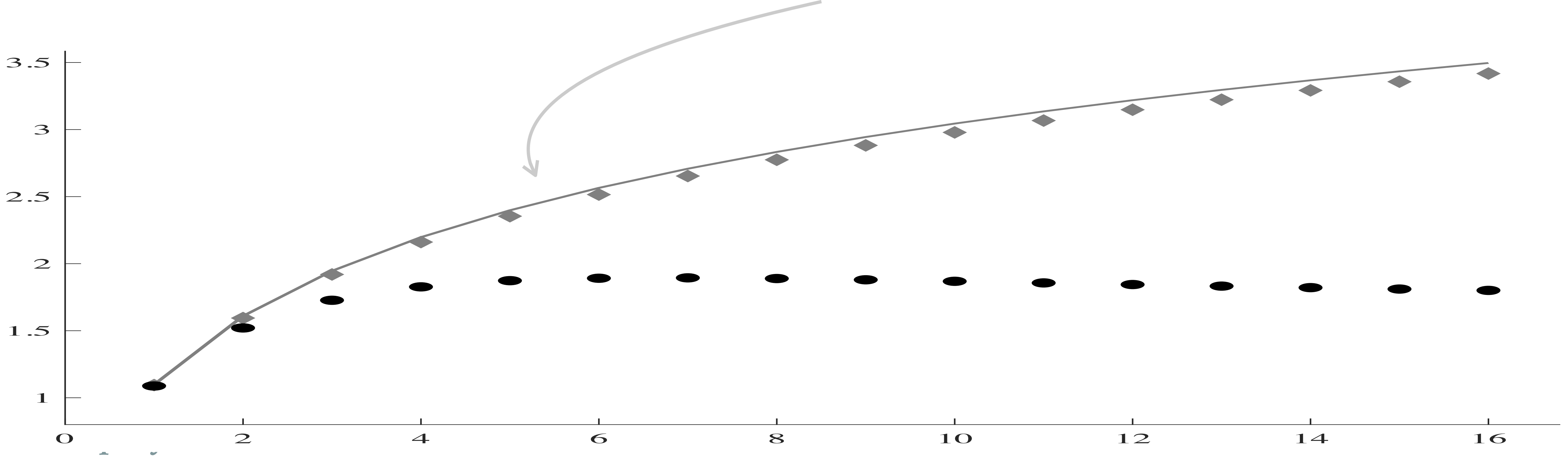
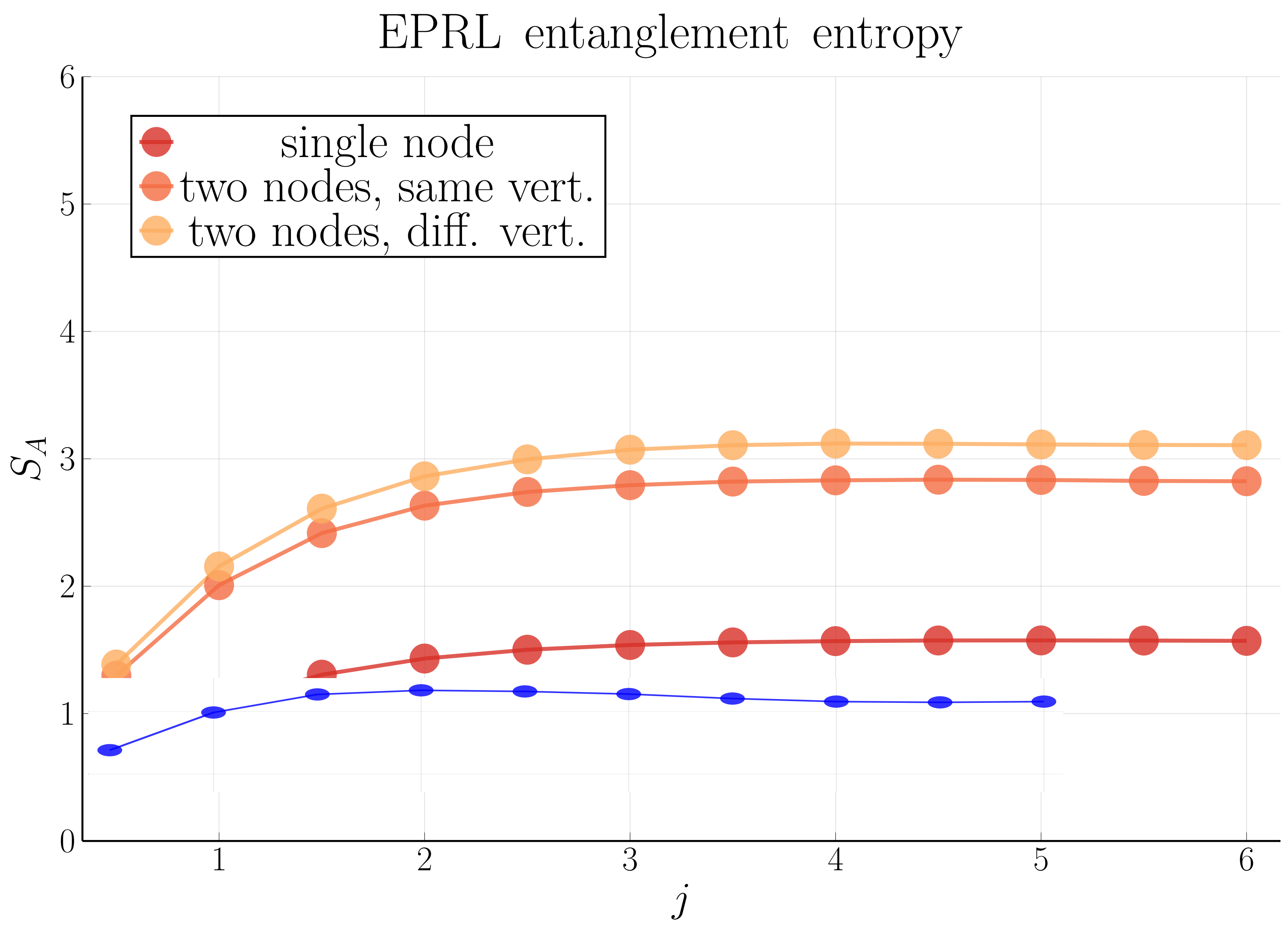
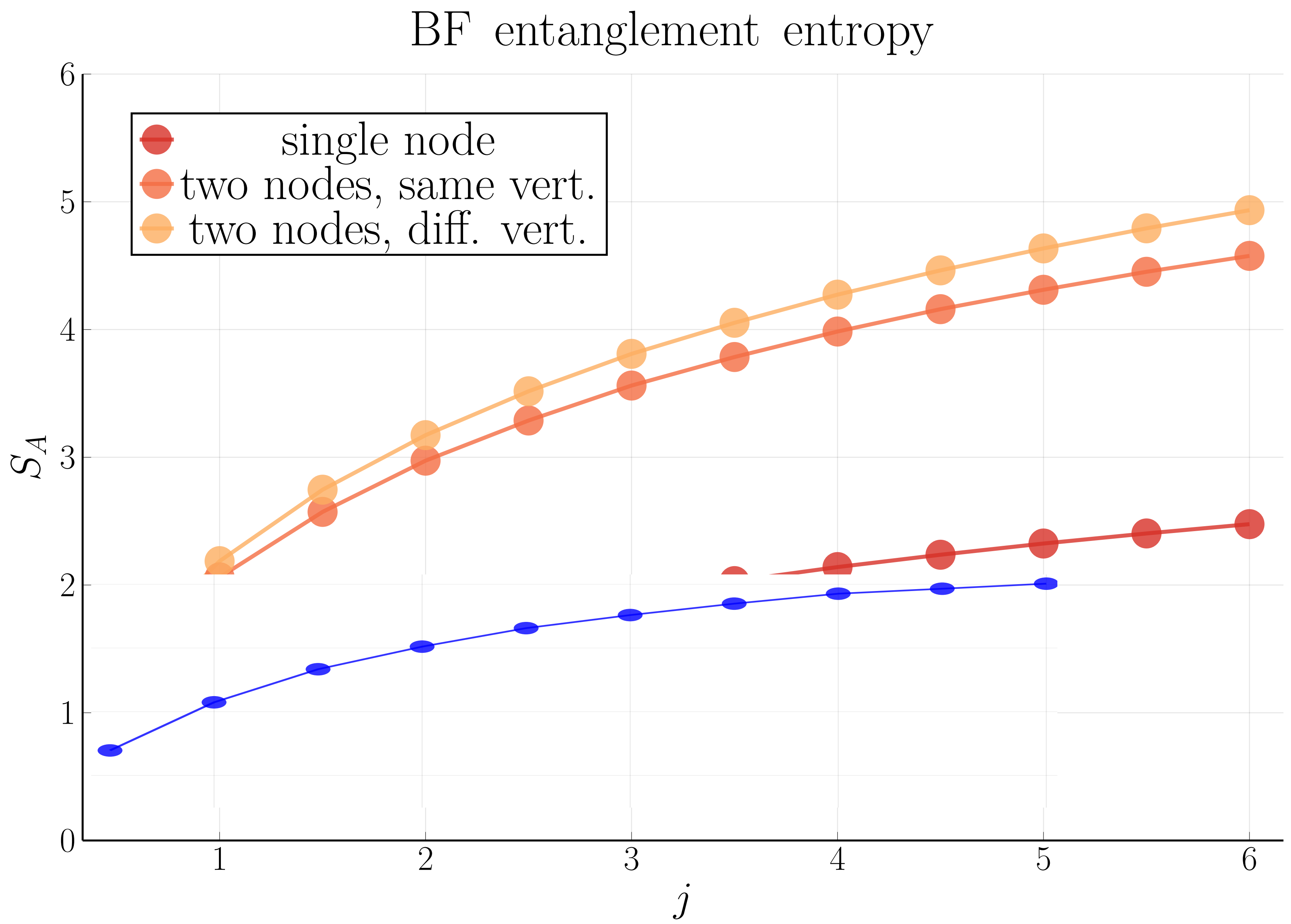
Bianchi, Donà, Vilensky *"Entanglement entropy of Bell-network states in LQG"*

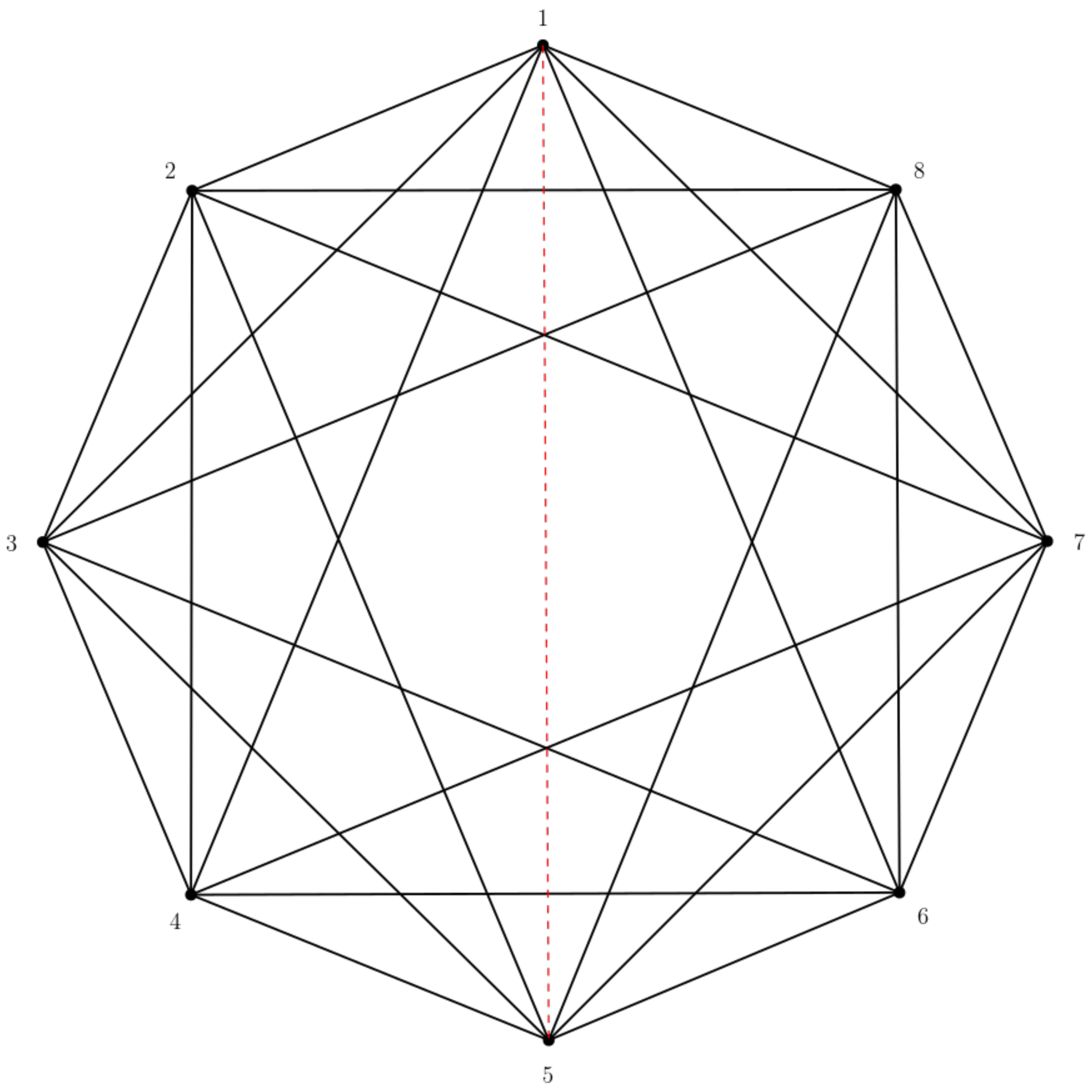


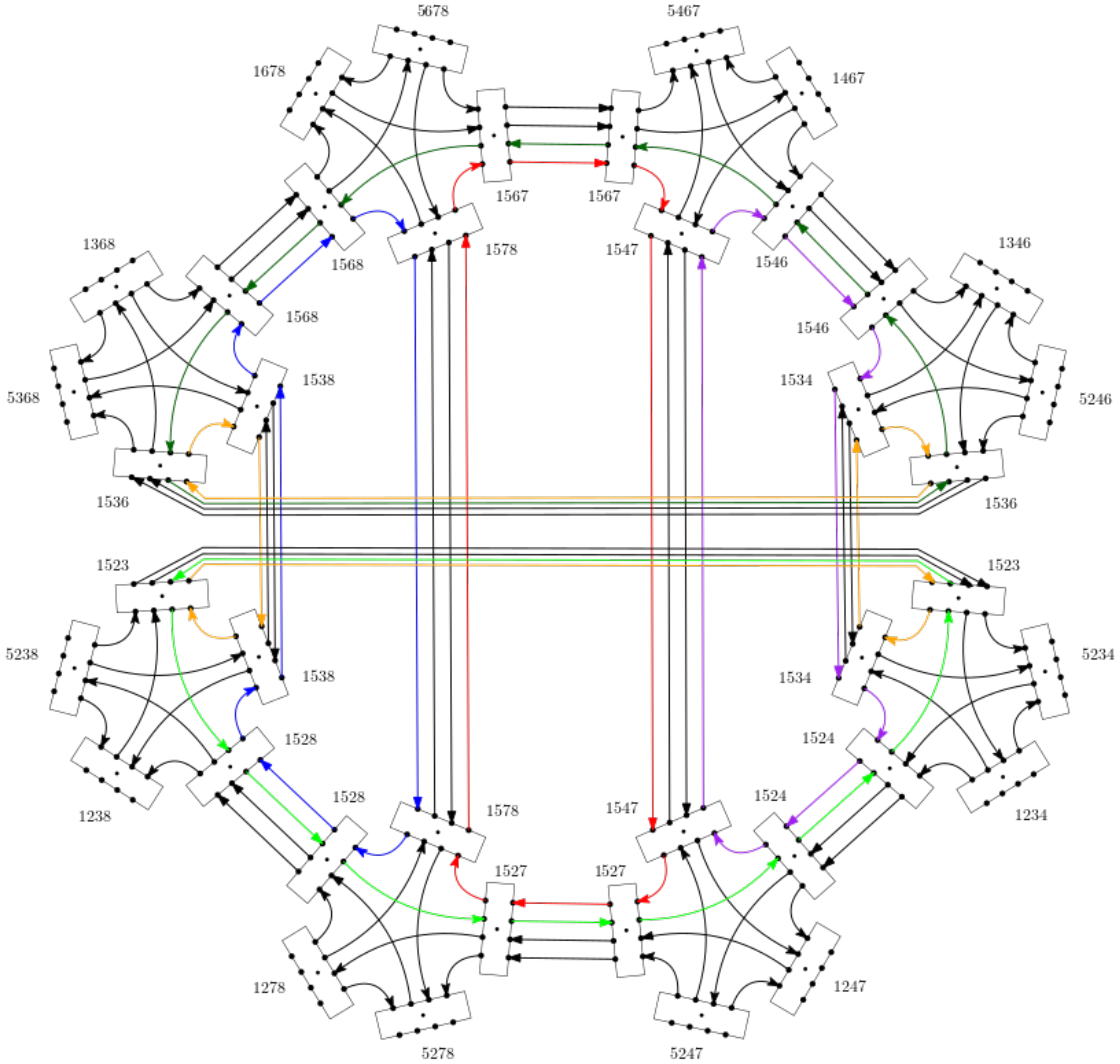




BF 16-CELL MODEL

[Frisoni, Gozzini, Vidotto '23]





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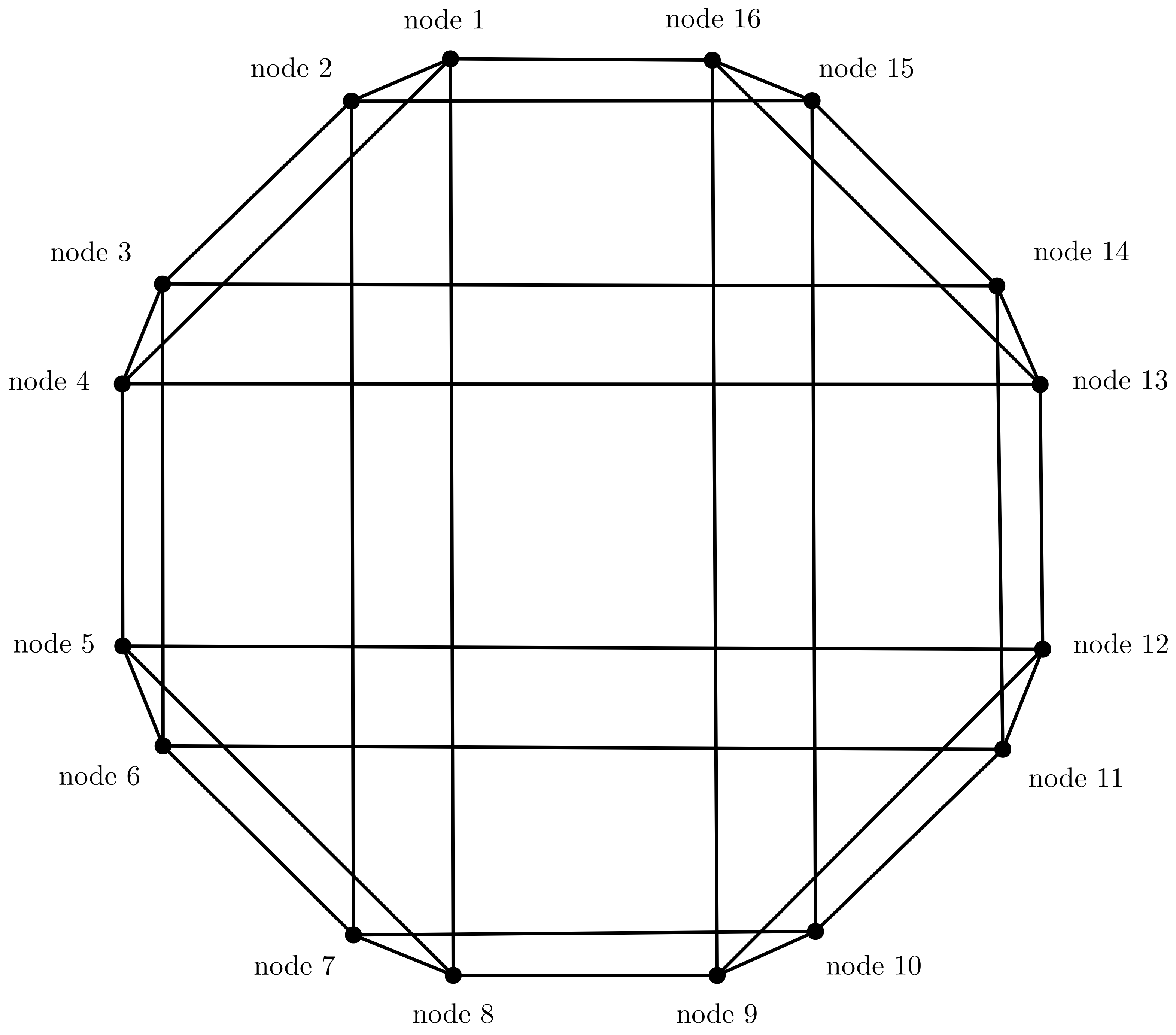

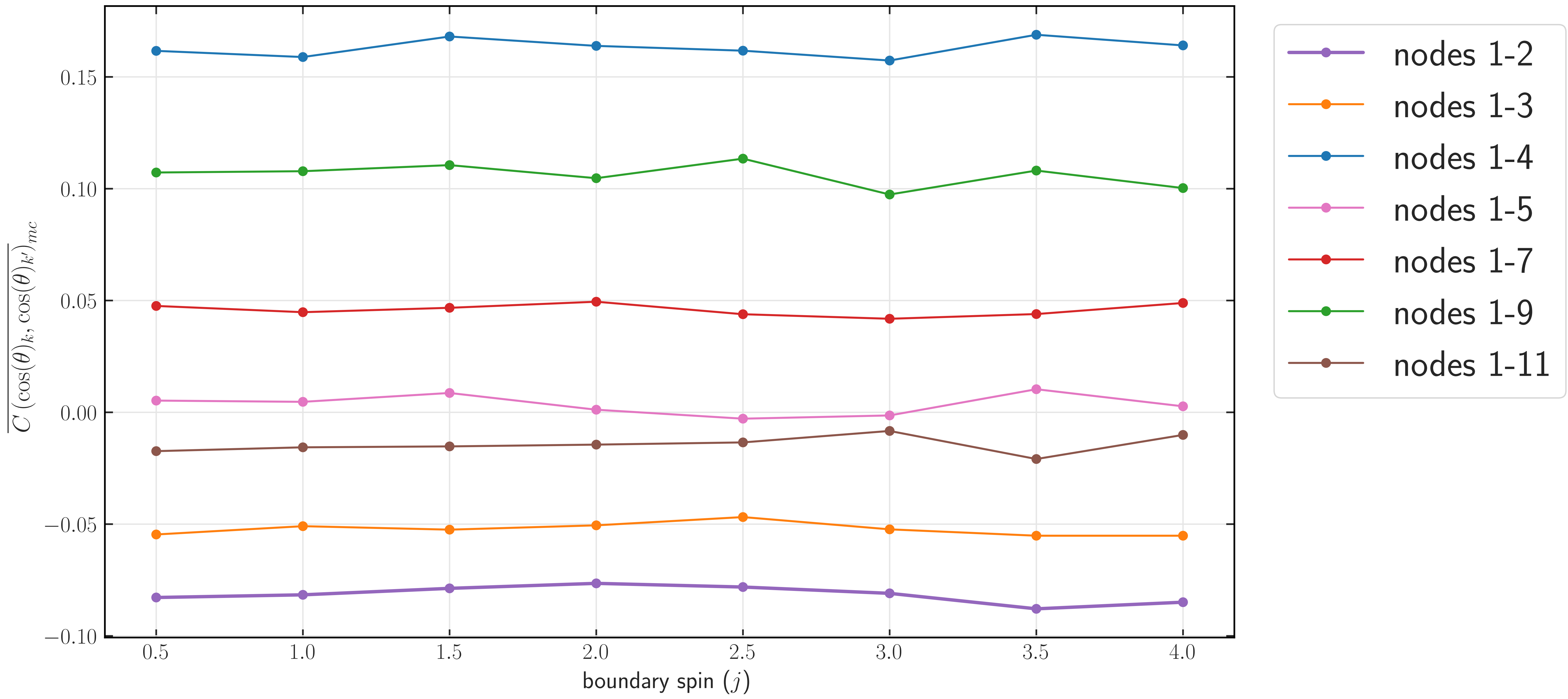

SUMMARY

- Computing primordial quantum fluctuations from the full theory is one of the main goals of a quantum theory of gravity!
- Proposal: use Spinfoam Hartle-Hawking States
- Graph truncation: 5-cell (full) ✓, 20-cell (refinement) ✓, 16-cell (topological) ✓
- Computational challenge: compute expectation values for observables
- Results:
  1. emerging S_3 geometry
  2. large fluctuations
  3. large correlations (for adjacent nodes) $\longrightarrow$ 16-cell needed for richer structure

COLLABORATIONS AND FUTURE DIRECTIONS

FIRST SIMPLE MODEL

- 1 vertex
- 5-cells boundary graph
- computation of observables
- high correlations

with Francesco Gozzini

RELATION TO COSMOLOGICAL VACUUM

- properties of standard cosmological vacua
- QFT on a triangulated 3-sphere
- entanglement entropy

with Sofie Ried

MORE COMPLEX RELIABLE MODELS

- 1 vertex, 6 vertices
- 16-cells and 20-cells boundary graphs
- MCMC to compute observables
- rich behaviour of correlations

with Pietropaolo Frisoni

NON-INFLATIONARY MODELS

- cosmological perturbations from an effective highly-correlated vacuum states
- matter bounce as an alternative to the inflationary models

with Mateo Pascual

COSMOLOGY SUMMARY

- THEORY: *Covariant* Loop Quantum Gravity (*Spinfoam*)
- STATE: Cosmological Lorentzian Spinfoam State — with Rovelli and Bianchi
- BOUNCE: Semiclassical techniques — with Han, Liu, Qu, and Zhang
- QUANTUM FLUCTUATIONS: Numerical Evaluation — with Gozzini and Frisoni
- FUTURE ROADMAP: *a lot of things to do!*

www.cpt.univ-mrs.fr/~rovelli/IntroductionLQG.pdf