Towards canonical LQG with neural networks

From the basics to 3d gravity in Smolin's weak coupling limit

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[arXiv: 2402.10622, 2405.00661]

ERLANGEN CENTRE FOR ASTROPARTICLE PHYSICS

Learned some deep things about QG from LQG

- Microstates of black holes
- Singularity avoidance
- …

One big open question (in my mind):

(Physical!) quantum state of $(1Å)^4$ of spacetime in this room? ?

(—> A. Perez: Planck scale DOF)

Idea

Solve constraints of canonical LQG numerically, without symmetry assumption.

Problems:

- Cutoff: Single graph γ?
- Cutoff: j_{max} ?
- Exponential growth of Hilbert space:

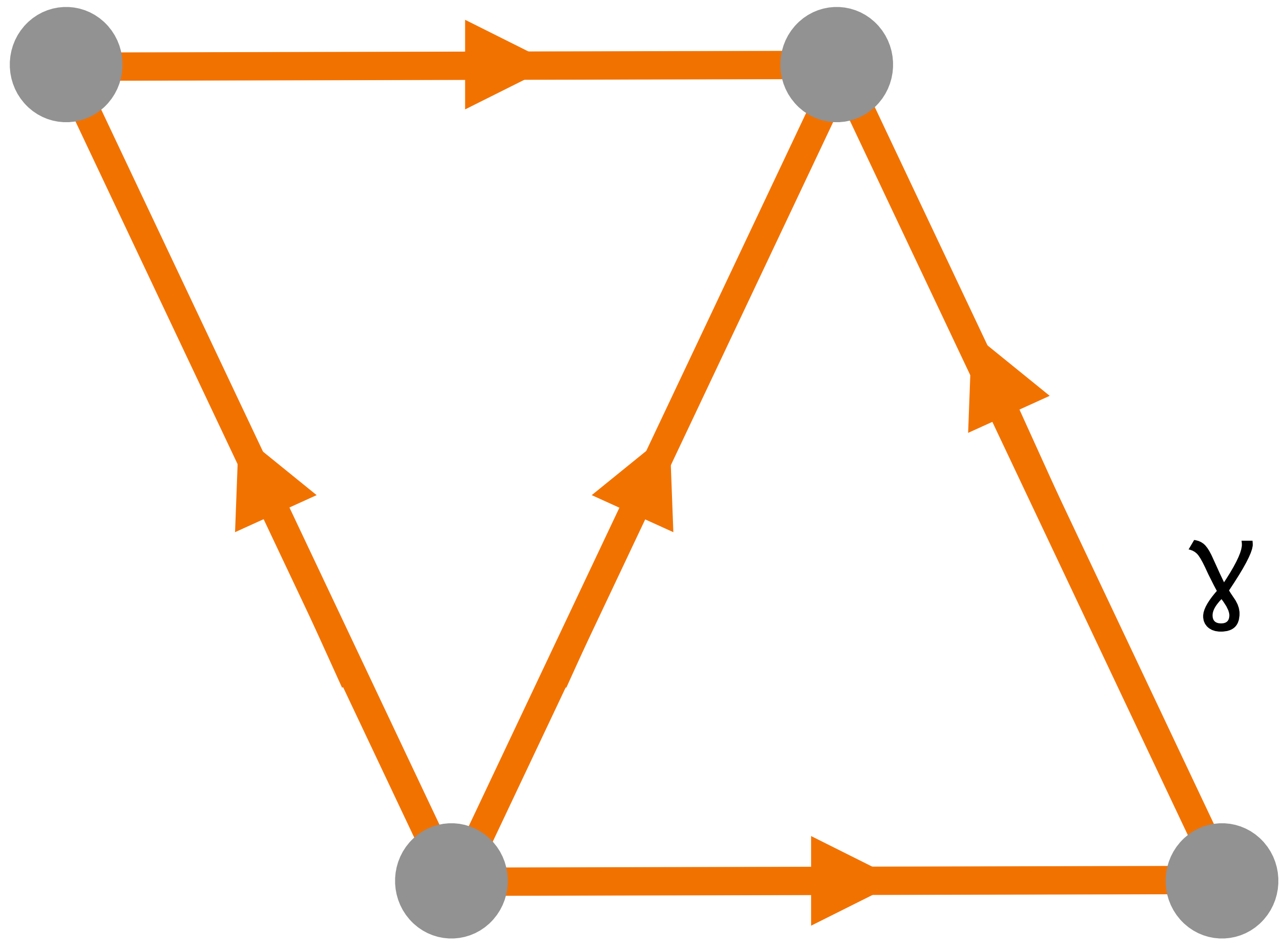
$$\dim \mathcal{H}_{\mathrm{kin}} \sim (2j_{\mathrm{max}} + 1)^{2|\gamma|}$$

Picture: solid state physics!

In this talk

Proof of principle, using

- U(1) BF-Theory [2402.10622]
- 3d Euclidean gravity in Smolin small coupling limit SU(2) —> $U(1)^3$ [2405.00661]
- Simple graph



Of course: Can be solved analytically (but…)

Exponential growth

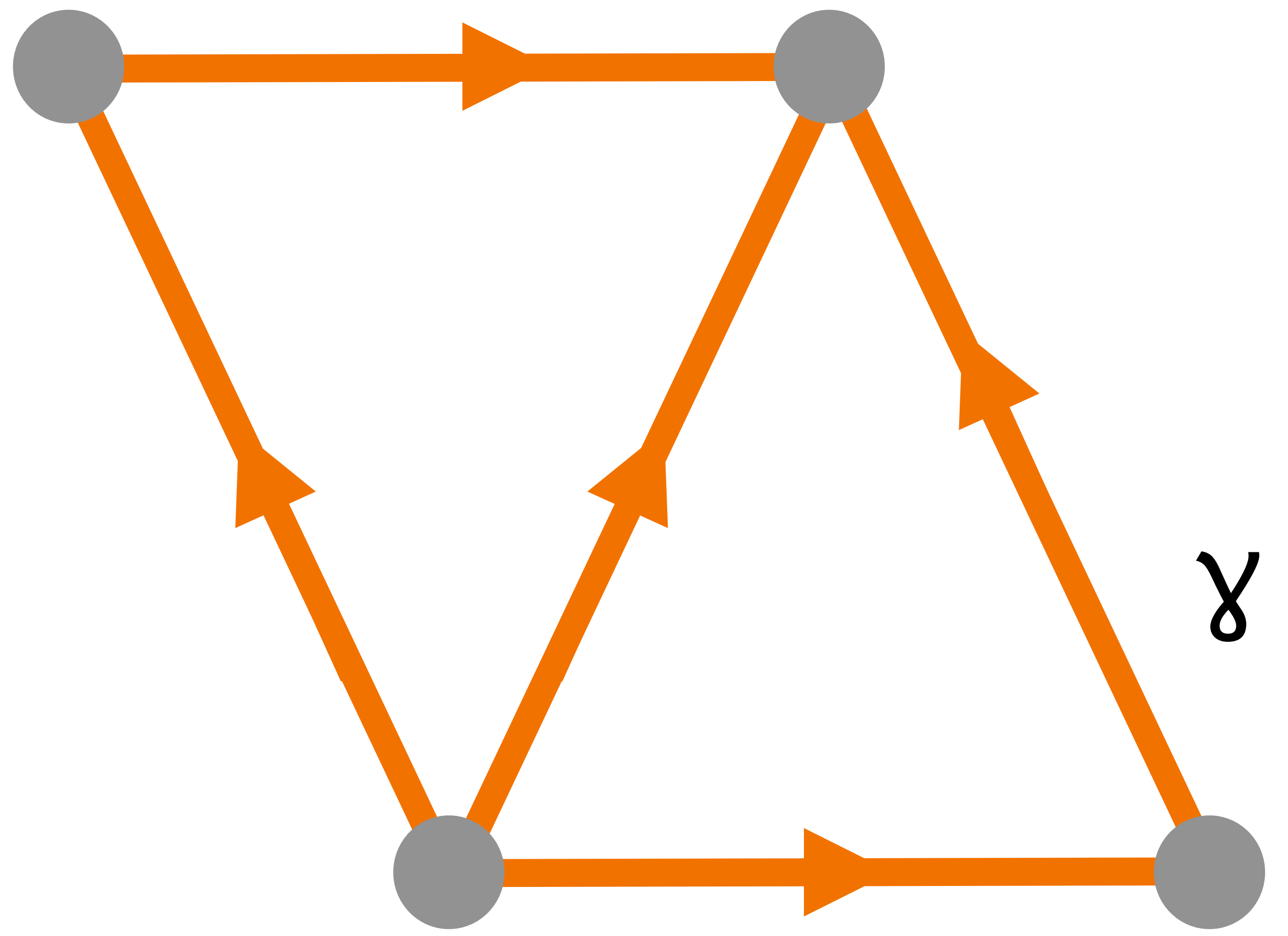
	dim H	size Ψ	size *C*
$m_{max} = ½$	10^4	100 kB	1 GB
$m_{max} = 1$	10^7	100 MB	1 PB
$m_{max} = 2$	10^{10}	10 GB	1 ZB

- Need smart Ansatz for the state
- Need sophisticated software tools

U(1),cutoff:

charges $m \in \{-m_{max},\ldots,m_{max}\}$

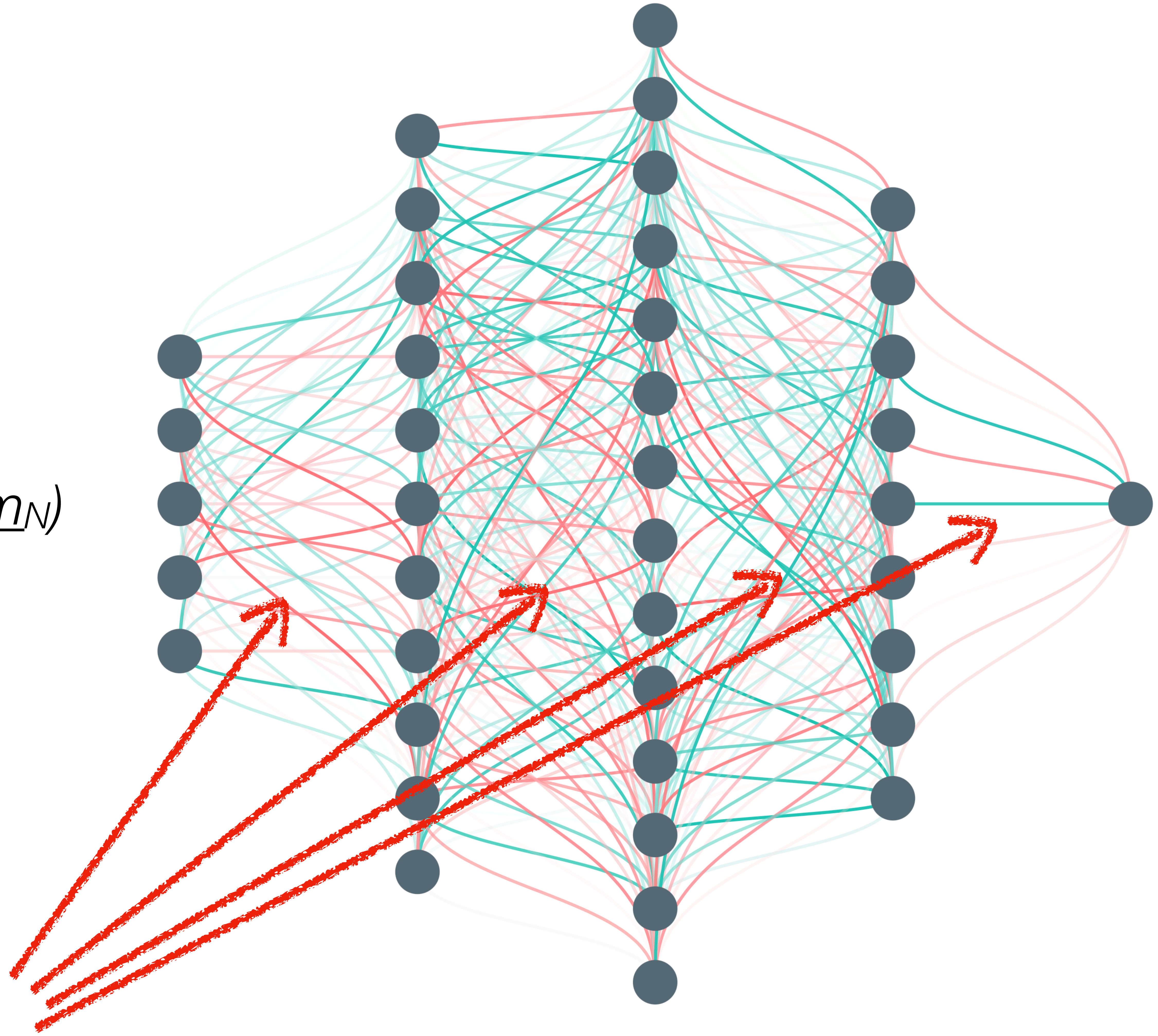
$$dim\ H = (2\ m_{max} + 1)^{15}$$



Neural network quantum states

In principle

$(\underline{m}_1, \underline{m}_2, \ldots, \underline{m}_N)$

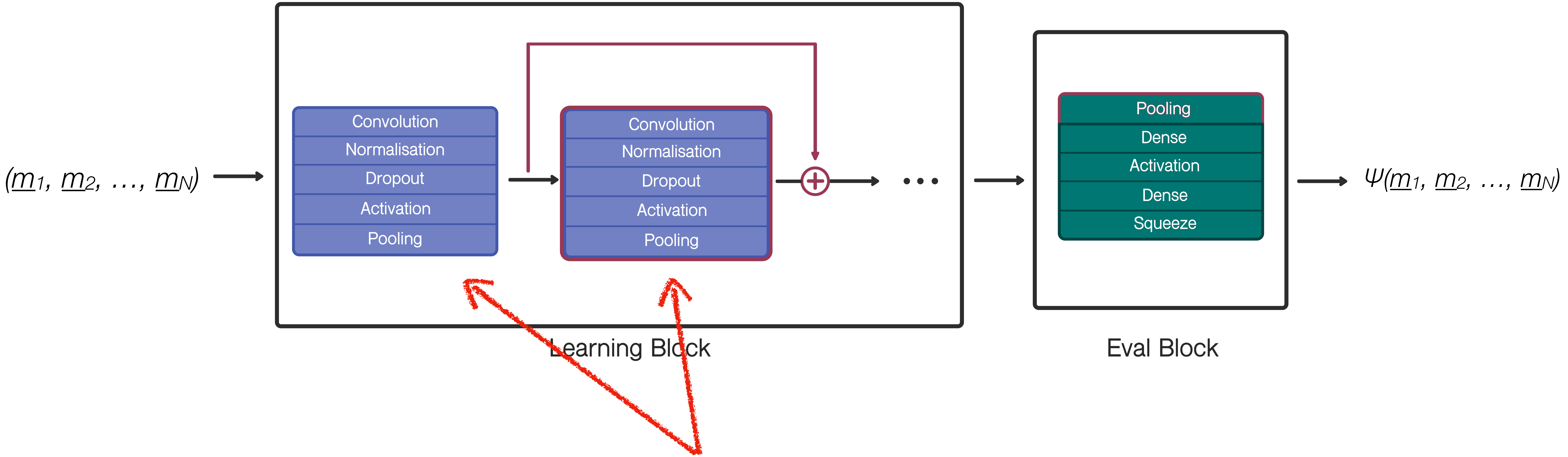


$\Psi(\underline{m}_1, \underline{m}_2, \ldots, \underline{m}_N)$

Weights w_{ij}

Neural network quantum states

In practice



Number of blocks scale with size of graph

Other methods: Tensor network states, ex. [Cunningham,Dittrich,Steinhaus]

$$\langle C \rangle_{\Psi_{w}} \stackrel{!}{=} \min$$

Prosaic interpretation: Variational Ansatz for physical state

Romantic interpretation: Special kind of brain learns about gauge invariance etc.

NetKet

- Differentiable programming for gradient descent
- Markov-chain Monte Carlo to compute $\langle C \rangle_{\Psi_w}$
- No need to hold entire C in memory

We did:

- Custom representation of graphs, holonomies, fluxes
- Spatial volume operator, constraint operators

Plan: eventually release software package. Dream: Network weights as essence

Quantum constraints

Master constraints:

$$\hat{G}|_\gamma = \sum_{v\in V(\gamma)} \sum_{i=1}^{3} (\hat{E}_{S(v),i})^2$$

(Gauß)

$$\hat{F}_\gamma = \sum_{\alpha\in L(\gamma)} \mathrm{tr}\left[\left(\hat{h}_\alpha - \mathbb{1}\right)\left(\hat{h}_\alpha^\dagger - \mathbb{1}\right)\right]$$

(Curvature)

Thiemann-regularized Hamilton constraint: (TRC) [Thiemann: QSD IV]

$$\hat{H}_{T(\gamma)}(N) = \frac{2}{\hbar^2} \sum_{\Delta,\Delta'\in T,v} \epsilon^{ij}\epsilon^{kl} N(v)\, \mathrm{tr}(\hat{h}_{\alpha_{ij}(\Delta')} \hat{h}_{s_k(\Delta)} [\hat{h}^{-1}_{s_k(\Delta)}, \sqrt{\hat{V}_v}] \hat{h}_{s_l(\Delta)} [\hat{h}^{-1}_{s_l(\Delta)}, \sqrt{\hat{V}_v}])$$

Cutoffs

Representations:

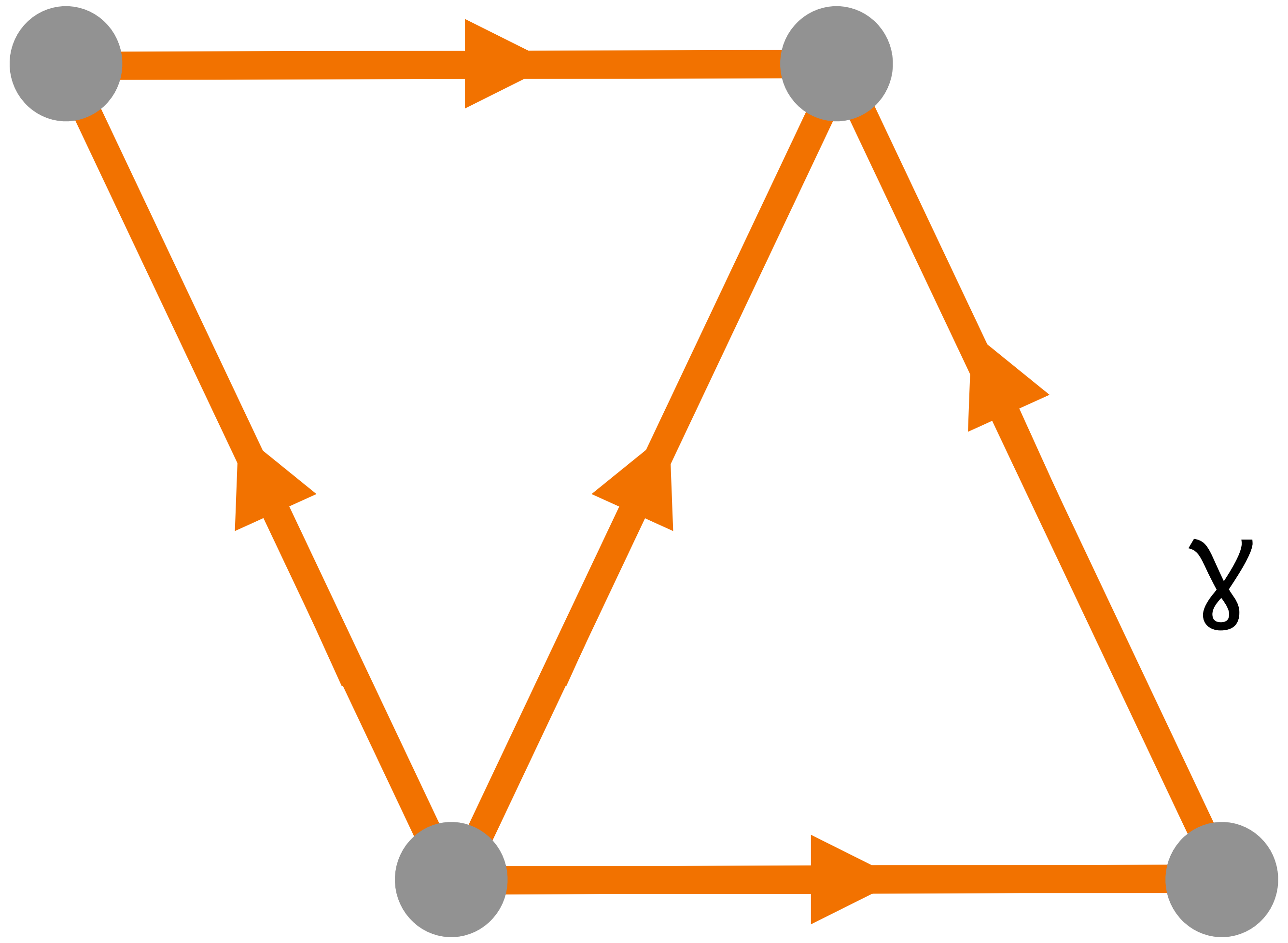
$$\mathrm{U}(1)^3 \longrightarrow \mathrm{U}(1)_q^3$$

$$\text{charges} \in \{-m_{\max}, -m_{\max}+1, \ldots, m_{\max}\}$$

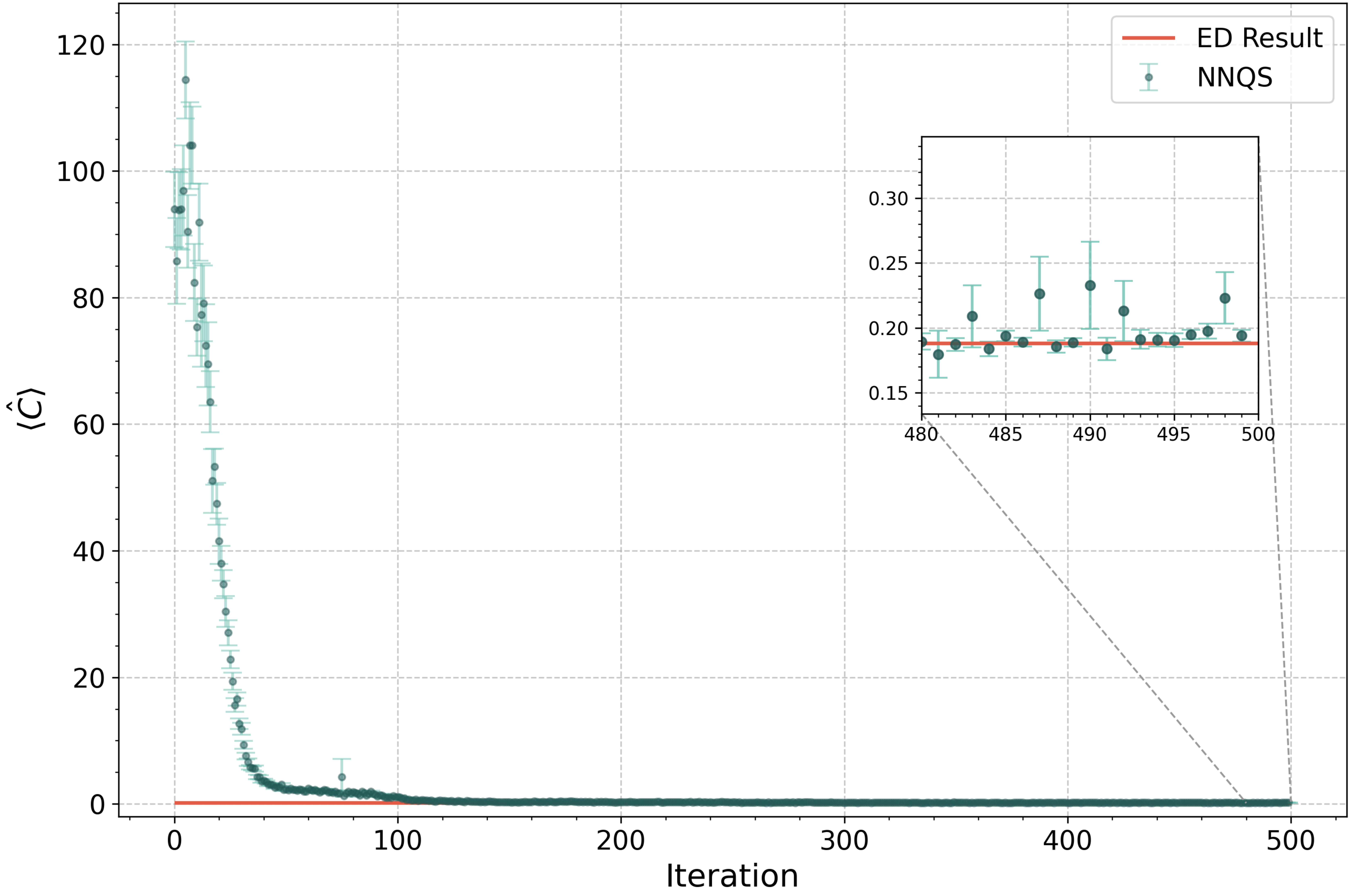
- tr(h_e) not gauge invariant
- Non-trivial theory

!

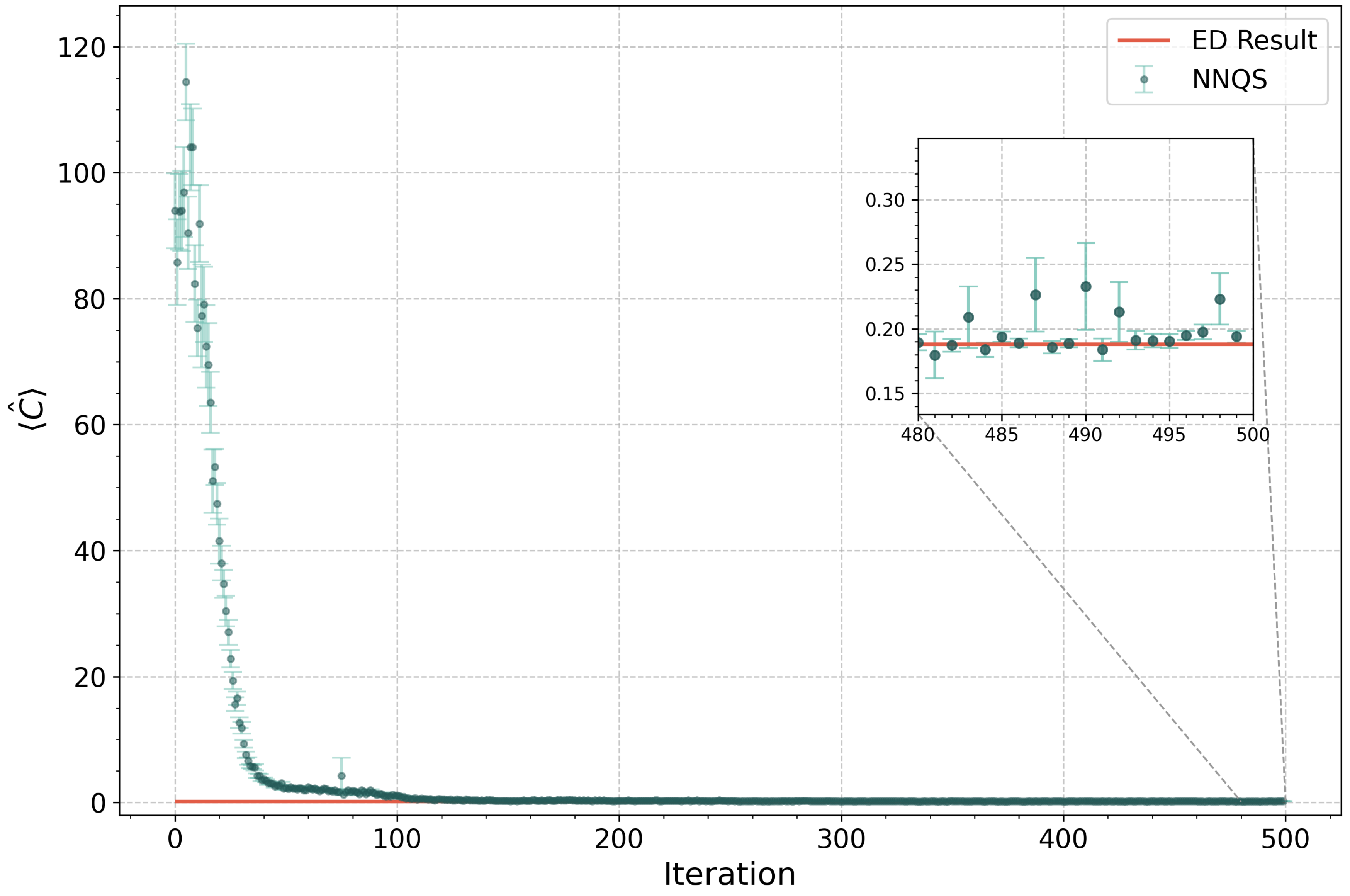
Graph:



Sanity checks: U(1) BF



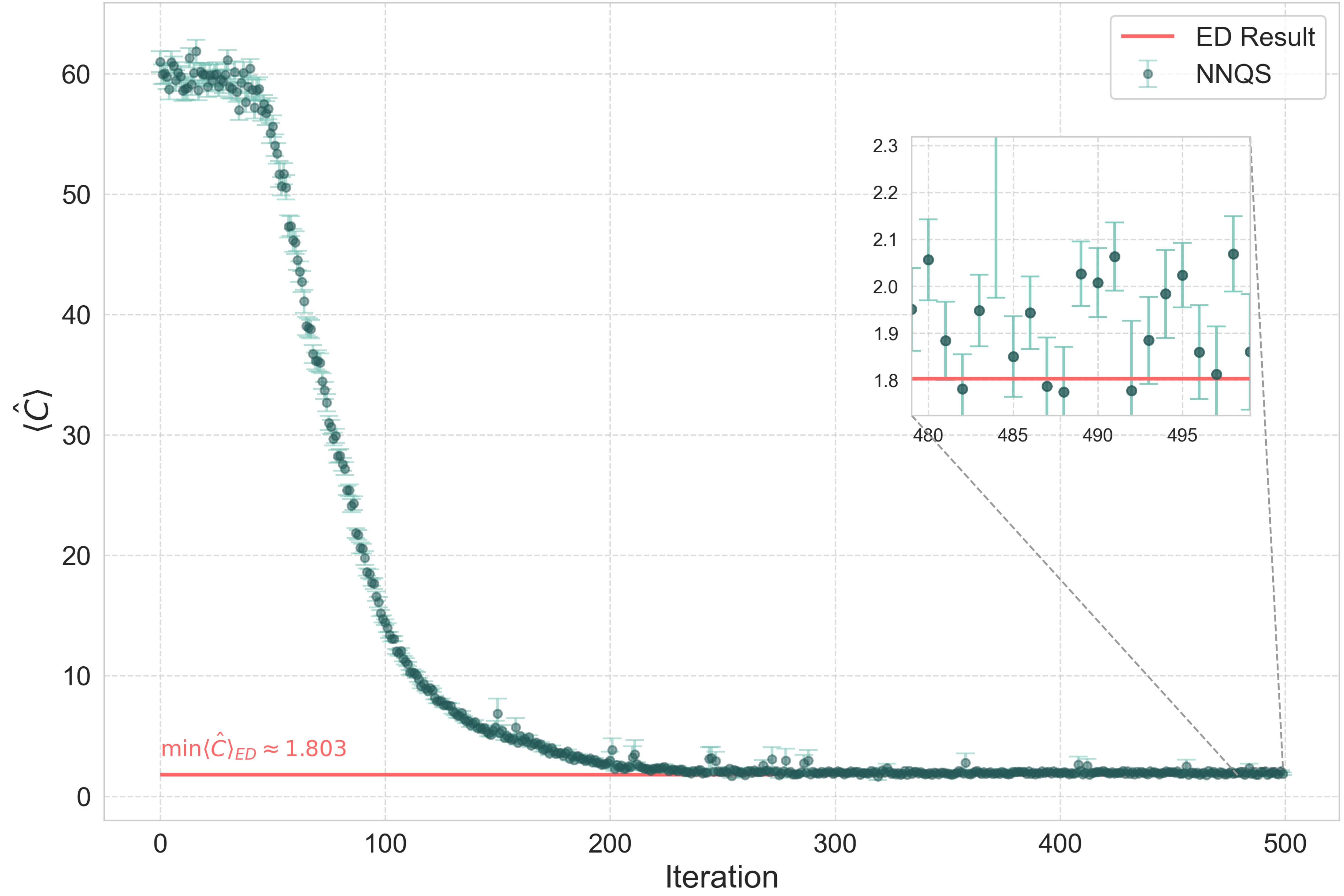
Sanity checks: U(1) BF



$m_{\max}$	$\min \langle \hat{C} \rangle_{\text{ED}}$	$\min \langle \hat{C} \rangle_{\text{NN}}$	Accuracy (%)	$\langle \Psi^{(\text{NN})} \| \Psi^{(\text{ED})} \rangle^2$
1	0.835968	0.99866 ± 0.00028	80.538	0.9895
2	0.601165	0.625 ± 0.0013	96.034	0.9979
3	0.389553	0.3942 ± 0.0045	98.818	0.996
4	0.263623	0.2648 ± 0.0013	99.539	0.9906
5	0.187973	0.1882 ± 0.0017	99.853	0.989
6	0.140084	0.1412 ± 0.0035	99.189	0.9834
7	0.108159	0.1133 ± 0.0066	95.218	0.983
8	0.085918	0.0833 ± 0.0079	96.931	0.9598

- Good approximation
- $m_{max} \rightarrow \infty$ looks reasonable
- $m_{max} = 8$: $\dim H = 1.4 * 10^6$
  #(weights) = $4 * 10^4$

U(1)³ with master constraints



m_{max}	$\min\langle\hat{C}\rangle^{**}_{(ED)}$	$\min\langle\hat{C}\rangle_{(NN)}$	Accuracy (%)
1	2.507903	2.998 ± 0.017	80.441
2	1.803495	1.74 ± 0.16	96.286
3	1.168658	1.12 ± 0.11	96.069
4	0.790868	0.84 ± 0.21	93.788

- Convergence!
- Same architecture works
- Way beyond exact diagonalization

- *m_{max} = 2: dim H = 3 * 10^{10}*
  #(weights) = 6 * 10^3

$U(1)^3$ with master constraints

$m_{max} = 4$ entails:

- dim H = 10^{12}
- Constraint represented naively as matrix would be 10^{18} TB

?

But:

- 42 000 weights
- 30 Minutes as base level HPC job

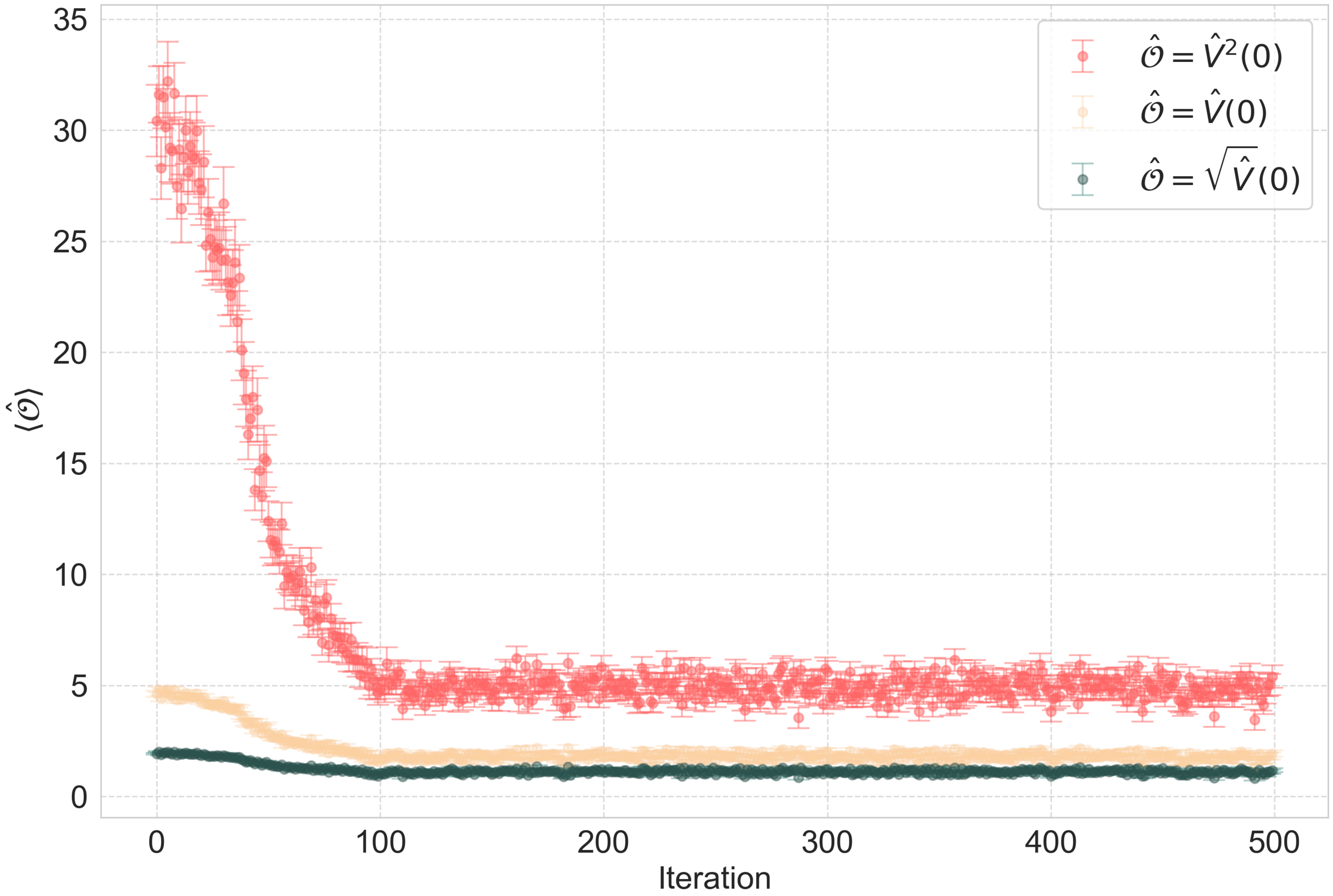
!

U(1)^3: Operators

Volume [Thiemann: QSD IV]

$$\hat{V}(B) := \sum_{v \in V(\gamma)\cap B} \hat{V}_v\,, \qquad \hat{V}_v := \sqrt{\sum_I \left(\sum_{e,e' \text{ at } v} \text{sign}(e,e')\epsilon_{IJK} X_e^J X_{e'}^K \right)^2}$$

- Can deal with complicated operators
- Have checked expected behavior



$U(1)^3$: Thiemann-regulated constraint [Thiemann: QSD IV]

$$\hat{H}_{T(\gamma)}(N) = \frac{2}{\hbar^2} \sum_{\Delta,\Delta' \in T,v} \epsilon^{ij}\epsilon^{kl} N(v)\, \mathrm{tr}(\hat{h}_{\alpha_{ij}(\Delta')}\hat{h}_{s_k(\Delta)}[\hat{h}^{-1}_{s_k(\Delta)}, \sqrt{\hat{V}_v}]\hat{h}_{s_l(\Delta)}[\hat{h}^{-1}_{s_l(\Delta)}, \sqrt{\hat{V}_v}])$$

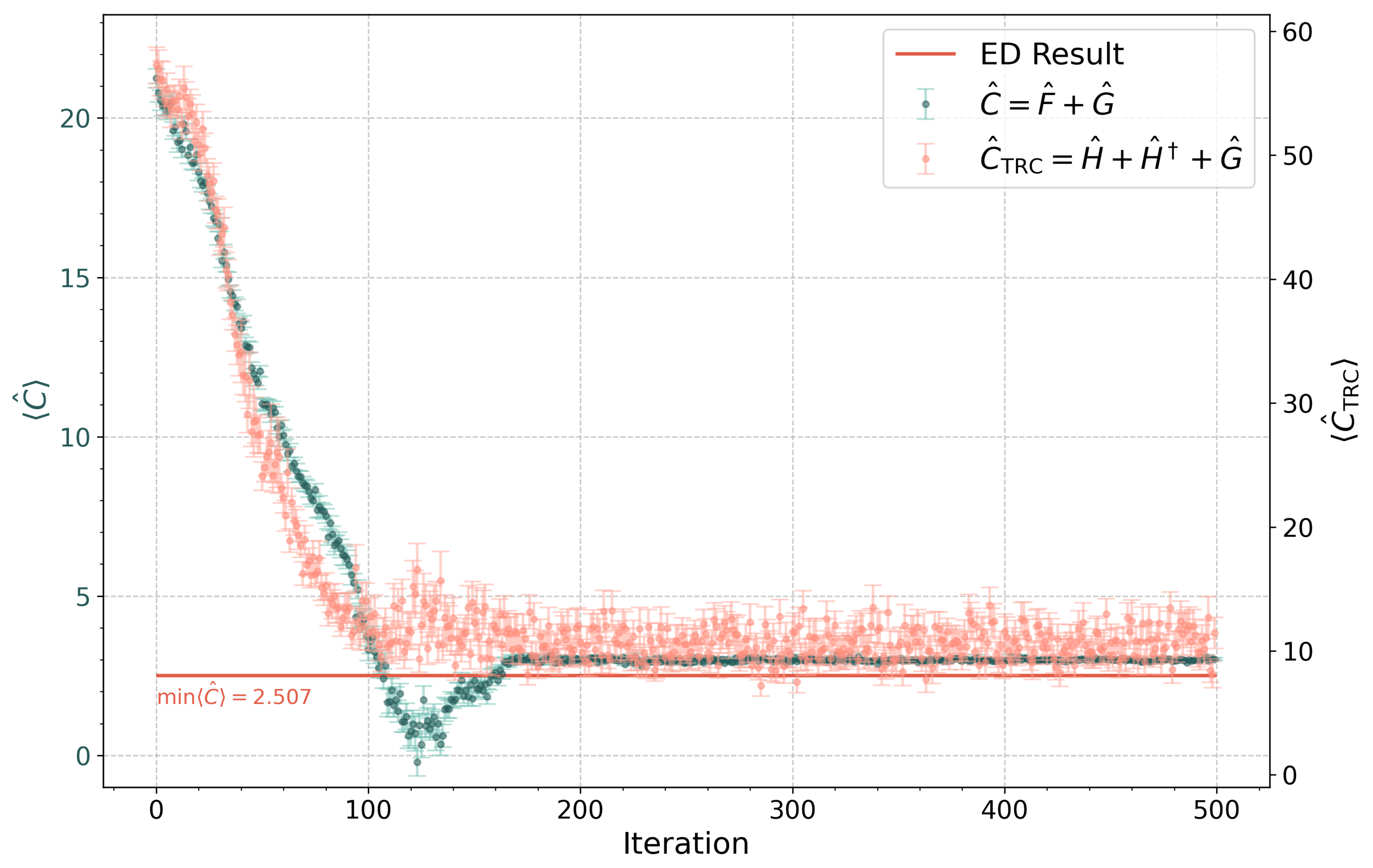
$$\hat{C}_{\mathrm{TRC}} = \hat{H} + \hat{H}^\dagger + \hat{G}.$$

Cautious remarks

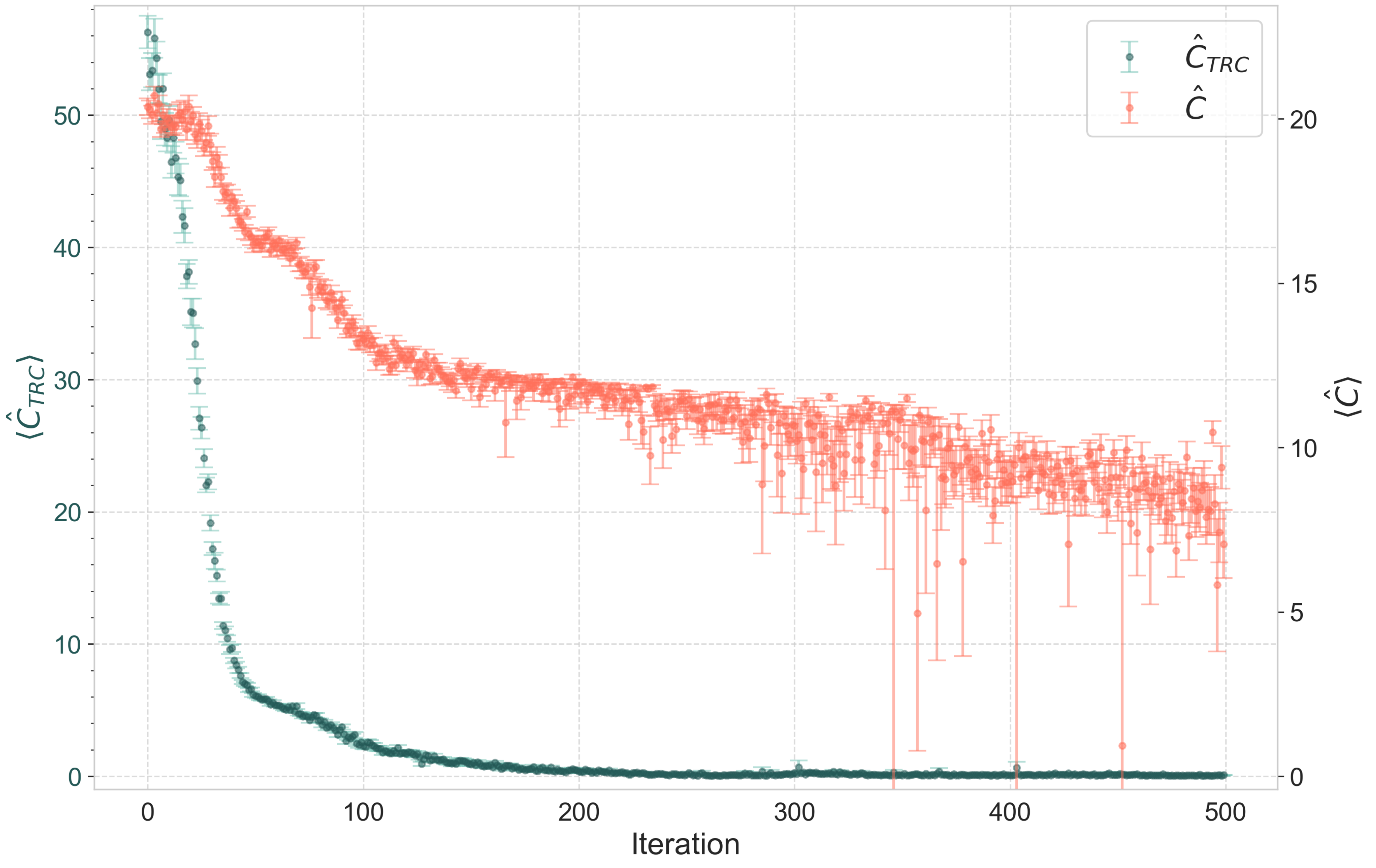
- More complicated than 4d!
- Not master constraint $\hat{H}\hat{H}^\dagger + G$
- Just approximate solution
- $\sqrt{\hat{V}}$ via Taylor expansion
- (Mostly) no diffeo constraint

Solving C, C_{TRC}

C, C_{TRC} look very different. So, do they have anything in common?



Solving C, observing C_{TRC}



Solving C_{TRC}, observing C

The solutions have a small overlap:

$$|\langle \Psi_{\hat{C}_{\text{TRC}}} \mid \Psi_{\hat{C}} \rangle|^2 \approx 0.0308 \quad , \quad \arccos|\langle \Psi_{\hat{C}_{\text{TRC}}} \mid \Psi_{\hat{C}} \rangle| \approx 1.394\text{rad}$$

However, this is not an accident. If they were

$$P_N^{\mathbb{C}}(|\langle \Psi_{\hat{C}_{\text{TRC}}} \mid \Psi_{\hat{C}} \rangle|^2 \geq 0.0308) \sim 10^{-194953}$$

$$P_{NG}^{\mathbb{C}}(|\langle \Psi_{\hat{C}_{\text{TRC}}} \mid \Psi_{\hat{C}} \rangle|^2 \geq 0.0308) \sim 10^{-10}$$

$$P_{NG}^{\mathbb{R}}(|\langle \Psi_{\hat{C}_{\text{TRC}}} \mid \Psi_{\hat{C}} \rangle|^2 \geq 0.0308) \sim 10^{-5}$$

Preliminary:

Implementation of diffeo = average over graph symmetries

- Overlap gets slightly bigger

Summary

- Neural networks can in principle par
- NetKet:
  - Strong enough to represent LQG-t
  - Powerful numerical capabilities
- Exponential growth of dim H always
  - Cutoffs too severe?
  - Start from gauge invariant descript
- Need to go to 4d and SU(2), need (r
- We live exciting times

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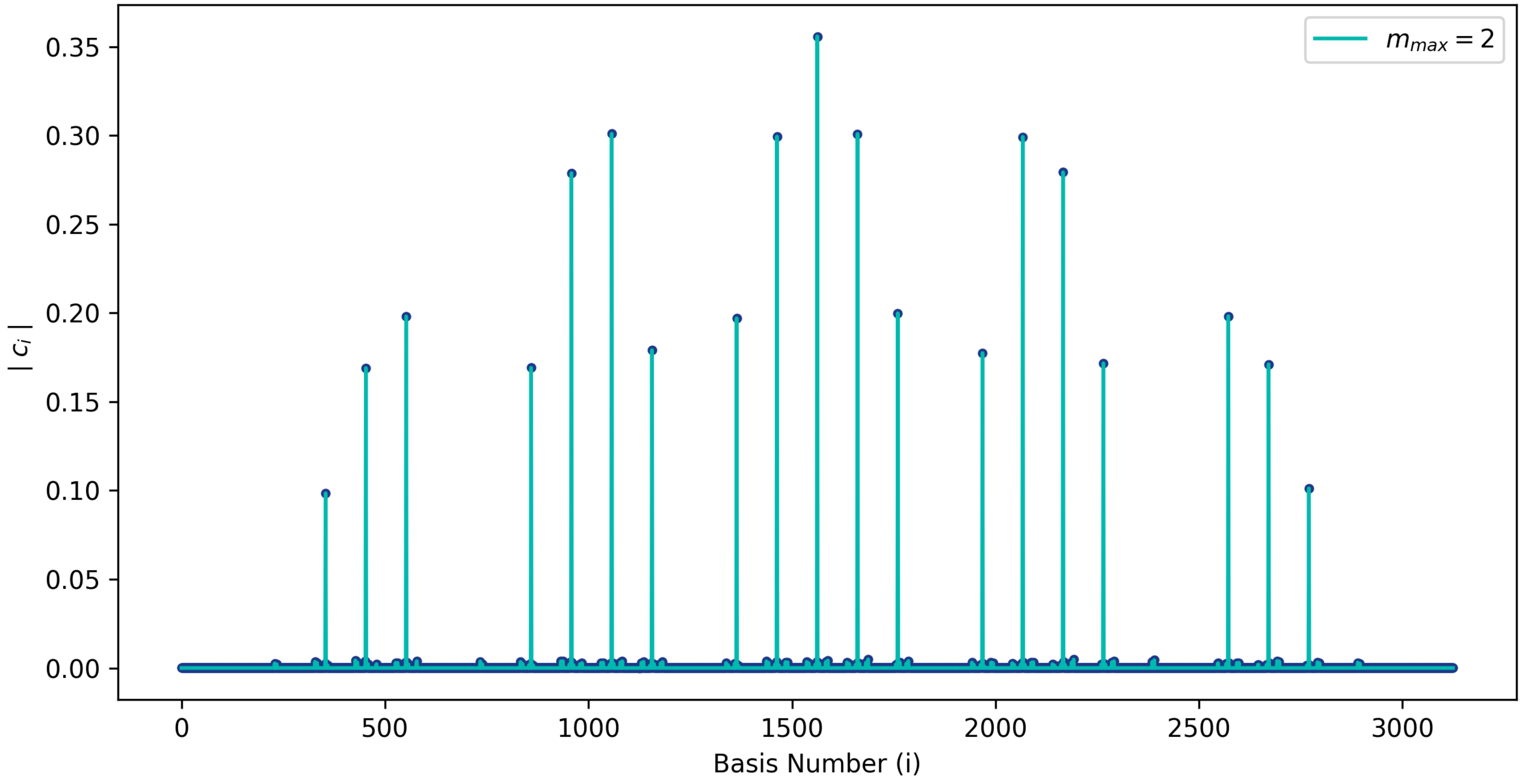
Sanity checks: U(1) BF



Gauge invariant subspace is selected