

Understanding gravitationally induced decoherence parameters in neutrino oscillations using a microscopic quantum mechanical model



Friedrich-Alexander-Universität
Erlangen-Nürnberg



joint work with Alba Domi, Thomas Eberl, Max Fahn, Lukas Hennig, Uli Katz,
Roman Kemper, Michael Kobler,
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Kristina Giesel, Lehrstuhl für Quantengravitation, ECAP
FAU Erlangen-Nürnberg

Motivation

Open Quantum System: interesting from a quantum gravity perspective

Gravitationally induced decoherence in linearised gravity

[Oníga, Wang 2016], [Anastopoulos, Hu 2013] [Lagouvardos, Anastopoulos 2021] [Fahn, K.G., Kemper, Kobler '22+'24]

Intersection of quantum gravity and quantum information

QM toy model for applications in neutrino oscillations

[Blencowe, Xu '22]

[Domí, Eberl, Fahn, K.G., Hennig, Katz, Kemper, Kobler '24]

Phenomenological models: Decoherence in neutrino oscillations

Phenomenological models

Take Lindblad equation no Lamb shift $H_S = H_{\text{vac}} + U^\dagger H_{\text{matt}} U$ in mass basis

$$\frac{\partial \rho_\nu(t)}{\partial t} = -\frac{i}{\hbar} [H_S, \rho_\nu(t)] + \mathcal{D}[\rho_\nu(t)] \quad H_{\text{matt}} = \pm \sqrt{2} G_f N_e \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \text{ in flavor basis}$$

effective mass basis $\tilde{H} = \tilde{U}^\dagger H \tilde{U} = \text{diag}(\tilde{E}_1, \tilde{E}_2, \tilde{E}_2)$

Do not choose Lindblad operators but parametrise dissipator by 45 = 8 · 9/2 decoherence parameters, additional assumption reduce to 2-3 parameters

$$\mathbf{D} = -\text{diag}(\Gamma_{21}, \Gamma_{21}, 0, \Gamma_{31}, \Gamma_{31}, \Gamma_{32}, \Gamma_{32}, 0) \quad \Gamma_{ij}(E) = \gamma_{ij} E^n$$

Additional damping term

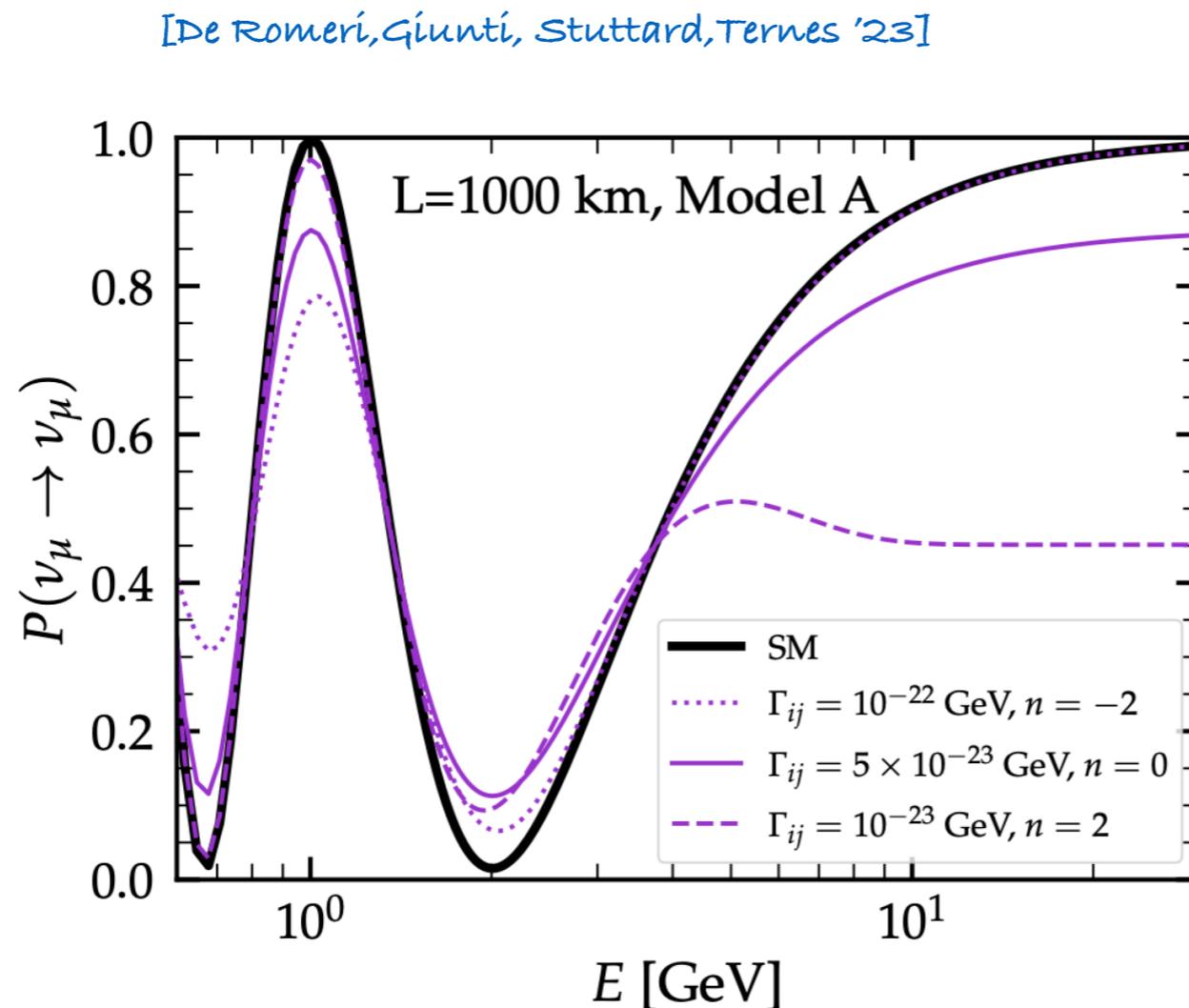
$$\tilde{\rho}_{ij} = \tilde{\rho}_{ij}(0) e^{-\frac{i}{\hbar}(\tilde{E}_i - \tilde{E}_j)t - \Gamma_{ij}t} \quad \text{effective mass basis}$$

Oscillation probability

$$P(\nu_\alpha \rightarrow \nu_\beta) = \text{Tr} \left[\hat{\rho}^{(\alpha)}(t) \hat{\rho}^{(\beta)}(0) \right] = \sum_{i,j} \tilde{U}_{\alpha i} \tilde{U}_{\beta i}^* \tilde{U}_{\alpha j}^* \tilde{U}_{\beta j} e^{-\frac{i}{\hbar}(\tilde{E}_i - \tilde{E}_j)t - \Gamma_{ij}t}$$

[Ellis, Lopez, Mavromatos, Nanopoulos 1996]; [Benatti, Floreanini 1999]; [Lisi, Marrone, Montanino 2000];
[Guzzo, de Holanda, Oliveira 2016]; [Gomes, Forero, Guzzo, de Holanda, Oliveira 2019]

Characteristic of long-baseline accelerator neutrino experiments



Model A: $\gamma_{21} = \gamma_{31} = \gamma_{32}$

Probabilities for oscillations change depending on the values for Γ_{ij} and choice of power n

$$\Gamma_{ij}(E) = \gamma_{ij}(E_0) \left(\frac{E}{E_0} \right)^n$$

Can we obtain some better physical intuition for decoherence parameters Γ_{ij} from a given microscopic model?

QM toy model for gravitationally induced decoherence

Microscopic model with QFT inspired coupling [Blencowe, Xu '22]

Microscopic model: $H_{\text{tot}} = H_S + H_\varepsilon + H_{\text{int}}$ $H_S^{(C)} = \sum_{i=1}^N g_i^2 \frac{\hbar}{\omega_i^2} (H_S^{(0)})^2$

$$H = \underbrace{(H_S^{(0)} + H_S^{(C)})}_{H_S} \otimes \mathbb{1}_\varepsilon + \mathbb{1}_S \otimes \underbrace{\left[\frac{1}{2} \sum_{i=1}^N (\hat{p}_i^2 + \omega_i^2 q_i^2) \right]}_{H_\varepsilon} - \underbrace{H_S \otimes \sum_{i=1}^N g_i q_i}_{H_{\text{int}}}$$

We consider a Gibbs state for the gravitational environment

$$\frac{d}{dt} \rho_S(t) = -\frac{i}{\hbar} [H_S, \hat{\rho}_S(t)] + \frac{i\Lambda(t-t_0)}{\hbar^2} \left[(H_S^{(0)})^2, \rho_S(t) \right] \leftarrow \text{lamb shift contribution}$$

$$+ \frac{\Gamma(t-t_0)}{\hbar^2} \left(H_S^{(0)} \rho_S(t) H_S^{(0)} - \frac{1}{2} \left\{ (H_S^{(0)})^2, \rho_S(t) \right\} \right)$$

Lindblad equation

Lindblad equation

- (ii). Markov approx: $t_0 \rightarrow -\infty$ $C_{\alpha\beta}(\xi)$ peaked
- spectral density $H_{\text{int}}(t) = gS(t) \otimes \sum_{\alpha} E_{\alpha}(t)$ continuous $J(\omega)$

Lindblad operators L_k LS correction H_{LS} γ_k L_k time-independ.

$$\frac{\partial}{\partial t} \rho_S(t) = \frac{1}{i\hbar} [H_S + H_{\text{LS}}, \rho_S(t)] + \sum_k \gamma_k \left(L_k \rho_S(t) L_k^\dagger - \frac{1}{2} \{ L_k^\dagger L_k, \rho_S(t) \} \right)$$

For given H_S model characterised by choice of L_k, γ_k ← QG

H_{LS} ← includes for 2nd order TCL second powers of L_k , depends on $J(\omega)$ + regulator, coupling g^2

γ_k ← depends on $J(\omega)$, if we use a Gibbs state depends on temperature parameter, Θ , coupling g^2

+ plus additional constants like c, k_B, \hbar are involved

Comparison with phenom. models

Do we see deviations in the probabilities for neutrino oscillations when we compare these two different couplings?

Phenol models (PQD)

Gravit. induc. decoh. (GQD)

$$\tilde{\rho}_{ij} = \tilde{\rho}_{ij}(0) e^{-\frac{i}{\hbar}(\tilde{E}_i - \tilde{E}_j)t - \Gamma_{ij}(E)t}$$

$$\tilde{\rho}_{ij}(t) = \tilde{\rho}_{ij}(0) \cdot e^{-\frac{i}{\hbar}(\tilde{H}_i - \tilde{H}_j)t - \frac{4\eta^2 k_B T}{\hbar^3}(\tilde{H}_i - \tilde{H}_j)^2 t}$$

$$\Gamma_{ij}(E) = \gamma_{ij} E^n$$



$$\frac{4\eta^2 k_B T}{\hbar^3} (\tilde{H}_i - \tilde{H}_j)^2 t \quad \eta, T$$

For oscillations in vacuum exact match

$$\Gamma_{ij}(E) = \gamma_{ij} E^{-2} \quad \gamma_{ij} = \frac{\eta^2 c^8 k_B T}{\hbar^3} (\Delta m_{ij}^2)^2 \quad \text{and} \quad n = -2$$

For oscillations in matter matching not possible if constant decoherence parameters assumed that do not include matter effects [\[Carpio, Massoni, Gargó '18\]](#)

Existing bounds for constant Γ_{ij} cannot be used to constrain η, T

Setting some of the γ_{ij} equal and/or zero might be inconsistent in grav. decoh.

Comparison with phenom. models

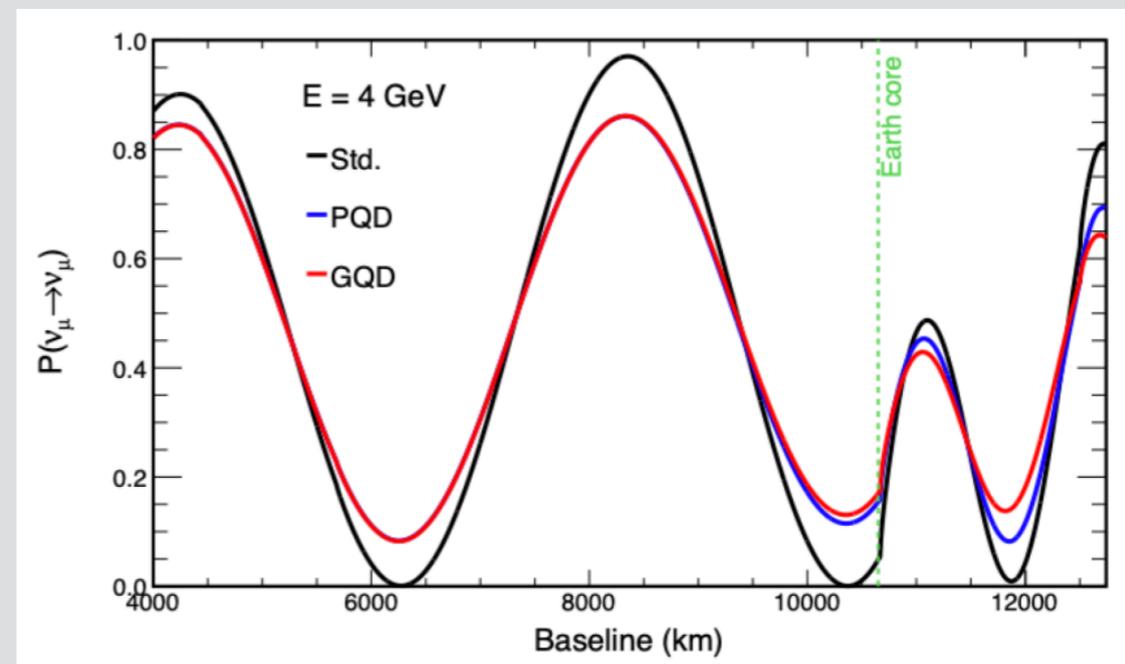
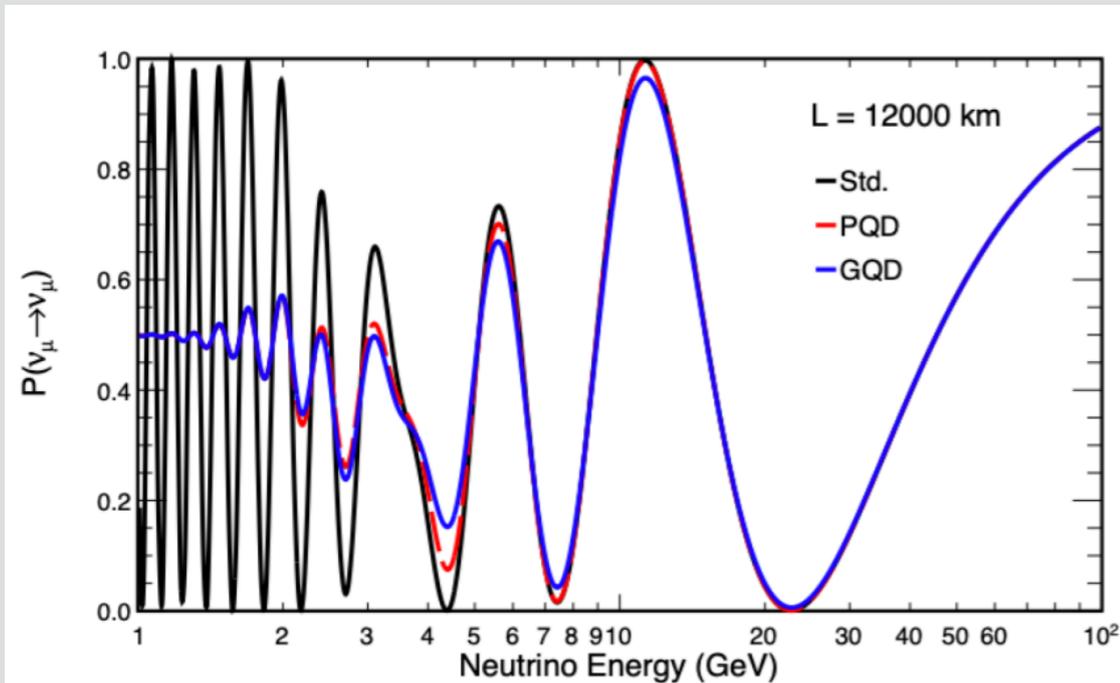
At the level of the Lindblad equation

$$\frac{d}{dt} \hat{\rho}_S(t) = -\frac{i}{\hbar} \left[\hat{H}_S^{(0)}, \hat{\rho}_S(t) \right] + \Gamma_M \left(\hat{L} \hat{\rho}_S(t) \hat{L}^\dagger - \frac{1}{2} \left\{ \hat{L}^\dagger \hat{L}, \hat{\rho}_S(t) \right\} \right)$$

Still models are not identically in non-vacuum case $L = H_S^{(0)}$ $L = H_{\text{vac}}$

—————→ different coupling to the gravitational environment

Deviations in the probabilities for neutrino oscillations $\eta = 10^{-8} \text{ s}, T = 0.9 \text{ K}$



Summary & Conclusions

Considered open quantum system for gravitationally induced decoherence

Strategy: Learn from open QFT models and consider toy models

→ Linearised gravity: gravity as an environment for matter systems

Aim: understand physical interpretation of decoherence parameters

Results: microscopic model can for vacuum oscillations be matched with subclass of phenom. models, in particular $n = -2$

→ importance of chosen coupling to the environment

Deviations to phenomenological models for oscillations in matter

→ gravitational decoherence favours non-constant decoherence parameters γ_{ij}

Only first steps in this direction

plan to investigate bounds for decoherence parameters in this model for neutrino telescopes → constrain η, T in the model

consider LQG inspired quantisation for gravitational environment and effects in neutrino oscillations.

Thank you for your attention!

QM toy model for gravitationally induced decoherence

Lamb shift contribution and decoherence term

$$\Lambda(t - t_0) = 2 \int_0^{t-t_0} d\tau \int_0^\infty \sin(\omega\tau) J(\omega)$$

$$\Gamma(t - t_0) = 4 \int_0^{t-t_0} d\tau \int_0^\infty \cos(\omega\tau) \coth\left(\frac{\beta\hbar\omega}{2}\right) J(\omega)$$

Spectral density

$$J(\omega) = \frac{1}{2} \sum_{i=1}^N g_i^2 \frac{\hbar^2}{\omega_i} \delta(\omega - \omega_i)$$

One considers continuous $J(\omega)$ linear in ω , different regulators

$$J(\omega) = \frac{2}{\pi} \eta^2 \omega \frac{\Omega^2}{\Omega^2 + \omega^2} \quad \text{Lorentz-Drude}$$

$$J(\omega) = \frac{2}{\pi} \eta^2 \omega e^{-\frac{2\omega}{\pi\Omega}} \quad \text{exponential}$$

$$J(\omega) = \frac{2}{\pi} \eta^2 \omega \frac{(2\Omega)^4}{((2\Omega)^2 + \omega^2)^2} \quad \text{quartic}$$

$$J(\omega) = \frac{2}{\pi} \eta^2 \omega e^{-\frac{\omega^2}{\pi\Omega^2}} \quad \text{Gaussian}$$

For all cases we find in the Markovian limit

$$\Lambda := \Lambda(\infty) = 2\Omega\eta^2 \quad \Gamma := \Gamma(\infty) = \frac{8\eta^2}{\beta\hbar} \quad \Lambda \text{ still depends on cutoff frequency, diverges when regulator is removed}$$

QM toy model for gravitationally induced decoherence

Need counter term to renormalise neutrino Hamiltonian

$$H_S^{(C)} = \sum_{i=1}^N g_i^2 \frac{\hbar}{\omega_i^2} \left(H_S^{(0)} \right)^2 \quad H_S^{(C)} = \frac{2}{\hbar} \int_0^\infty d\omega \frac{J(\omega)}{\omega} \left(H_S^{(0)} \right)^2 = \frac{2\Omega\eta^2}{\hbar} \left(H_S^{(0)} \right)^2$$

We end up with (non-physical lamb shift is removed)

$$\frac{d}{dt} \rho_S(t) = -\frac{i}{\hbar} \left[H_S^{(0)}, \rho_S(t) \right] + \frac{8\eta^2}{\hbar^2} \frac{1}{\hbar\beta} \left(L\rho_S(t)L^\dagger - \frac{1}{2} \{L^\dagger L, \rho_S(t)\} \right) \quad \begin{array}{l} L = H_S^{(0)} \\ H_S = H_{\text{vac}} + U^\dagger H_{\text{matt}} U \end{array}$$

[Benatti., Floreanini '01]

Massless neutrino oscillations not possible here

here counter term higher order in coupling, can be neglected in coupling to environment

often LS + counter term neglected, works here but not in any model Caldeira-Leggett

