The Bounce in the Bianchi Models

as a quantum scattering

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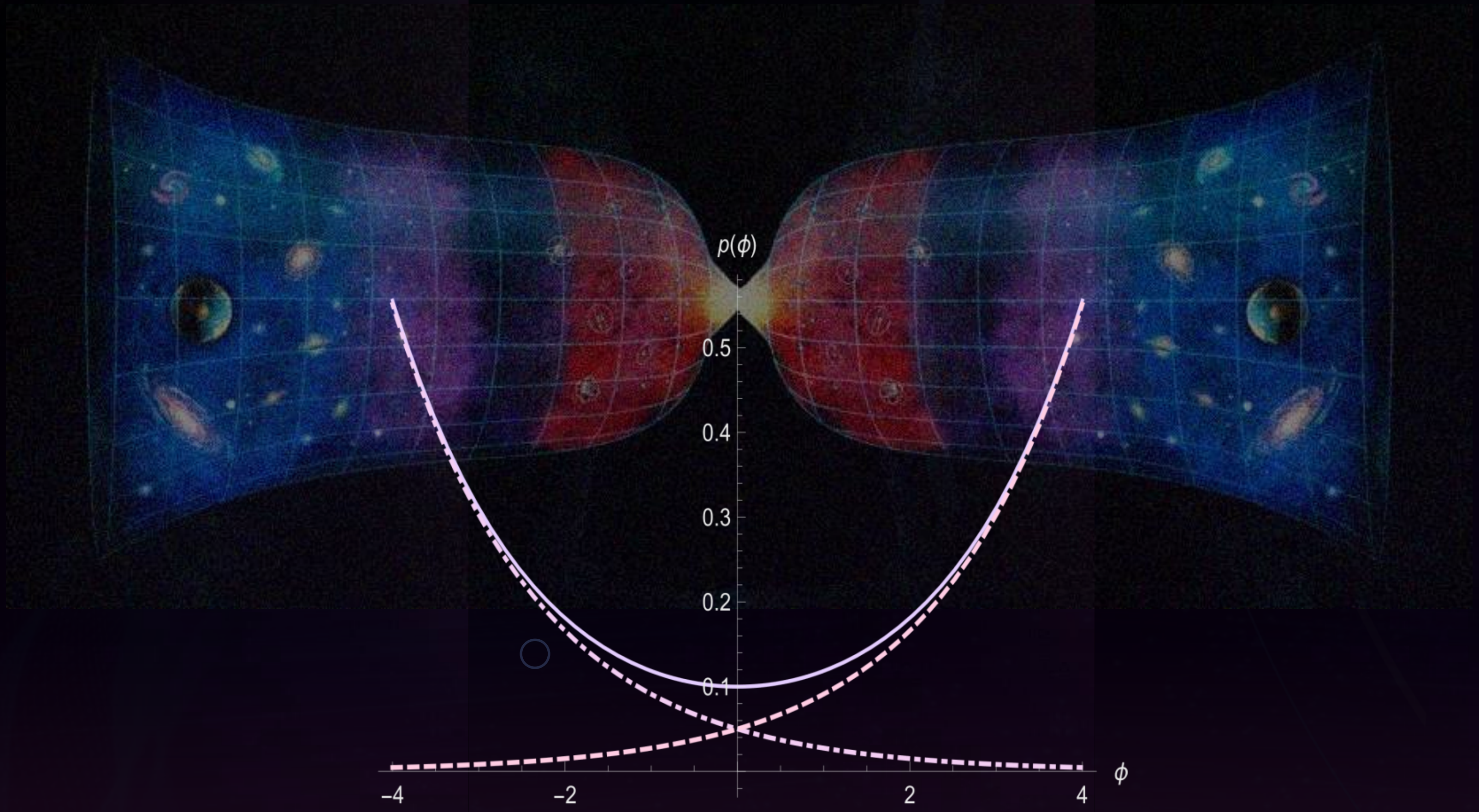
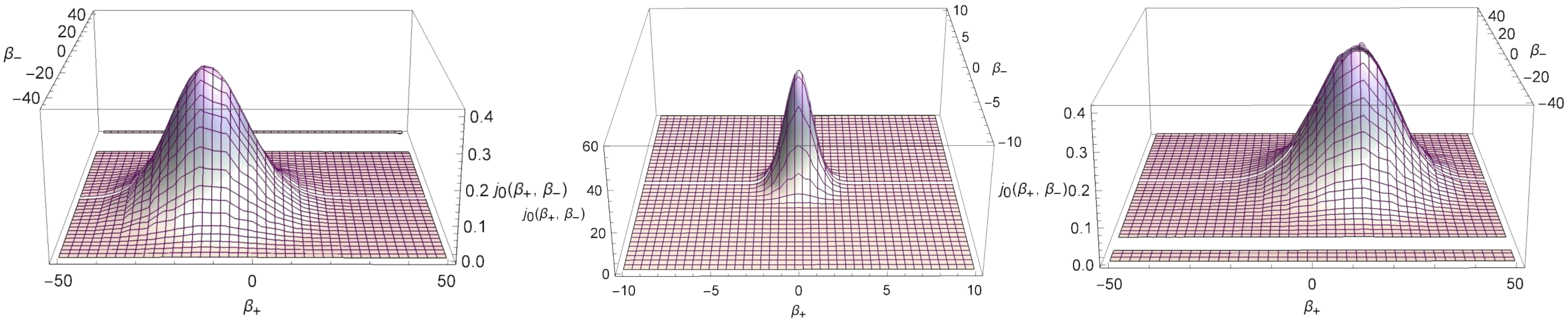
What is the Big Bounce?



PROBLEM	BIG BOUNCE AS A QUANTUM PROCESS
CONTEXT	WHEELER-DE WITT FORMULATION OF THE BIANCHI MODELS → ANALOGY WITH THE KLEIN-GORDON EQUATION
PROPOSAL	SCATTERING AMPLITUDE AS DESCRIBED IN RELATIVISTIC QUANTUM MECHANICS

PROBLEM

- The Big Bounce is a Planckian phenomenon, so the semiclassical description is not satisfactory since quantum effects are not negligible.
- When considering generic cosmological models before the CMB, anisotropies arise and hence the hypothesis of localized wave packets is violated.



3D-profiles of the Bianchi I wave packet for $\alpha = -10, 0, 10$ respectively.

CONTEXT

Classical world

Bianchi I Hamiltonian in the Misner variables

$$H = Ce^{-3\alpha}\left[-p_\alpha^2 + p_+^2 + p_-^2\right] = 0$$

collapsing and expanding singular solutions

$$\dot{\alpha} = -2NCe^{-3\alpha}p_\alpha$$

Quantum world

Wheeler-DeWitt equation

$$\hat{H}\psi = \Box\psi(\alpha,\beta_\pm) = [\partial_\alpha^2 - \partial_{\beta_+}^2 - \partial_{\beta_-}^2]\psi(\alpha,\beta_\pm) = 0$$

separation of frequencies

$$\psi_{\omega_k}^{\pm}(\alpha,\beta_\pm) = e^{\mp i\omega_k\alpha}e^{i(k_+\beta_+ + k_-\beta_-)}\ , \ \omega_k \equiv \sqrt{k_+^2 + k_-^2}$$

- analogy with a massless Klein-Gordon equation
- α emerges as time at a quantum level (different signature)
- the positive frequency solutions $\psi_{\omega_k}^{+}$ describe an expanding Universe,

whereas the negative frequency ones $\psi_{\omega_k}^{-}$ describe a collapsing Universe (see the eigenvalues of $\hat{p}_\alpha = -i\partial_\alpha$)

PROPOSAL

$$\hat{H}\psi = \left[\partial_\alpha^2 - \partial_{\beta_+}^2 - \partial_{\beta_-}^2 + \lambda e^{-3\varepsilon\alpha}\right]\psi(\alpha, \beta_\pm) = 0$$

Wheeler-DeWitt equation for the Bianchi I model with an ekpyrotic-like matter term

$\psi(\alpha, \beta_\pm)$

in-going wave packet

the collapsing solution that emerges from the interaction with the time-dependent potential → is a mixed state

$V(\alpha)$

time-dependent potential

quantum interacting term responible for the transition

$\chi(\alpha, \beta_\pm)$

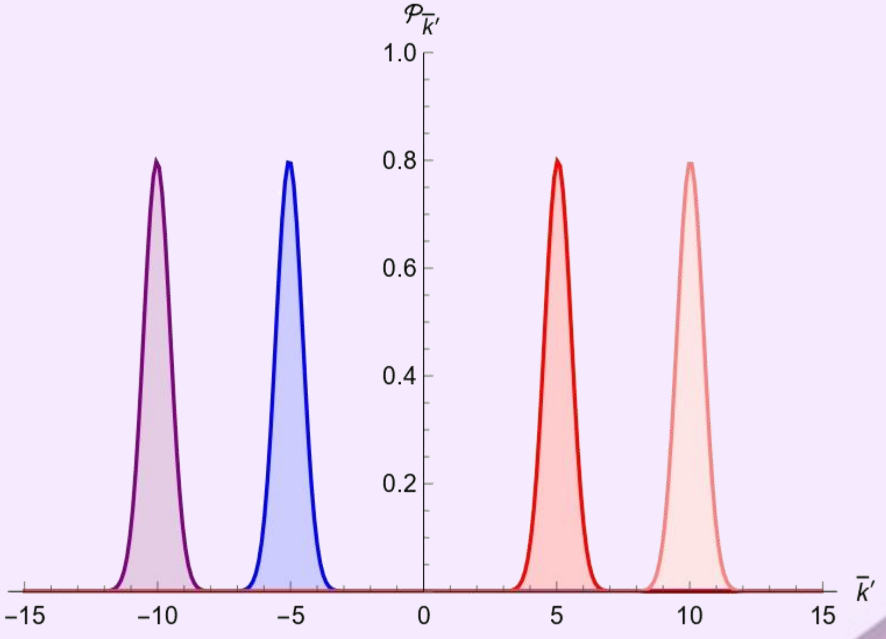
out-going wave packet

the expanding solution → is a free state

SCATTERING AMPLITUDE

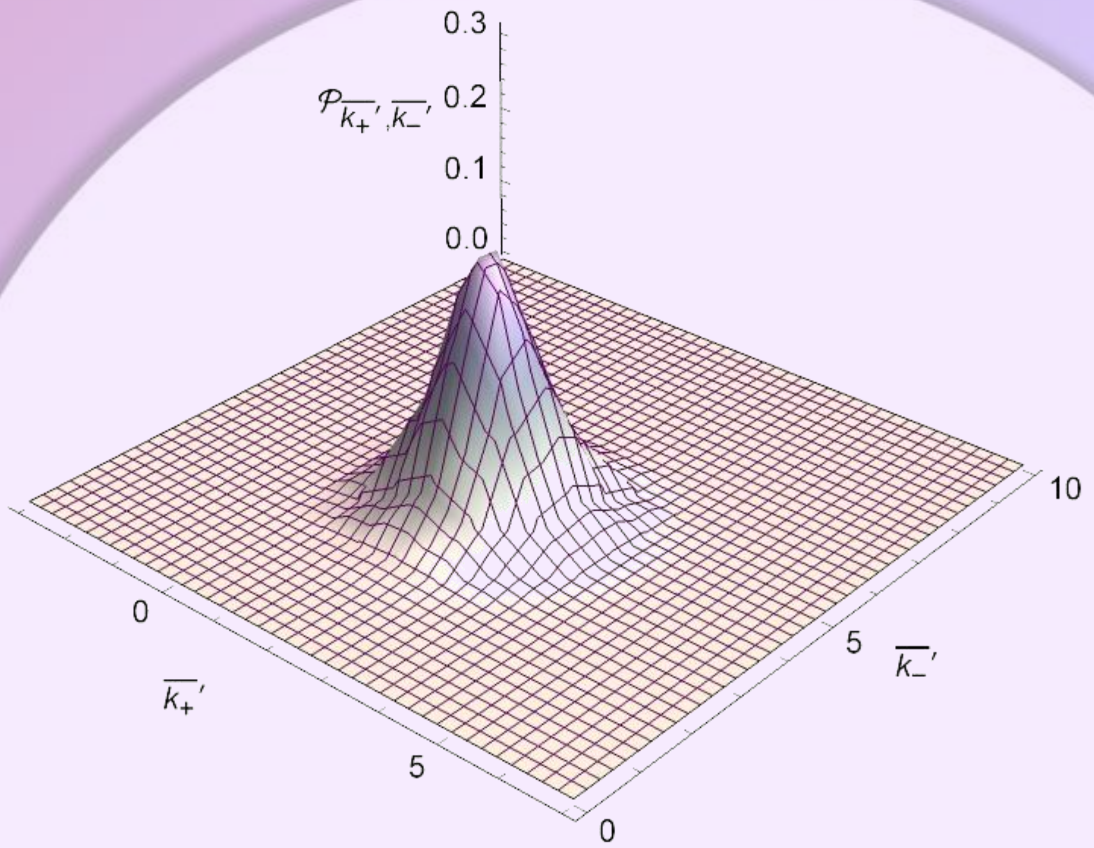
$$S_{Bounce} = -i \iiint_{-\infty}^{+\infty} d\alpha\, d\beta_+ d\beta_-\, \chi^*(\alpha, \beta_\pm) V(\alpha) \psi(\alpha, \beta_\pm)$$

The Big Bounce as a probabilistic process



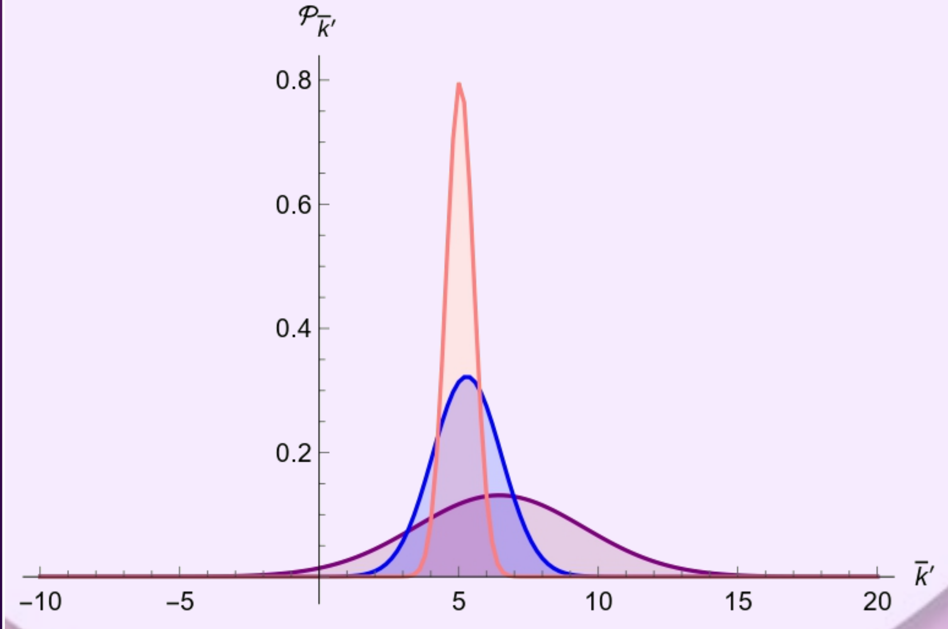
Transition probability of the Quantum Big Bounce

$$\mathcal{P} = |S_{Bounce}(\bar{k}'_+, \bar{k}'_-, \bar{k}_+, \bar{k}_-)|^2$$

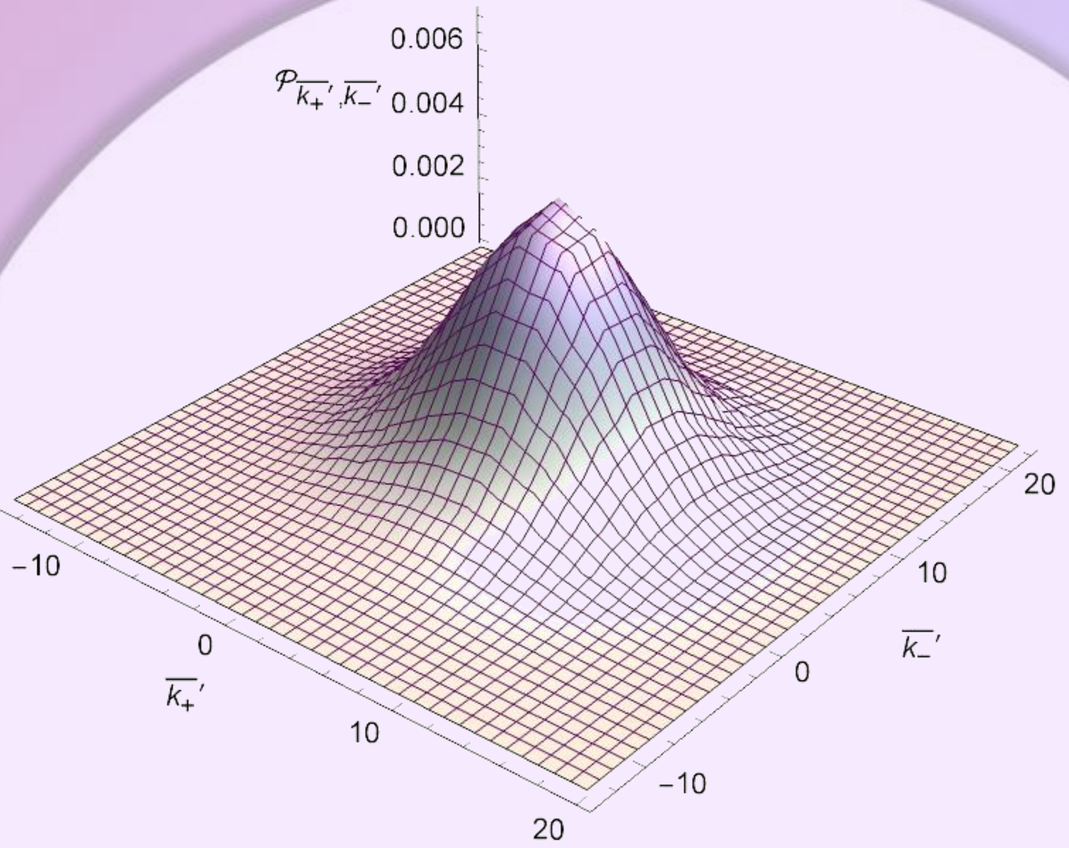


The plots highlight the Gaussian shape of the probability density.

What is a Quantum Big Bounce?



The bigger the variance of the wave packet is, the more appreciable the shift of the peak $\bar{k}'_+, \bar{k}'_-$ from $\bar{k}_+, \bar{k}_-$ is.



MAIN RESULT: for highly-localized wave packets this probability density reproduces the same symmetrical reconnection of the semiclassical Big Bounce.

UPGRADE: the Dirac approach!

- well-defined norm
- suitable to describe the Kasner transition a quantum level

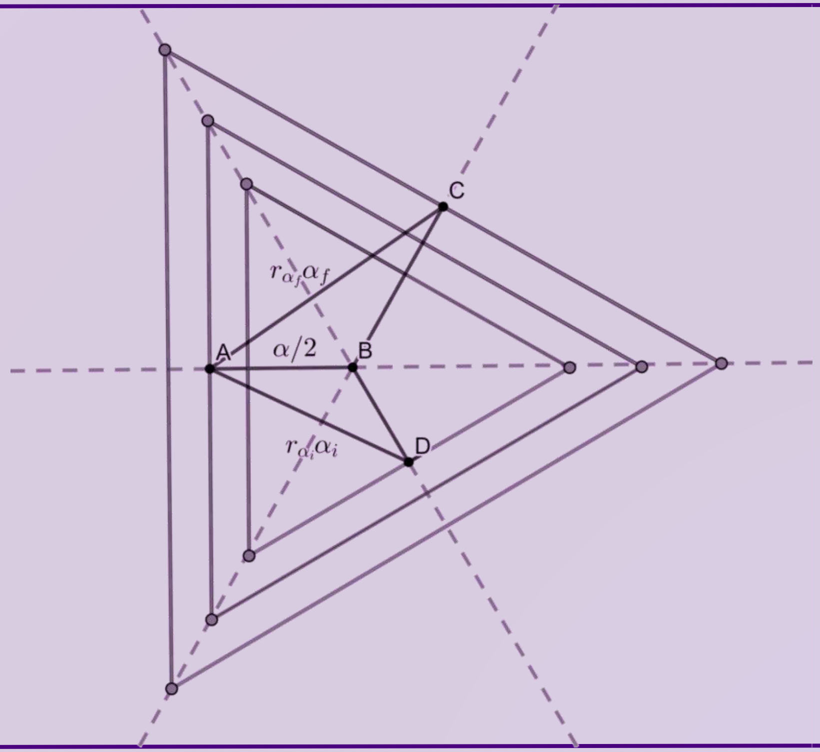
Classical world

Bianchi II Hamiltonian in the Misner variables

$$H = -p_\alpha^2 + p_+^2 + p_-^2 + \frac{3(4\pi)^4}{(8\pi G)^2} e^{4(\alpha - 2\beta_+)} = 0$$

ADM reduction → particle in a potential well

$$-p_\alpha = \sqrt{p_+^2 + p_-^2 + \frac{3(4\pi)^4}{(8\pi G)^2} e^{4(\alpha - 2\beta_+)}}$$



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Quantum world

Klein-Gordon equation

$$\hat{H}\psi(\alpha, \beta_\pm) = \left[\partial_\alpha^2 - \partial_+^2 - \partial_-^2 + \frac{3(4\pi)^4}{(8\pi G)^2} e^{4(\alpha - 2\beta_+)}\right]\psi(\alpha, \beta_\pm)$$

Dirac equation

$$\left[i\sigma^\mu \partial_\mu - \sigma_3 \frac{\sqrt{3}(4\pi)^2}{8\pi G} e^{2(\alpha - 2\beta_+)}\right]\psi(\alpha, \beta_\pm) = 0$$

σ^μ → usual Pauli matrices + the identity matrix

- $\psi(\alpha, \beta_\pm)$ is a two-components object!
- Reference: "*Emergent spin and clock variable in Bianchi type-I quantum cosmology*" , V. and M. K. Nandy https://arxiv.org/pdf/2402.13839.pdf

UPGRADED PROPOSAL

PRELIMINARY RESULTS

Dirac equation for the Bianchi I model with an ekpyrotic-like matter term

$$\left[i\sigma^\mu\partial_\mu - \sigma_3\lambda e^{-3\varepsilon\alpha}\right]\psi(\alpha,\beta_\pm) = 0$$

$$v(k_+,k_-) = \begin{pmatrix} k_0 \\ -(k_+ + ik_-) \end{pmatrix}$$

- out-going wave packet $\Psi^{exp} = \iint_{-\infty}^{+\infty} dk_+ dk_- u(k_+,k_-) N e^{-(k_0-\bar{k}_0^{exp})^2} e^{-i(k_\alpha\alpha - k_+\beta_+ - k_-\beta_-)}$

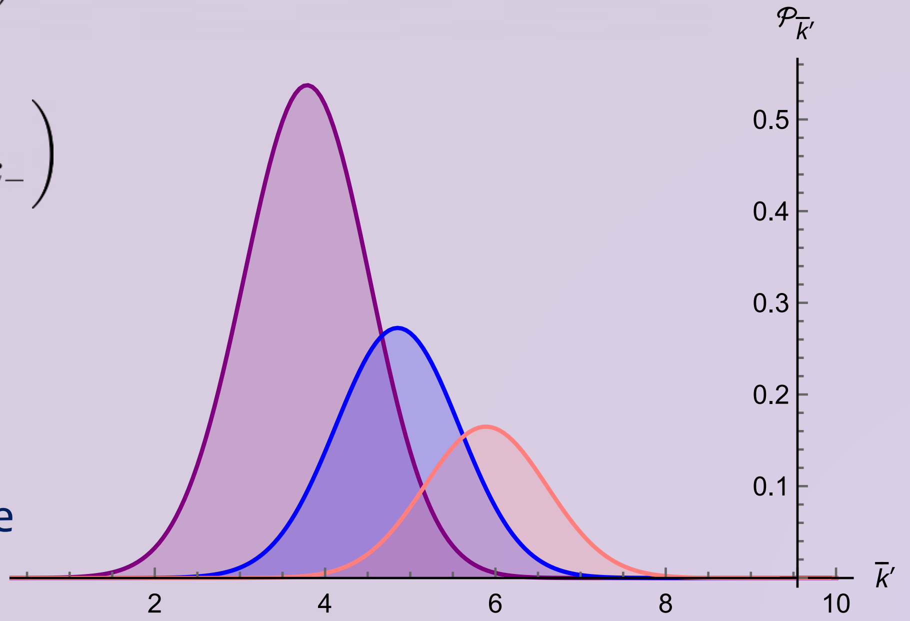
$$k_0 = \sqrt{k_+^2 + k_-^2}$$

- in-going wave packet $\Psi^{coll} = \iint_{-\infty}^{+\infty} dk_+ dk_- v(k_+,k_-) N e^{-(k_0-\bar{k}_0^{coll})^2} e^{i(k_\alpha\alpha - k_+\beta_+ - k_-\beta_-)}$

$$u(k_+,k_-) = \begin{pmatrix} k_0 \\ k_+ + ik_- \end{pmatrix}$$

SCATTERING AMPLITUDE

$$S_{Bounce} = -i\iiint_{-\infty}^{+\infty} d\alpha d\beta_+ d\beta_- \bar{\Psi}^{coll}\lambda e^{-3\varepsilon\alpha}\Psi^{exp}$$



MAIN RESULT: the probability density seems to have the same properties of the KG case.

CONCLUSIONS & PERSPECTIVES

- Analogy between **collapsing/expanding** Bianchi I Universes and **positive/negative** frequency solutions of the **KG formalism**.

- The **ekpyrotic-like matter term** creates a mixed state near the singularity, thus making possible the transition from a collapsing to an expanding Universe.

- The **Quantum Big Bounce** probability density shows a **symmetrical reconnection** of the collapsing and expanding branches for semiclassical states, as it happens in the semiclassical Big Bounce.

STAY TUNED!

- The same scheme can be reproduced using the **Dirac formalism**.

- In the Dirac approach we deal with a **well-defined norm**, on the other hand it is difficult to find the analytic expression of the mixed state.

- The Dirac approach is well-suited to describe the **BKL map at a quantum level** → application to the Bianchi IX model and its chaotic bounces towards the singularity.

THANK YOU FOR THE ATTENTION!

REFERENCES

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- "*Is Bianchi I a bouncing cosmology in the Wheeler-DeWitt picture?*",
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- "*An Overview on the Nature of the Bounce in LQC and PQM*",
  G. Barca, E. Giovannetti and G. Montani,
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If you have questions, please ask or email me at
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BACKUP SLIDES

BIANCHI I

Two-dimensional relativistic particle

$$(\partial_\alpha^2 - \partial_{\beta_+}^2 - \partial_{\beta_-}^2)\psi(\alpha, \beta_\pm) = 0$$

Non-linear dispersion relation

$$\omega_k = \sqrt{k_+^2 + k_-^2}$$

FLRW

One-dimensional relativistic particle

$$(\partial_\alpha^2 - \partial_\phi^2)\psi(\alpha, \phi) = 0$$

Linear dispersion relation

$$\omega_k = k_\phi$$

In the case of the **Bianchi I** model the non-zero **second derivative of the dispersion relation** enters in the variance of the Gaussian wave packet through a linear term in ***α*** that produces the **spreading phenomenon**.

Klein-Gordon equation

$$(\Box + m^2)\varphi(x) = 0$$

→ free relativistic particles of zero spin

$$f_{\mathbf{p}}^{(\pm)}(x) = \frac{e^{\mp ip\cdot x}}{\sqrt{(2\pi)^2 2\omega_{\mathbf{p}}}}$$

that form an orthonormal set

$$\int d^2x\, f_{\mathbf{p}'}^{(\pm)*}(x) i\overleftrightarrow{\partial_0} f_{\mathbf{p}}^{(\pm)}(x) = \pm\delta^2(\mathbf{p}-\mathbf{p}'),$$
$$\int d^2x\, f_{\mathbf{p}'}^{(\pm)*}(x) i\overleftrightarrow{\partial_0} f_{\mathbf{p}}^{(\mp)}(x) = 0.$$

Interaction potential

$$(\Box + m^2 + V(x))\phi(x) = 0$$

→ general solution in terms of the Feynman propagator (by iteration)

$$\phi(x) = \varphi(x) - \int d^3y\, \Delta_F(x-y) V(y)\phi(y)$$

"Relativistic quantum mechanics" by James D. Bjorken and Sidney D. Drell

Transition amplitude in the wave function formalism

Particles scattering	***Pair annihilation***
$S_{\mathbf{p}'_+,\mathbf{p}_+} = \delta^2(\mathbf{p}'_+ - \mathbf{p}_+) - i\int d^3y\, f^{(+)*}_{\mathbf{p}'_+}(y)V(y)\phi(y)$	$S_{\mathbf{p}_-,\mathbf{p}_+} = -i\int d^3y\, f^{(-)*}_{\mathbf{p}'_-}(y)V(y)\phi(y)$
Antiparticles scattering	***Pair production***
$S_{\mathbf{p}'_-,\mathbf{p}_-} = \delta^2(\mathbf{p}'_- - \mathbf{p}_-) - i\int d^3y\, f^{(-)*}_{\mathbf{p}'_-}(y)V(y)\phi(y)$	$S_{\mathbf{p}_+,\mathbf{p}_-} = -i\int d^3y\, f^{(+)*}_{\mathbf{p}'_+}(y)V(y)\phi(y)$

The transition probability is the square modulus of *S*.

In-going wave packet

$$\psi(\alpha, \beta_\pm) = \iint_{-\infty}^{+\infty} dk_+ dk_- \, A(k_+, k_-)\varphi(\alpha) e^{ik_+\beta_+} e^{ik_-\beta_-}$$

the collapsing solution
that emerges from the interaction
with the time-dependent potential ***V(α)***

(Bessel function of the first kind!)

Out-going wave packet

$$\chi(\alpha, \beta_\pm) = \iint_{-\infty}^{+\infty} dk'_+ dk'_- \, A'(k'_+, k'_-) e^{-i\omega_{k'}\alpha} e^{ik'_+\beta_+} e^{ik'_-\beta_-}$$

the free expanding solution

Scattering amplitude

$$S_{Bounce} = -i \iiint_{-\infty}^{+\infty} d\alpha \, d\beta_+ d\beta_- \, \chi^*(\alpha, \beta_\pm) V(\alpha) \psi(\alpha, \beta_\pm)$$