Entanglement in QFT: Lessons from Minkowski and deSitter space

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Loops 24, May 9th 2024

Work in collaboration with B. Bonga, P. Calizaya-Cabrera, B. Elizaga-Navascués, E. Martin-Martinez, S. Nadal-Gisvert, P. Ribes-Metidieri, K. Yamaguchi.

Goal:

Understand/quantify the entanglement content of QFT's, its spatial distribution, and its relation to curvature.

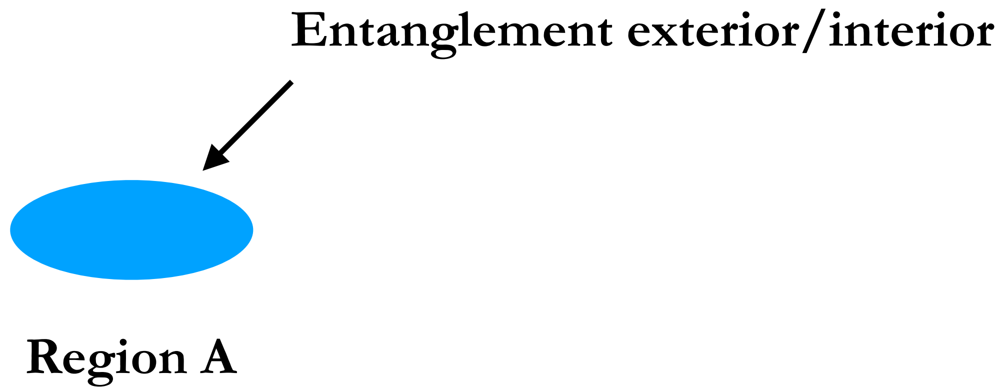
Interesting applications:

- Hawking radiation in evaporating scenarios (Cf. Beatriz Elizaga-Navascues', and Paula Calizaya-Cabrera's talks)
- de Sitter (cosmology)
- Connection with quantum gravity

The approach

Entanglement is all around in QFT

Simplest example: Free scalar field, Minkowski st, Minkowski vacuum.



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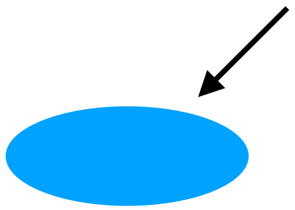
Entanglement exterior/interior



Region A

But region A hosts infinitely many field degrees of freedom.

"Who is entangled with whom?"

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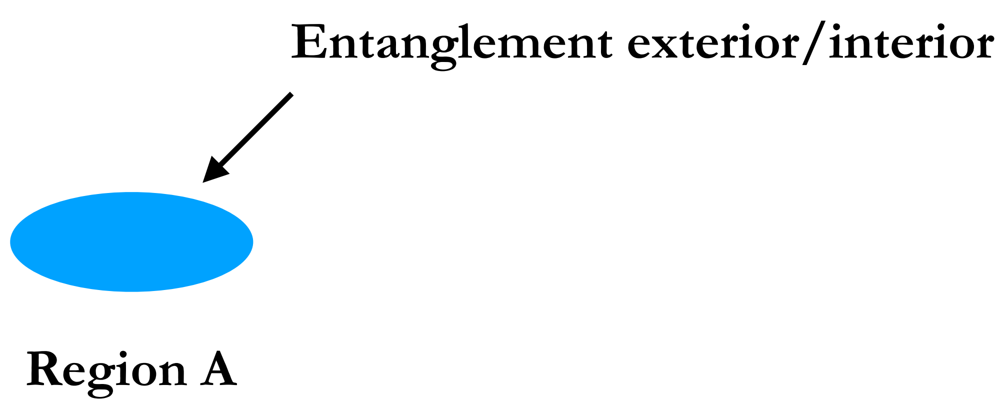


But region A hosts infinitely many field degrees of freedom.

“Who is entangled with whom?”

von Neumann entropy a region diverges. Cut-off makes it finite. But then, unclear interpretation.

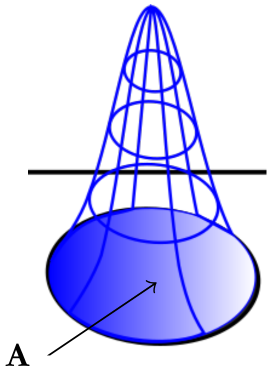
A complementary approach:

Study entanglement between an *a priori* specified set of finitely many field d.o.f.

(For similar lines of thought see e.g. Bianchi-Satz 2019)

Some relevant concepts

Free massless scalar field in 3+1 dim



Single-mode subsystem:

Consider a complex solution of the Klein-Gordon eqn. $f(x)$ such that $\langle f|f\rangle \neq 0$

Define the operator: $\hat{O}_f = \langle f|\hat{\Phi}\rangle$

$$\Longrightarrow \quad [\hat{O}_f, \hat{O}_f^\dagger] = \langle f|f\rangle \neq 0$$

Single-mode subsystem = sub-ablgebra generated from $\hat{O}_f$ and $\hat{O}_f^\dagger$

Notation: $\{f\}$ = Single-mode subsystem

$\{\cdot\}$ indicates $g = \alpha\, f + \beta\, f^*$ with $|\alpha|^2 - |\beta|^2 = 1$ defines the same single-mode subsyst.

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Simple way of computing $\hat{\rho}_f^{\rm red}$: take advantage it is a Gaussian state

$$\sigma_f = \begin{pmatrix} \langle\{\hat{O}_f, \hat{O}_f\}\rangle & \langle\{\hat{O}_f, \hat{O}_f^\dagger\}\rangle \\ \langle\{\hat{O}_f, \hat{O}_f^\dagger\}\rangle & \langle\{\hat{O}_f^\dagger, \hat{O}_f^\dagger\}\rangle \end{pmatrix} \qquad \Omega_f = \begin{pmatrix} \Omega(f,f) & \Omega(f,f^*) \\ \Omega(f,f^*) & \Omega(f^*,f^*) \end{pmatrix}$$

→ state independent

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Ω_f → state independent

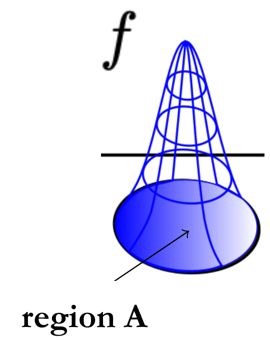
Example: entropy and purity

$\pm i\nu$ = eigenvalues of $\sigma_f \cdot \Omega_f$

$$S[\hat{\rho}_f^{\text{red}}] = \left(\frac{\nu+1}{2}\right)\log\left(\frac{\nu+1}{2}\right) - \left(\frac{\nu-1}{2}\right)\log\left(\frac{\nu-1}{2}\right)$$ entropy of $\hat{\rho}_f^{\text{red}}$

➡ $\hat{\rho}_f^{\text{red}}$ is pure iff the eigenvalues of $\sigma_f \cdot \Omega_f$ are equal to $\pm i$

If $\{f\}$ **mixed** → $\{f\}$ **must be entangled with other field modes**



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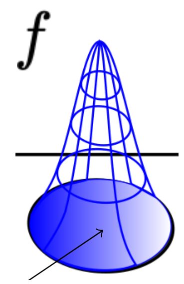

It turns out that one can find a single-mode subsystem $\{\bar{f}\}$ encoding all entangled with $\{f\}$

$\{\bar{f}\}$ = Partner of $\{f\}$

[Hotta, Schützhold, Unruh 2015; Trevison, Yamaguchi, Hotta 2019; Hackl, Johnson 2019]

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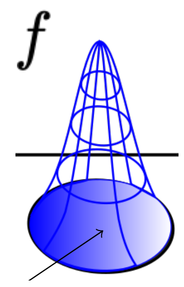

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Partner from the complex structure J:

[Agullo, Martin-Martinez, Nadal-Gisvert, Yamaguchi, to appear]

$$\bar{f} = \Pi_f^{\top}(J\, f)$$

Where

- J complex structure of $|0\rangle$
- $\Pi_f^{\top} = 1 - \Pi_f = 1 - \left(f\,\langle f, \cdot\rangle - f^*\,\langle f^*, \cdot\rangle\right)$: projector orthogonal to subsystem $\{f\}$

Partner of $\{f\}$

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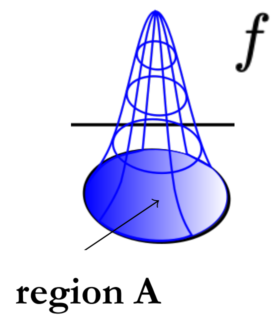
For f compactly supported, $\bar{f}(x)$ falls off at spatial infinity at a rate determined by the two-point function $\langle\hat{\Phi}(x)\hat{\Phi}(x')\rangle$

[Ribes-Metidieri, Agullo]

Minkoswki spacetime

Two-point function: $$\langle \hat{\Phi}(x)\hat{\Phi}(x') \rangle \sim \frac{1}{r^2}$$



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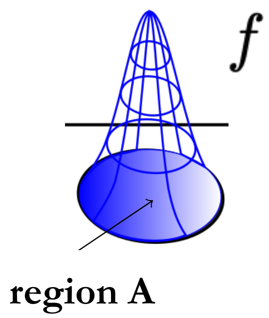

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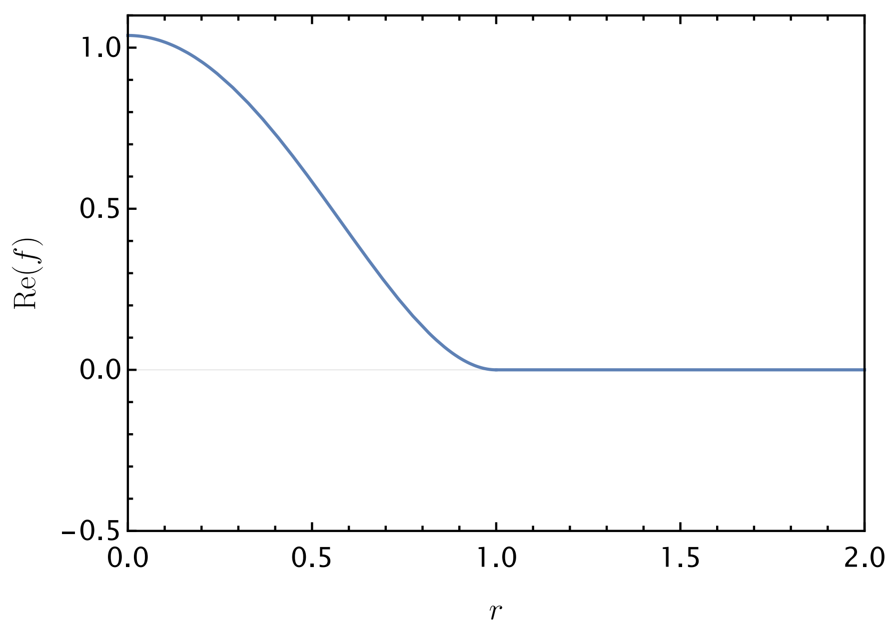
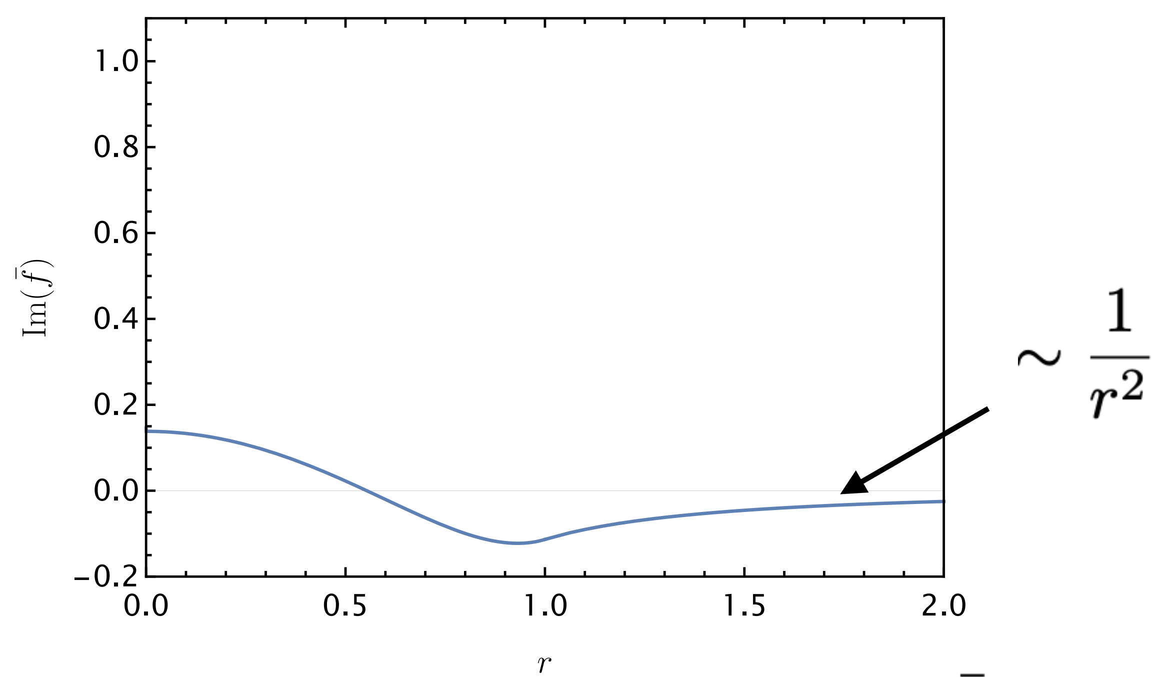
Example: $f(\vec{x}) = N\left(1-\left(\frac{r}{R}\right)^2\right)^2 \times \Theta\left(1-\frac{r}{R}\right)$



Spatial profile mode f
(compactly supported)



Spatial profile of the partner $\bar{f}$
(Not compactly supported)

[Agullo, Martin-Martinez, Nadal-Gisvert, Yamaguchi, to appear]

The spatial support of the partner serves to quantify the spatial distribution of entanglement

de Sitter spacetime

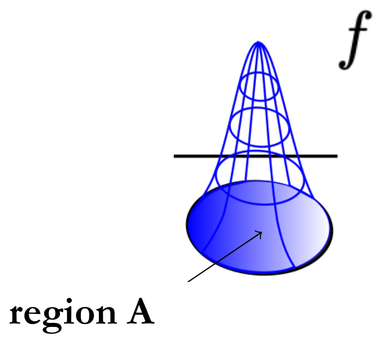
(Cosmological patch)

Massive scalar field, with small mass: $\mu^2 \equiv \frac{m^2}{3H^2} \ll 1$

Bunch-Davies vacuum: $|0\rangle$

$$\langle \hat{\Phi}(x)\hat{\Phi}(x')\rangle \sim \frac{1}{r^{2\mu}}$$

Almost scale invariant



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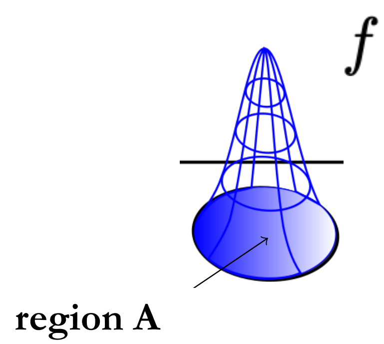
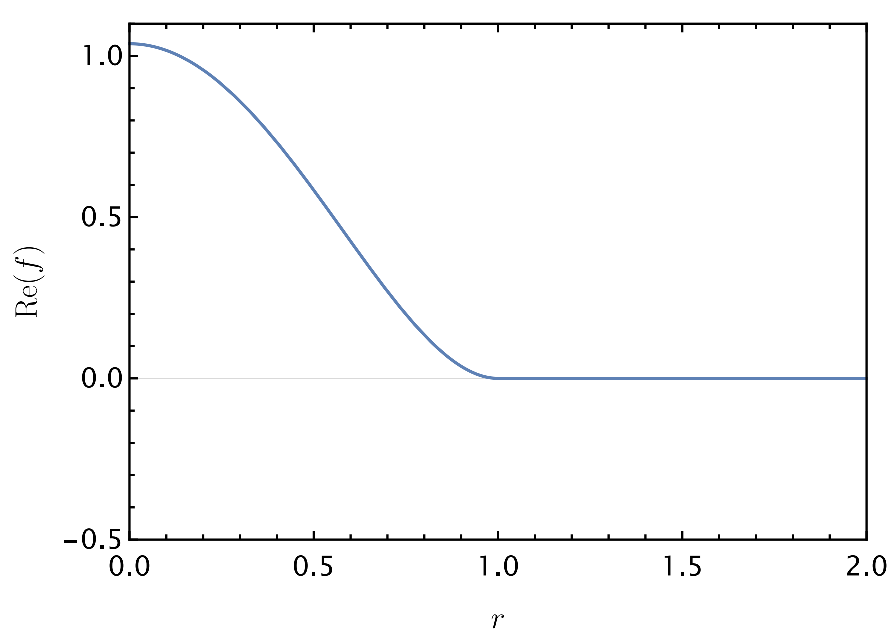


we expect → $\bar{f} \propto \frac{1}{r^{2\mu^2}}$

Example:



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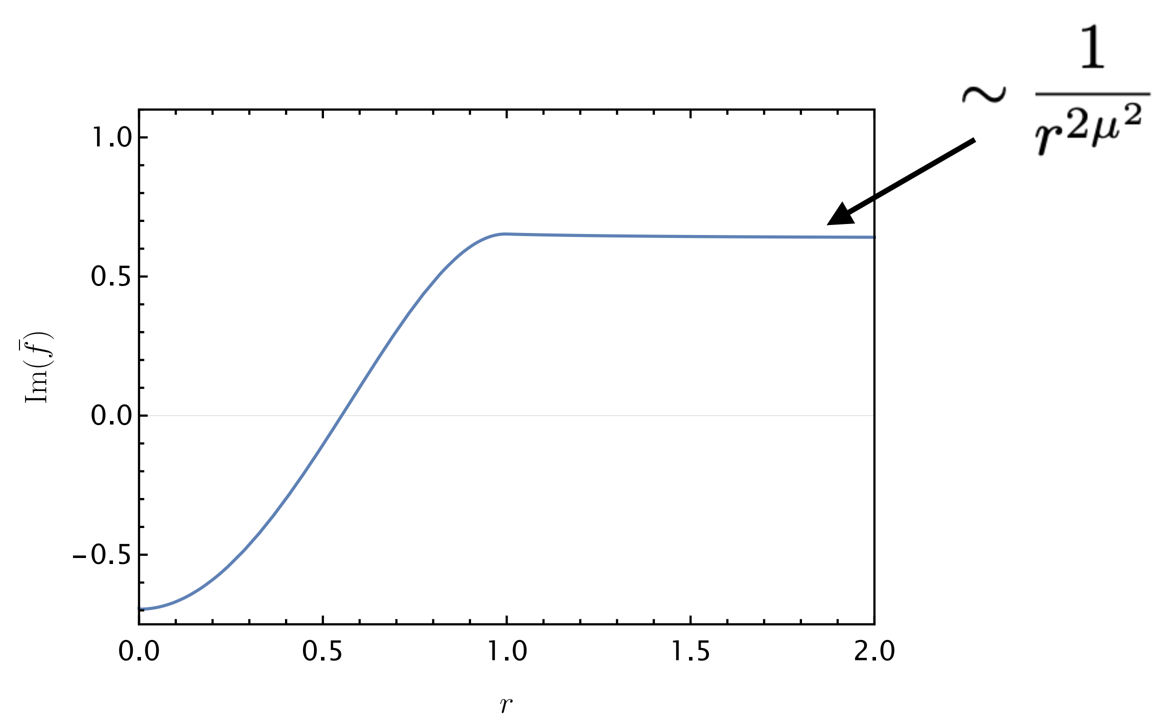

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A few results: [Agullo, Bonga, Ribes-Metidieri]

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- **Entropy of $\{f\}$ is larger in dSitter than in Minkwowski** ➡ **BD vacuum in dS space is more entangled than Minkowski vac.!**

(Compatible with many previous results for the entropy of regions; see e.g. Maldacena 2013)

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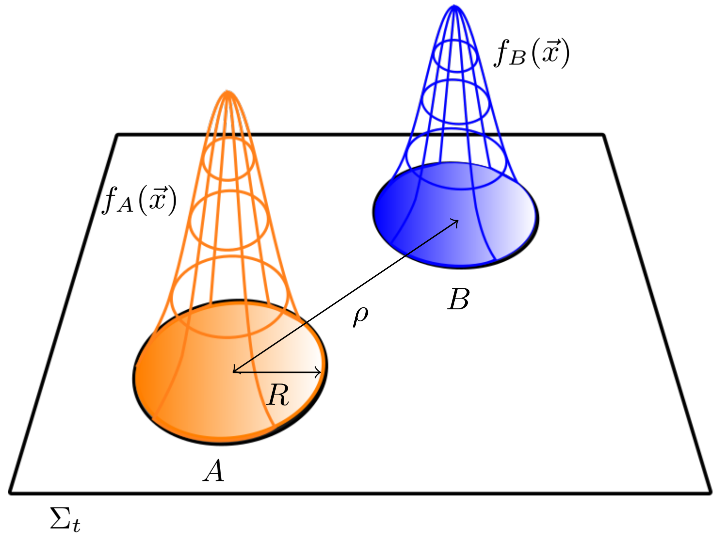
- **Entropy of $\{f\}$ is larger in dSitter than in Minkowski** → **BD vacuum in dS space is more entangled than Minkowski vac.!**

(Compatible with many previous results for the entropy of regions; see e.g. Maldacena 2013)

- **But entanglement distributed very differently. Spread across much larger distances in dS.**

Consequences:

Given two modes of compact support in dS



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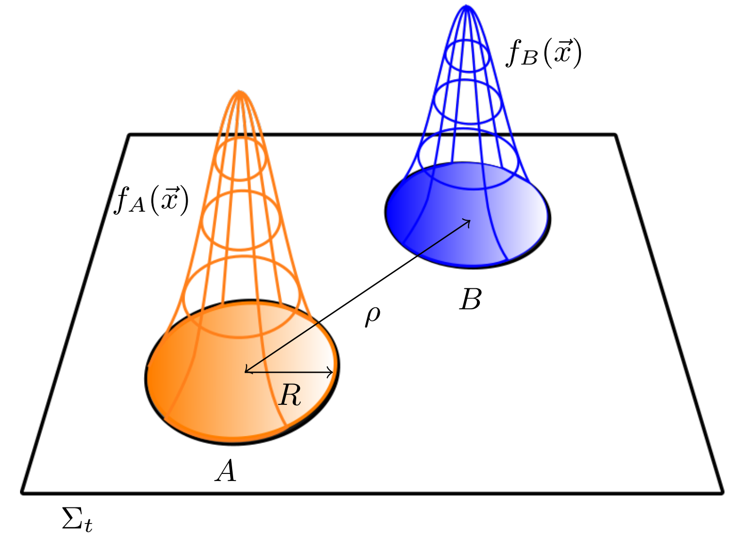
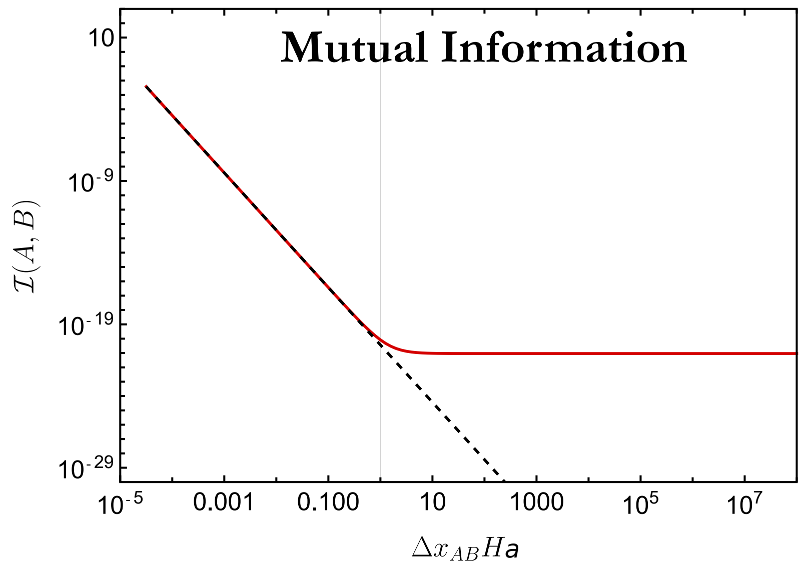


- They are correlated with each other more than they would be in Minkowski



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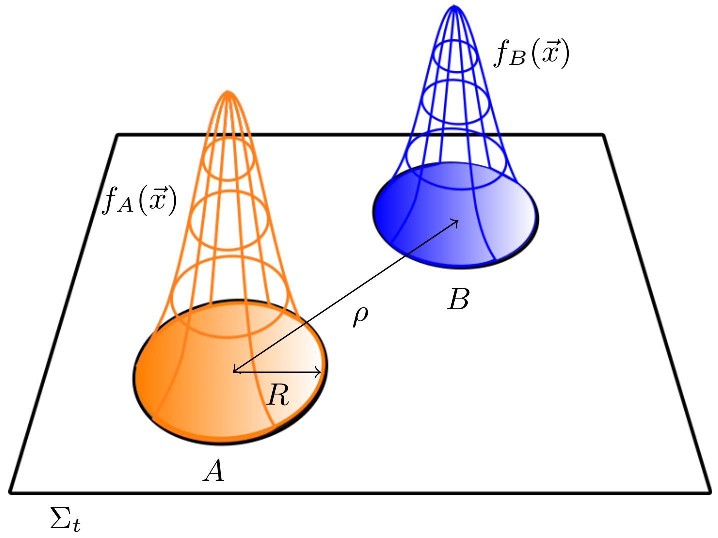

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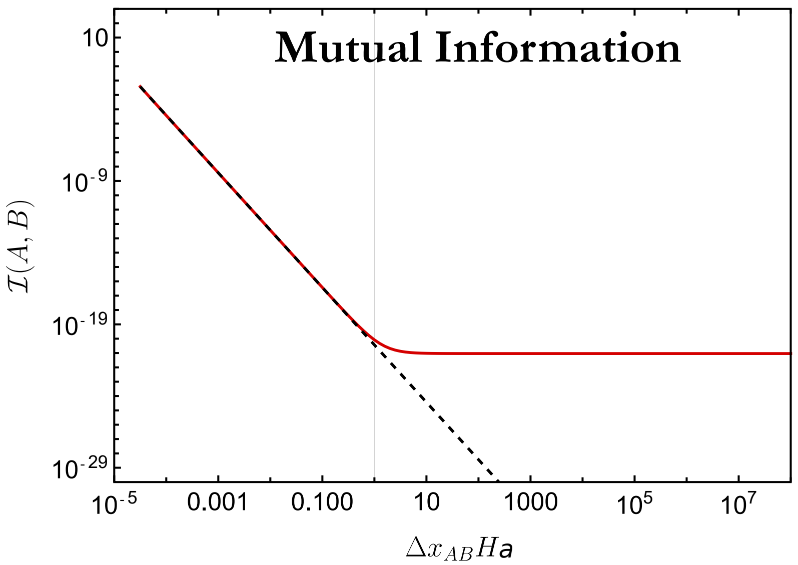

- Individually, they are more entangled with their corresponding partner

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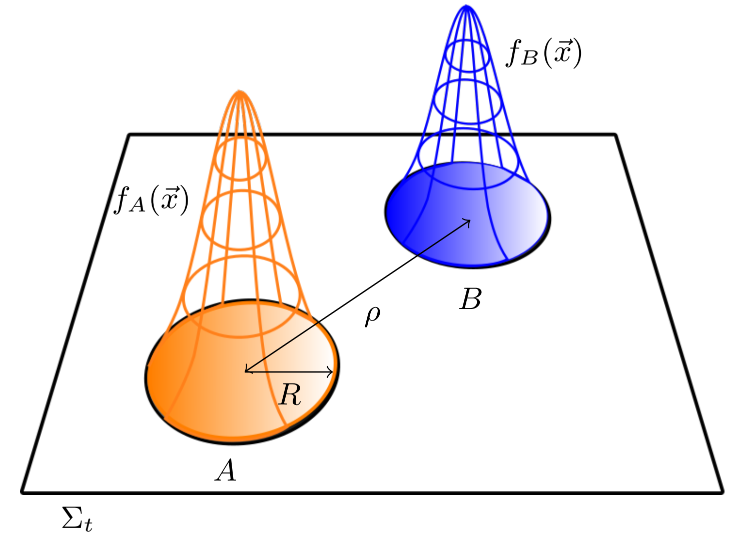

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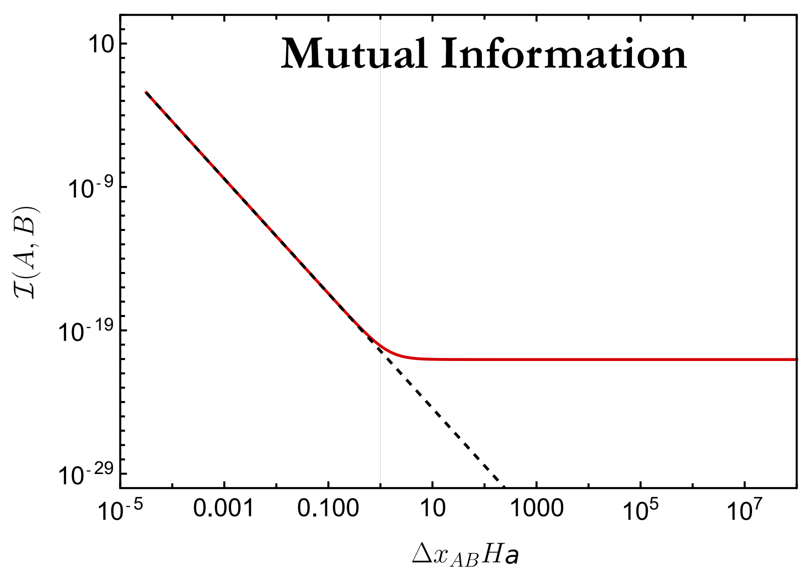
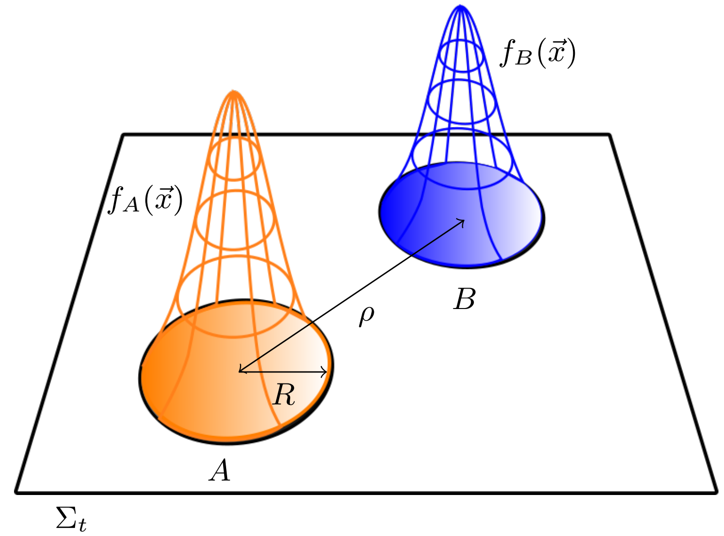


- Individually, they are more entangled with their corresponding partner

- Individually, they are more mixed

Given two modes of compact support in dS



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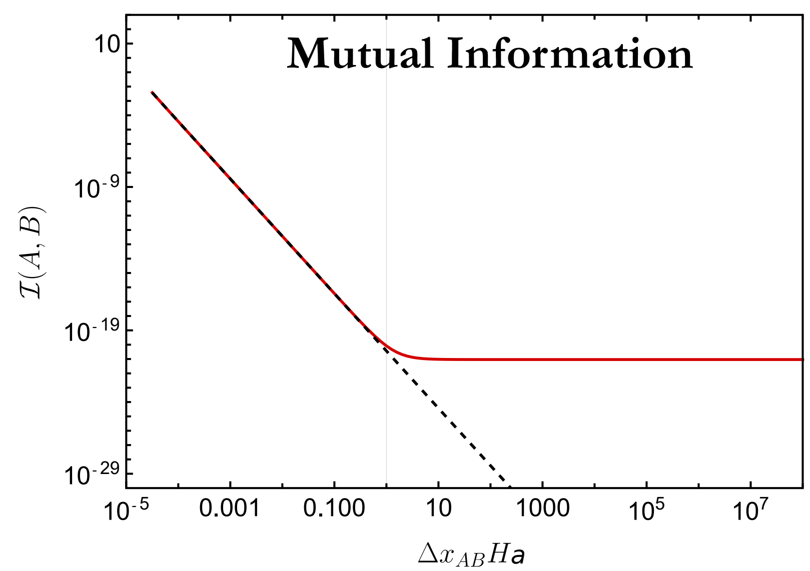


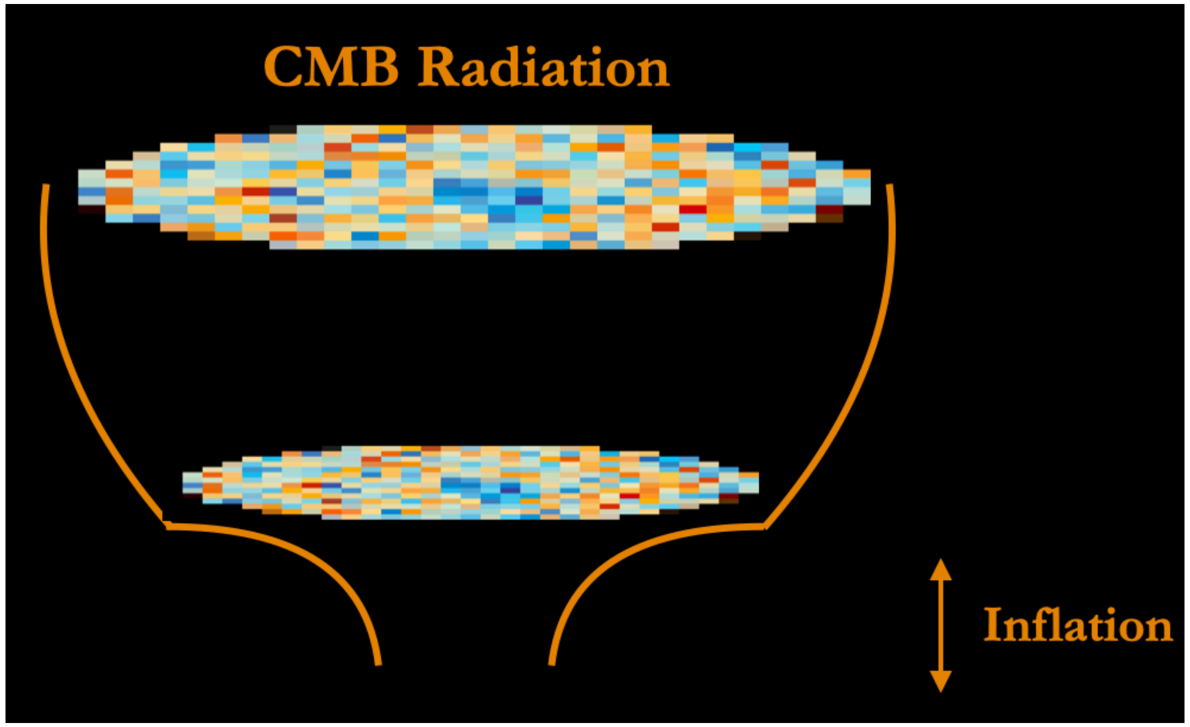
- Individually, they are more entangled with their corresponding partner
- Individually, they are more mixed
- We find they are less entangled with each other than they would be in Minkoswki st !

(Intuition: more entanglement with the partner is detrimental for entanglement with other modes)

Interesting consequences for cosmology

Does Inflation generates entanglement??

(Long debate)



My answer: No if we only have access to local observables