Uniqueness Fock quantization of a massless scalar field inside of a nonrotating black hole

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- Can we extend the prescription to the KS-anisotropic scenario?

Overview of the prescription

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  - How to identify the high energy sector? $\rightarrow$ CT!

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- (3). Imposition of unitary quantum dynamics.

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$$\tilde{\mathbf{H}} = \tilde{\sum_{nlm}} \frac{b_{\hat{l}}}{2} \left[\tilde{\Pi}_{nlm}^2 + \left(k^2 + s_{\hat{l}}(\tau)\right) \tilde{\phi}_{nlm}^2\right]. \qquad (3)$$

  - $s_{\hat{l}}(\tau) = \frac{3(b'_{\hat{l}})^2}{4b_{\hat{l}}^4} - \frac{b''_{\hat{l}}}{2b_{\hat{l}}^3}$, Mass function.

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    - Spacetime evolve in time: Motivation to introduce a

      time-evolution splitting

      Heissenberg evolution – Background evolution

  (unfixed and come back later).

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    - For unitary dynamics we use a result: antilinear part of $\mathcal{B}_{nlm}(\tau, \tau_0)$ be square summable.
- Unitary dynamics prescription successfully selects a family of unitary equivalent representations.

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  - Time-independent functions:
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  - Time-dependent functions: - change of the dynamics by the time-dependent CT:
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  - Remaining freedom $\rightarrow$ extra criterion: Self-interaction terms become zero order by order.

Asymptotic Hamiltonian diagonalization

- Proceed recursively. Each subdominant term will be smaller in k. In the ultraviolet limit, we obtain:

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- The time-dependent part of the scalar field:

$$\Phi = \tilde{\sum_{nlm}} A(s_{\hat{l}}) e^{-i \int d\bar{\tau}\, b_{\hat{l}} \Lambda_{nl}} \; a_{nlm}(\tau_0) + \text{h.c.},$$

where the time evolution splitting becomes evident.

Conclusions

- By the unitary quantum dynamics prescription we were able to select a unitary equivalent class of representations.
- We were able to eliminate all possible ambiguities in the process and obtained a preferred proposal for a unique choice of a vacuum.
- By introducing an asymptotic Hamiltonian diagonalization, we could completely fix the time-evolution splitting of the scalar field and have good physical properties of the resulting Hamiltonian.