Thermodynamics from Entanglement in QFT and Black Holes

Erick Muiño

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[Work in progress with E. Bianchi]

IGC

PennState
Eberly College of Science

Can we obtain BH Thermodynamics from Entanglement?

Entanglement across the horizon?

- Vacuum: $S_A(|0\rangle) \sim \frac{A}{\epsilon^2} \xrightarrow[\epsilon \to 0]{} \infty$

It diverges because correlations probe all energies.

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What if we fix the energy?

Claim: Entanglement entropy gives thermodynamic entropy!

Subalgebras and energy constraint

System:

- Composite Hilbert space: $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$.
- Algebra of observables: $\mathcal{A} = \mathcal{L}(\mathcal{H}_A \otimes \mathcal{H}_B)$.
- Hamiltonian: $H = H_A \otimes \mathbf{1}_B + \mathbf{1}_A \otimes H_B$.

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Subsystems:

- Subalgebra of observables that preserves the energy: $\mathcal{A}_{HA} = \{O_A \in \mathcal{L}(\mathcal{H}_A \otimes \mathbf{1}_B) : [O_A, H] = 0\}$.
- Commutant: $(\mathcal{A}_{HA})' = \{O \in \mathcal{A} : [O, O_A] = 0, \forall O_A \in \mathcal{A}_A\}$.
- Center: $H_A \otimes \mathbf{1}_B \in \mathcal{A}_{HA} \cap (\mathcal{A}_{HA})'$.

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Constraint:

- $\mathcal{H}(E) = \bigoplus_\varepsilon \Big(\mathcal{H}_A(\varepsilon) \otimes \mathcal{H}_B(E-\varepsilon)\Big)$.

Thermodynamics for random pure states

The typical entanglement entropy is obtained by averaging over random pure states in the sector $\mathcal{H}(E)$ [Bianchi & Donà, PRD(2019)]:

$$\langle S_A \rangle_E = \sum_{\varepsilon=0}^{E} \frac{d_A d_B}{D} \left[\Psi(D+1) - \Psi(\max(d_A, d_B)+1) - \min\left(\frac{d_A - 1}{2d_B}, \frac{d_B - 1}{2d_A} \right) \right].$$

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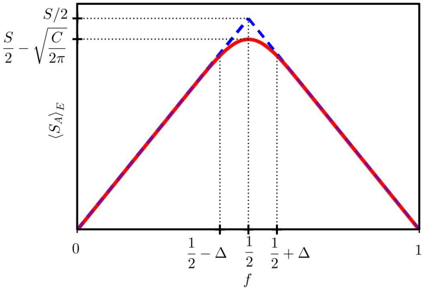


Application in QFT

How to build subsystems preserving the energy constraint?

- Pick a set of modes $A = \{\vec{k}_i\}$.

Fock space decomposition

$$\mathcal{H} = \otimes_{\vec{k}} \mathcal{H}_{\vec{k}} = \mathcal{H}_A \otimes \mathcal{H}_B .$$

General free QFT Hamiltonian

$$H = \sum_{\lambda, s} \int \frac{d^3 k}{(2\pi)^3} \, \omega_{\vec{k}} \; a^{\dagger}_{\vec{k},\lambda,s} a_{\vec{k},\lambda,s} = H_A + H_B .$$

Constraint the Hilbert space to those states at energy E:

$$\mathcal{H}(E) = \bigoplus_{\varepsilon} \Big(\mathcal{H}_A(\varepsilon) \otimes \mathcal{H}_B(E - \varepsilon) \Big).$$

The black body

How to obtain thermodynamics?

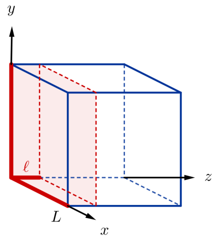
- Pick a set of modes $A = \{\vec{k}_i\}$.
- Make sure A is extensive.

Electromagnetic quantum field at total energy E, in a volume L^3.

Hamiltonian: $H = \sum_m \hbar\omega_m a_m^\dagger a_m$.

The set of modes A that couple to the antenna is extensive in the energy:

$$\bar{\varepsilon} = \frac{E}{V} V_A.$$



Thermodynamics from entanglement

Black body thermodynamics:

$$\langle S_A \rangle_E \approx \frac{4\sqrt{\pi}}{3(15)^{\frac{1}{4}}} \left(\frac{E/V}{c\hbar} \right)^{\frac{3}{4}} V_A \approx \frac{4\pi^2}{45} k_B \left(\frac{k_B T}{c\hbar} \right)^3 V_A$$

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But, what about black holes?

- Can we obtain thermodynamics from random pure states in $\mathcal{H}(E_{ADM})$?
- What is a horizon subsystem that preserves E_{ADM}?