

Pilot-wave approach to key issues in non-perturbative quantum gravity

Dr. Indrajit Sen

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1. I. Sen, S. Alexander and J. Dressel. A realist interpretation of unitarity in quantum gravity. *Class. Quantum Grav.* 41, 115005 (2024).
2. I. Sen. Physical interpretation of non-normalizable harmonic oscillator states and relaxation to pilot-wave equilibrium. *Nat. Sci. Rep.* 14, 669 (2024).





1 The problem of time



- 1 The problem of time
- 2 The problem of non-normalizability.



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- 2 The problem of non-normalizability.
- 3 The problem of classical limit.



- 1 The problem of time
- 2 The problem of non-normalizability.
- 3 The problem of classical limit.
- 4 The problem of reality conditions.



All major approaches to resolving these problems based on orthodox quantum mechanics.

- 1 The problem of time
- 2 The problem of non-normalizability.
- 3 The problem of classical limit.
- 4 The problem of reality conditions.



New conceptual approach based on pilot-wave theory.

- 1 The problem of time ✓
- 2 The problem of non-normalizability. ✓
- 3 The problem of classical limit. ✓
- 4 The problem of reality conditions. ✓

Pilot-wave theory

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Quantum State: $\hat{H}j \ i = i\text{-@}_tj \ i$

Pilot-wave theory

Quantum State: $\hat{H} \psi = i \hbar \partial_t \psi$

Configuration: $\mathbf{v} = \frac{j}{j^2}$

Pilot-wave theory

Quantum State: $\hat{H} \psi = i \hbar \partial_t \psi$

Configuration: $\mathbf{v} = \frac{j}{j^2}$

Pilot-wave theory

$$\left. \begin{array}{l} \text{Quantum State: } \hat{H} \psi = i \hbar \frac{\partial \psi}{\partial t} \\ \text{Configuration: } \mathbf{v} = \frac{\hbar \nabla \psi}{m \psi} \end{array} \right\} \text{Laws of nature}$$

Pilot-wave theory

$$\left. \begin{array}{l} \text{Quantum State: } \hat{H} \psi = E \psi \\ \text{Configuration: } \mathbf{v} = \frac{j}{j^2} \end{array} \right\} \text{Laws of nature}$$

Conceptual approach to key issues

Pilot-wave theory

$$\left. \begin{array}{l} \text{Quantum State: } \hat{H} \psi = E \psi \\ \text{Configuration: } \psi = \frac{1}{j} \psi^2 \end{array} \right\} \text{Laws of nature}$$

Conceptual approach to key issues

1. The problem of time \longleftrightarrow t parameterizes variation of the configuration.

Pilot-wave theory

$$\left. \begin{array}{l} \text{Quantum State: } \hat{H} \psi = E \psi \\ \text{Configuration: } \psi = \frac{1}{\sqrt{N}} \sum_j \psi_j \end{array} \right\} \text{Laws of nature}$$

Conceptual approach to key issues

1. The problem of time \longleftrightarrow t parameterizes variation of the configuration.
2. The problem of non-normalizability \longleftrightarrow locally conserved current.

Pilot-wave theory

$$\left. \begin{array}{l} \text{Quantum State: } \hat{H} \psi = E \psi \\ \text{Configuration: } \psi = \frac{1}{j} \psi^2 \end{array} \right\} \text{Laws of nature}$$

Conceptual approach to key issues

1. The problem of time \longleftrightarrow t parameterizes variation of the configuration.
2. The problem of non-normalizability \longleftrightarrow locally conserved current.
3. The problem of classical limit \longleftrightarrow quantum corrections in the guidance equation.

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Conceptual approach to key issues

1. The problem of time \longleftrightarrow t parameterizes variation of the configuration.
2. The problem of non-normalizability \longleftrightarrow locally conserved current.
3. The problem of classical limit \longleftrightarrow quantum corrections in the guidance equation.
4. The problem of reality conditions \longleftrightarrow implement at the level of guidance equation.

Pilot-wave non-perturbative quantum gravity

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Total constraint for interacting gravitational-fermionic system ¹:

$$\int_M (N\hat{H} + N^a\hat{V}_a) [A;] = 0$$

¹S. Alexander et al., Phys. Rev. D 2022, 106, 106012.

Pilot-wave non-perturbative quantum gravity

Total constraint for interacting gravitational-fermionic system:

$$Z \left(N \hat{H} + N^a \hat{V}_a \right) [A;] = 0$$

↙
↘

Lapse function
Shift vector

Pilot-wave non-perturbative quantum gravity

Total constraint for interacting gravitational-fermionic system:

$$\begin{array}{c}
 Z \\
 \downarrow \\
 (N\hat{H} + N^a\hat{V}_a) [A;] = 0 \\
 \begin{array}{l}
 \swarrow \quad \searrow \\
 \text{Lapse function} \quad \text{Shift vector}
 \end{array}
 \end{array}
 \begin{array}{l}
 \rightarrow \text{Ashtekar connection} \\
 \rightarrow \text{2-component Weyl Spinor}
 \end{array}$$

Pilot-wave non-perturbative quantum gravity

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Lapse function
Shift vector
Ashtekar connection
2-component Weyl Spinor

$$\hat{H} = \frac{1}{2} \epsilon^{ijk} \hat{E}^{bj} \hat{E}^{ai} F_{ab}^k + \frac{1}{3} \epsilon^{abc} \hat{E}^{ck} + (\hat{D}_a)_A \hat{E}^{AB ai} b_B + \hat{E}^{ai} (\hat{D}_a)_A \hat{E}^{AB} b_B$$

$$\hat{V}_a = \frac{i}{2} F_{ab}^k \hat{E}_k^b + (\hat{D}_a)_B \hat{E}^{AB}$$

Total constraint for interacting gravitational-fermionic system:

$$\begin{aligned}
 & \int_M \left[\frac{1}{2} \frac{N^2}{A_{ai}} \frac{ijk}{2} \frac{1}{A_{bj}} F_{ab}^k + \frac{1}{3} abc \frac{1}{A_{ck}} \right] 2(D_a)_A{}^{AB} i \frac{1}{B} [A;] \\
 & + \int_M i N_b \frac{1}{A_{ai}} \frac{F_{ai}^b}{2} [A;] \int_M i N_b \frac{1}{2} \frac{F_{ai}^b}{A_{ai}} \\
 & = \int_M N^b (D_b)_B{}^{Bi} \frac{1}{B} [A;]
 \end{aligned}$$

Where is **time**?

Total constraint for interacting gravitational-fermionic system:

$$\begin{aligned}
 & \int_M \left[\frac{1}{2} \frac{F_{ab}^k}{A_{ai} A_{bj}} + \frac{1}{3} \epsilon^{abc} \frac{F_{ab}^k}{A_{ck}} \right] \int_B 2(\mathcal{D}_a)_A \mathcal{D}_i^{AB} \int_{[A;]} \\
 & + \int_M i N_b \frac{F_{ai}^b}{A_{ai}} \int_{[A;]} \int_M i N_b \frac{F_{ai}^b}{A_{ai}} \\
 & = \int_M N^b (\mathcal{D}_b)^{Bi} \int_B [A;]
 \end{aligned}$$

Where is **time**?

→ Kodama State

Use ansatz $[A;] = \kappa[A][A;]$

Total constraint for interacting gravitational-fermionic system:

$$\int_M \frac{1}{\kappa} \sqrt{|A_{ai}|} \left(\frac{1}{2} \epsilon^{ijk} \sqrt{|A_{bj}|} F_{ab}^k + \frac{1}{3} \epsilon^{abc} \sqrt{|A_{ck}|} \right) + i N_b \frac{F_{ai}^b}{2} \kappa [A;]$$

$$+ \int_M \frac{1}{\sqrt{|A_{ai}|}} 2N^a (\mathcal{D}_a)_A \psi_i^{AB} = i \frac{\mathcal{H}[A;]}{\mathcal{H}}$$

Where is time?

Use ansatz $[A;] = \kappa[A][A;]$ → Kodama State

Total constraint for interacting gravitational-fermionic system:

$$\begin{aligned}
 & \int_M \frac{1}{K} \frac{1}{A_{ai}} \left(\int_N \frac{ijk}{2} \frac{1}{A_{bj}} F_{ab}^k + \frac{1}{3} abc \frac{1}{A_{ck}} + i N_b \frac{F_{ai}^b}{2} \right) \kappa [A;] \\
 & + \int_M \frac{1}{A_{ai}} 2N(\mathcal{D}_a)_{AB} i \frac{1}{B} [A;] = i \frac{\mathcal{H}[A;]}{\mathcal{H}}
 \end{aligned}$$

Where is time?

→ Kodama State

Use ansatz $[A;] = \int_M \frac{1}{t} \frac{1}{B} [A;]$

$$\frac{\mathcal{H}[A;]}{\mathcal{H}} = \int_M \frac{1}{t} \frac{1}{B} [A;]$$

Total constraint for interacting gravitational-fermionic system:

$$\int_M \frac{1}{\kappa} \sqrt{|A_{ai}|} \left(N \frac{ijk}{2} \sqrt{|A_{bj}|} F_{ab}^k + \frac{1}{3} abc \sqrt{|A_{ck}|} + i N_b \frac{F_{ai}^b}{2} \right) \kappa [A;]$$

$$+ \int_M \sqrt{|A_{ai}|} \left(2N (\mathcal{D}_a)_A \right)_i^{AB} \frac{1}{\sqrt{|B|}} [A;] = i \frac{\mathcal{H}[A;]}{\mathcal{H}}$$

Where is time?

→ Kodama State

Use ansatz $[A;] = \frac{1}{\kappa[A]} [A;]$

$$\frac{\mathcal{H}[A;]}{\mathcal{H}} = \int_M \frac{1}{t} \sqrt{|B|} \frac{1}{\sqrt{|B|}} [A;]$$

$$\frac{1}{t} \left(N^b (\mathcal{D}_b)_B + 2 \sqrt{2} \frac{1}{\text{Pl}} \frac{1}{\sqrt{|A_{ai}|}} N (\mathcal{D}_a)_A \right)_i^{AB}$$

Total constraint for interacting gravitational-fermionic system:

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$$+ \int_M \sqrt{|A_{ai}|} \left(2N (\mathcal{D}_a)_A \right)^{AB} i \sqrt{|A_{bi}|} \frac{\mathcal{D}_B [A;]}{\mathcal{D}_A [A;]} = 0$$

Where is time?

Kodama State

Use ansatz $[A;] = \int_M \kappa[A] \mathcal{D}_A [A;]$

$$\frac{\mathcal{D}_B [A;]}{\mathcal{D}_A [A;]} \int_M \frac{1}{t} \sqrt{|A_{bi}|} \mathcal{D}_B [A;] \text{ implicitly depends on } [A;]$$

does not **absolute simultaneity**



$$\int_M \frac{1}{t} \sqrt{|A_{bi}|} \left(N^b (\mathcal{D}_b)_B + 2 \sqrt{|A_{ai}|} \frac{1}{\kappa} N (\mathcal{D}_a)_A \right)^{AB}$$



Continuity equation

Continuity equation

$$\frac{\partial (j^j)}{\partial t} + r^{ai} J_{ai} + \bar{r}^{ai} \bar{J}_{ai} = 0$$

$$r^{ai} R_M = A_{ai}$$

Continuity equation

$$\frac{\partial (j^i j^2)}{\partial t} + r^{ai} J_{ai} + \bar{r}^{ai} \bar{J}_{ai} = 0$$

$$r^{ai} R_M = A_{ai}$$

Constraint on :

$$\frac{2}{A_{ai} A_{bj}} \frac{i N_{ijk}}{2} F_{ab}^k + c:c + \frac{3}{A_{ai} A_{bj} A_{ck}} \frac{i N_{ijk}}{2} \frac{1}{3} abc + c:c$$

$$N_b \frac{F_{ai}^b}{2} + c:c = 0$$

Continuity equation

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$$[A; \bar{A}] = \frac{1}{k K}$$

Continuity equation

$$\frac{\partial (j^i j^2)}{\partial t} + r^{ai} J_{ai} + \bar{r}^{ai} \bar{J}_{ai} = 0$$

$$r^{ai} \frac{R}{M} = A_{ai}$$

$$[A; \bar{A}] = \frac{1}{\kappa K}$$

Implications for Unitarity :

κ naturally factored out ! $[A;] = \frac{[A;]}{\kappa [A;]}$ may be normalizable.

Non-perturbative **inner product**.

Continuity equation

$$\frac{\partial (j^i j^2)}{\partial t} + r^{ai} J_{ai} + \bar{r}^{ai} \bar{J}_{ai} = 0$$

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Guidance equation

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$$\frac{A_{ai}}{t} \frac{J_{ai}}{j^2} = \frac{iN_{Pl}^2}{2} \frac{\ln \kappa}{A_{bj}} \text{ijk} 2F_{ab}^k + \frac{2}{Pl} \text{abc} \frac{\ln \kappa}{A_{ck}} N_b F_{ai}^b + 2N_{Pl}^2 (\Phi_a)_{A_i}^{AB} \frac{\ln}{B}$$

$$+ \frac{iN_{Pl}^2}{2} \text{ijk} 2F_{ab}^k \frac{\ln}{A_{bj}} + \frac{2}{3} \text{abc} 2 \frac{\ln}{A_{bj}} \frac{\ln \kappa}{A_{ck}} + \frac{1}{A_{ck} A_{bj}}^2$$

Guidance equation

$$\frac{A_{ai}}{t} \frac{J_{ai}}{j} \frac{1}{j^2} = \frac{i\hbar^2}{2} \frac{\nabla_{Pl}^2}{A_{bj}} \frac{\ln K}{ijk} + 2F_{ab}^k + \frac{\hbar^2}{3} \frac{\nabla_{Pl}^2}{abc} \frac{\ln K}{A_{ck}} + N_b F_{ai}^b + 2\hbar^2 \nabla_{Pl}^2 (\Phi_a)_{A_i} \frac{AB}{i} \frac{\ln}{B}$$

$$+ \frac{i\hbar^2}{2} \frac{\nabla_{Pl}^2}{ijk} + 2F_{ab}^k \frac{\ln}{A_{bj}} + \frac{\hbar^2}{3} \frac{\nabla_{Pl}^2}{abc} + 2 \frac{\ln}{A_{bj}} \frac{\ln K}{A_{ck}} + \frac{1}{A_{ck} A_{bj}^2}$$

Classical solution:

$$A_{ai} = \frac{i\hbar^2}{2} E^{bj} \frac{ijk}{2F_{ab}^k + abc E^{ck}} + N_b F_{ai}^b + 2\hbar^2 (\Phi_a)_{A_i} \frac{AB}{i} B$$

Guidance equation

$$\frac{A_{ai}}{t} \frac{J_{aj}}{j^2} = \frac{iN_{Pl}^2}{2} \frac{\ln \kappa}{A_{bj}} \text{ijk} \left(2F_{ab}^k + \frac{2}{3} \text{abc} \frac{\ln \kappa}{A_{ck}} \right) N_b F_{ai}^b + 2N_{Pl}^2 (\mathcal{D}_a)_A \frac{AB}{i} \frac{\ln}{B}$$

$$+ \frac{iN_{Pl}^2}{2} \text{ijk} \left(2F_{ab}^k \frac{\ln}{A_{bj}} + \frac{2}{3} \text{abc} \frac{\ln \kappa}{A_{ck}} \right) + \frac{1}{A_{ck} A_{bj}} \frac{2}{A_{ck} A_{bj}}$$

Classical solution:

$$A_{ai} = \frac{iN_{Pl}^2}{2} E^{bj} \text{ijk} \left(2F_{ab}^k + \frac{2}{3} \text{abc} E^{ck} \right) N_b F_{ai}^b + 2N_{Pl}^2 (\mathcal{D}_a)_A \frac{AB}{i} \frac{B}{B}$$

For self-dual solutions,

$$F_{ik}^j = \frac{1}{3} \text{ikb} E^{bj}$$

Guidance equation

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$$+ \frac{iN_{Pl}^2}{2} \text{ijk} \left(2F_{ab}^k \frac{\ln}{A_{bj}} + \frac{1}{3} \frac{\ln \kappa}{A_{ck}} \right) 2 \frac{\ln}{A_{bj}} \frac{\ln \kappa}{A_{ck}} + \frac{1}{A_{ck} A_{bj}}^2$$

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$$\frac{1}{3} \frac{\ln \kappa}{A_{bj}} = E^{bj}$$

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$$+ \frac{iN_{Pl}^2}{2} \text{ijk} \left(2F_{ab}^k \frac{\ln}{A_{bj}} + \frac{1}{3} \frac{\ln \kappa}{A_{ck}} \right) + \frac{1}{A_{ck} A_{bj}}$$

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$$\frac{A_{ai}}{t} \frac{J_{ai}}{j} \frac{1}{j^2} = \frac{iN_{Pl}^2}{2} \frac{\ln \kappa}{A_{bj}} \text{ijk} \left[2F_{ab}^k + \frac{1}{3} \frac{1}{A_{ck}} \right] + N_b F_{ai}^b + 2N_{Pl}^2 (\nabla_a)_A \frac{AB}{i} \frac{\ln}{B}$$

$$+ \frac{iN_{Pl}^2}{2} \text{ijk} \left[2F_{ab}^k \frac{\ln}{A_{bj}} + \frac{1}{3} \frac{1}{A_{ck}} \right] + 2 \frac{\ln}{A_{bj}} \frac{\ln \kappa}{A_{ck}} + \frac{1}{A_{ck} A_{bj}} \frac{1}{A_{ck} A_{bj}}$$

Classical limit is obtained when $\hbar \rightarrow 0$ [1].

Guidance equation

$$\frac{A_{ai}}{t} \frac{J_{ai}}{j^2} = \frac{iN_{Pl}^2}{2} \frac{\ln K}{A_{bj}} \text{ijk} 2F_{ab}^k + \frac{N_{Pl}^2}{3} \text{abc} \frac{\ln K}{A_{ck}} N_b F_{ai}^b + 2N_{Pl}^2 (\Phi_a)_A \frac{AB}{i} \frac{\ln}{B}$$

$$+ \frac{iN_{Pl}^2}{2} \text{ijk} 2F_{ab}^k \frac{\ln}{A_{bj}} + \frac{N_{Pl}^2}{3} \text{abc} 2 \frac{\ln}{A_{bj}} \frac{\ln K}{A_{ck}} + \frac{1}{A_{ck} A_{bj}}^2$$

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Reality Conditions ($A_{ai} + \bar{A}_{ai} = 2 A_{ai}$, $E_{ai} = \bar{E}_{ai}$) implemented at the level of guidance equation.

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$$+ \frac{i\hbar^2}{2} \text{ijk} \left[2F_{ab}^k \frac{\ln}{A_{bj}} + \frac{\hbar^2}{3} \text{abc} \left(2 \frac{\ln}{A_{bj}} \frac{\ln K}{A_{ck}} + \frac{1}{A_{ck} A_{bj}} \right) \right]$$

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Reality Conditions ($A_{ai} + \bar{A}_{ai} = 2 \text{Re} A_{ai}$, $E_{ai} = \bar{E}_{ai}$) implemented at the level of guidance equation.

$$\text{Re} A_{ai} = \frac{1}{2} \text{ijk} E^{bk} E_{a;b}^j E_{b;a}^j + E_j^c E_a^l E_{c;b}^l + \frac{1}{4} \text{ijk} E^{bk} \left[2E_a^j \frac{E_{;b}}{E} - E_b^j \frac{E_{;a}}{E} \right]$$

Continuity equation

$$\frac{\partial (j^2)}{\partial t} + r^{ai} J_{ai} + \bar{r}^{ai} \bar{J}_{ai} = 0$$

$$r^{ai} \frac{R}{M} = A_{ai}$$

$$[A; \bar{A}] = \frac{1}{\kappa K}$$

Implications for Unitarity :

κ naturally factored out ! $= \frac{1}{\kappa}$ may be normalizable.

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PW Unitarity: $\frac{\partial}{\partial t} + r^{ck} \left(\frac{A_{ck}}{t} \right) + \bar{r}^{ck} \left(\frac{\bar{A}_{ck}}{t} \right) + r_B \left(\frac{B}{t} \right) + \bar{r}_B \left(\frac{-B}{t} \right) = 0$

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Probabilities stay normalized in minisuperspace.

Conclusions

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Global time in pilot-wave approach \longleftrightarrow Relational time approaches.

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Future directions:

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Future directions:

1. Quantum fluctuations on de-Sitter space.

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THANK YOU !

Experimental implications

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Low-energy physics: Emergence of quantization in the early universe.

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$$\psi(\mathbf{y}; t) = e^{iA_0 t} \psi(\mathbf{y}) + \frac{i}{\hbar} \int_0^t dt e^{iA_0(t-t')} V(\mathbf{y}; t') e^{iA_0 t'} \psi(\mathbf{y}) + O(\hbar^2)$$

Experimental implications

Low-energy physics: Emergence of quantization in the early universe.

$$\psi(\mathbf{y}; t) = e^{iA_0 t - K(\mathbf{y})} \frac{ie^{-iA_0 t} \int_0^t dt^0 e^{iA_0 t^0} V(\mathbf{y}; t^0) e^{iA_0 t^0 - K(\mathbf{y})} + O(V^2)}{\sim}$$

For realistic perturbations

$$\psi(\mathbf{y}; t) = e^{iE_K t - K(\mathbf{y})} \frac{ie^{-iA_0 t} \int_0^t dt^0 \sum_j e^{i(E_j - E_K)t^0} c_j(t^0) \psi_j(\mathbf{y}) + O(V^2)}{\sim}$$

normalizable branches of the quantum state

Experimental implications

Low-energy physics: Emergence of quantization in the early universe.

$$\psi(\mathbf{y}; t) = e^{-iH_0 t} \psi(\mathbf{y}; t^0) + \frac{i}{\hbar} \int_{t^0}^t dt e^{-iH_0(t-t^0)} V(\mathbf{y}; t) e^{-iH_0(t-t^0)} \psi(\mathbf{y}; t^0) + O(V^2)$$

For realistic perturbations

$$\psi(\mathbf{y}; t) = e^{-iE_K t} \psi(\mathbf{y}; t^0) + \frac{i}{\hbar} \int_{t^0}^t dt e^{-i(E_j - E_K)(t-t^0)} c_j(t^0) \psi_j(\mathbf{y}; t^0) + O(V^2)$$

normalizable branches of the quantum state

Quantization due to **decoherence**.

Equilibrium density?

Equilibrium density?

Normalizable case: $\int |\psi|^2$

Equilibrium density?

Non-normalizable case: $\bar{\rho} ! \quad ?$

Equilibrium density?

\ Clearly, there is no lower bound on H and so there is no physical equilibrium state.²

²A. Valentini, arXiv:2104.07966 2021.

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Pilot-wave equilibrium

Pilot-wave equilibrium

$$H_q = \int_C \mathcal{L}(\mathbf{y}) \ln \frac{\mathcal{L}(\mathbf{y})}{\int \mathcal{L}(\mathbf{y}) d\mathbf{y}} d\mathbf{y}$$

Pilot-wave equilibrium

$$H_q = \int_C \psi \ln \frac{\psi}{j|\psi|^2} d\psi$$

not a well-defined relative entropy

Pilot-wave equilibrium

$$H_q = \int_C \rho(\mathbf{y}) \ln \frac{\rho(\mathbf{y})}{j(\mathbf{y})} d\mathbf{y}$$

not a well-defined relative entropy

$$H_{pw} = \int_C \rho(\mathbf{y}) \ln \frac{\rho(\mathbf{y})}{\rho_{pw}(\mathbf{y})} d\mathbf{y}$$

Pilot-wave equilibrium

$$H_q = \int_C^Z (\psi) \ln \frac{(\psi)}{j(\psi)j^2} d\psi$$

not a well-defined relative entropy

$$H_{pw} = \int_C^Z (\psi) \ln \frac{(\psi)}{pw(\psi)} d\psi$$

where,

$$pw(\psi) = \begin{cases} j(\psi)j^2 = N & , \text{ for } \psi \geq 2 \\ 0 & , \text{ for } \psi \geq C/n \end{cases}$$

$$\text{and } N = \int_C^R j(\psi)j^2 d\psi.$$

Pilot-wave equilibrium

$$H_q = \int_C \rho(\mathbf{y}) \ln \frac{\rho(\mathbf{y})}{\int_C \rho(\mathbf{y}) j^2 d\mathbf{y}} d\mathbf{y}$$

not a well-defined relative entropy

$$H_{pw} = \int_C \rho(\mathbf{y}) \ln \frac{\rho(\mathbf{y})}{\rho_{pw}(\mathbf{y})} d\mathbf{y}$$

where,

$$\rho_{pw}(\mathbf{y}) = \begin{cases} \int_C \rho(\mathbf{y}) j^2 d\mathbf{y} = N & , \text{ for } \mathbf{y} \in C \\ 0 & , \text{ for } \mathbf{y} \in C^c \end{cases}$$

$$\text{and } N = \int_C \rho(\mathbf{y}) j^2 d\mathbf{y}.$$

compact support of ρ on C

Pilot-wave equilibrium

$$H_q = \int_C \psi \ln \frac{\psi}{\int_C \psi^2} d\psi$$

not a well-defined relative entropy

$$H_{pw} = \int_C \psi \ln \frac{\psi}{p_w(\psi)} d\psi$$

where,

$$p_w(\psi) = \begin{cases} \int_C \psi^2 = N & , \text{ for } \psi \geq 0 \\ 0 & , \text{ for } \psi \in C \setminus \mathbb{R}^+ \end{cases}$$

$$\text{and } N = \int_C \psi^2 d\psi.$$

compact support of ψ on C

Pilot-wave equilibrium

$$H_q = \int_C \psi \ln \frac{\psi}{\int_C \psi^2} d\psi$$

not a well-defined relative entropy

$$H_{pw}(t) = \int_C \psi(t) \ln \frac{\psi(t)}{\int_C \psi^2(t)} d\psi$$

where,

$$\psi(\psi) = \begin{cases} \int_C \psi^2 = N & , \text{ for } \psi \in C \\ 0 & , \text{ for } \psi \in C^c \end{cases}$$

$$\text{and } N = \int_C \psi^2 d\psi.$$

compact support of ψ on C

Pilot-wave equilibrium

$$H_q = \int_C \rho(\mathbf{y}) \ln \frac{\rho(\mathbf{y})}{\int_C \rho(\mathbf{y}) j^2 d\mathbf{y}} d\mathbf{y}$$

not a well-defined relative entropy

$$H_{pw}(t) = \int_C \rho(\mathbf{y}; t) \ln \frac{\rho(\mathbf{y}; t)}{\int_C \rho(\mathbf{y}; t) j^2 d\mathbf{y}} d\mathbf{y}$$

where,

$$\rho(\mathbf{y}; t) = \begin{cases} \int_C \rho(\mathbf{y}; t) j^2 d\mathbf{y} = N(t) & , \text{ for } \mathbf{y} \in C \\ 0 & , \text{ for } \mathbf{y} \in C^c \end{cases}$$

$$\text{and } N(t) = \int_C \rho(\mathbf{y}; t) j^2 d\mathbf{y}$$

compact time-dependent support of ρ on C

Pilot-wave equilibrium

$$H_q = \int_C \rho(\mathbf{y}) \ln \frac{\rho(\mathbf{y})}{\int_C \rho(\mathbf{y}) j^2 d\mathbf{y}} d\mathbf{y}$$

not a well-defined relative entropy

$$H_{pw}(t) = \int_C \rho(\mathbf{y}; t) \ln \frac{\rho(\mathbf{y}; t)}{\int_C \rho(\mathbf{y}; t) j^2 d\mathbf{y}} d\mathbf{y}$$

where,

$$\rho(\mathbf{y}; t) = \begin{cases} \int_C \rho(\mathbf{y}; t) j^2 d\mathbf{y} = N(t) & , \text{ for } \mathbf{y} \in C_t \\ 0 & , \text{ for } \mathbf{y} \in C \setminus C_t \end{cases}$$

$$\text{and } N(t) = \int_{C_t} \rho(\mathbf{y}; t) j^2 d\mathbf{y}$$

compact time-dependent support of ρ on C

Pilot-wave equilibrium

$$H_q = \int_C \rho(\mathbf{y}) \ln \frac{\rho(\mathbf{y})}{\int_C \rho(\mathbf{y}) j^2 d\mathbf{y}} d\mathbf{y}$$

not a well-defined relative entropy

$$H_{pw}(t) = \int_C \rho(\mathbf{y}; t) \ln \frac{\rho(\mathbf{y}; t)}{\int_C \rho(\mathbf{y}; t) j^2 d\mathbf{y}} d\mathbf{y}$$

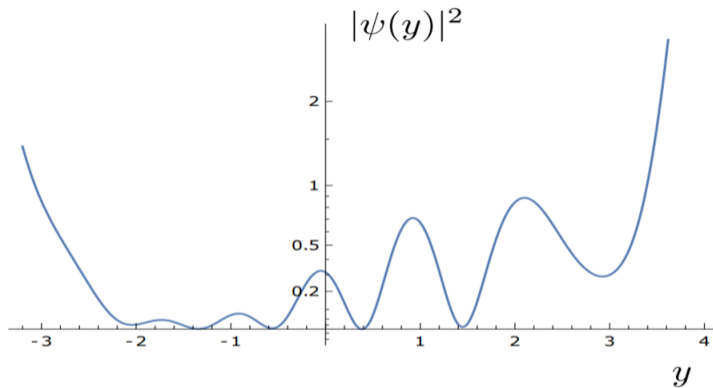
where,

$$\rho(\mathbf{y}; t) = \begin{cases} \int_C \rho(\mathbf{y}; t) j^2 d\mathbf{y} = N(t) & , \text{ for } \mathbf{y} \in \text{supp}(\rho) \\ 0 & , \text{ for } \mathbf{y} \in C \setminus \text{supp}(\rho) \end{cases}$$

$$\text{and } N(t) = \int_C \rho(\mathbf{y}; t) j^2 d\mathbf{y}.$$

compact time-dependent support of ρ on C

$$N(t) = \int_C \rho(\mathbf{y}; t) j^2 d\mathbf{y} = N(0)$$



$$N(t) = \int_{\Omega_t}^Z (y; t) j^2 dy = N(0)$$