# Full spacetime of a minimal-uncertainty black hole

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# Classical Schwarzschild interior

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- Isometric to Kantowski-Sachs (KS) metric with globally hyperbolic topology $\mathbb{R} \times \mathbb{S}^2$

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- Gravitational Hamiltonian can be written in terms of Ashtekar variables adapted to the KS spacetime

**[Ashtekar & Bojowald 06']**

$$\tilde{H} = -\frac{\tilde{N}}{2G\gamma^2}\left[2\tilde{b}\tilde{c}\sqrt{\tilde{p}_c} + \left(\tilde{b}^2 + \gamma^2\right)\frac{\tilde{p}_b}{\sqrt{\tilde{p}_c}}\right]$$

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- We choose a lapse which effectively decouples the canonical variables

$$N(T) = \frac{\gamma\sqrt{p_c(T)}}{b(T)} \qquad H = -\frac{1}{2G\gamma}[(b^2+\gamma^2)\frac{p_b}{b} + 2cp_c]$$

# Classical Schwarzschild interior - canonical variables

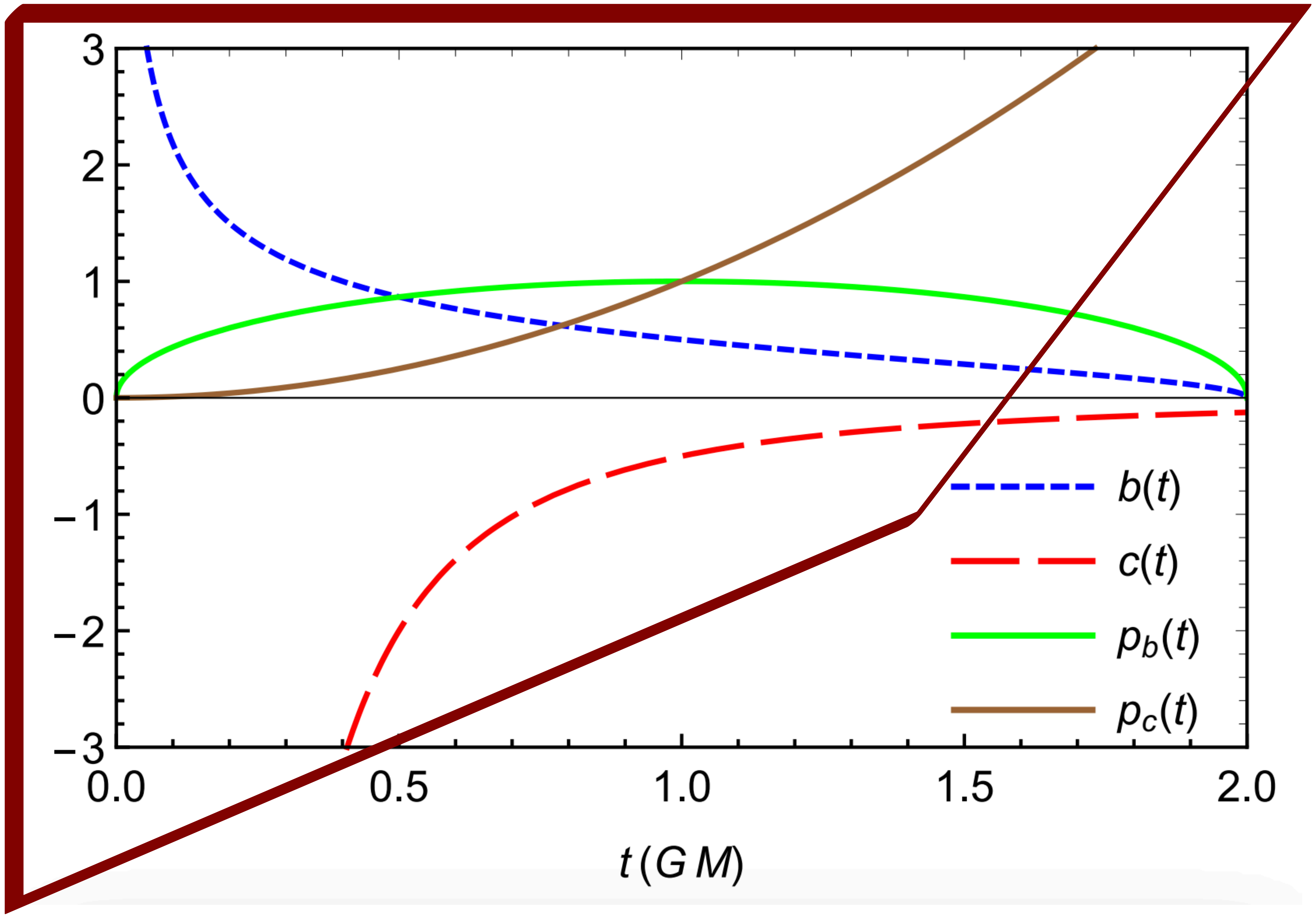
- Equations of motion for canonical variables

$$\frac{db}{dT} = \{b, H\} = -\frac{1}{2}\left(b + \frac{\gamma^2}{b}\right)$$

$$\frac{dp_b}{dT} = \{p_b, H\} = \frac{p_b}{2}\left(1 - \frac{\gamma^2}{b^2}\right)$$

$$\frac{dc}{dT} = \{c, H\} = -2c$$

$$\frac{dp_c}{dT} = \{p_c, H\} = 2p_c$$




* In Schwarszchild time $t = e^T$

# Deformation of Poisson algebra

- Many theories of quantum gravity predict a deviation from the standard Heisenberg uncertainty principle at high energies/momenta

**[Garay 95']**

**[Kempf 96']**

**[Scardigli 99']**

- Modified Poisson algebra in the classical theory

$$\{b, p_b\} = G\gamma F_1(b, p_b, c, p_c, \beta_b, \beta_c) \quad \{c, p_c\} = 2G\gamma F_2(b, p_b, c, p_c, \beta_b, \beta_c)$$

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- Configuration-dependent modification (commonly found in minimal-uncertainty theories)

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- Implies a minimal-uncertainty relation in the quantum theory

$$[b, p_b] = iG\gamma\left(1 + \beta_b b^2\right) \qquad \Delta b \Delta p_b \geq \frac{G\gamma}{2}\left[1 + \beta_b (\Delta b)^2\right]$$

$$[c, p_c] = i2G\gamma\left(1 + \beta_c c^2\right) \qquad \longrightarrow \qquad \Delta c \Delta p_c \geq G\gamma\left[1 + \beta_c (\Delta c)^2\right]$$

- We then re-solve the EOMs for the canonical variables to yield the effective metric

# Full spacetime extension

[Bosso 23']

- Now we have an effective Schwarzschild interior metric with a resolved singularity

$$ds^2 = -\frac{\gamma^2 \tilde{p}_c(t)}{t^2 \tilde{b}(t)^2} dt^2 + \frac{\tilde{p}_b(t)^2}{L_0^2 \tilde{p}_c(t)} dr^2 + \tilde{p}_c(t)(d\theta^2 + \sin^2\theta d\phi^2)$$

- But what happens when we extend to the full spacetime?

$$ds^2 = \frac{\tilde{p}_b(r)^2}{L_0^2 \tilde{p}_c(r)} dt^2 - \frac{\gamma^2 \tilde{p}_c(r)}{r^2 \tilde{b}(r)^2} dr^2 + \tilde{p}_c(r)(d\theta^2 + \sin^2\theta d\phi^2)$$

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- Classical limits

$$\lim_{\beta_b, \beta_c \to 0} \tilde{g}_{00} = -\left(1 - \frac{R_s}{r}\right)$$

$$\lim_{\beta_b, \beta_c \to 0} \tilde{g}_{11} = \left(1 - \frac{R_s}{r}\right)^{-1}$$

$$\lim_{\beta_b, \beta_c \to 0} \tilde{g}_{22} = r^2$$

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$Q_b = \mathrm{sgn}\beta_b |\beta_b| \gamma^2 \quad Q_c = \mathrm{sgn}\beta_c |\beta_c| \gamma^2$

- Krestchmann scalar

$$K \propto \frac{1}{r^4}$$

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# Full spacetime extension - improved ($\overline{\beta}$) scheme

- Inspired by similar issues in LQC, we make the quantum parameters momentum-dependent

$$\beta_b \to \overline{\beta_b} = \frac{L_0^4 \beta_b}{p_b^2} \qquad \beta_c \to \overline{\beta_c} = \frac{L_0^4 \beta_c}{p_c^2}$$

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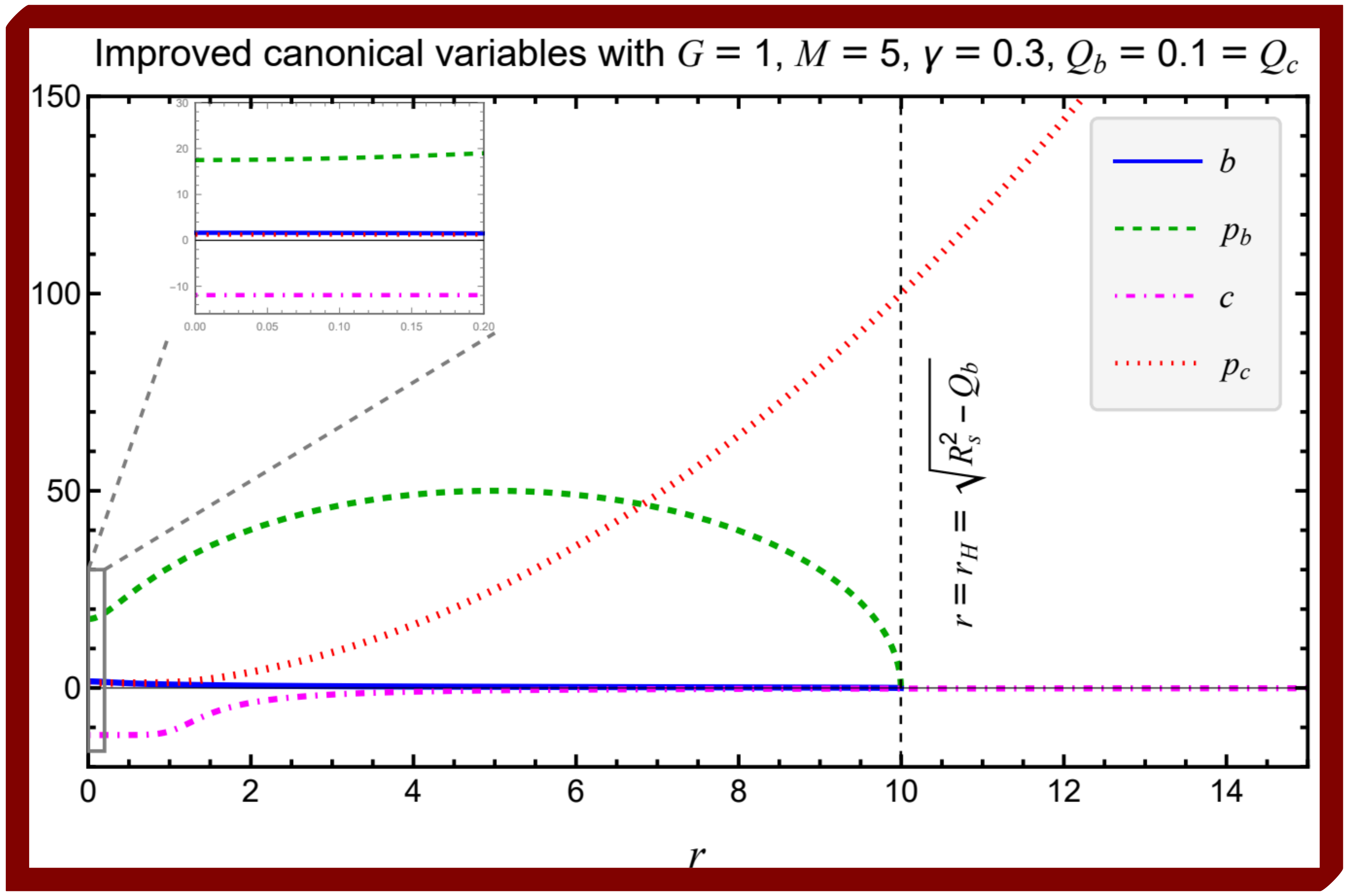
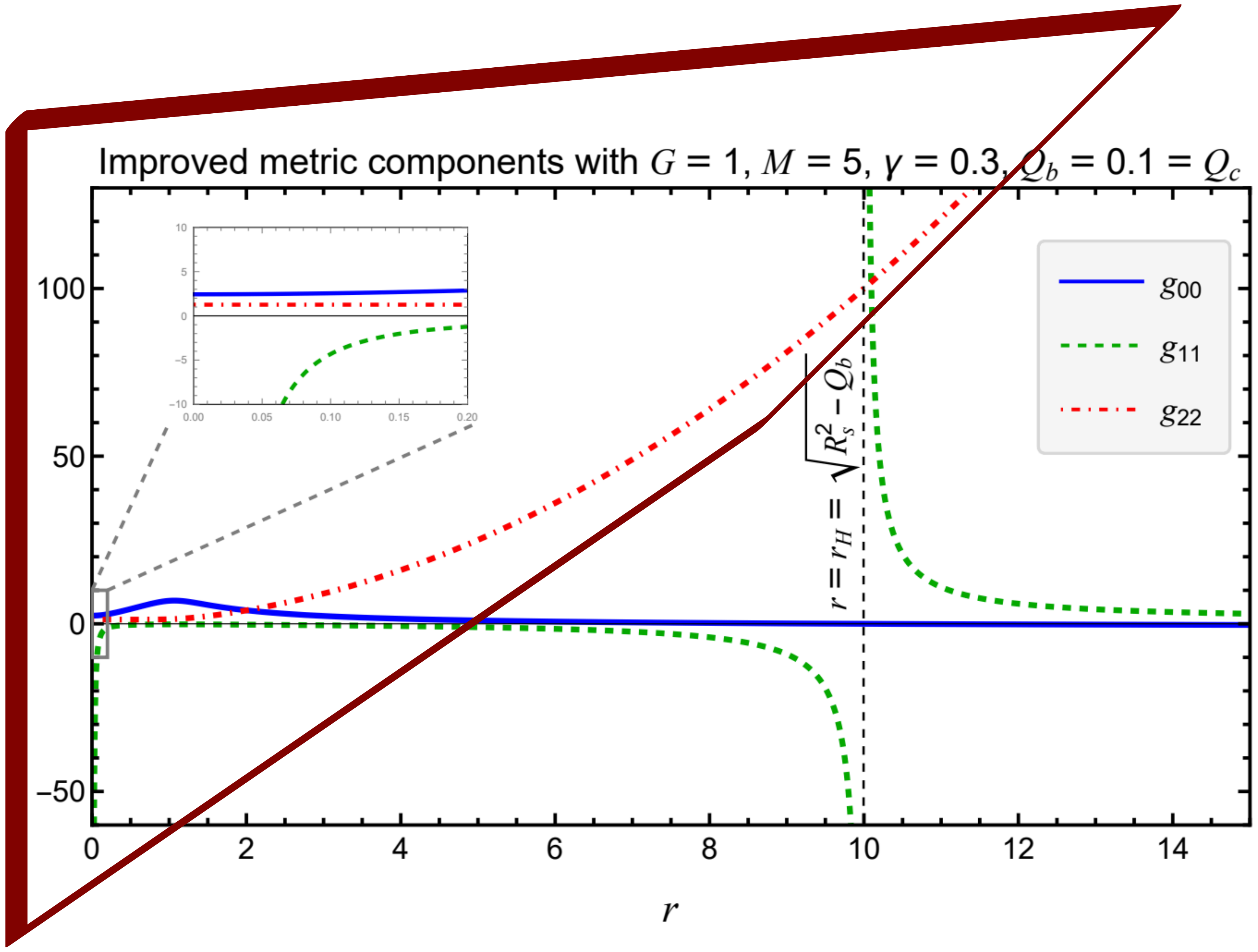
- Re-solve canonical variable EOMs, sub into metric and swap t and r to get exterior metric



# $\bar{\beta}$-scheme metric limits and expansions

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# Horizon and entropy

- We can find the position of event horizon by solving $g^{11} = 0$ , yielding

$$r_H = \sqrt{R_s^2 - Q_b} = R_s - \frac{1}{2}\frac{Q_b}{R_s} + \mathcal{O}(Q_b^2)$$

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- The horizon modification implies a modification to the entropy of the BH

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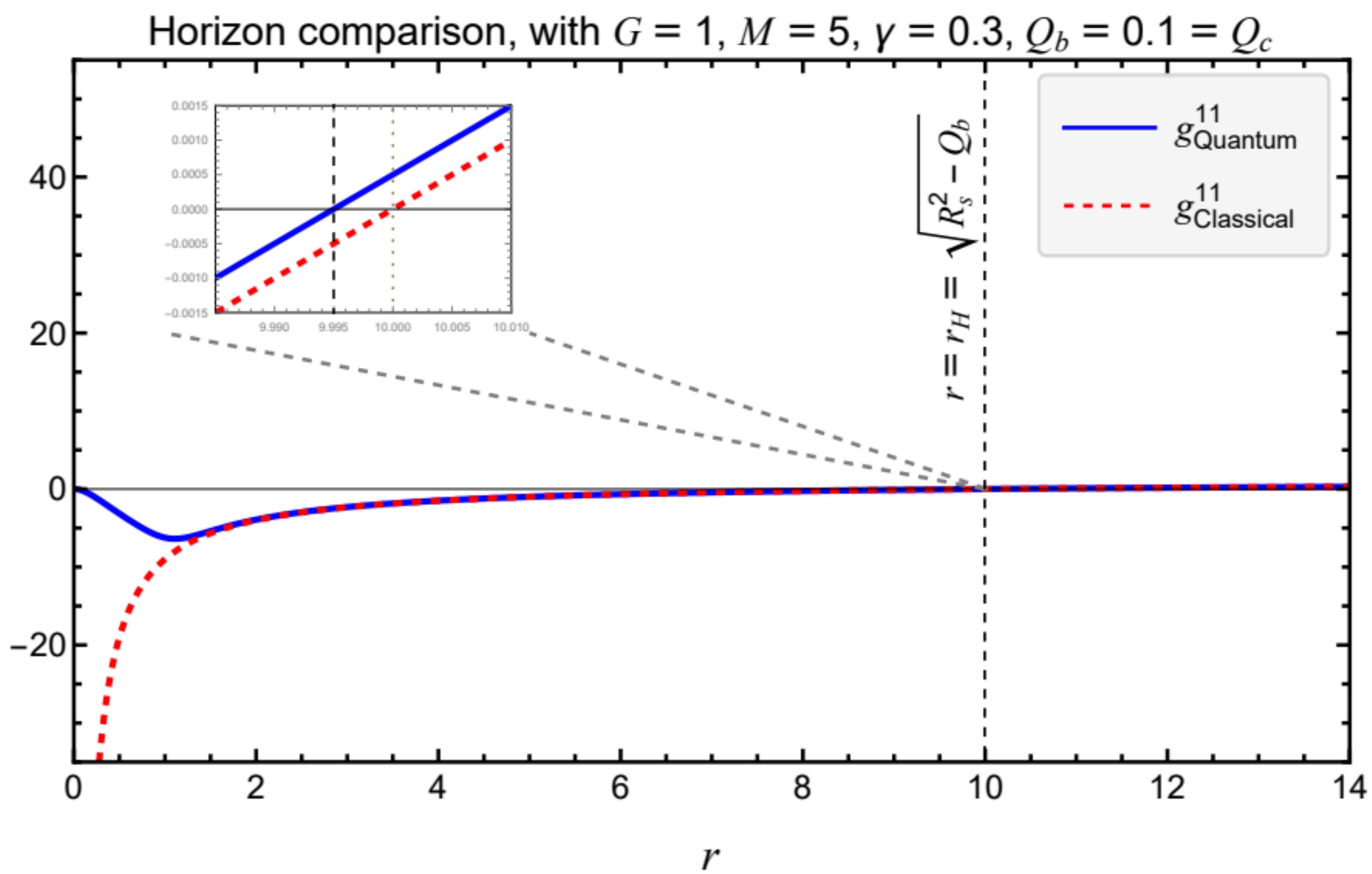
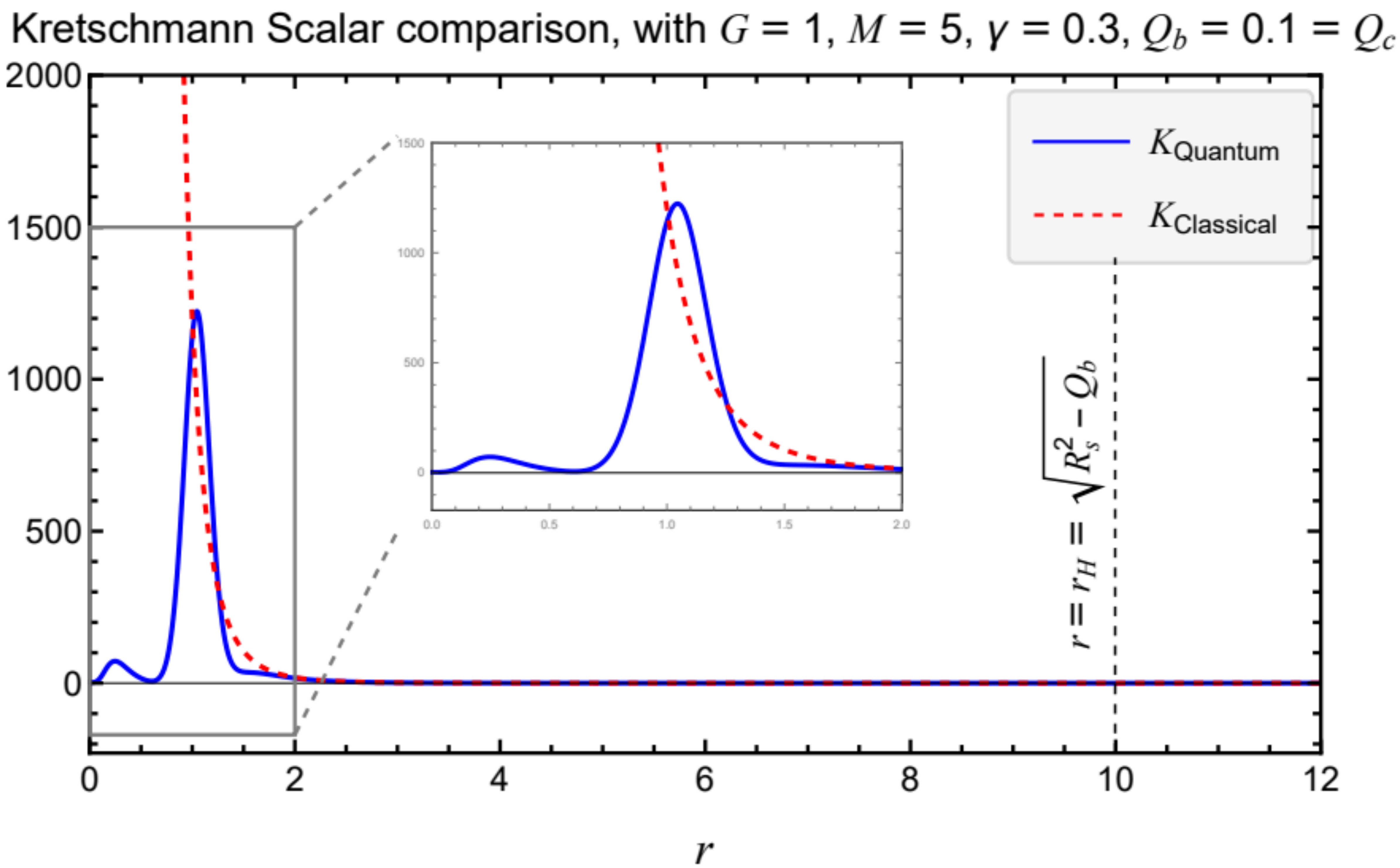


# Krestchmann scalar

- The Kretschmann scalar now has all of the desired properties

$$K\Big|_{r\to\infty} = \frac{12R_s^2}{r^6} + \mathcal{O}\left(\frac{1}{r^7}\right) \qquad \lim_{r\to\infty} K = 0 \qquad \lim_{r\to 0^+} K = K(r=0) = \frac{4}{\sqrt{\rho}}\Big|_{r=0} = \frac{8}{R_s\sqrt{Q_c}}$$



# Effective potential

- Two conserved quantities (energy and angular momentum) associated with the Killing vector fields corresponding to time translation and rotational symmetry

$$E = -g_{\mu\nu} K^{\mu} \frac{dx^{\nu}}{d\lambda} = \sqrt{\frac{\nu}{\rho}} (\sqrt{\nu} - R_s) \frac{dt}{d\lambda} \qquad L = g_{\mu\nu} R^{\mu} \frac{dx^{\nu}}{d\lambda} = \rho^{\frac{1}{4}} \frac{d\phi}{d\lambda}$$

$$\Xi = -g_{\mu\nu} \frac{dx^{\mu}}{d\lambda} \frac{dx^{\nu}}{d\lambda} = \begin{cases} 0, & \text{null} \\ 1, & \text{timelike} \end{cases}$$

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- Yields an EOM for a test particle

$$\frac{\nu}{r^2}\left(\frac{dr}{d\lambda}\right)^2 + V_{\text{eff}} = \mathfrak{E}$$

$$V_{\text{eff}} = -g_{00}\left[\frac{L^2}{g_{22}} + \Xi\right] = \sqrt{\frac{\nu}{\sqrt{\rho}}}\left(\sqrt{\nu} - R_s\right)\left[\frac{L^2}{\rho^{\frac{1}{4}}} + \Xi\right]$$

$$\mathfrak{E} = E^2$$

$$\nu = r^2 + Q_b$$

$$\rho = r^8 + \frac{1}{4}Q_c R_s^2$$

# Photon spheres

- Extrema of the null effective potential determine the location of photon spheres

$$r_{ph}^{eff} = \frac{3R_s}{2} - \frac{7Q_b}{9R_s} + \frac{64Q_c}{6561R_s^5}$$

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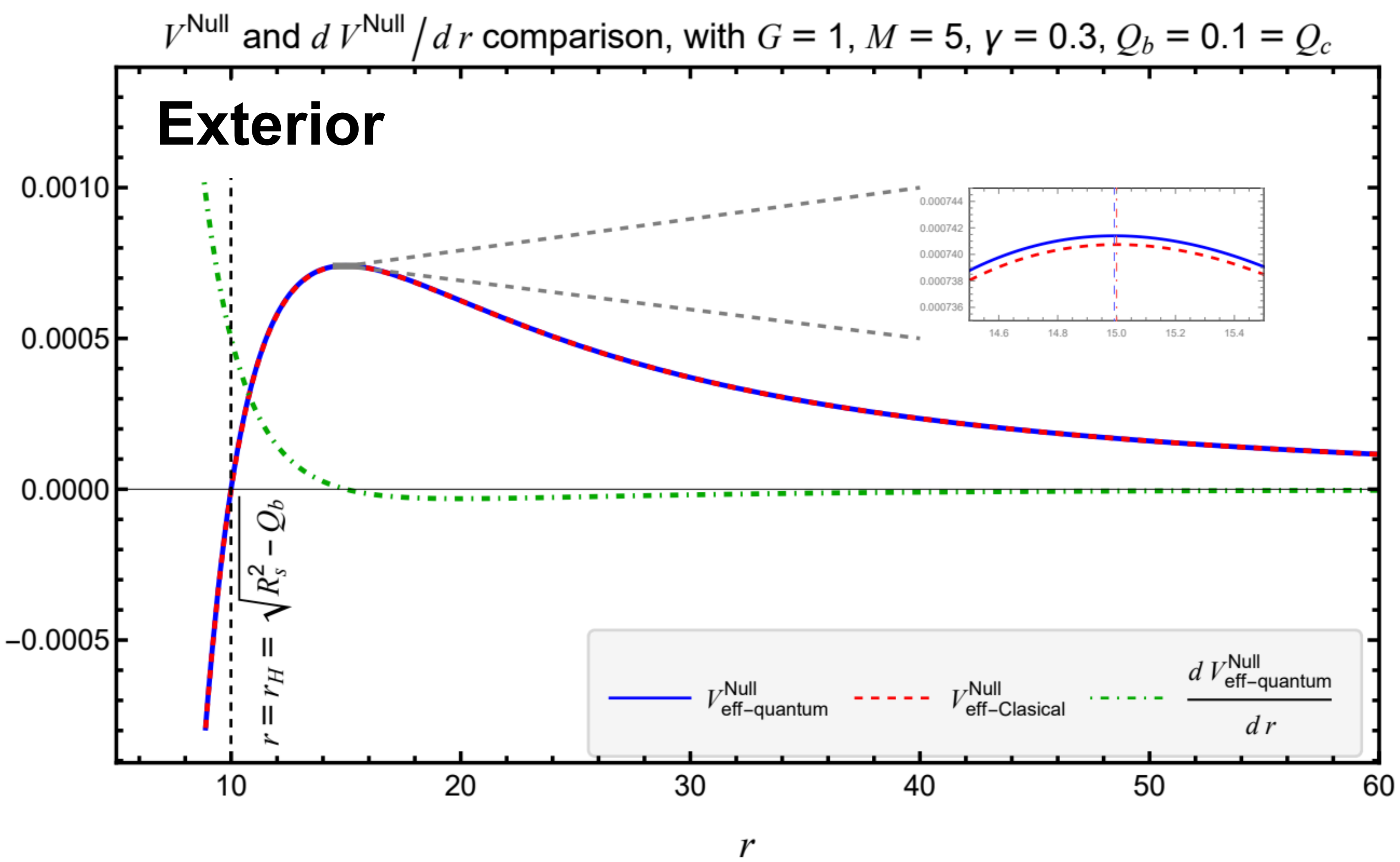


**Unstable**

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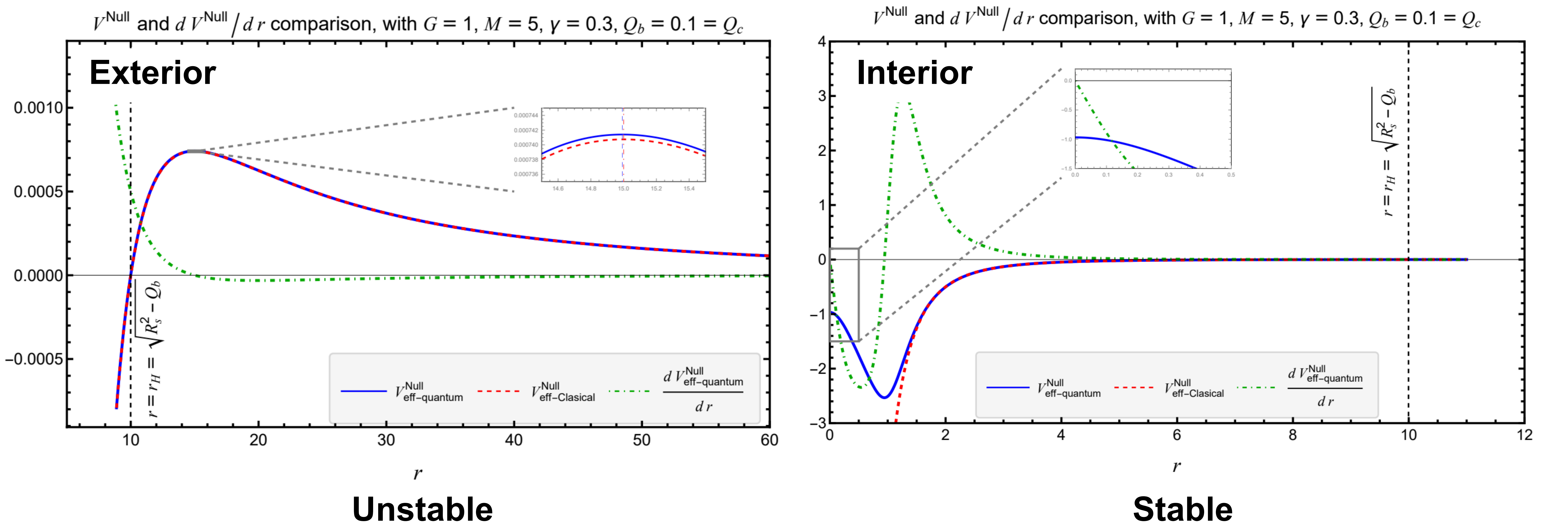


- We find similar results for timelike geodesics

# Expansion and Raychaudhuri equation

- Singularity resolution can be further explored via the Raychaudhuri equation

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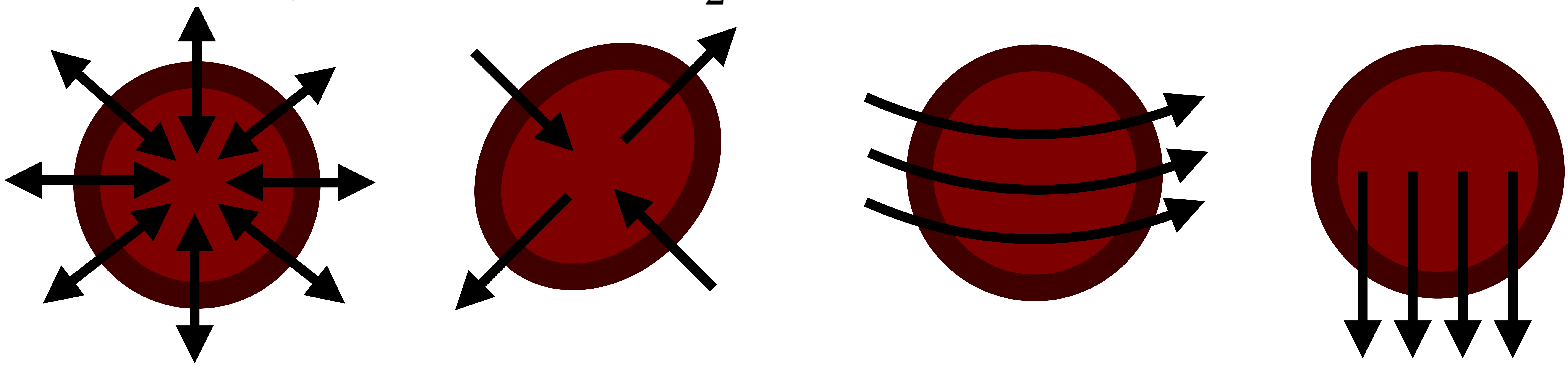
| Expansion | Shear | Vorticity | Tides |
|---|---|---|---|
| $\theta = \widehat{B}^{\mu}{}_{\mu}$ | $\sigma_{\mu\nu} = \widehat{B}_{(\mu\nu)} - \frac{1}{2}\theta Q_{\mu\nu}$ | $\omega_{\mu\nu} = \widehat{B}_{[\mu\nu]}$ | $R_{\mu\nu}k^{\mu}k^{\nu}$ |



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Expansion

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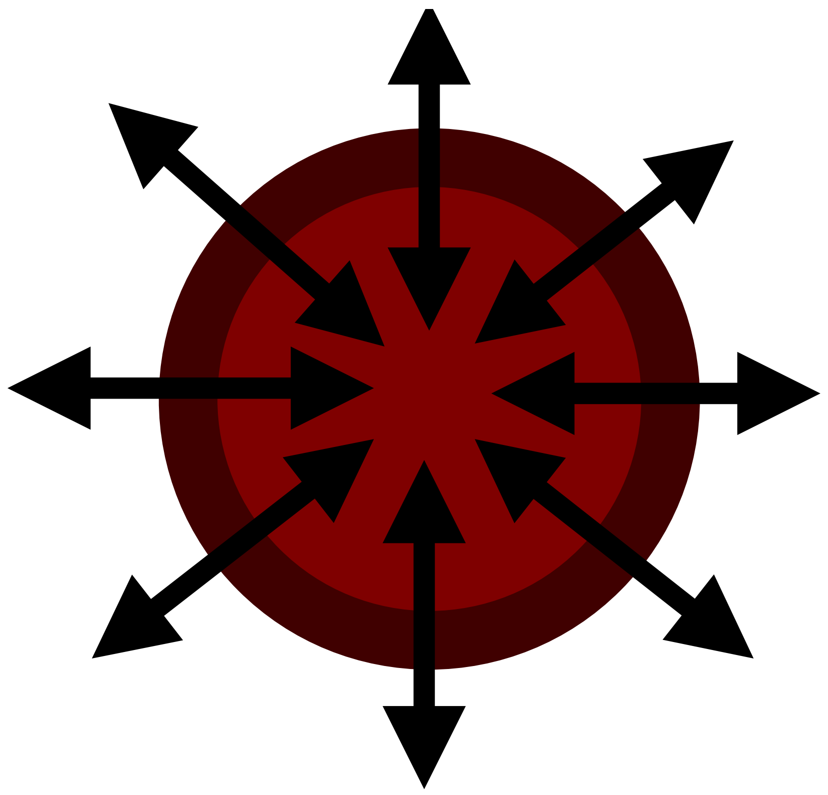
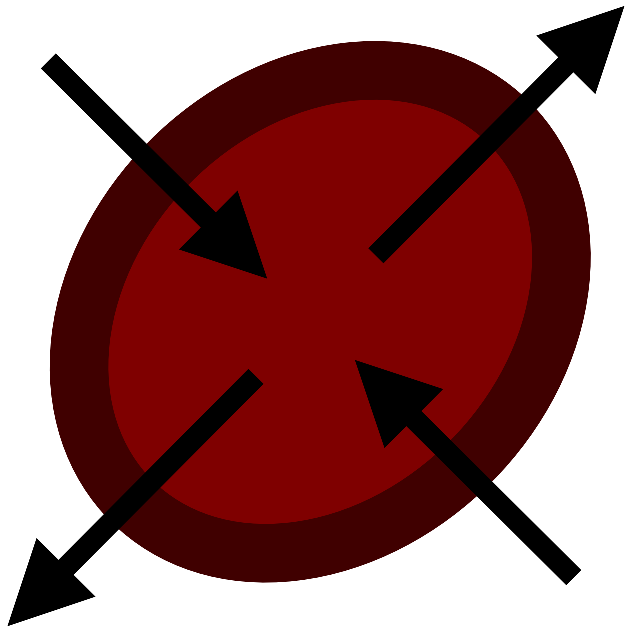


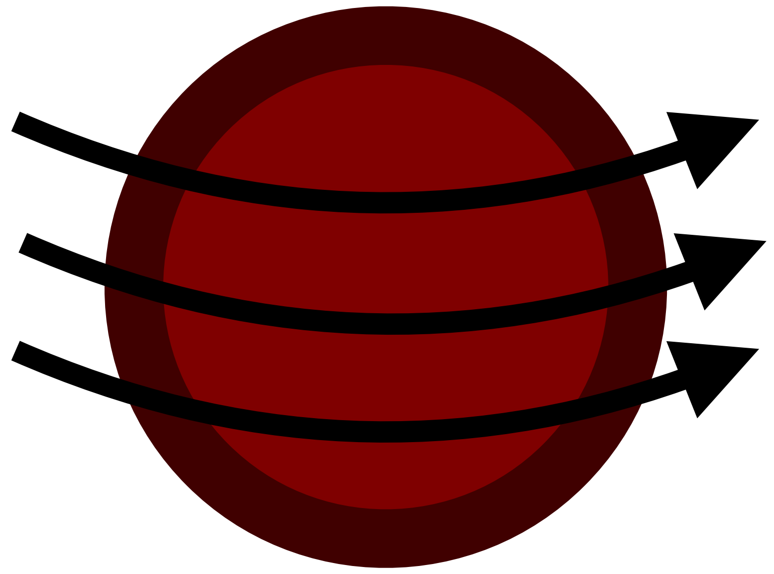
Shear

$$\sigma_{\mu\nu} = \widehat{B}_{(\mu\nu)} - \frac{1}{2}\theta Q_{\mu\nu}$$



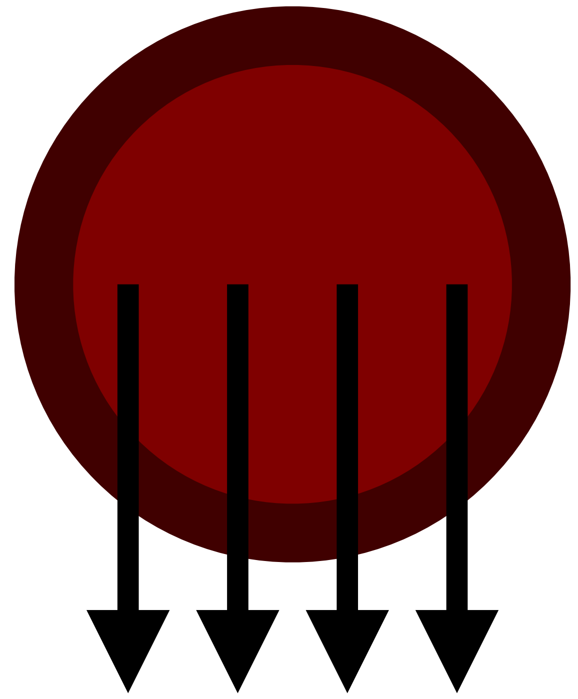
Vorticity

$$\omega_{\mu\nu} = \widehat{B}_{[\mu\nu]}$$



Tides

$$R_{\mu\nu}k^{\mu}k^{\nu}$$

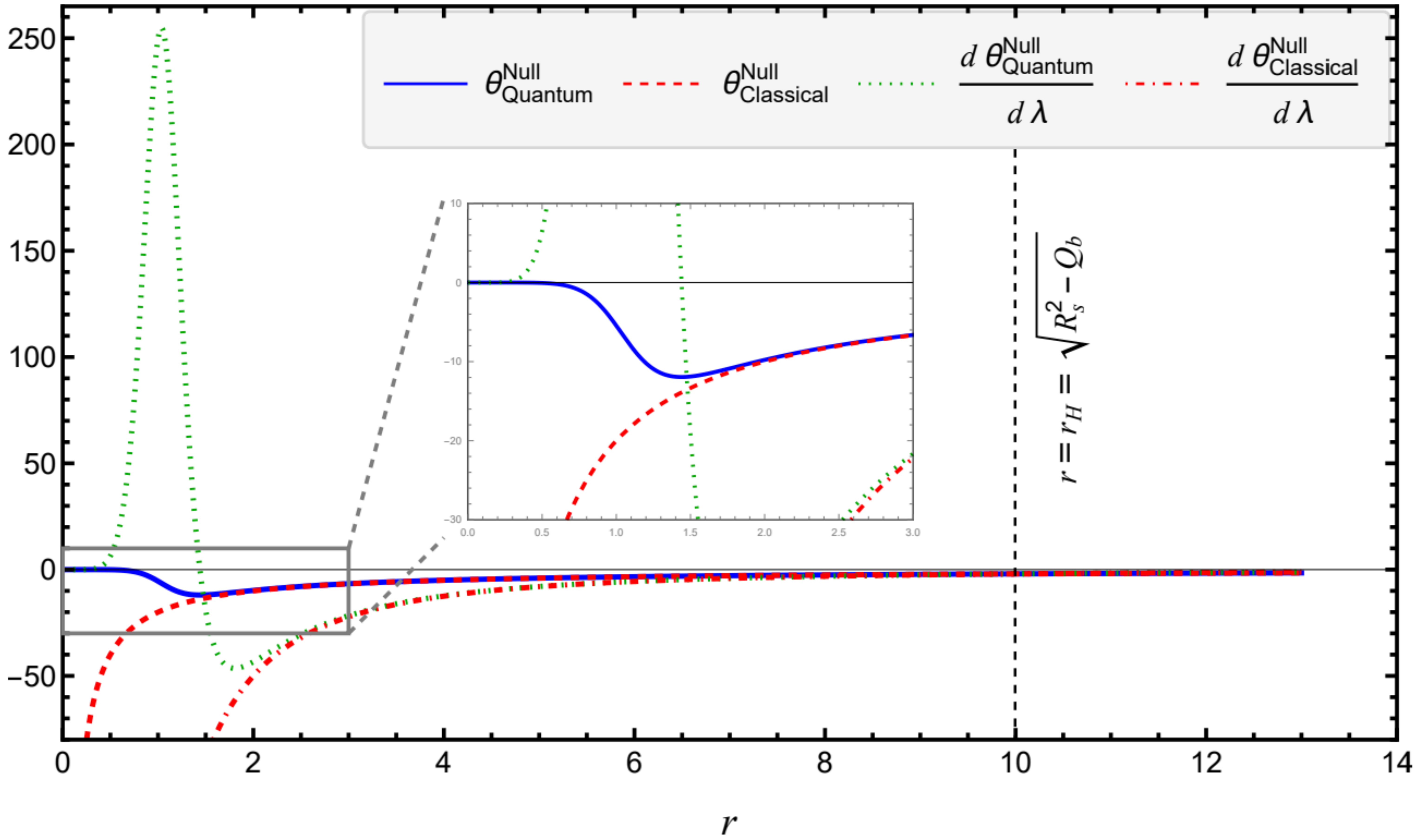


**Raychaudhuri**

$$\frac{d\theta}{d\tau} = -\frac{1}{2}\theta^2 - \sigma_{\mu\nu}\sigma^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} - R_{\mu\nu}k^{\mu}k^{\nu}$$

# Expansion and Raychaudhuri equation

$\theta$ and $\frac{d\theta}{d\lambda}$ classical vs. quantum, with $G = 1$, $M = 5$, $\gamma = 0.3$, $Q_b = 0.1 = Q_c$, $E = 10$



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  - One unstable circular orbit in the exterior (slightly closer in than a classical BH photon sphere)
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- Current / future work :

  - Find rotating solution using the Newman-Janis algorithm
  - Compute full coupled geodesic equations, greybody factors, quasinormal modes, shadow etc.

# Thank you!

# Backups

$$Q_{\mu\nu} = g_{\mu\nu} + k_\mu l_\nu + k_\nu l_\mu$$

$$\widehat{B}^\mu{}_\nu = Q^\mu{}_\alpha Q^\beta{}_\nu B^\alpha{}_\beta$$

$$B^\mu{}_\nu = \nabla_\nu k^\mu$$

# Expansion and Raychaudhuri equation

- Singularity resolution can be further explored via the Raychaudhuri equation
- Defining the tangent vector field $U^\mu$ to a timelike geodesic congruence

Projection

$$P_{\mu\nu} = g_{\mu\nu} + U_\mu U_\nu$$

Shear

$$\sigma_{\mu\nu} = B_{(\mu\nu)} - \frac{1}{3}\theta P_{\mu\nu}$$

Vorticity

$$\omega_{\mu\nu} = B_{[\mu\nu]}$$

Deviation

$$B_{\mu\nu} = \nabla_\mu U_\nu$$

Expansion

$$\theta = \nabla_\mu U^\mu$$

**Raychaudhuri**

$$\frac{d\theta}{d\tau} = -\frac{1}{3}\theta^2 - \sigma_{\mu\nu}\sigma^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} - R_{\mu\nu}U^\mu U^\nu$$

# Classical Schwarzschild interior

- Symplectic form

$$\mathbf{\Omega} = \frac{1}{8\pi G\gamma}\int_{\mathcal{I}\times\mathbb{S}^2} d^3x dA_a^i(\mathbf{x})\wedge d\tilde{E}_i^a(\mathbf{y}) \longrightarrow \mathbf{\Omega} = \frac{1}{2G\gamma}(dc\wedge dp_c + 2db\wedge dp_b)$$

- Yields the reduced Poisson brackets for the canonical variables $b,\ c,\ p_b,\ p_c$

$$\{c, p_c\} = 2G\gamma \qquad \{b, p_b\} = 2G\gamma$$

- The KS-adapted Ashtekar variables along with $qq^{ab} = \delta^{ij}\tilde{E}_i^a\tilde{E}_j^b$ yields the metric

$$ds^2 = -N(T)^2 dT^2 + \frac{p_b(T)^2}{L_0^2 p_c(T)}dr^2 + p_c(T)(d\theta^2 + \sin^2\theta d\phi^2)$$

- We choose a lapse which effectively decouples the canonical variables

$$N(T) = \frac{\gamma\sqrt{p_c(T)}}{b(T)} \qquad H = -\frac{1}{2G\gamma}[(b^2+\gamma^2)\frac{p_b}{b} + 2cp_c]$$

# Classical Schwarzschild interior - canonical variables

$$ds^2 = -\frac{\gamma^2 p_c(T)}{b(T)^2}dT^2 + \frac{p_b(T)^2}{L_0^2 p_c(T)}dr^2 + p_c(T)(d\theta^2 + \sin^2\theta d\phi^2)$$

- Equations of motion for canonical variables

$$\frac{db}{dT} = \{b, H\} = -\frac{1}{2}\left(b + \frac{\gamma^2}{b}\right) \qquad \frac{dc}{dT} = \{c, H\} = -2c$$

$$\frac{dp_b}{dT} = \{p_b, H\} = \frac{p_b}{2}\left(1 - \frac{\gamma^2}{b^2}\right) \qquad \frac{dp_c}{dT} = \{p_c, H\} = 2p_c$$

- Interpretation of canonical variables follows from these e.o.m and weakly vanishing of the Hamiltonian constraint

$$A_{x\theta} = A_{x\phi} = 2\pi L_0\sqrt{g_{xx}g_{\Omega\Omega}} = 2\pi p_b\ , \quad b = \frac{\gamma}{2}\frac{1}{\sqrt{p_c}}\frac{dp_c}{d\tau} = \frac{\gamma}{\sqrt{\pi}}\frac{d}{d\tau}\sqrt{A_{\theta\phi}}$$

$$A_{\theta\phi} = \pi g_{\Omega\Omega} = \pi p_c\ , \quad c = \gamma\frac{d}{d\tau}\left(\frac{p_b}{\sqrt{p_c}}\right) = \gamma\frac{d}{d\tau}(L_0\sqrt{g_{xx}})$$

# Classical Schwarzschild interior - canonical variables

Also, note that in the fiducial volume, we can consider three surfaces $S_{x,\theta}$, $S_{x,\phi}$, and $S_{\theta,\phi}$, respectively, bounded by $\mathcal{I}$ and a great circle along a longitude of $V_0$, $\mathcal{I}$ and the equator of $V_0$, and the equator and a longitude with areas [9]

$$A_{x,\theta} = A_{x,\phi} = 2\pi L_0 \sqrt{g_{xx} g_{\Omega\Omega}} = 2\pi p_b, \tag{2.22}$$

$$A_{\theta,\phi} = \pi g_{\Omega\Omega} = \pi p_c, \tag{2.23}$$

with the volume of the fiducial region $\mathcal{I} \times \mathbb{S}^2$ given by [9]

$$V = \int \mathrm{d}^3x \sqrt{|\det \tilde{E}|} = 4\pi L \sqrt{g_{xx}} g_{\Omega\Omega} = 4\pi p_b \sqrt{p_c}, \tag{2.24}$$

where $\sqrt{\det |\tilde{E}|} = \sqrt{q}$ with $q$ being the determinant of the spatial metric.