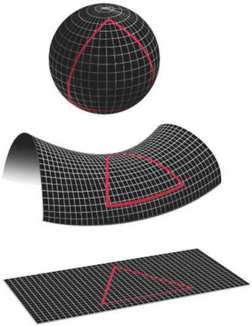
Inflationary and pre-inflationary scalar perturbations on closed universes in loop quantum cosmology

Nelson Yokomizo
(UFMG)

[In collaboration with L. Montese]

Loops'24
Fort Lauderdale, May 2024



Closed: $\Omega_k < 0$

Flat: $\Omega_k = 0$

Open: $\Omega_k > 0$

Constraints on curvature from Planck:

The combination of the *Planck* temperature and polarization power spectra give

$$\Omega_K = -0.056^{+0.028}_{-0.018} \quad (68\%,\ \textit{Planck}\ \text{TT+lowE}), \tag{46a}$$

$$\Omega_K = -0.044^{+0.018}_{-0.015} \quad (68\%,\ \textit{Planck}\ \text{TT,TE,EE+lowE}), \tag{46b}$$

an apparent detection of curvature at well over 2σ. The 99% probability region for the TT,TE,EE+lowE result is $-0.095 < \Omega_K < -0.007$, with only about 1/10 000 samples at $\Omega_K \geq 0$.

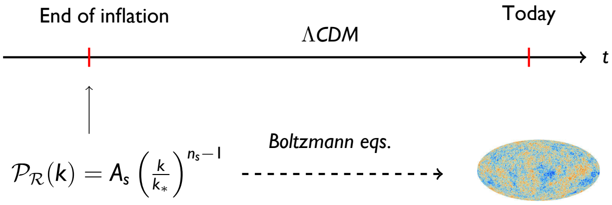
The constraint can be further sharpened by combining the *Planck* data with BAO data; this convincingly breaks the geometric degeneracy to give

$$\Omega_K = 0.0007 \pm 0.0019 \quad (68\%,\ \text{TT,TE,EE+lowE+lensing+BAO}). \tag{47b}$$

[Planck 2018 VI. Cosmological parameters]

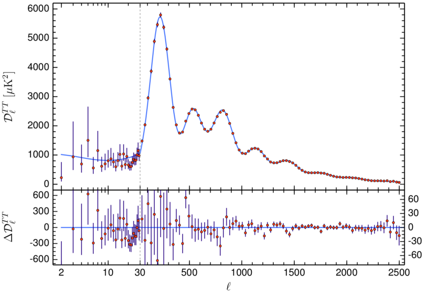
Introduction

- Primordial curvature perturbations modelled by a power law spectrum.
- Evolution of cosmological perturbations in the ΛCDM model described by Boltzmann equations that include curvature terms.
- Predictions for CMB compared with observations.

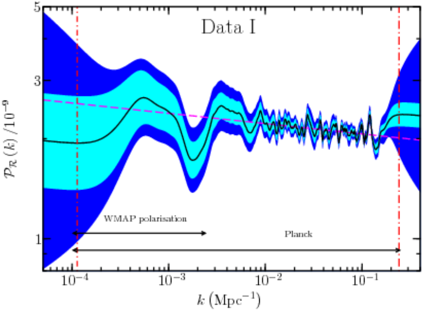


Spatial curvature is then bounded by : $|\Omega_k| \lesssim 0.005$.

Introduction



Power spectrum of temperature fluctuations [Planck 2018]

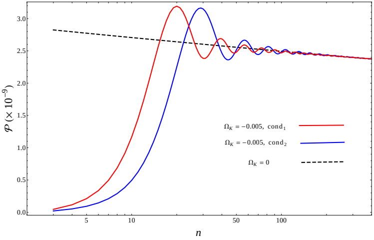


Reconstruction of power spectrum of primordial perturbations [Hunt and Sarkar, JCAP 12 (2015) 052]

- Predictions fit observations for small angular scales.
- Anomalies observed for large angular scales.

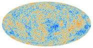
Physical mechanisms leading to large scale anomalies? Spatial curvature.

[Bonga, Gupt and NY, JCAP 1610 (2016) 031 and JCAP 1705 (2017) 021]



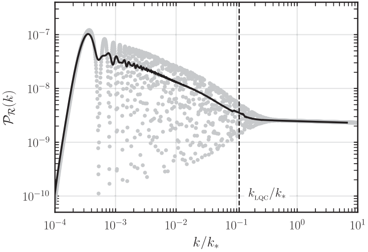
Onset of inflation — $\phi,\ \Omega_k$ — End of inflation — $\Lambda CDM,\ \Omega_k$ — Today — t

$|0\rangle$ --- *QFT in CST* ---> $\mathcal{P}(k) \neq A_s \left(\frac{k}{k_*}\right)^{n_s - 1}$ --- *Boltzmann eqs.* --->



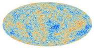
Physical mechanisms leading to large scale anomalies? Quantum gravity effects.

[Agullo, Morris PRD 92 (2015); Agullo, Bolliet, Sreenath PRD 97 (2018)]



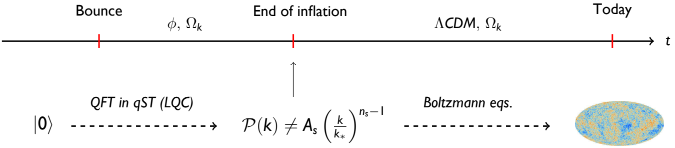
Bounce — ϕ — End of inflation — ΛCDM — Today → t

$|0\rangle$ - - - *QFT in qST (LQC)* - - → $\mathcal{P}(k) \neq A_s \left(\frac{k}{k_*}\right)^{n_s - 1}$ - - - *Boltzmann eqs.* - - →



Primordial power spectrum predicted by loop quantum cosmology for a closed universe?

Quantum gravity effects in the pre-inflationary regime + spatial curvature.



Background dynamics

- Modified Friedmann equation in the presence of spatial curvature (LQC)
  [Ashtekar, Pawlowski,Singh, Vandersloot PRD 75 (2007); Motaharfar, Singh PRD 104 (2021)]

$$H^2 = \frac{8\pi G}{3}(\rho - \rho_{min})\left[1 - \frac{\rho - \rho_{min}}{\rho_c}\right], \quad \rho_c = 3/(8\pi G\gamma^2\lambda^2)$$
$$\simeq \frac{8\pi G}{3}\rho - \frac{1}{a^2}, \qquad (\rho \ll \rho_c)$$

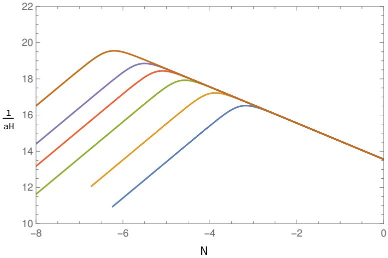
- Single scalar field, Starobinsky potential.

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0$$

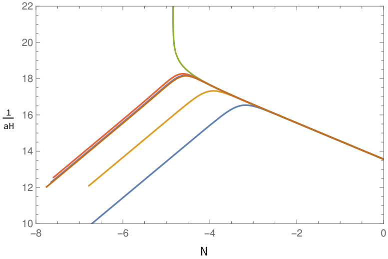
- Initial conditions: Initial time t_*, during inflation ($k_* = 0.002\ \mathrm{Mpc}^{-1}$).

  a_*, ϕ_* and $\dot{\phi}_*$ determined from observational estimates for A_s, n_s and the duration of inflation from t_* until the end of inflation.

Background dynamics



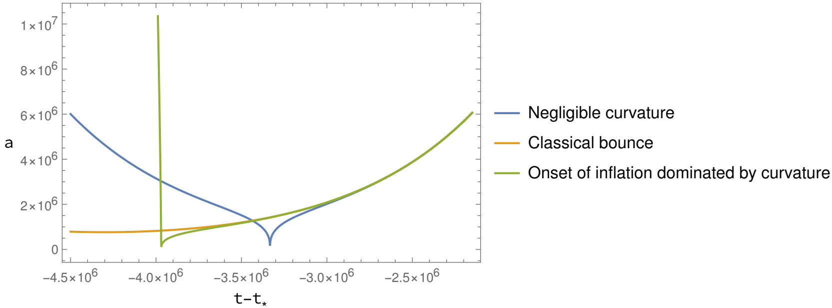
Flat universe



Closed universe, $\Omega_k = -0.005$

Background evolution near the onset of inflation for distinct initial conditions at t_* $(N = 0)$ in a flat and a closed universe.

Background dynamics



Three kinds of bounce in a closed universe.

Scalar perturbations and primordial power spectrum

Dynamics of scalar modes

- ADM formalism for the gravitational field coupled to the scalar field.
- Hamiltonian expressed in terms of background quantities and linear perturbations, keeping terms up to second order in the perturbations.
- Choice of gauge: Spatially spherical gauge, analogous to spatially flat gauge. Scalar cosmological perturbations described by field perturbations $\delta\phi$.
- Initial conditions set before the bounce. 4-th order adiabatic vacuum.
- Perturbations evolved until the end of inflation and translated to comoving curvature perturbations $\mathcal{R}$.

Scalar perturbations and primordial power spectrum

Field and momentum perturbations:

$$\delta\phi = \sum_{n=2}^{\infty}\sum_{lm} f_{nlm} Q_{nlm}$$

$$\delta\tilde{\pi}_\phi = \sum_{n=2}^{\infty}\sum_{lm} \pi^f_{nlm} Q_{nlm}\sqrt{\Omega}$$

$$\left[(n^2-1)/r_0^2 \to k^2\right]$$

Quadratic Hamiltonian:

$$H^{(2)}_{n\ell m} = \frac{c_1(n)}{2}(\pi^f_{nlm})^2 + \frac{c_2(n)}{2}(f_{nlm})^2$$

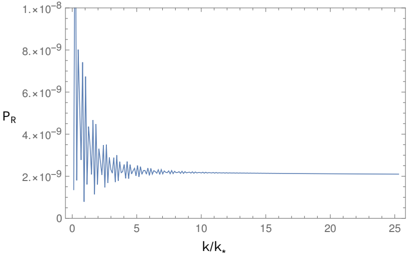
Equation of motion:

$$\ddot{f} - \frac{\dot{c}_1}{c_1}\dot{f} + c_1 c_2 f = 0$$

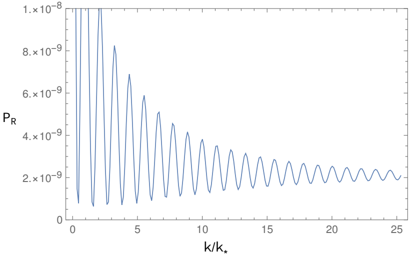
End of inflation:

$$\mathcal{R} = -\frac{H}{\dot{\phi}_0}\delta\phi \quad \to \quad \mathcal{P}_{\mathcal{R}} \propto (\delta\phi)^2$$

Scalar perturbations and primordial power spectrum



$|\Omega_k| = 10^{-8}, N_e = 13.7$



$|\Omega_k| = 10^{-6}, N_e = 12.2$

Conclusion

1. Analysis of effective dynamics for backgrounds that satisfy observational constraints on inflation, for a closed universe in loop quantum cosmology. For non-negligible spatial curvatures, there can be a classical bounce (small interval of curvatures) or a quantum bounce (generically).

2. The duration of the inflationary regime is determined by the spatial curvature. Larger curvatures correspond to shorter inflationary regimes.

3. Quadratic Hamiltonian and equations of motion determined for linear scalar perturbations in closed universes in the spatially spherical gauge.

4. Primordial power spectrum of scalar perturbations at the end of inflation was numerically determined. Considerable corrections are present for observable modes even for curvatures $|\Omega_k| \sim 10^{-6}$.