Entanglement aspects of evaporating black holes

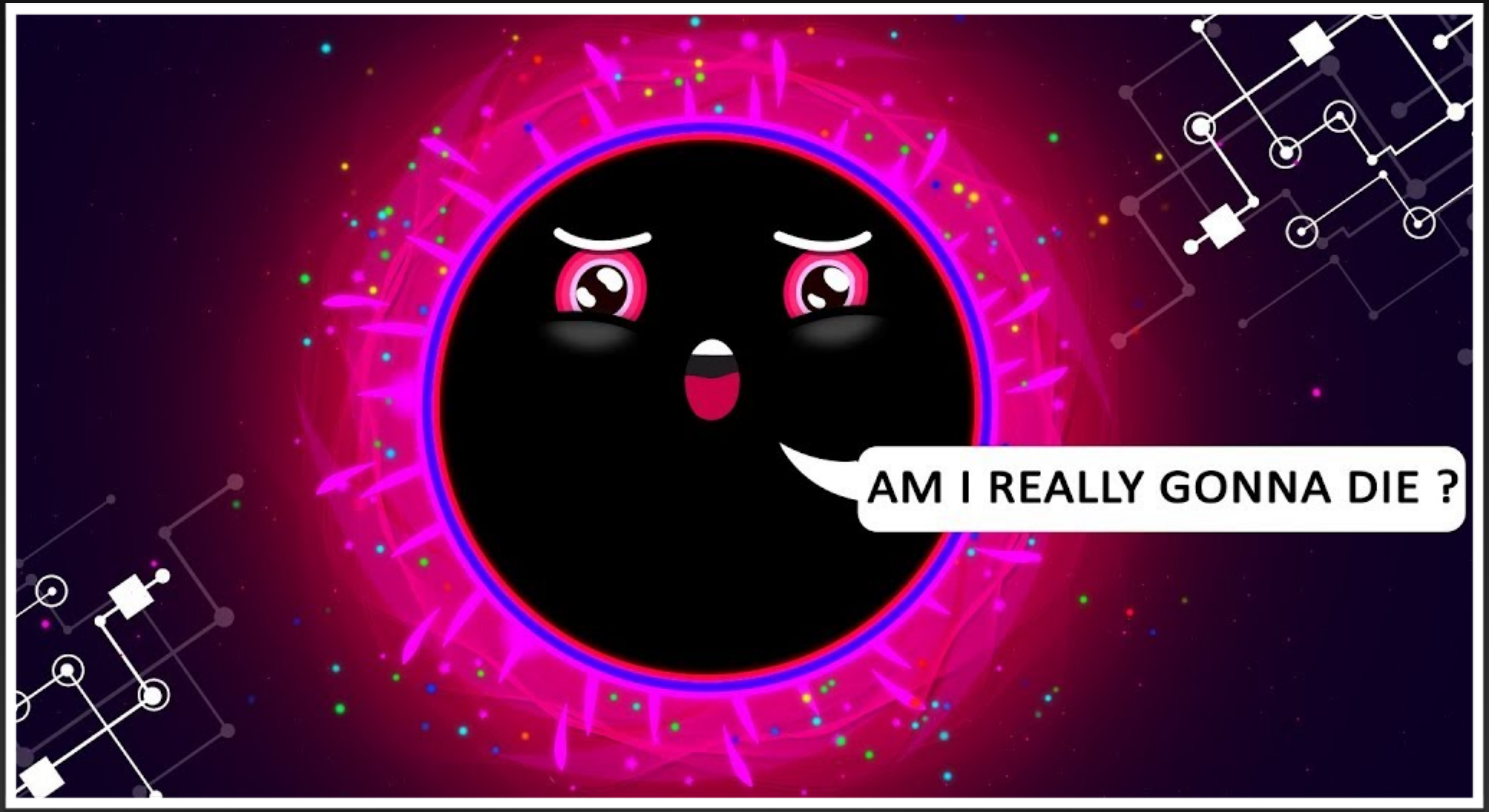
[arXiv:2405.xxxxx]

$(G = \hbar = c = k_B = 1)$

Loops'24

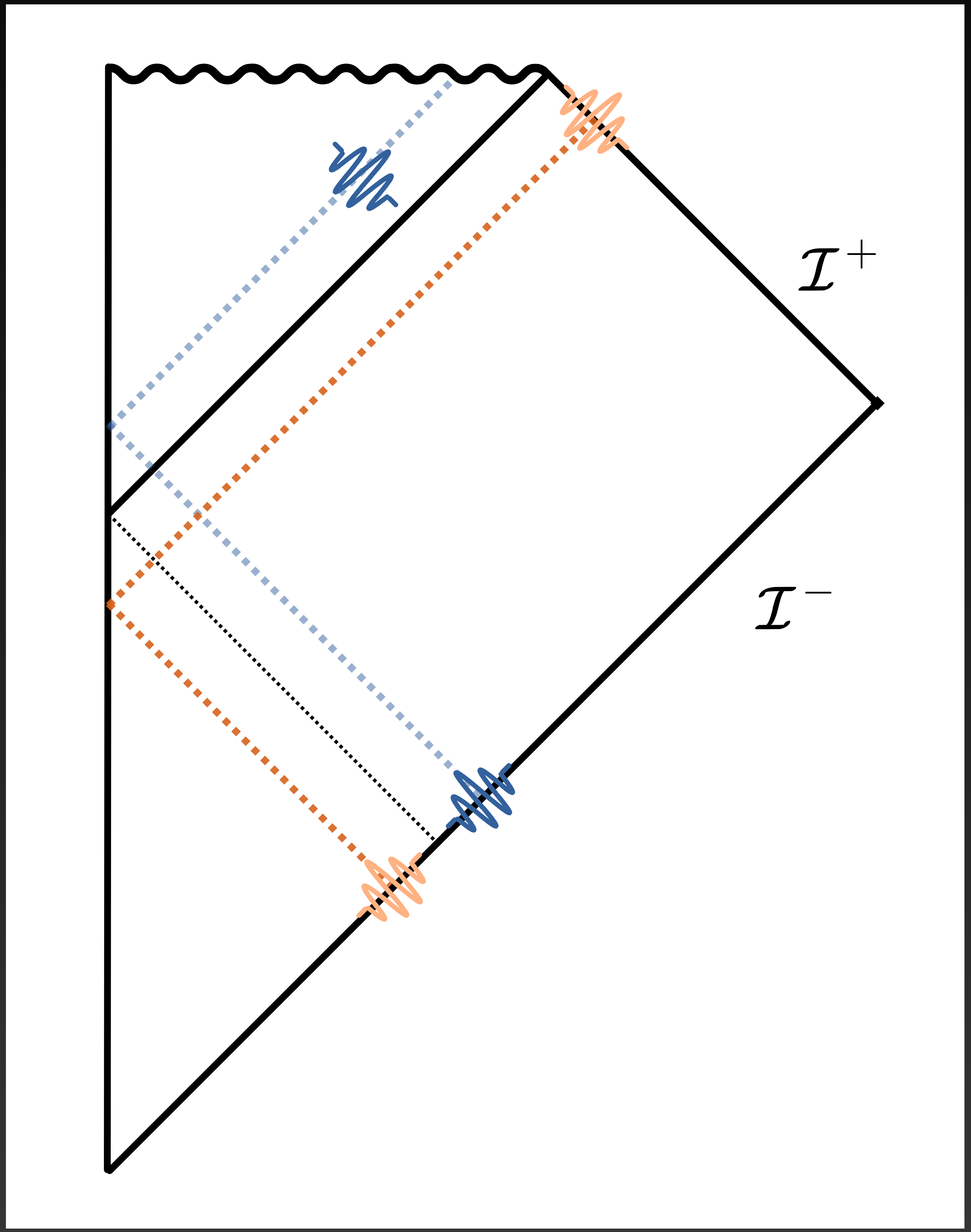
Beatriz Elizaga Navascués, May 8th 2024
In collaboration with Ivan Agullo and Paula Calizaya Cabrera

Quantum physics & black holes



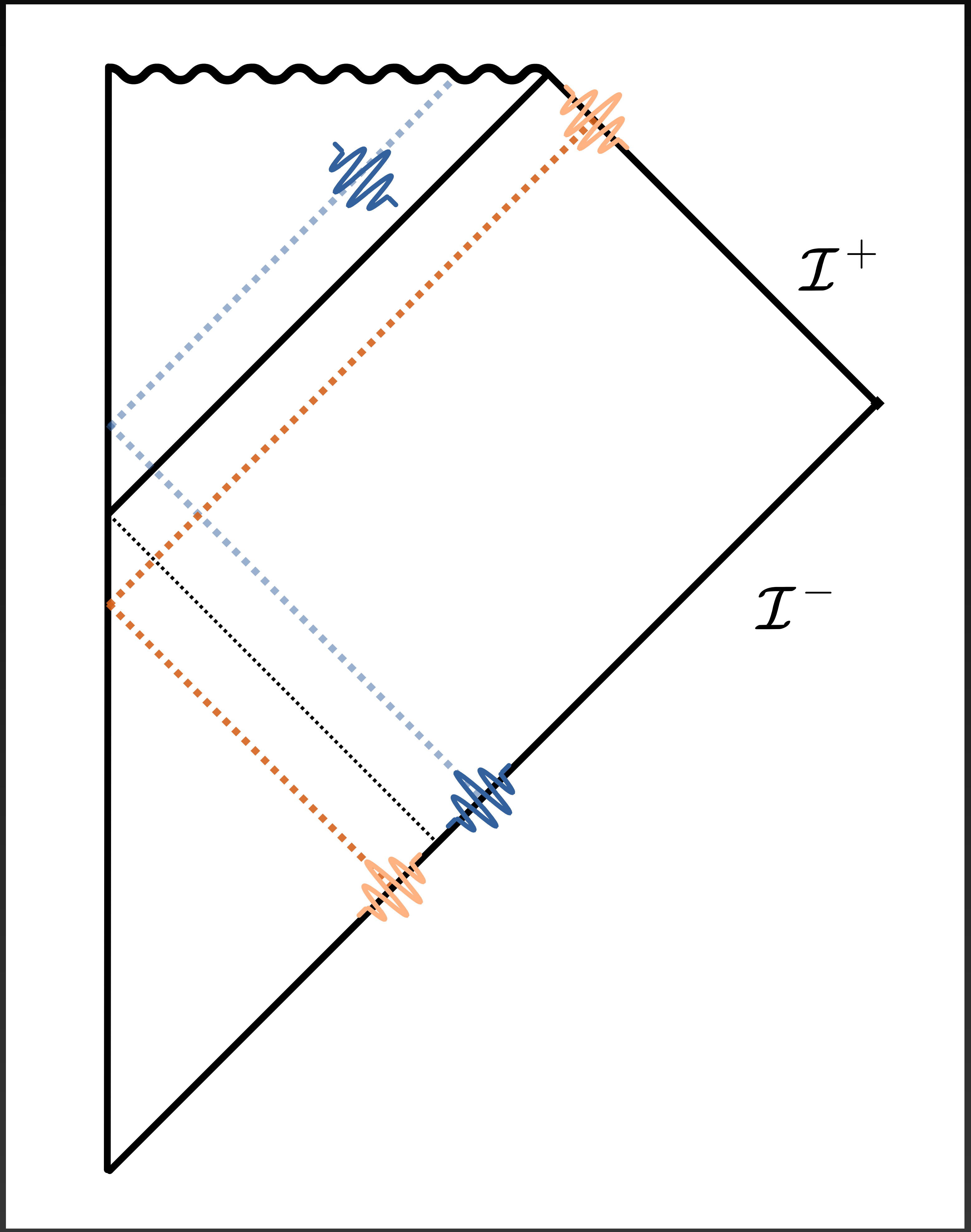
Source: Kurzgesagt

The Hawking effect



- Massless test field in gravitational collapse.
- Hawking modes: Inertial particles at $\mathscr{I}^+$.
- Natural vacuum at $\mathscr{I}^-$ shows [Hawking1975, Wald1975]
  - ✹ Hawking excitations in thermal state.

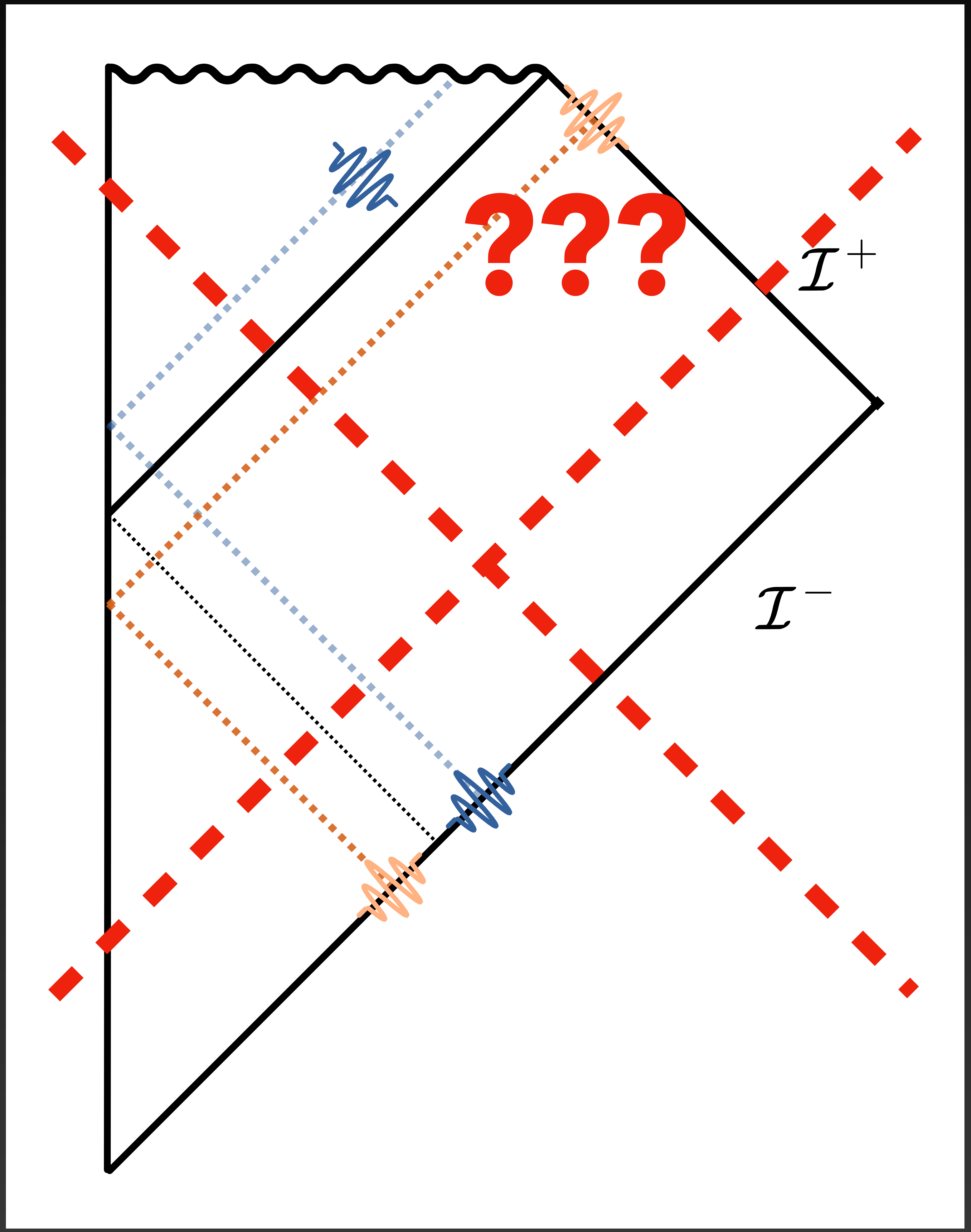
The Hawking effect



- Massless test field in gravitational collapse.
- Hawking modes: Inertial particles at $\mathscr{I}^+$.
- Natural vacuum at $\mathscr{I}^-$ shows [Hawking1975, Wald1975]
  - Hawking excitations in thermal state.
  - Pairwise entanglement with "partners".
- Partners: Reflection across **event** horizon at $\mathscr{I}^-$.

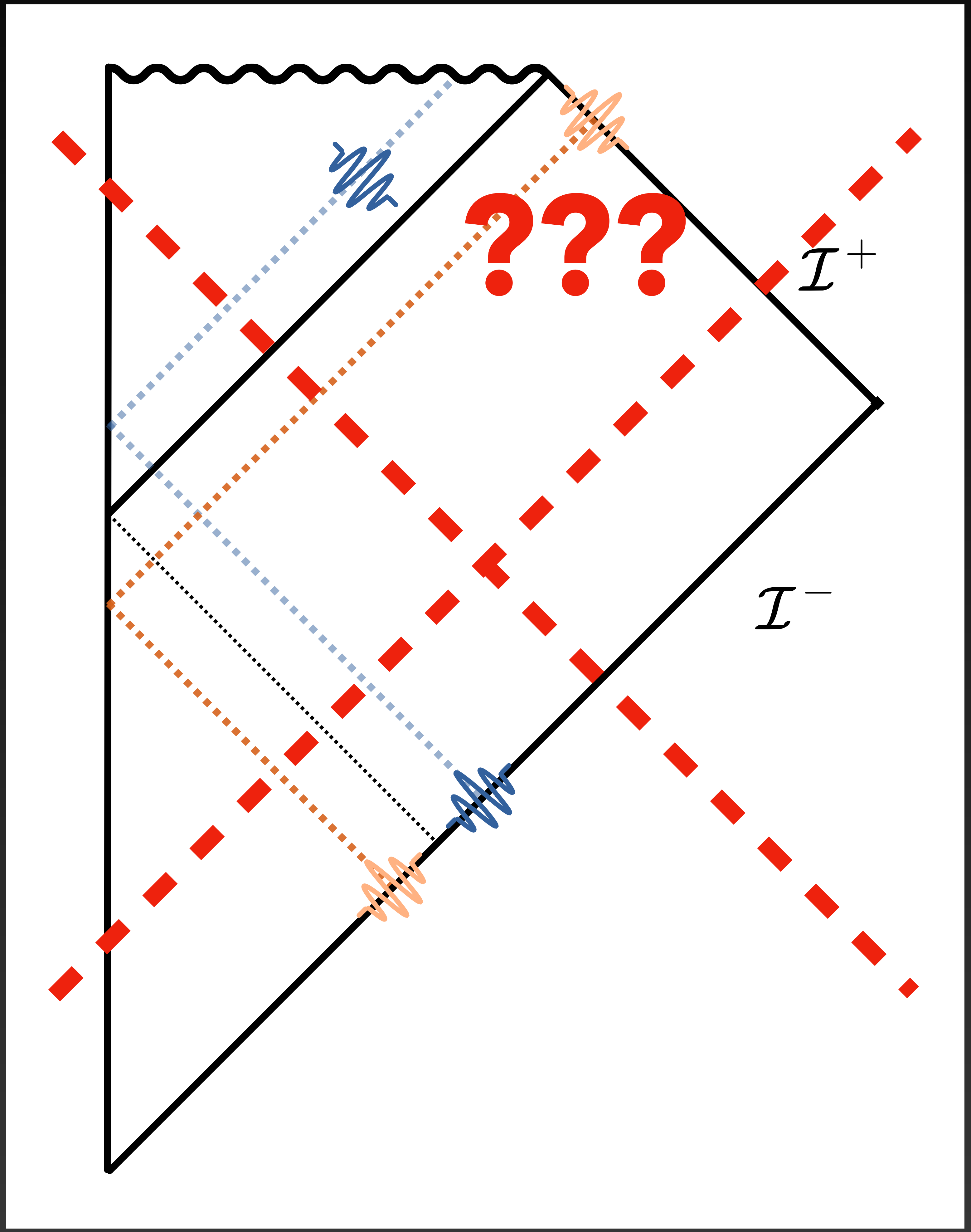
[Wald1975]

With back-reaction…



- At $\mathscr{I}^+$, thermal radiation with $T = (4M)^{-1}$.
- Energy considerations ⟶ Mass loss over time.
  [Hawking1975, Page1976]
- As black hole evaporates, **fate of information?**

With back-reaction…



- At $\mathscr{I}^+$, thermal radiation with $T = (4M)^{-1}$.
- Energy considerations ⟶ Mass loss over time.

  [Hawking1975, Page1976]
- As black hole evaporates, **fate of information?**
- An avenue for answers: **black holes in LQG.**

[Alesci, Alonso-Bardaji, Ashtekar, Bahrami, Bianchi, Bobula, Boehmer, Bodendorfer, Bojowald, Brannlund, Brizuela, Campiglia, Cartin, Chiou, Christodoulo, Corichi, Cortez, Cuervo, De Benedictis, Dadhich, D'Ambrosio, Elizaga Navascués, Fazzini, Gambini, García-Quismondo, Giesel, Haggard, Han, Hergott, Hussain, Joe, Kelly, Khanna, Kloster, Lewandowski, Liu, Ma, Martin-Dussaud, Mele, Mena Marugán, Mínguez-Sánchez, Modesto, Morales-Técotl, Münch, Pawlowski, Olmedo, Perez, Pullin, Pranzetti, Qu, Rastgoo, Rovelli, Ruelas, Sabharwal, Saini, Santacruz, Singh, Soltani, Song, Speziale, Vandersloot, Vera, Vidotto, Viollet, Weigl, Wang, Wilson-Ewing, Yang, Yonika, Zhang…]

Description of Planck regime needed (e.g. singularity),

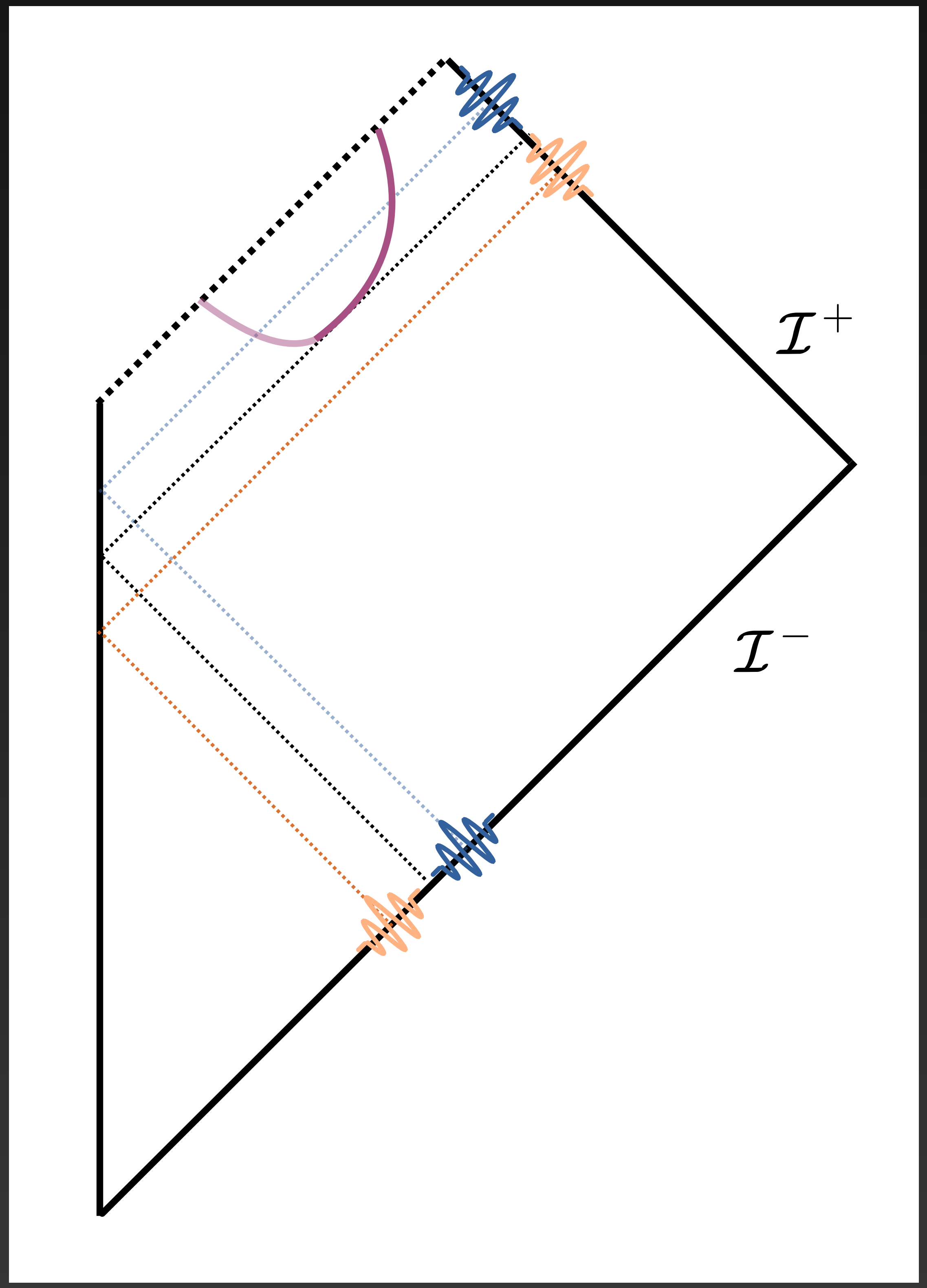
but…

Is it crucial for the ultimate fate of Hawking partners?

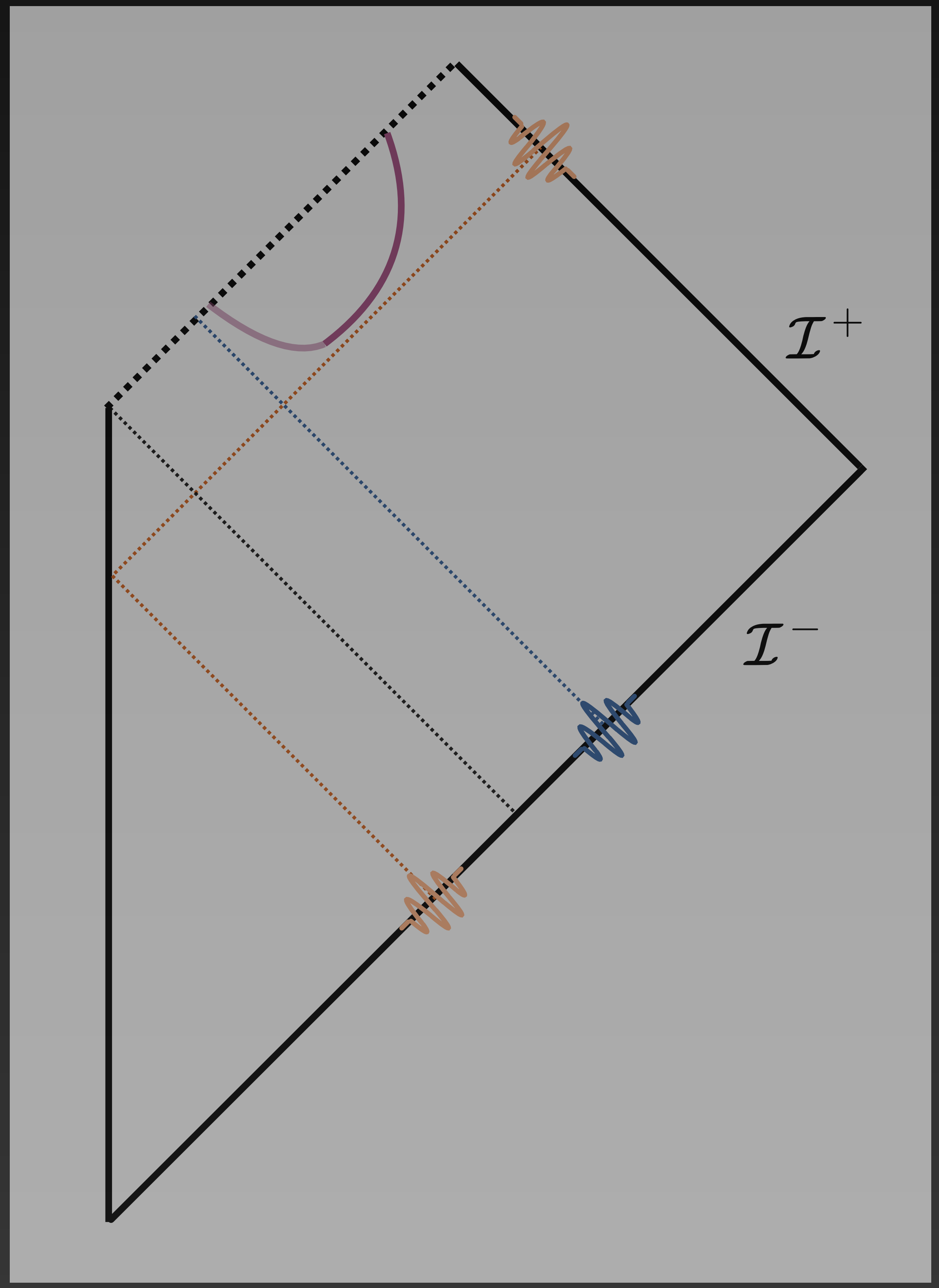
Possibilities

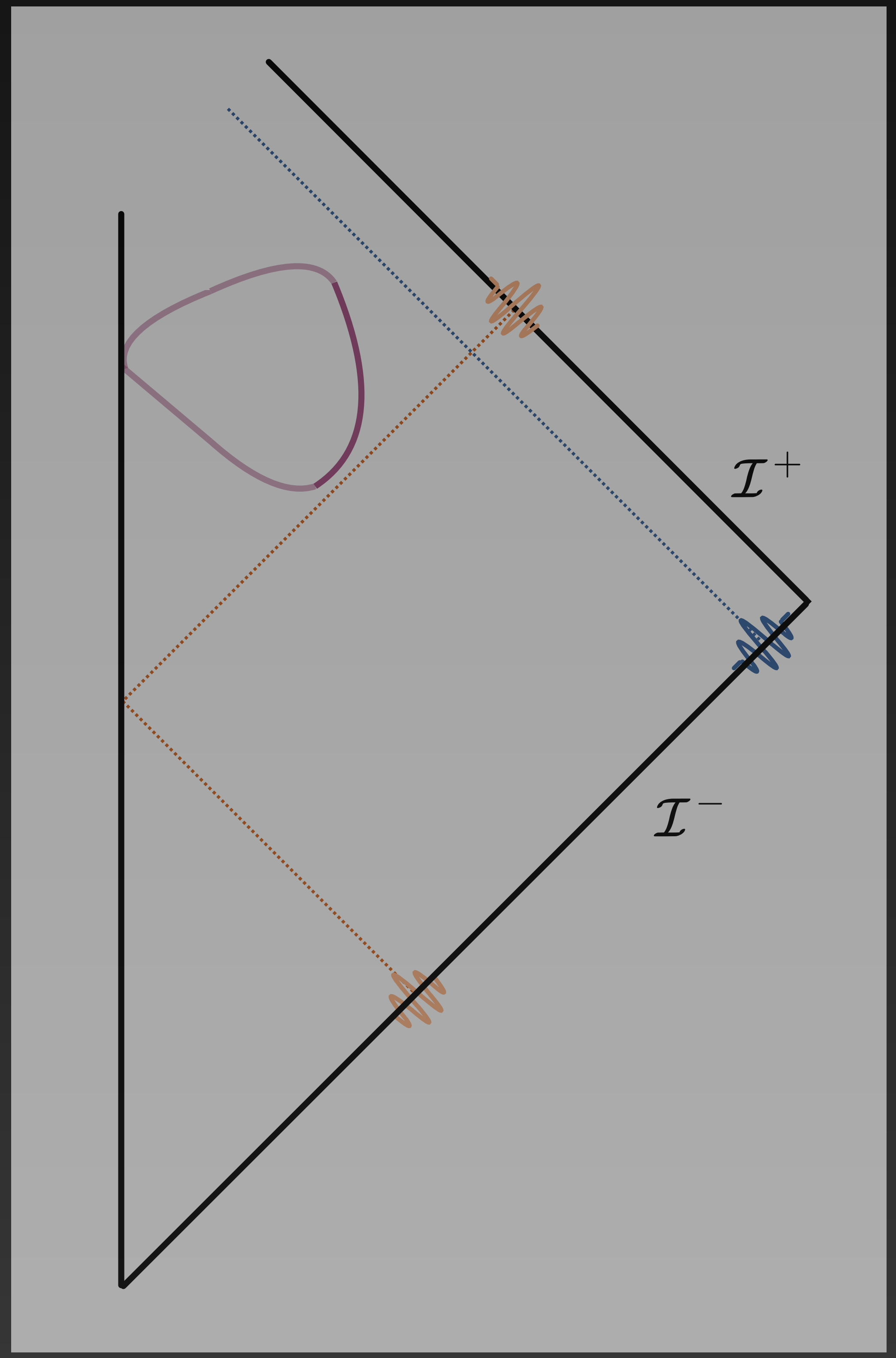
e.g. [Ashtekar, Hayward, Krishnan, Lewandowski…]

- Upon evaporation: Event horizon**?!** → Transient, quasi-local horizons.
- Properties of partners depend on **nonlocal** structures (unlike radiation at $\mathscr{I}^+$).



e.g. [Hayward2005]





Possibilities

e.g. [Ashtekar, Hayward, Krishnan, Lewandowski…]

- Upon evaporation: Event horizon**?!** → Transient, quasi-local horizons.

- Properties of partners depend on **nonlocal** structures (unlike radiation at $\mathscr{I}^+$).

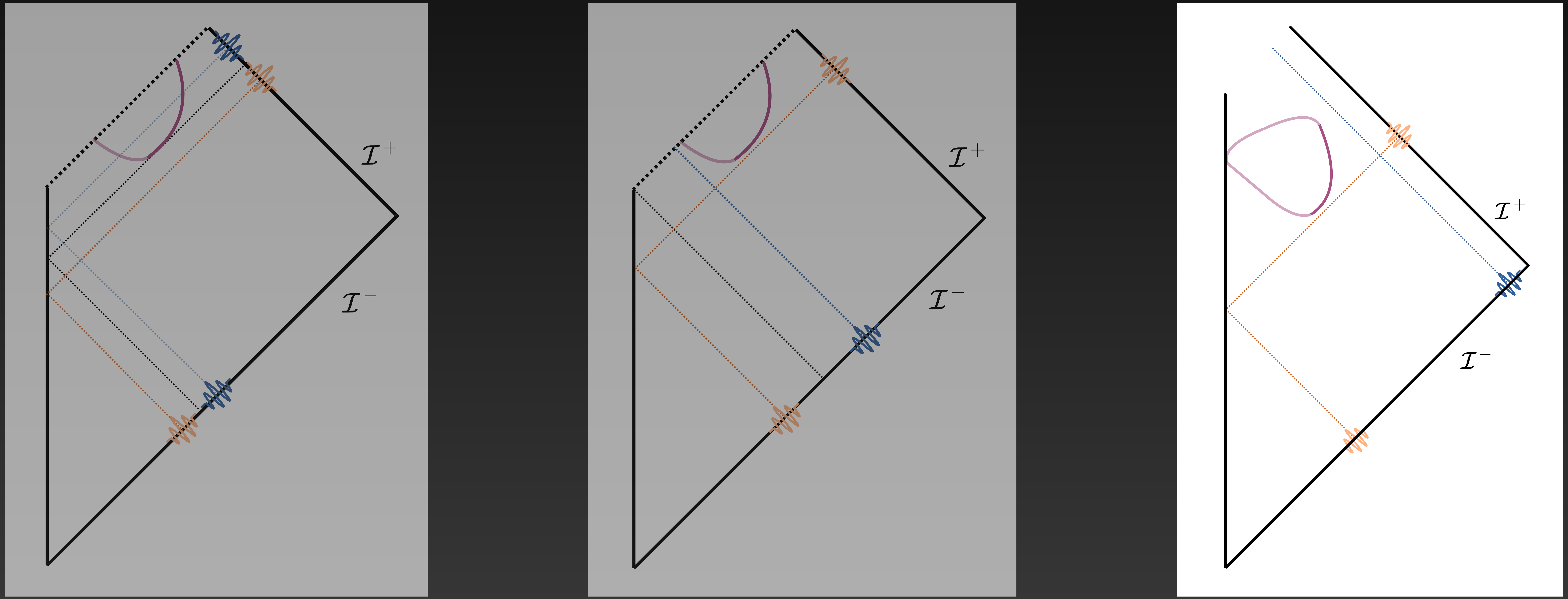
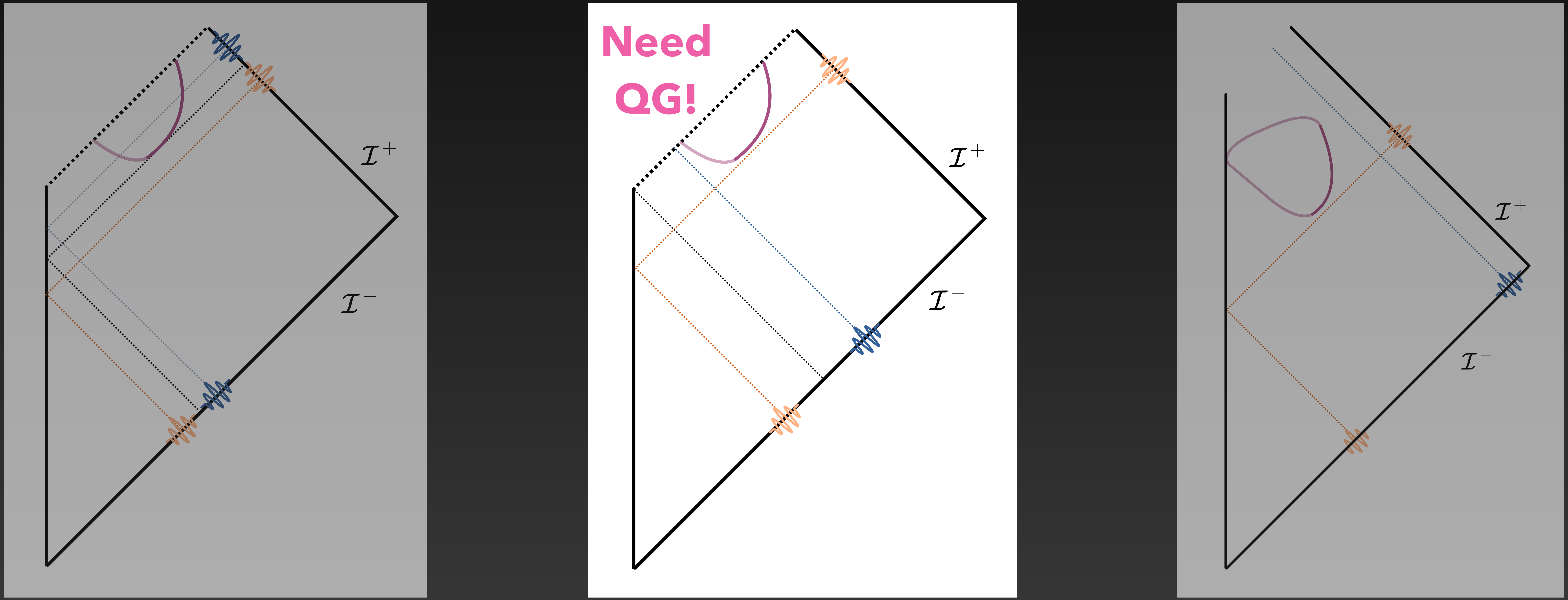


Diagram in the spirit of [Ashtekar&Bojowald2005;Rovelli&Vidotto2014,Haggard&Rovelli2015] and many posterior developments

Possibilities

e.g. [Ashtekar, Hayward, Krishnan, Lewandowski…]

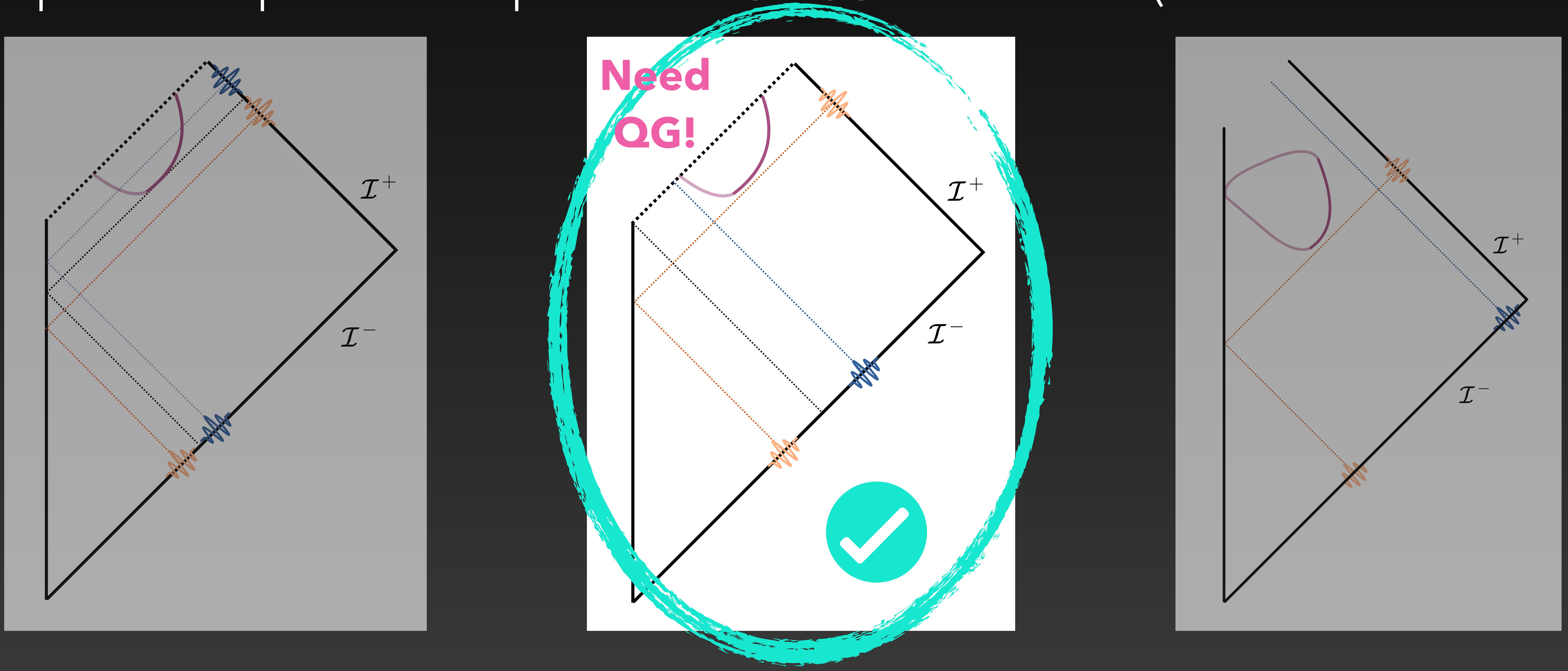
- Upon evaporation: Event horizon?! → Transient, quasi-local horizons.
- Properties of partners depend on **nonlocal** structures (unlike radiation at $\mathscr{I}^+$).



Possibilities

e.g. [Ashtekar, Hayward, Krishnan, Lewandowski…]

- Upon evaporation: Event horizon**?!** → Transient, quasi-local horizons.
- Properties of partners depend on **nonlocal** structures (unlike radiation at $\mathscr{I}^+$).



Modeling evaporation

Importance of null rays

- Asymptotically flat spacetime, $v, u \to$ radial null coords. at $\mathscr{I}^-, \mathscr{I}^+$.

- Null rays naturally define a map $v = p(u)$ between $\mathscr{I}^-$ and $\mathscr{I}^+$.

e.g. [Hajicek 1987; Hu 1996; Visser 2003; Frolov, Zelnikov 2018]

  ✺ Long believed to determine particle content of quantum fields at $\mathscr{I}^+$.

Importance of null rays

- Asymptotically flat spacetime, $v, u \to$ radial null coords. at $\mathscr{I}^-, \mathscr{I}^+$.

- Null rays naturally define a map $v = p(u)$ between $\mathscr{I}^-$ and $\mathscr{I}^+$.

e.g. [Hajicek 1987; Hu 1996; Visser 2003; Frolov, Zelnikov 2018]

  ✹ Long believed to determine particle content of quantum fields at $\mathscr{I}^+$.

- **Important observation**: Any map that locally behaves as

$$p(u) \approx v_\star^{(H)} - A_\star e^{-\kappa_\star u},$$

leads to Hawking radiation at $\mathscr{I}^+$ with temperature $\kappa_\star$. [Barceló, Liberati, Sonego, Visser 2011]

Evaporating black holes

- Our physical hypotheses:

  1. Global dynamics of fields is ruled by $v = p(u)$, up to back-scattering.

  2. There is "time-dependent" Hawking radiation at $\mathscr{I}^+$ for $u \in [u_0, u_{Pl}]$:

$$T(u) \approx \frac{1}{4M(u)}, \qquad \dot{M}(u) = -\frac{\alpha}{M(u)^2}, \qquad \alpha \sim 10^{-4}$$

value of α : [Page2013]

- Mathematically, this means that we locally require

$$p(u) \approx v_\star^{(H)} - A_\star e^{-\kappa_\star u}, \qquad \kappa_\star = \frac{1}{4M(u_\star)}$$

on any interval around $u = u_\star$, longer than $\kappa_\star^{-1}$, where $M(u) \approx M(u_\star)$.

Ray tracing for $u \in [u_0, u_{Pl}]$

- Result, up to generalizations that do not change the conclusions of this work:

$$-\frac{\ddot{p}(u)}{\dot{p}(u)} = \kappa(u), \qquad \kappa(u) = \frac{1}{4M(u)}$$

$$p(u) = v_0 + 4\dot{v}_0\, e^{-M_0^2/(8\alpha)} \left\{ M_0\, e^{M_0^2/(8\alpha)} - M(u)\, e^{M(u)^2/(8\alpha)} + \sqrt{2\pi\alpha}\left[\mathrm{erfi}\left(\frac{M(u)}{2\sqrt{2\alpha}}\right) - \mathrm{erfi}\left(\frac{M_0}{2\sqrt{2\alpha}}\right)\right]\right\}$$

Ray tracing for $u \in [u_0, u_{Pl}]$

- Result, up to generalizations that do not change the conclusions of this work:

$$-\frac{\ddot{p}(u)}{\dot{p}(u)} = \kappa(u), \qquad \kappa(u) = \frac{1}{4M(u)}$$

$$p(u) = v_0 + 4\dot{v}_0\, e^{-M_0^2/(8\alpha)} \left\{ M_0\, e^{M_0^2/(8\alpha)} - M(u)\, e^{M(u)^2/(8\alpha)} + \sqrt{2\pi\alpha} \left[\mathrm{erfi}\left(\frac{M(u)}{2\sqrt{2\alpha}} \right) - \mathrm{erfi}\left(\frac{M_0}{2\sqrt{2\alpha}} \right) \right] \right\}$$

- In this case, $p(u) \approx v_\star^{(H)} - A_\star e^{-\kappa_\star u}$ for $\left| u - u_\star \right| \ll M(u_\star)^2/\sqrt{\alpha}$.

"Instantaneous would-be horizon"

Partner modes

Single-mode subsystems

- Let $|0\rangle$ be the "inertial" vacuum of a massless scalar at $\mathscr{I}^-$.

- Single-mode subsystem: Algebra generated by any pair $(\hat{a}_A, \hat{a}_A^\dagger)$ s.t.

$$\hat{a}_A = \int_0^\infty d\omega \left[\alpha_\omega \hat{a}_\omega + \beta_\omega \hat{a}_\omega^\dagger\right] \qquad \left(\int_0^\infty d\omega \left|\beta_\omega\right|^2 < \infty\right)$$

✵ Any complex solution f_A of e.o.m. defines a single mode.

✵ Single-mode subsystems are invariant under symplectic transformations.

Partners in general

e.g. [Hotta, Schützhold, Unruh 2015; Trevison, Yamaguchi, Hotta 2019; Hackl, Johnson 2019]

- Let $|0\rangle$ be the "inertial" vacuum of a massless scalar at $\mathscr{I}^-$.
- Trace of $|0\rangle$ over all d.o.f. but one single-mode subsystem A can be mixed.
- If the reduced state is mixed, unique single mode-subsystem that purifies it:

$$f_{A_P} = N\,\Pi_A^{\perp}(J_o f_A)$$

Complex structure of $|0\rangle$

Projector on orthogonal complement of A, w.r.t. symplectic structure

This formula: [Agullo, Martín-Martínez, Nadal-Gisbert, Yamaguchi, to appear]

Partners in evaporating black holes

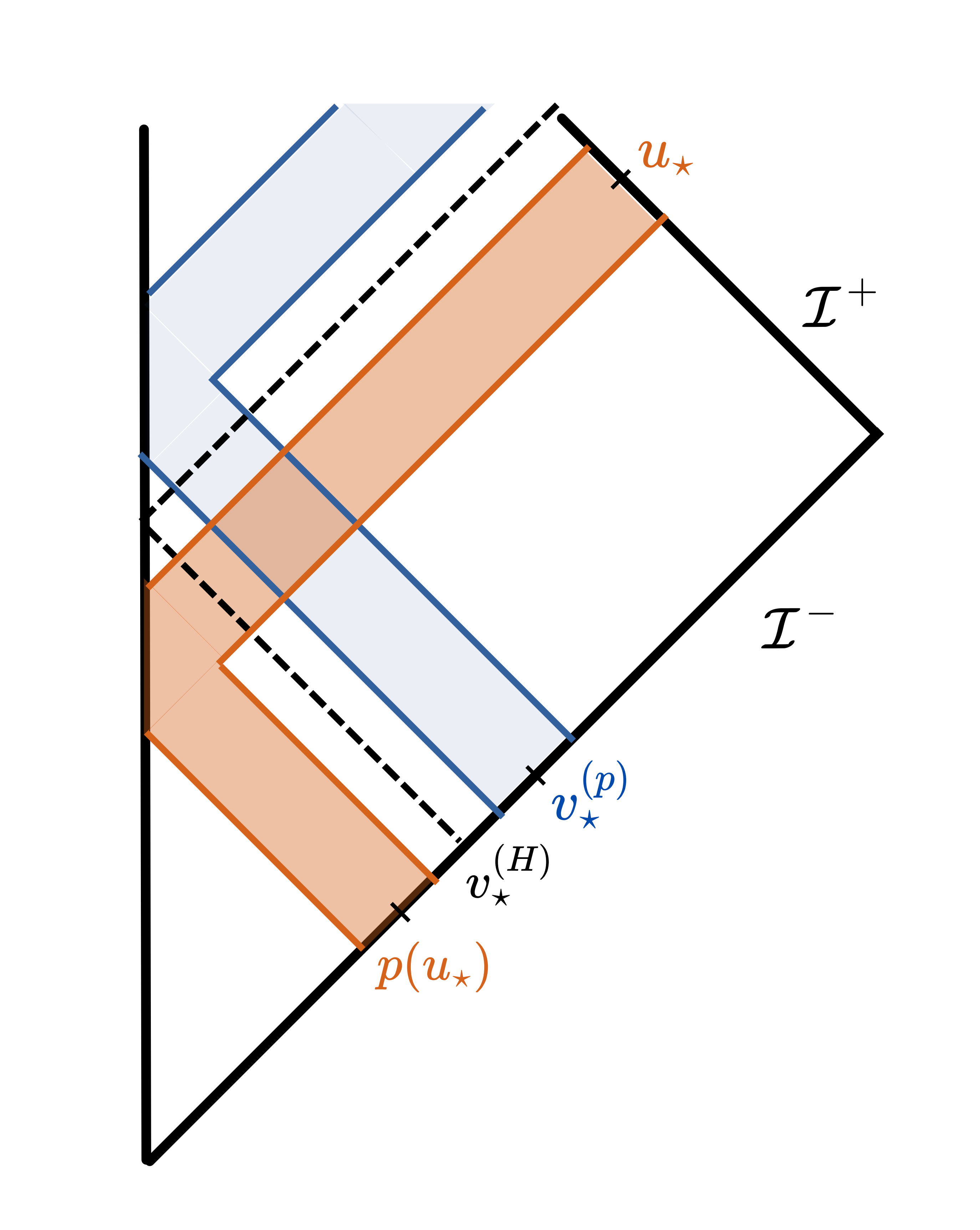
- Our definition of Hawking mode $f_\star$ at $\mathscr{I}^+$:

  ✺ Truncated "+ve-freq." wave-packet, C_0^∞ .

  ✺ Support within exponential approximation.

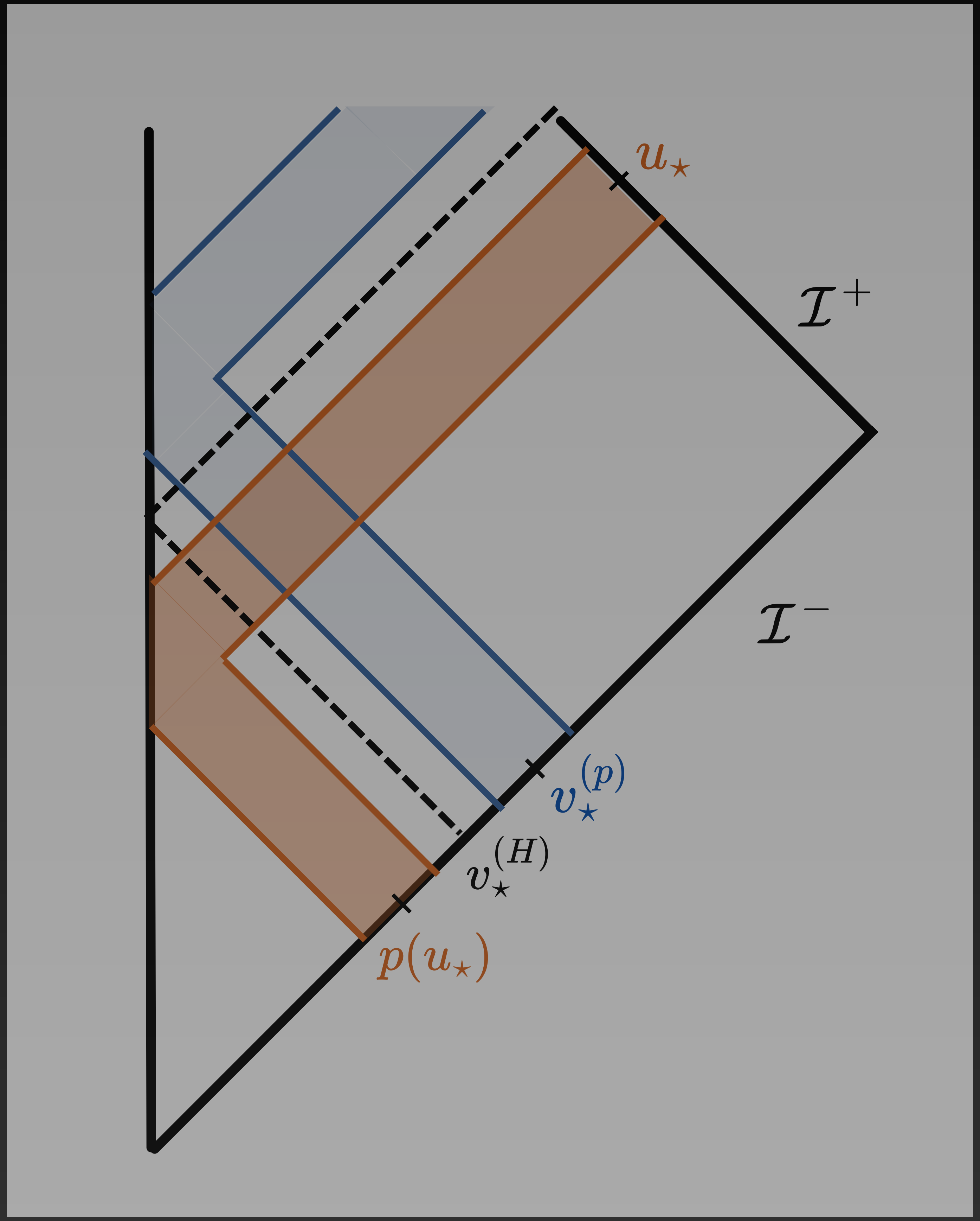
- Evolution to $\mathscr{I}^-$: Geometric optics.

- Partner $f_{\star_P} \approx$ Reflection of $f_\star$ across $v_\star^{(H)}$.



Partners in evaporating black holes

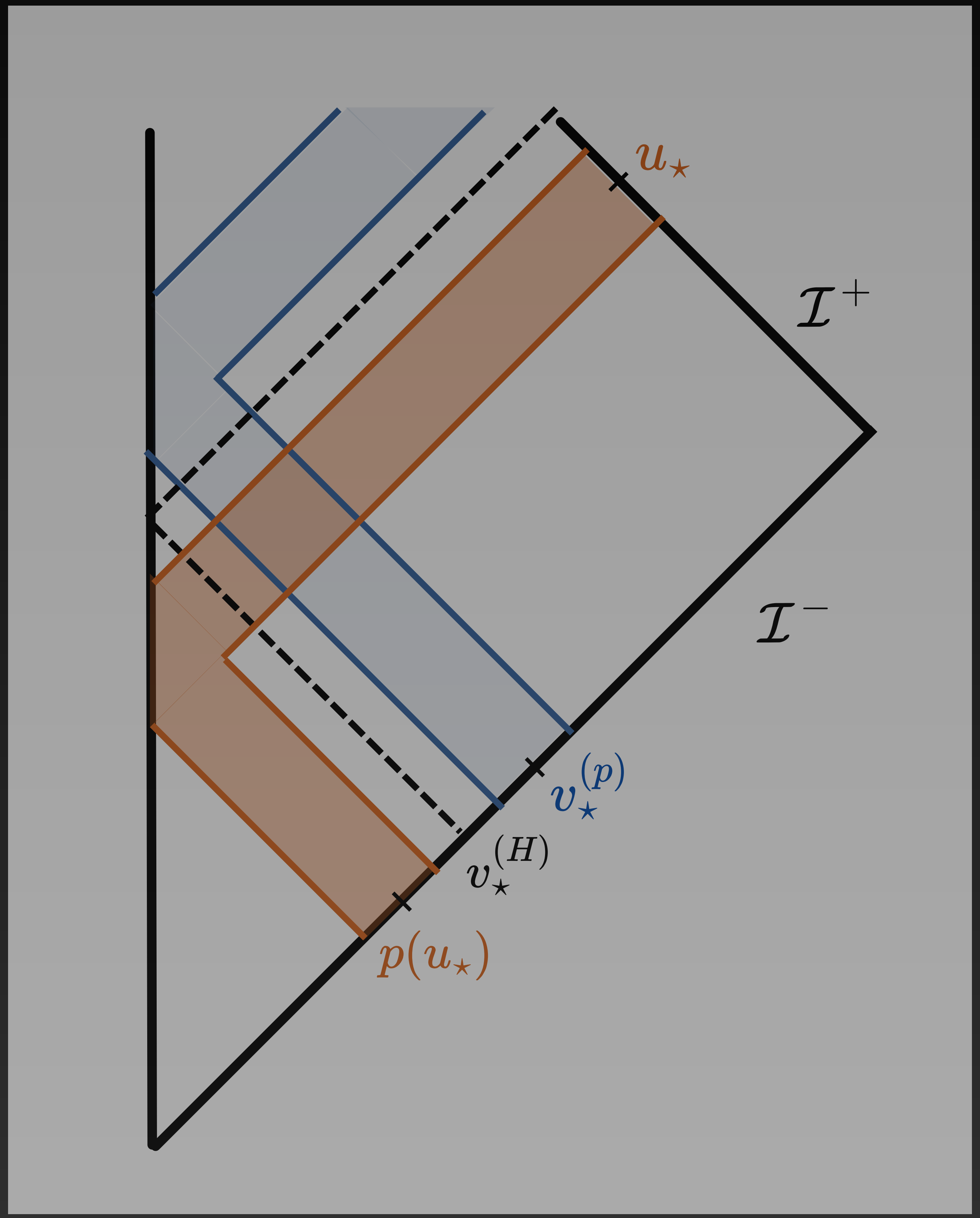
- Our definition of Hawking mode $f_\star$ at $\mathscr{I}^+$:
  - Truncated "+ve-freq." wave-packet, C_0^∞ .
  - Support within exponential approximation.
- Evolution to $\mathscr{I}^-$: Geometric optics.
- Partner $f_{\star_P} \approx$ Reflection of $f_\star$ across $v_\star^{(H)}$.



- Errors of the same order as in the original computation by Wald. ✓

Partners in evaporating black holes

- Our definition of Hawking mode $f_\star$ at $\mathscr{I}^+$:
  - ✺ Truncated "+ve-freq." wave-packet, C_0^∞ .
  - ✺ Support within exponential approximation.
- Evolution to $\mathscr{I}^-$: Geometric optics.
- Partner $f_{\star_P} \approx$ Reflection of $f_\star$ across $v_\star^{(H)}$.



✺ In progress: Numerical computations using the exact formula.

Physical consequences

$f_{\star_p} \approx$ **Reflection of** $f_\star$ **across** $v_\star^{(H)}$

Where are the partners centered at $\mathscr{I}^-$?

$$v_\star^{(p)} = 2v_\star^{(H)} - p(u_\star)$$

Results on location of partners

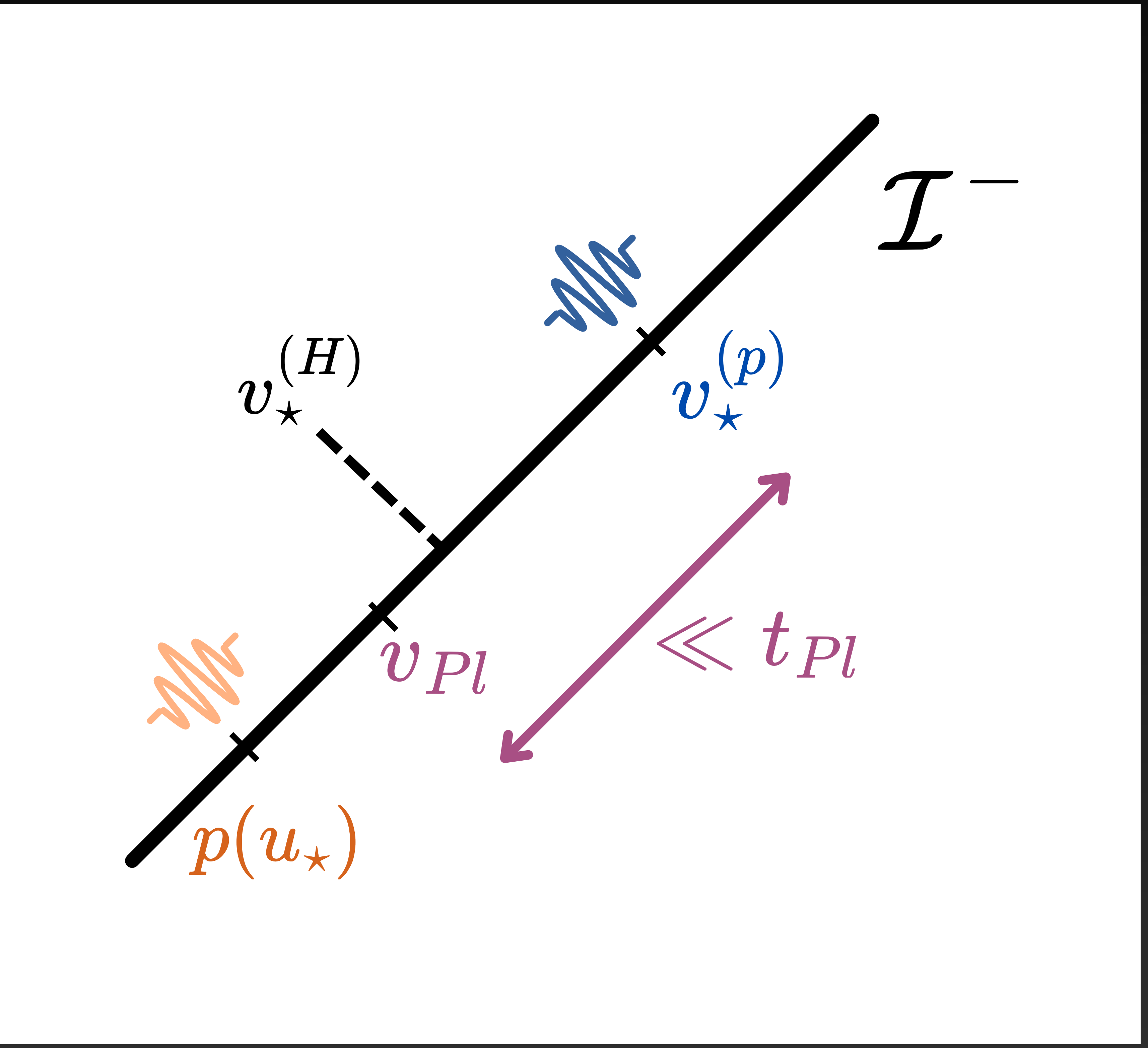
- Known relation $v = p(u)$ for $u \in [u_0, u_{Pl}]$.

- Instantaneous would-be horizon:

$$v_\star^{(H)} = p(u_\star) + \dot{p}(u_\star)\kappa_\star^{-1}$$

- With this we can show that

$$\boxed{0 < v_\star^{(p)} - v_{Pl} \ll t_{Pl}} \qquad v_{Pl} \equiv p(u_{Pl})$$



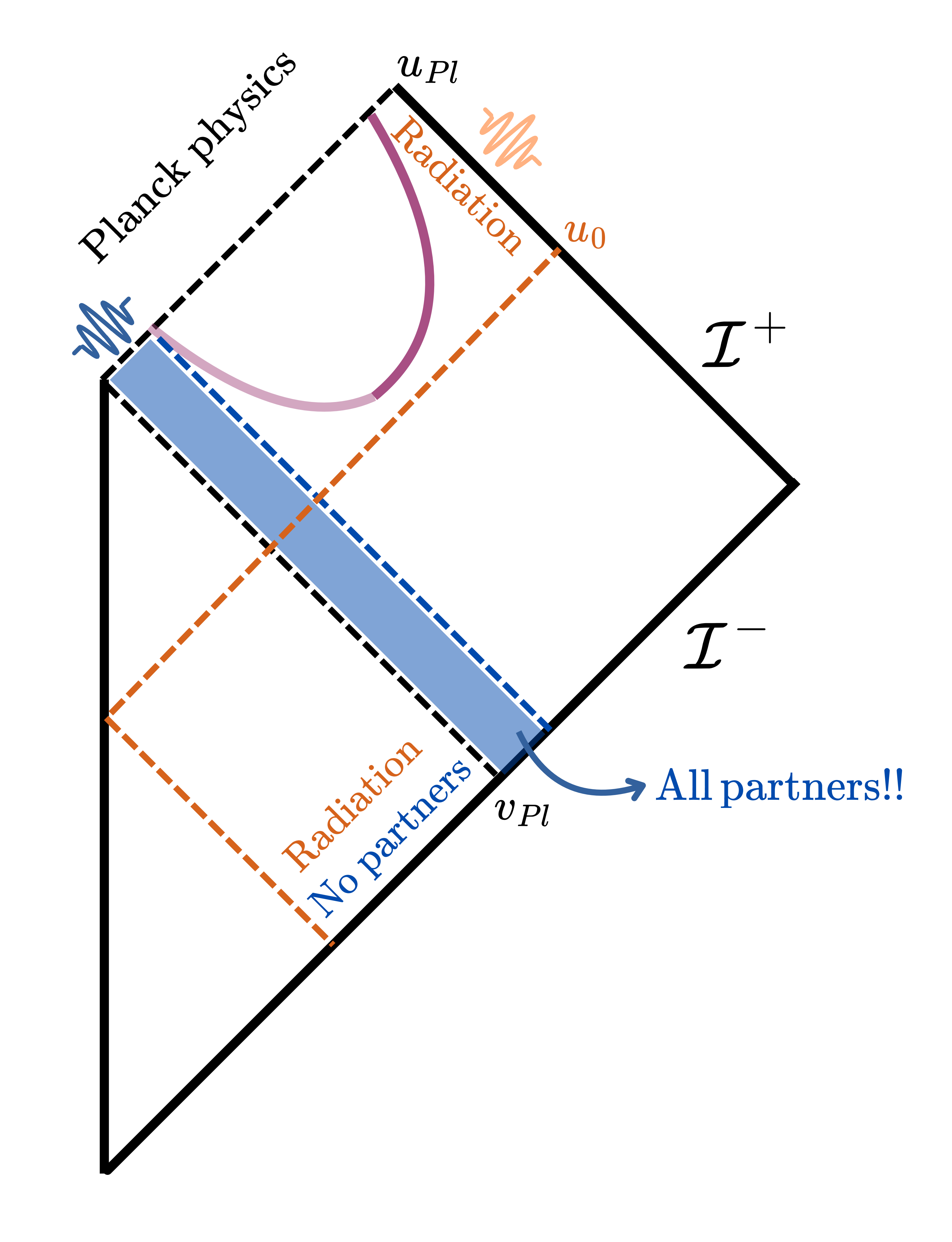
- Result is robust under allowed generalizations of $-\ddot{p}(u)/\dot{p}(u) = \kappa(u)$.

Planck regime of spacetime
may be crucial!

$$0 < v_{\star}^{(p)} - p(u_{Pl}) \ll t_{Pl}$$

Partners leave $\mathscr{I}^-$ after the last ray
that explores low curvatures!

Realistic scenario



- Partners cannot leak out during evaporation.
- They must explore the Planck regime if:
  - ❋ Ray $v = p(u_{Pl})$ traverses a trapped region.
  - ❋ Standard GR holds in collapsing region.
  - ❋ Redshift for outgoing light in collapsing region.
- Consistent with dynamical horizon pictures.

Conclusions

- General & conservative QFT study.
- Recipe for partners in evap. BHs.
- Info cannot escape semiclassically.
- All partners enter Planck regime.

Quantum gravity to the rescue?!

Source: Kurzgesagt