Quantum Geometry of the Light Cone

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FAU Erlangen-Nuremberg

[ww, arXiv:2402.12578]
[ww, arXiv:2401.17491]
[ww, JHEP 2021, arXiv:2104.05803]
[ww, Class. Quant. Grav 34 2017, arXiv:1704.07391]
[ww, Ann. Henri Poincaré 18 (2017), arXiv:1706.00479]

A simple Observation

Planck luminosity

In $D = 4$ spacetime dimensions, the Planck power is independent of $\hbar$.

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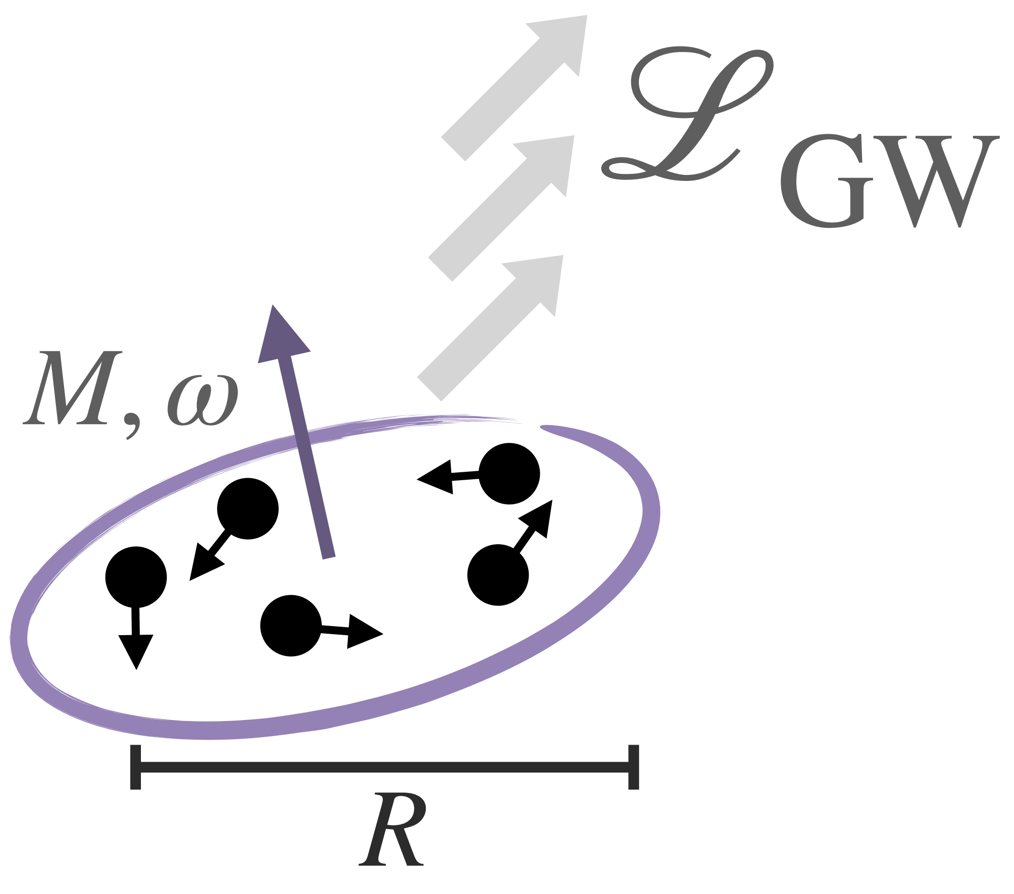

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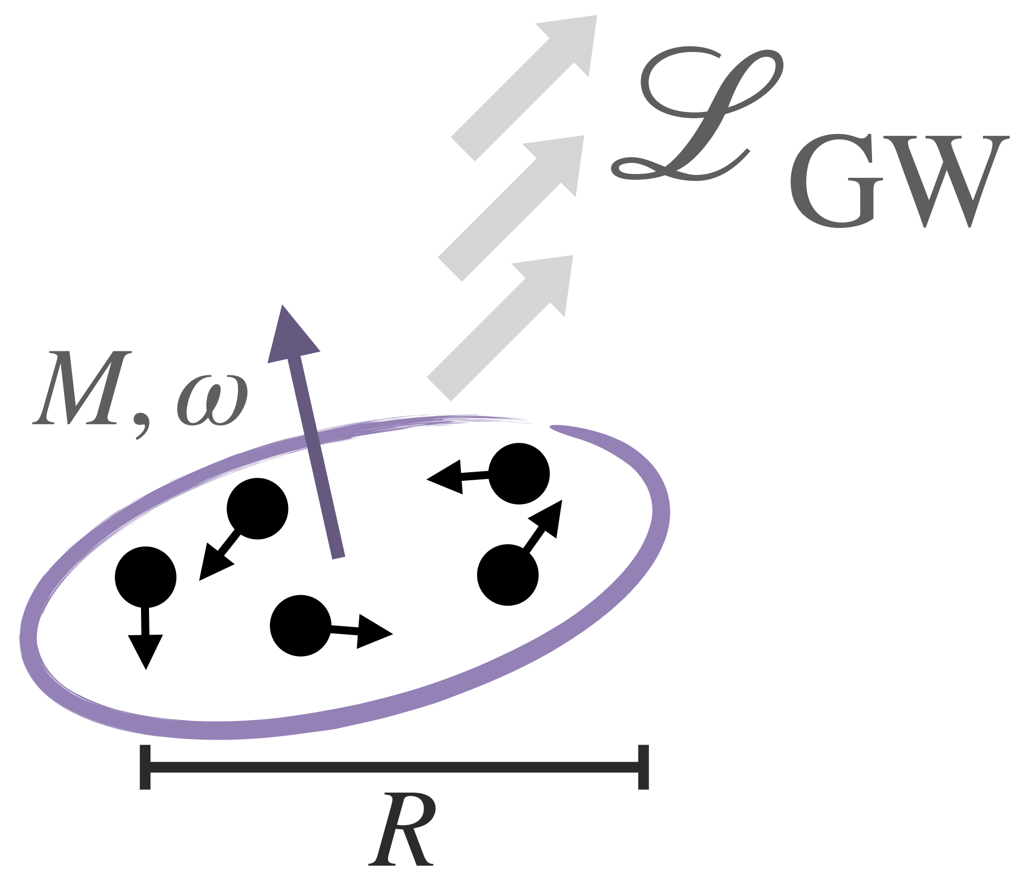

[Misner, Thorne Wheeler]

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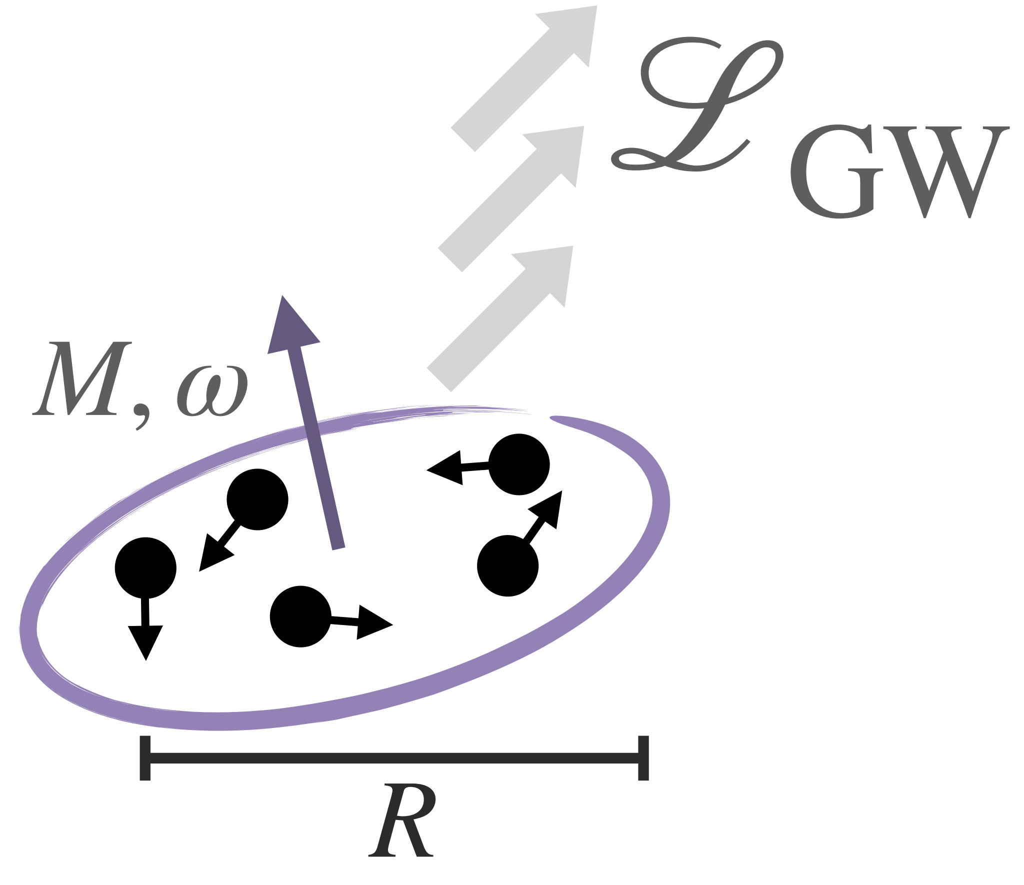

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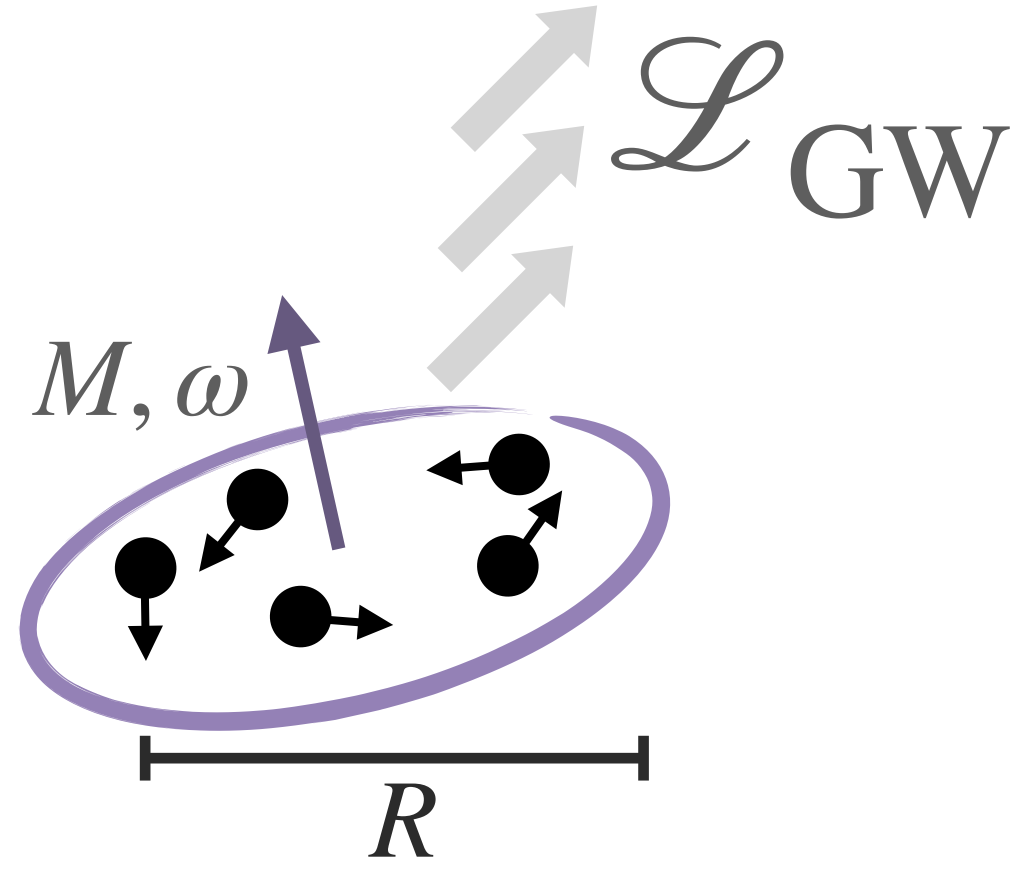

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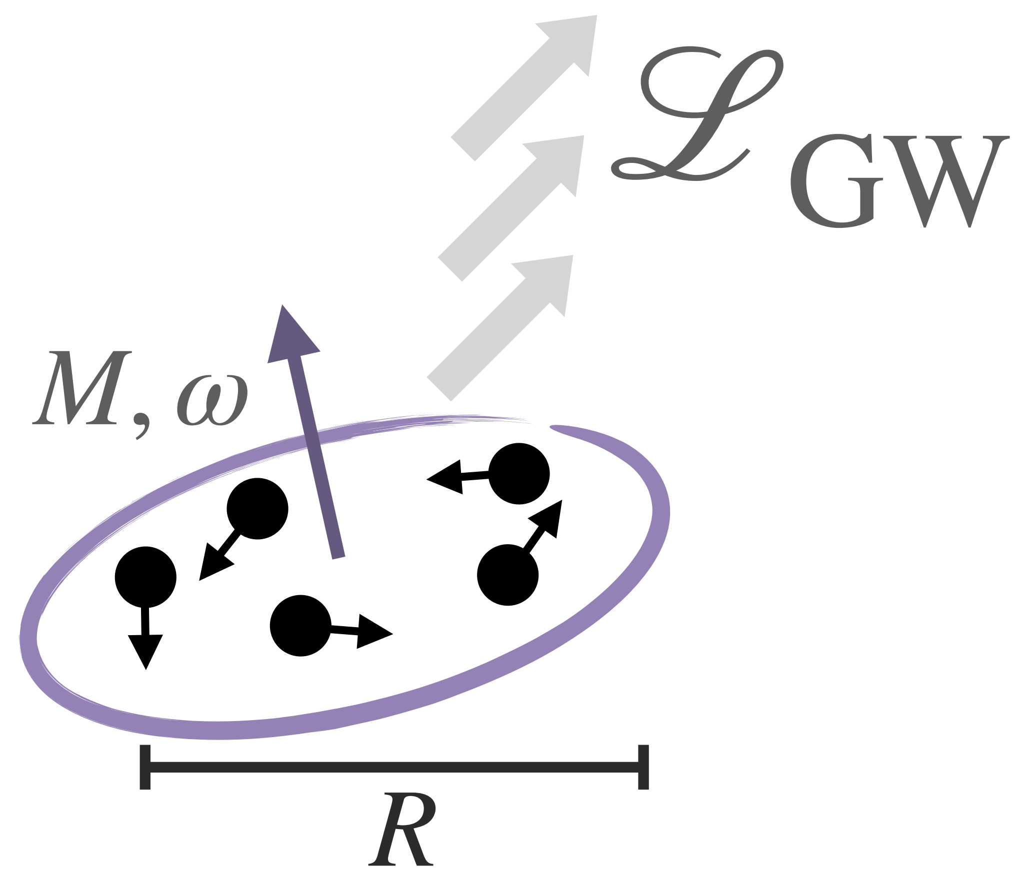

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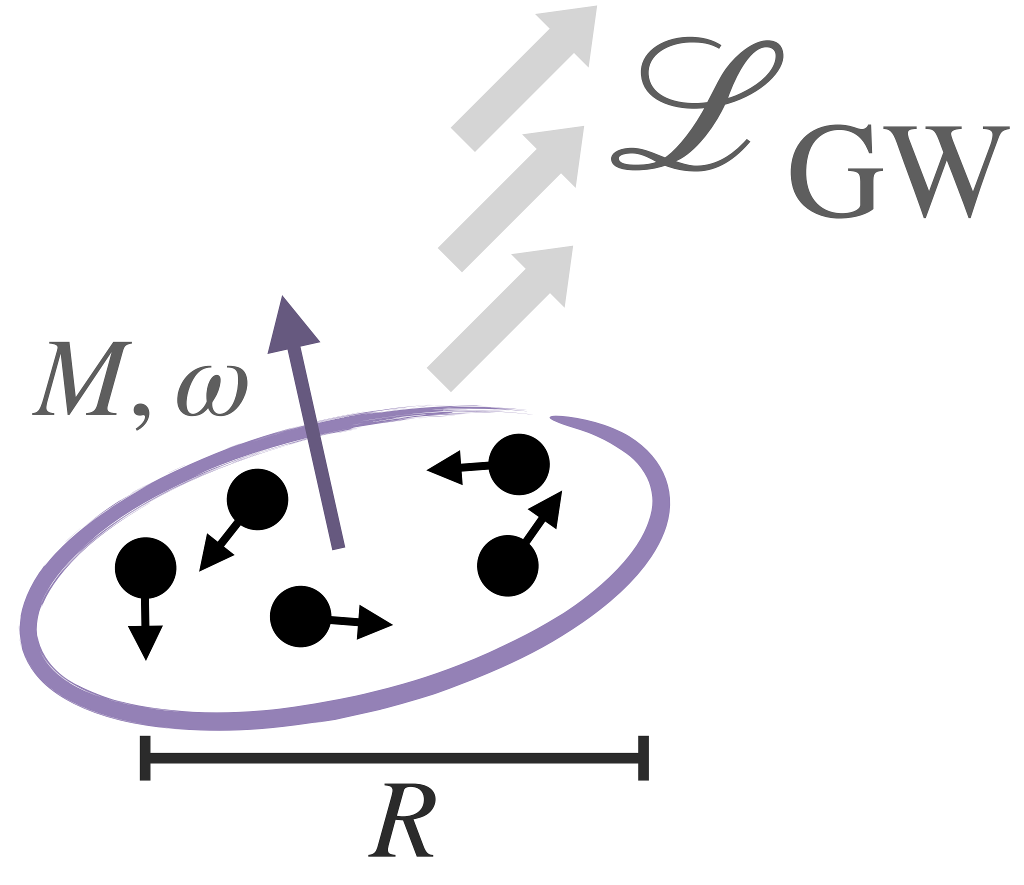

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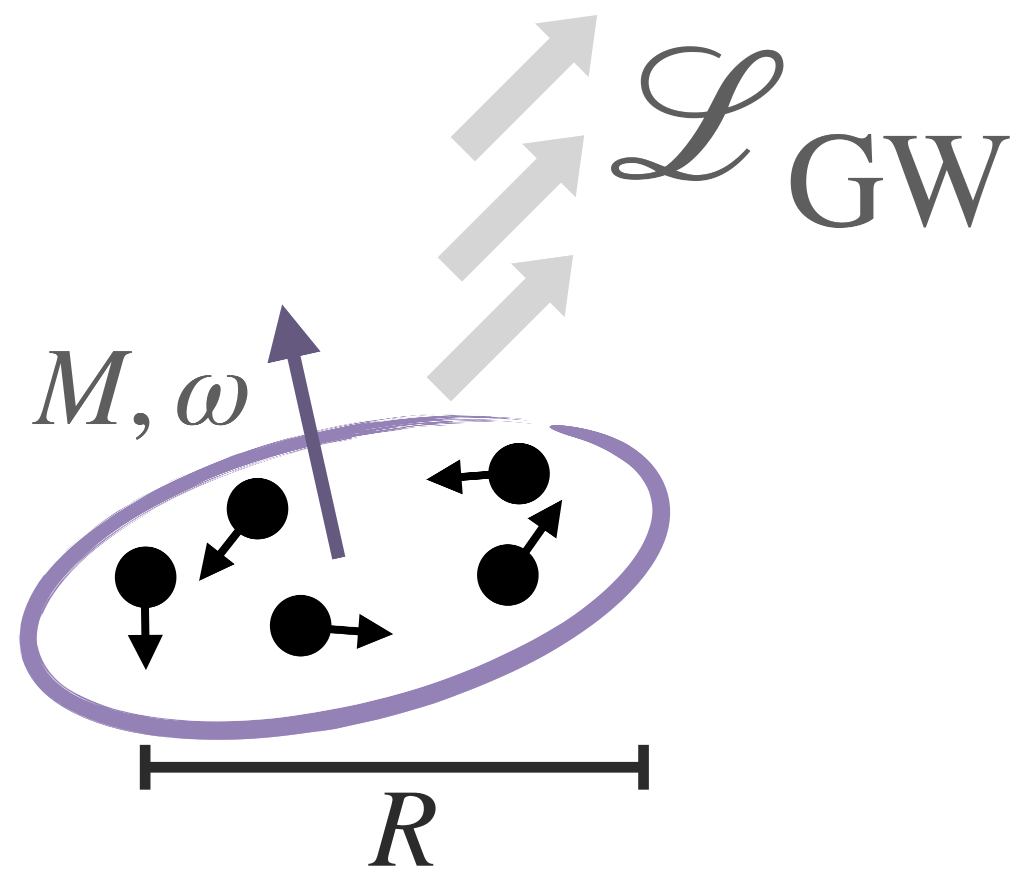

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[Freidel, Livine, Girelli, Smolin, Kowalski-Glikmann, Amelino-Camelia, ..., Corichi, Ashtekar, Varadarajan, ...]

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This talk: Quantum gravity could place a bound on gravitational wave luminosity. Conjecture based on new results.

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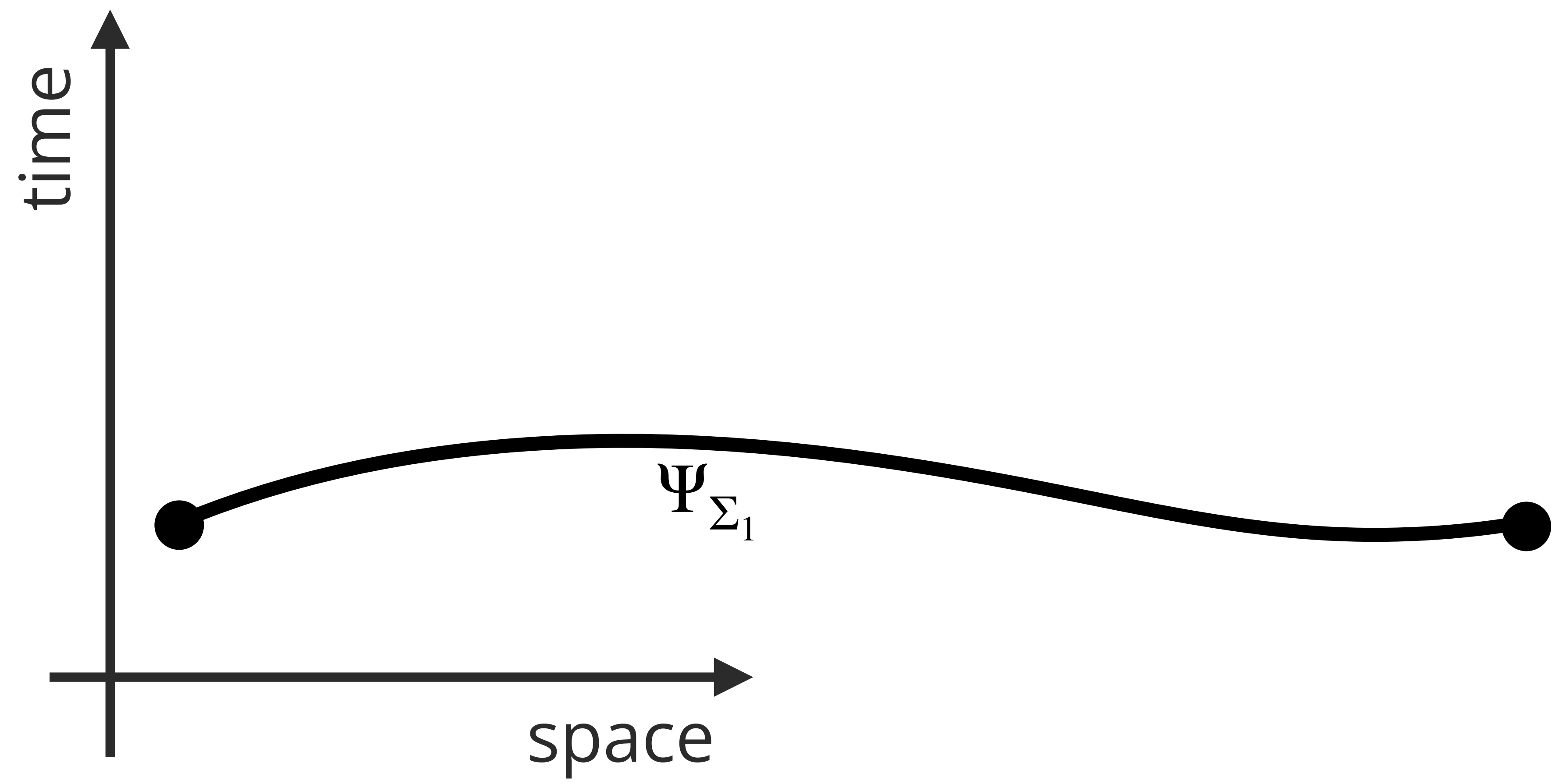
$$\mathscr{L}_{Bondi} = \frac{c^5}{4\pi G} \oint_{S_2} d^2\Omega \, |\dot{\sigma}^{(0)}|^2$$

free data

How to explore such a bound

Register radiation at null boundary



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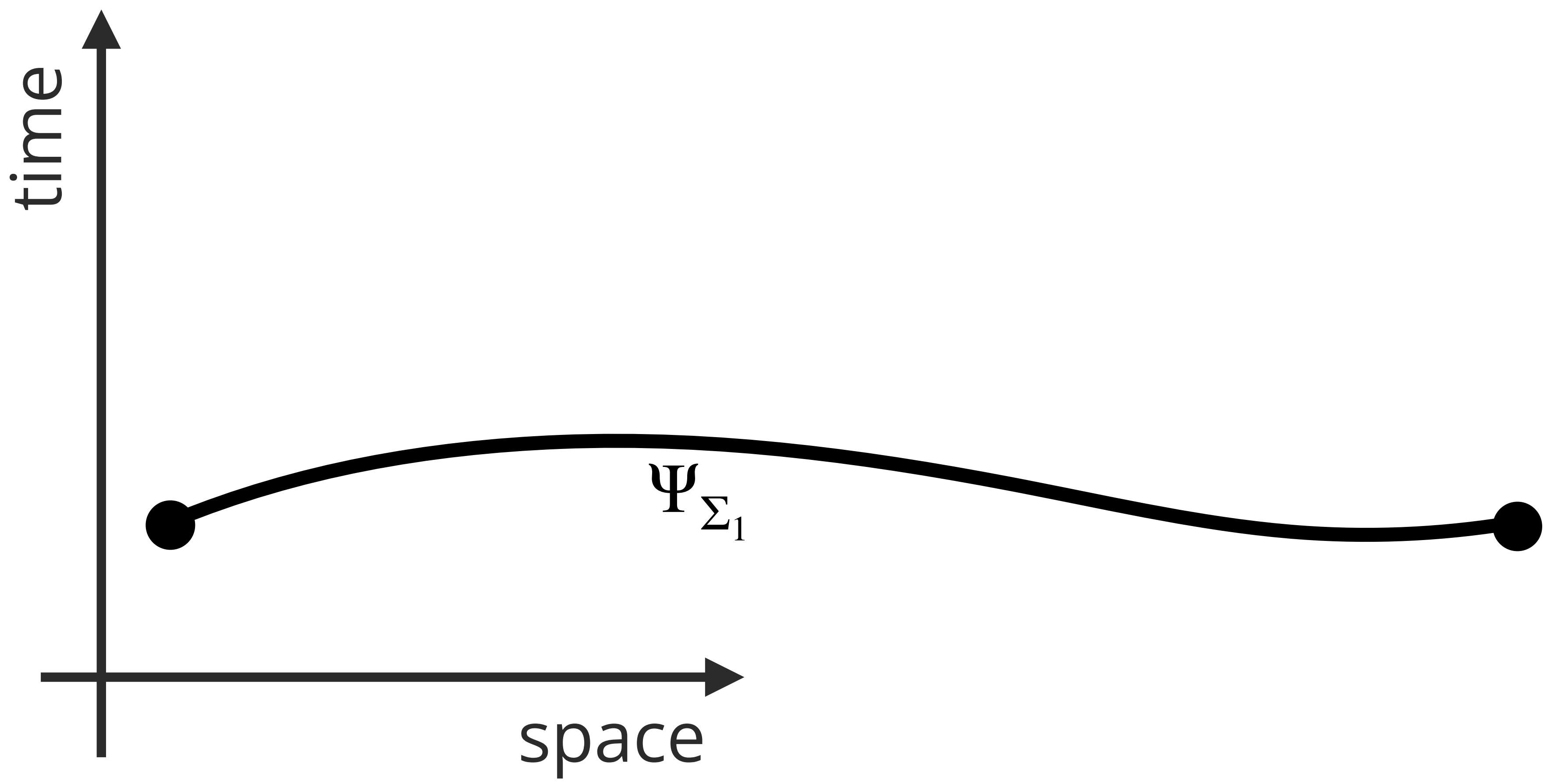
Why null boundaries?



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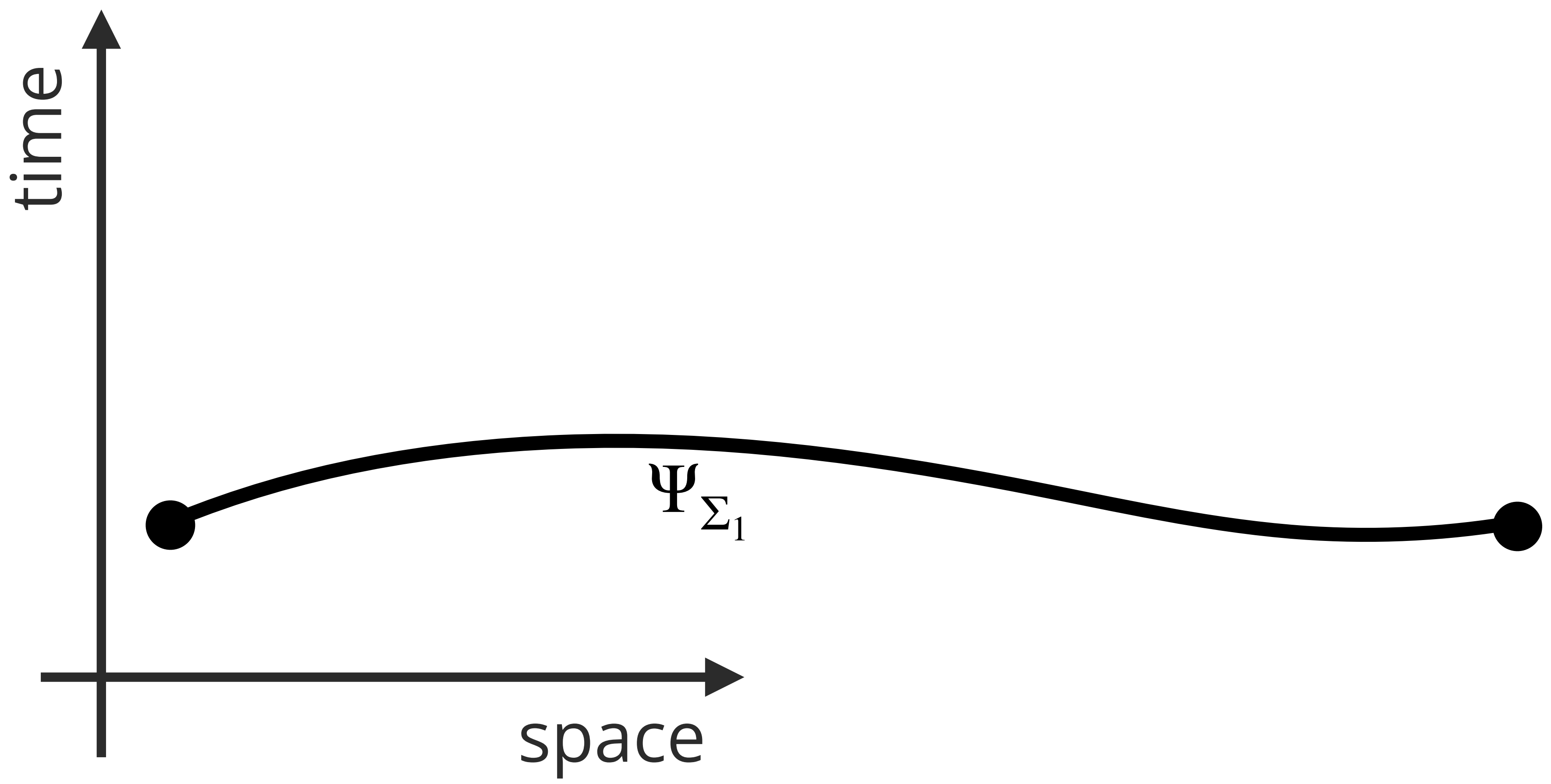
- Take ADM initial data. Constraints $\mathscr{H}_a = D_b(K^{ab} - h^{ab}K)$, $\mathscr{H} = K_{ab}K^{ab} - K^2 - {}^{(3)}R[h]$ generate gauge redundancies.



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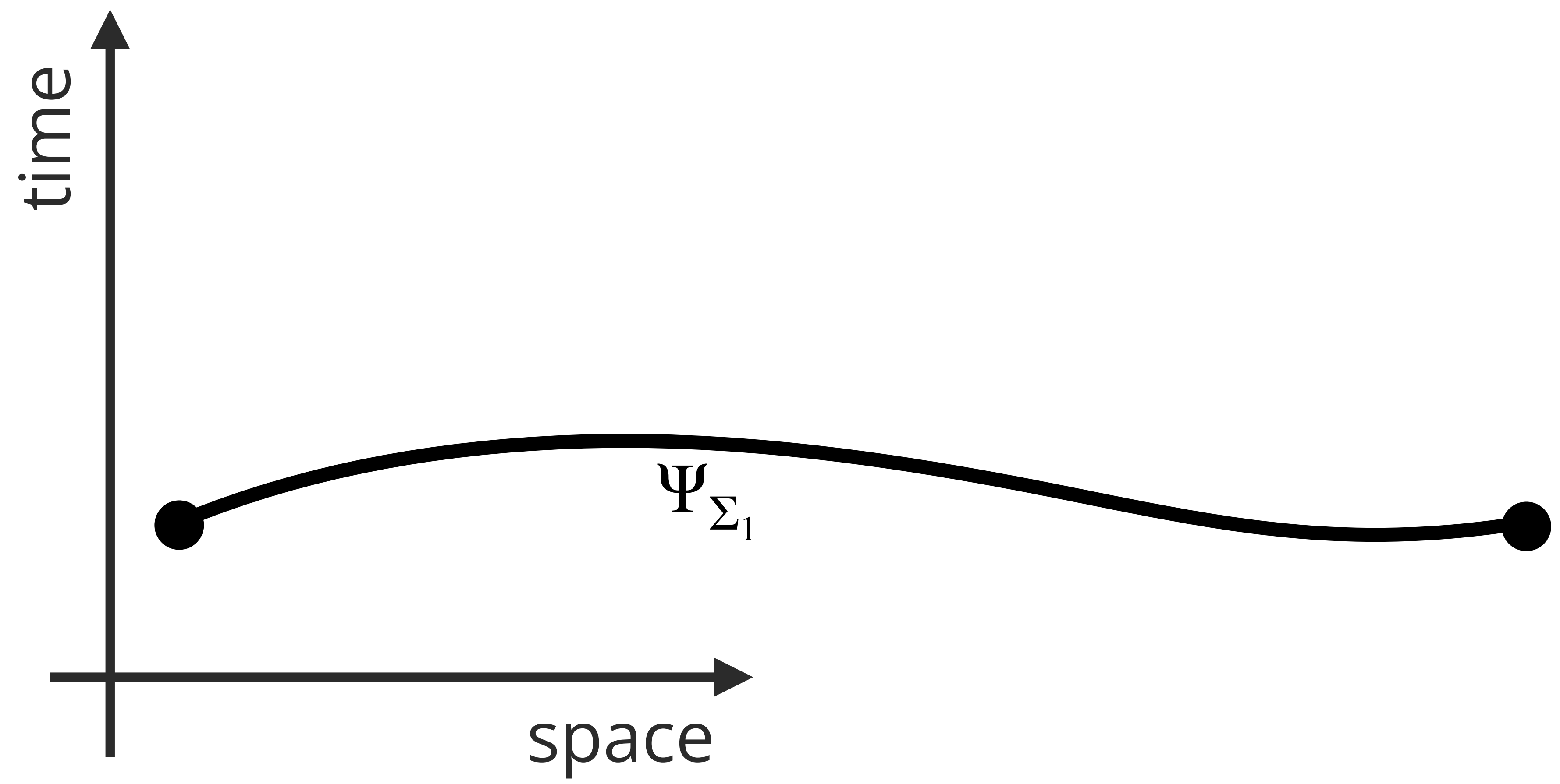

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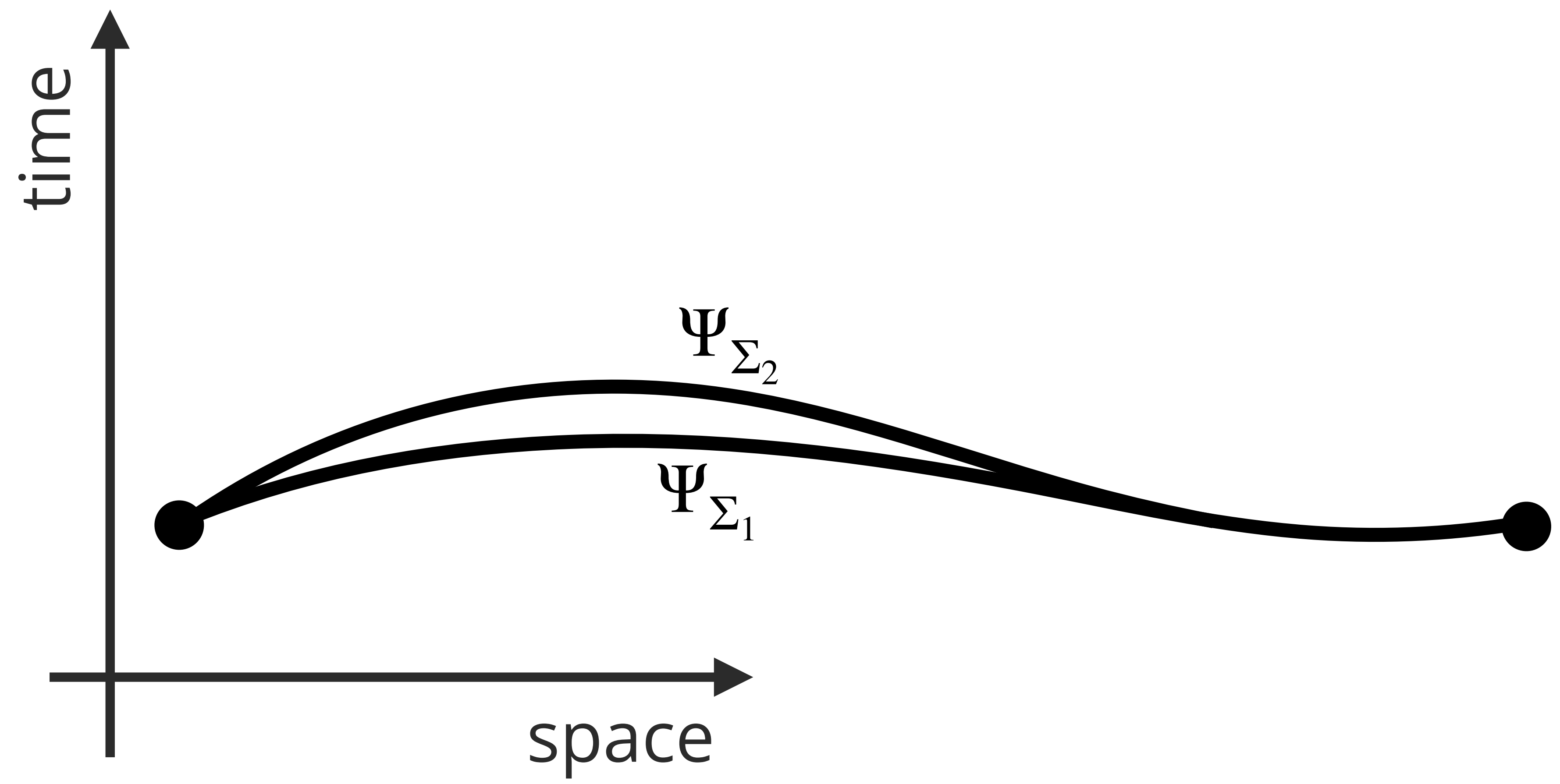

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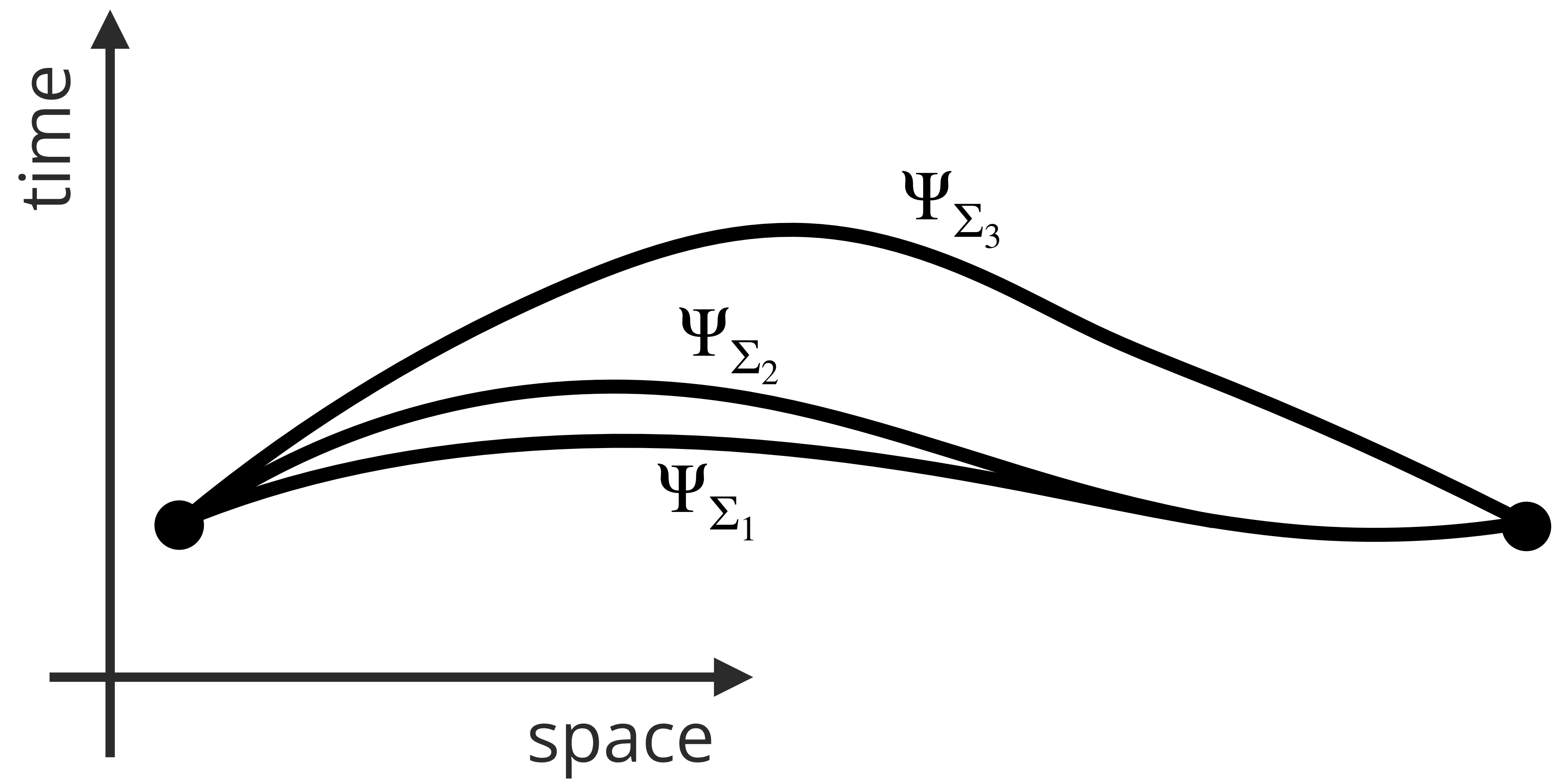

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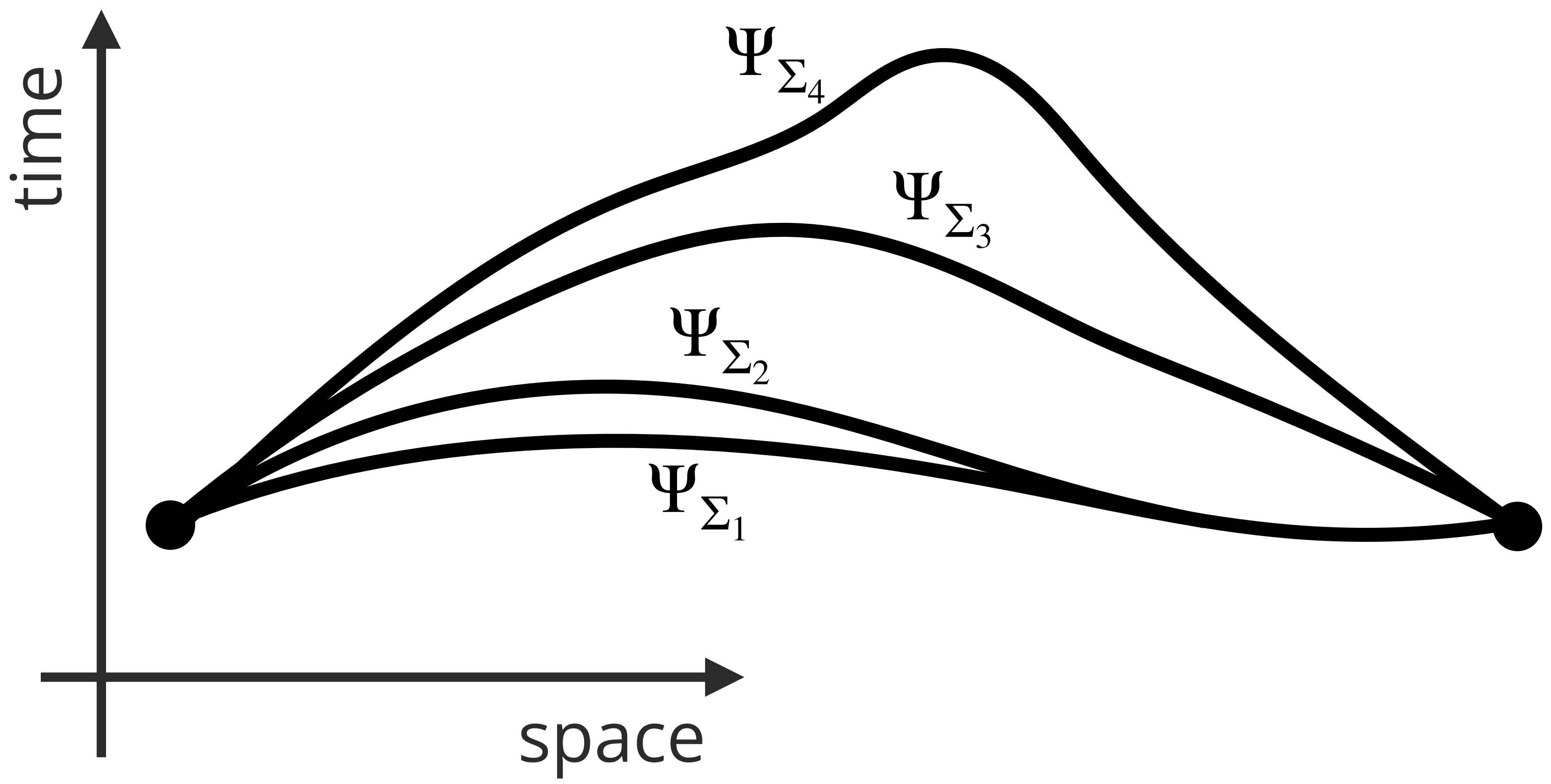

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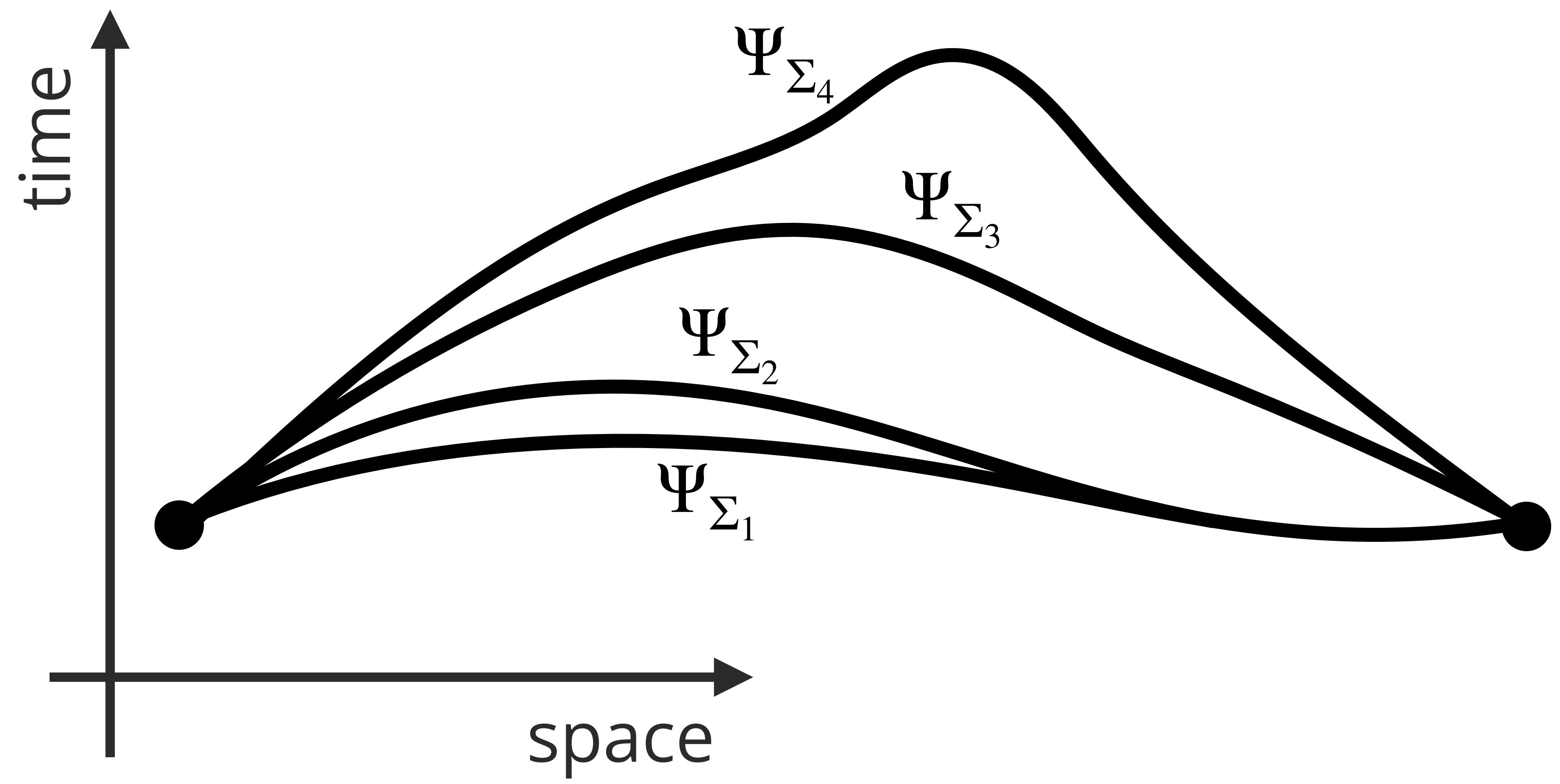
  - Push gauge evolution to its extreme.



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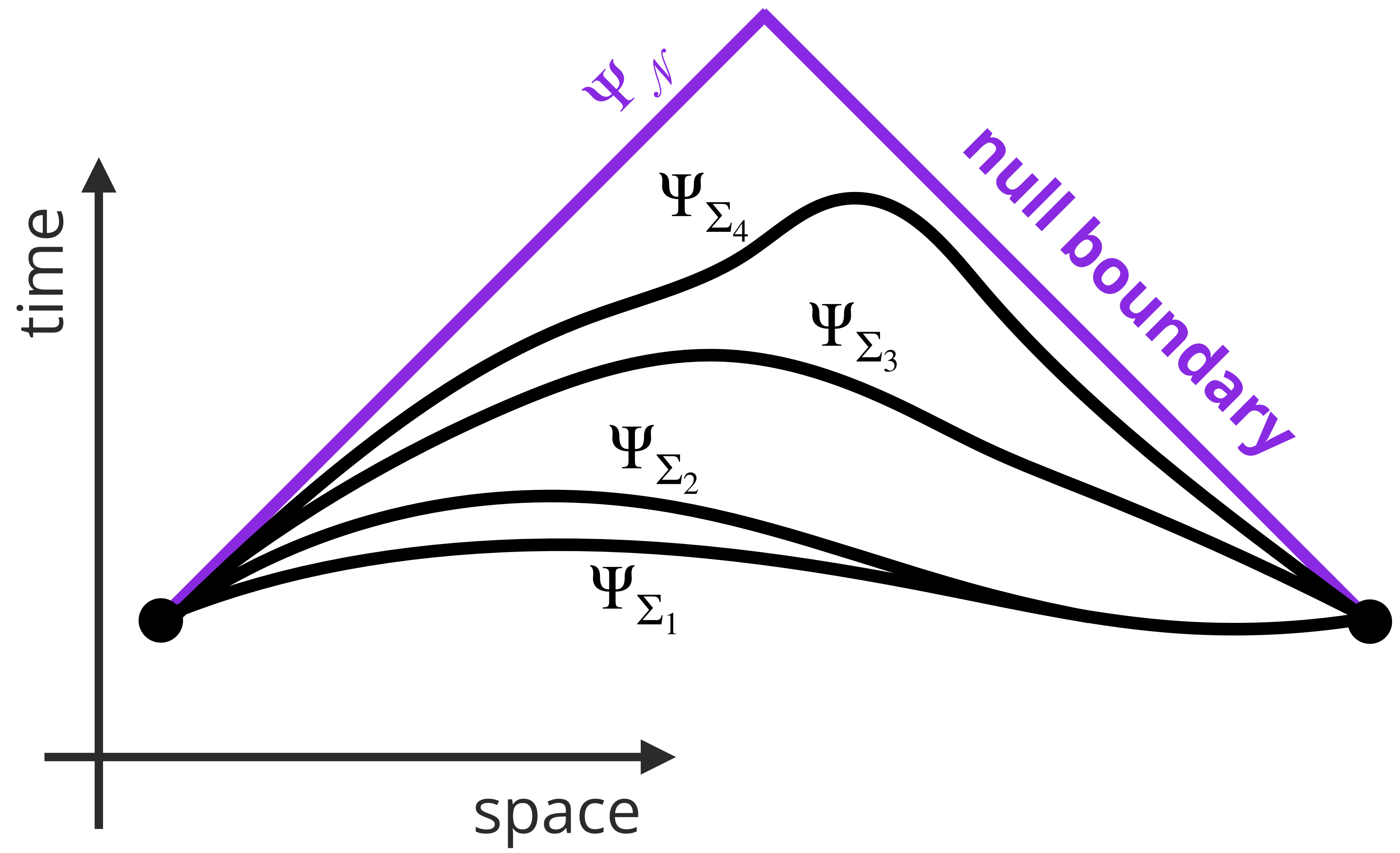

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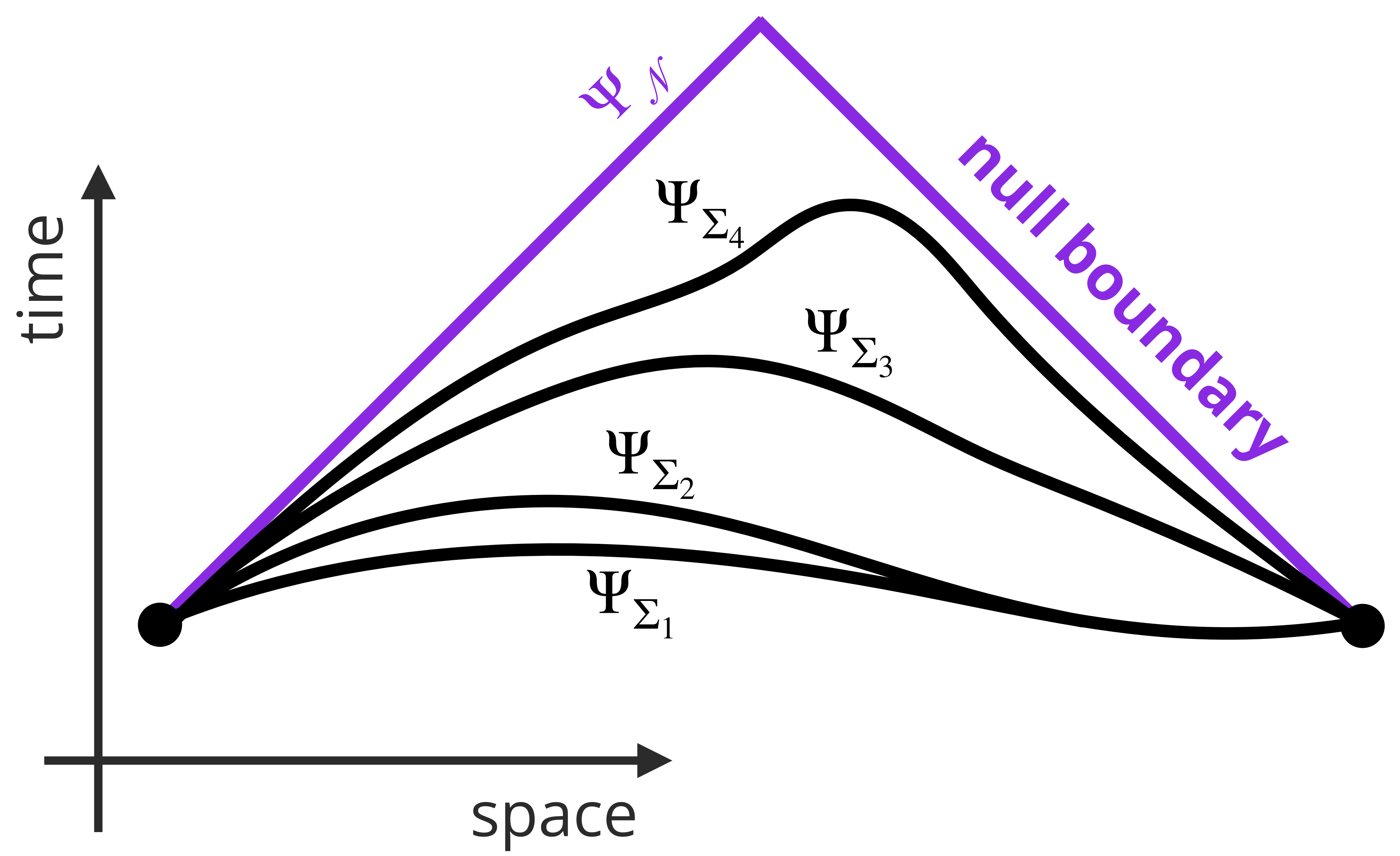

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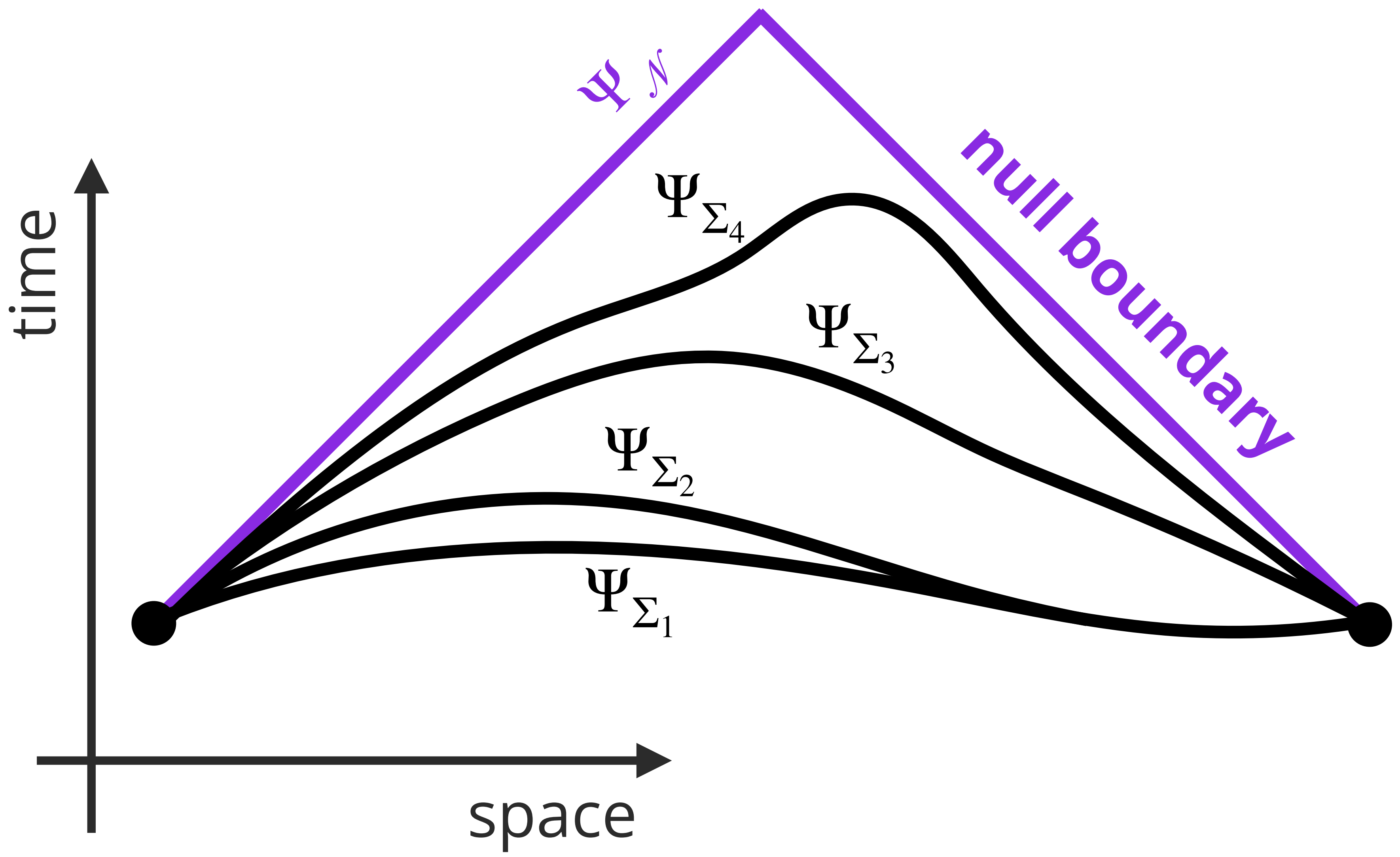

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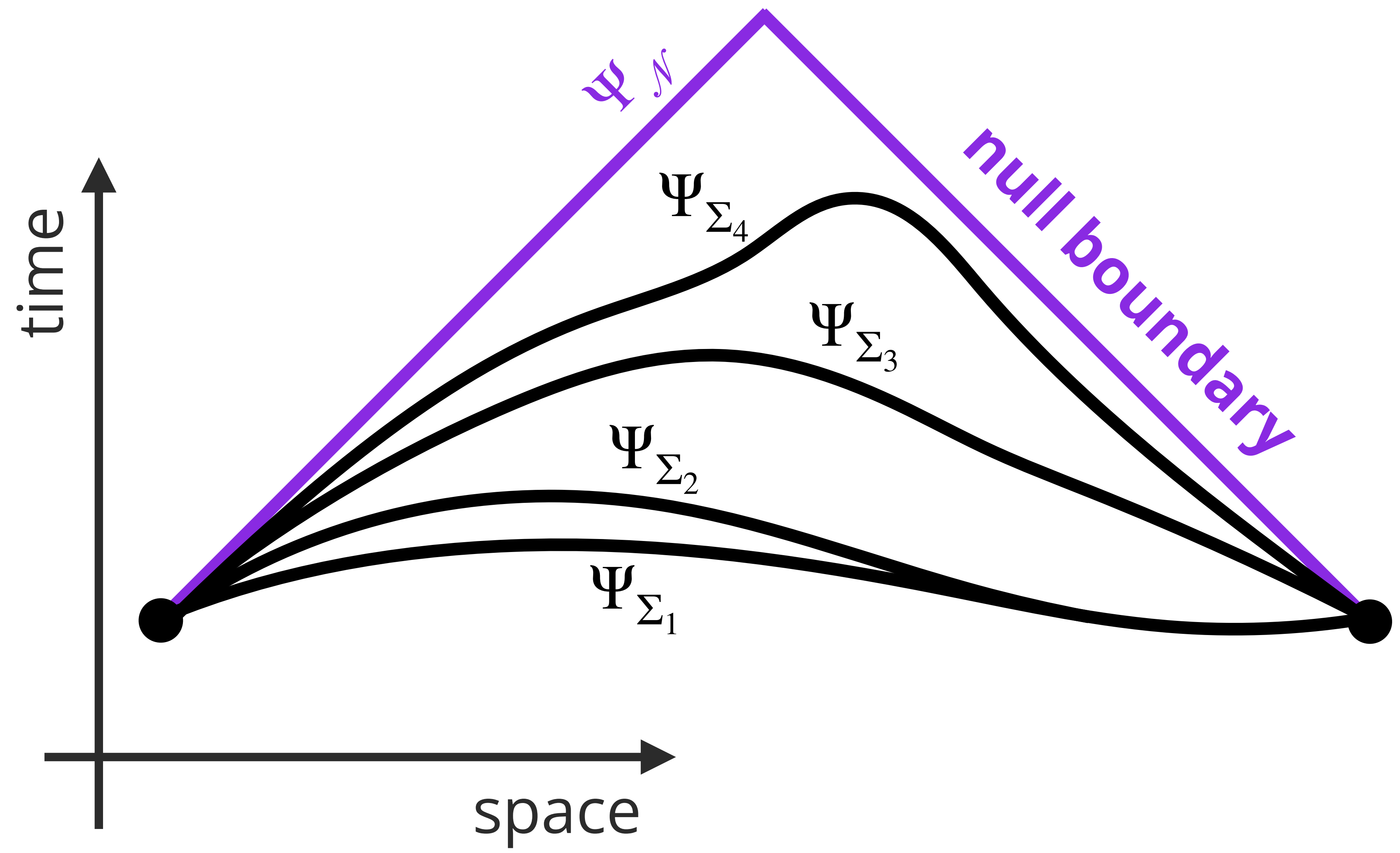

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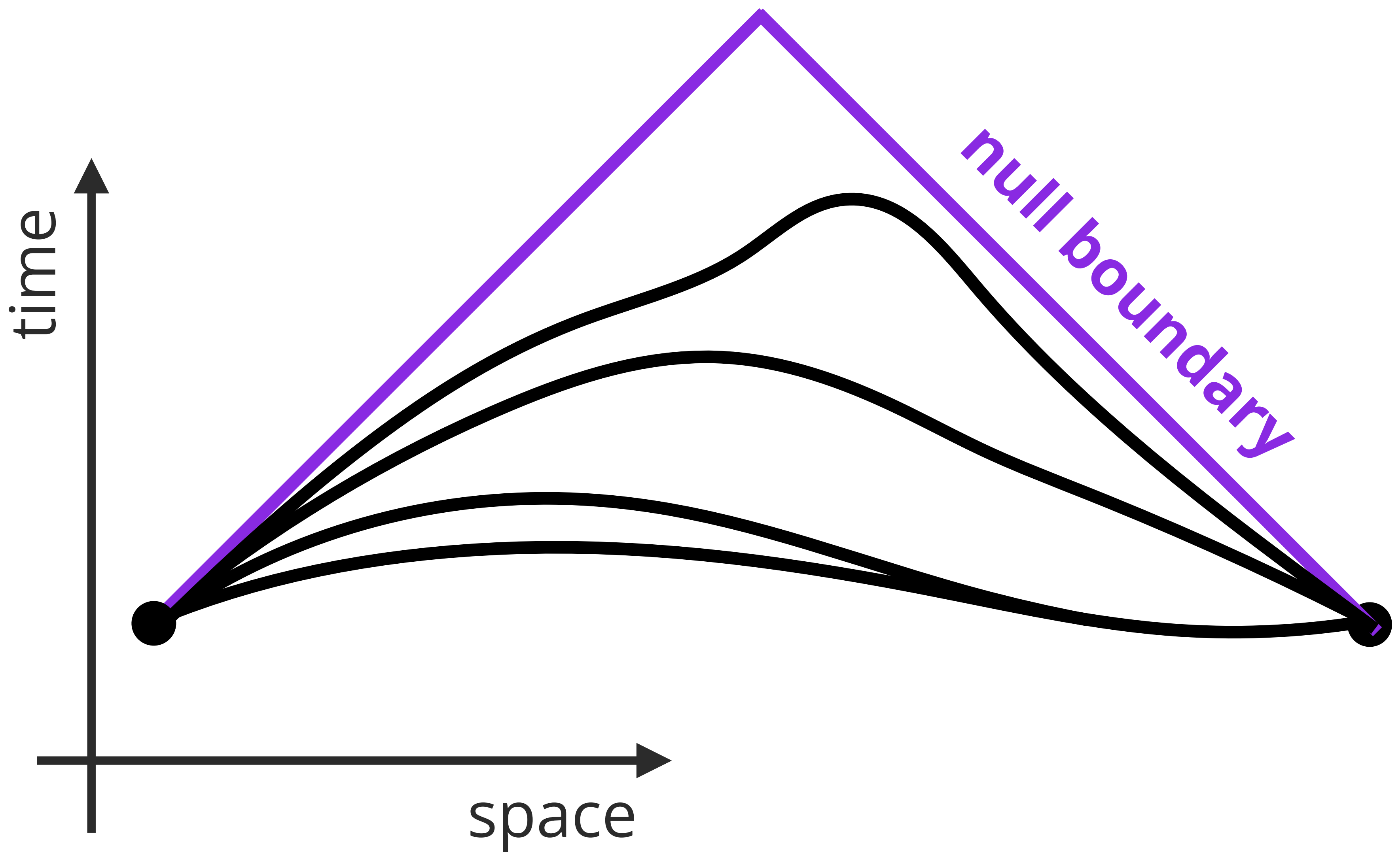
- Register radiation at null surface boundary.

Quantum gravity in finite regions

- Register radiative modes at null boundary



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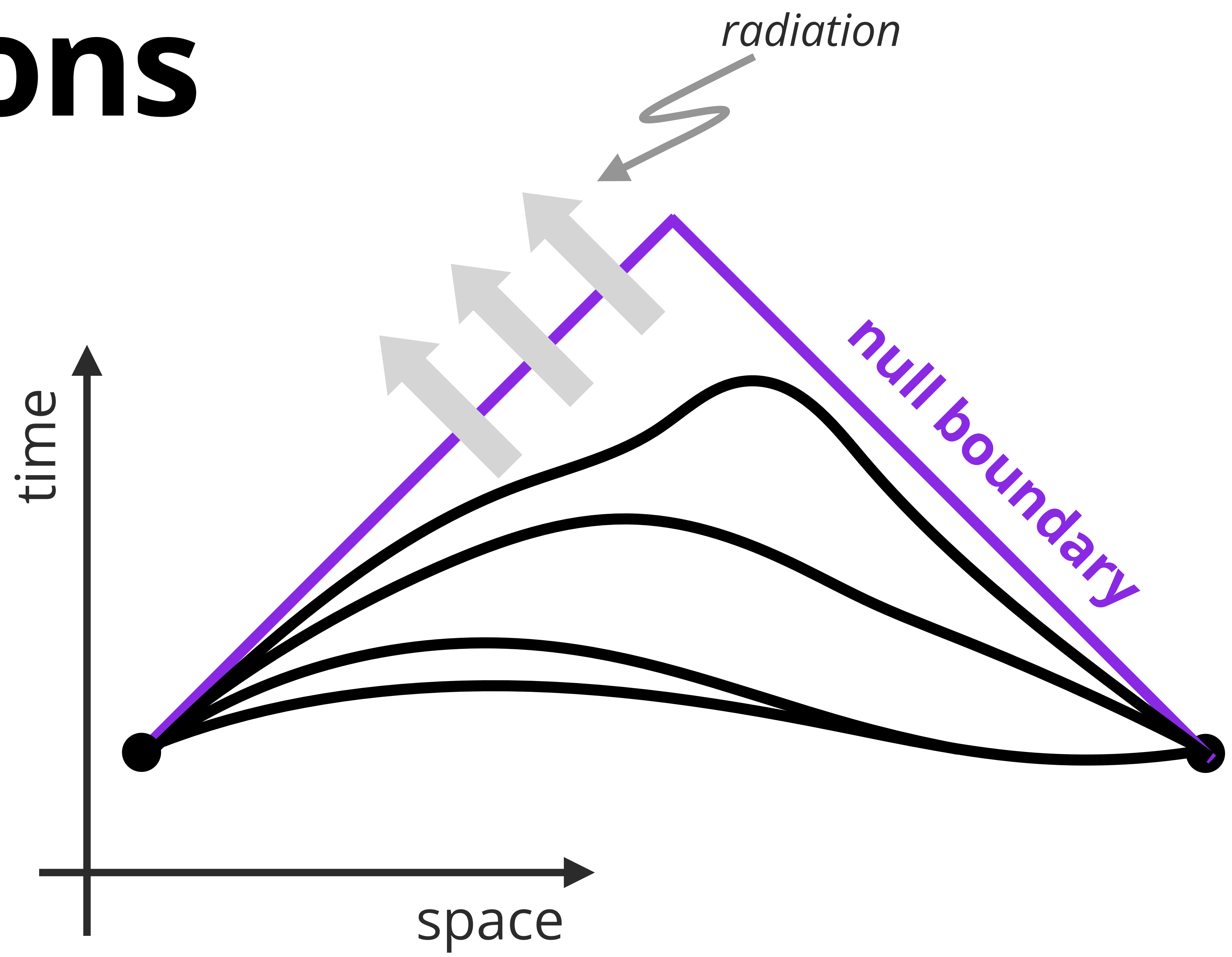

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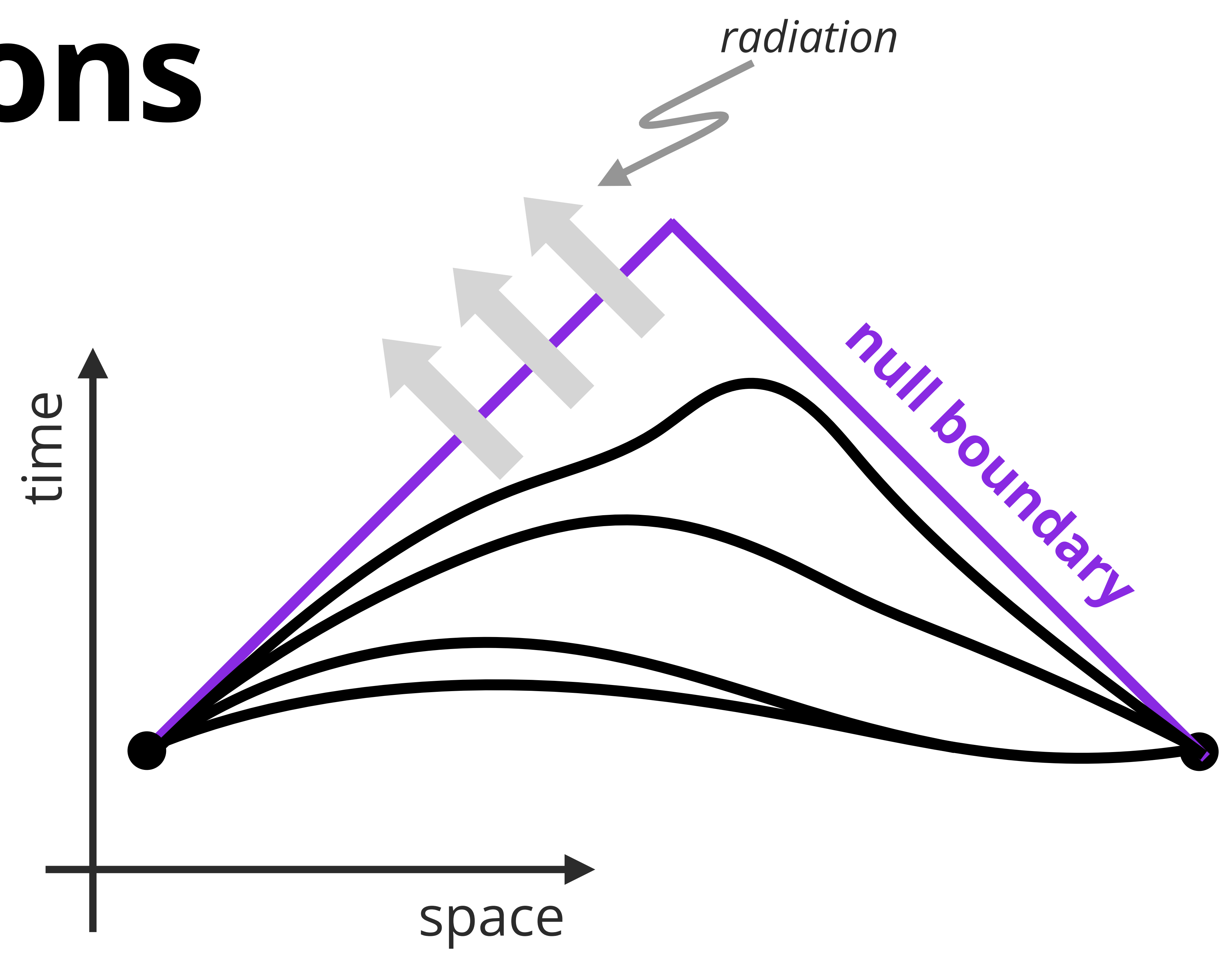
- Besides radiation, we have corner data



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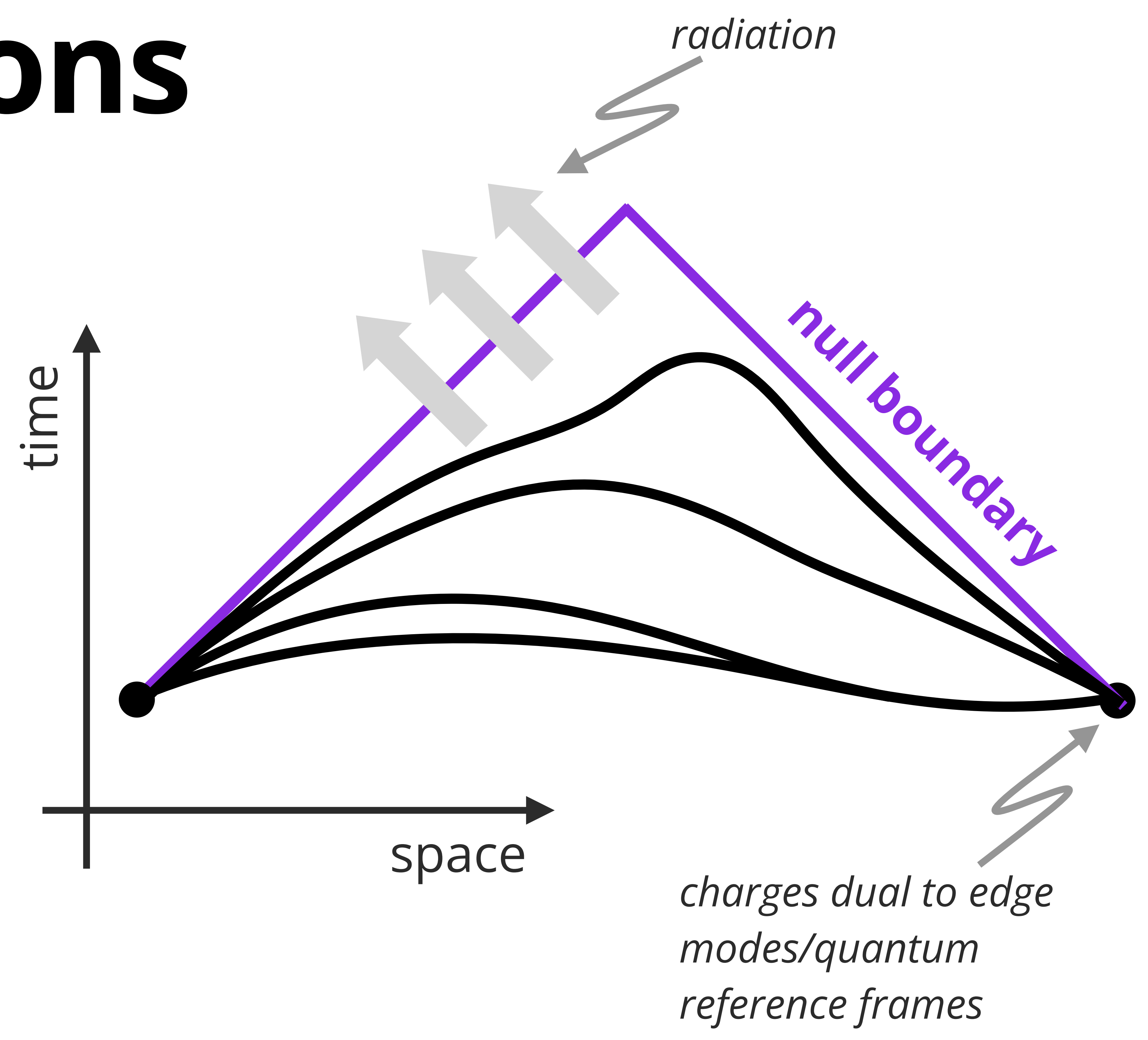

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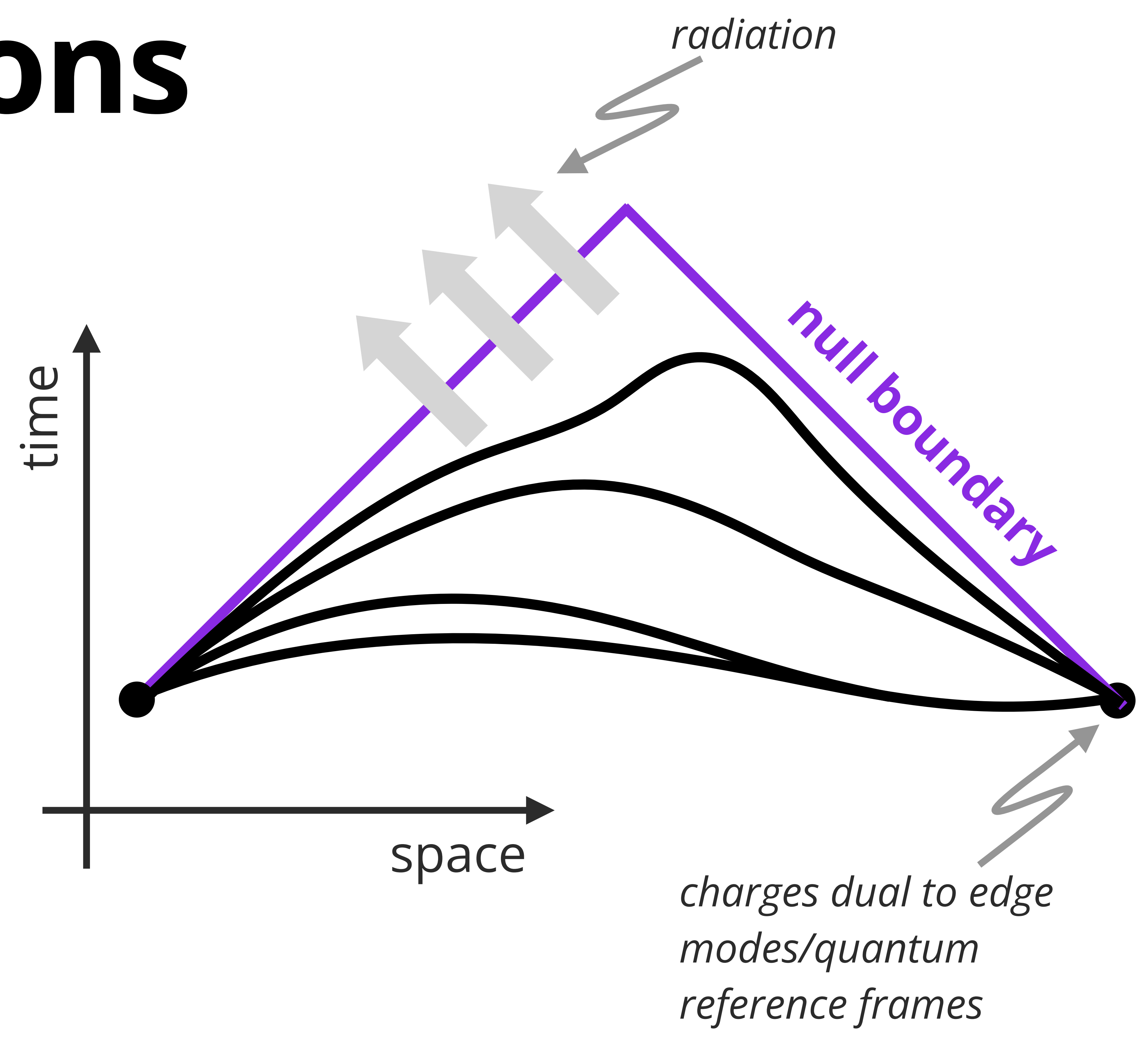

[Sachs, Ashtekar, Lewandowski, Freidel, Ciambelli, Leigh, Reisenberger, Geiller, Pranzetti, Chandrasekaran, Flanaghan, Prabhu, Oliveri, Pranzetti, Speziale, ww, ...]

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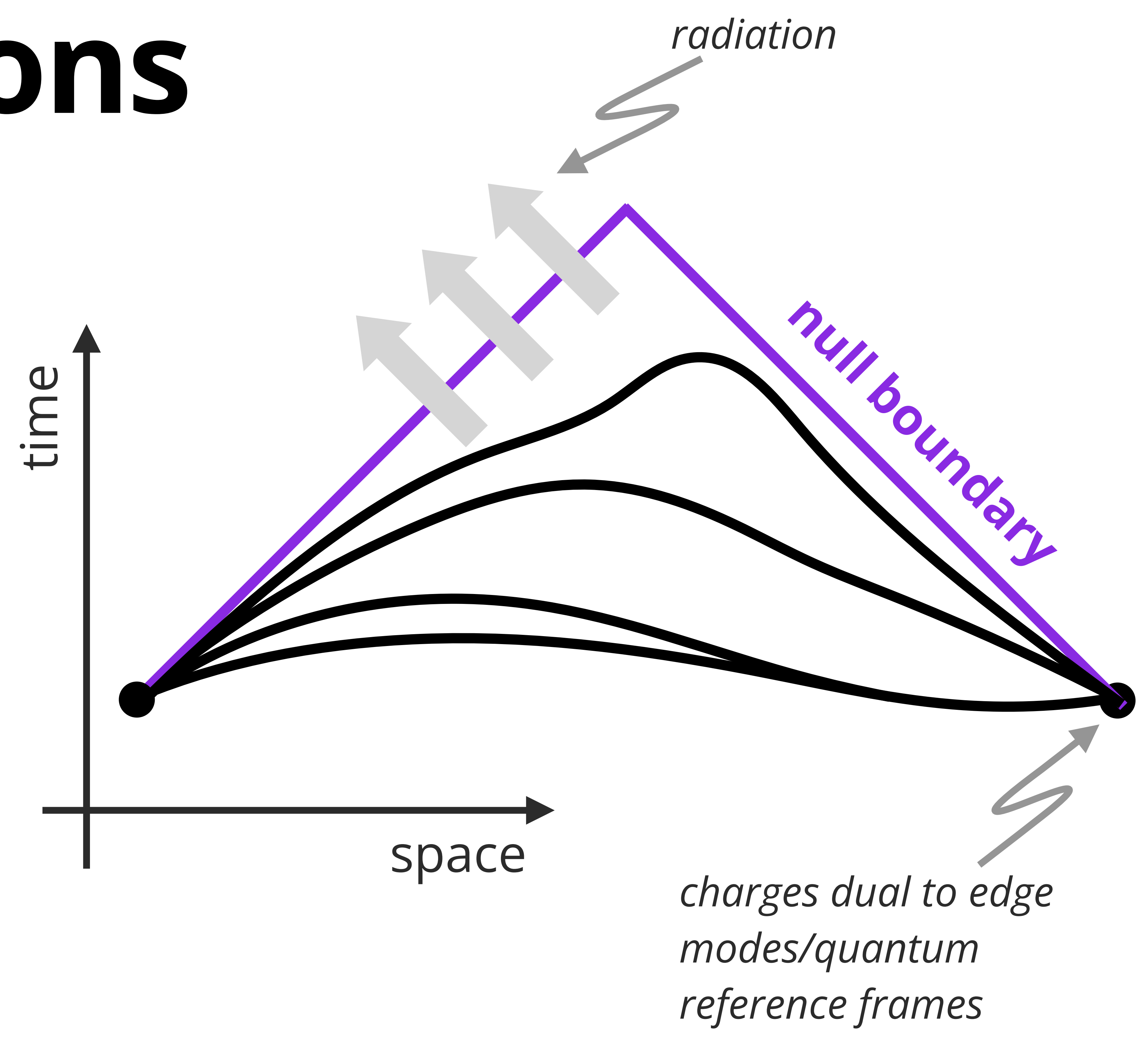

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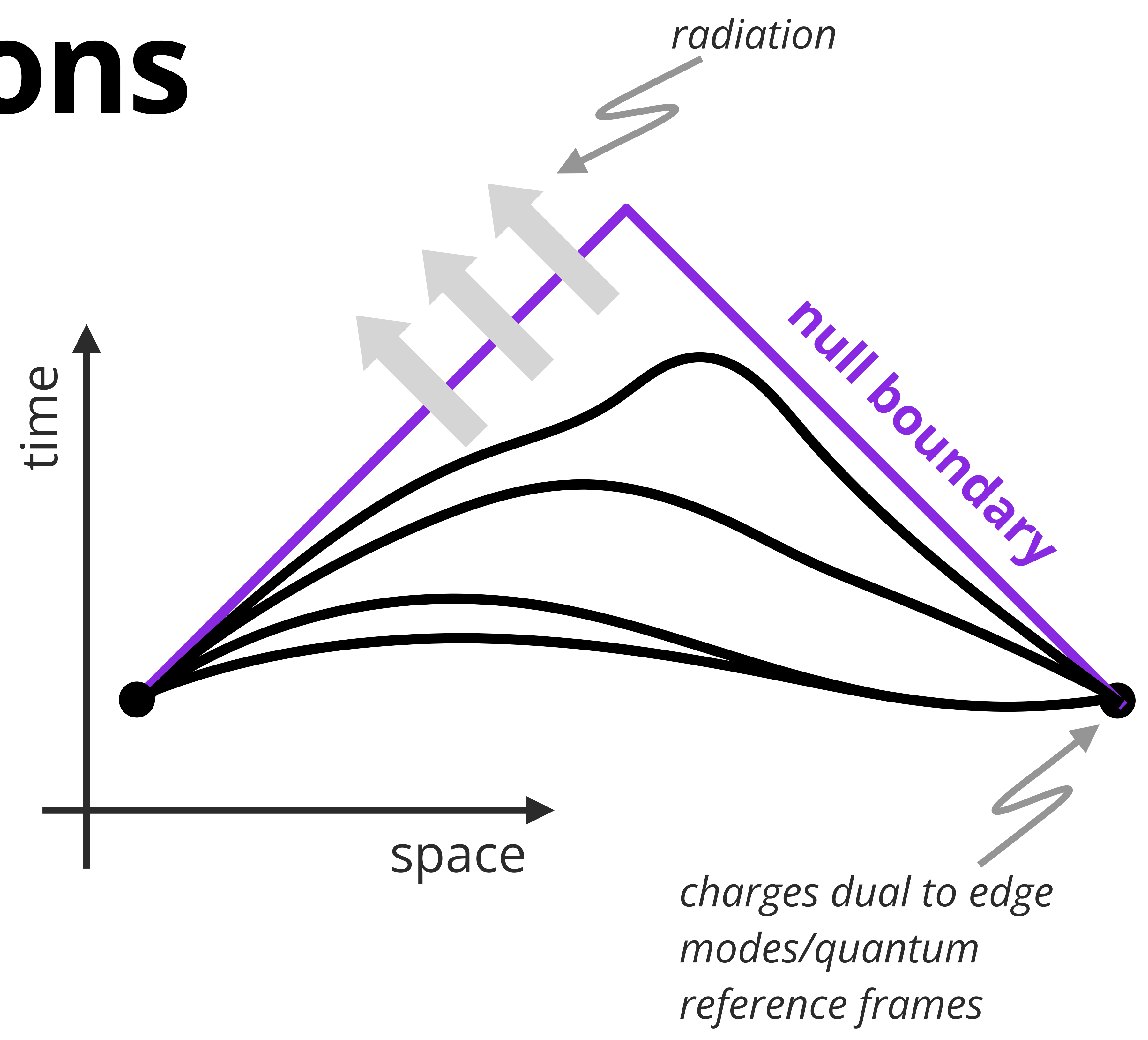

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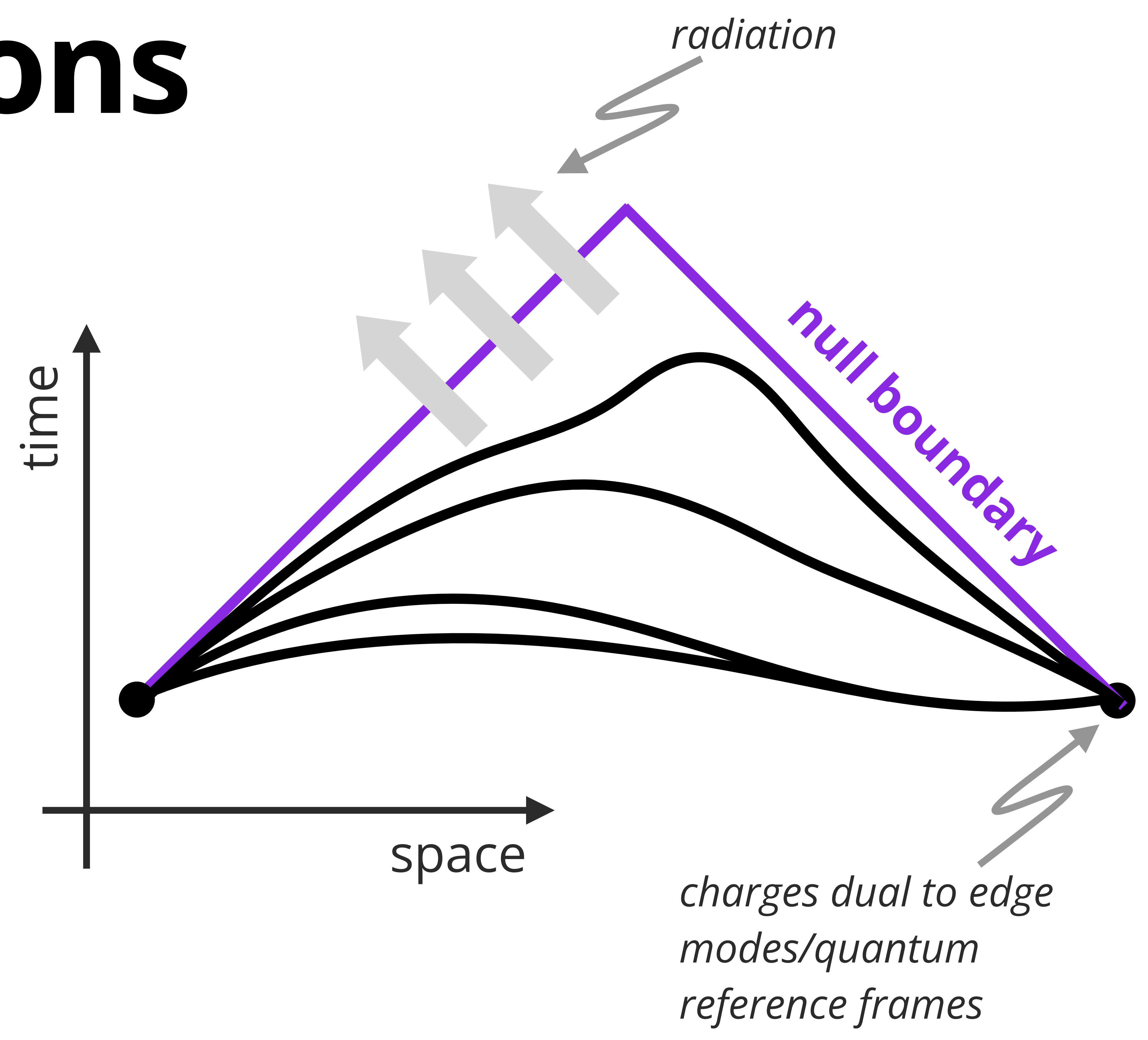

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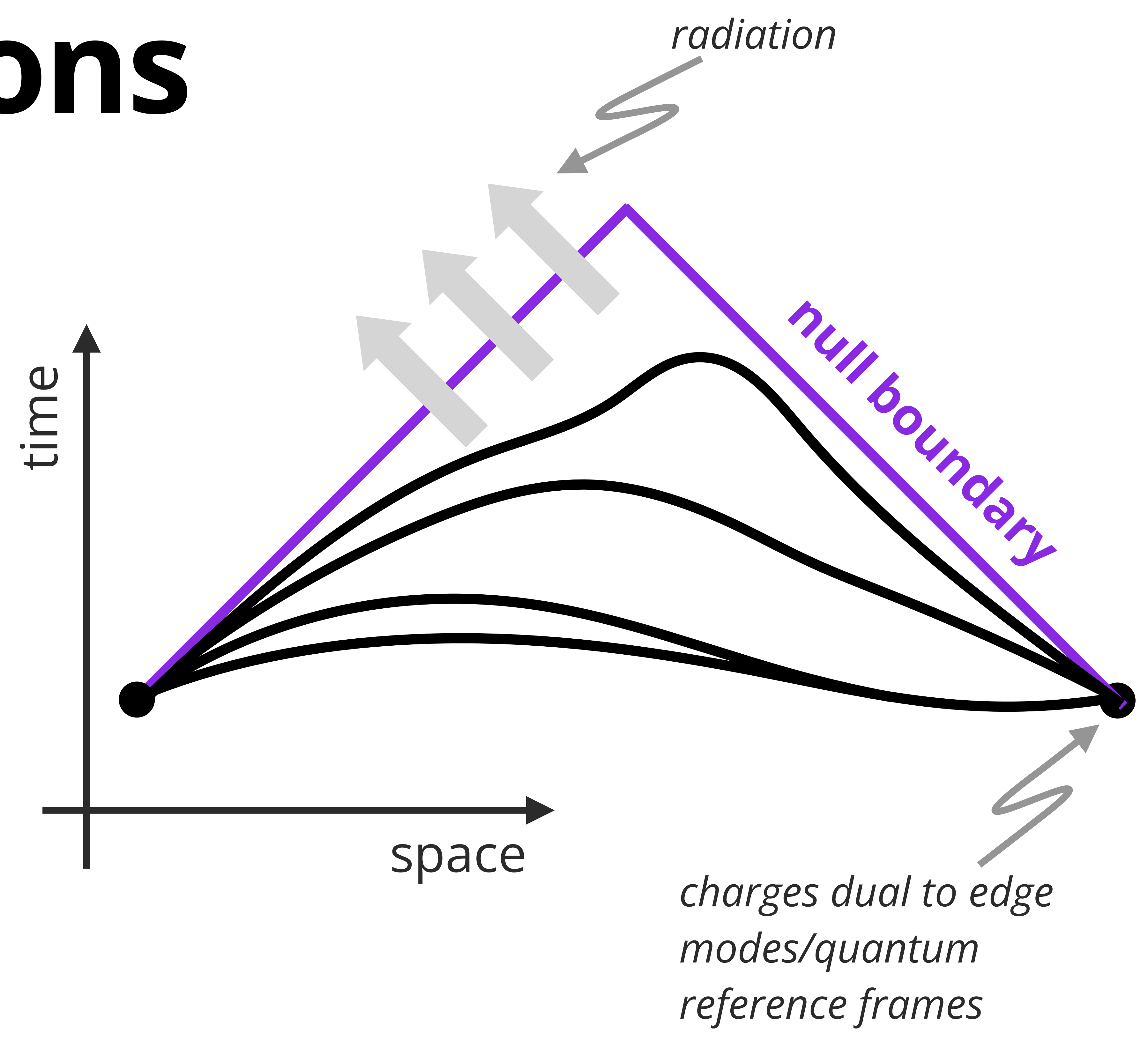
  - Truncation + quantisation

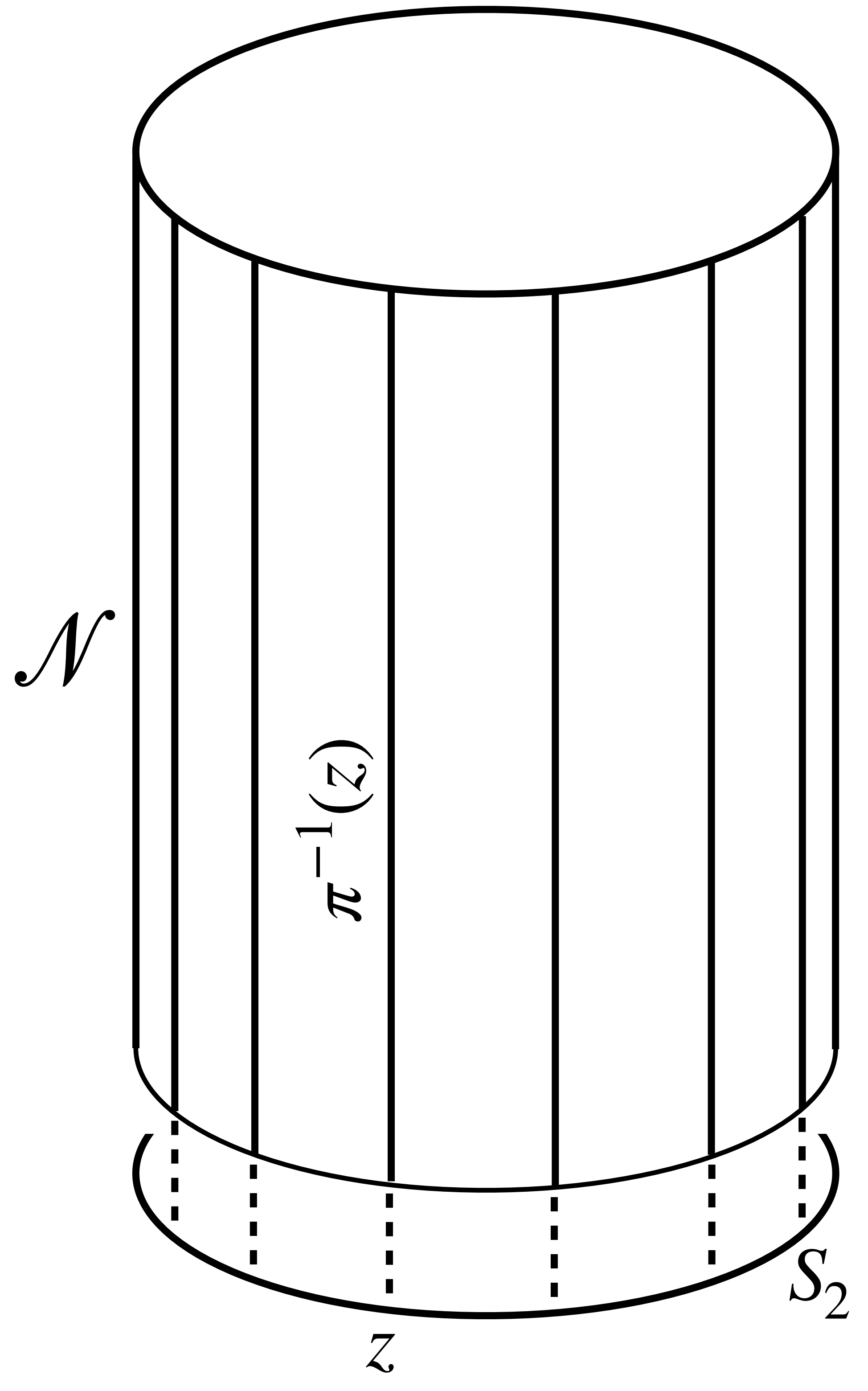


[Sachs, Ashtekar, Lewandowski, Freidel, Ciambelli, Leigh, Reisenberger, Geiller, Pranzetti, Chandrasekaran, Flanaghan, Prabhu, Oliveri, Pranzetti, Speziale, ww, ...]

Step 1: Null surface geometry

Null surface geometry

- Signature $(0++)$ metric



Null surface geometry

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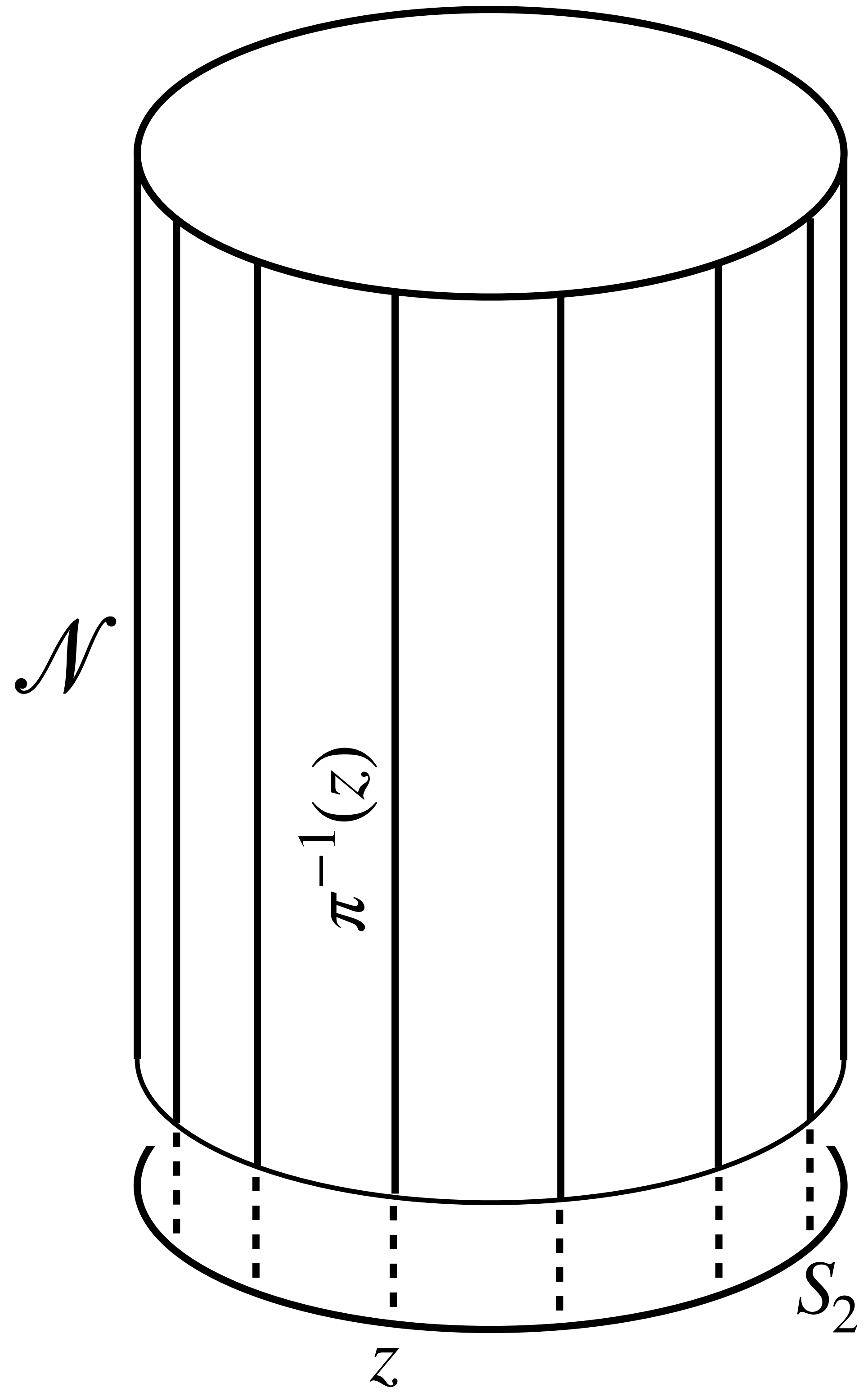
$$\boxed{\varphi^*_{\mathcal{N}} g_{ab} = q_{ab} = \delta_{ij} e^i{}_a e^j{}_b, \quad i,j = 1,2}$$



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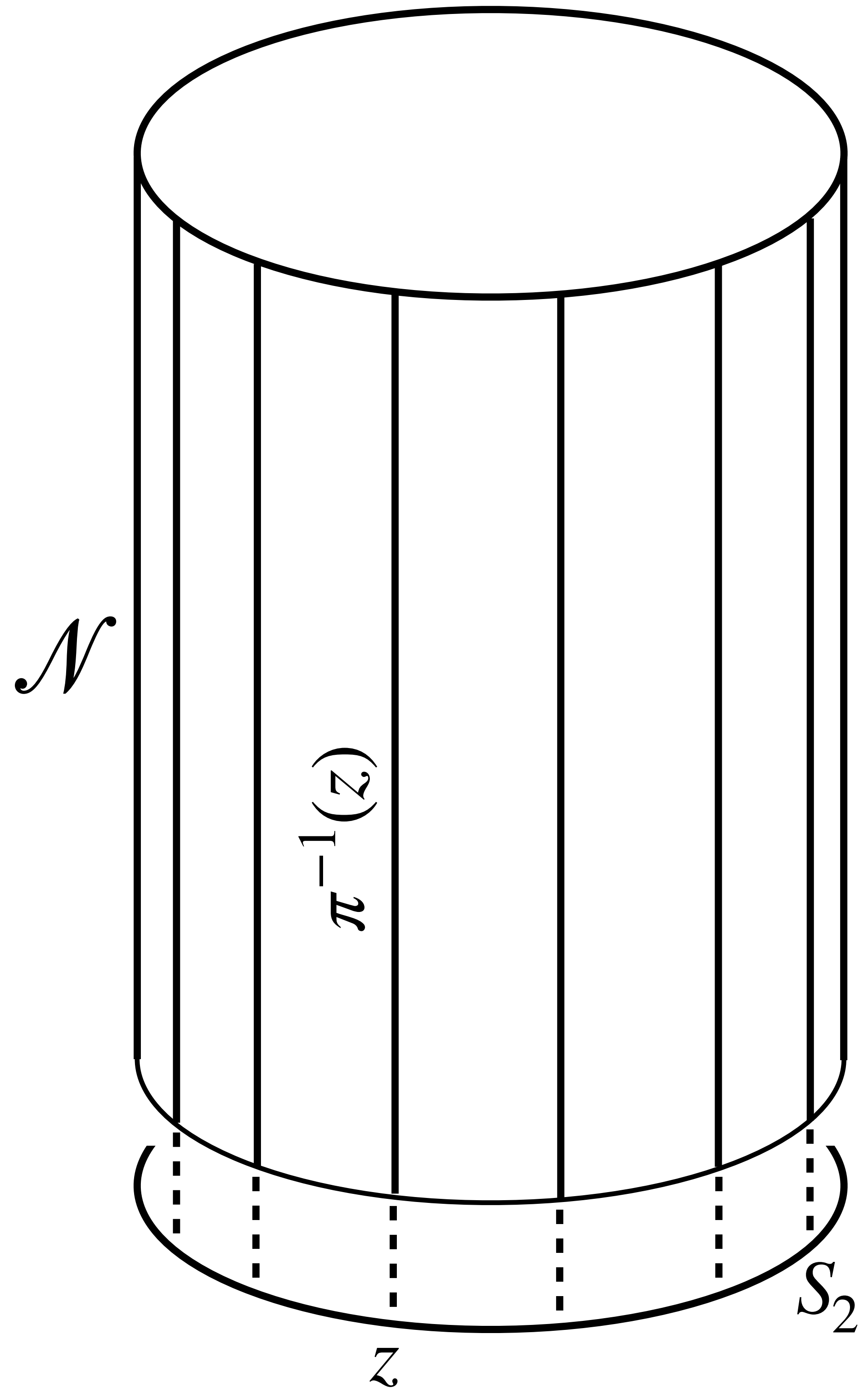

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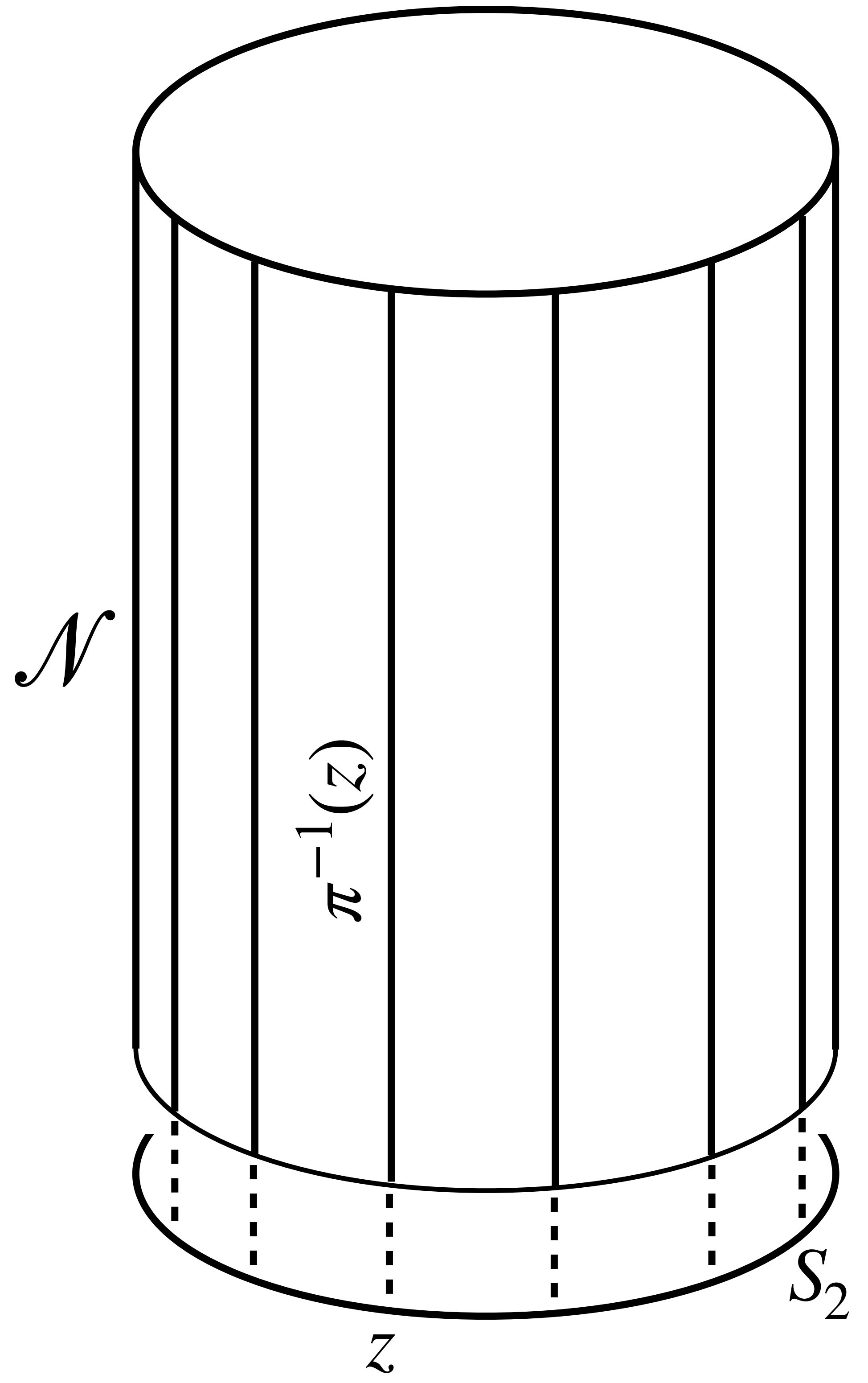
  - Conformal factor Ω

$$e^i = \Omega\, S^i{}_m \, {}^{(o)}e^m$$



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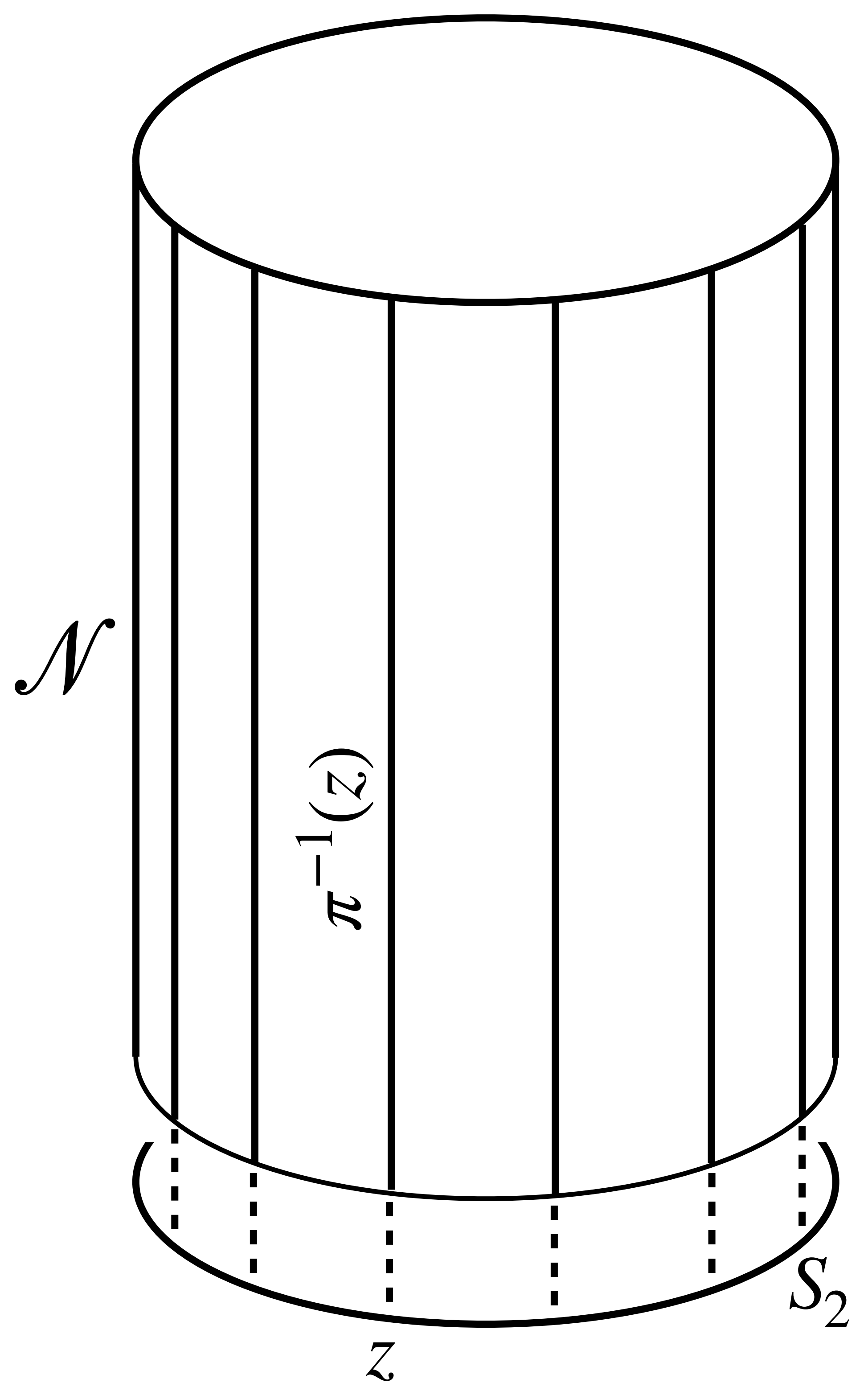

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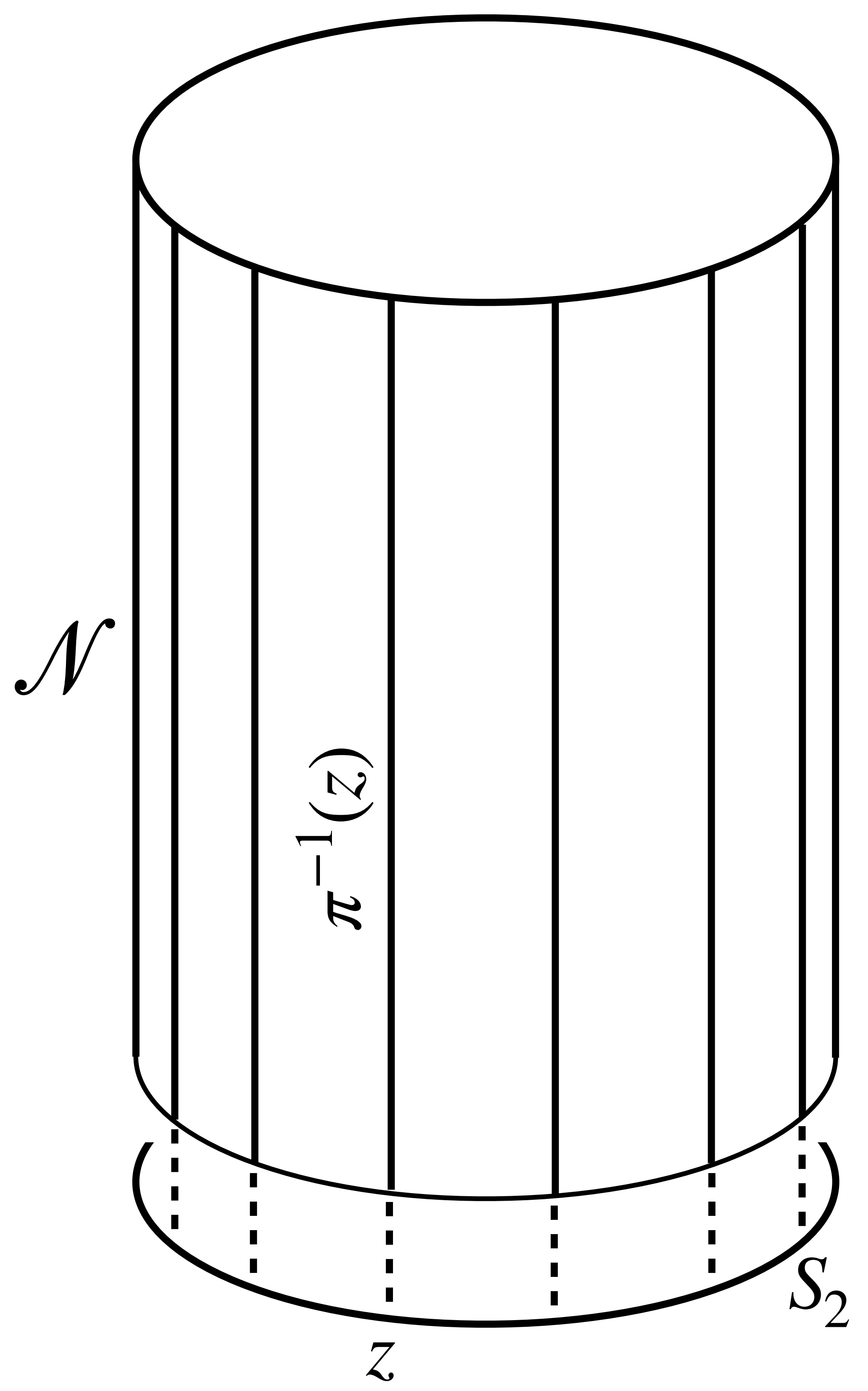

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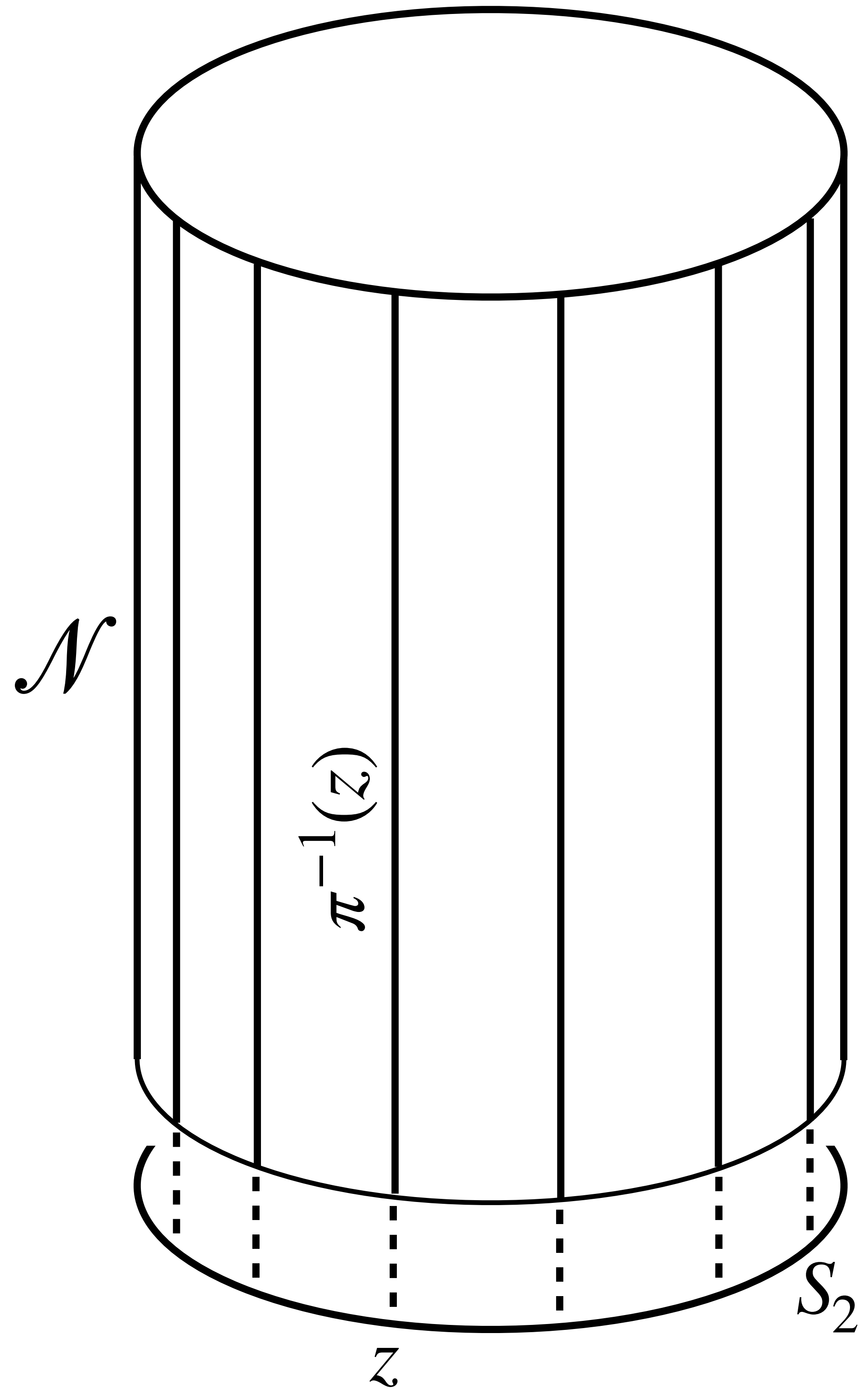
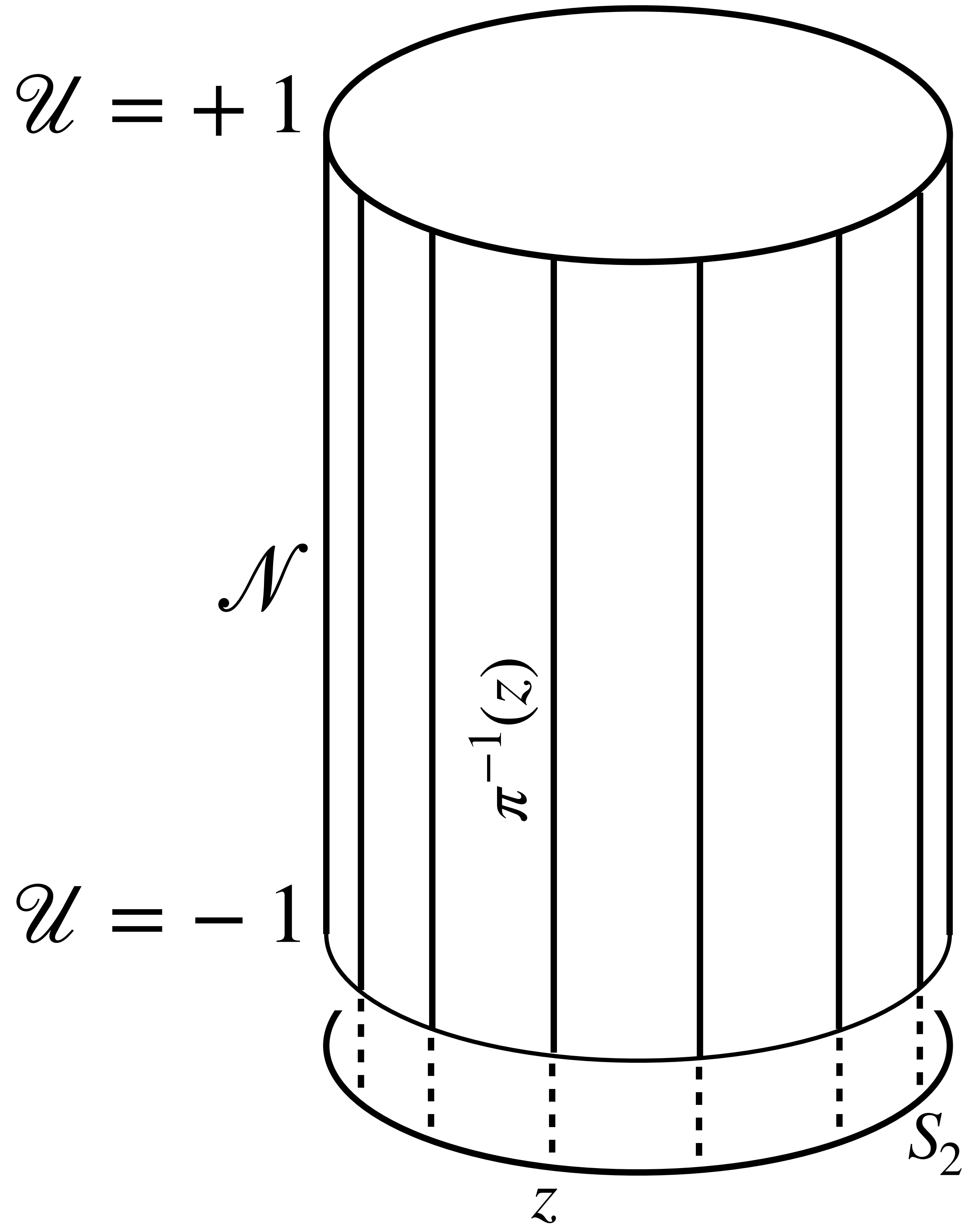


[ww 2017]

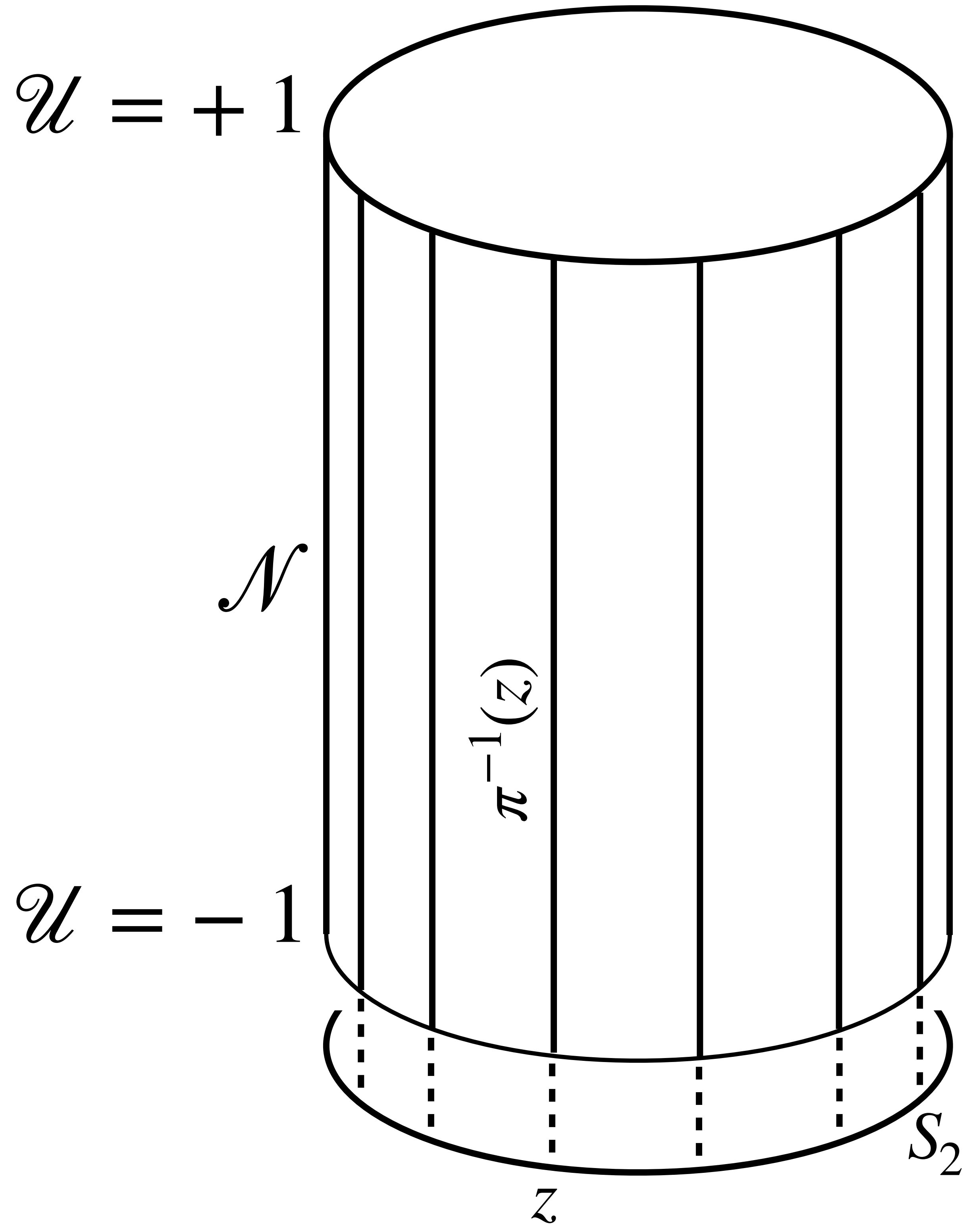
Teleological clock

- We look at abstract null boundary with initial and final cuts



Teleological clock

- We look at abstract null boundary with initial and final cuts
- Choice of teleological clock



Teleological clock

- We look at abstract null boundary with initial and final cuts
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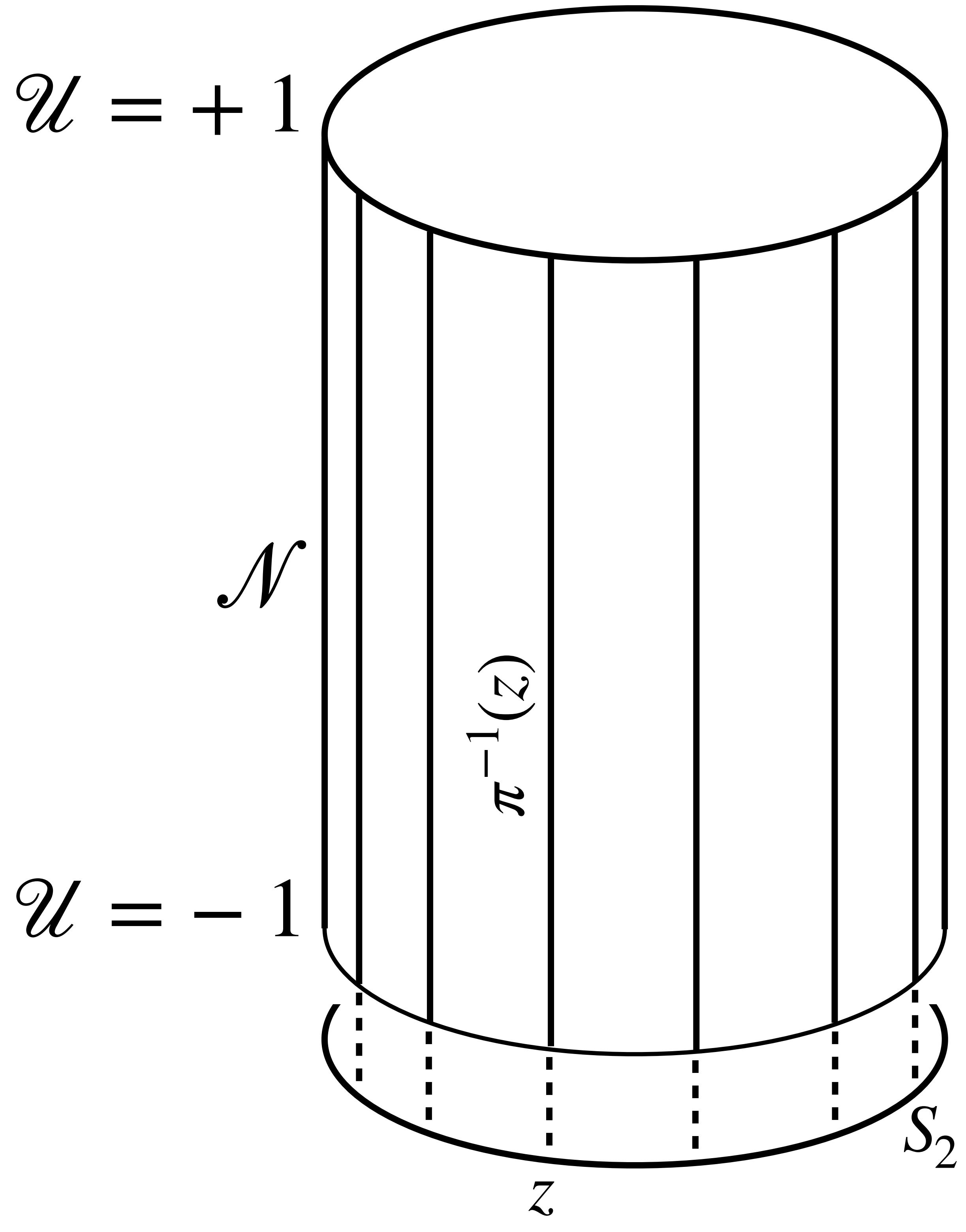
$$\partial^b_{\mathscr{U}} \nabla_b \partial^a_{\mathscr{U}} \Big|_{\mathscr{N}} = -\frac{1}{2}(\Omega^{-2}\frac{\mathrm{d}}{\mathrm{d}\mathscr{U}}\Omega^2)\partial^a_{\mathscr{U}}$$

$$\mathscr{U}\Big|_{\mathscr{C}_\pm} = \pm 1$$



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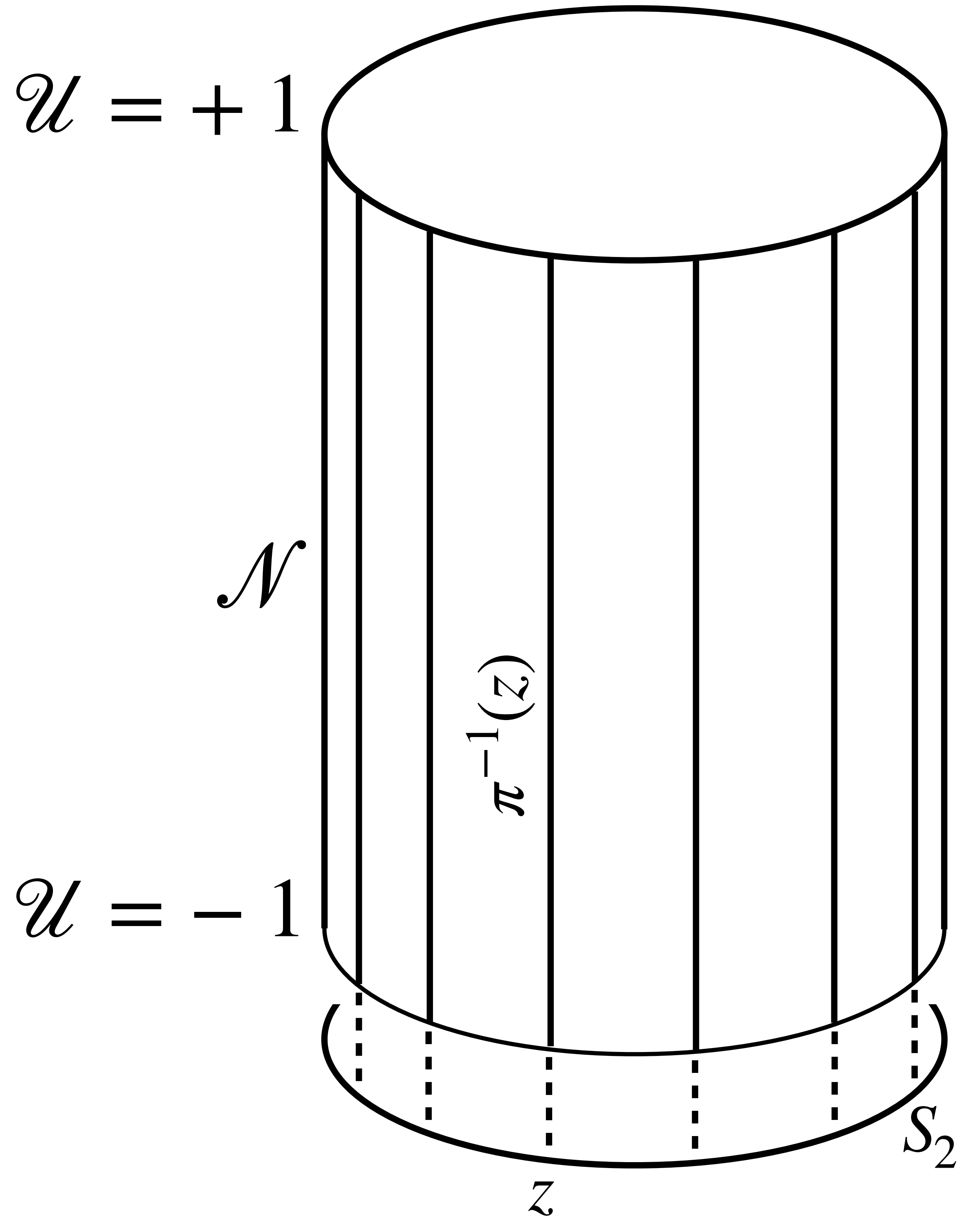

[ww, 2017]
[Ciambelli, Leigh, Freidel, 2023]

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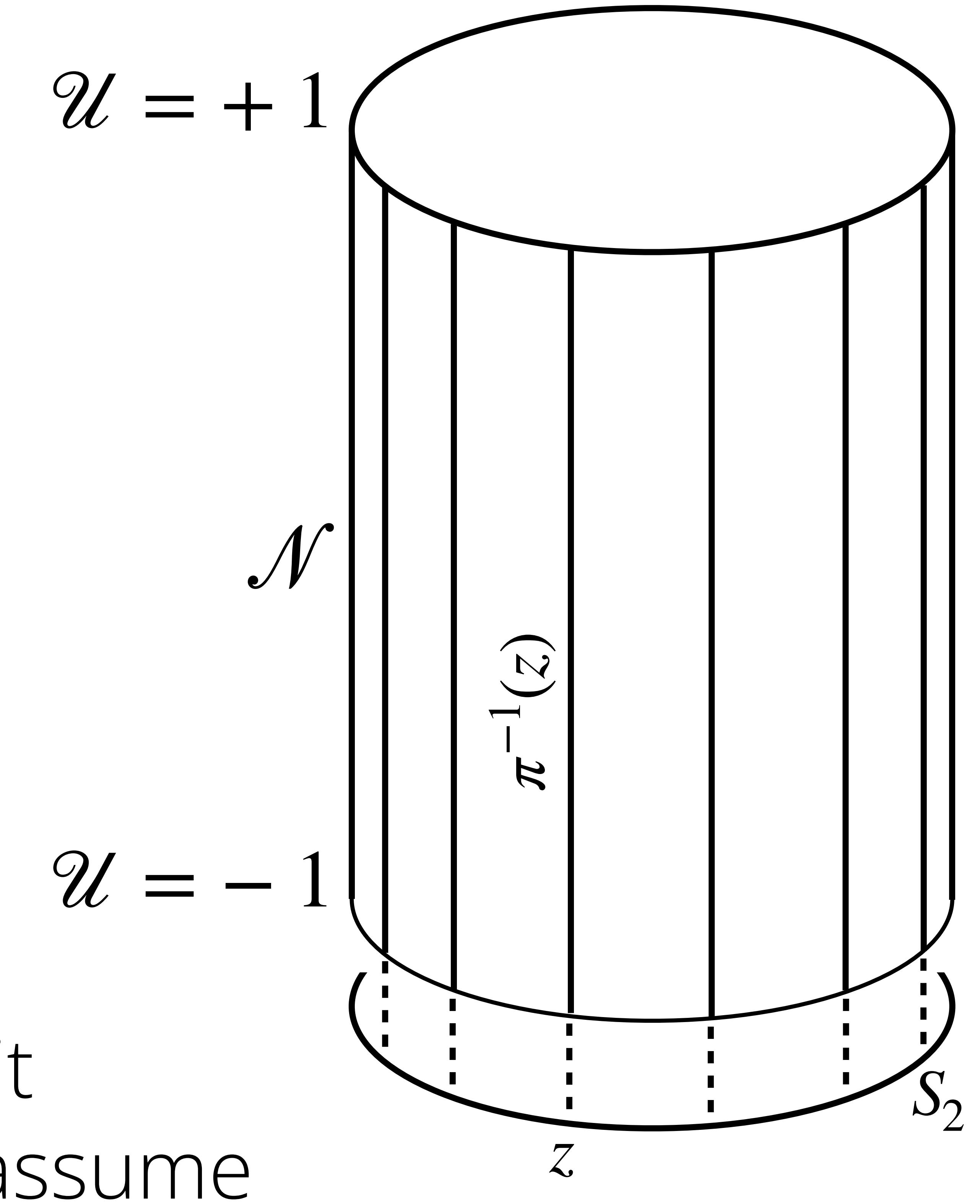
- teleological, because it knows that it should assume specific boundary values.



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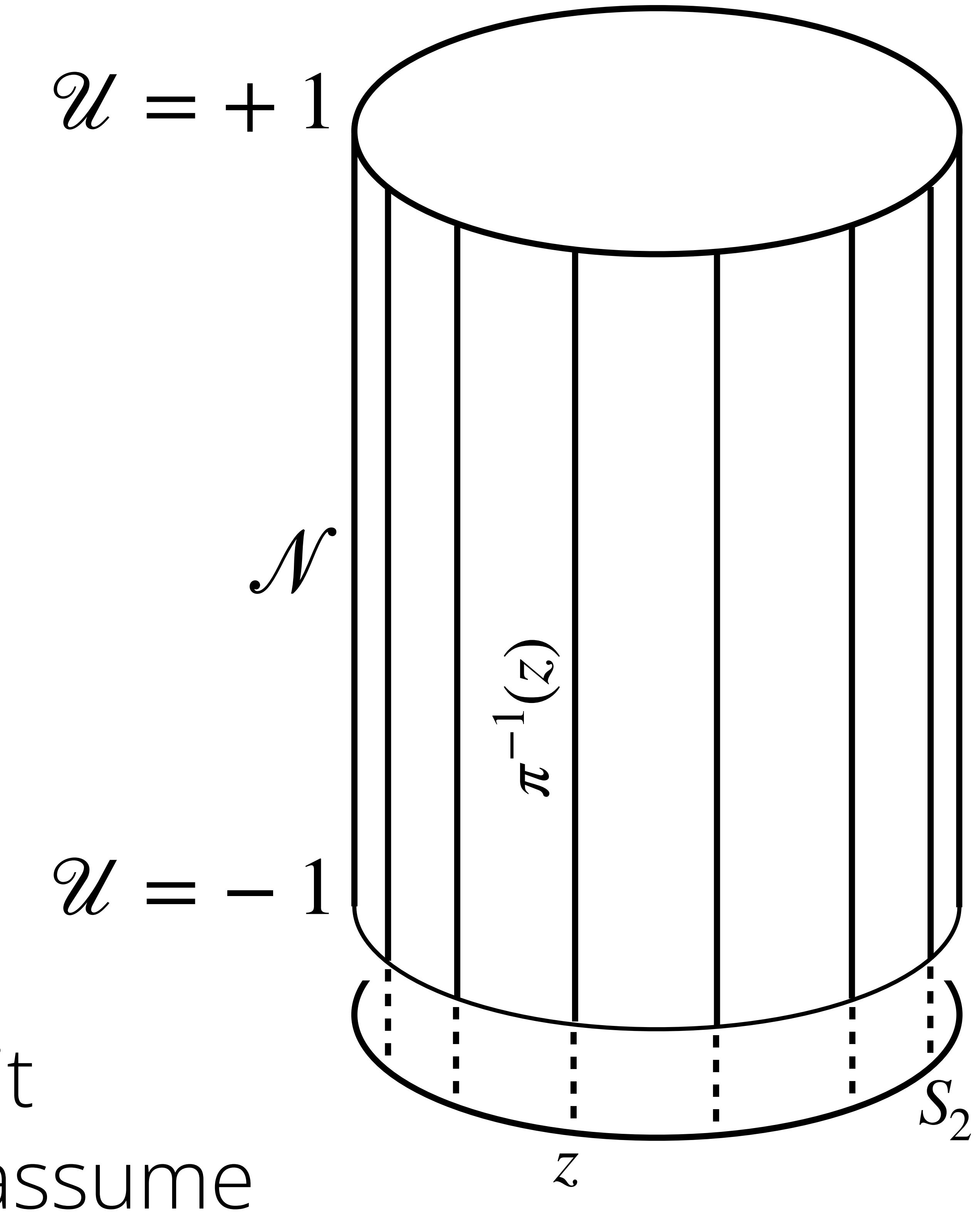

[ww, 2017]
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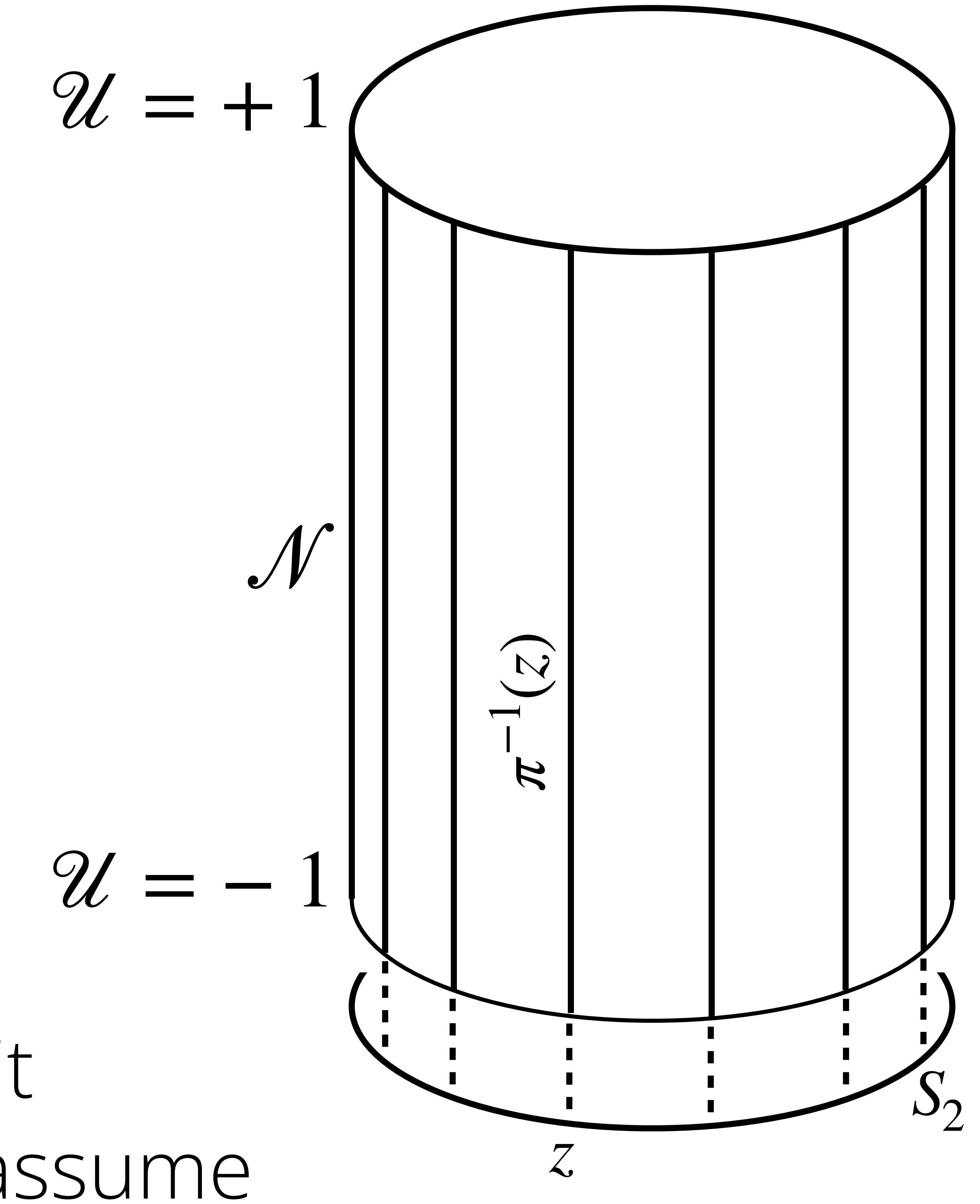
- teleological, because it knows that it should assume specific boundary values.
- Affinity proportional to expansion.
- On phase space, $\delta\mathscr{U} \neq 0$.



[ww, 2017]
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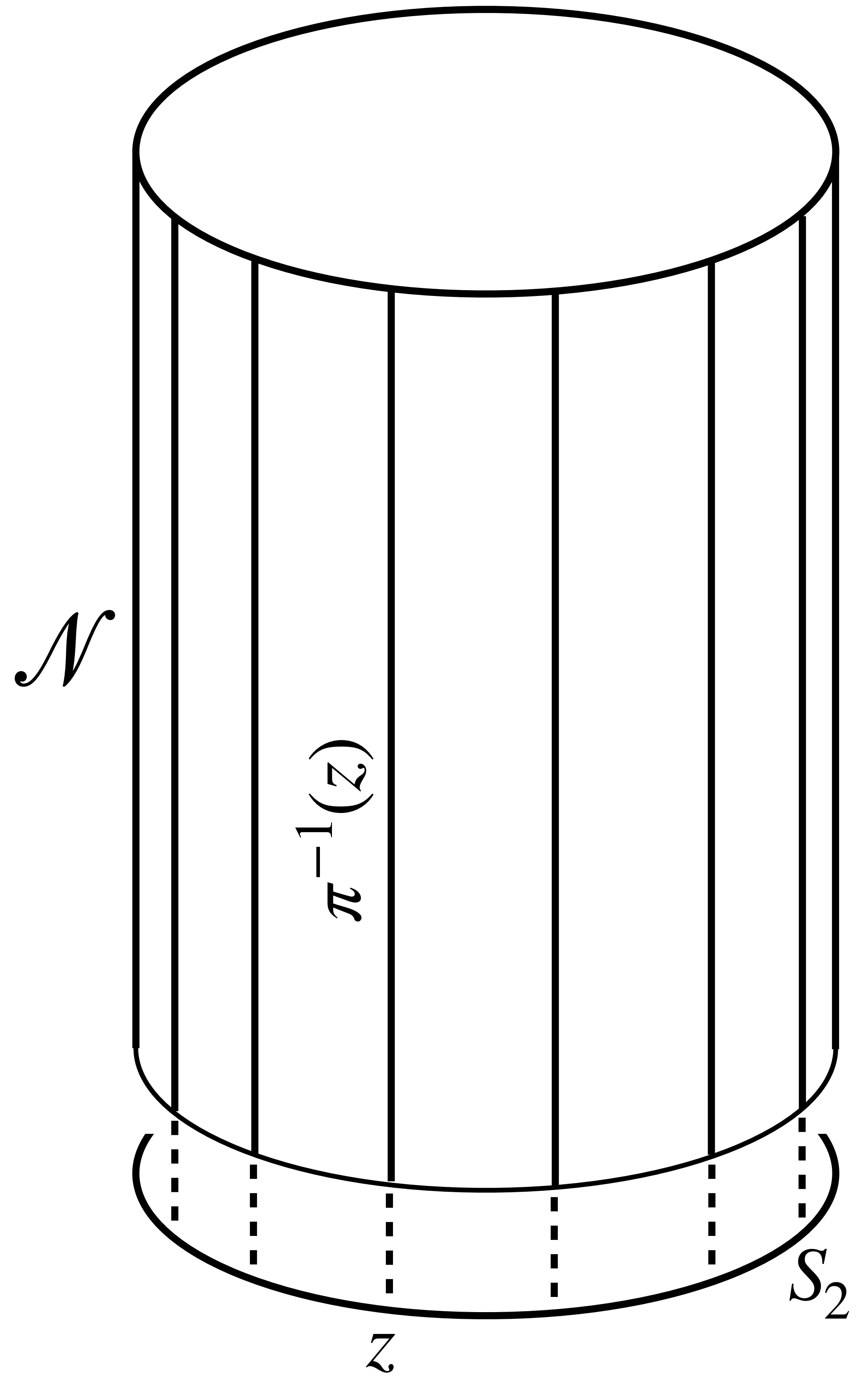
Constraints

Given such data, Einstein's equations and torsionless equation impose two constraints



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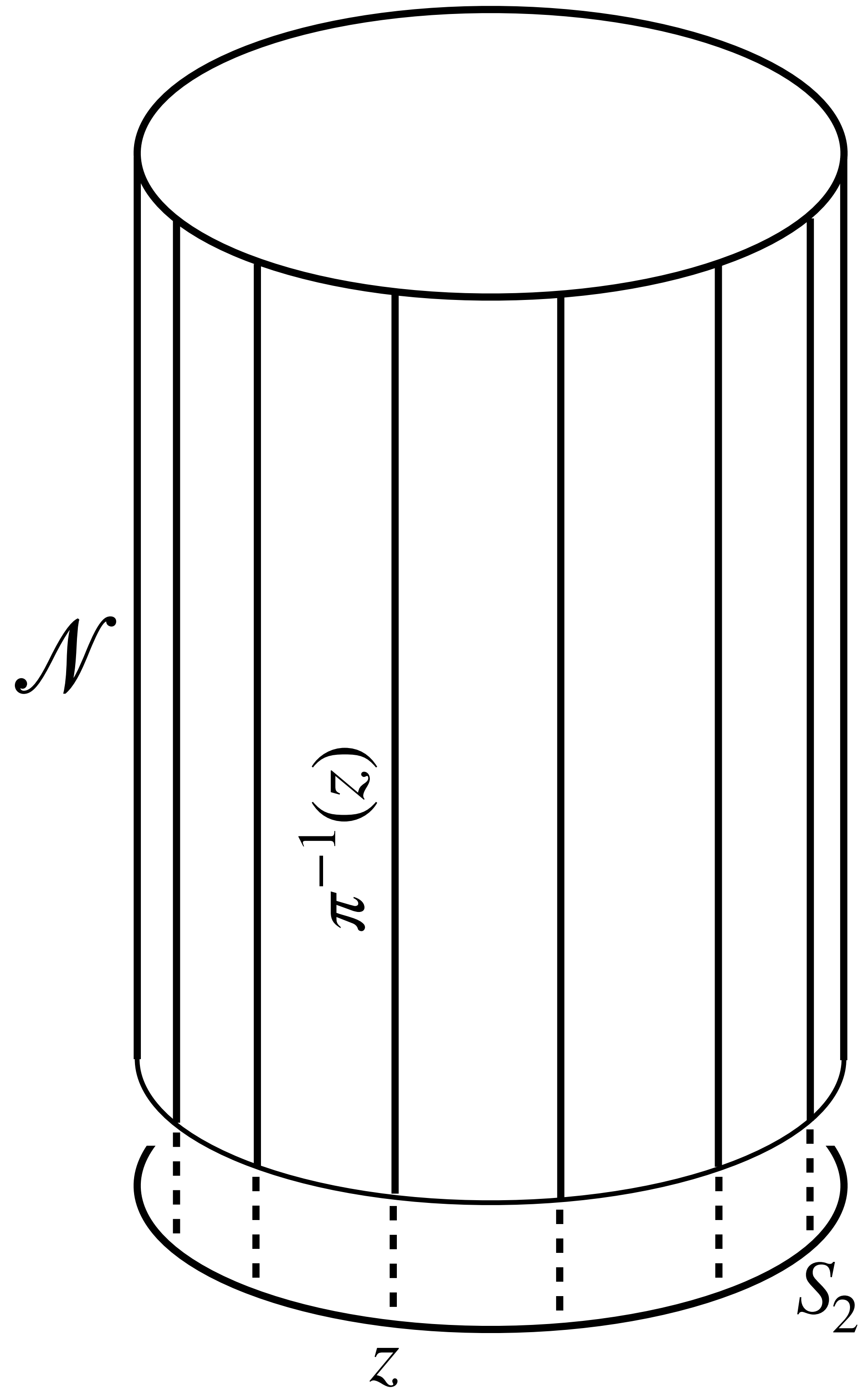

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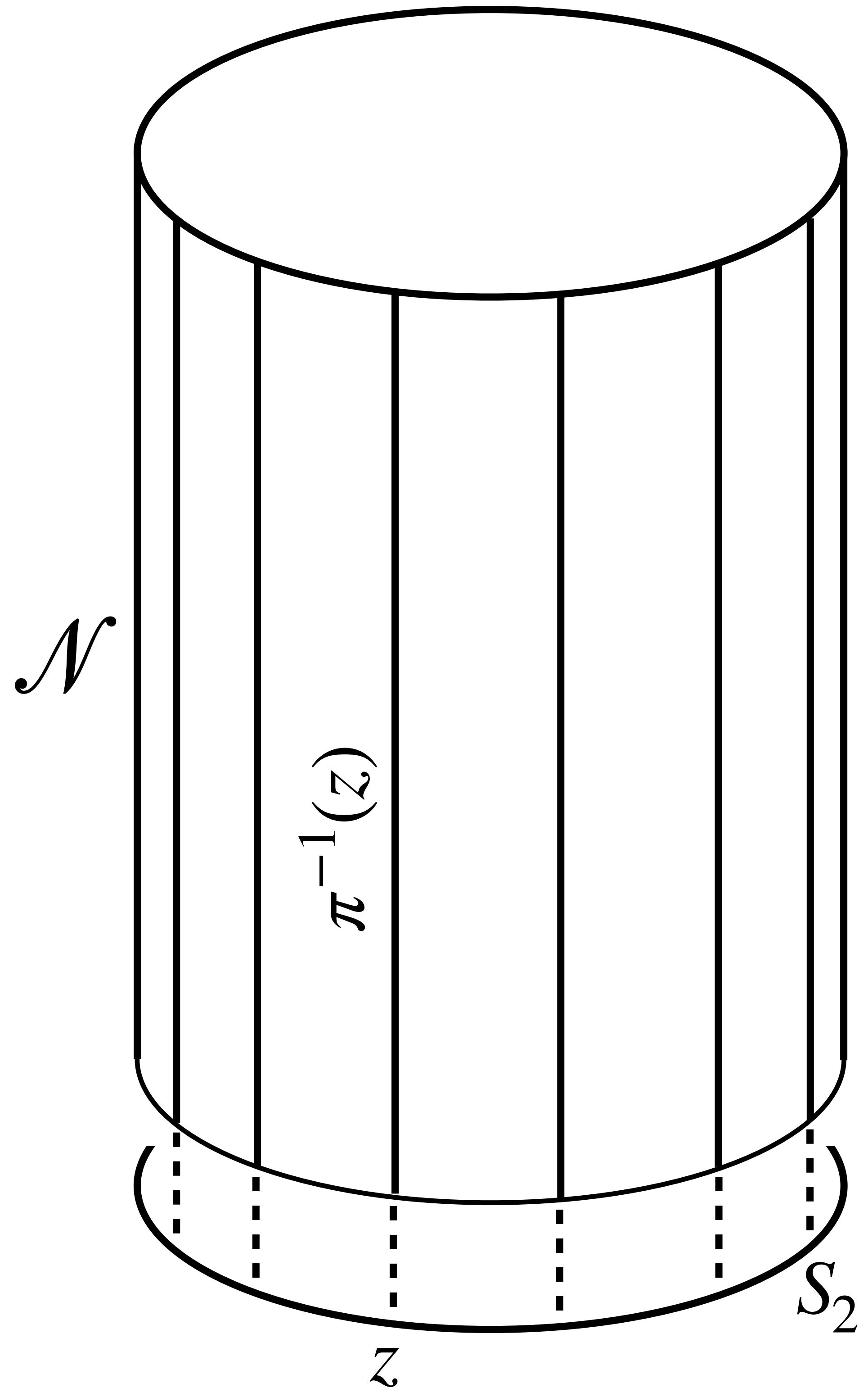
Raychaudhuri equation:
$G_{ab}\ell^a\ell^b = 0$



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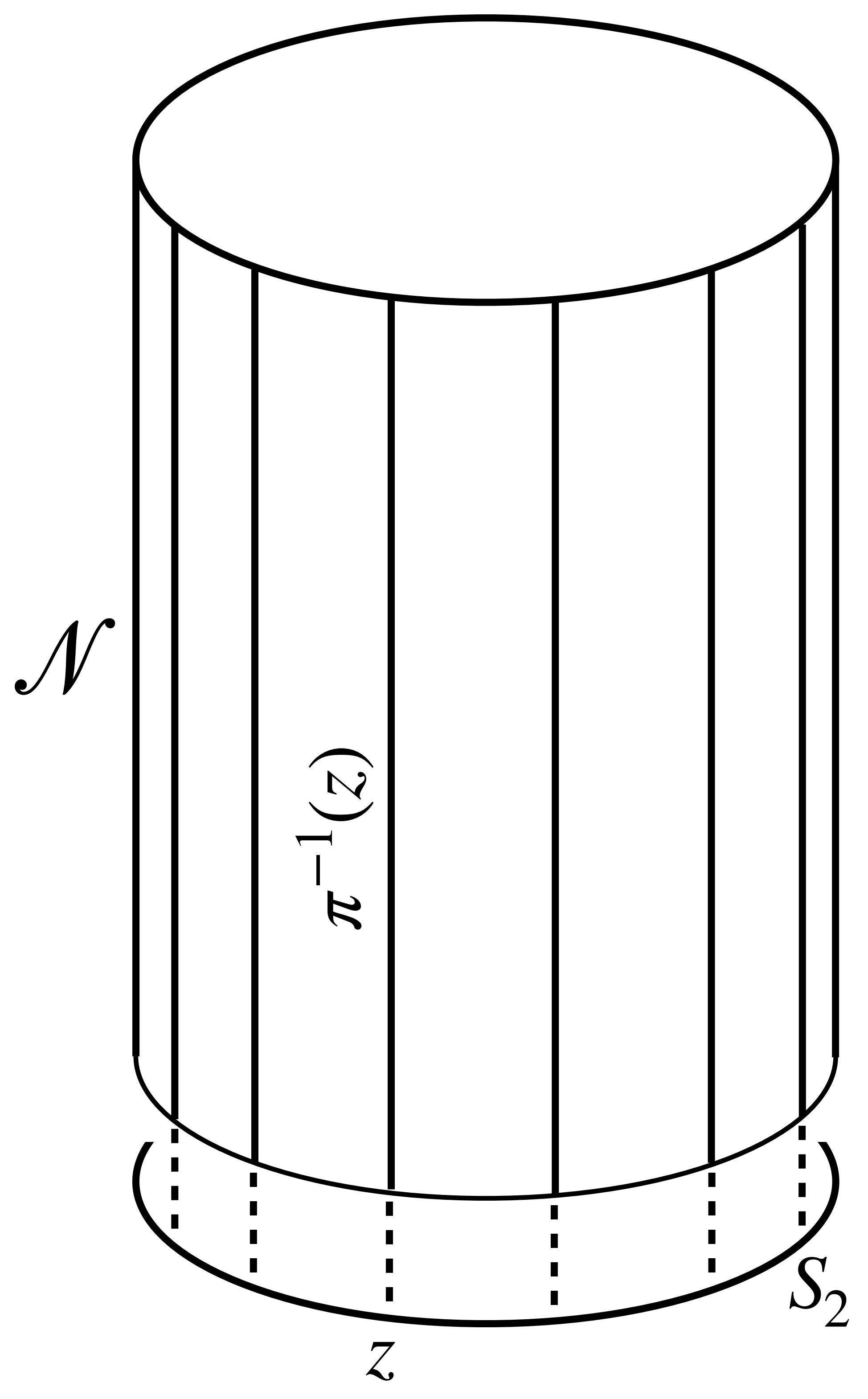

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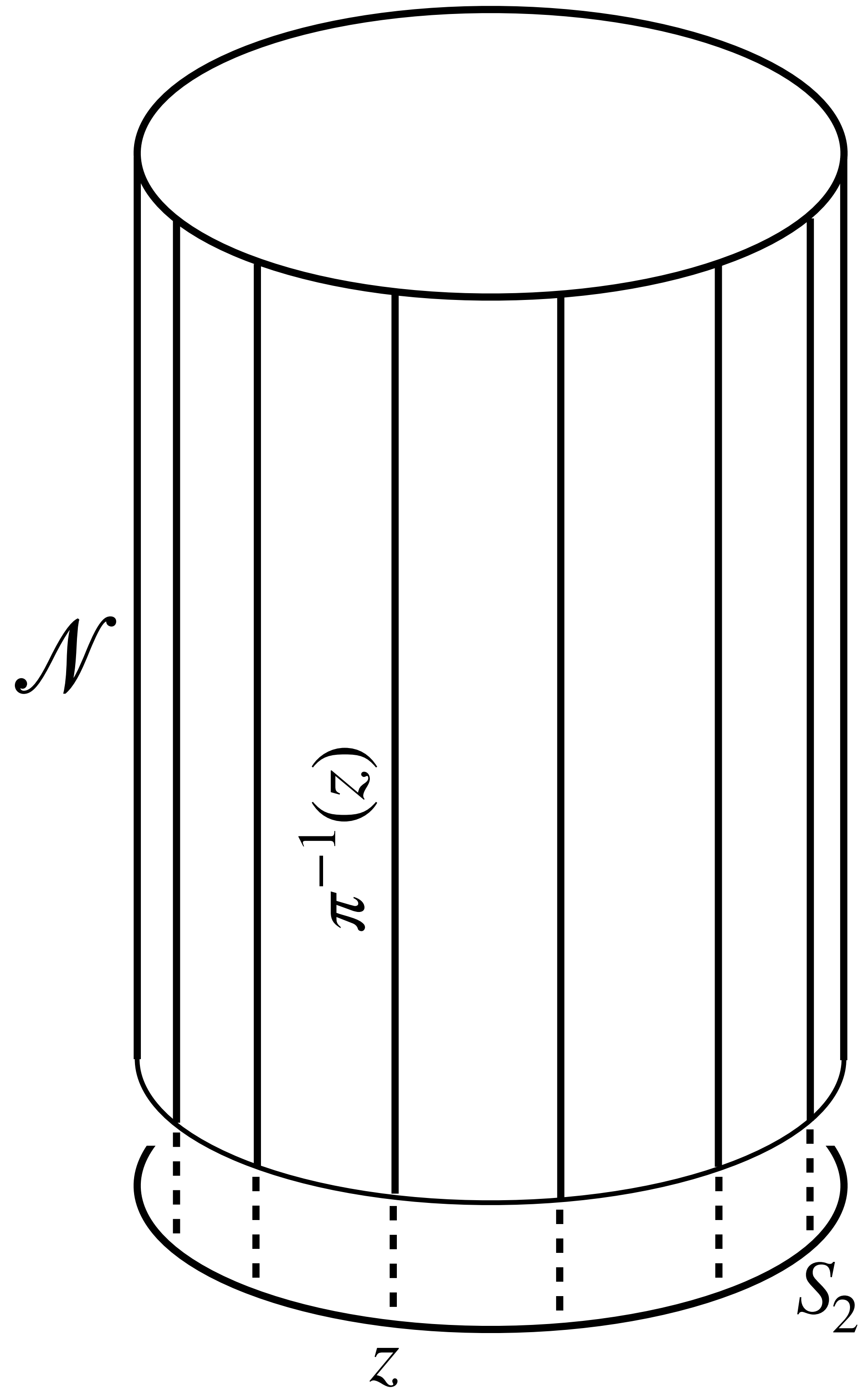
shear



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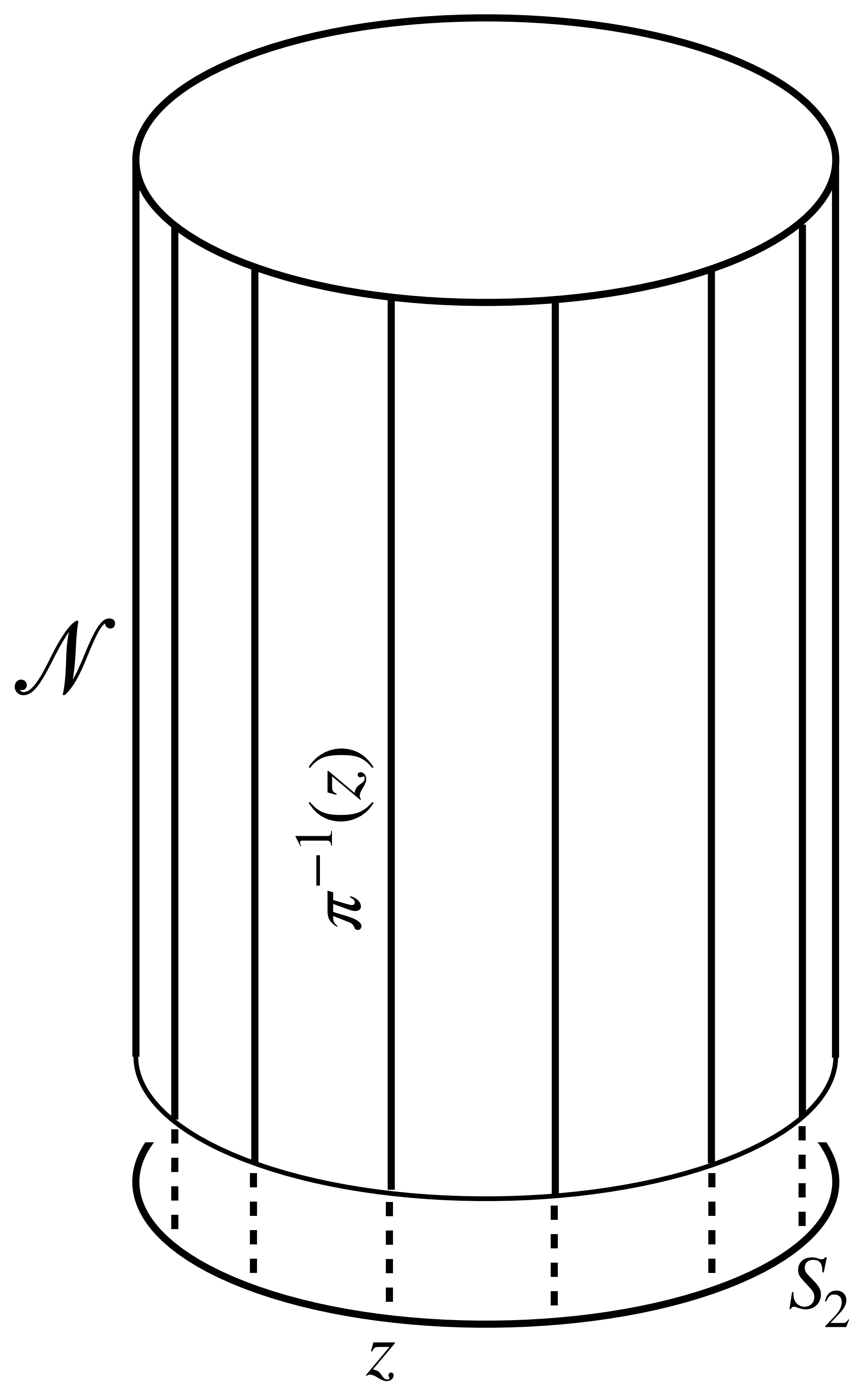
Transport equation:



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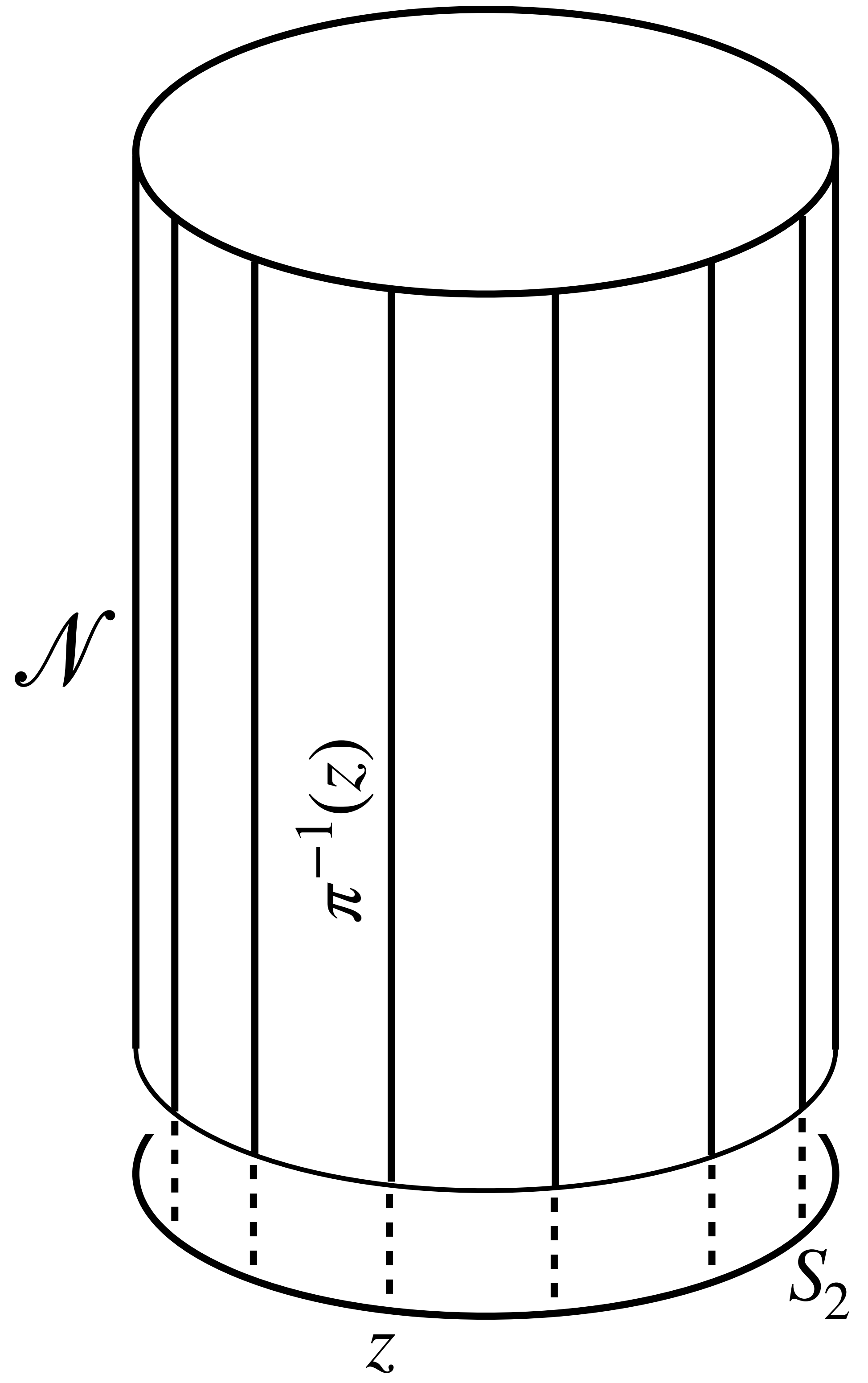
shear

Transport equation:
- *φ is a U(1) connection on $\mathcal{N}$*



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Transport equation:
- φ *is a U(1) connection on* $\mathcal{N}$
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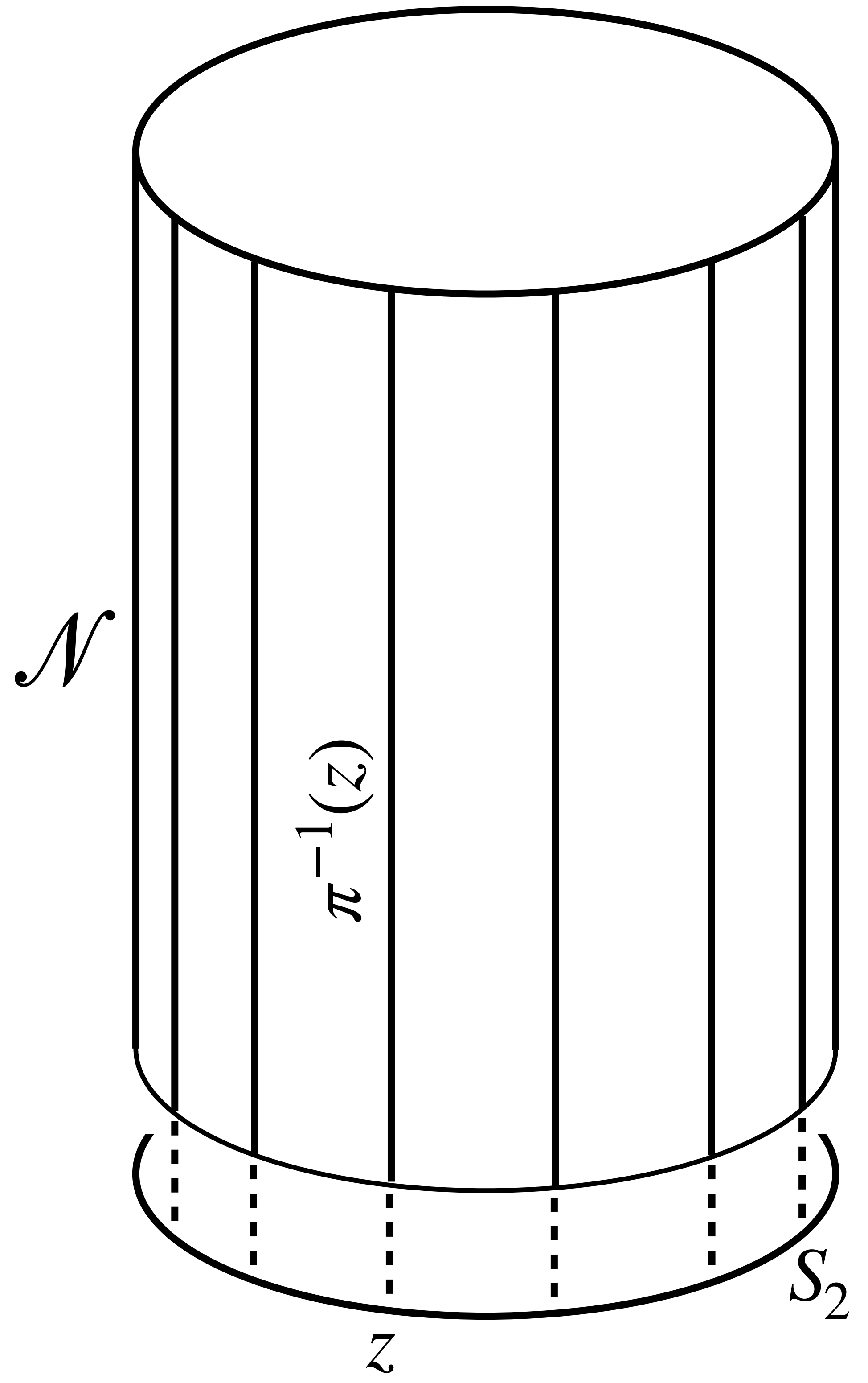

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shear

Transport equation:
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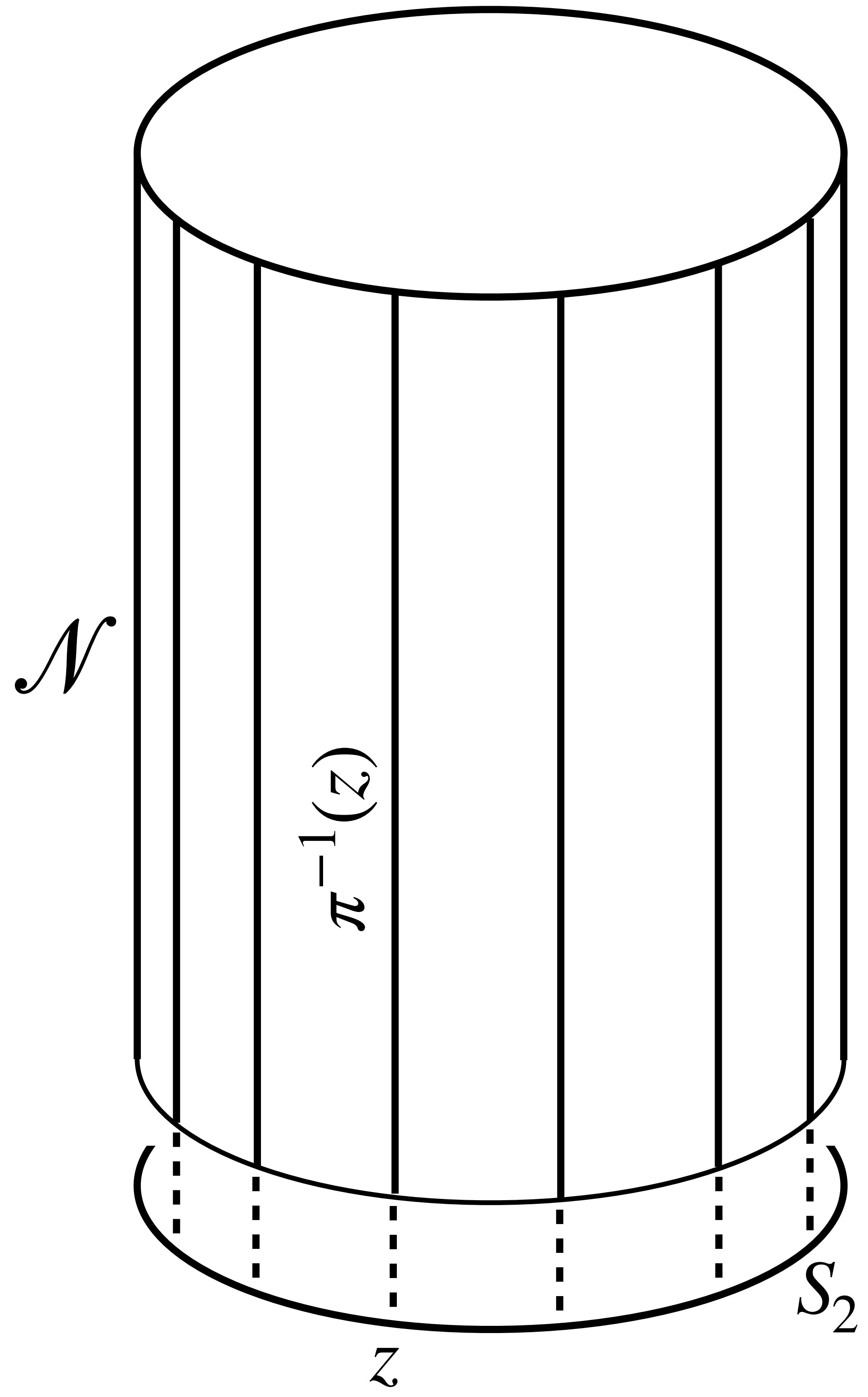
- $J, X, \bar{X}$ *are* $\mathfrak{sl}(2,\mathbb{R})$ *generators*
- $[J,X] = -\,2\mathrm{i}\,X, \quad [X,\bar{X}] = \mathrm{i}\,J$



Step 2: Null symplectic structure

Null symplectic structure

Boundary symplectic structure for γ-action

Null symplectic structure

Boundary symplectic structure for γ-action

$$S[A,e] = \frac{1}{16\pi G}\int_{\mathscr{M}} \left(*\,(e_\alpha \wedge e_\beta) - \frac{1}{\gamma}(e_\alpha \wedge e_\beta) \right) \wedge F^{\alpha\beta}[A]$$

Null symplectic structure

Boundary symplectic structure for γ-action

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Symplectic potential $\Theta = p\mathbb{d}q$ determines classical $(\hbar \to 0)$ algebra of observables.

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Boundary symplectic structure for γ-action

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Corner phase space: initial data for Raychaudhuri equation

Null symplectic structure

Boundary symplectic structure for γ-action

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Corner phase space: *initial data for Raychaudhuri equation*

Free radiative data: *shear* σ_I

Null symplectic structure

Boundary symplectic structure for γ-action

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Corner phase space: *initial data for Raychaudhuri equation*

Free radiative data: *shear* σ_I

- *dressed* $\mathbb{D}$*-differential*

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Corner phase space: *initial data for Raychaudhuri equation*

Free radiative data: *shear* σ_I

- *dressed* $\mathbb{D}$*-differential*

$$\mathbb{D} = \mathbb{d} - \mathbb{d}\mathcal{U}\frac{\mathrm{d}}{\mathrm{d}\mathcal{U}}$$

Null symplectic structure

Boundary symplectic structure for γ-action

$$S[A,e] = \frac{1}{16\pi G}\int_{\mathscr{M}}\left(*(e_\alpha \wedge e_\beta) - \frac{1}{\gamma}(e_\alpha\wedge e_\beta)\right)\wedge F^{\alpha\beta}[A]$$

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Corner phase space: initial data for Raychaudhuri equation

Free radiative data: shear σ_I

- *dressed* $\mathbb{D}$*-differential* $\mathbb{D} = \mathbb{d} - \mathbb{d}\mathscr{U}\dfrac{\mathrm{d}}{\mathrm{d}\mathscr{U}}$
- $U(1)$*-interaction picture for* σ_I

Null symplectic structure

Boundary symplectic structure for γ-action

$$S[A,e] = \frac{1}{16\pi G}\int_{\mathcal{M}}\left(*(e_\alpha\wedge e_\beta) - \frac{1}{\gamma}(e_\alpha\wedge e_\beta)\right)\wedge F^{\alpha\beta}[A]$$

Symplectic potential $\Theta = p\mathbb{d}q$ determines classical ($\hbar \to 0$) algebra of observables.

$$\Theta_{\mathcal{N}} = -\frac{1}{16\pi\gamma G}\int_{\partial\mathcal{N}} d^2v_o\,\Omega^2\,\mathrm{Tr}\big(J\mathbb{d}SS^{-1}\big) + \\ -\frac{1}{8\pi G}\int_{\mathcal{N}} \mathrm{d}\mathcal{U}\wedge d^2v_o\,\Omega^2\,\mathrm{Tr}\big((\sigma_I\bar{X}+\bar{\sigma}_I X)\mathbb{D}S_I S_I^{-1}\big) + \\ -\frac{1}{8\pi G}\int_{\mathcal{N}} \mathrm{d}\mathcal{U}\wedge d^2v_o\,\mathbb{d}\mathcal{U}\left(\frac{\mathrm{d}^2}{\mathrm{d}\mathcal{U}^2}\Omega^2 + 2\sigma_I\bar{\sigma}_I\Omega^2\right).$$

Corner phase space: initial data for Raychaudhuri equation

Free radiative data: shear σ_I

- *dressed* $\mathbb{D}$*-differential* $\mathbb{D} = \mathbb{d} - \mathbb{d}\mathcal{U}\frac{\mathrm{d}}{\mathrm{d}\mathcal{U}}$
- $U(1)$*-interaction picture for* σ_I

Clock momentum: Raychaudhuri constraint coupling σ_I *and* Ω.

Step 3: Quantum impulsive null geometries

Impulsive null initial data

Replace smooth profile by series of step functions

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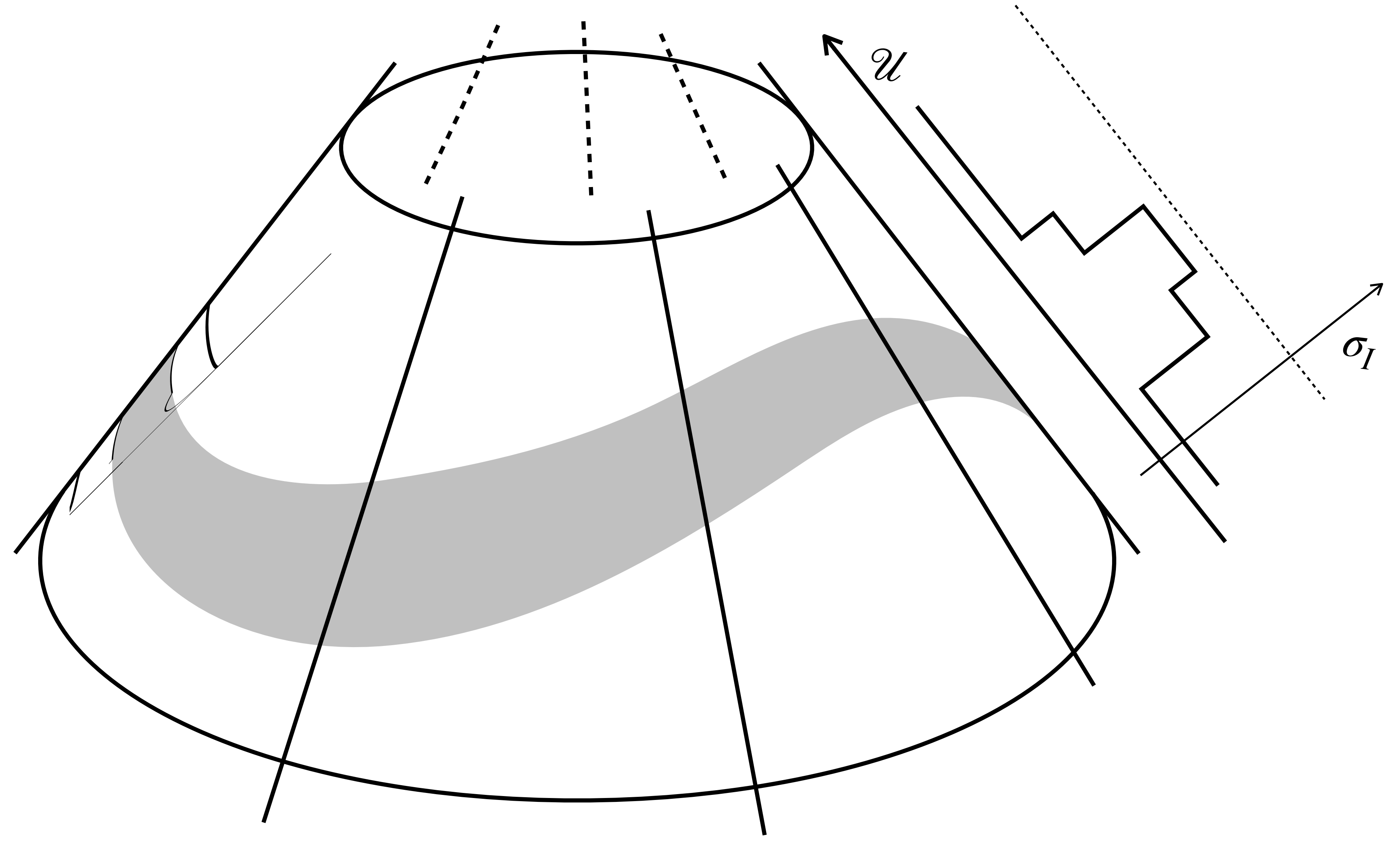

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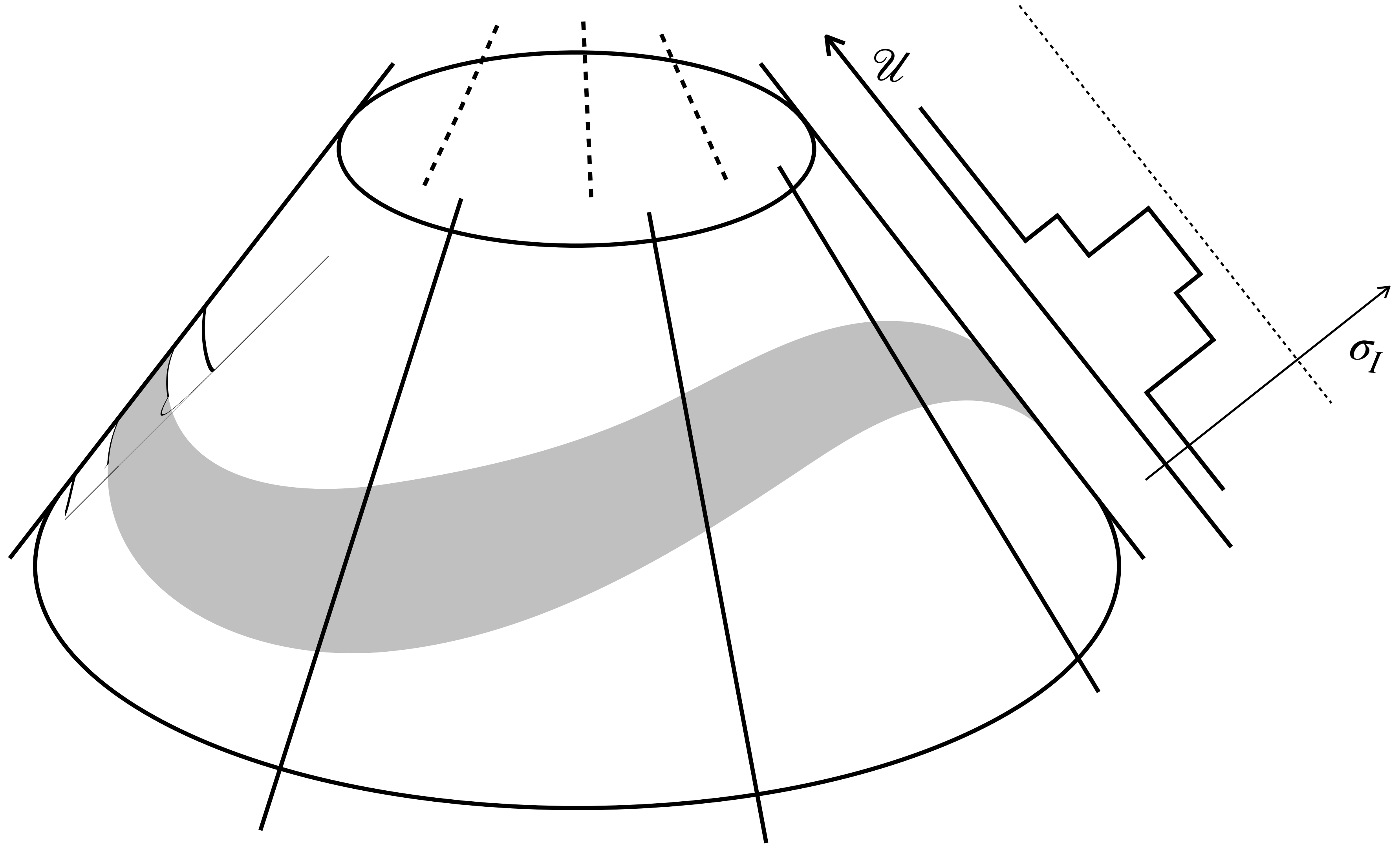

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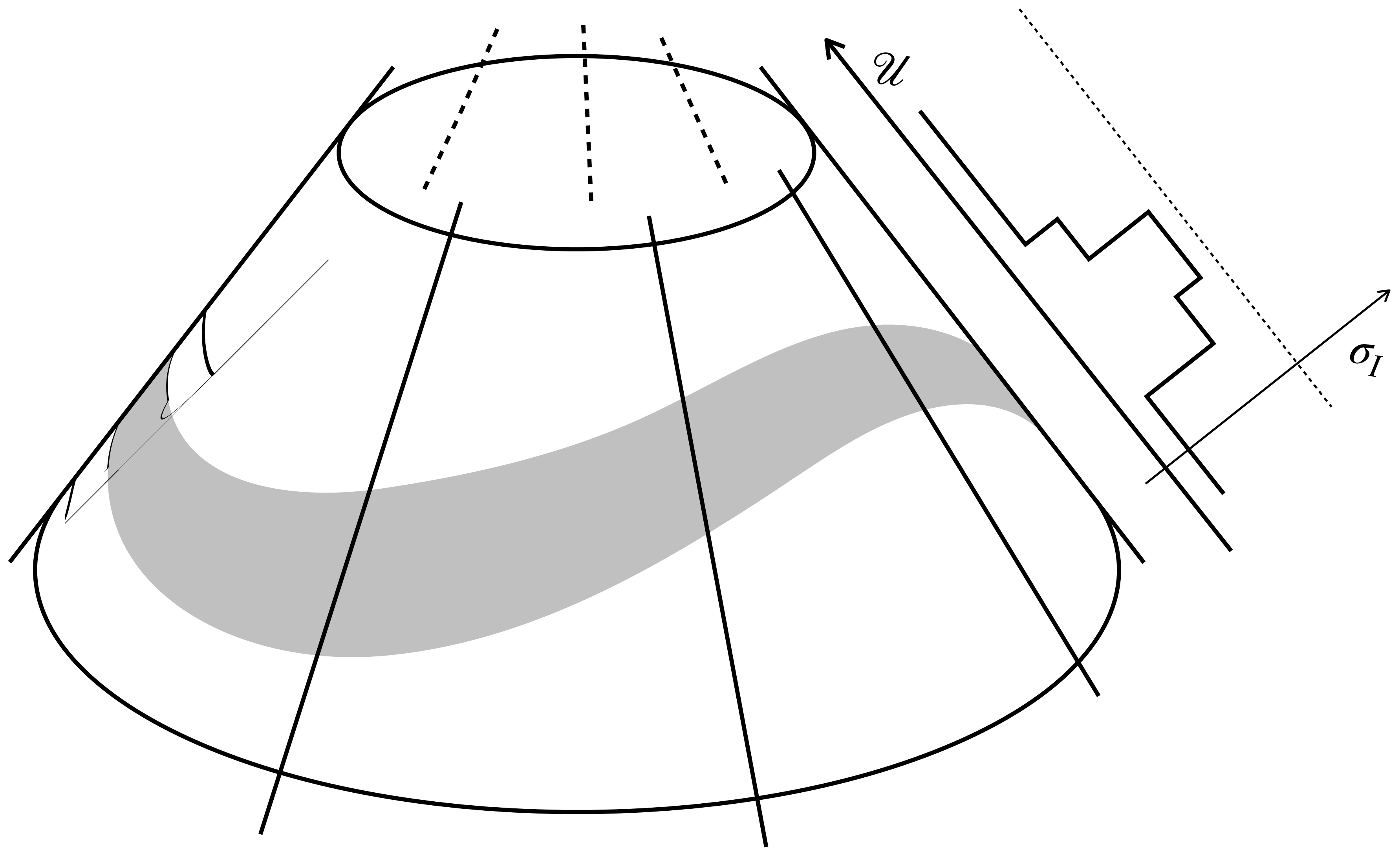

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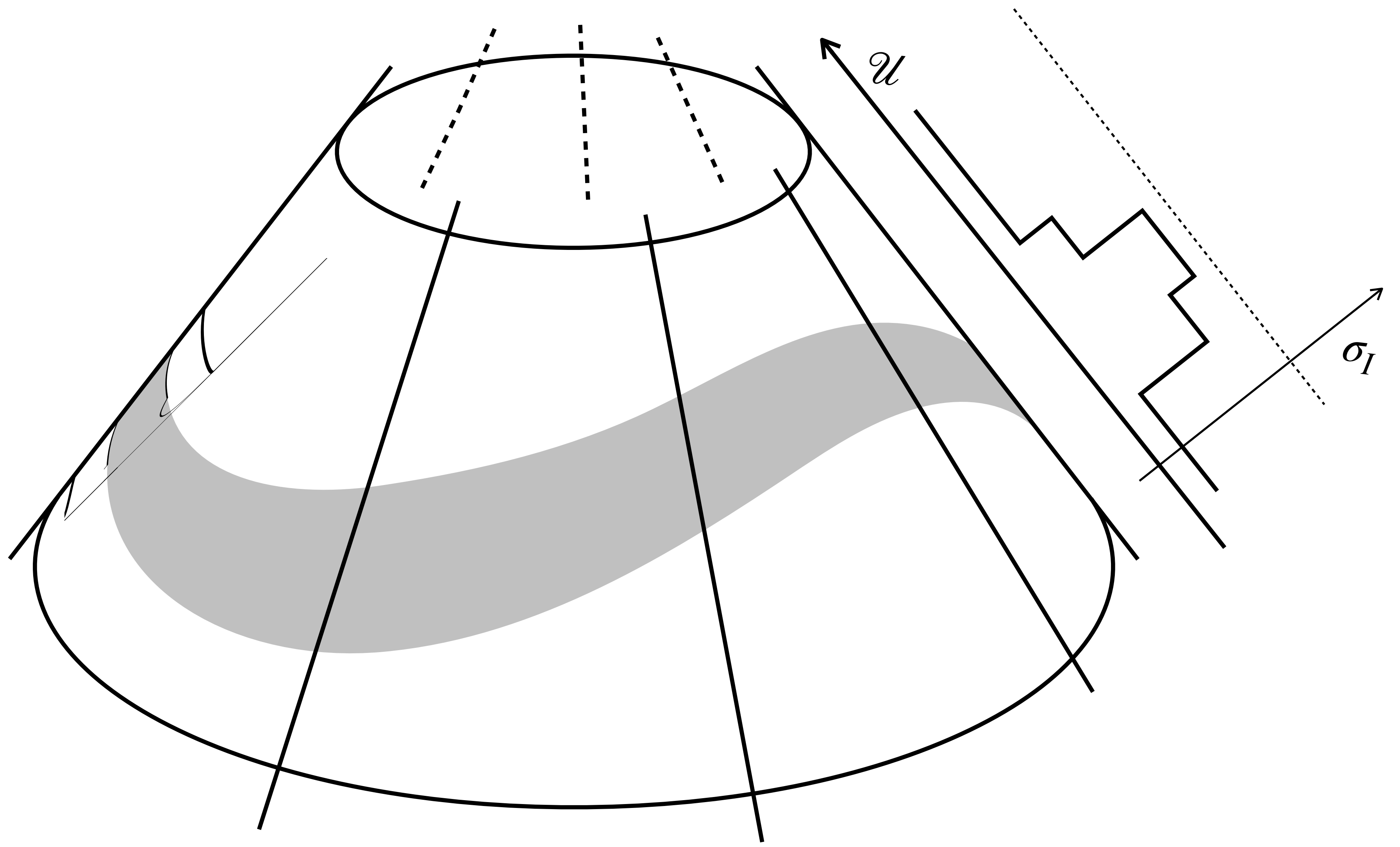

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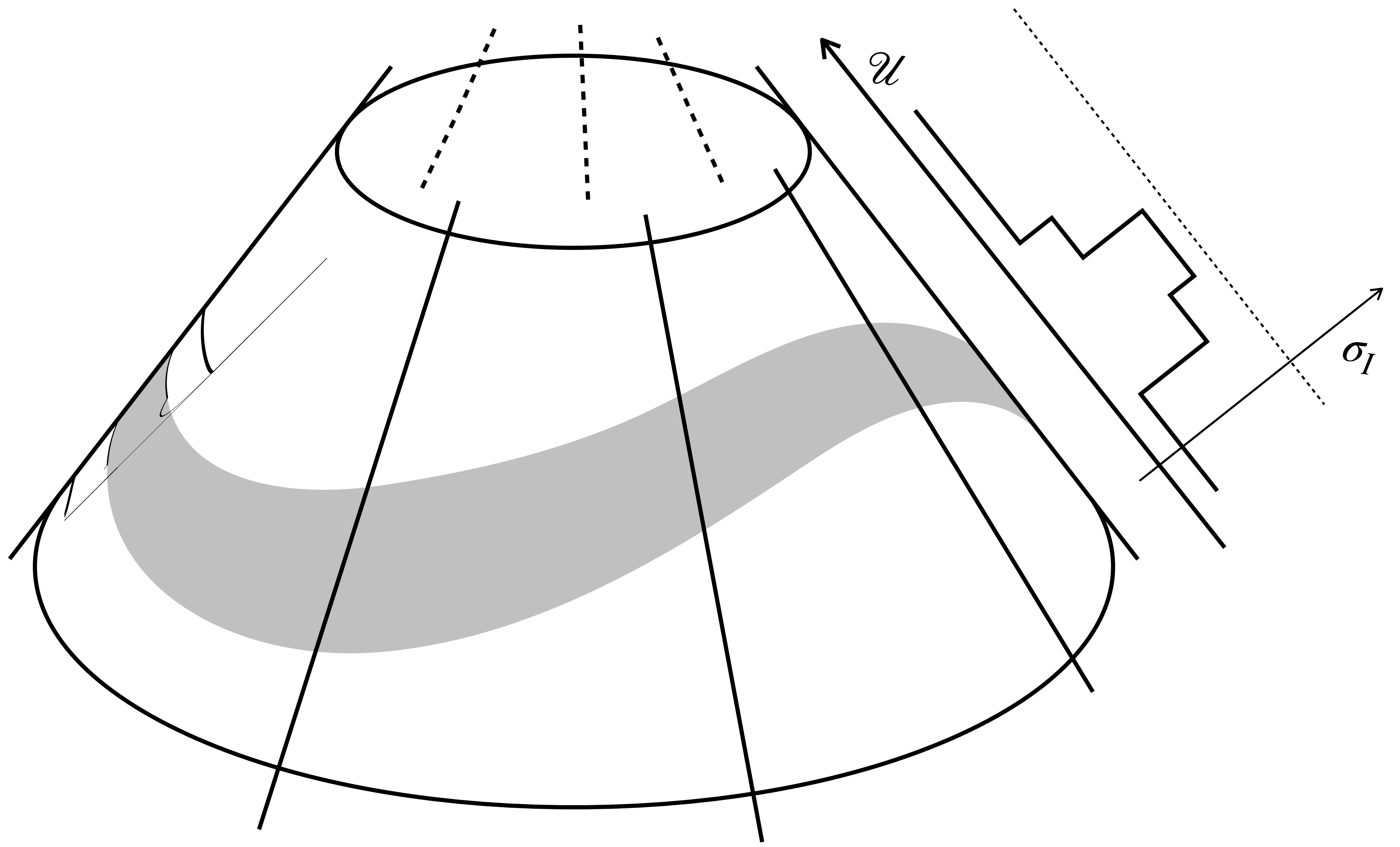
- Quantize each pulse, then glue many such pulses back together.



Solving the constraints for impulsive data

Constant $U(1)$ dressed shear: $\dfrac{\mathrm{d}}{\mathrm{d}\mathscr{U}}\sigma_I = 0$

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- Role of γ-parameter: mixing the two (recall $A = \Gamma + \gamma K$)

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Kinematical phase space of a single pulse

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- Two angle-dependent Heisenberg charges (edge modes)

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Highly non-linear relation to geometric data

$$\Omega_-^2 + \Omega_+^2 = 16\pi\gamma G\,(L + a\bar{a})$$
$$\Omega_-^2 - \Omega_+^2 = 16\pi\gamma G\,\left(L + b\bar{b}\right)$$

$$\mathrm{th}\left(2\sqrt{\sigma\bar{\sigma}}\right) = \sqrt{\frac{\bar{b}b}{\bar{a}a}}$$
$$U = \mathrm{e}^{\gamma \ln\left(\tan\left(\sqrt{2\sigma\bar{\sigma}}\right)/\sqrt{2\sigma\bar{\sigma}}\right)J} S_-$$

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$$L^2 - c\bar{c} = \frac{1}{8\pi G}\left[\frac{1}{4\gamma^2}(\Omega_-^2 - \Omega_+^2)^2 + -\frac{1}{\gamma^2}\sinh^2\left(2\sqrt{\sigma\bar{\sigma}}\right)\Omega_+^2\Omega_-^2 - \frac{1}{2}(\Omega_+^2 + \Omega_-^2)^2\tan^2\left(\sqrt{2\sigma\bar{\sigma}}\right)\right]$$

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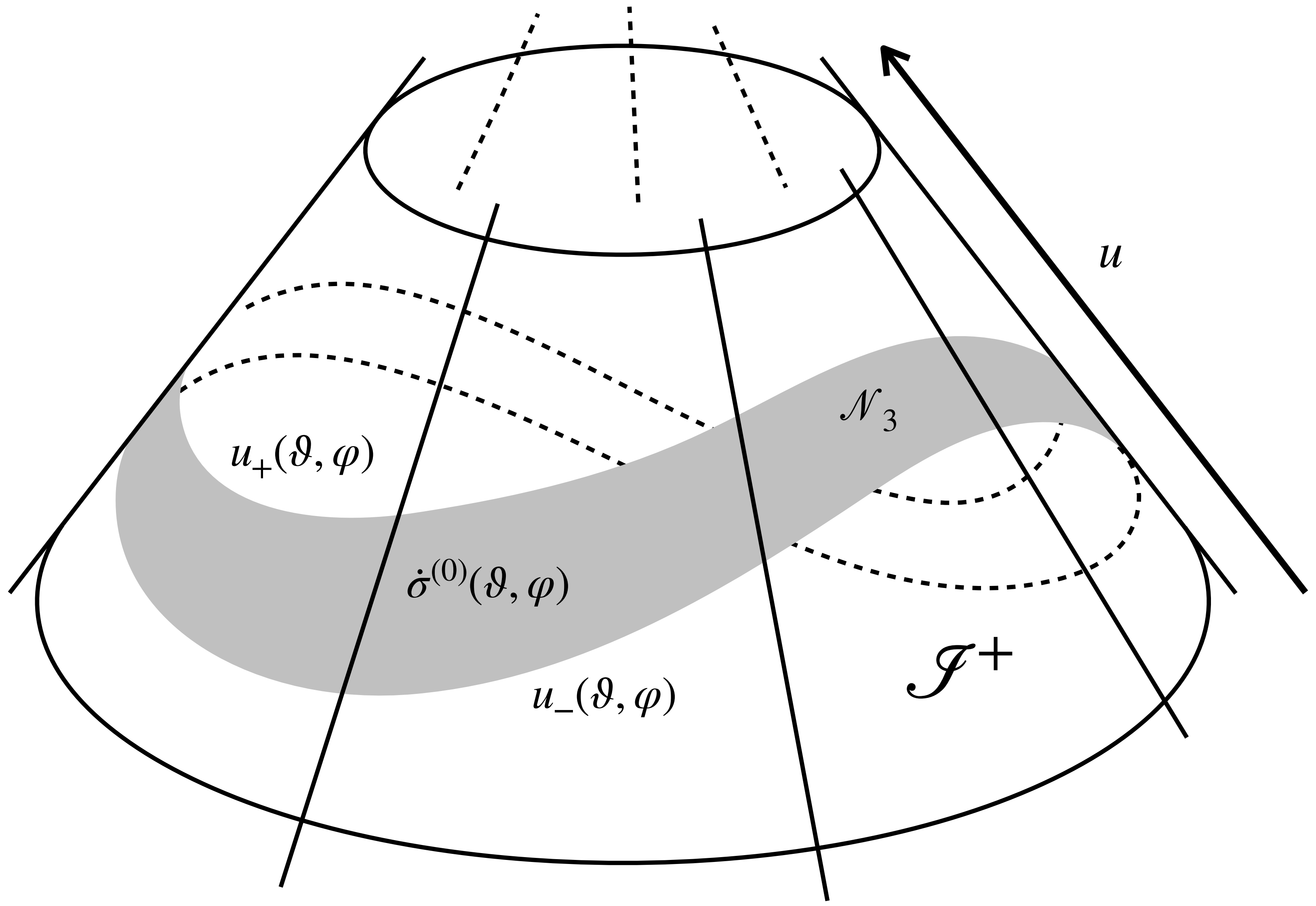
$$\boxed{|\sigma_{crit.}|^2 = \frac{1}{4}\frac{(\Omega_-^2 - \Omega_+^2)^2}{\gamma^2(\Omega_+^2 + \Omega_-^2)^2 + 4\Omega_+^2\Omega_-^2} + \mathcal{O}(|\sigma_{crit}|^3)}$$

Planck luminosity

When do we go from discrete to continuous representations?



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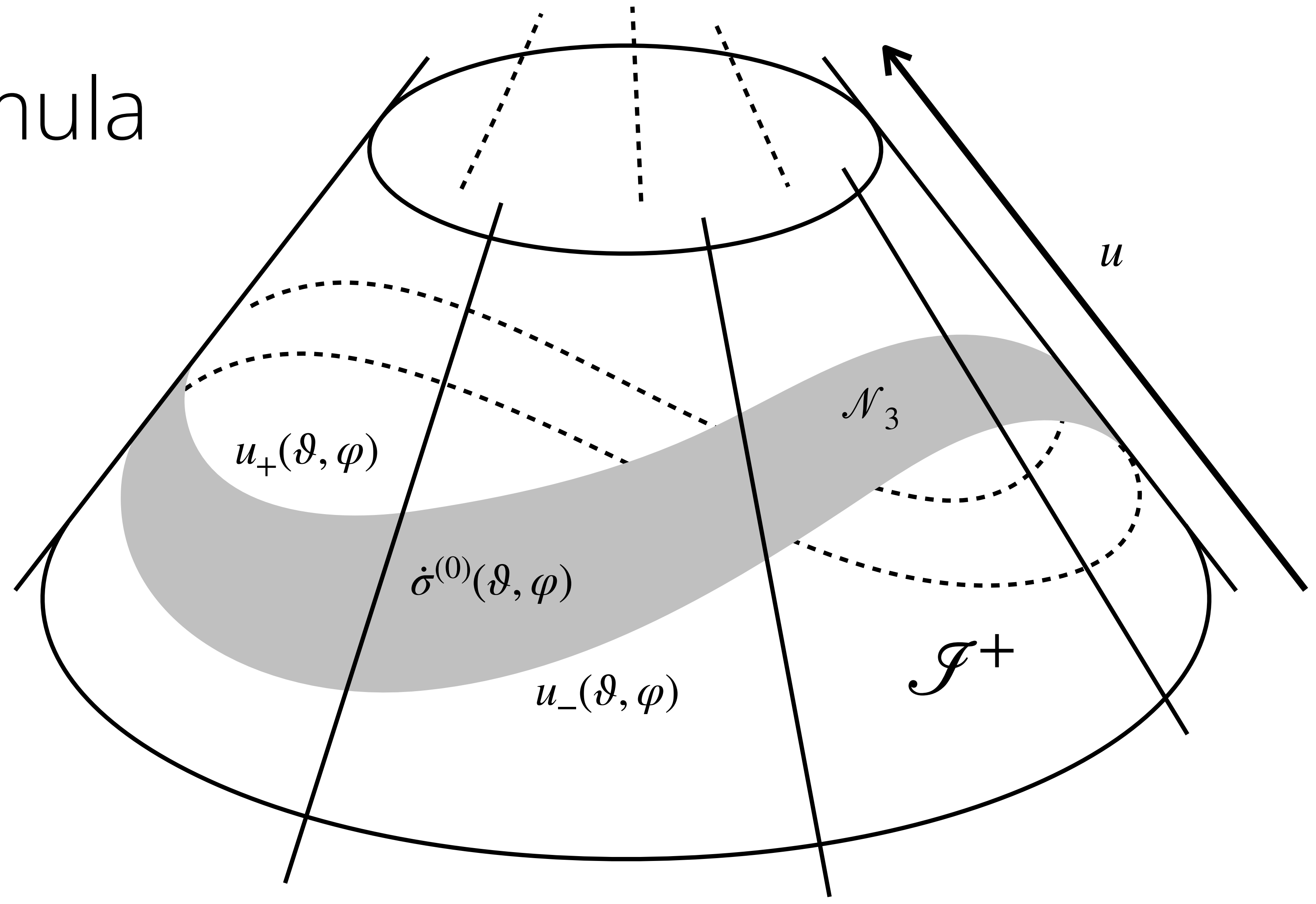

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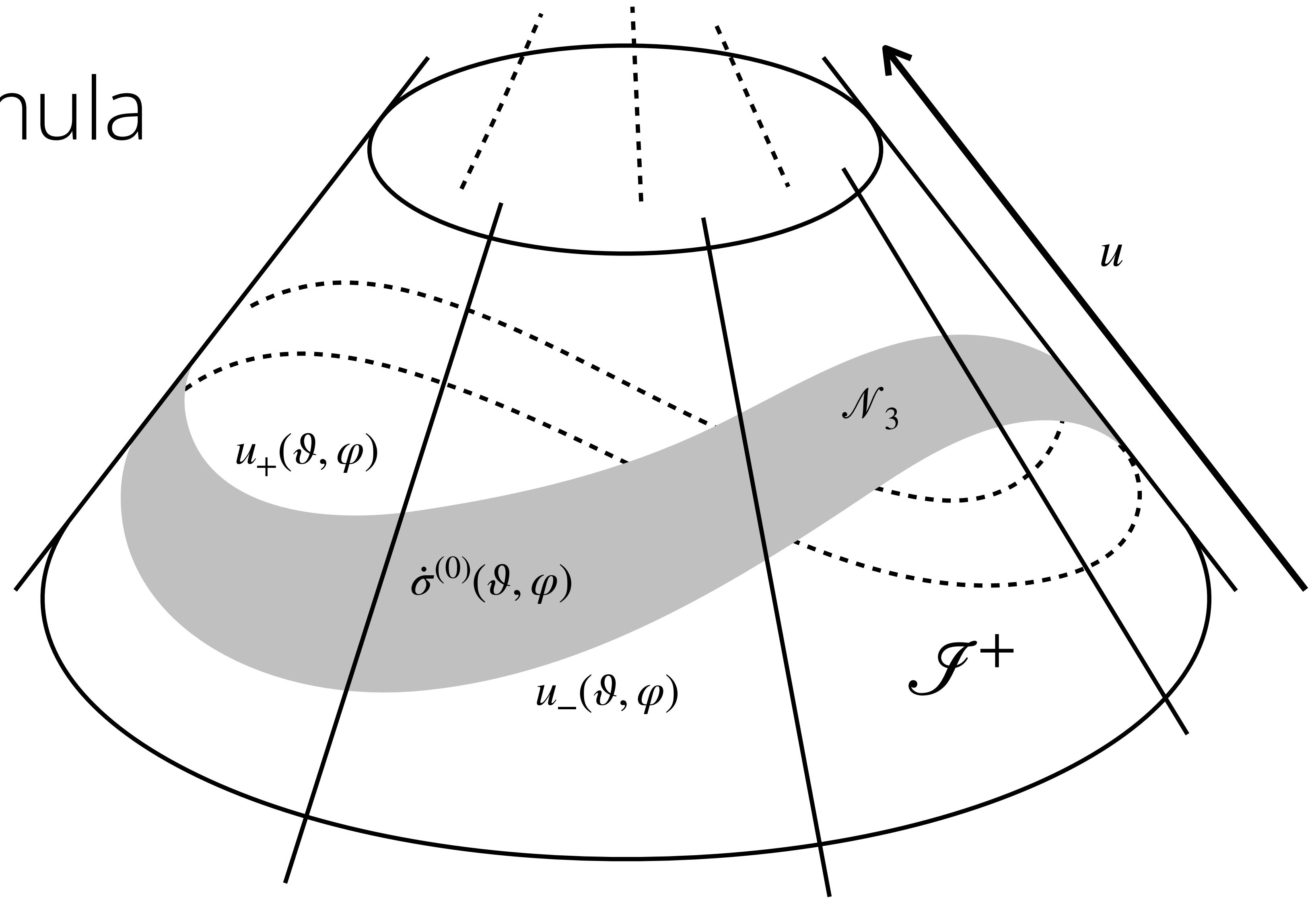

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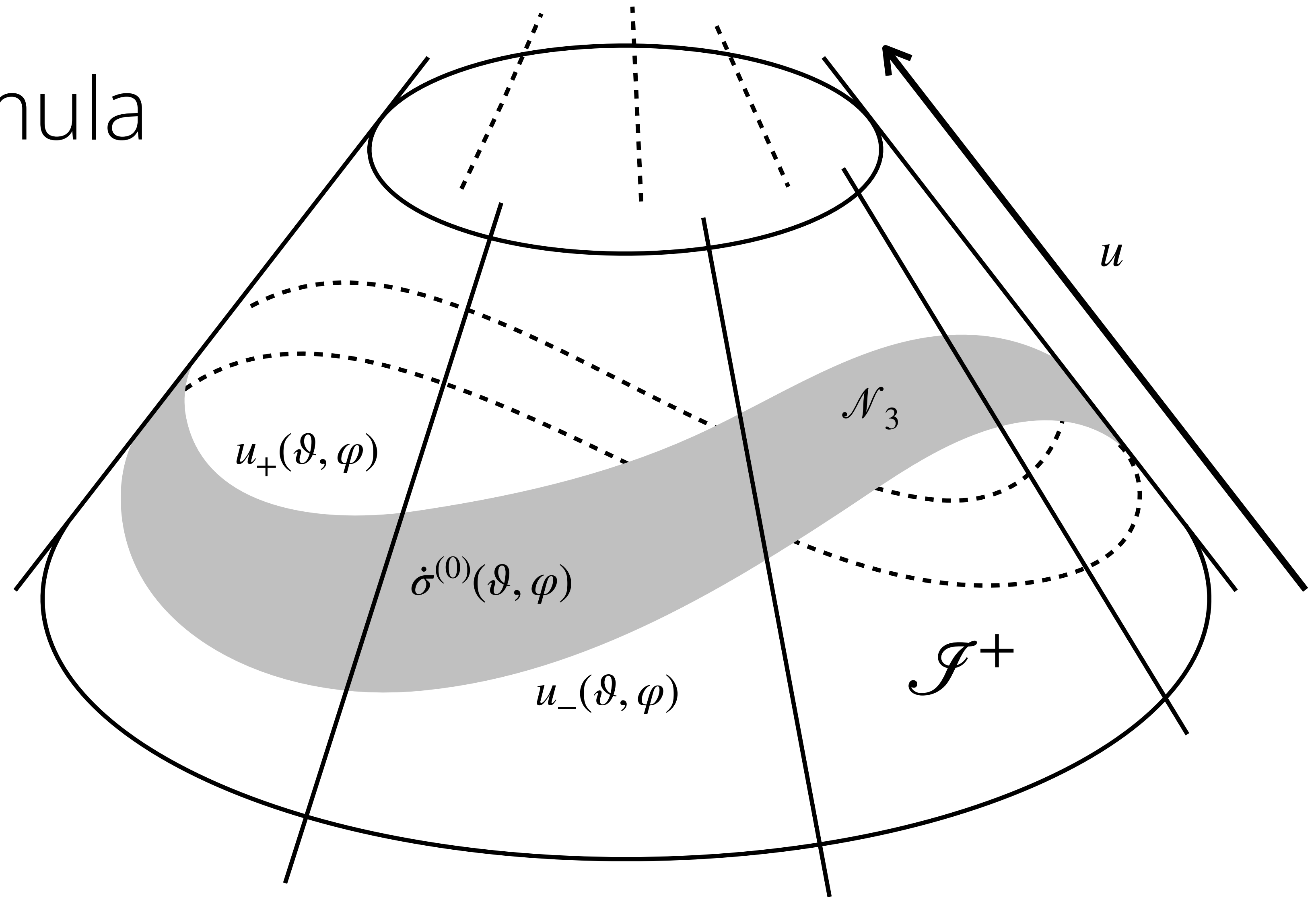

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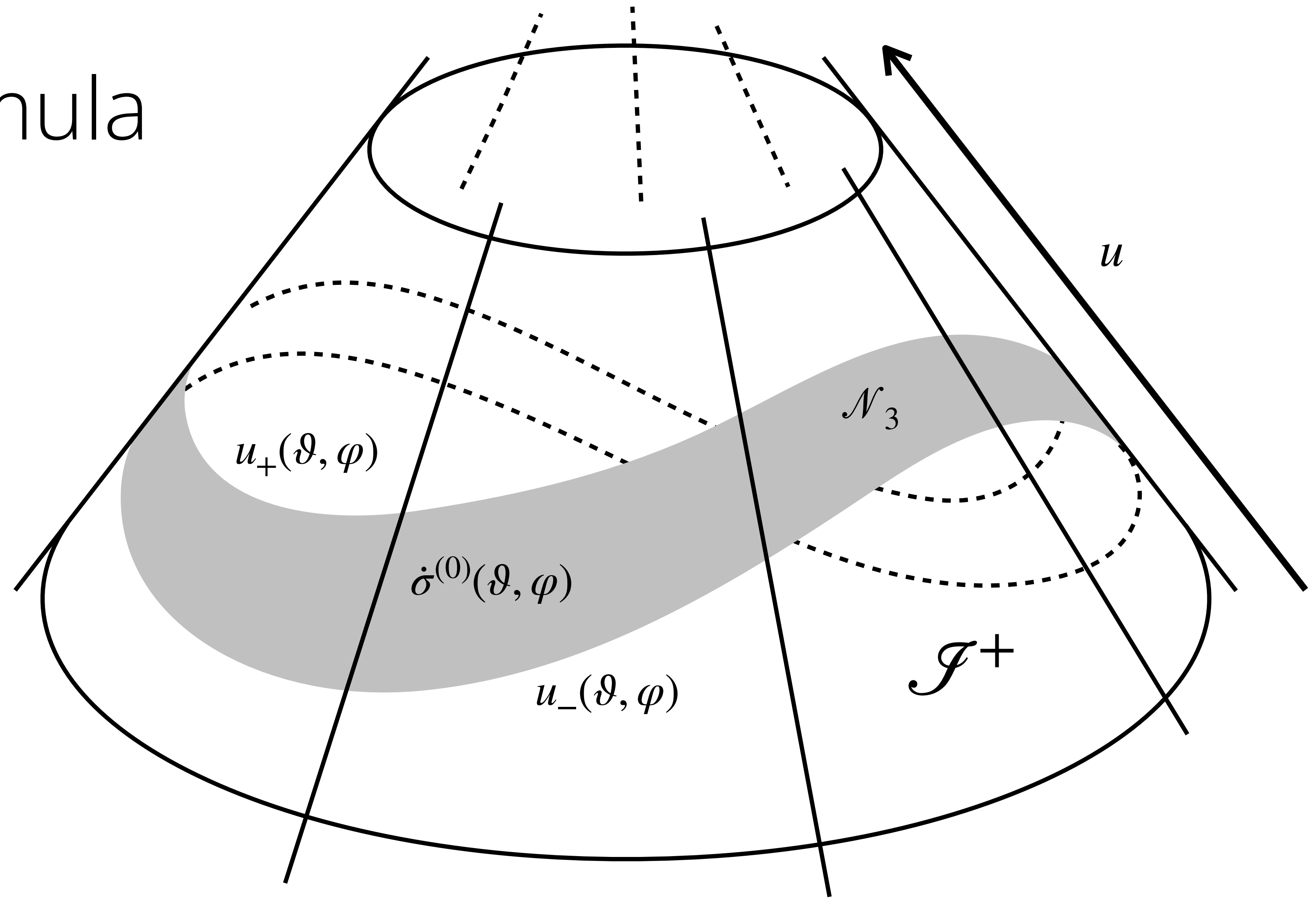

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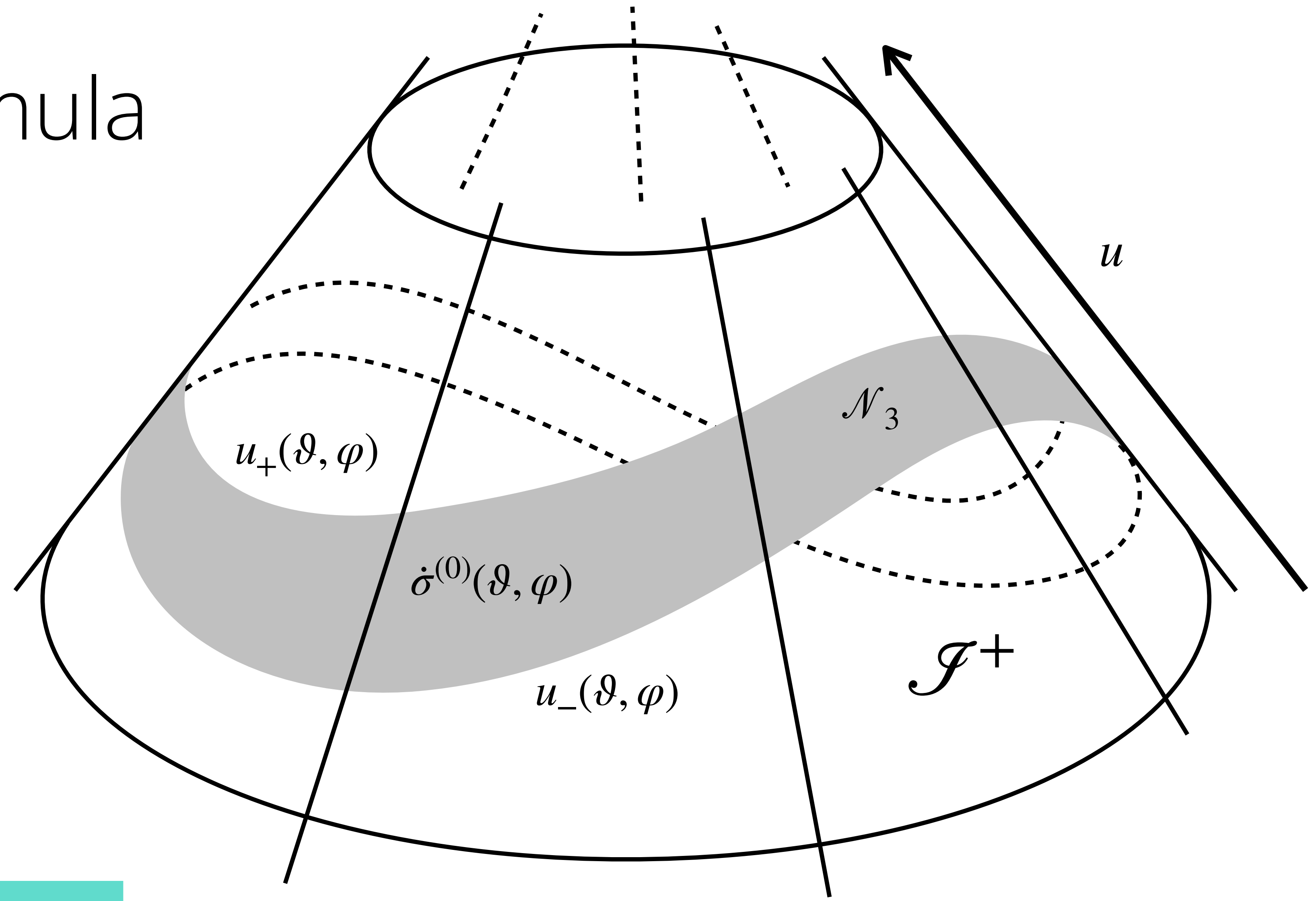
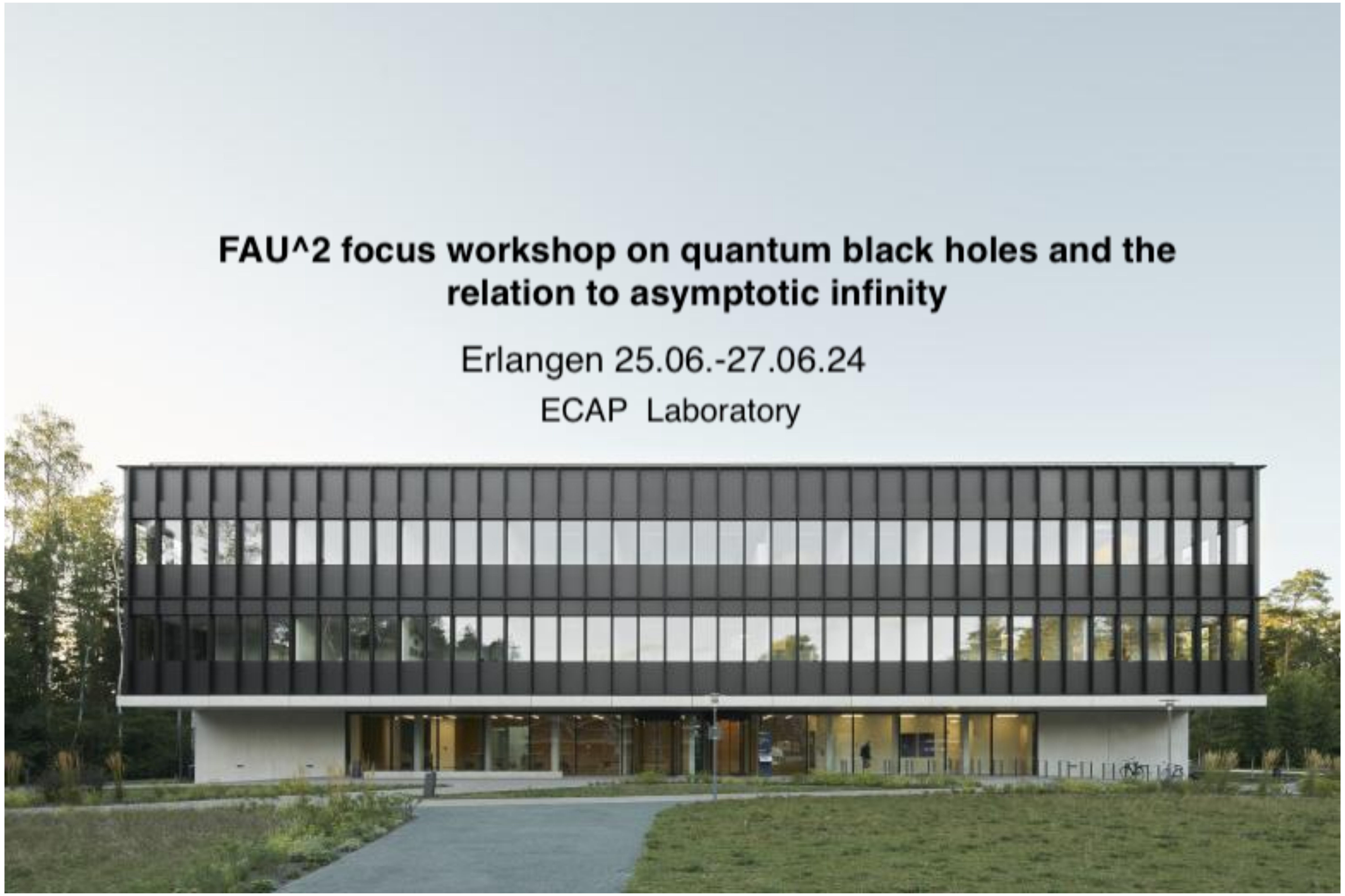


$$\boxed{\mathscr{L}_{crit.} = \frac{c^5}{4\pi G} \oint_{S_2} d^2\Omega \, |\, \dot{\sigma}^{(0)}_{crit.} |^2 = \frac{\mathscr{L}_{\mathrm{P}}}{\gamma^2 + 1}}$$

Summary and conclusion


FAU^2 focus workshop on quantum black holes and the relation to asymptotic infinity
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