Boundary symmetries in classical and quantum gravity

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Motivations

What is quantum gravity?

- What are the fundamental degrees of freedom? What is quantum geometry?
- What is the origin of black hole entropy? What are the microstates?
- What are the observables? What is the S-matrix?
- What is quantum general covariance? What are quantum reference frames?
- What are the UV and IR behaviors? What happens to singularities?
- What is the role of matter?
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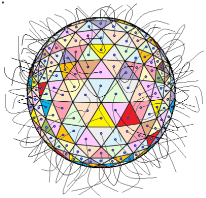
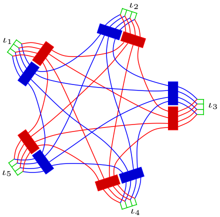
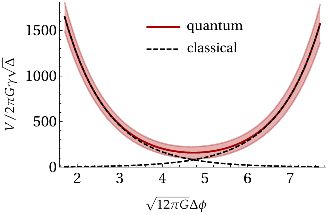


Loop quantum gravity

- LQG provides partial answers to these questions

$$Z^E_{eprl}(\Delta) = \sum_{j_f}\sum_{\iota_e}\prod_{f\in\Delta^\star} \mathrm{d}_{|1-\gamma|\frac{j}{2}}\mathrm{d}_{(1+\gamma)\frac{j}{2}}\prod_{v\in\Delta^\star}$$





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- LQG takes seriously the classical structure handed to us by Einstein's general relativity
- In particular, a strong emphasis is put on symmetries
  - background independence and diffeomorphism invariance
  - local Lorentz symmetry, leading to $\mathrm{SU}(2)$ spin network states and $\mathrm{SL}(2,\mathbb{C})$ amplitudes

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- Asking what are the symmetries of gravity is more subtle and rich than it appears
  - known old results (Noether's theorem) coming back in fashion in the last $\sim$10 years
  - what does this tell us about LQG, and what does LQG say about this?

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  - Carter's constant for Kerr and relation to Killing–Stäckel and Killing–Yano tensors
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  - these boundary may be at infinity (e.g. $\mathcal{I}^+$), finite distance (BH), or entangling surfaces
  - this mechanism is key to the distinction between gauge and physical charges
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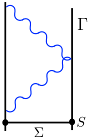
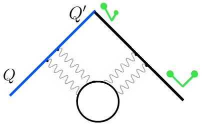
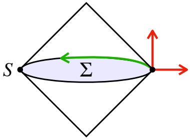
Natural questions arise

- Can we classify these boundary symmetry groups?
- Can we quantize/represent them?
- Which new insights do they give into classical and quantum gravity? [W. Wieland's talk]
- Is gravity holographic (or tomographic)? [S. Raju's talk, A. Ashtekar's talk]







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- **If these are features of classical gravity, should LQG implement or recover them?**

Outine

1. Symmetries in minisuperspace models
2. Symmetries of finite subregions
3. Symmetries of asymptotic boundaries
4. Perspectives

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Minisuperspace models

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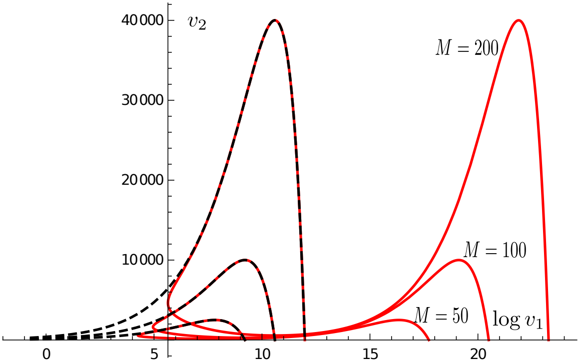

Figure: Effective bouncing trajectories $(\log v_1, v_2)$ (red) versus classical trajectories (dark dashed)

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- One can (should?) also find polymerization schemes which respect the symmetry [H. Liu's talk]
- Many generalizations (e.g. including Schrödinger algebra) [Ben Achour, Livine, Oriti, Piani]
  - these symmetries (and larger ones) arise from homothetic Killing vectors in field space
  - exists for Bianchi, FLRW with scalar field, Kantowski–Sachs with Λ, ...

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Charges and symmetries

- In gauge theories, Noether's theorem assigns codimension-2 charges to symmetries

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- Allows to distinguish gauge (vanishing) and physical (non-vanishing) charges
- A powerful technical tool is the covariant phase space formalism $\delta L = \text{EOM} \cdot \delta\Phi + \mathrm{d}\theta$
[Anderson, Ashtekar, Barnich, Brandt, Crnkovic, Henneaux, Kijowski, Lee, Wald, Witten, Zoupas]
- Lots of subtleties: integrability, conservation, bracket, renormalization, corner terms, ...
[Chandrasekaran, Ciambelli, Compère, Flanagan, Freidel, Fiorucci, MG, Harlow, Margalef-Bentabol, Oliveri, Pranzetti, Rignon-Bret, Ruzziconi, Speranza, Speziale, Villaseñor, Wieland, Wu, ...]

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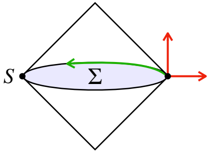
- General transformation of the charges

$$\begin{array}{ccccc} \delta_{\xi_1} Q_{\xi_2} & = & Q_{[\xi_1,\xi_2]} & + & \delta_{\xi_2} \cdot Q_{\text{flux}} \\ \text{evolution} & = & \text{rotation} & + & \text{dissipation} \end{array}$$

- Corner symmetry group [Ciambelli, Donnelly, Freidel, MG, Leigh, Pranzetti]

$$\begin{array}{ccccc} G_S & = & \big(\text{Diff}(S) \ltimes H\big) & \ltimes & \mathbb{R}^2 \\ \text{group} & = & \text{kinematical} & \ltimes & \text{dynamical} \end{array}$$



- Similar to coulombic vs radiative split [A. Ashtekar's talk]

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Formulation-dependence

- For a formulation F of gravity, the symplectic structure and kinematical symmetry group are

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- Different formulations have different symmetry groups → **inequivalent quantizations**

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Einstein–Hilbert	$\mathsf{SL}(2,\mathbb{R})_\perp$
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Discreteness at the classical and continuum level

- LQG symplectic structure

$$\Omega_{\mathsf{LQG}} = \Omega_{\mathsf{ADM}} + \mathrm{d}\big(\,\delta E_I\,\delta n^I + \gamma\,\delta e_I \wedge \delta e^I\,\big)$$

- The fluxes E_I form the familiar $\mathfrak{su}(2)$ algebra of LQG
- Tangential metric $q_{ab} = e^I_a e^J_b \eta_{IJ}$ on S forms an $\mathfrak{sl}(2,\mathbb{R})$ algebra

$$\big\{q_{ab}(\mathsf{x}), q_{cd}(\mathsf{y})\big\} = -\gamma\big(q_{ac}\epsilon_{bd} + q_{bc}\epsilon_{ad} + q_{ad}\epsilon_{bc} + q_{bd}\epsilon_{ac}\big)(\mathsf{x})\delta^2(\mathsf{x}-\mathsf{y})$$

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- Casimirs related by $\mathcal{C}_{\mathsf{SL}(2,\mathbb{R})} = -\big(\gamma^{-1}\sqrt{q}\big)^2 = \mathcal{C}_{\mathsf{SU}(2)} \rightarrow$ **quantization of area element**

$$\sqrt{q}(\mathsf{x}) = \gamma\ell^2_{\mathsf{Pl}} \sum_i \sqrt{j_i(j_i+1)}\,\delta^2(\mathsf{x}-\mathsf{x}_i)$$

Symmetries of finite subregions

Formulation-dependence

- For a formulation F of gravity, the symplectic structure and kinematical symmetry group are

$$\Omega_{\mathsf{F}} = \Omega_{\mathsf{ADM}} + \mathrm{d}\Omega_{\mathsf{F/ADM}} \qquad G_{\mathsf{kin}} = \mathsf{Diff}(S) \ltimes H$$

- Different formulations have different symmetry groups $\rightarrow$ **inequivalent quantizations**

formulation	H
ADM	$\emptyset$
Einstein–Hilbert	$\mathsf{SL}(2,\mathbb{R})_\perp$
Einstein–Cartan	$\mathsf{SL}(2,\mathbb{R})_\parallel \times$ (boosts)
Einstein–Cartan–Holst	$\mathsf{SL}(2,\mathbb{R})_\parallel \times \mathsf{SL}(2,\mathbb{C})$

Discreteness at the classical and continuum level

- LQG symplectic structure

$$\Omega_{\mathsf{LQG}} = \Omega_{\mathsf{ADM}} + \mathrm{d}\big(\delta E_I\,\delta n^I + \gamma\,\delta e_I \wedge \delta e^I\big)$$

- The fluxes E_I form the familiar $\mathfrak{su}(2)$ algebra of LQG
- Tangential metric $q_{ab} = e^I_a e^J_b \eta_{IJ}$ on S forms an $\mathfrak{sl}(2,\mathbb{R})$ algebra

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- Should we represent the whole quasi-local corner symmetry group G_S? (note BMS $\subset G_S$)

Symmetries of asymptotic boundaries

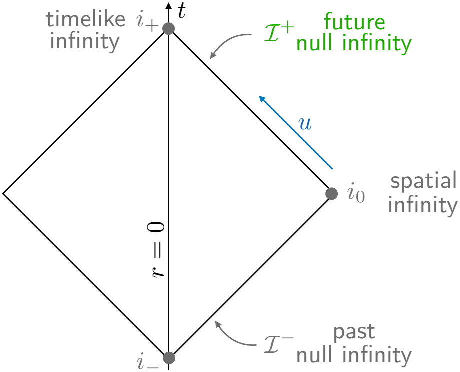
1. Symmetries in minisuperspace models

2. Symmetries of finite subregions

3. Symmetries of asymptotic boundaries

4. Perspectives

Symmetries of asymptotic boundaries

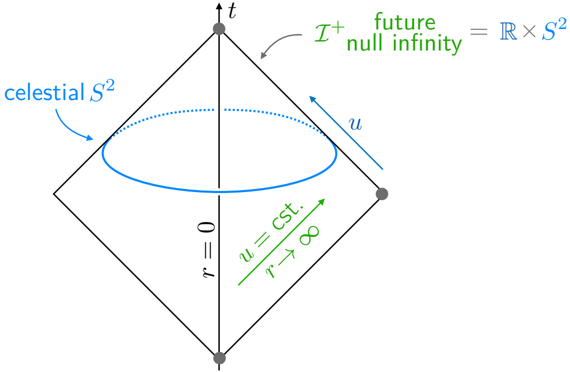


- Consider Minkowski in retarded null coordinates

$$\mathrm{d}s^2 = -\mathrm{d}u^2 - 2\mathrm{d}u\,\mathrm{d}r + r^2 q_{ab}\mathrm{d}x^a\mathrm{d}x^b$$

- The spacetime has 5 boundaries $= i_0 \cup i_+ \cup i_- \cup \mathcal{I}^- \cup \mathcal{I}^+$

Symmetries of asymptotic boundaries



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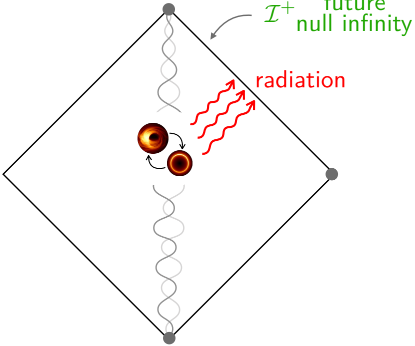
- Future null infinity $\mathcal{I}^+$ is the ideal region where to read off gravitational radiation [Ashtekar, Bondi, Geroch, Hansen, Metzner, Newman, Penrose, Sachs, Trautman, van der Burg]

Symmetries of asymptotic boundaries



- Consider Minkowski + radiation

$$\mathrm{d}s^2 = -\mathrm{d}u^2 - 2\mathrm{d}u\,\mathrm{d}r + r^2 q_{ab}\mathrm{d}x^a\mathrm{d}x^b + \mathcal{O}(r^{-1})$$

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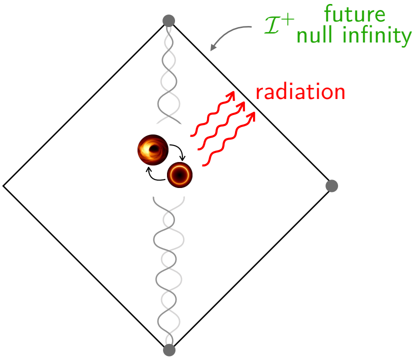
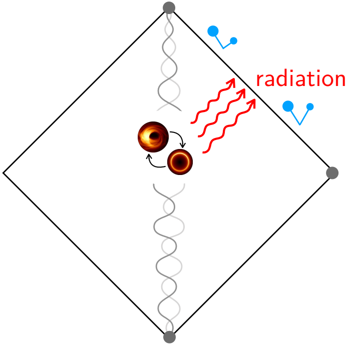
  [Ashtekar, Bondi, Geroch, Hansen, Metzner, Newman, Penrose, Sachs, Trautman, van der Burg]
- This is described by the notion of radiative asymptotically-flat spacetimes

Symmetries of asymptotic boundaries

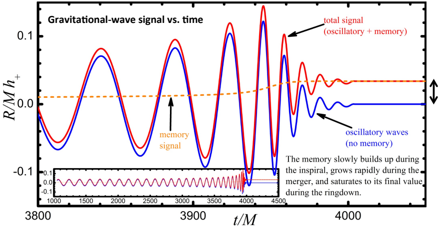


- Radiative asymptotically-flat spacetimes have very interesting properties

Symmetries of asymptotic boundaries



- Radiative asymptotically-flat spacetimes have very interesting properties
  - memory effects
    [Marc Favata]



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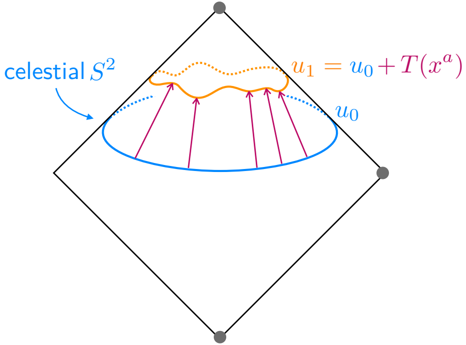
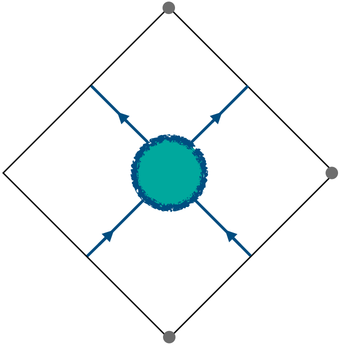


- Radiative asymptotically-flat spacetimes have very interesting properties
  - memory effects [Blanchet, Christodoulou, Damour, Polnarev, Thorne, Zel'dovich]
  - ∞-dimensional asymptotic symmetries [Bondi, Metzner, Sachs, van der Burg]

$$\xi = T\partial_u + Y^a\partial_a + D_aY^a(u\partial_u - r\partial_r) + \mathcal{O}(r^{-1}) \qquad \rightarrow \qquad \text{BMS} = \text{Diff}(S^2) \ltimes \mathbb{R}$$

Symmetries of asymptotic boundaries



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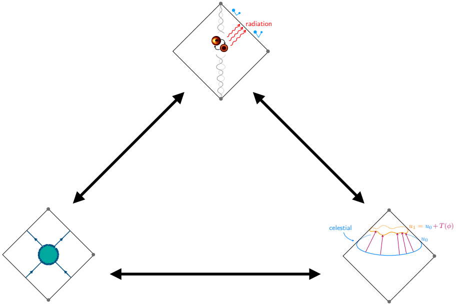
  - link with the S-matrix and soft theorems [Weinberg, Low]

$$\mathcal{A}_{n+1}(p_1,\dots,p_n,\omega q) = \sum_{n=-1}^{\infty} \omega^n S_n(p_1,\dots,p_n,q)\mathcal{A}_n(p_1,\dots,p_n) + \dots$$

Symmetries of asymptotic boundaries



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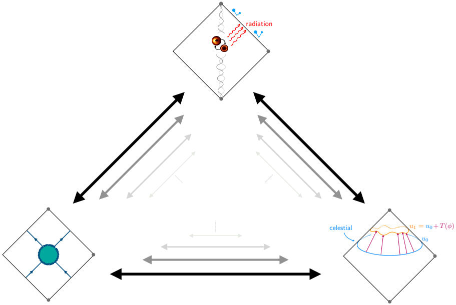
- All these aspects are connected through the so-called infrared triangle [Strominger et al.]

Symmetries of asymptotic boundaries



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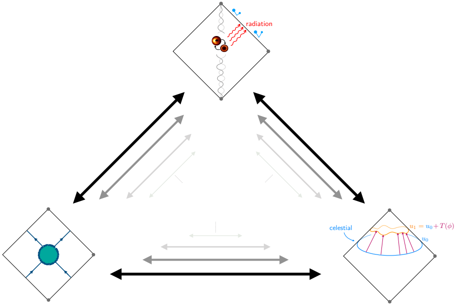
- There is now strong (yet subtle) evidence that many such subleading triangles exist in gravity

[Barnich, Cachazo, Campiglia, Compère, Conde, Fiorucci, Laddha, Mao, Ruzziconi, Strominger, Troessaert,. . .]

Symmetries of asymptotic boundaries



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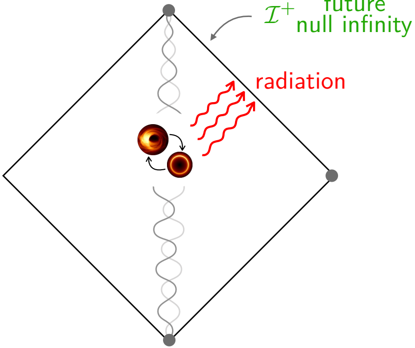
- **What is the symmetry interpretation of this subleading structure?**

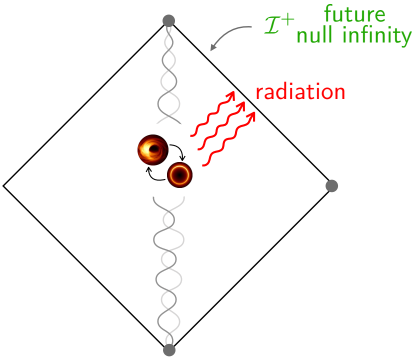
Symmetries of asymptotic boundaries



- Near $\mathcal{I}^+$ it is convenient to work in the Bondi gauge

$$\mathrm{d}s^2 = \left(-1 + \frac{M(u,x^a)}{r} + \dots\right)\mathrm{d}u^2 - (2 + \dots)\mathrm{d}u\,\mathrm{d}r + \left(\frac{P_a(u,x^a)}{r} + \dots\right)\mathrm{d}u\,\mathrm{d}x^a + g_{ab}\mathrm{d}x^a\mathrm{d}x^b$$

Symmetries of asymptotic boundaries



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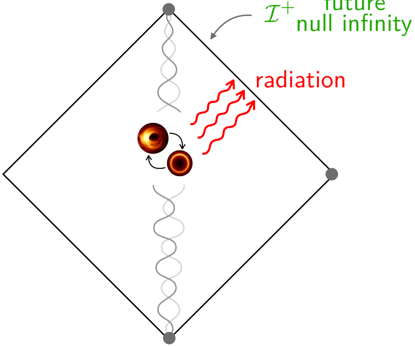
$$ds^2 = \left(-1 + \frac{M(u,x^a)}{r} + \dots\right) du^2 - (2 + \dots) du\, dr + \left(\frac{P_a(u,x^a)}{r} + \dots\right) du\, dx^a + g_{ab} dx^a dx^b$$

$$g_{ab} = r^2 \; q_{ab} \; + r\, C_{ab} + D_{ab} + \frac{1}{r} E^1_{ab} + \mathcal{O}(r^{-2})$$

Symmetries of asymptotic boundaries



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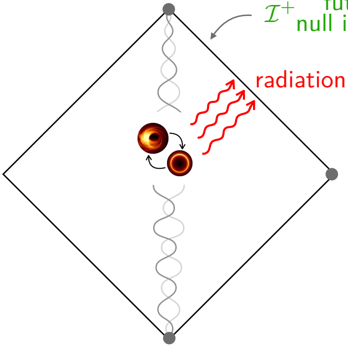

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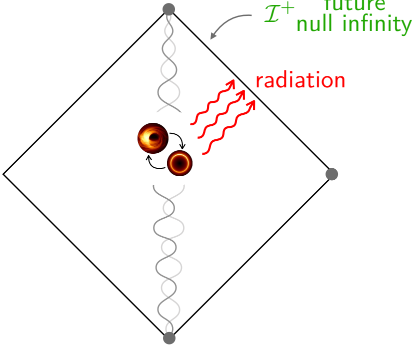
- 2 types of data: C_{ab} free on $\mathcal{I}^+$ and ∞-amount of data $(M, P_a, E^1_{ab}, \dots)$ satisfying EOMs

Symmetries of asymptotic boundaries



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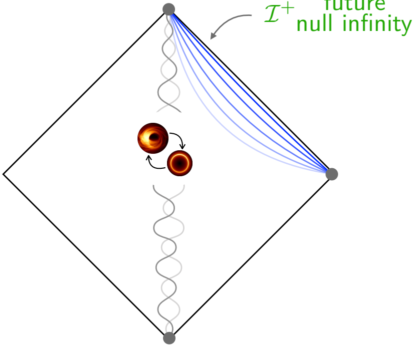
- 2 types of data: C_{ab} free on $\mathcal{I}^+$ and ∞-amount of data $(M, P_a, E^1_{ab}, \dots)$ satisfying EOMs
- The first flux balance laws is the Bondi–Trautman mass loss $\dot{M} = -N_{ab}N^{ab} + D_a D_b N^{ab}$

Symmetries of asymptotic boundaries



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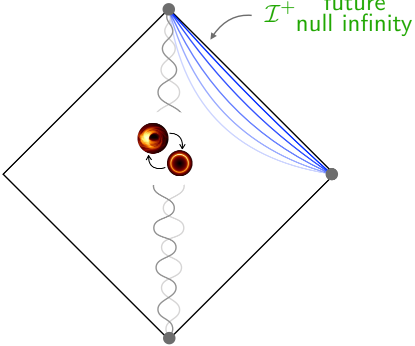
$$g_{ab} = r^2\ q_{ab}\ + r\,C_{ab} + D_{ab} + \frac{1}{r}\,E^1_{ab} + \mathcal{O}(r^{-2})$$

- We want to understand the subleading structure of the evolution equations for (M, P_a, E^n_{ab})

Symmetries of asymptotic boundaries



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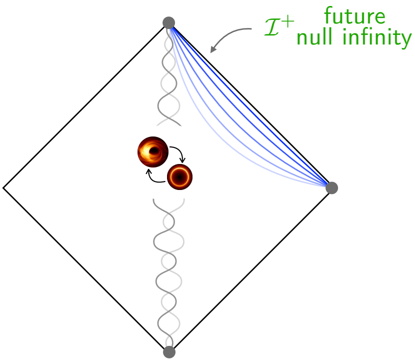
$$g_{ab} = r^2\ q_{ab}\ + r\, C_{ab} + D_{ab} + \frac{1}{r}\, E^1_{ab} + \mathcal{O}(r^{-2})$$

- M = spin 0 ↔ sub^0-leading soft graviton theorem ↔ supertranslations
- P_a = spin 1 ↔ sub^1-leading soft graviton theorem ↔ superrotations
- E^1_{ab} = spin 2 ↔ sub^2-leading soft graviton theorem ↔ non-local spin 2 symmetry
  [Weinberg] [Cachazo, Strominger] [Campiglia, Laddha] [Freidel, Pranzetti, Raclariu]

Symmetries of asymptotic boundaries



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$$g_{ab} = r^2\ q_{ab}\ + r\,C_{ab} + D_{ab} + \frac{1}{r}\,E^1_{ab} + \mathcal{O}(r^{-2})$$

- The EOMs for E^n_{ab} come from the Einstein equations $G^{\mathsf{TF}}_{ab} = 0$
- The study of the data (M, P_a, E^n_{ab}) is much easier in the Newman–Penrose formalism ...

Symmetries of asymptotic boundaries

Setup

- Using a null tetrad $e_i = (\ell, n, m, \bar{m})$, one builds the spin coefficients $\gamma_{ijk} = e_j^\mu e_k^\nu \nabla_\nu e_{i\mu}$ and the Weyl scalars (here labelled by their spin/helicity)

Symmetries of asymptotic boundaries

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$$\Psi_0 = \frac{Q_2}{r^5} + \mathcal{O}(r^{-6})$$

$$\Psi_1 = \frac{Q_1}{r^4} + \mathcal{O}(r^{-5})$$

$$\Psi_2 = \frac{Q_0}{r^3} + \mathcal{O}(r^{-4})$$

$$\Psi_3 = \frac{Q_{-1}}{r^2} + \mathcal{O}(r^{-3})$$

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- In terms of the previous Bondi free and initial data we have

$$Q_{-2}(\dot{N}_{ab}) \qquad Q_{-1}(N_{ab}) \qquad Q_0(M, C_{ab}) \qquad Q_1(P_a, C_{ab}) \qquad Q_2(E^1_{ab}, C_{ab})$$

Symmetries of asymptotic boundaries

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- NB: one should choose a tetrad such that the spin coefficients are $\kappa = \pi = \epsilon = 0$ and $\rho = \bar{\rho}$

Symmetries of asymptotic boundaries

Sub-leading expansion of the Weyl scalars

- Using a null tetrad $e_i = (\ell, n, m, \bar{m})$, one builds the spin coefficients $\gamma_{ijk} = e_j^\mu e_k^\nu \nabla_\nu e_{i\mu}$ and the Weyl scalars (here labelled by their spin/helicity)

$$
\begin{aligned}
\Psi_0 &= \frac{Q_2}{r^5} + \mathcal{O}(r^{-6}) \\
\Psi_1 &= \frac{Q_1}{r^4} - \frac{\bar{\eth} Q_2}{r^5} + \mathcal{O}(r^{-6}) \\
\Psi_2 &= \frac{Q_0}{r^3} - \frac{\bar{\eth} Q_1}{r^4} + \mathcal{O}(r^{-5}) \\
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\Psi_4 &= \frac{Q_{-2}}{r^1} - \frac{\bar{\eth} Q_{-1}}{r^2} + \mathcal{O}(r^{-3})
\end{aligned}
$$

- In terms of the previous Bondi free and initial data we have

$$
Q_{-2}(\dot{N}_{ab}) \qquad Q_{-1}(N_{ab}) \qquad Q_0(M, C_{ab}) \qquad Q_1(P_a, C_{ab}) \qquad Q_2(E^1_{ab}, C_{ab})
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- NB: one should choose a tetrad such that the spin coefficients are $\kappa = \pi = \epsilon = 0$ and $\rho = \bar{\rho}$

Symmetries of asymptotic boundaries

Introducing the higher spin charges

- Using a null tetrad $e_i=(\ell,n,m,\bar{m})$, one builds the spin coefficients $\gamma_{ijk}=e^\mu_j e^\nu_k \nabla_\nu e_{i\mu}$ and the Weyl scalars (here labelled by their spin/helicity)

$$
\begin{aligned}
\Psi_0 &= \frac{Q_2}{r^5}-\frac{\bar{\eth}Q_3}{r^6}+\frac{\bar{\eth}^2Q_4+\dots}{r^7}+\mathcal{O}(r^{-8})\\
\Psi_1 &= \frac{Q_1}{r^4}-\frac{\bar{\eth}Q_2}{r^5}+\mathcal{O}(r^{-6})\\
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\end{aligned}
$$

- In terms of the previous Bondi free and initial data we have

$$
Q_{-2}(\dot{N}_{ab}) \qquad Q_{-1}(N_{ab}) \qquad Q_0(M,C_{ab}) \qquad Q_1(P_a,C_{ab}) \qquad Q_2(E^1_{ab},C_{ab})
$$

- NB: one should choose a tetrad such that the spin coefficients are $\kappa=\pi=\epsilon=0$ and $\rho=\bar{\rho}$
- One can introduce by hand higher spin charges $Q_{s\geq3}$ in the expansion for Ψ_0
- In terms of the previous Bondi data we have $Q_{s\geq2}(E^{s-1}_{ab},C_{ab})$

Interpretation

- Using a null tetrad $e_i = (\ell, n, m, \bar{m})$, one builds the spin coefficients $\gamma_{ijk} = e_j^\mu e_k^\nu \nabla_\nu e_{i\mu}$ and the Weyl scalars (here labelled by their spin/helicity)

$$
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- Q_0 and Q_1 are the leading BMS_4 charges (essentially mass and angular momentum)

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- Using a null tetrad $e_i = (\ell, n, m, \bar{m})$, one builds the spin coefficients $\gamma_{ijk} = e_j^\mu e_k^\nu \nabla_\nu e_{i\mu}$ and the Weyl scalars (here labelled by their spin/helicity)

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Symmetries of asymptotic boundaries

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- $Q_{s\geq 2} \sim$ Newman–Penrose charges [Newman, Penrose]

  $\sim$ subleading BMS charges [Godazgar, Godazgar, Long] [MG]

  $\sim$ canonical multipole moments [Compère, Oliveri, Seraj]

Symmetries of asymptotic boundaries

Evolution equations

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- Introducing $C := C_{ab} m^a m^b = h_\times + i h_+$, the asymptotic Einstein equations can be written as

$$\boxed{\partial_u Q_s = \eth Q_{s-1} - (s+1) C Q_{s-2}}$$

Symmetries of asymptotic boundaries

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Linearized bracket

- Integrate the equations of motion to write iteratively $Q_s = \partial_u^{-1} \eth Q_{s-1} - (s+1) \partial_u^{-1} \big(C Q_{s-2} \big)$
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$$q_s = \sum_{k=1}^{k_{\text{max}}} q_s^k = \underbrace{q_s^1}_{\text{soft}} + \underbrace{q_s^2}_{\text{hard}} + \mathcal{O}(C^3)$$

- Use the Ashtekar–Streubel symplectic structure $\big\{ C(u,z), \bar{N}(u',z') \big\} = \delta(u-u')\delta(z-z')$ to compute the linearized bracket

$$\big\{ q_{s_1}, q_{s_2} \big\}^{(1)} = \big\{ q^1_{s_1}, q^2_{s_2} \big\} + \big\{ q^2_{s_1}, q^1_{s_2} \big\}$$

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- After a daunting calculation, one arrives at the $w_{1+\infty}$ loop algebra [Adamo, Ball, Freidel, MG, Guevara, Mason, Narayanan, Pranzetti, Raclariu, Salzer, Sharma, Strominger]

$$\boxed{\big\{q_{s_1}(Z_1), q_{s_2}(Z_2)\big\}^{(1)} = -\,q^1_{s_1+s_2-1}\Big((s_1+1)Z_1\eth Z_2 - (s_2+1)Z_2\eth Z_1\Big)}$$

Symmetries of asymptotic boundaries

Interpretation

- NP version of a result obtained from twistor theory and from the celestial soft graviton OPE [Adamo, Ball, Donnay, Freidel, Guevara, Herfray, Himwich, Mason, Narayanan, Pate, Pranzetti, Raclariu, Ruzziconi, Salzer, Sharma, Strominger, Yelleshpur Srikant]
- Symmetry algebra which governs the subleading structure of (self-dual?) gravity

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- The quasi-conserved soft charges correspond to temporal moments of the news

$$q^1_s(u,z) = \eth^{s+2} \bar{N}_s(u,z) \qquad\qquad \bar{N}_s(u,z) = \frac{(-1)^s}{s!} \int_{-\infty}^{u} \mathrm{d}u' \, u'^s \bar{N}(u',z)$$

and their flux-balance laws give rise to the so-called higher memory effects [Grant, Nichols] [Grant, Mitman] [Flanagan, Grant, Harte, Nichols] [Compère, Oliveri, Seraj]

Perspectives

1. Symmetries in minisuperspace models

2. Symmetries of finite subregions

3. Symmetries of asymptotic boundaries

4. Perspectives

Perspectives

Surprizing symmetry structures in gravity

- Already in minisuperspaces: coincidences or features?
- At finite distance
  - quasi-local corner symmetry group G_S related to kinematics and dynamics
  - LQG has states labelled by a subgroup of G_S: should we represent all of G_S?
- At infinity
  - $w_{1+\infty}$ algebra controls the subleading structure of asymptotically-flat spacatimes
  - can we use this algebraic structure to inform numerical codes and extract physics?
  - are the higher spin symmetries related to hidden symmetries (e.g. Killing tensors?)
  - what happens in dS: radiation, symmetries, cosmological memories?
  - logarithmic soft theorems [Choi, Das, MG, Laddha, Puhm, Sahoo, Saha, Sen, Zwikel] and loss of peeling at $\mathcal{I}^+$ [Bieri, Blanchet, Christodoulou, Chrusciel, Damour, Friedrich, Gajic, Kehrberger, Klainerman, Kroon, Laddha, MacCallum, Masaood, Singleton, Winicour]

Opportunities for LQG

- LQG quantization of quasi-local symmetry group?
- LQG quantization of null infinity? [A. Ashtekar's talk, W. Wieland's talk]
- Possible observational signatures of tetrad variables and Barbero–Immirzi parameter?

ISLQG - Thank you to all the members and candidates

Ballot for president and president elect

- Guillermo A. Mena Marugán (CSIC)
- Hanno Sahlmann (FAU Erlangen-Nürnberg)

Ballot for the board

- Ivan Agullo (Louisiana State University)
- Kristina Giesel (FAU Erlangen-Nürnberg)
- Florian Girelli (University of Waterloo)
- Hal Haggard (Bard College)
- Muxin Han (Florida Atlantic University)
- Jerzy Lewandowski (Uniwersytet Warszawski)
- Etera Livine (CNRS, ENS de Lyon - Laboratoire de Physique)
- Yongge Ma (Beijing Normal University)
- Mercedes Martín-Benito (Universidad Complutense de Madrid)
- Daniele Oriti (Universidad Complutense de Madrid)
- Francesca Vidotto (Western University)
- Anzhong Wang (Baylor University)
- Wolfgang Wieland (FAU Erlangen-Nürnberg)
- Edward Wilson-Ewing (University of New Brunswick)