# Ultracompact stars with polynomial complexity by gravitational decoupling

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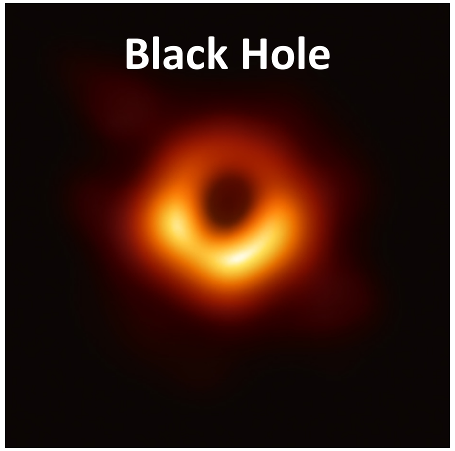
# Contents

# MOTIVATION

- Buchdahl's Limit: $\frac{M}{R} < \frac{4}{9}$ → What if we surpass this limit? → **ULTRACOMPACT STAR**




EHT, 2019

↓

Singularity

- Special Case: Mazur & Mottola (2004) – uniform density spherical star

Let the interior be described by

$$ds^2 = e^{\nu}dt^2 - e^{\lambda}dr^2 - r^2(d\theta^2 + \mathrm{Sin}^2\theta d\phi^2),$$ where

$$e^{\nu} = \frac{1}{4}\left(3\sqrt{1 - H^2R^2} - \sqrt{1 - H^2r^2}\right)^2,$$

$$e^{-\lambda} = 1 - H^2r^2,$$

$$H^2 = \frac{2M}{R^3},$$

# MOTIVATION

Consider uniform density $\rho = \rho_0$

$$\Rightarrow p = \rho_0 \left( \frac{1 - H^2 r^2 - \sqrt{1 - H^2 R^2}}{3\sqrt{1 - H^2 R^2} - \sqrt{1 - H^2 r^2}} \right)$$

regular except at $R_0 = 3R\sqrt{1 - \frac{8}{9}\frac{R}{R_S}}$

In the ultracompact limit: $R = R_0 = R_S = 2M$ → $p = -\rho_0 = constant$

Mazur and Mottola model:

| | | | |
|---|---|---|---|
| I. | Interior: | $0 \leq r < r_1,$ | $\rho = -p,$ |
| II. | Thin Shell: | $r_1 < r < R,$ | $\rho = +p$ |
| III. | Exterior: | $r > R,$ | $\rho = p = 0.$ |

# GRAVITATIONAL DECOUPLING

- Einstein Field Equations: $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T_{\mu\nu}$ with $T_{\mu\nu} = T_{\mu\nu}^{(s)} + \alpha\theta_{\mu\nu}$ (1)

$\boldsymbol{r < R}$

- Consider a static spherically symmetric spacetime: $ds^2 = e^{\nu} dt^2 - e^{\lambda} dr^2 - r^2(d\theta^2 + \mathrm{Sin}^2\theta d\phi^2)$ (2)

$$T_{\nu}^{\mu(s)} = \begin{pmatrix} \rho^{(s)} & 0 & 0 & 0 \\ 0 & -p_r^{(s)} & 0 & 0 \\ 0 & 0 & -p_{\perp}^{(s)} & 0 \\ 0 & 0 & 0 & -p_{\perp}^{(s)} \end{pmatrix} \qquad \theta_{\nu}^{\mu} = \begin{pmatrix} \theta_0^0 & 0 & 0 & 0 \\ 0 & \theta_1^1 & 0 & 0 \\ 0 & 0 & \theta_2^2 & 0 \\ 0 & 0 & 0 & \theta_2^2 \end{pmatrix} \tag{3}$$

$$\rho = \rho^{(s)} + \alpha\theta_0^0 \qquad p_r = p_r{}^{(s)} - \alpha\theta_1^1 \qquad p_{\perp} = p_{\perp}{}^{(s)} - \alpha\theta_2^2 \tag{4}$$

$\boldsymbol{r < R}$

# GRAVITATIONAL DECOUPLING

- Einstein Field Equations:

$$\kappa\rho = \frac{1}{r^2} + e^{-\lambda}\left(\frac{\lambda'}{r} - \frac{1}{r^2}\right)$$

$$\kappa p_r = -\frac{1}{r^2} + e^{-\lambda}\left(\frac{\nu'}{r} + \frac{1}{r^2}\right) \qquad (5)$$

$$\kappa p_\perp = \frac{1}{4}e^{-\lambda}\left(2\nu'' + \nu'^2 - \lambda'\nu' + 2\frac{\nu' - \lambda'}{r}\right)$$

- Minimal Geometric Deformation (MGD)

$$\nu \rightarrow \xi + \alpha g \qquad (6)$$

$$e^{-\lambda} \rightarrow e^{-\mu} + \alpha f \qquad (7)$$

MGD $\Rightarrow$ $g = 0, f \neq 0$

$Re$: $T_{\mu\nu} = T^{(s)}_{\mu\nu} + \alpha\theta_{\mu\nu}$

**EFE split into 2 parts**

Seed sector, $T^{(s)}_{\mu\nu}$

New sector, $\theta_{\mu\nu}$

$r > R$

# GRAVITATIONAL DECOUPLING

**MODIFIED VACUUM**

$$ds^2 = \left(1 - \frac{2M}{r}\right)dt^2 - \left(1 - \frac{2M}{r} + \alpha g(r)\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (8)$$

- Matching Conditions

$$e^{\nu}|_{\Sigma^-} = \left(1 - \frac{2M}{r}\right)|_{\Sigma^+} \quad (9)$$

$$e^{\lambda}|_{\Sigma^-} = \left(1 - \frac{2M}{r} + \alpha g(r)\right)^{-1}|_{\Sigma^+} \quad (10)$$

$$p_r(r)|_{\Sigma^-} = p_r(r)|_{\Sigma^+} \quad (11)$$

**3 EFE for 5 unknowns,**

$\{\nu, \lambda, \rho, p_r, p_\perp\}$

↓ MGD & known seed sector

$\{\lambda, \rho, p_r, p_\perp\}$

↓ Relating the unknown sector

$\{f, \rho, p_r, p_\perp\}$

3 EFE for 4 unknowns ➡ Add information: **COMPLEXITY!**

# COMPLEXITY OF COMPACT STARS

- Least complex gravitational system: homogeneous energy density and isotropic pressure.
- Scalar associated: **complexity factor,**

$$Y_{\mathrm{TF}} = 8\pi \Pi - \frac{4\pi}{r^3} \int_0^r \tilde{r}^3 \rho' \, \mathrm{d}\tilde{r} \qquad (11)$$

with

$$\Pi \equiv p_r - p_\perp \qquad (12)$$

- Tolman mass:

$$m_T = (m_T)_\Sigma \left(\frac{r}{r_\Sigma}\right)^3 + r^3 \int_r^{\mathrm{r}\Sigma} \frac{\mathrm{e}^{(\nu+\lambda)/2}}{\tilde{r}} Y_{\mathrm{TF}} \, \mathrm{d}\tilde{r} \qquad (13)$$

# ULTRACOMPACT STAR BY GRAVITATIONAL DECOUPLING

Let

$$e^{\nu} = \frac{1}{4}(1 - H^2 r^2) \quad (14)$$

$$e^{-\lambda} = 1 - H^2 r^2 + \alpha f(r) \quad (15)$$

$$H^2 = \frac{2M}{R^3} \quad (16)$$

4 unknows and 3 equations ➡ add information ➡ $f(r)$

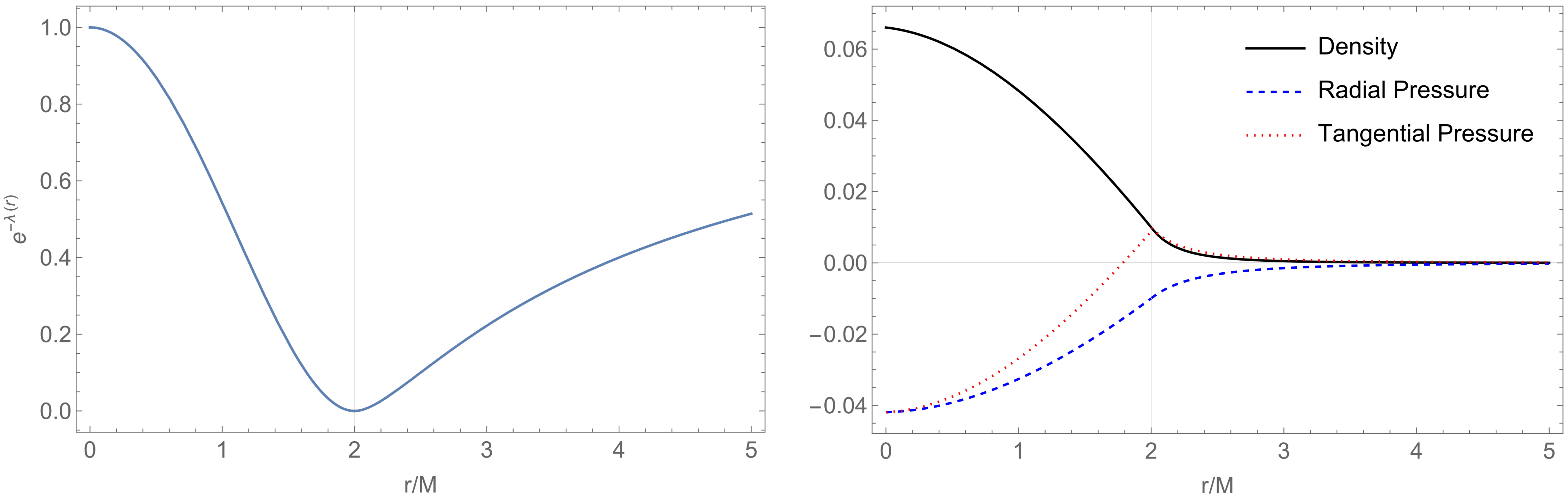
**COMPLEXITY**

$$Y_{TF} = \sum_{i=0}^{N} a_i r^i = 8\pi\Pi - \frac{4\pi}{r^3}\int_0^r \tilde{r}^3 \rho' \, d\tilde{r} \quad (17)$$

| **INTERIOR:** complexity | **EXTERIOR:** 2 deformed vacuums |
|---|---|
| With $a_0 = a_1 = 0,\ N = 3$ | $g(r) = \left(1 - \frac{2M}{r}\right)\left(\frac{\ell}{2r - 3M}\right) \quad (19)$ |
| ➡ $f(r) = \frac{2}{\alpha H^2}(1 - H^2 r^2)\left(\frac{a_2 r^2}{2} + \frac{a_3 r^3}{3}\right) \quad (18)$ | $g(r) = \left(1 - \frac{2M}{r}\right)\left(\frac{1}{(r+M)^{2(a-1)}}\right) \quad (20)$ |

# ULTRACOMPACT STAR BY GRAVITATIONAL DECOUPLING

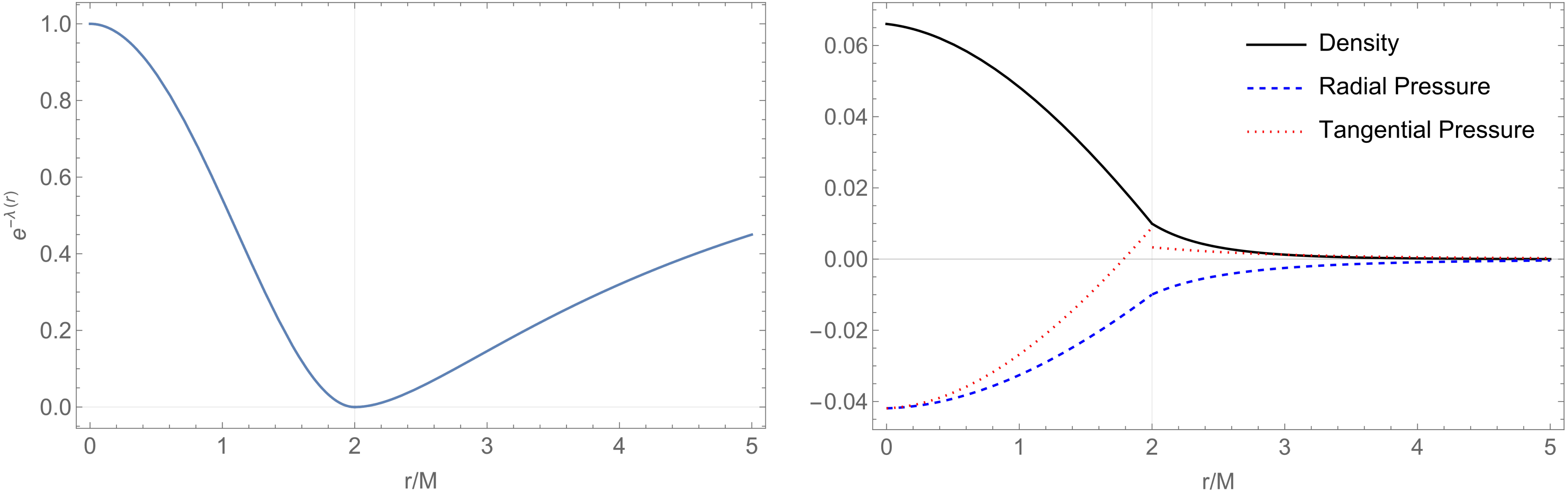
**EXTERIOR 1**




$$a_3 = 0.01, \ell = -1$$

# ULTRACOMPACT STAR BY GRAVITATIONAL DECOUPLING

**EXTERIOR 2**




$$a_3 = 0.01, \beta = -9, a = 2$$

# CONCLUSIONS

- New model for an ultracompact anisotropic star using MGD approach.
- System is closed by the complexity factor.
- Interior solution well matched with 2 different modified vacuum.
- Solution fulfill all the requirements.
  - Regular at the origin.
  - Positive density and decreases monotonously.
  - Radial pressure is monotonic and non uniform.


THANKS
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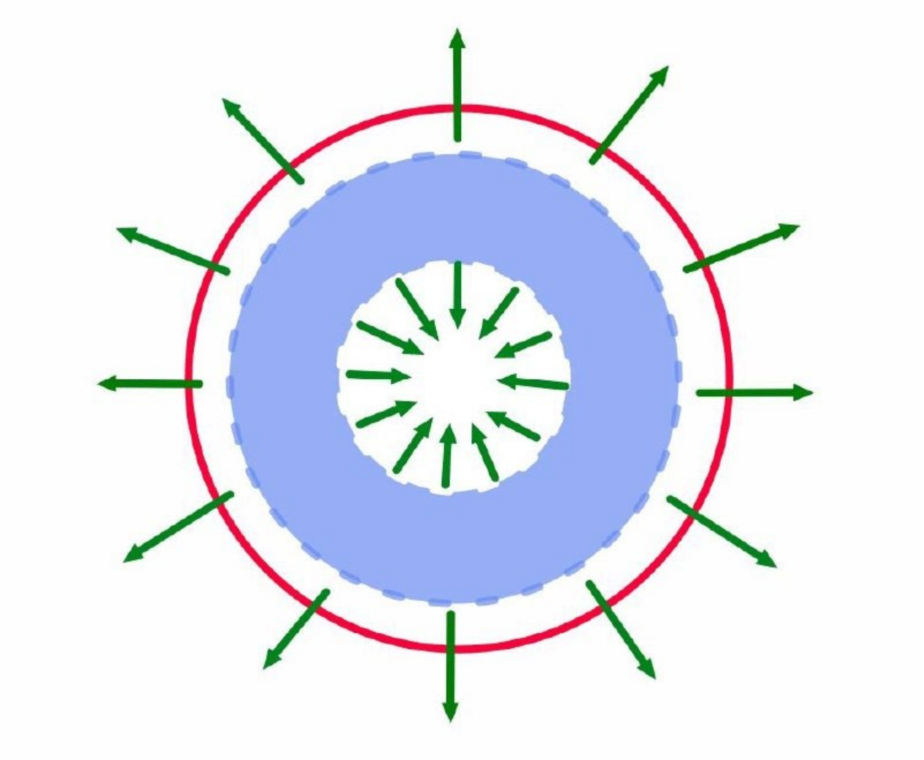
# GRAVASTAR (p<0)

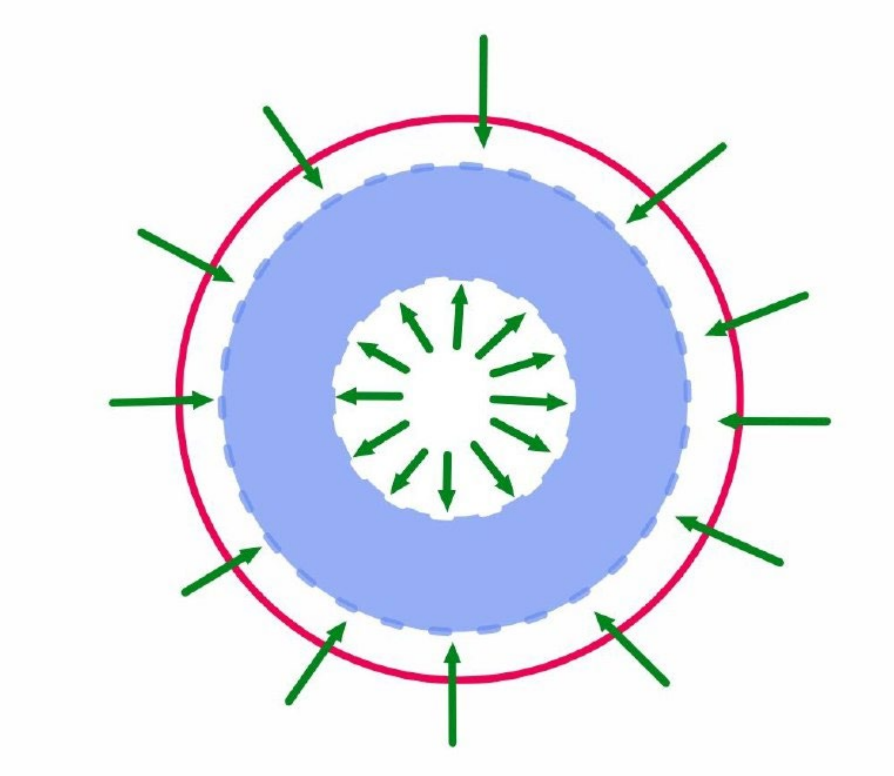


$$P(r+\delta r)-P(r)>0$$

$$P(r)-P(r)+\frac{dp}{dr}\delta r>0$$

$$\boxed{\frac{dp}{dr}>0}$$

# STAR (p>0)



$$P(r)-P(r+\delta r)>0$$

$$P(r)-P(r)-\frac{dp}{dr}\delta r>0$$

$$\boxed{\frac{dp}{dr}<0}$$

# ULTRACOMPACT STAR BY GRAVITATIONAL DECOUPLING

**INTERIOR**

$$f(r) = \frac{2}{\alpha H^2}(1 - H^2 r^2)\left(\frac{a_2 r^2}{2} + \frac{a_3 r^3}{3}\right)$$

$$e^{\nu} = \frac{1}{4}(1 - H^2 r^2)$$

$$e^{-\lambda} = (1 - H^2 r^2)\left[1 - \frac{2}{H^2}\left(\frac{a_2}{2}r^2 + \frac{a_3}{3}r^3\right)\right]$$

Metric Components

$$\rho = -\frac{9a_2 + 8a_3 r}{24\pi H^2} + \frac{15a_2 r^2 + 12a_3 r^3}{24\pi} + \frac{3H^2}{8\pi}$$

$$p_r = \frac{3a_2 + 2a_3 r}{24\pi H^2} - \frac{9a_2 r^2 + 6a_3 r^3}{24\pi} - \frac{3H^2}{8\pi}$$

$$p_\perp = \frac{a_2 + a_3 r}{8\pi H^2} - \frac{5a_2 r^2 + 4a_3 r^3}{8\pi} - \frac{3H^2}{8\pi}$$

Matter Sector

# ULTRACOMPACT STAR BY GRAVITATIONAL DECOUPLING

**EXTERIOR 1**

$$g(r) = \left(1 - \frac{2M}{r}\right)\left(\frac{\ell}{2r - 3M}\right)$$

$$e^{\nu} = 1 - \frac{2M}{r}$$

$$e^{-\lambda} = \left(1 - \frac{2M}{r}\right)\left(1 + \frac{\ell}{2r - 3M}\right)$$

$$\rho = -\frac{\ell M}{8\pi r^2 (3M - 2r)^2}$$

$$p_r = -\frac{\ell}{24\pi M r^2 - 16\pi r^3}$$

$$p_\perp = \frac{\ell (M - r)}{8\pi r^2 (3M - 2r)^2}$$

$$a_2 = -\frac{128 a_3 M^6 + 3\ell + 9M}{96 M^5}$$

**EXTERIOR 2**

$$g(r) = \left(1 - \frac{2M}{r}\right)\left[\frac{1}{(r + M)^{2(a-1)}}\right]$$

$$e^{\nu} = 1 - \frac{2M}{r}$$

$$e^{-\lambda} = \left(1 - \frac{2M}{r}\right)\left[1 + \frac{\beta}{(r + M)^{2(a-1)}}\right]$$

$$\rho = \frac{\beta (M + r)^{1-2a}[(3 - 4a)M + (2a - 3)r]}{8\pi r^2}$$

$$p_r = \frac{\beta (M + r)^{2-2a}}{8\pi r^2}$$

$$p_\perp = \beta \frac{(a - 1)(M - r)(M + r)^{1-2a}}{8\pi r^2}$$

$$a_2 = -\frac{1}{32} 3^{-2a-1} M^{-2a-4} (128\, 3^{2a} a_3 M^{2a+5} + 3^{2a+2} M^{2a} + 27\beta M^2)$$

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# Ultracompact stars with polynomial complexity by gravitational decoupling

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**Abstract** In this work we construct an ultracompact star configuration in the framework of Gravitational Decoupling by the Minimal Geometric Deformation approach. We use the complexity factor as a complementary condition to close the system of differential equations. It is shown that for a polynomial complexity the resulting solution can be matched with two different modified-vacuum geometries.

## 1 Introduction

traversable wormholes [10–13] and ultracompact stars [14], among others, and in this work we shall focus our attention in the latter.

As it is well known, Buchdahl limit relies on the hypothesis of isotropy and entails that the maximum compactness of a self-gravitating, isotropic, spherically-symmetric object of mass $M$ and radius $R$ has an upper bound given by $M/R = 4/9$ (for modifications of the Buchdahl's limits induced by the presence of the cosmological constant see [15–18], for example ). In this regard, anisotropic self-