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Grand Unification of All Discrete Wigner Functions on $d \times d$ Phase Space

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Discrete Wigner functions (DWFs) are central tools for visualising states, signifying nonclassicality, and supporting quantitative analysis in quantum information, yet many inequivalent constructions coexist for each Hilbert-space dimension. This fragmentation obscures which features are fundamental and which are artefacts of representation, and it impedes quantitative comparison of operational properties such as negativity. We present a unifying, dimension-preserving framework that exhausts all possible $d \times d$ DWFs for a single qudit. The key result (Stencil Theorem) shows that every valid DWF arises by cross-correlating a single parent function—the “doubled” Wigner function defined on a $2d \times 2d$ phase space—with a suitable stencil, and that all valid choices of stencils are characterised by simple projection criteria. This construction also yields explicit, invertible linear maps connecting any two DWF definitions at fixed d , enabling representation-independent benchmarking of resource measures and side-by-side comparison of physical predictions, further unifying the landscape.

We illustrate the approach with concrete stencils. For odd d , a reduction stencil reproduces standard frames (Wootters, Leonhardt, Gross). For even d , a coarse-grain stencil averages neighbouring cells—to remove existing redundant information within the doubled phase space—and generates a novel $d \times d$ DWF that lies outside previously studied families. A third, Dirichlet-kernel stencil produces a DWF valid for odd- d yet distinct from Gross’ construction.

Beyond unification, stencils convert redundancy removal from a nuisance into a design choice. The framework organises the landscape of DWFs to a landscape of easily constructible stencils. Furthermore, by relaxing certain criteria, our framework also extends to other quasidistributions (e.g., Kirkwood–Dirac). Overall, this work clarifies what is truly representation-independent at fixed dimension and opens new avenues for studying dimension-agnostic features in discrete phase space.

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