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International Joint Workshop on the Standard Model and Beyond 2024 & 3rd Gordon Godfrey Workshop on Astroparticle Physics

1. https://www.sydneynewyearseve.com/

Delicate Cancellations in Kaluza-Klein Dark Matter Calculations

#1
Phys. Rev. D 108, L051702, arXiv: 2303.04329

J.G., Sengupta, Williams

See also: G. Sanamyan (Next), R. S. Chivukula and E. Simmons (Friday Afternoon)

12/12/24

Outline

- **Motivation**
  - Gravity as **massless spin-2** - making it **massive and issues**
  - Published **incorrect scaling** - lower EFT scale, implications for phenom/colliders/DM
- **Delicate Cancellation Example**
  - **Massive Spin-2 Graviton/Photon Production**
- (Briefly) **Geometric Higgs Mechanism**
  - Spontaneous Symmetry Breaking, hierarchy solution, infinite tower of massive states, unitary and Goldstone Equivalence
- **Summary**

Gravity as Spin-2 Field

- General Relativity: EFT up to Planck Mass (classical field theory with dim-6 operator)

$$S = \frac{M_{\mathrm{Pl}}^2}{4} \int d^4x \sqrt{|\det g|}\, R$$

- Weak field expansion - generates kinetic energy term for spin-2 field $h_{\mu\nu}$

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa\, h_{\mu\nu}, \qquad \kappa \propto 1/M_{\mathrm{Pl}}$$

- **GR is the fully self-interacting theory of massless spin-2 field!**
- Only two propagating degrees of freedom
- What if it was massive - how does it all change?

Massive (Spin-2) Gravity

- Add a mass term in the Lagrangian by hand - recovers GR in massless limit

$$S_{\mathrm{Mass}} = -\frac{1}{4}\int d^4x \sqrt{|\det g|}\, M_G^2 \left(h_{\mu\nu}h^{\mu\nu} - h^2\right)$$

- Massive spin-1 polarisation vectors:

$$\varepsilon_0^\mu(k_2) = \frac{E_{k_2}}{M_G}\left(\sqrt{1-\frac{M_G^2}{E_{k_2}^2}},\, \hat{k}\right).$$

- Now have **FIVE** propagating degrees of freedom c.f. **TWO** in massless GR

$$\begin{aligned}
\lambda_G &= \pm 2, \ \varepsilon^{\mu\nu}_{\pm 2} = \varepsilon^\mu_{\pm 1}\varepsilon^\nu_{\pm 1}, \\
\lambda_G &= \pm 1, \ \varepsilon^{\mu\nu}_{\pm 1} = \frac{1}{\sqrt{2}}\left[\varepsilon^\mu_{\pm 1}\varepsilon^\nu_0 + \varepsilon^\mu_0\varepsilon^\nu_{\pm 1}\right], \\
\lambda_G &= 0, \quad \varepsilon^{\mu\nu}_0 = \frac{1}{\sqrt{6}}\left[\varepsilon^\mu_{+1}\varepsilon^\nu_{-1} + \varepsilon^\mu_{-1}\varepsilon^\nu_{+1} + 2\varepsilon^\mu_0\varepsilon^\nu_0\right],
\end{aligned}$$

- **Massless limit is discontinuous for both ± 1 and 0 modes!**

Stuckelberg Trick

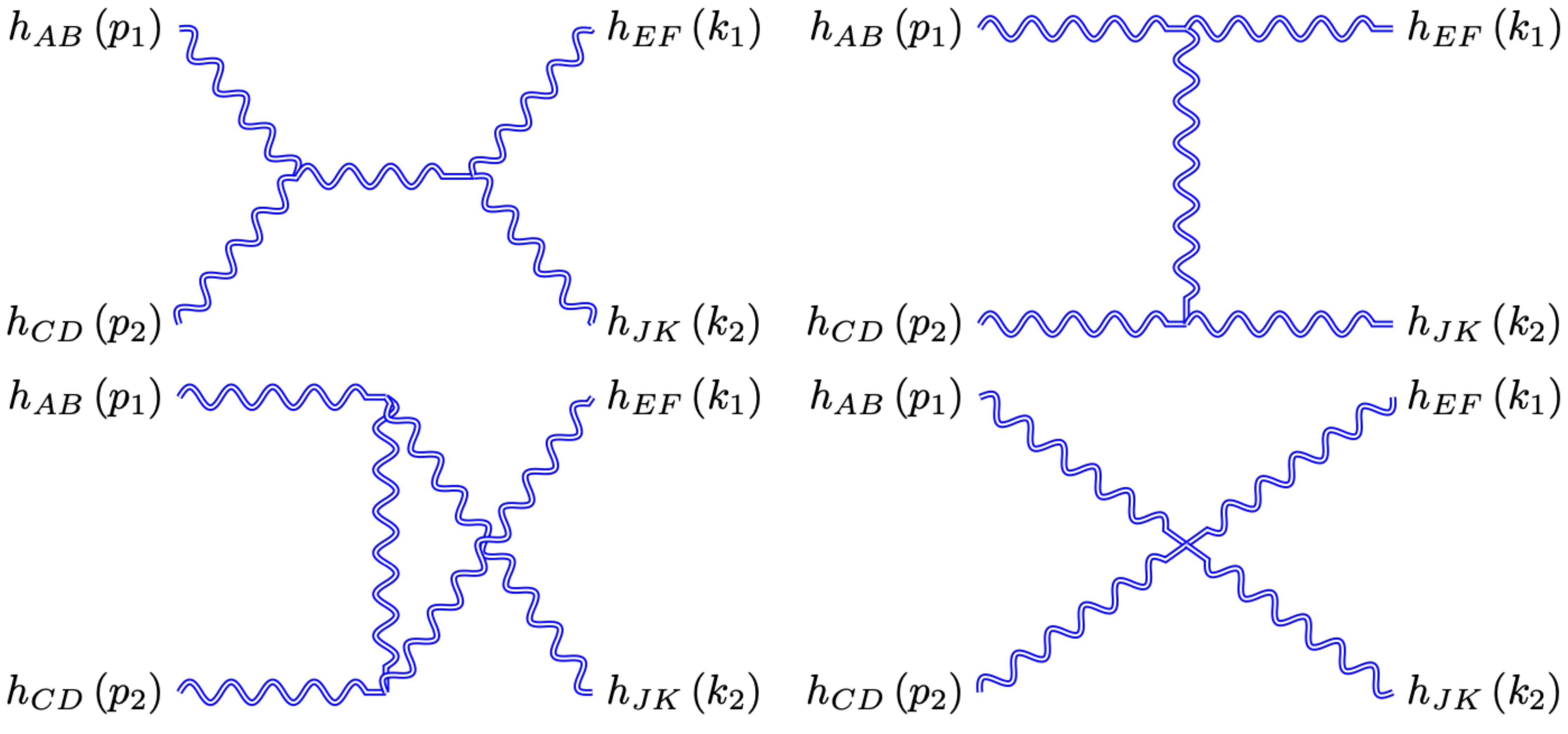
- Splitting the spin-2 field into each of its polarisations

- The 0 mode couples to the trace of stress-energy tensor

  - **Does not decouple in massless limit**:

    - van Dam-Veltman-Zakharov (vDVZ) discontinuity

- **Resolution**: Vainshtein mechanism - linear theory not enough



- Naive power counting: $\mathscr{M} \propto \kappa^2 E^{10}/M_G^8$

- dRGT Model (adding finely-tuned potentials): $\mathscr{M} \propto \kappa^2 E^6/M_G^4$

- **EFT scale now drastically reduced!**

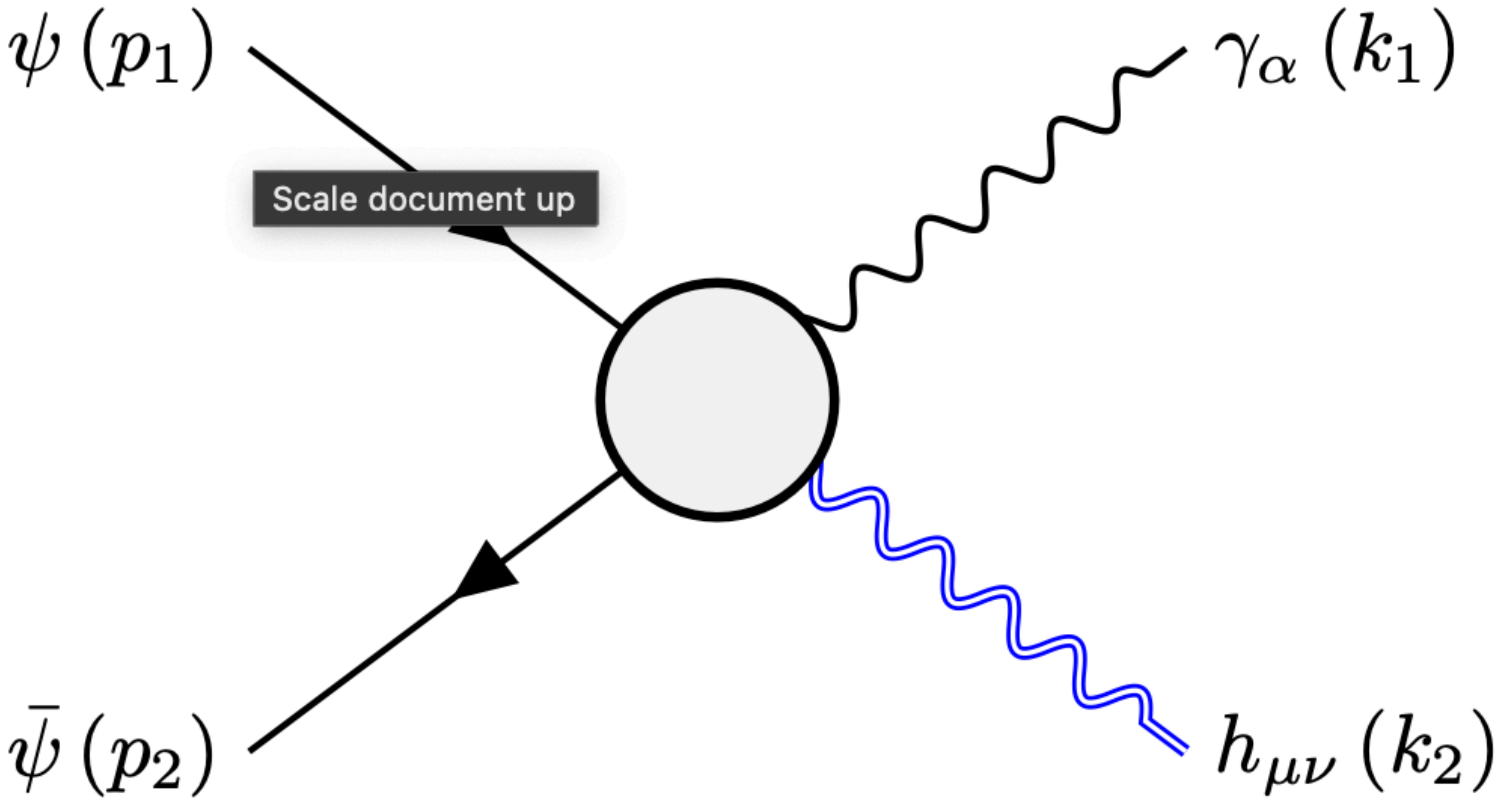
Note: for massive gauge field - longitudinal mode couples to $\partial_\mu J^\mu$ and vanishes for conserved currents

A Model - 1 Massive Graviton Leg

- Take $\psi + \bar{\psi} \to \gamma + h_{\mu\nu}$
- PRL **128**, 081806 (gluon) found

$$\left|\mathcal{M}\right|^2 \propto m_q^4 / M_G^4$$

- Agnostic to mass generation
- If graviton - GW experiments set upper bound $M_G \leq \mathcal{O}\left(10^{-23}\right)$ eV/c^2
  - **Heavily constrain gravitational coupling**



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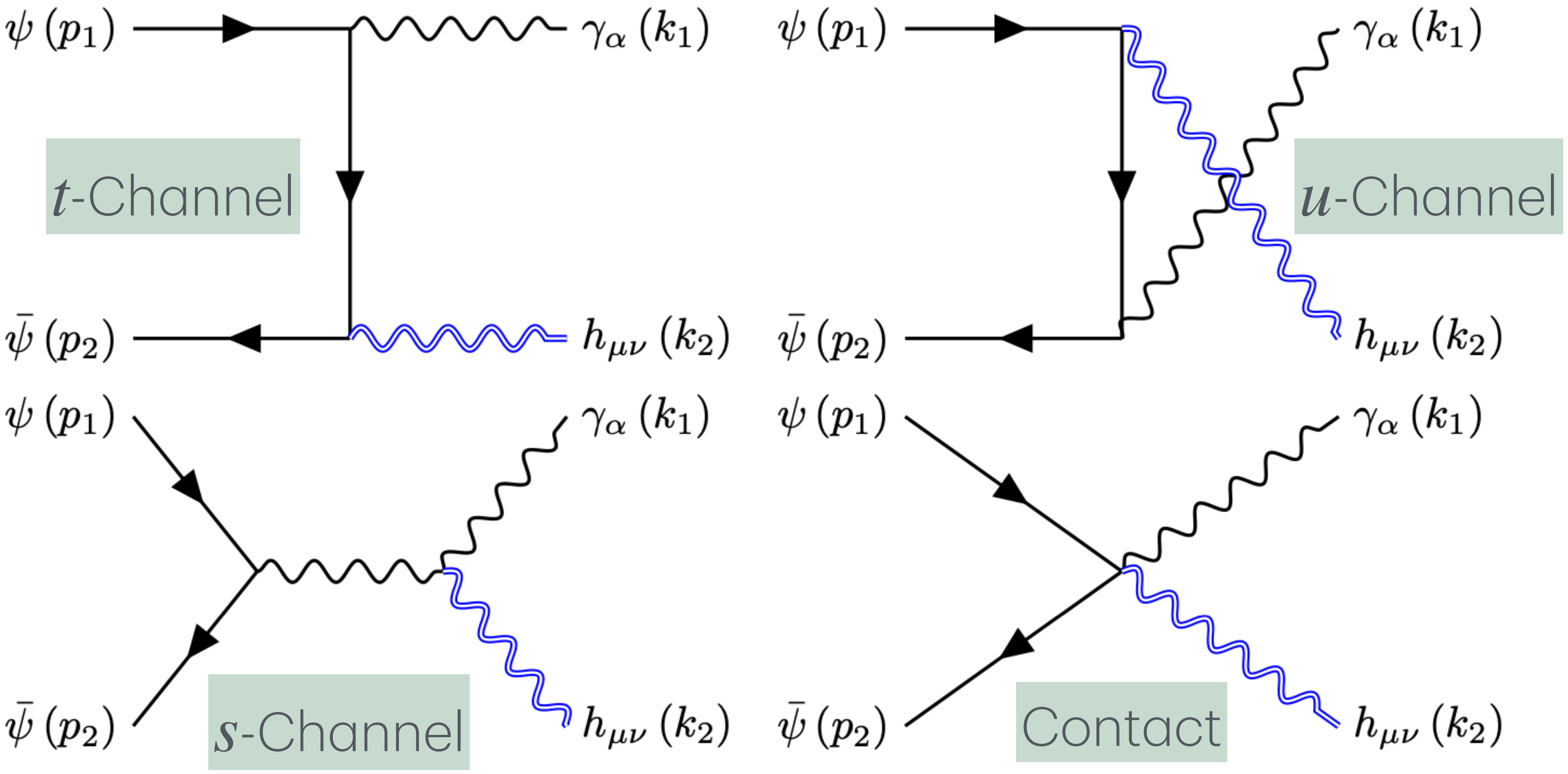
- **To test this, we work in full generality - no shortcuts or simplifying tricks**

Tree-Level Diagrams



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Investigation

- Want to check: $1/M_G^2$ behaviour for longitudinal (0) mode.

- Check spin-by-spin of fermions and polarisation-by-polarisation of photon and ...

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Investigation

- Want to check: $1/M_G^2$ behaviour for longitudinal (0) mode.
- Check spin-by-spin of fermions and polarisation-by-polarisation of photon and ...
- **Each diagram has this behaviour**

$$\mathcal{M}_t = \frac{\kappa g_\psi e}{2}\left\{\frac{s\,\sin\theta}{\sqrt{3}}\right\}\left\{1+\cos\theta\sqrt{1-\frac{4m_\psi^2}{s}}\right\}\left(\frac{m_\psi}{M_G^2}\right)+\mathcal{O}\left(M_G^0\right),$$

$$\mathcal{M}_u = \frac{\kappa g_\psi e}{2}\left\{\frac{s\,\sin\theta}{\sqrt{3}}\right\}\left\{1-\cos\theta\sqrt{1-\frac{4m_\psi^2}{s}}\right\}\left(\frac{m_\psi}{M_G^2}\right)+\mathcal{O}\left(M_G^0\right),$$

$$\mathcal{M}_s = -\kappa g_\psi e\left\{\frac{s\,\sin\theta}{\sqrt{3}}\right\}\left(\frac{m_\psi}{M_G^2}\right)+\mathcal{O}\left(M_G^0\right),$$

$$\mathcal{M}_c = \mathcal{O}\left(M_G^0\right).$$

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Investigation

- Want to check: $1/M_G^2$ behaviour for longitudinal (0) mode

The sum of them exactly cancels!

- **Each diagram has this behaviour**

$$\mathcal{M}_t = \frac{\kappa g_\psi e}{2}\left\{\frac{s\,\sin\theta}{\sqrt{3}}\right\}\left\{1 + \cos\theta\sqrt{1-\frac{4m_\psi^2}{s}}\right\}\left(\frac{m_\psi}{M_G^2}\right) + \mathcal{O}\left(M_G^0\right),$$

$$\mathcal{M}_u = \frac{\kappa g_\psi e}{2}\left\{\frac{s\,\sin\theta}{\sqrt{3}}\right\}\left\{1 - \cos\theta\sqrt{1-\frac{4m_\psi^2}{s}}\right\}\left(\frac{m_\psi}{M_G^2}\right) + \mathcal{O}\left(M_G^0\right),$$

$$\mathcal{M}_s = -\kappa g_\psi e\left\{\frac{s\,\sin\theta}{\sqrt{3}}\right\}\left(\frac{m_\psi}{M_G^2}\right) + \mathcal{O}\left(M_G^0\right),$$

$$\mathcal{M}_c = \mathcal{O}\left(M_G^0\right).$$

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Investigation

- Checking for every polarisation of fermions, photons and the graviton
  - Graviton: longitudinal 0, vector ± 1 and transverse ± 2 polarisations

Amplitudes grew no faster than $\mathcal{M} \propto \kappa E$

External graviton leg behaves as a scalar!

Process is unitary to Planck scale $\kappa \propto M_{\text{Pl}}$

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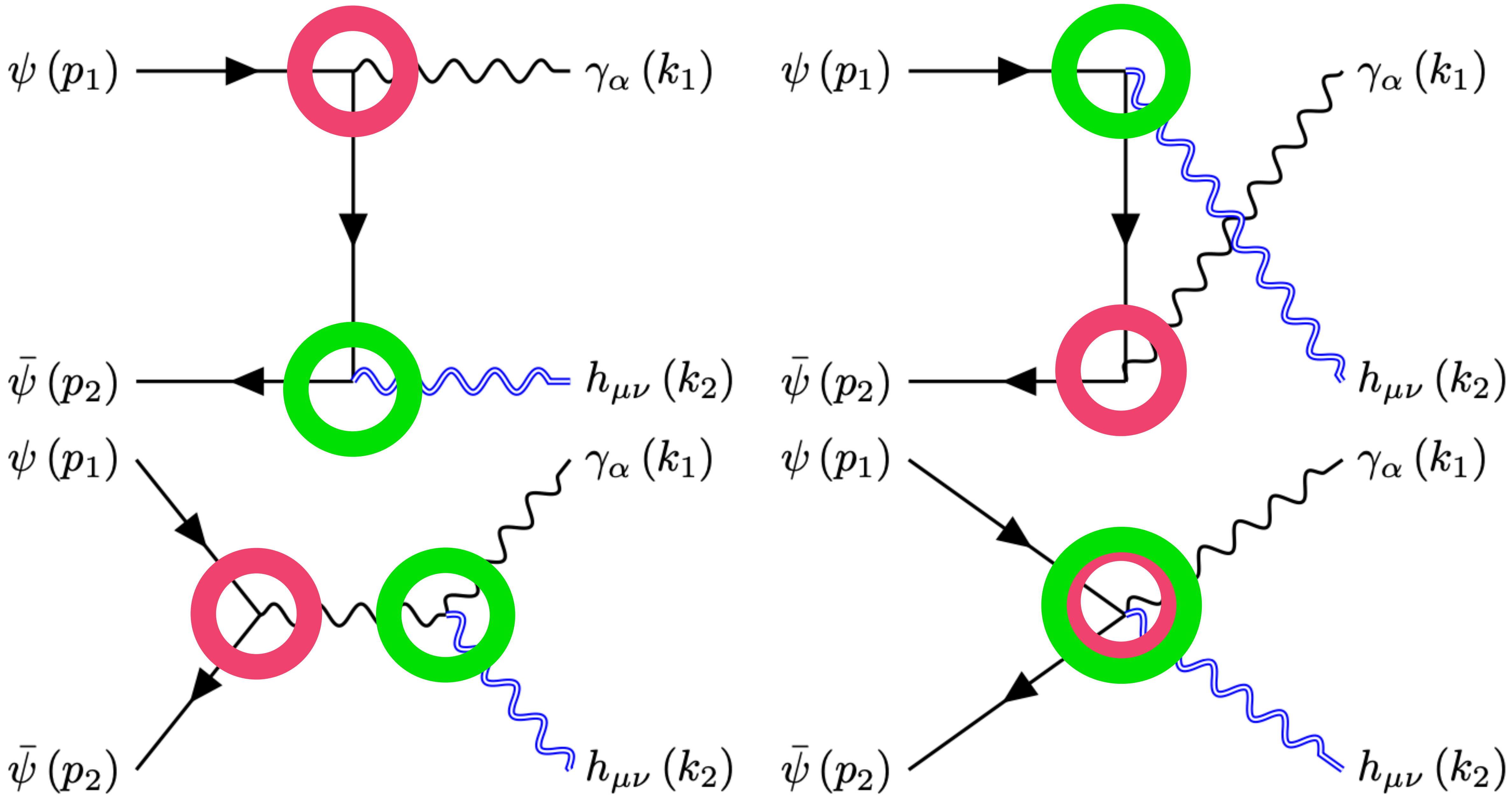
Spin-2 Ward Identity?

- Pathologies in massive gravity typically from propagators - none at tree-level here
- Inclusion of external graviton would not alter global $U(1)$ symmetry
  - Effective QED Ward identity
- Computed QED (red) and gravitational (green) Ward identities hold
- $\mathscr{L}_{\text{int}} \propto h_{\mu\nu} T^{\mu\nu}$, conserved $T^{\mu\nu}$ → Ward Identity



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Remarks

- Generalisable to incoming quarks and outgoing gluon, and all crossing symmetries
- No mass scale dependence in amplitude
- **No large cross-section → no spike in relic density**
- **Null detections does not heavily constrain the coupling**
- Mass generation mechanism (by hand or otherwise) does not influence result
- Gravitational Ward Identity hold in general?
- Phenomenological implications for graviton (single spin-2 external state):
  - Production at LHC, direct detection and relic density calculations

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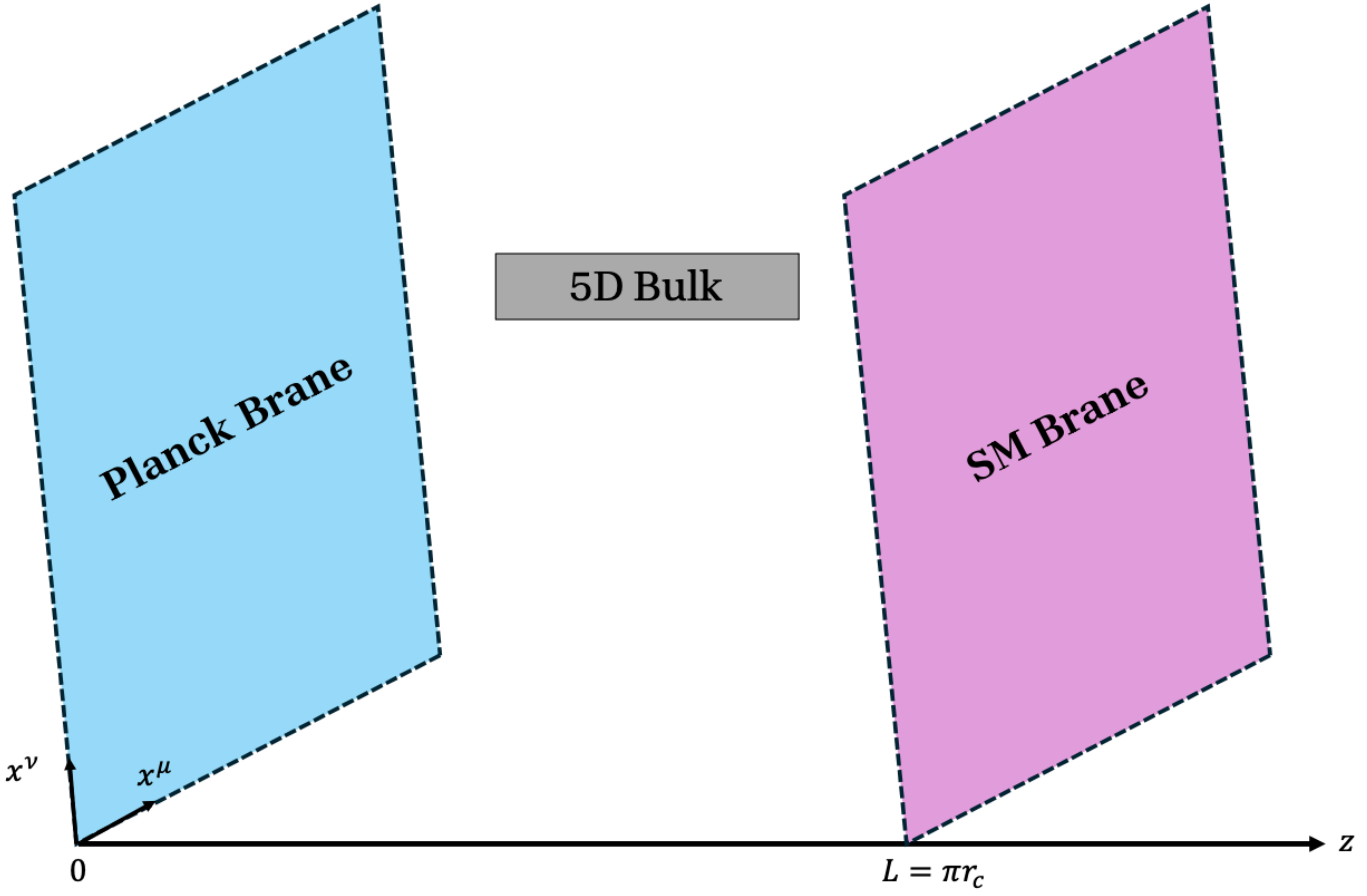
Spontaneous Mass Generation

- Massive spin-2 **explicitly breaks diffeomorphism**
- GR is diffeomorphism invariant in N-dim
- **Massive 4D spin-2 = 5 DoF = massless 5D spin-2**
- 'Softly' broken 5D GR (compactified) will generate massive 4D spin-2
  - **Exactly 1 massless spin-2 field (GR)**
  - **Infinite tower of massive spin-2 states**

$$h_{\mu\nu}(x,z) \propto \sum_{n=0}^{\infty} h_{\mu\nu}^{(n)}(x)\,\psi_n(z)$$



Hierarchy Resolution
SM scaled down,
Gravity unchanged

$$\langle\Phi\rangle_{\mathrm{SM}} = e^{-kL}\langle\Phi\rangle_{\mathrm{Pl}} \qquad \kappa_5 \sim \kappa_4$$

Kaluza Klein Gravity - Unitary

- Kaluza Klein decomposition of spin-2 in compactifified extra-dim
- Unitarity tests:
  - **KK Graviton Self-scattering (Phys. Rev. D 101, 075013)**
  - **Stabilised KK Graviton Self Scattering (Phys. Rev. D 103, 095024)**
  - **Brane and Bulk Matter Scattering to KK Gravitons (Phys. Rev. D 109, 015033)**
  - etc...
- **Goldstone Equivalence Theorem for KK Scattering (Phys. Rev. D 109, 075016)**

Summary Future

See also:
- **G. Sanamyan (DM in Stabilised KK Portal)**
- **R. S. Chivukula / E. Simmons (Extra-Dim Gravity and KK DM Portal Models)**

- **Gravitational calculations are tedious**
  - Beyond first order amplitudes become intangible - algebraic tools (xAct etc)
- **Care**: momenta and Dirac structure - one small error and cancellation not realised
- Here, **no large constraints placed on coupling** because no scale dependence
- **Immediate work - KK modes as dark matter, KK portal between dark sector.**
- **Future Work - Calculations beyond tree-level? Gravitational Ward Identity? Gravity as Double Copy of Gauge Field?**

Extra Slides

#2
Phys. Rev. D 109, 015033
arXiv: 2311.00770
Chivukula, J.G., Mohan, Sengupta, Simmons, Wang

#3
Phys. Rev. D 109, 075016
arXiv: 2312.08576
Chivukula, J.G., Mohan, Sengupta, Simmons, Wang

Extra-Dim Gravity

- Look to extra-dimensions, in particular the Degrees of Freedom (DoF)
- **DoF for massive 4D spin-2 = 5 = DoF for massless 5D spin-2**
- Consider **massless 5-Dim GR and decompose into Fourier modes:**

$$S = \frac{M_{\mathrm{Pl},5}^2}{4}\int d^5x\sqrt{\left|\det g\right|}R^{(5)}, \qquad h_{AB}\left(x,y\right) = \sum_{n=0}^{\infty} h_{AB}^{(n)}(x)\,\psi_n\left(y\right)$$

- The extra-dim profile $\psi_n\left(y\right)$ satisfy a Sturm-Liouville equation:
  - Boundary conditions generate unique eigenvalues and ortho-normal states
- The eigenvalues are the factor to a quadratic term in the 4D effective spin-2 field $h_{AB}^{(n)}(x)$

'Natural' process to generate exactly one massless mode, and a tower of massive spin-2 states

Randall-Sundrum 1 Model

- B.C.s set by 4D slices (branes)
- Warped 4D (AdS) geometry

$$G_{AB} = \begin{pmatrix} e^{-2(k|z|+\hat{u})}\left(\eta_{\mu\nu} + \kappa_5 \hat{h}_{\mu\nu}\right) & 0 \\ 0 & -(1+2\hat{u})^2 \end{pmatrix}$$

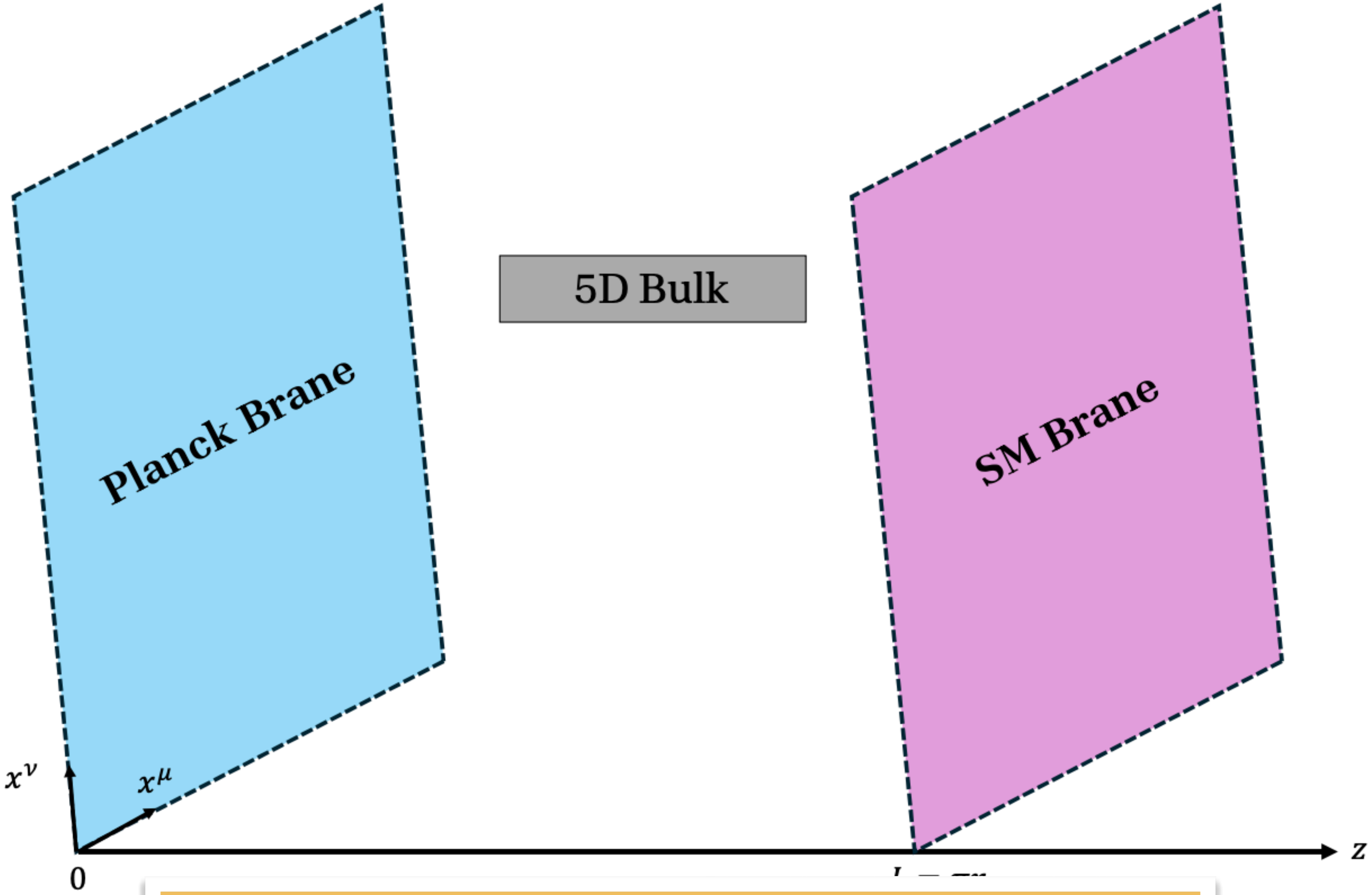
$$\hat{u}(x,\, z) \equiv \frac{\kappa_5 \hat{r}(x,\, z)}{2\sqrt{6}} e^{k(2|z|-\pi r_c)}$$

- ‘Solution’ to EW hierarchy

$$\langle \Phi \rangle_{\mathrm{SM}} = e^{-kL} \langle \Phi \rangle_{\mathrm{Pl}}$$

$$\kappa_5 \sim \kappa_4$$

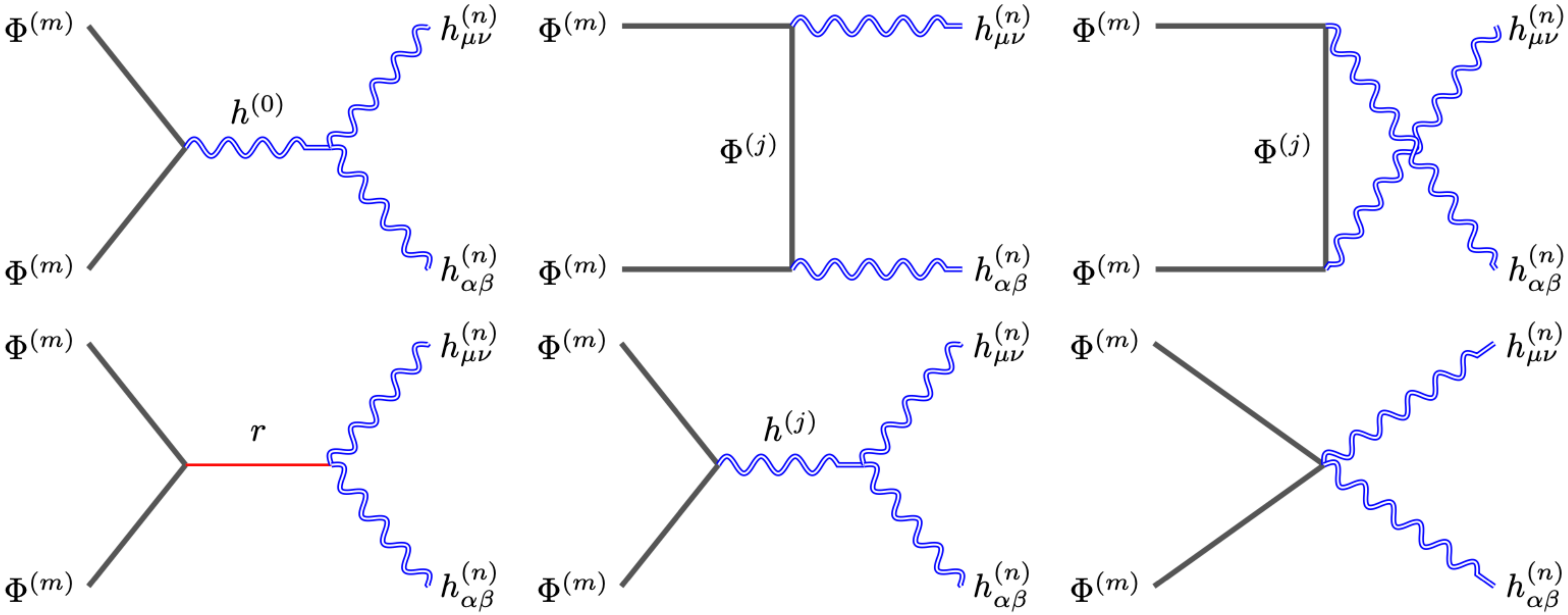
- SM scaled down, GR unscaled



Note: **Introduced massless scalar field (radion) and propagates a long-range force → unstable geometry (more on this later)**

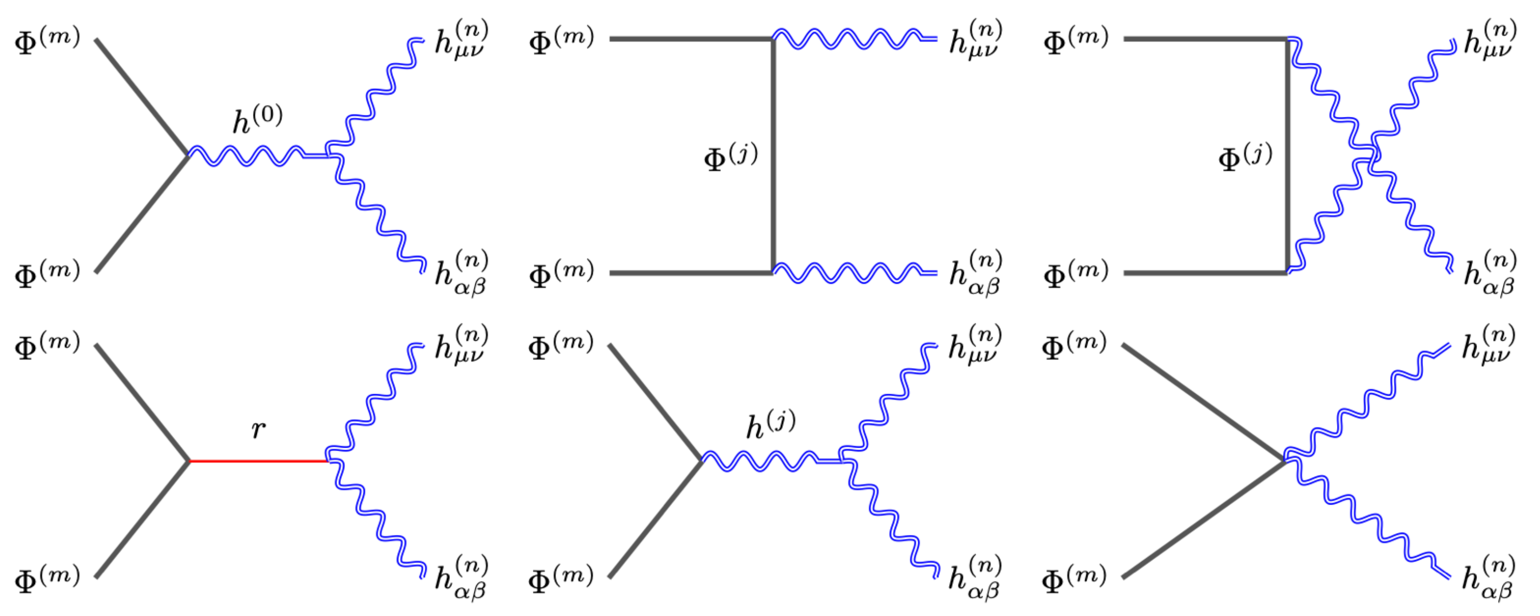
#2: Check Unitary

- Compute amplitudes for brane-localised (no tower) and bulk matter (also infinite tower)
- No explicit symmetry breaking - expect $\mathcal{M} \propto E^2/M_{\rm Pl}^2$
- Explicitly chosen to work in unitary gauge and elastic scattering



#2
Phys. Rev. D 109, 015033
arXiv: 2311.00770

#2: E^6 Results



- Vertices proportional to overlap integrals of extra-dim profiles
- A set of 'sum-rules' determined -> consequence of S-L
- Order $E^6 = s^3$ cancels due to completeness

$$\widetilde{\mathcal{M}}^{(6)} = \frac{\kappa^2(1-\cos 2\theta)}{192 m_n^4}\left[\left(f^{(n)}(\bar{z})\right)^2 - \sum_{j=0}^{\infty} a_{nnj} f^{(j)}(\bar{z})\right]$$

$$\begin{aligned}\sum_{j=0}^{\infty} a_{nnj} f^{(j)}(\bar{z}) &= \sum_{j=0}^{\infty}\left[\int_{z_1}^{z_2} dz\; e^{3A(z)} f^{(n)}(z) f^{(n)}(z) f^{(j)}(z)\right] f^{(j)}(\bar{z}) \\ &= \int_{z_1}^{z_2} dz\; f^{(n)}(z) f^{(n)}(z)\delta(z-\bar{z}) \\ &= \left[f^{(n)}(\bar{z})\right]^2.\end{aligned}$$

#2
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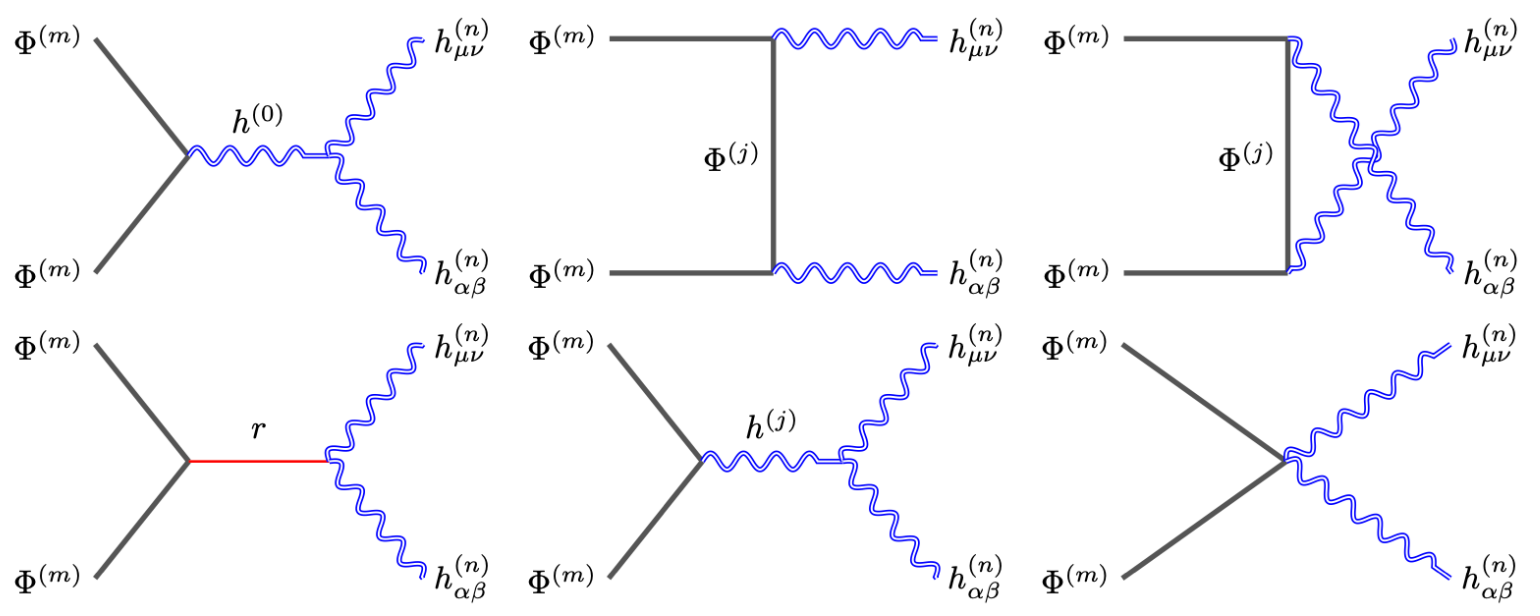


#2: E^4 and E^2 Results

- The radion kicks in at this level

- Order $E^4 = s^2$ cancels due by using:

  - Eiegn-equation → relates propagator mass to external mass

  - Neumann B.C. → First deriv. vanishes on brane

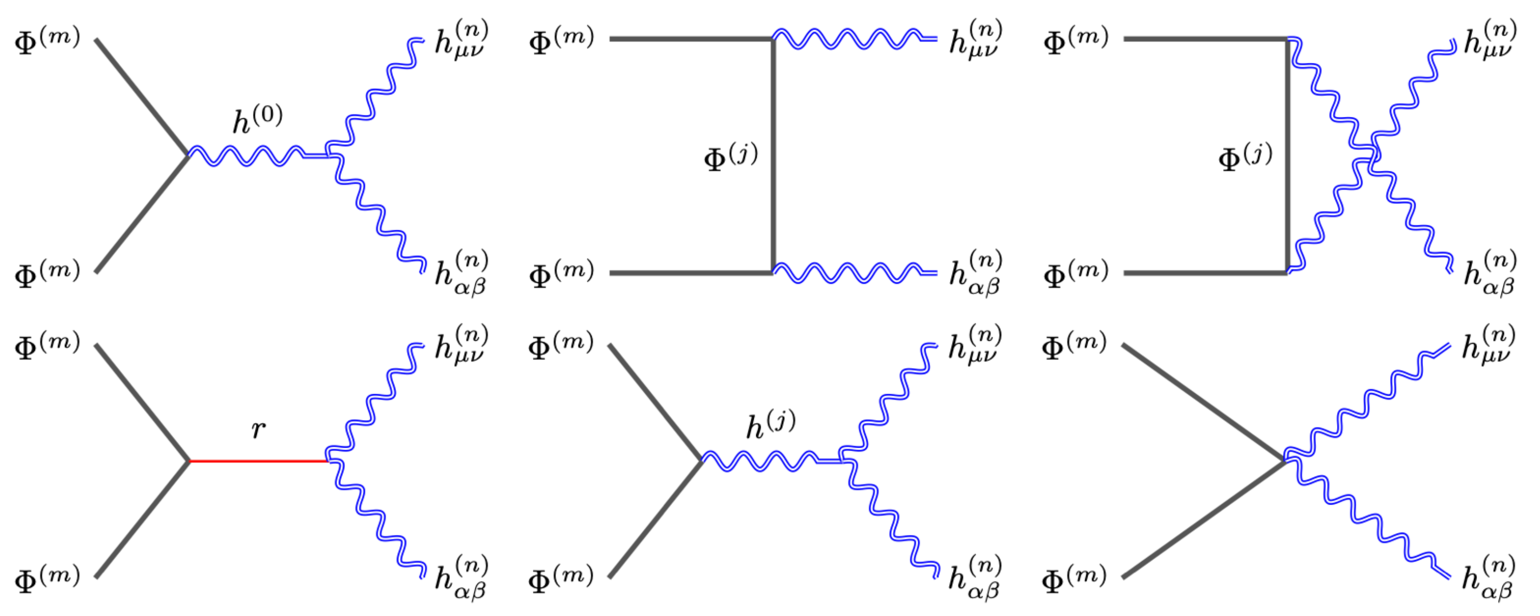
  - $N = 2$ SUSY relations (algebraic relations to ensure no kinetic mixing)

- The remaining $E^2 = s$ contribution simplifies to

$$\widetilde{\mathcal{M}}^{(2)} = -\frac{\kappa^2(3\cos 2\theta + 1)}{96}\left[f^{(n)}(\bar{z})\right]^2$$

#2
Phys. Rev. D 109, 015022
arXiv: 2311.00770

#2: E^4 and E^2 Results



- The radion kicks in here
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  - Eiegn-equation → relates propagator mass to external mass
  - Neumann B.C. → First deriv. vanishes on brane
  - $N = 2$ SUSY relations (algebraic relations to ensure no kinetic mixing)
- The remaining $E^2 = s$ contribution simplifies to - **Scalar Wavefunction**

$$\widetilde{\mathcal{M}}^{(2)} = -\frac{\kappa^2(3\cos 2\theta + 1)}{96}\left[k^{(n)}(\bar{z})\right]^2$$

#2
Phys. Rev. D 109, 015033
arXiv: 2311.00770

#2 - Remarks

- Results hold for all types of brane-localised and bulk matter (scalar, fermion and vector)
- Elastic scattering **incoming** spin-2 modes (Phys. Rev. D **101**, 075013) - same results applied
- Brane matter:
    - Branes break diffeo sym. however, remnant 5D diffeo sym
    - Spin-2 sector still invariant for gauge choice which leaves brane fixed
    - As long as **matter localised on the boundary** - it too has remnant 5D diffeo sym
    - See Phys. Rev. D **106**, 035026 for details
- **Bulk matter - same procedure and unitary results up to M_{Pl}**

#2 - Remarks

- Results hold for all types of brane-localised and bulk matter (scalar, fermion and vector)
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- Bro
    - E
    - S
    - A
    - See Phys. Rev. D **106**, 035026 for details

Goldstone equivalence theorem appears!

Each amplitudes behaves $\propto$ kinetic factors multiplied by overlap of incoming states with Goldstone Scalar Bosons!

- **Bulk matter - same procedure and unitary results up to M_{Pl}**

#3 - Goldstone Equivalence

- Following the GRavitational Equivalence Theorem (GRET) by Hang and He
- Transparent power counting using Ward Identities
  - Introduce 't Hooft-Feynman Gauge
  - Apply Ward Identities to polarisation vectors
  - **No bad-high energy behaviour from the beginning!**
- Explicitly shown for:
  - Two bulk scalars → two KK graviton states
  - Two KK graviton states → two KK graviton states

#3 - Specifics

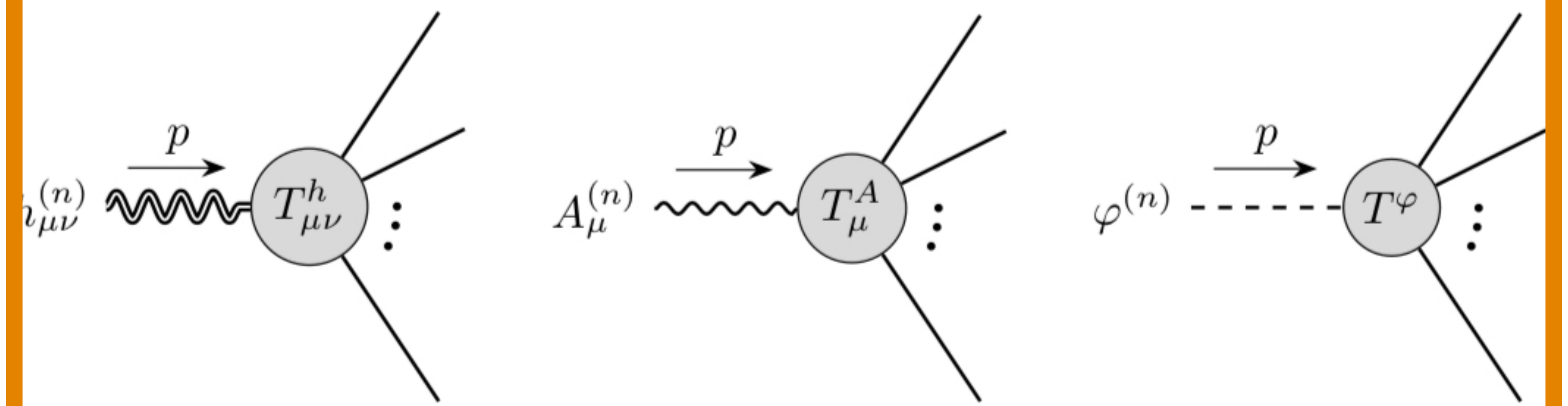
$$\mathcal{L}_{\rm GF} = \sum_n F^{(n)\mu} F^{(n)}_\mu - F^{(n)}_5 F^{(n)}_5,$$

$$F^{(n)}_\mu = -\left(\partial^\nu h^{(n)}_{\mu\nu} - \frac{1}{2}\partial_\mu h^{(n)} + \frac{1}{\sqrt{2}} m_n A^{(n)}_\mu\right),$$

$$F^{(n)}_5 = -\left(\frac{1}{2} m_n h^{(n)} - \frac{1}{\sqrt{2}}\partial^\mu A^{(n)}_\mu + \sqrt{\frac{3}{2}} m_n \varphi^{(n)}\right)$$

$$\begin{aligned} T^h_{\mu\nu} &= \mathcal{N}\int d^4x\, e^{ipx}\, D^h_{\mu\nu\rho\sigma}\langle \mathbf{T}\, h^{(n)}_{\rho\sigma}(x)\Phi\rangle \\ &= \mathcal{N}\int d^4x\, e^{ipx}\, (-\Box - m_n^2)\langle \mathbf{T}\left(h^{(n)}_{\mu\nu}(x) - \frac{1}{2}\eta_{\mu\nu}h^{(n)}(x)\right)\Phi\rangle \\ T^A_\mu &= \mathcal{N}\int d^4x\, e^{ipx}\, D^A_{\mu\nu}\langle \mathbf{T}\, A^{(n)}_\nu(x)\Phi\rangle = \mathcal{N}\int d^4x\, e^{ipx}\,(-\Box - m_n^2)\langle \mathbf{T}\left(-A^{(n)}_\mu(x)\right)\Phi\rangle \\ T^\varphi &= \mathcal{N}\int d^4x\, e^{ipx}\, D^\varphi\langle \mathbf{T}\,\varphi^{(n)}(x)\Phi\rangle = \mathcal{N}\int d^4x\, e^{ipx}\,(-\Box - m_n^2)\langle \mathbf{T}\,\varphi^{(n)}(x)\Phi\rangle \end{aligned}$$

$$\langle \mathbf{T} F^{(n)}_\mu(x)\Phi\rangle = \langle \mathbf{T} F^{(n)}_5(x)\Phi\rangle = 0,$$



$$\frac{i}{2}p^\nu\left(T^h_{\mu\nu} + T^h_{\nu\mu}\right) - \frac{1}{\sqrt{2}} m_n T^A_\mu = 0,$$

$$-\frac{1}{2} m_n T^{h\mu}_\mu + \frac{i}{\sqrt{2}} p^\mu T^A_\mu + \sqrt{\frac{3}{2}} m_n T^\varphi = 0.$$

$$T^h_{\mu\nu}\epsilon^{\mu\nu}_0 = T^\varphi - i\sqrt{3}\, T^A_\mu \tilde{\epsilon}^\mu_0 + T^h_{\mu\nu}\tilde{\epsilon}^{\mu\nu}_0\,.$$

$$T^h_{\mu\nu}\epsilon^{\mu\nu}_{\pm 1} = -iT^A_\mu \epsilon^\mu_\pm + T^h_{\mu\nu}\tilde{\epsilon}^{\mu\nu}_{\pm 1},$$

#3 - GRET - Main Takeaway

Longitudinal mode of spin-2 field appears as Goldstone Scalar Boson in high energy limit

Made clear using Ward Identities in a general 't Hooft-Feynman Gauge

$$\mathcal{M}\left[h_L^{(n_1)} h_L^{(n_2)} \cdots\right] = \mathcal{M}\left[\varphi^{(n_1)} \varphi^{(n_2)} \cdots\right] + \mathcal{O}(s^0),$$