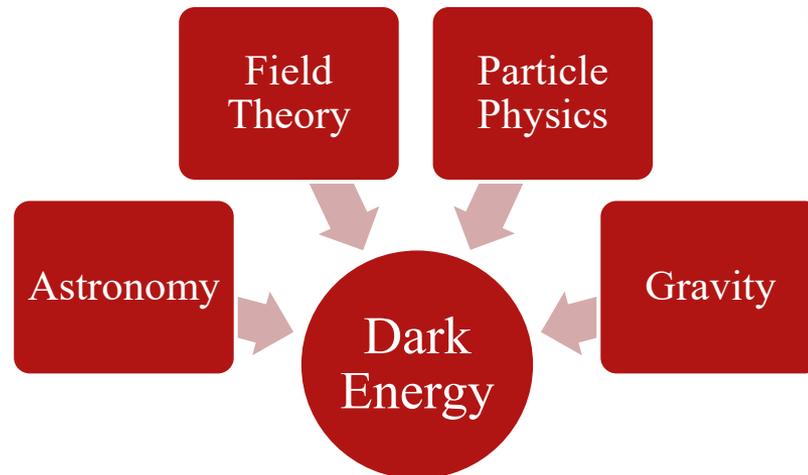# A Beyond Standard Model Approach to Dark Energy and its Multiple Observational Implications

- Prof. Anupam Singh, LNMIIT.

Dark Energy arose out of observations and lies at the crossroads of field theory, particle physics and gravity. It has illuminated our universe and continues to shed light both on fundamental physics and consequences for our universe.

Outline

The topics we plan to discuss are:

- Introduction:
    - Dark Energy (DE) : Key requirements
- Dark Energy Implications (other than rapid expansion):
    - Collapse of DE and Supermassive Black Hole Formation
    - Gravitational Radiation from DE
    - Gravitational Wave Observations and Ice Age Periodicity
    - Neutrino Masses, Early Structure Formation and James Webb Space Telescope Observations
    - Gauging Family Symmetries: Matter-Anti Matter Asymmetry from Higher Dimensions
- Conclusions

Dark Energy: An invention driven by necessity (astronomical observations)

Small nonvanishing cosmological constant from vacuum energy: Physically and observationally desirable

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(Received 2 December 1994)

Increasing improvements in the independent determinations of the Hubble constant and the age of the universe now seem to indicate that we need a small nonvanishing cosmological constant to make the two independent observations consistent with each other. The cosmological constant can be physically interpreted as due to the vacuum energy of quantized fields. To make the cosmological observations consistent with each other we would need a vacuum energy density $\rho_v \sim (10^{-3} \text{ eV})^4$ today (in the cosmological units $\hbar = c = k = 1$). It is argued in this paper that such a vacuum energy density is natural in the context of phase transitions linked to massive neutrinos. In fact, the neutrino masses required to provide the right vacuum energy scale to remove the age versus Hubble constant discrepancy are consistent with those required to solve the solar neutrino problem by the MSW mechanism.

Introduction

- Cosmological Constant was first introduced by Albert Einstein in his General Theory of Relativity

- The exact magnitude of the Cosmological Constant and its Physical Role have been unclear until recently

- Observations in the mid 1990s shed new light on the magnitude of the Cosmological Constant and its Physical Role.

Introduction

- Dark Energy is the dominant component of the Energy Density of the Universe.
- Most Natural Candidate for Dark Energy is the Energy Density due to fields in Curved Space-time
- Specific Particle Physics candidates exist which can be characterized as Pseudo Nambu Goldstone Bosons with a well-defined potential [Singh, Holman & Singh, Gupta, Hill, Holman & Kolb]

The Implications

- Einstein's Equations can be solved for Cosmological Space Time to obtain the following relationship:

$$t_o = \frac{2}{3} H_o^{-1} \Omega_{vac}^{-1/2} \ln \left[\frac{1 + \Omega_{vac}^{1/2}}{(1 - \Omega_{vac})^{1/2}} \right]$$

Where t_o is the age of the Universe, H_o is the Hubble Constant and Ω_{vac} is the Vacuum Energy Density of the Universe.

The Vacuum Energy Density

- From the relationship between the Age of the Universe, the Hubble Constant and the Vacuum Energy Density together with the observed values of the Age and Hubble Constant it can be inferred that

$$\Omega_{vac} \sim 0.7$$

- In Units of eV the Vacuum Energy Density is then given by $\rho_v \sim (10^{-3} eV)^4$

Physical Origin of Vacuum Energy

- The Energy Density of quantum fields propagating in curved space time can be re-interpreted as the cosmological constant (vacuum energy).

- To compute its value one has to calculate the Effective Potential from first principles.

The Effective Potential

- The detailed expression for the Effective Potential incorporating non-vanishing neutrino masses has been calculated by us from first principles.

- This Effective Potential calculated by us is able to explain the modern observations.

Summary on Dark Energy: Why do we need it and a field theory approach to what it might be

Why do we need it: Observations (Hubble Constant and Age of Universe)

What can it be: Maybe the Energy density of fields if neutrinos have small masses as implied by observations – this is well motivated and gives the right phenomenology to explain observations.

We will now look at some of the observational implications implied by this approach to Dark Energy.

Updating status on the Plan

The topics we will discuss are:

- Introduction:

    - √ Dark Energy: Why do we need it and what is it? [PRD1995] Done

- Dark Energy Implications (other than rapid expansion):

    - Collapse of DE and Supermassive Black Hole Formation[PASCOS, JETP1]

    - Gravitational Radiation from DE[JETP2]

    - Gravitational Wave Observations and Ice Age Periodicity[PLB]

    - Neutrino Masses, Early Structure Formation and James Webb Space Telescope Observations

    - Gauging Family Symmetries: Matter – Anti Matter Asymmetry from Higher Dimensions

- Conclusions

Implications: Collapse of Dark Energy Field Configurations and formation of Super Massive Black Holes (SMBHs).

NUCLEI, PARTICLES, FIELDS, GRAVITATION, AND ASTROPHYSICS

Gravitational Collapse of Dark Energy Field Configurations and Supermassive Black Hole Formation[1]

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Abstract—Dark energy is the dominant component of the total energy density of our Universe. The primary interaction of dark energy with the rest of the Universe is gravitational. It is therefore important to understand the gravitational dynamics of dark energy. Since dark energy is a low-energy phenomenon from the perspective of particle physics and field theory, a fundamental approach based on fields in curved space should be sufficient to understand the current dynamics of dark energy. Here, we take a field theory approach to dark energy. We discuss the evolution equations for a generic dark energy field in curved space-time and then discuss the gravitational collapse for dark energy field configurations. We describe the 3 + 1 BSSN formalism to study the gravitational collapse of fields for any general potential for the fields and apply this formalism to models of dark energy motivated by particle physics considerations. We solve the resulting equations for the time evolution of field configurations and the dynamics of space-time. Our results show that gravitational collapse of dark energy field configurations occurs and must be considered in any complete picture of our Universe. We also demonstrate the black hole formation as a result of the gravitational collapse of the dark energy field configurations. The black holes produced by the collapse of dark energy fields are in the supermassive black hole category with the masses of these black holes being comparable to the masses of black holes at the centers of galaxies.

Introduction & Motivation

- Need to understand the dynamics of the Dark Energy fields

- For cosmology: need to understand the gravitational dynamics of the Dark Energy fields

- Now we will describe the formalism for understanding the gravitational dynamics of the Dark Energy fields for any general potential for the Dark Energy fields.

Introduction & Motivation

- The set of evolution equations describing the time evolution of the Dark Energy fields coupled with gravity is a set of coupled Partial Differential Equations.

- These equations can be solved numerically and this has been done by us.

- Our results demonstrate the gravitational collapse of Dark Energy field configurations.

Evolution Equations

- Interested in studying gravitational dynamics of Dark Energy field configurations.

- In addition to the time evolution of the Field we need to study the time evolution of space-time which is described by the metric:

$$ds^2 = dt^2 - U(r,t)dr^2 - V(r,t)\left[d\theta^2 + \sin^2\theta d\phi^2\right]$$

Evolution Equations

metric, is of course a generalization of the usual FRW metric used to study cosmological space-times. Note that the functions $U(r, t)$ and $V(r, t)$ are functions of both space and time and can capture both homogeneous cosmological expansion as well as inhomogeneous gravitational collapse under appropriate circumstances.

We of course also want to study the time evolution of the field for which we need the Lagrangian for the field.

Lagrangian L given by

$$L = \frac{1}{2} \partial^\mu \Phi \partial_\mu \Phi - \mathcal{V}(\Phi) \tag{2}$$

where $\mathcal{V}$ is the potential for the field Φ and is for now a general function. Later, when we consider the physically motivated PNGB models this potential will take on a specific functional form.

Evolution Equations

$$\ddot{V} = 2\left[-1 + \frac{V''}{2U} - \frac{V'U'}{4U^2} - \frac{\dot{V}\dot{U}}{4U} + 8\pi GV\left(\frac{\rho}{2} - \frac{P}{2} - \frac{(\Phi')^2}{3U}\right)\right] \tag{3}$$

$$\ddot{U} = 2U\left[-\frac{\ddot{V}}{V} + \frac{\dot{U}^2}{4U^2} + \frac{\dot{V}^2}{2V^2} - 4\pi G\left(\rho + 3P\right)\right] \tag{4}$$

$$\ddot{\Phi} = \frac{\Phi''}{U} - \dot{\Phi}\left[\frac{\dot{V}}{V} + \frac{\dot{U}}{2U}\right] + \frac{\Phi'}{U}\left[\frac{V'}{V} - \frac{U'}{2U}\right] - \frac{\partial\mathcal{V}(\Phi)}{\partial\Phi} \tag{5}$$

where a dot represents a partial derivative w.r.t. t and a prime represents a partial derivative w.r.t. r. Further,

$$\rho = \frac{1}{2}\dot{\Phi}^2 + \mathcal{V}(\Phi) + \frac{(\Phi')^2}{2U} \tag{6}$$

and

$$P = \frac{1}{2}\dot{\Phi}^2 - \mathcal{V}(\Phi) + \frac{(\Phi')^2}{6U}. \tag{7}$$

Evolution Equations

The above equations are true for any general potential $\mathcal{V}(\Phi)$. One can of course write down the corresponding equations for PNGB fields. The simplest potential one can write down for the physically motivated PNGB fields [6] can be written in the form:

$$\mathcal{V}(\Phi) = m^4 \left[K - \cos(\frac{\Phi}{f}) \right] \tag{8}$$

- The above potential can thus be substituted in the general equations given on the previous slide to get the full system of evolution equations.

- These are coupled Partial Differential Equations which can be solved numerically to obtain the results of interest to us.

Solutions to the Evolution Equations

- Key issue we want to understand is the _timescale for the gravitational collapse_ for dark energy fields. If this timescale is larger than the age of the Universe then this gravitational collapse has no significance today. On the other hand, _if gravitational collapse occurs on timescales less than the age of the Universe then the gravitational collapse of Dark Energy fields must be considered._

Solution to the Evolution Equations

Guided by the evolution equations given in the previous section we define dimensionless quantities such that the field is measured in units of f and time and space are measured in units of $\frac{f}{m^2}$.
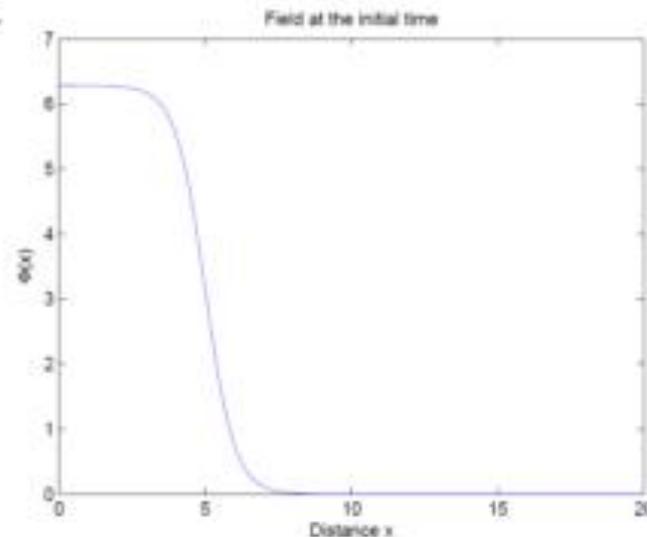

Figure 1: Initial Field configuration

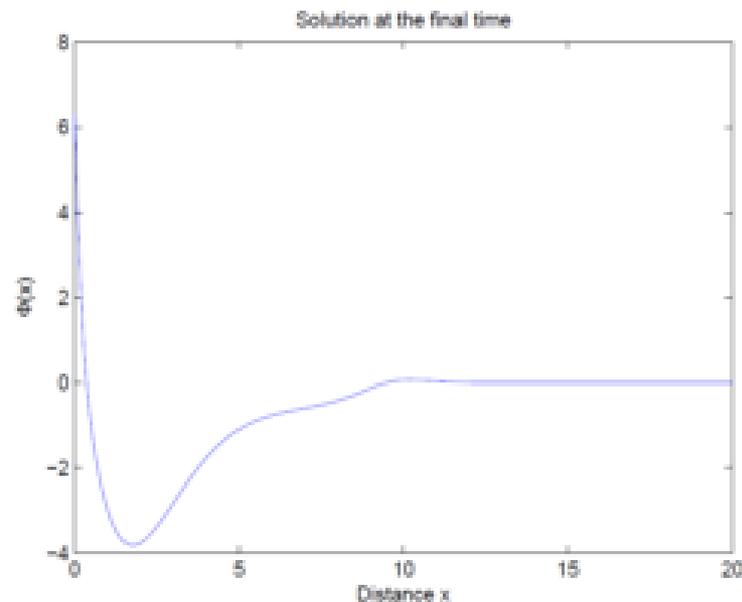# Solution to the Evolution Equations

Figure 2: Final Field configuration

Gravitational Collapse of Field Configuration has occurred.
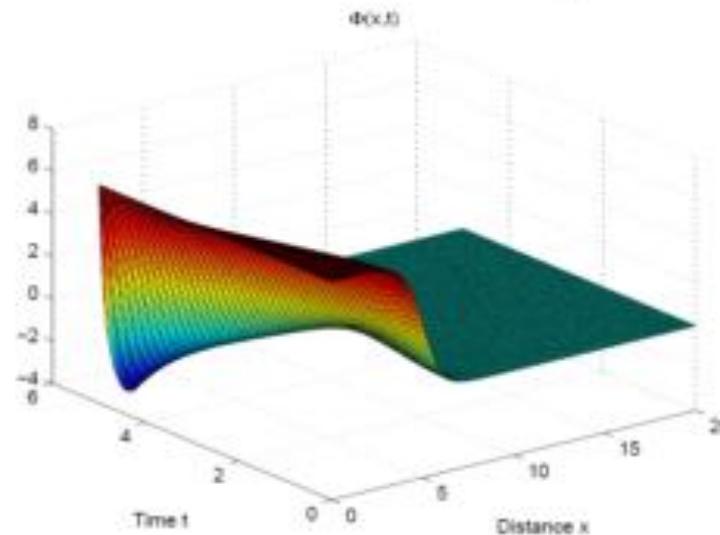
Solution to the Evolution Equation

Figure 3: Field configuration in space-time

From this it can be clearly seen that field configuration has collapsed and the timescale for collapse can be seen by studying the figure 3. Since the units of time are given by $\frac{f}{m^2}$, we note that gravitational collapse happens on timescales of $\sim \frac{f}{m^2}$. This timescale is much shorter than the age of the Universe.

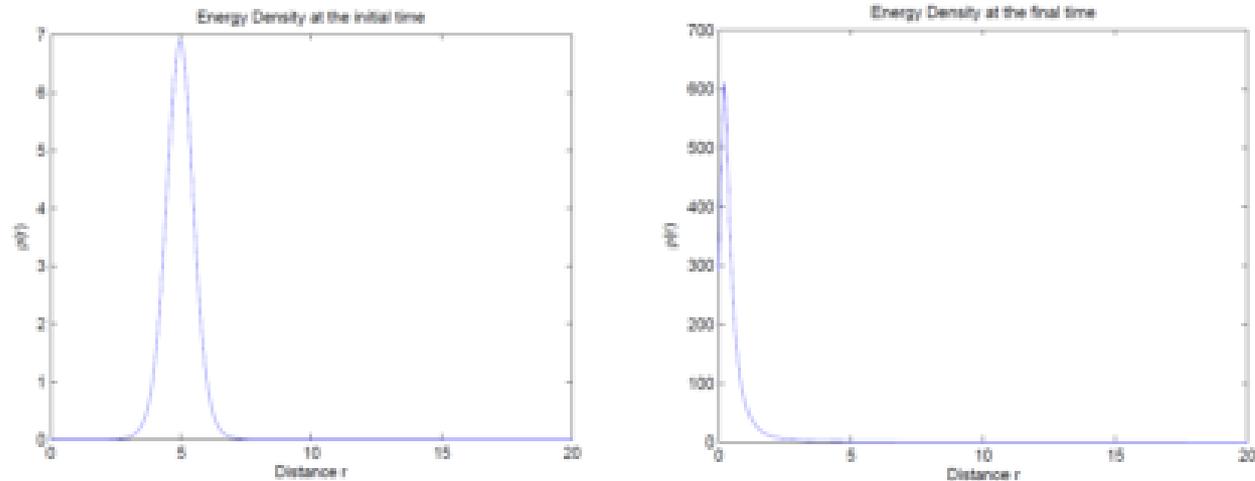# Time Evolution of the Energy Density and the Masses of Collapsed Objects

The Energy Density is given by:

$$\rho = \frac{1}{2}\dot{\Phi}^2 + \mathcal{V}(\Phi) + \frac{(\Phi')^2}{2U}$$

This can be plotted as a function of space and time and can also be integrated to obtain the Masses of Collapsed Objects.

Let us first show the time evolution of the energy density to demonstrate the formation of collapsed objects.

Time Evolution of the Energy Density and the Masses of Collapsed Objects

Energy Density moves radially inwards as collapse occurs.

Black Hole formation and masses of BHs formed

We used the 3+1 BSSN formalism to study Black Hole formation and were able to demonstrate the formation of a Black Holes as result of the dark enegy scalar field collapse.

Further, we were able to obtain the masses of Black Holes formed

$$M_{BH} = \frac{R_0^2}{(50 \text{ kpc})^2} \times 2.5 \times 10^6 M_{\text{solar}}$$

where R_0 is the physical size of the initial configuration that collapses.

This is comparable to the masses of Black Holes at the Center of Galaxies

Summary for Dark Energy Collapse.

Finally, we wish to emphasize the important results discussed and reported in this article.

1—The dynamical length scale f/m^2 of the dark energy field was shown to be comparable to the galaxy length scale.

2—Dark energy fields collapse on a timescale of 2×10^5 years, which is much shorter than the age of the Universe and hence this dynamics cannot be neglected.

3—The collapse of dark energy fields results in black hole formation.

4—We computed the masses of these black holes and determined that these are supermassive black holes with masses comparable to the masses of the black holes at the centers of galaxies.

Updating status on the Plan

The topics we will discuss are:

- Introduction:
  - √ Dark Energy: Why do we need it and what is it? [PRD1995] Done

- Dark Energy Implications (other than rapid expansion):
  - √ Collapse of DE and Supermassive Black Hole Formation[PASCOS, JETP1]   Done
  - Gravitational Radiation from DE[JETP2]
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- Conclusions

Gravitational Radiation from Dark Energy Dynamics

NUCLEI, PARTICLES, FIELDS, GRAVITATION, AND ASTROPHYSICS

Gravitational Wave Formation from the Collapse of Dark Energy Field Configurations

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Abstract—Dark energy is the dominant component of the energy density in the Universe. In a previous paper, we have shown that the collapse of dark energy fields leads to the formation of supermassive black holes with masses comparable to the masses of black holes at the centers of galaxies. Thus, it becomes a pressing issue to investigate the other physical consequences of the collapse of dark energy fields. Given that the primary interactions of dark energy fields with the rest of the Universe are gravitational, it is particularly interesting to investigate the gravitational wave signals emitted during the collapse of dark energy fields. This is the focus of the current work described in this paper. We describe and use the 3+1 BSSN formalism to follow the evolution of the dark energy fields coupled with gravity and to extract the gravitational wave signals. Finally, we describe the results of our numerical computations and the gravitational wave signals produced by the collapse of dark energy fields.

Equations and Initial Conditions

- Used coupled evolution equations for scalar field and gravity (in the 3+1 BSSN formalism for numerical stability)

- Initial conditions determined by the Dark Energy field configuration:

$$\Phi(r, \theta, \phi) = \phi(r)[1 + \epsilon \operatorname{Re}(Y_{20}(\theta, \phi))],$$

$$\phi(r) = \pi[1 - \tanh(r - r_0)],$$

$$\operatorname{Re}(Y_{20}(\theta, \phi)) = \frac{1}{4}\sqrt{\frac{5}{\pi}}(3\cos^2(\theta) - 1),$$

Gravitational Waves

- We are interested now in studying gravitational waves $h_{\mu\nu}$

- They are the perturbations of the full metric $g_{\mu\nu}$

about the background metric $g^0_{\mu\nu}$

- The relationship between these quantities is:

$$g_{\mu\nu} = g^0_{\mu\nu} + h_{\mu\nu}$$

Computing the Gravitational Wave strength

- To proceed, we decompose the gravitational waves into odd and even multipoles and introduce Regge-Wheeler functions as:

decomposing the metric perturbation $h_{\mu\nu}$ into odd even multipoles, i.e., we can write

$$h_{\mu\nu} = \sum [(h_{\mu\nu}^{\ell m})^{(o)} + (h_{\mu\nu}^{\ell m})^{(e)}].$$

$$Q_{\ell m}^{\times} \equiv \sqrt{\frac{2(\ell+1)!}{(\ell-2)!}} \frac{1}{r} \left(1 - \frac{2M}{r}\right)$$

$$\times \left[(h_1^{\ell m})^{(o)} + \frac{r^2}{2} \partial_r \left(\frac{(h_2^{\ell m})^{(o)}}{r^2} \right) \right]$$

$$Q_{\ell m}^{+} \equiv \sqrt{\frac{2(\ell+1)!}{(\ell-2)!}} \frac{r q_1^{\ell m}}{\Lambda[r(\Lambda-2) + 6M]}$$

The odd-and even-parity master functions, Eqs. (19) and (20), can be straightforwardly related to the gravitational-wave strain and are given by

$$h_+ - ih_\times = \frac{1}{\sqrt{2}r} \sum_{\ell,m} \left(Q_{\ell m}^{+} - i \int_{-\infty}^{t} Q_{\ell m}^{\times}(t')dt' \right)$$

$$\times {}_{-2}Y^{\ell m}(\theta,\phi) + \mathbb{O}\left(\frac{1}{r^2}\right), \quad (26)$$

where ${}_{-2}Y^{\ell m}(\theta,\phi)$ are the spherical harmonics with spin weight $s = -2$.

Gravitational Wave Plots

- Gravitational Wave results can be computed numerically (we used the publically available Einstein Toolkit for doing this). Sample Results:

the time is measured in units of f/m^2. It should also be noted that f/m^2 is the fundamental time scale of the dynamics as determined by the evolution equations.

Fig. 3. Even Q_{lm} with $l = 2$, $m = 0$, $\epsilon = 0.01$.

Gravitational Wave Results

Note, in particular, that the time period of the produced gravitational waves is comparable to the fundamental time scale of the dynamics.

The scale f is the high-energy symmetry breaking scale in PNGB models. In the see-saw model of neutrino masses [3] this corresponds to the heavy symmetry breaking scale. Whereas f has a range of possible values, the typical value of f in the see-saw model of neutrino masses is $f \sim 10^{13}$ GeV. The typical value of m is $\sim 10^{-3}$ eV. It should also be noted that so far we have been working in the particle physics and cosmology units with $\hbar = c = k = 1$. It is straightforward to convert from these units to more familiar units using standard conversion factors [8]. Thus, 1 GeV^{-1} = 1.98 × 10^{-14} cm and 1 GeV^{-1} = 6.58 × 10^{-25} s.

Using these conversion factors, we see that the fundamental time scale of the dynamics corresponding to f/m^2 is 2 × 10^5 years.

The Time Period of Gravitational Waves produced by Dark Energy Dynamics is about 100,000 years.

Updating status on the Plan

The topics we will discuss are:

- Introduction:
    - √ Dark Energy: Why do we need it and what is it? [PRD1995] Done

- Dark Energy Implications (other than rapid expansion):
    - √ Collapse of DE and Supermassive Black Hole Formation[PASCOS, JETP1]   Done
    - √ Gravitational Radiation from DE[JETP2]   Done
    - Gravitational Wave Observations and Ice Age Periodicity[PLB]
    - Neutrino Masses, Early Structure Formation and James Webb Space Telescope Observations
    - Gauging Family Symmetries: Matter Anti Matter Asymmetry from Higher Dimensions

- Conclusions

Gravitational Wave Observations and Ice Age Periodicity

Dark energy gravitational wave observations and ice age periodicity

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A B S T R A C T

Dark Energy is the dominant component of the energy density of the Universe. However, it is also very elusive since its interaction with the rest of the Universe is primarily gravitational. Since Dark Energy is a low energy phenomenon from the perspective of particle physics and field theory, a fundamental approach based on fields in curved space is sufficient to understand the current dynamics of Dark Energy. The key issue is to understand the gravitational dynamics of Dark Energy and its observational consequences. However, finding the observational consequences of Dark Energy dynamics has been a very challenging task. For something which is the dominant component of the energy density of the Universe, Dark Energy appears to be very distant and reclusive. Here we show that the Dark Energy dynamics results in the production of gravitational waves which produce the ellipticity variation in earth's orbit that results in the periodicity of the Ice Ages observed and documented by geologists and climatologists. Previously, no observational signature of gravitational waves produced by Dark Energy dynamics has been reported. Further, no interpretation of the ellipticity variation of the earth's orbit due to gravitational waves or the linking of such gravitational waves to the Ice Age periodicity has been reported previously. We hope that the current work will lead to some fresh insights and some more interesting work.

Dark Energy Gravitational Wave Observations: Context and Challenge

In our quest for interactions of the Dark Energy with the rest of the Universe, our first attempts perhaps justifiably focus on the gravitational interactions of the Dark Energy, since it is the dominant interaction of Dark Energy with the rest of the Universe. One new signal that one can perhaps hope to measure is the gravitational waves emitted by Dark Energy. Thus, a study was made of the gravitational waves emitted by Dark Energy [12]. One of the key results obtained in that paper is that the time period of the gravitational waves emitted is $\sim 10^5$ years.

At first, one may be filled with dismay that with the time period being $\sim 10^5$ years and the human lifetime being $\sim 10^2$ years, the chances of detecting such gravitational waves are slim to none. However, nature has been kind to us.

Dark Energy Gravitational Wave Observations: Path Forward.

To find a pathway to the detection of these gravitational waves, it is perhaps worth recalling that a gravitational wave periodically turns circles into ellipses and vice versa as it passes through a region of space. For an easily accessible discussion, please see Gravitational Waves, Sources, and Detectors [15]. Thus, we expect a periodic oscillation in the ellipticity of orbits as a signature of gravitational waves.

Nature has in fact been kind to us in that it has not only detected these gravitational waves but maintained a record of it for us to uncover and interpret.

For this, we turn to a description of Ice Age periodicity, the underlying geological data and its interpretation in the terms of the periodic oscillations in the ellipticity of earth's orbit.

Period matching for Gravitational Waves and Ice Age Periodicity

It turns out that geologists and climatologists have known for about a century that the periodicity in the ice ages can be linked to what they call insolation which essentially quantifies the heat received from the sun. This insolation has been tied to the variations in the Earth's orbit. It further turns out that the dominant driver of this phenomena is the variation in ellipticity on the timescale of $\sim 10^5$ years. For more details on this please see Variations in the Earth's Orbit: Pacemaker of the Ice Ages [7] and Climate and atmospheric history of the past 420,000 years from the Vostok ice core, Antarctica [6] and references therein. These articles bring out the role played by periodic variations in the eccentricity of the Earth's orbit on the time scale of $\sim 10^5$ years in driving the periodicity of the Ice Ages.

Physical origin for the matching of periods: Resonance

The equation of motion of the gravitational waves in vacuum is:

$$\Box h_{\mu\nu} = 0 \tag{24}$$

which follows from the Einstein Equation:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa T_{\mu\nu} \tag{25}$$

with the source term zero i.e. when $T_{\mu\nu} = 0$. See for example [28] for an excellent discussion on this and related topics.

When the source term is not zero, the equation for the gravitational wave components $h_{\mu\nu}$ is given by:

$$\Box h_{\mu\nu} = S_{\mu\nu} \tag{26}$$

The non-zero source term $S_{\mu\nu}$ is present for example if you have non-trivial scalar field configurations present.

Resonance between scalar field dynamics and gravity waves

For a scalar field source with the scalar field having a natural frequency ω (related to the mass m of the scalar field), the source term can be expressed as:

$$S_{\mu\nu} = S_{\mu\nu}^{(0)} e^{i\omega t} \tag{27}$$

Putting this into the equation for the gravitational wave componenets we get:

$$\Box h_{\mu\nu} = S_{\mu\nu}^{(0)} e^{i\omega t} \tag{28}$$

This has the form of a driven harmonic oscillator which can be seen much more explicitly by going into Fourier space.

As is well known from the solution of the driven harmonic oscillator, this implies two things:

(1) The resonance picks out some special frequencies and enhances their amplitude. This is the reason that the time period of the Gravitational Waves matches the time period of scalar field dynamics as already observed.

(2) The amplitude at resonance can grow to be large.

Looking at the equation for the gravitational wave components in Fourier space, we get:

$$\frac{d^2 h_{\mu\nu}}{dt^2} + k^2 h_{\mu\nu} = S_{\mu\nu}^{(0)} e^{i\omega t} \tag{29}$$

Amplitude of the gravity waves

We now want to push these ideas one step further by using the energetics to arrive at an estimate of the amplitude of the gravitational waves produced by dark energy dynamics. In order to do this, we first note that the system of interest for us here consists of 2 important sub-components: namely dark energy and gravitational waves. These two sub-components can exchange energy as they interact dynamically. Initially, when the gravity wave amplitudes are very small, the direction of transfer of energy will be from the dark energy field to the gravitational wave field. However, as the gravitational wave amplitude grows as a result of the resonance phenomenon, it will also start dynamically transferring some energy back to the dark energy field. Since at this point there is a significant flow of energy in both directions (from and towards the dark energy field) the long term state of the system of interest will be dynamically driven towards the point where the energy density in gravitational waves ρ_{gw} is equal to the energy density in the dark energy field ρ_{DE}. Thus we can use the condition (which we henceforth refer to as the equipartition condition):

$$\rho_{gw} \simeq \rho_{DE} \tag{31}$$

to determine the amplitude of the gravitational waves at long times. One can arrive at the equipartition condition from a Sta-

Energy density of Dark Energy

In order to do this we consider the observational data. We need to start with the Dark Energy density. We now know that the Dark Energy density is the dominant component of the energy density of the Universe. For estimating the gravitational wave amplitude inside our galaxy we need to start with the energy density within our galaxy. This can be estimated fairly robustly from available data (see for example [30]). We can summarize this data here by noting that a mass $\sim 10^{12} M_\odot$ is confined to a volume with linear dimensions ~ 10 kpc. From this data we can estimate that on galaxy length scales (within our galaxy) the dark energy density is $\rho_{DE} \sim 10^{-1}$ erg/cm^3.

Gravity wave energy density and amplitude of gravity wave

For a gravitational wave with amplitude h_0 and angular frequency ω, the energy density in gravitational waves ρ_{gw} is given by (See for example [28] for an excellent discussion on this and related topics.):

$$\rho_{gw} \simeq \frac{c^2 \omega^2 h_0^2}{(32\pi G)} \tag{32}$$

We note that the angular frequency is related to the time period T by $\omega = 2\pi/T$. Both from the resonance condition as well as the observational data on the time period T we get that $T \sim 10^5 \, years$. Inserting this into the equation for the gravitational wave energy density and using the equipartition condition we get $h_0 \sim 10^{-2}$ as the amplitude for the gravitational waves produced by the dark energy dynamics.

Matches observations!!

It is worth pointing out at this juncture that the value of the gravitational wave amplitude as calculated above is of the correct magnitude to explain the data on the periodic eccentricity variation of earth's orbit as documented by Hinnov in the Ann. Rev. Earth Planet. Sci. (2000) [8].

Thus, we see that the resonance analysis helps us not only to intuitively understand and gain insights into the basic underlying physics, but also helps us to quantitatively arrive at the correct magnitude for both frequency and amplitude of the gravitational waves produced by the dark energy dynamics which matches the observational data on the Ice Age periodicity as given for example by Hinnov [8].

Summary

In summary, we see that the gravitational waves produced by dark energy dynamics can produce the periodic variations in the Earth's orbital eccentricity resulting in the Ice Age periodicity.

In the current work, we have described how gravitational waves with an amplitude $h_0 \sim 10^{-2}$ and with a time period $T \sim 10^5$ years which corresponds to a frequency of $\sim 10^{-13}$ Hz can arise out of dark energy dynamics. The value of $h_0 \sim 10^{-2}$ at the low frequencies of $\sim 10^{-13}$ Hz is compatible with other constraints on gravitational waves – please see for example [31] for

Updating status on the Plan

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- Conclusions

Dark Energy Black Holes with Intermediate Masses at High Redshifts: an earlier generation of Quasars and observations.

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Dark Energy is the largest fraction of the energy density of our Universe - yet it remains one of the enduring enigmas of our times. Here we show that Dark Energy can be used to solve 2 tantalizing mysteries of the observable universe. We build on existing models of Dark Energy linked to neutrino masses. In these models Dark Energy can undergo Phase Transitions and form Black Holes. Here we look at the implications of the family structure of neutrinos for the phase transitions in dark energy and associated peaks in black hole formation. It has been previously shown that one of these peaks in Black Hole formation is associated with the observed peak in Quasar formation. Here, we predict that there will also be an earlier peak in the Dark Energy Black Holes at high redshifts. These Dark Energy Black Holes formed at high redshifts are Intermediate Mass Black Holes (IMBHs). These Dark Energy Black Holes at large redshift can help explain both the EDGES observations and the observations of large Supermassive Black Holes (SMBHs) at redshifts $z \sim 7$ through the accretion onto the Dark Energy IMBHs at high redshifts. Thus, these Dark Energy Black Holes solve 2 of the most puzzling mysteries of the observable early universe. Not only does the existence of an earlier phase of Dark Energy Black Holes take care of some current challenges to theory implied by existing astronomical data, it also helps us actively look for these Dark Energy Black Holes at these high redshifts as predicted here through targeted searches for these Black Holes at the redshifts $z \sim 18$. There is a slight dependence of the location of the peak on the lightest neutrino mass - so the peak may be located at a slightly lower value of the redshift. This may actully enable a measurement of the lightest neutrino mass - something which has eluded us so far. Finding these Dark Energy Black Holes of Intermediate Mass should be within the reach of upcoming observations - particularly with the James Webb Space Telescope - but perhaps also through the use of other innovative techniques focusing specifically on the redshifts around $z \sim 18$.

Black Hole Formation due to an earlier Phase Transition linked to the 3rd generation of neutrinos and JWST observations.

- Dark Energy can solve 2 further tantalizing mysteries in Dark Energy models linked to neutrino masses.

- In these models Dark Energy can undergo Phase Transitions and form Black Holes.

- The family structure of neutrinos then imply peaks in black hole formation.

- Previously shown that one of these peaks in Black Hole formation gives the observed peak in Quasar formation.

- Predicted that there will also be an earlier peak in the Dark Energy Black Holes at high redshifts.

Black Hole Formation due to an earlier Phase Transition linked to the 3rd generation of neutrinos and JWST observations.

- Dark Energy Black Holes at large redshift can help explain both the EDGES observations and the observations of large Supermassive Black Holes (SMBHs) at redshifts z > 7

- Thus, these Dark Energy Black Holes solve 2 of the most puzzling mysteries of the observable early universe.

- JWST has already started seeing Galaxies at large redshift as expected in the model described here.

- Further, the ΛCDM model currently has challenges in explaining the large number of such galaxies at high redshift.

- Whereas in our model we expect an earlier peak in structure formation at z ≈ 18.

Black Hole Formation due to an earlier Phase Transition linked to the 3rd generation of neutrinos and JWST observations.

- There is a slight dependence of the location of the peak on the lightest neutrino mass - so the peak may be located at a slightly lower value of the redshift. **This may actually enable a measurement of the lightest neutrino mass - something which has eluded us so far.**

- Finding these Dark Energy Black Holes of Intermediate Mass should be within the reach of upcoming observations - particularly with the James Webb Space Telescope - but perhaps also through the use of other innovative techniques

Phase Transitions, Neutrino Masses and Peak Redshifts

We now wish to use this to relate the epochs at which the Phase Transitions occur resulting in the peaks in the associated Black Hole formation. We can label the three families of light neutrinos as 1, 2 and 3 in increasing order of mass, so they will have masses denoted by:

by m_1, m_2 and m_3. Let us now label the critical temperatures associated with the phase transitions as T_1, T_2, T_3 and scale factors associated with these phase transitions as R_1, R_2, R_3 respectively. The scale factors can themselves be associated with the corresponding redshifts z_1, z_2, z_3 respectively.

Phase Transitions, Neutrino Masses and Peak Redshifts

Using the physics of the phase transitions and the evolution of temperature, scale factor and redshift in an expanding universe we note that:

$$\frac{T_3}{T_2} = \frac{m_3}{m_2}$$

$$\frac{T_3}{T_2} = \frac{R_2}{R_3}$$

and

$$\frac{R_2}{R_3} = \frac{1+z_3}{1+z_2}.$$

From observations of the Quasar distribution with redshift, we get: $z_2 \sim 2.5.$

If the lightest neutrino mass is insignificant comapared to the masses of the heavier neutrinos (i.e. $m_1 \ll m_2, m_3$), then we get: $z_3 \sim 18.$

Phase Transitions, Neutrino Masses and Peak Redshifts

The value of z_3 has a dependence on the lightest neutrino mass m_1 and so the result is displayed as a plot of z_3 versus m_1 in the figure below.

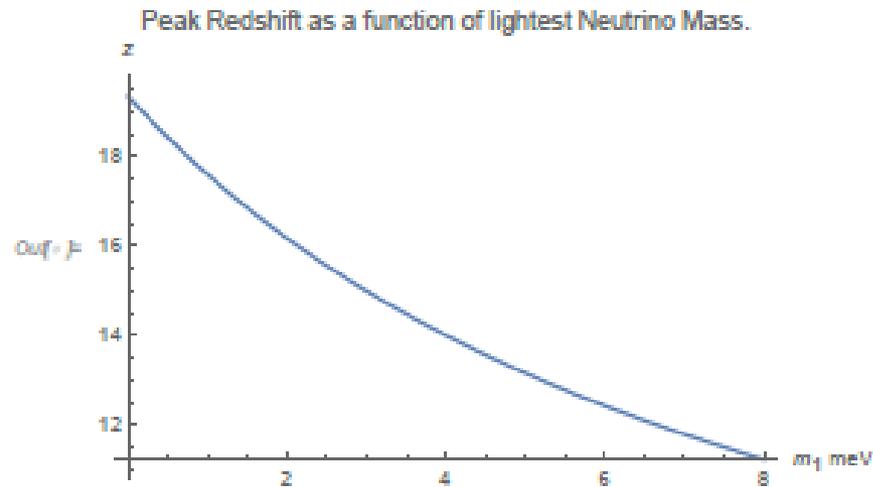Peak Redshift as a function of lightest Neutrino Mass.

FIG. 1. Peak Redshift as a funcion of m_1.

Figure 1 displays the loaction of the Peak in redshift (z) as function of the mass of the lightest neutrino (m_1 in meV).

Updating status on the Plan

The topics we will discuss are:

- Introduction:

  - √ Dark Energy: Why do we need it and what is it? [PRD1995] Done

- Dark Energy Implications (other than rapid expansion):

  - √ Collapse of DE and Supermassive Black Hole Formation[PASCOS, JETP1]   Done

  - √ Gravitational Radiation from DE[JETP2]   Done

  - √ Gravitational Wave Observations and Ice Age Periodicity[PLB] Done

  - √ Neutrino Masses, Early Structure Formation and James Webb Space Telescope Observations

  - Gauging Family Symmetries: Matter – Anti Matter Asymmetry from Higher Dimensions

- Conclusions

Gauging Flavor Symmetries of the Standard Model: from Dark Energy to Matter - Antimatter Asymmetry from Higher Dimensions in the Early Universe.

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Abstract. The Standard Model of Elementary Particle Physics has a global $U_1(1) \times U_2(1) \times U_3(1)$ flavor symmetry (often called an accidental symmetry because it was not put in by hand). Here the subscripts $i = 1, 2, 3$ refer to the indices corresponding to the 3 families in the Standard Model. It has previously been shown that the breaking of these symmetries at low energy may result in producing Dark Energy which is the dominant component of the energy density of the Universe. It has also previously been shown that this model of Dark Energy not only explains the accelerated expansion of the universe but has additional observational consequences consistent with observations and also makes predictions which may be verified in the near future. Thus, this model of Dark Energy allows a space and time dependent Dark Energy as indicated by recent observations. Further, in this model of Dark Energy the collapse of space dependent Dark Energy configurations can lead to the formation of Dark Energy Black Holes in both the Supermassive Black Hole category and the Intermediate Mass Black Hole category leading to predictions that may soon be verified by observations. Here we wish to examine the potential implication of these symmetries at high energies and in the Early Universe. In particular, we consider what might happen if these global symmetries get gauged at higher energies and that the global symmetries today are just a consequence of the gauge fields acquiring a constant vacuum expectation value below some energy scale v. Since, we have not yet seen signatures of such gauge fields at any of our particle accelerators, this would imply that v is greater than the electroweak scale. Thus, in the early Universe at the time when the $U_i(1)$ gauge fields would be dynamical, the Higgs would not yet have acquired a non-zero vacuum expectation value and all Standard Model particles would be massless. The gauging of these global symmetries is desirable from a high energy physics and quantum gravity perspective. The chiral dynamics has a natural interpretation in terms of D Branes and our 4-dimensional Universe embedded in higher dimensions. Anomaly cancellation in this scenario results from the sum of the cubic terms vanishing as described below. An important point to note is that the analysis and discussion in this article can provide an explanation for one of the long standing mysteries of the natural world: the matter - antimatter asymmetry observed in our Universe.

Gauging Family Symmetries: Matter – Anti Matter Asymmetry from Higher Dimensions

- The <u>breaking of the Global</u> $U_1(1) \times U_2(1) \times U_3(1)$ <u>Family Flavor Symmetries</u> by non-vanishing neutrino masses is <u>sufficient for the Dark Energy</u> predictions discussed earlier. (Here the subscripts are the family indices.)

- We now consider the <u>high energy implications</u> of these symmetries: in particular, <u>the gauging of these Family Flavor Symmetries.</u>

Motivations for Gauging Family Symmetries.

- There are long standing concerns about global symmetries at high energies arising out of quantum gravity.

- Further, N Coincident D-Branes give rise to U(N) Gauge theories whereas the Standard Model and its extensions such as the Pati-Salam Model and SU(5) GUTs etc. use SU(N) Gauge theories leaving some extra U(1) Gauge symmetries.

- If we can Gauge the U(1) Global Flavor symmetries, then this can get rid of the mystery of the missing U(1) Gauge fields

Symmetries, Anomalies and Anomaly Cancellation

The $U_i(1)$ flavor symmetries of the Standard Model that we are discussing are also called the Lepton Family symmetries L_i for each of the 3 families in the Standard Model labeled by the 3 family indices $i = 1, 2, 3$. The electron family has the index $i = 1$, the muon family has the index $i = 2$ and tau family has the index $i = 3$.

The corresponding symmetry currents are given by

$$j_{i\mu} = \bar{\psi}_i \gamma_\mu \psi_i$$

and the axial or chiral current is given by

$$j_{i\mu}^5 = \bar{\psi}_i \gamma_\mu \gamma_5 \psi_i$$

Conservation of the symmetry current $j_{i\mu}$ is given by the equation $\partial^\mu j_{i\mu} = 0$ and gives us the non-zero conserved charges Q_i as per the Standard Model:

Table 1: Charges Q_i

Particles	Q_1	Q_2	Q_3
Electron	+1	0	0
Positron	−1	0	0
Electron Neutrino	+1	0	0
Electron Antineutrino	−1	0	0
Muon	0	+1	0
Anti-Muon	0	−1	0
Muon Neutrino	0	+1	0
Muon Antineutrino	0	−1	0
Tau	0	0	+1
Anti-Tau	0	0	−1
Tau Neutrino	0	0	+1
Tau Antineutrino	0	0	−1

Please note that for all other particles the charges $Q_i = 0$.

it is only the gauge anomaly which is proportional

to the sum over the Q_i^3 which needs to vanish. Thus, in the case we are interested in, since the charges come in $+1, -1$ pairs or are 0 (please see Table 1 above), the sum over the Q_i^3 actually vanishes and hence we can gauge the the $U_i(1)$ symmetries as there in no obstruction from anomalies.

Thus, we can consistently build a picture where at low energies the flavor symmetries are global symmetries but at high energies these flavor symmetries become gauge symmetries. Essentially, this means that above some high energy scale which we will denote by v, the gauge fields are dynamical and can have a non-trivial space-time dependence. However, below the energy scale v, these gauge fields acquire their uniform space independent vacuum expectation values. We further note that the scale v must be above the electroweak symmetry breaking scale because otherwise we would have seen the dynamical signature of these extra gauge fields at existing particle accelerators such as the LHC.

Symmetries, Anomalies and Anomaly Cancellation

Dirac Equation and D - Branes

The Dirac equation can be written in the form:

$$(i\gamma^\mu \partial_\mu - m)\, \psi = 0 \tag{5}$$

The discussion that follows is conveniently carried out in terms of the states ψ_L and ψ_R, where we have introduced the left-handed projections: $\psi_L = (1 - \gamma^5)\psi/2$ and right-handed projection: $\psi_R = (1 + \gamma^5)\psi/2$.

First, we note that in the massless $m = 0$ limit the 2 states ψ_L and ψ_R decouple and are independent of each other.

Moreover, we note that the ψ_L states are positive energy solutions to the Dirac Equation and represent particle states or matter and the ψ_R states are negative energy solutions and represent anti-particle states or anti-matter.

It is a fact of nature that it is only the ψ_L states which interact with the electroweak gauge bosons. Furthermore, in the D Brane picture of the Standard Model - please see e.g. the work of Leontaris et. al. -[22] - the electroweak gauge bosons and ψ_L are confined to the 2 coincident branes giving rise to the $SU(2)$ gauge fields of the Standard Model. After the Higgs field acquires a non-zero vacuum expectation value, the mass term connects the ψ_L to the ψ_R - this happens after the electroweak symmetry breaking.

Dirac Equation and D - Branes

The discussion above leads to the point of view that we live on the Left-handed Brane on which all the matter particles that we detect as well as the electroweak gauge bosons live. This statement may raise a concern about the fact that we do occasionally come across anti-matter particles. But this concern is immediately alleviated by noting that the appearance of the anti-matter particles is determined precisely by the Dirac equation given above. Thus, from this perspective, the anti-matter particles are visitors to our Left-handed Brane and their appearance is strictly regulated by the Dirac equation. Since the Dirac equation has been verified a very large number of times and there is no known violation of this equation this puts the point of view put forth here on a firm phenomenological footing.

As we raise the temperature of our Universe and effectively travel into it's past, first we cross the electroweak phase transition - at higher energy densisties - the Higgs vacuum expectation value is zero and all Standard Model particles are massless at this point. At the electroweak phase transition, the flavor symmetries are still global - otherwise we would have seen the signature of the extra gauge bosons at existing accelerators. However, at some higher energy scale v the flavor symmetries can become dynamical gauge fields. At this point, since the flavor symmetries are carried by $U_i(1)$ gauge fields, we have extra copies of electromagnetic type forces - except the charges are not the usual electromagnetic charges but instead given by the charges as already enumerated in Table 1 above.

Further, we re-iterate that all matter particles carry $+1$ charge and are located on L Branes all anti-matter particle (anti-particles) carry -1 charge and are located on R Branes.

Thus, even if the L and R were located close together in the early universe as one might expect, the L - R symmetry can be easily broken by an additional Brane approaching or colliding with the L and R Brane pair initially located close to each other.

Brane Collisions and Matter – Anti Matter Asymmetry from Brane Collisions in the Early Universe

Conclusions

Dark Energy has come a long way from its birth in 1995. It has important observable implications for our universe:

1. **Removes discrepancy between Hubble Constant and Age of the Universe**: determines dark energy density

2. **Dark Energy collapse can produce Super Massive Black Holes**: the masses of Black Holes produced is comparable to the masses of Black Holes found at the centers of galaxies.

3. **Dark Energy dynamics can produce Gravitational Waves with interesting observational consequences**: The frequency and amplitude of the gravitational waves produced is just right to explain the periodicity of Ice Ages on earth.

4. **Phase Transitions in Dark Energy can give us early structure formation as observed by JWST .** This may also enable the first measurement of the lightest neutrino mass.

5. **Dark Energy fields arise naturally from non-vanishing neutrino masses as implied by observations:** they provide insights into the **particle physics beyond the standard model** of particle physics.

6. **Gauging of the Family Symmetries that give rise to Dark Energy can explain the Matter – Anti Matter Asymmetry.**

Key References

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- Dark Energy Black Holes and Broken Family Flavor Symmetries due to non-zero neutrino masses:
  Anupam Singh, doi:10.1142/S0217732323501353, https://doi.org/10.1142/S0217732323501353 Mod. Phys. Lett. A , Volume **38**, 2350135 (2023); arXiv:2207.02143 (2022).

Gauging Family Symmetries and Matter-Anti Matter Asymmetry from Higher Dimensions: arXiv: 2411.17714, Anupam Singh (2024)

Back up slides only to be used if needed.

FRW metric and Age of Universe

Quasar Formation

JWST and Early Formation of Structure

Neutrino Masses, Effective Potential and Phase Transitions

3+1 BSSN formalism

Space-time Geometry

Gravitational Waves

Spacetime Metric and Age of Universe

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2\right],$$

$$(1)$$

where (t, r, θ, ϕ) are the comoving coordinates describing a space-time point and $R(t)$ is the cosmic scale factor. Also, $k = +1, -1,$ or 0 depending on when the universe is closed, open, or flat.

The present expansion age of a matter dominated universe can be evaluated in a Robertson-Walker universe. General fine-tuning arguments as well as the inflationary picture gives us a preference for a flat universe, $\Omega_0 = 1$. In this case, $t_0 = \frac{2}{3}H_0^{-1}$. For, $\Omega_0 \simeq 1$, one can expand the above expressions in a Taylor expansion:

$$t_0 = \frac{2}{3}H_0^{-1}\left[1 - \frac{1}{5}(\Omega_0 - 1) + \cdots\right].$$

$$(2)$$

We can also determine the present age of the universe containing both matter and vacuum energy such that $\Omega_{\text{vac}} + \Omega_{\text{matter}} = 1$:

$$t_0 = \frac{2}{3}H_0^{-1}\Omega_{\text{vac}}^{-1/2}\ln\left[\frac{1 + \Omega_{\text{vac}}^{1/2}}{(1 - \Omega_{\text{vac}})^{1/2}}\right].$$

$$(3)$$

This will give us much longer lifetimes as can be seen most dramatically by examining the limit $\Omega_{\text{vac}} \to 1$ in which case $t_0 \to \infty$. Indeed having an $\Omega_{\text{vac}} \sim 0.8$ is

R'[t]/R[t] (in units of 75 km/(sec Mpc))

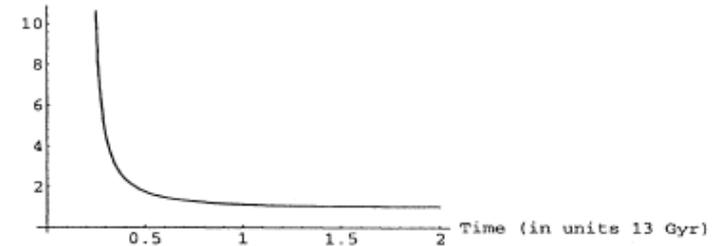FIG. 1. Time evolution of the Hubble parameter in the LTPT model: $\dot{R}(t)/R(t)$ vs time.

Quasar Formation

Quasar production: Topological defect formation due to a phase transition linked with massive neutrinos

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Recent observations of the space distribution of quasars indicate a very notable peak in space density at a redshift of 2 to 3. It is pointed out in this article that this may be the result of a phase transition which has a critical temperature of roughly a few meV (in the cosmological units $h = c = k = 1$). It is further pointed out that such a phase transition is natural in the context of massive neutrinos. In fact, the neutrino masses required for quasar production and those required to solve the solar neutrino problem by the Mikheyev-Smirnov-Wolfenstein mechanism are consistent with each other.

Zero Temperature Potential

Consider now the three light families of neutrinos which we will denote by ν_j with j being the family index

$$\mathcal{L} = \frac{1}{2}\partial^\mu \Phi_j \partial_\mu \Phi_j + \bar{\nu}_j i\partial\!\!\!/ \, \nu_j + m_j \bar{\nu}_{jL}\nu_{jR} + g_j \bar{\nu}_{jL}\Phi_j \nu_{jR} + \text{H.c.}$$

where repeated indices are summed over and Φ_j are $U(1)$ complex scalar fields that develop vacuum expectation values (VEVs) given by:

$$\langle \Phi_j \rangle = \frac{f e^{i\phi_j/f}}{\sqrt{2}}$$

The zero temeperature part of the potential

$$V(\phi_1, \phi_2, \phi_3) = -\frac{M_j^4}{16\pi^2}\ln(M_j^2) + K$$

where K is a constant and

$$M_j^2 = m_j^2[1 + \epsilon_j^2 + 2\epsilon_j Cos(\phi_j/f)]$$

and we have defined ϵ_j as $\epsilon_j = \frac{g_i f}{m_j \sqrt{2}}$.

Finite Temperature Potential

$$\Delta V_T(\phi_1, \phi_2, \phi_3) = -4 \frac{T^4}{2\pi^2} \sum_{j=1}^{3} \int_0^\infty dx x^2 \ln \left[1 + \exp -\sqrt{x^2 + \frac{M_j^2}{T^2}} \right]$$

$$V_{Full}(\phi_1, \phi_2, \phi_3) = V(\phi_1, \phi_2, \phi_3) + \Delta V_T(\phi_1, \phi_2, \phi_3)$$

model has $U_1(1) \times U_2(1) \times U_3(1)$ symmetry

When $m_i \neq 0$, the Goldstone Boson ϕ_i corresponding to $U_i(1)$ when $m_i \to 0$, picks up a non-trivial potential and becomes a Pseudo Nambu Goldstone Boson (PNGB).

Now the full potential of the scalar fields can be used to determine the presence of phase transitions and the critical temperatures of the phase transitions. It turns out that there are 3 phase transitions in this model and as might be expected the critical temperatures of these 3 phase transitions T_1, T_2 and T_3 are determined by the 3 masses m_1, m_2 and m_3. In particular, we note for use in the following sections that $T_3/T_2 = m_3/m_2$ and that all the 3 phase transitions in this model are second order phase transitions.

Phase Transitions

The 3+1 BSSN Formalism.

To obtain numerical solutions, it is convenient to use the $3 + 1$ decomposition of the Einstein equations, for which the line element can be written as

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt), \quad (4)$$

where γ_{ij} is the 3-dimensional metric. The Latin indices label the three spatial coordinates. The functions α and β^i in Eq. (4) are gauge parameters, respectively known as the lapse function and the shift vector. The determinant of the 3-metric is γ. The Greek indices range from 0 to 3 and the Latin indices range from 1 to 3.

For the purpose of numerical evolution, the Klein-Gordon equation can be written as a first-order system. This is done by first splitting the scalar field into the real and imaginary parts as $\Phi = \phi_1 + i\phi_2$ and then defining variables in terms of combinations of their derivatives:

$$\Pi = \pi_1 + i\pi_2, \quad \psi_a = \psi_{1a} + i\psi_{2a}.$$

Here,

$$\pi_1 = (\sqrt{\gamma}/a)(\partial_t \phi_1 - \beta^c \partial_c \phi_1), \quad \psi_{1a} = \partial_a \phi_1,$$

and we can similarly replace the subscript 1 with 2 to obtain the remaining quantities of interest. With this notation, the evolution equations become

$$\partial_t \phi_1 = \frac{\alpha}{\sqrt{\gamma}}\pi_1 + \beta^j \psi_{1j},$$

$$\partial_t \psi_{1a} = \partial_a \left(\frac{\alpha}{\gamma^{\frac{1}{2}}}\pi_1 + \beta^j \psi_{1j} \right), \quad (5)$$

$$\partial_t \pi_1 = \partial_j(\alpha\sqrt{\gamma}\phi_1^j) - \frac{1}{2}\alpha\sqrt{\gamma}\frac{\partial V}{\partial|\Phi|^2}\phi_1.$$

Again, we can replace the subscript 1 with 2 to obtain the remaining quantities of interest. On the other hand, the geometry of the space-time is evolved using the BSSN formulation of the $3 + 1$ decomposition. According to this formulation, the variables to be evolved are

$$\Psi = \ln(\gamma_{ij}\gamma^{ij})/12, \quad \tilde{\gamma}_{ij} = e^{-4\Psi}\gamma_{ij}, \quad K = \gamma^{ij}K_{ij},$$

$$\tilde{A}_{ij} = e^{-4\Psi}(K_{ij} - \gamma_{ij}K/3),$$

and the contracted Christoffel symbols $\tilde{\Gamma}^i = \tilde{\gamma}^{jk}\Gamma^i_{jk}$, instead of the ADM variables γ_{ij} and K_{ij}. The equations for the BSSN variables are described in [10, 11]:

$$\partial_t \Psi = -\frac{1}{6}\alpha K, \quad (6)$$

Black Hole formation details

We were also able to demonstrate that the collapse of the dark energy field leads to black hole formation. For this, we used an additional component of the Einstein Toolkit [12, 13] which is also publicly available.

An apparent horizon satisfies the equation

$$\partial_t \tilde{\gamma}_{ij} = -2\alpha \tilde{A}_{ij}, \tag{7}$$

$$H \equiv \nabla_i n^i + K_{ij} n^i n^j - K = 0, \tag{12}$$

$$\partial_t K = -\gamma^{ij} D_i D_j \alpha$$
$$+\alpha \left[\tilde{A}_{ij} \tilde{A}^{ij} + \frac{1}{3} K^2 + \frac{1}{2}(-T^t{}_t + T) \right], \tag{8}$$

where n^i is the outward-pointing unit normal to the horizon and all the field variables are evaluated on the horizon surface.

$$\partial_t \tilde{A}_{ij} = e^{-4\Psi}[-D_i D_j \alpha + \alpha(R_{ij} - T_{ij})]^{TF}$$
$$+\alpha(K\tilde{A}_{ij} - 2\tilde{A}_{il}\tilde{A}^l_j), \tag{9}$$

As previously shown [13], Newton's method provides an excellent and efficient horizon-finding algorithm by numerically finding the roots of the above

$$\frac{\partial}{\partial t} \tilde{\Gamma}^i = -2\tilde{A}^{ij}\alpha_{,j}$$
$$+2\alpha \left(\tilde{\Gamma}^i_{jk} \tilde{A}^{kj} - \frac{2}{3} \tilde{\gamma}^{ij} K_{,j} - \tilde{\gamma}^{ij} T_{jt} + 6\tilde{A}^{ij}\phi_{,j} \right) \tag{10}$$
$$-\frac{\partial}{\partial x^j} \left(\beta^l \tilde{\gamma}^{ij}_{,l} - 2\tilde{\gamma}^{m(j}\beta^{i)}_{,m} + \frac{2}{3} \tilde{\gamma}^{ij}\beta^l_{,l} \right),$$

equation. By numerically executing the horizon-finding algorithm, we were able to demonstrate the formation of black holes resulting from the collapse of dark energy. Further, we computed the mass M_{BH} of the

where D_i is the covariant derivative on the spatial hypersurface, T is the trace of stress-energy tensor (3), and the label TF denotes the trace-free part of the quantity in brackets.

Space-time Geometry

The metric g_{ik} which determines the geometry of space-time in general is given by:

$$ds^2 = g_{ik}dx^i dx^k$$

Parallel transport in curved space time is determined by the Christoffel symbols:

$$\Gamma_{kl}^i = \frac{1}{2} g^{im} \left(\frac{\partial g_{mk}}{\partial x^l} + \frac{\partial g_{ml}}{\partial x^k} - \frac{\partial g_{kl}}{\partial x^m} \right)$$

The Curvature of Space-time is expressed in terms of the Curvature Tensor

$$R_{ik} = \frac{\partial \Gamma_{ik}^l}{\partial x^l} - \frac{\partial \Gamma_{il}^l}{\partial x^k} + \Gamma_{ik}^l \Gamma_{lm}^m - \Gamma_{il}^m \Gamma_{km}^l$$

Dynamics of Space-Time: The Einstein Equation

The Einstein Equation involves in addition to Curvature Tensor, the Curvature Scalar: $R = g^{ik}R_{ik}$

We can now express the Einstein Equation as:

$$R_{ik} - \frac{1}{2}g_{ik}R = \frac{8\pi k}{c^4}T_{ik}$$

Where T_{ik} is the energy momentum tensor and can be expressed in terms of the energy density, pressure and velocity as: $T_{ik} = (p + \varepsilon)u_i u_k - p g_{ik}$

Gravitational Waves

Consider fluctuations of the metric around a background value expressed as: $g_{ik} = g_{ik}^{(0)} + h_{ik}$

We can impose supplementary conditions to get rid of the arbitrariness of gauge (corresponds to freedom in choosing coordinate systems), thus we can impose conditions such as :

$$\frac{\partial \psi_i^k}{\partial x^k} = 0, \quad \psi_i^k = h_i^k - \frac{1}{2} \delta_i^k h$$

The curvature tensor then becomes: $R_{ik} = \frac{1}{2} \Box h_{ik}$

Inserting this into the vacuum Einstein Equation give us the gravitational wave equation: $\Box h_i^k = 0$

Or in the familiar form of a wave equation: $\left(\frac{\partial^2}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) h_i^k = 0$