Anomaly calculations

in curved spacetime

17th February 2023, Murdock Grewar

Gauge-covariant, non-chiral quantisation

Measure needs regulation

Gauge-covariant, non-chiral quantisation

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- Measure is ill-defined; divergent integral: $\int \mathcal{D}\overline{\psi}\mathcal{D}\psi \exp\left\{iS[\overline{\Psi}, \Psi]\right\}$
- A definite quantisation is fixed by: (1) choice of basis, (2) order of summation
- Regulator R_Λ determines the basis; Λ-dependence determines order:

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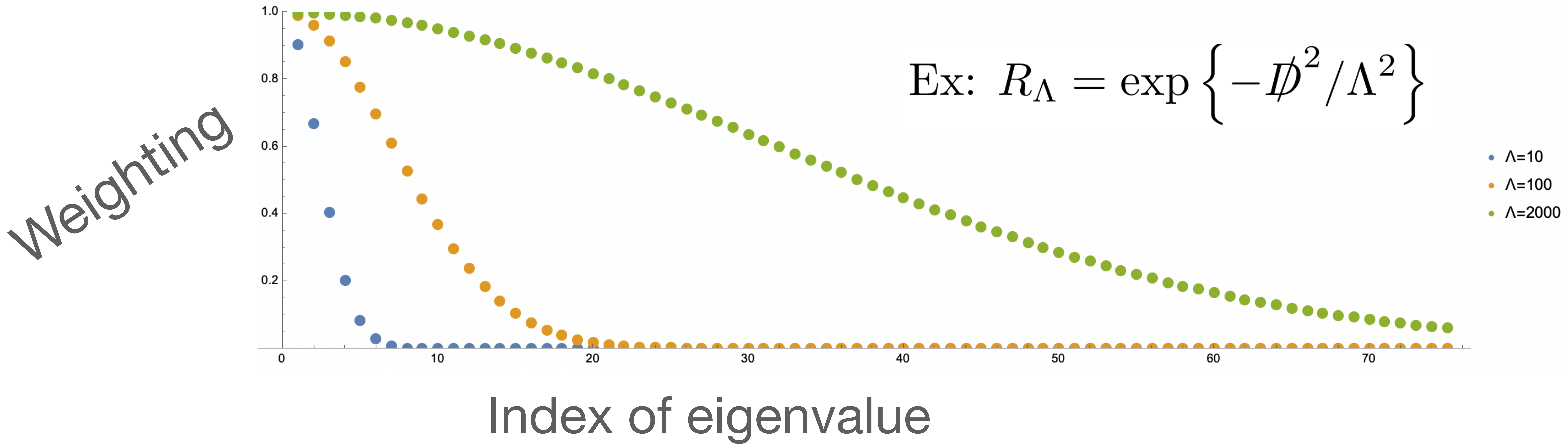

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- $\Gamma^\alpha{}_{\beta\mu}U_\alpha{}^\beta$: Ordinary GR
- $A_\mu^a T^a$: Gauge freedom

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$$\mathrm{Tr}_{\mathrm{reg.}}\left[\hat{X}\right] = \lim_{\Lambda\to 0} \mathrm{Tr}\left[\hat{X}\, f\big((\Lambda\not{D})^2\big)\right] \tag{4.15}$$

$$= \lim_{\Lambda\to 0} \int \mathrm{d}^4x\,\sqrt{g}(x)\mathrm{tr}\left[\left\langle x\middle|\hat{X} f\big((\Lambda\not{D})^2\big)\middle|x\right\rangle\right] \tag{4.16}$$

$$= \lim_{\Lambda\to 0} \int \mathrm{d}^4x\,\sqrt{g}(x)X(x)\mathrm{tr}\left[\langle x| f\big((\Lambda\not{D})^2\big)|x\rangle\right] \tag{4.17}$$

$$= \lim_{\Lambda\to 0} \int \mathrm{d}^4x\sqrt{g}(x)X(x) \int \frac{\mathrm{d}^4k}{(2\pi)^4}\,\mathrm{tr}\left[\langle x| f\big((\Lambda\not{D})^2\big)|k\rangle\,\langle k|x\rangle\right] \tag{4.18}$$

$$= \lim_{\Lambda\to 0} \int \mathrm{d}^4x\sqrt{g}(x)X(x) \int \frac{\mathrm{d}^4k}{(2\pi)^4}\,\mathrm{tr}\left[\langle k|x\rangle\,\langle x| f\big((\Lambda\not{D})^2\big)|k\rangle\right] \tag{4.19}$$

$$= \lim_{\Lambda\to 0} \int \mathrm{d}^4x\sqrt{g}(x)X(x) \int \frac{\mathrm{d}^4k}{(2\pi)^4}\,\mathrm{tr}\left[e^{-ikx} f\big((\Lambda\not{D})^2\big)\, e^{ikx}\right]\,. \tag{4.20}$$

$$= \lim_{\Lambda\to 0} \int \mathrm{d}^4x\sqrt{g}(x)X(x) \int \frac{\mathrm{d}^4k}{(2\pi)^4}\,\mathrm{tr}\left[f\big((\Lambda\not{D})^2\big)\,[1]\right]\,. \qquad \partial_\mu \mapsto \partial_\mu + ik_\mu\,.$$

Weyl Anomaly (fermion sector)

Long calculation - substitute expression for covariant derivative

Weyl Anomaly (fermion sector)

Long calculation - substitute expression for covariant derivative

Anomaly is computed w.r.t. regulated measure: $\mathrm{Tr}\,[T] \equiv \lim_{\Lambda \to 0} \mathrm{Tr}\left[w(x) f((\not{D}\Lambda)^2)\right]$

Weyl Anomaly (fermion sector)

Long calculation - substitute expression for covariant derivative

Anomaly is computed w.r.t. regulated measure: $\mathrm{Tr}\,[T] \equiv \lim_{\Lambda\to 0} \mathrm{Tr}\left[w(x) f((\not{D}\Lambda)^2)\right]$

$$
\begin{aligned}
&\lim_{\Lambda\to 0} \mathrm{Tr}\left[w(x)\,\mathbf{Id}\, f\big((\Lambda\not{D})^2\big)\right] \\
&= \lim_{\Lambda\to 0} \int \mathrm{d}^4x\, w(x) \int \frac{\mathrm{d}^4k}{(2\pi)^4}\left\{\mathrm{tr}\left[f\Big(\Lambda^2\Big((D_\mu + ik_\mu)^2 + F + G^\nu(D_\nu + ik_\nu)\Big)\Big)[1]\right]\right\} \\
&= \lim_{\Lambda\to 0} \int \mathrm{d}^4x\, w(x)\,\Lambda^{-4} \int \frac{\mathrm{d}^4(\Lambda k)}{(2\pi)^4}\left\{\mathrm{tr}\left[f\Big(\Lambda^2\Big((D_\mu + ik_\mu)^2 + F + G^\nu(D_\nu + ik_\nu)\Big)\Big)[1]\right]\right\} \\
&= \lim_{\Lambda\to 0} \int \mathrm{d}^4x\, w(x)\,\Lambda^{-4} \int \frac{\mathrm{d}^4k}{(2\pi)^4}\left\{\mathrm{tr}\left[f\Big((\Lambda D_\mu + ik_\mu)^2 + \Lambda^2 F + G^\nu(\Lambda^2 D_\nu + \Lambda ik_\nu)\Big)[1]\right]\right\}.
\end{aligned} \tag{4.23}
$$

Weyl Anomaly (fermion sector)

Long calculation - Taylor expansion

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Weyl Anomaly (fermion sector)

Long calculation - Taylor expansion

Anomaly is computed w.r.t. regulated measure: $\mathrm{Tr}\,[T] \equiv \lim_{\Lambda\to 0} \mathrm{Tr}\left[w(x) f((\not{D}\Lambda)^2)\right]$

Taylor expanding $f(\dots)$:

$$f\Big((\Lambda D_\mu + ik_\mu)^2 + \Lambda^2 F + G^\nu(\Lambda^2 D_\nu + \Lambda i k_\nu)\Big) \tag{4.24a}$$

$$= f\Big(-k_\mu k^\mu + \Lambda i k_\mu\,(2D^\mu + G^\mu) + \Lambda^2\,(G^\mu D_\mu + D_\mu D^\mu + F)\Big) \tag{4.24b}$$

$$\equiv \sum_{m=0}^{\infty} \frac{1}{m!} f^{(m)}\Big(-k_\mu k^\mu\Big)\Big(\Lambda i k_\mu\,(2D^\mu + G^\mu) + \Lambda^2\,(G^\mu D_\mu + D_\mu D^\mu + F)\Big)^m \tag{4.24c}$$

$$= f(-k_\mu k^\mu) \tag{4.24d}$$

$$+ \Lambda\Big(f'(-k_\mu k^\mu)(ik_\nu(2D^\nu + G^\nu))\Big) \tag{4.24e}$$

$$+ \Lambda^2\left(\frac{1}{2} f''(-k_\mu k^\mu)(ik_\nu(2D^\nu + G^\nu))^2 + f'(-k_\mu k^\mu)\left(D^2 + F + G^\mu D_\mu\right)\right) \tag{4.24f}$$

Weyl Anomaly (fermion sector)

Long calculation - Taylor expansion

Anomaly is computed w.r.t. regulated measure: $$\mathrm{Tr}\left[T\right] \equiv \lim_{\Lambda\to 0} \mathrm{Tr}\left[w(x) f((\not{D}\Lambda)^2)\right]$$

Taylor expanding $f(\dots)$:

$$\begin{aligned}
&f\Big((\Lambda D_\mu + ik_\mu)^2 + \Lambda^2 F + G^\nu(\Lambda^2 D_\nu + \Lambda i k_\nu)\Big) \\
&\qquad = f\Big(- k_\mu k^\mu + \Lambda i k_\mu \left(2D^\mu + G^\mu\right) + \Lambda^2 \left(G^\mu D_\mu + D_\mu D^\mu + F\right)\Big) \\
&\equiv \sum_{m=0}^{\infty} \frac{1}{m!} f^{(m)}\Big(- k_\mu k^\mu\Big)\Big(\Lambda i k_\mu \left(2D^\mu + G^\mu\right) + \Lambda^2 \left(G^\mu D_\mu + D_\mu D^\mu + F\right)\Big)^m \\
&= f(-k_\mu k^\mu) \\
&\quad + \Lambda\Big(f'(-k_\mu k^\mu)(ik_\nu(2D^\nu + G^\nu))\Big) \\
&\quad + \Lambda^2\left(\frac{1}{2} f''(-k_\mu k^\mu)(ik_\nu(2D^\nu + G^\nu))^2 + f'(-k_\mu k^\mu)\left(D^2 + F + G^\mu D_\mu\right)\right) \\
&\quad + \Lambda^3\bigg(\frac{1}{6} f'''(-k_\mu k^\mu)(ik_\nu(2D^\nu + G^\nu))^3 \\
&\qquad + \frac{1}{2} f''(-k_\mu k^\mu)\Big(\{ik_\nu(2D^\nu + G^\nu), D^2 + F + G^\mu D_\mu\}\Big)\bigg) \\
&\quad + \Lambda^4\bigg(\frac{1}{24} f''''(-k_\mu k^\mu)\Big((ik_\nu(2D^\nu + G^\nu))^4\Big) \\
&\qquad + \frac{1}{6} f'''(-k_\mu k^\mu)\Big(\\
&\qquad\quad (ik_\nu(2D^\nu + G^\nu))^2(D^2 + F) + (ik_\nu(2D^\nu + G^\nu))(D^2 + F)(ik_\nu(2D^\nu + G^\nu)) \\
&\qquad\quad + (D^2 + F)(ik_\nu(2D^\nu + G^\nu))^2\Big) \\
&\qquad + \frac{1}{2} f''(-k_\mu k^\mu)\left(G^\mu D_\mu + D^2 + F\right)^2\bigg) \\
&\quad + \mathcal{O}(\Lambda^5)\,.
\end{aligned}$$

Weyl Anomaly (fermion sector)

Long calculation - Taylor expansion

Anomaly is computed w.r.t. regulated measure: $\mathrm{Tr}\left[T\right] \equiv \lim_{\Lambda\to 0} \mathrm{Tr}\left[w(x) f((\not{D}\Lambda)^2)\right]$

Taylor expanding $f(\dots)$:

$$
\begin{aligned}
&f\Big((\Lambda D_\mu + ik_\mu)^2 + \Lambda^2 F + G^\nu(\Lambda^2 D_\nu + \Lambda i k_\nu)\Big) \\
&\qquad = f\Big(-k_\mu k^\mu + \Lambda i k_\mu\left(2D^\mu + G^\mu\right) + \Lambda^2\left(G^\mu D_\mu + D_\mu D^\mu + F\right)\Big) \\
&\equiv \sum_{m=0}^{\infty} \frac{1}{m!} f^{(m)}\Big(-k_\mu k^\mu\Big)\Big(\Lambda i k_\mu\left(2D^\mu + G^\mu\right) + \Lambda^2\left(G^\mu D_\mu + D_\mu D^\mu + F\right)\Big)^m \\
&= f(-k_\mu k^\mu) \\
&\quad + \Lambda\Big(f'(-k_\mu k^\mu)(ik_\nu(2D^\nu + G^\nu))\Big) \\
&\quad + \Lambda^2\left(\frac{1}{2} f''(-k_\mu k^\mu)(ik_\nu(2D^\nu + G^\nu))^2 + f'(-k_\mu k^\mu)\left(D^2 + F + G^\mu D_\mu\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
&+ \Lambda^3\left(\frac{1}{6} f'''(-k_\mu k^\mu)(ik_\nu(2D^\nu + G^\nu))^3\right. \\
&\qquad \left. + \frac{1}{2} f''(-k_\mu k^\mu)\Big(\{ik_\nu(2D^\nu + G^\nu), D^2 + F + G^\mu D_\mu\}\Big)\right) \\
&+ \Lambda^4\left(\frac{1}{24} f''''(-k_\mu k^\mu)\Big((ik_\nu(2D^\nu + G^\nu))^4\Big)\right. \\
&\qquad + \frac{1}{6} f'''(-k_\mu k^\mu)\Big(\\
&\qquad\quad (ik_\nu(2D^\nu + G^\nu))^2(D^2 + F) + (ik_\nu(2D^\nu + G^\nu))(D^2 + F)(ik_\nu(2D^\nu + G^\nu)) \\
&\qquad\quad + (D^2 + F)(ik_\nu(2D^\nu + G^\nu))^2\Big) \\
&\qquad \left. + \frac{1}{2} f''(-k_\mu k^\mu)\left(G^\mu D_\mu + D^2 + F\right)^2\right) \\
&+ \mathcal{O}(\Lambda^5)\,.
\end{aligned}
$$

After a number of partial integrations, and exploiting Lorentz symmetries in various integrals...

Weyl Anomaly (fermion sector)

Long calculation - penultimate formula

Weyl Anomaly (fermion sector)

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Anomaly is computed w.r.t. regulated measure: $\mathrm{Tr}\,[T] \equiv \lim_{\Lambda\to 0} \mathrm{Tr}\left[w(x) f((\not{D}\Lambda)^2)\right]$

Weyl Anomaly (fermion sector)

Long calculation - penultimate formula

Anomaly is computed w.r.t. regulated measure: $\mathrm{Tr}\,[T] \equiv \lim_{\Lambda\to 0} \mathrm{Tr}\left[w(x) f((\not{D}\Lambda)^2)\right]$

$$\lim_{\Lambda\to 0} \mathrm{Tr}\left[w(x)\,\mathbf{Id}\,f\big((\Lambda\not{D})^2\big)\right] = \lim_{\Lambda\to 0} \int \mathrm{d}^4x\, \frac{w(x)}{(4\pi)^2} \Bigg\{ \Lambda^{-4}\left(\int \mathrm{d}r\, r\, f(r)\right) \mathrm{tr}\,[\mathbf{Id}]\,[1] - \Lambda^{-2}\left(\int \mathrm{d}r\, f(r)\right)\mathrm{tr}\,[C]\,[1] + \mathrm{tr}\left[\frac{1}{12}[H_\alpha, H_\beta][H^\alpha, H^\beta] + \frac{1}{6}[H_\alpha,[H^\alpha, C]] + \frac{1}{2}C^2\right][1]\Bigg\}.$$

$$\begin{aligned}
\not{D}^2 &= D^2 + F + G^\nu D_\nu \\
F &= \frac{1}{4}[\gamma^\mu, \gamma^\nu][D_\mu, D_\nu] \\
G^\nu &= -\gamma^\mu e_m{}^\nu \left(iA^a_\mu[g_a, \gamma^m] + A^{mn}_\mu \gamma_n\right) \\
H^\nu &= D^\nu + \tfrac{1}{2}G^\nu \\
C &= F - \tfrac{1}{4}G^2 - \tfrac{1}{2}[H_\alpha, G^\alpha]\,.
\end{aligned}$$

Weyl Anomaly (fermion sector)

Long calculation - ultimate formula

Weyl Anomaly (fermion sector)

Long calculation - ultimate formula

Anomaly is computed w.r.t. regulated measure: $\mathrm{Tr}\left[T\right] \equiv \lim_{\Lambda \to 0} \mathrm{Tr}\left[w(x) f((\not{D}\Lambda)^2)\right]$

Weyl Anomaly (fermion sector)

Long calculation - ultimate formula

Anomaly is computed w.r.t. regulated measure: $\mathrm{Tr}\left[T\right] \equiv \lim_{\Lambda\to 0} \mathrm{Tr}\left[w(x) f((\not{D}\Lambda)^2)\right]$

Assuming a Levi-Civita connection...

Weyl Anomaly (fermion sector)

Long calculation - ultimate formula

Anomaly is computed w.r.t. regulated measure: $\mathrm{Tr}\,[T] \equiv \lim_{\Lambda\to 0} \mathrm{Tr}\left[w(x) f((\not{D}\Lambda)^2)\right]$

Assuming a Levi-Civita connection...

$$\lim_{\Lambda\to 0} \mathrm{Tr}\left[w(x)\,\mathbf{Id}\, f\left((\Lambda\not{D})^2\right)\right] = \lim_{\Lambda\to 0} \int \mathrm{d}^4x\, \frac{\sqrt{g}(x) w(x)}{(4\pi)^2} \Bigg\{ \Lambda^{-4} \left(\int \mathrm{d}r\, r\, f(r)\right) \mathrm{tr}\,[\mathbf{Id}]\,[1] - \Lambda^{-2} \left(\int \mathrm{d}r\, f(r)\right) \mathrm{tr}\,[F]\,[1] + \mathrm{tr}\left[\frac{1}{12}[D_\alpha, D_\beta][D^\alpha, D^\beta] + \frac{1}{6}[D_\mu, [D^\mu, F]] + \frac{1}{2}F^2\right][1] \Bigg\}.$$

Weyl Anomaly (fermion sector)

Long calculation - the wrong answer

Weyl Anomaly (fermion sector)

Long calculation - the wrong answer

Anomaly is computed w.r.t. regulated measure: $\mathrm{Tr}\,[T] \equiv \lim_{\Lambda\to 0} \mathrm{Tr}\left[w(x) f((\not{D}\Lambda)^2)\right]$

Weyl Anomaly (fermion sector)

Long calculation - the wrong answer

Anomaly is computed w.r.t. regulated measure: $\mathrm{Tr}\left[T\right] \equiv \lim_{\Lambda\to 0} \mathrm{Tr}\left[w(x) f((\not{D}\Lambda)^2)\right]$

My answer for QED appears to be wrong...

Weyl Anomaly (fermion sector)

Long calculation - the wrong answer

Anomaly is computed w.r.t. <u>regulated measure:</u> $\mathrm{Tr}\,[T] \equiv \lim_{\Lambda\to 0} \mathrm{Tr}\left[w(x) f((\not{D}\Lambda)^2)\right]$

My answer for QED appears to be wrong...

$$\lim_{\Lambda\to 0} \mathrm{Tr}\left[-2w(x)\,\mathbf{Id}\,f\big((\Lambda\not{D})^2\big)\right] = 2\lim_{\Lambda\to 0}\int \mathrm{d}^4x\,\frac{\sqrt{g}(x)w(x)}{(4\pi)^2}\Bigg\{$$

$$4\Lambda^{-4}\left(\int \mathrm{d}r\, r\, f(r)\right) + \Lambda^{-2}\left(\int \mathrm{d}r\, f(r)\right) R$$

$$+\, e^2\tfrac{2}{3}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{8}R^2 - \tfrac{1}{24}R^{abcd}R_{abcd} - \tfrac{1}{6}D_\mu D^\mu R\Bigg\}.$$

Restoring broken Weyl symmetry

Changing the measure

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Can change the model to be Weyl-invariant; loses coordinate-invariance.

E.g.

Restoring broken Weyl symmetry

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E.g.

$$\mathcal{D}\overline{\tilde{\psi}}\mathcal{D}\tilde{\psi} \longrightarrow \mathcal{D}\left(\overline{|g|^{n/4}\tilde{\psi}}\right)\mathcal{D}\left(|g|^{n/4}\tilde{\psi}\right)$$

Restoring broken Weyl symmetry

Changing the measure

Can change the model to be Weyl-invariant; loses coordinate-invariance.

E.g.

$$\mathcal{D}\overline{\tilde{\psi}}\mathcal{D}\tilde{\psi} \longrightarrow \mathcal{D}\left(\overline{|g|^{n/4}\tilde{\psi}}\right)\mathcal{D}\left(|g|^{n/4}\tilde{\psi}\right)$$

- Retains partial coordinate-invariance (isovolumetric/`unimodular' spacetime transformations).
- Next steps:
    1. Complete derivation of gravitational anomalies. (Maybe require Zeta-function reg.)
    2. Formulate Weyl-invariant theory. (Requires new gauge field: `dilaton'. May be `alive').
    3. Apply techniques of anomaly calculation to *coordinate anomalies*.
    4. Make phenomenological predictions based on new/broken symmetries.