Kayne West Reveals Why You Should Stop Using WIMPs as Dark Matter Candidates and Use Primordial Black Holes Instead! (OMFG!)

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Where Do Black Holes Come From?

Classically a black hole *is a region of spacetime where gravity is so strong that nothing — no particles or electromagnetic radiation — can escape*

Black holes are solutions to the Einstein Field Equations

$$S_{EH} = \int d^4x \sqrt{-g} \left[\frac{R}{2\kappa} + \mathcal{L}_M \right]$$
$$\Rightarrow R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa T_{\mu\nu}$$

- $\kappa = 8\pi G = 8\pi / M_p^2$
- R is the only independent scalar which we can construct (up to second derivatives) of the metric

Why Modify General Relativity?

General Relativity is the simplest theory coupling spacetime curvature to matter

Good reason to look at modified theories

- Interaction with quantum matter, should be a limit from any quantum theory of gravity

We can consider modified theories by adding terms to the Hilbert action, as long as they

- Are diffeomorphism invariant, scalar, etc
- Limit correctly to GR, and Newtonian gravity

Project Aim

To investigate the quantum correction of the Schwarzschild solution due to matter in R^2 gravity. And to see the stability of these black hole solutions.

$$S_{QEH} = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} + \beta R^2 + \gamma R_{\mu\nu}R^{\mu\nu} + \mathcal{L}_M\right]$$

where the R^2 terms are due to the quantisation of matter!

We are interested in spherically symmetric solutions and it's deformations due to quantum excitations of matter fields.

Quantum Corrected Schwarzschild Solution I

We choose the Kruskal–Szekeres coordinates (z^+, z^-) in which the metric takes the conformally flat form

$$ds^2 = e^{\sigma(z^+,z^-)} dz^+ dz^- - r^2(z^+, z^-)(d\theta^2 + \sin^2\theta d\phi^2)$$

The non-zero Ricci tensors in this metric are

$$\begin{aligned}
R_{+-} &= \partial_+\partial_-\sigma + \partial_+\partial_- \ln r^2 + \frac{1}{2}\partial_+ \ln r^2 \partial_- \ln r^2 \\
R_{\theta\theta} &= -1 - 2e^{-\sigma}\partial_+\partial_- r^2 \\
R_{\phi\phi} &= \sin^2\theta R_{\theta\theta} \\
R_{\pm\pm} &= \partial_\pm^2 \ln r^2 + \partial_\pm \ln r^2 \partial_\pm \ln r^2 - \partial_\pm\sigma\partial_\pm \ln r^2
\end{aligned}$$

Quantum Corrected Schwarzschild Solution II

By varying with action respect to the metric $g_{\mu\nu}$, a differential equation is obtained

$$r\frac{dg(r)}{dr} + g(r) - \left[1 + \frac{16\pi}{r^2}(2\beta + \gamma)\right] = 0$$

And consequently, one has the following black hole solution

$$g(r) = 1 - \frac{2GM}{r} - \frac{dG}{r^2}$$

where $d = 16\pi(2\beta + \gamma)$.

Primordial Black Holes

Primordial Black Holes are hypothetical black holes that were formed soon after the Big Bang due to highly density regions undergoing gravitational collapse.

They are a good candidate for Dark Matter as they are collision-less, stable, non-relativistic, and were formed in the early universe.

However a problem that arises for black holes with quantised matter is that they emit Hawking radiation and eventually disappear!

Black Hole Thermodynamics I

To find the stability of a black hole, we must first look at it's thermodynamical properties. The Hawking temperature is given as

$$T = \frac{\kappa}{2\pi} = \frac{1}{2\pi GM} \frac{\sqrt{1+q}}{(1+\sqrt{1+q})^2}$$

where $q = d/GM^2$, and κ is the surface gravity.

The heat capacity is given by

$$C = \frac{dQ}{dT}$$

The form is long but the point is that it is always negative, in contrast to a discontinuity for a RN black hole.

Black Hole Thermodynamics II

Finally the lifetime of the black hole is given by the Stephan-Boltzmann radiation law

$$\frac{dU}{dt} = A\sigma T^4$$
$$\Rightarrow \tau = \frac{80\pi G^2}{\hbar c^4} \frac{(1+\sqrt{1+q})^6}{(1+q)^2} M^3$$

where $\sigma = \pi^2 k_b^4 / 60\hbar^3 c^4$ is the Stephan-Boltzmann constant.

If one sets $q = 0$ (Schwarzschild black hole), the smallest stable black hole has a mass of $10^{-19} M_\odot$, any lower and the black hole evaporates due to Hawking radiation.

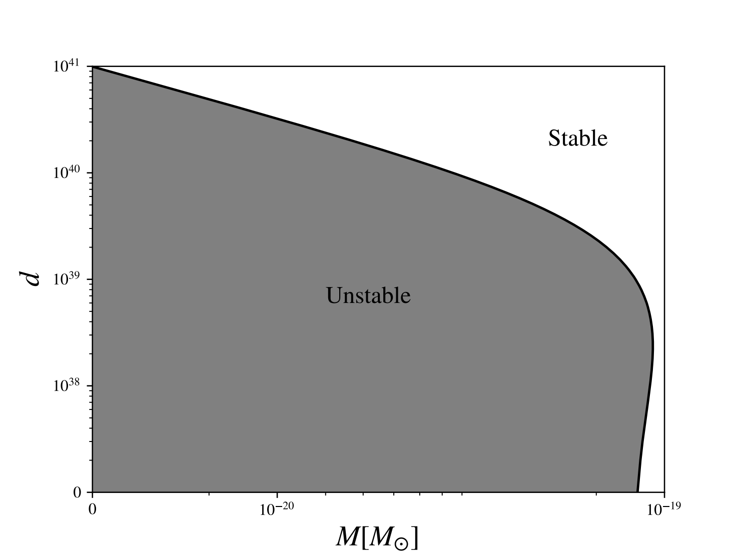
Stability of our Black Holes I

To find a stable (lifetime longer than the age of the universe) black hole, we can use the values $d \approx 10^{39}$ and $M \approx 10^{-19} M_{\odot}$

However, in our case we have another "dimension" to work with. The value d can theoretically be larger, and this in turn would decrease the lower bound for stable black holes

Thus we can use this black hole solution as our Primordial Black Holes

Stability of our Black Holes II



Summary

Found a modified Schwarzschild metric from quantum excitations of matter

Discovered stable lifetimes for these black hole solutions by using it's thermodynamics properties

Conjecture that the black hole solution we have is a candidates for Dark Matter

Will investigate more consequences of these black holes, e.g. microlensing

Two Dimensional Ricci Tensor

The non-zero Ricci tensors in this metric are

$$
\begin{aligned}
R_{+-} &= \partial_+\partial_-\sigma + \partial_+\partial_- \ln r^2 + \frac{1}{2}\partial_+ \ln r^2 \partial_- \ln r^2 \\
R_{\theta\theta} &= -1 - 2e^{-\sigma}\partial_+\partial_- r^2 \\
R_{\phi\phi} &= \sin^2\theta R_{\theta\theta} \\
R_{\pm\pm} &= \partial_\pm^2 \ln r^2 + \partial_\pm \ln r^2 \partial_\pm \ln r^2 - \partial_\pm\sigma\partial_\pm \ln r^2
\end{aligned}
$$

And the two dimensional metric

$$
\begin{aligned}
R &= \tilde{R} - \frac{2}{r^2}(\nabla r)^2 + \frac{2}{r^2}\Box r^2 + \frac{2}{r^2} \\
R^2 &= \frac{4}{r^2}\tilde{R} - \frac{8}{r^4}(\nabla r)^2 + \frac{8}{r^4}\Box r^2 + \frac{4}{r^4} \\
R_{\mu\nu}R^{\mu\nu} &= \frac{2}{r^4}\Box r^2 + \frac{2}{r^4}
\end{aligned}
$$

Modified Schwarzschild Solution

The metric can be written in the Schwarzschild gauge

$$ds^2 = g(r)dt^2 - \frac{G}{g(r)}dr^2$$

Note that using this metric we have $(\nabla r)^2 = -g/G$, and $\Box r = -g'/G$. Consequently a differential equation is obtained and solved to give the modified solution

$$r\frac{dg(r)}{dr} + g(r) - \left[U(r) + \frac{16\pi}{r^2}(2\beta + \gamma)\right] = 0$$

Black Hole Thermodynamics

For a general metric that is stationary, and spherically symmetric

$$ds^2 = h(r)dt^2 - \frac{1}{f(r)}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{1}$$

The location of the event horizon

$$f(r) = 0 \tag{2}$$

The surface gravity

$$\kappa = \frac{1}{2}\sqrt{h'(r_0)f'(r_0)} \tag{3}$$