A Parisian style guide to adding some chic to your neutrino simulations

(revamp your outdated numerical wardrobe in one step)

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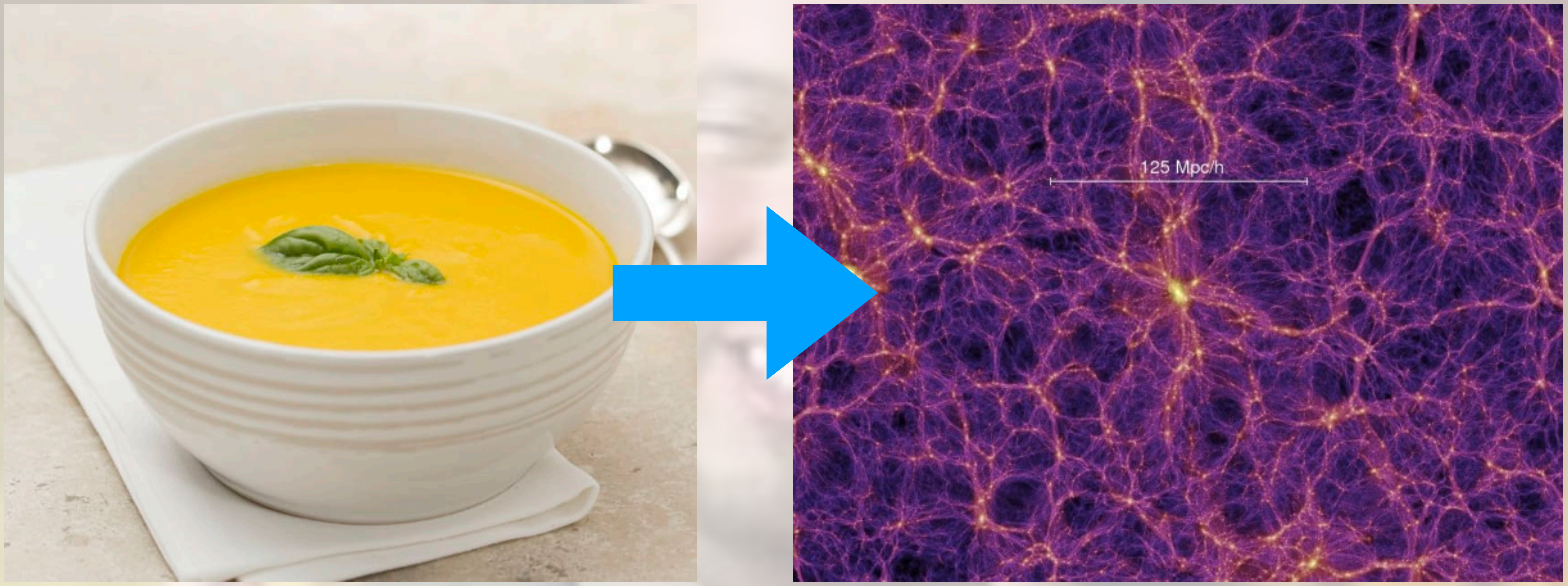
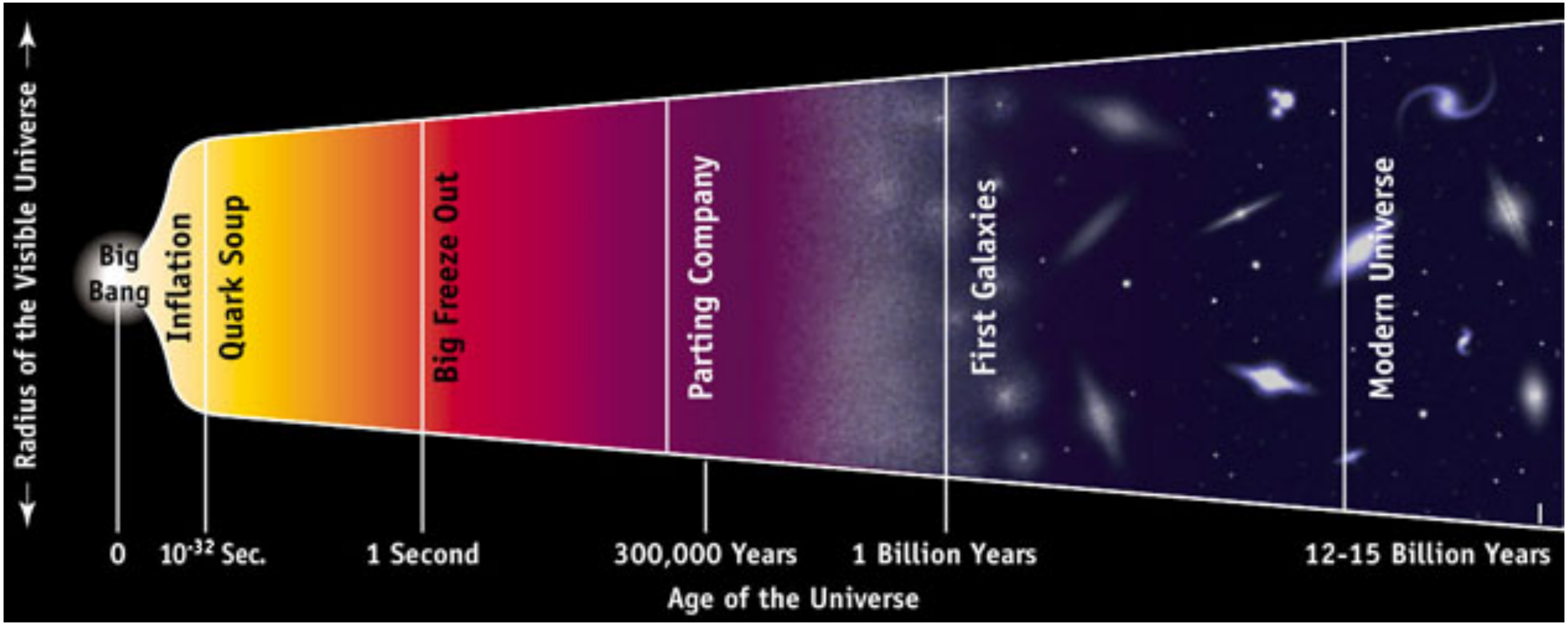
arXiv: 2011.12503

Main question at hand



Big Bang



Your guess is as good as mine

Cool high energy stuff

Basically just gravity (or is it?)

The Standard Model of Cosmology

- Cold dark matter
- Cosmological constant
- Massive standard model neutrinos
- (Baryons)

Focus: clustering behaviour

- Initial conditions: small inhomogeneities seeded by the quantum fluctuations.
- Described by density contrast: $\delta(\mathbf{x}, \tau) = \frac{\rho(\mathbf{x}, \tau) - \bar{\rho}(\tau)}{\bar{\rho}(\tau)}$.
- This will get affected by high energy physics during early universe, and as modes re-enter the horizon.
- After transition to matter domination, the magnitude of δ grows as over-dense regions accrete more material via gravitational attraction.

So what’s the issue?

- The evolution of δ can be solved for perturbatively at early times, when the universe is still relatively uniform, i.e. δ is small.

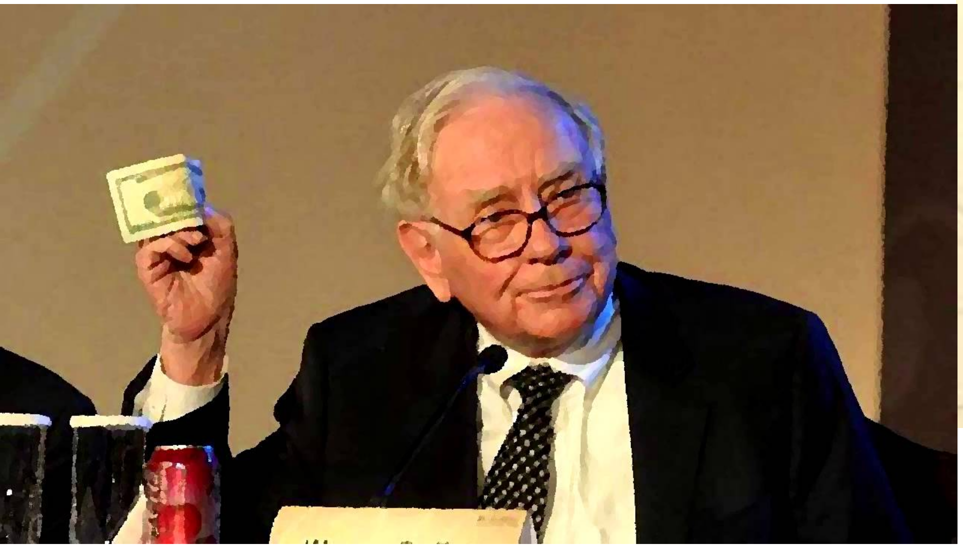
- At later times, δ will grow large enough that PT breaks down (our existence on a very dense rock floating in near empty space can testify to the fact).

- Understanding the nonlinear clustering of matter requires us to solve for the evolution of δ past linear (and even nonlinear) PT results.

Keynesian economics

- If you have a problem, throw money at it

N-body code
insert here



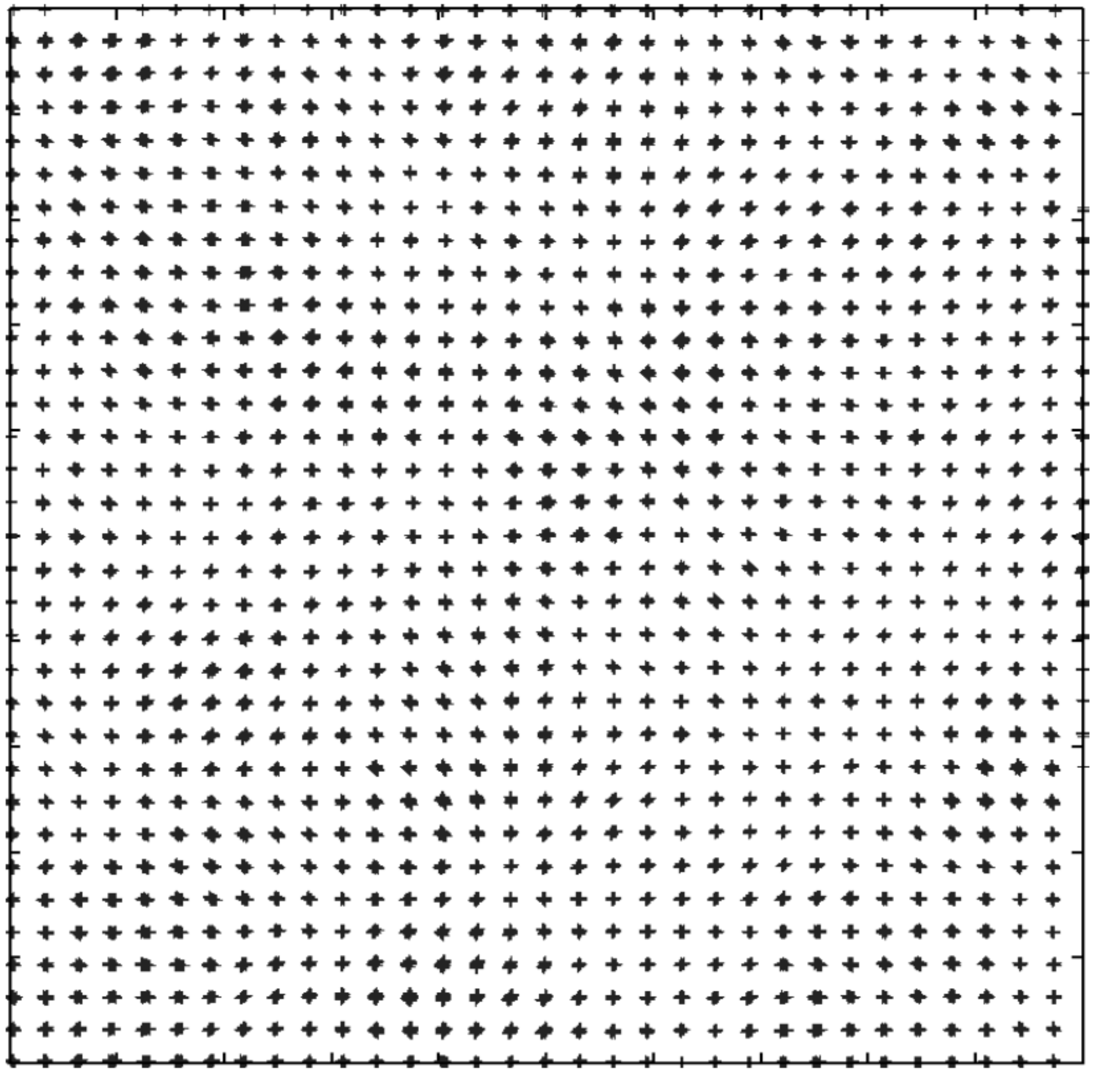
N-body simulations real quick summary

- Big box (Mpc to Gpc sizes depending on your needs)
- Tracer particles sampling the phase space
- Initialise at high redshift (z~100) when δ is still linear
- Evolve the gravitational forces between particles
- Output = nonlinear matter distribution at z=0

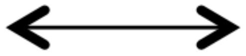
The cost - ΛCDM

- The cosmological constant is ‘free’, expansion of the box is easily incorporated.
- Cold dark matter is the only clustering component.
- Cold is good -> only have to sample the 3 spatial dimensions.
- Slow velocity -> reasonable time steps.

ΛCDM



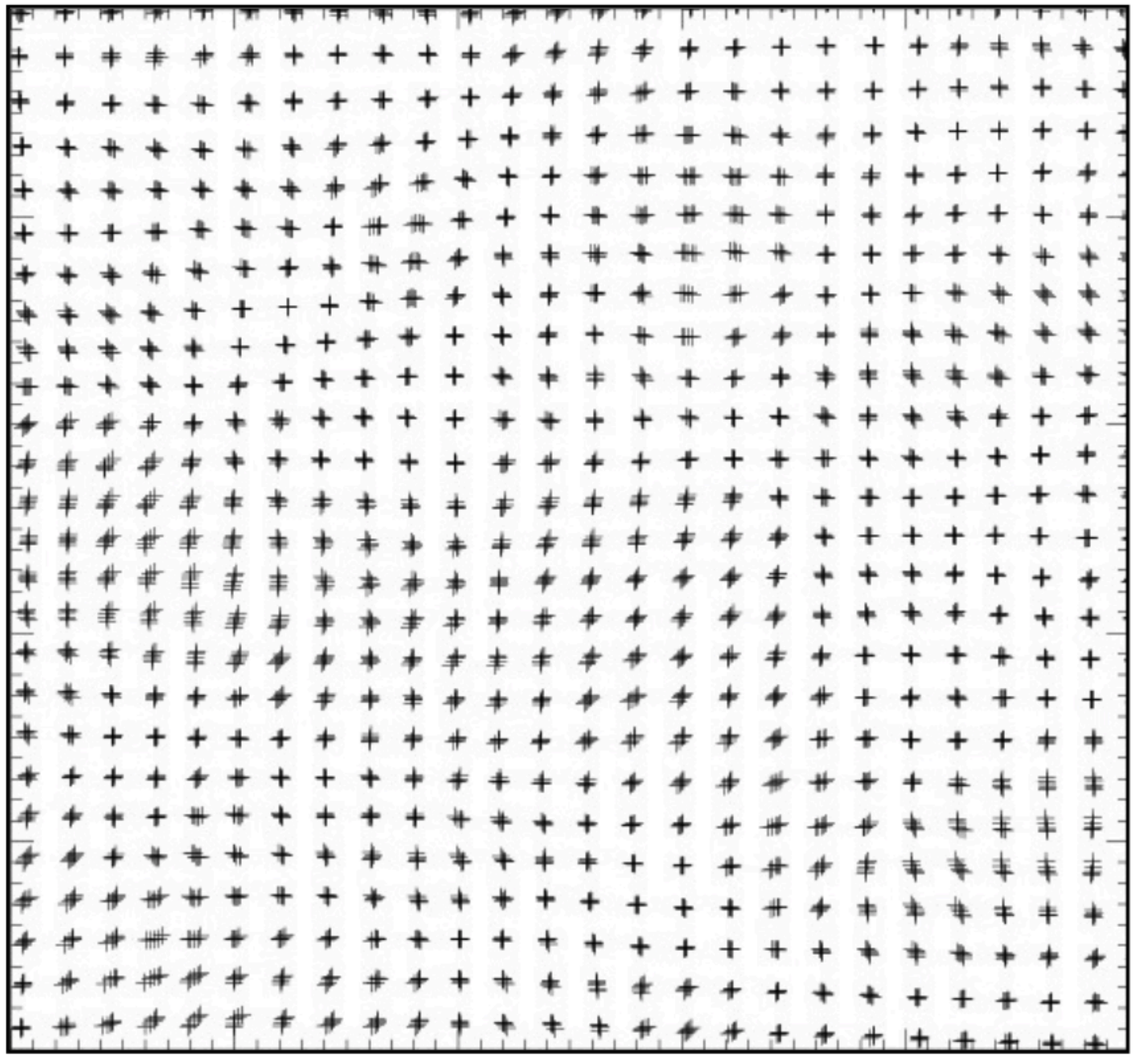
ZA procedure



$$\hat{\delta}(k) = \sqrt{P(k)}R_{\vec{k}}\, e^{i\varphi_{\vec{k}}}$$

$$R_{\vec{k}} e^{i\varphi_{\vec{k}}} = R_1 + iR_2$$

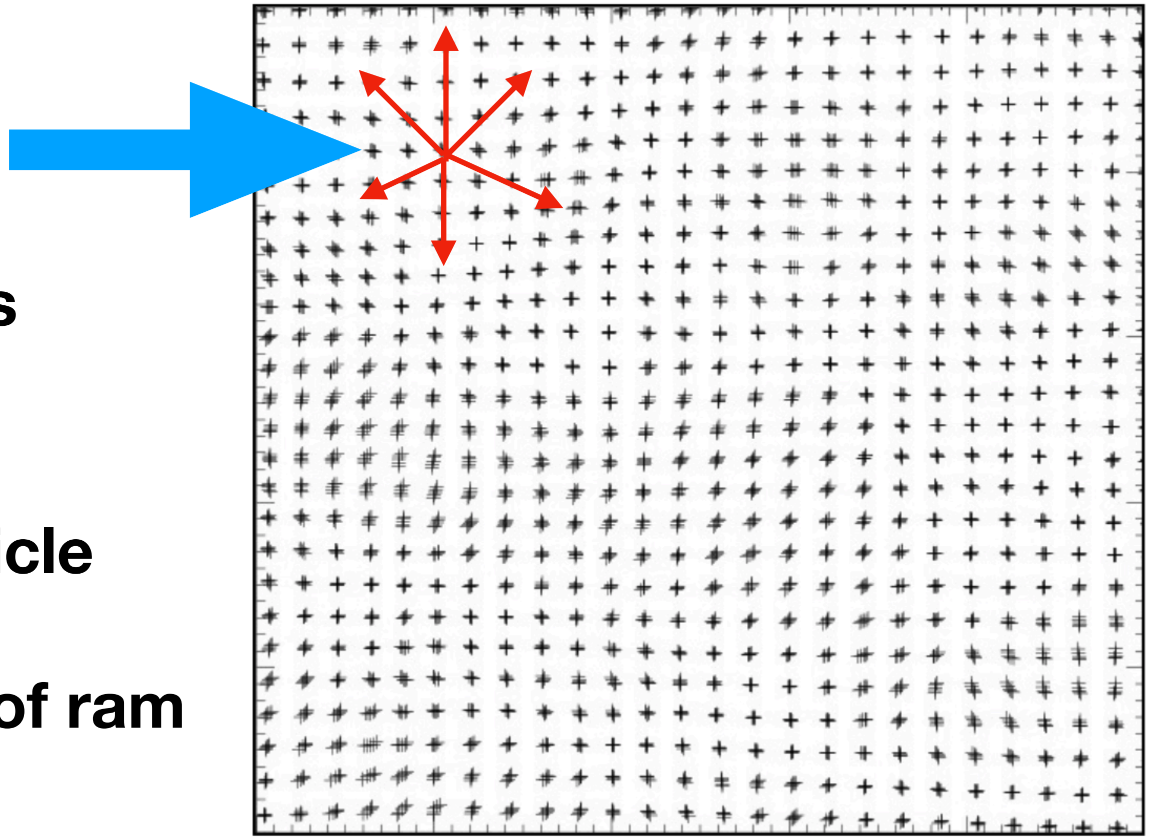
$$R_1, R_2 = \text{Gauss}(0,1)$$



Plot from Alexander Knebe, Universidad Autonoma de Madrid

The trouble with neutrinos

- Not cold, large thermal velocity at high z.
- Main issue: have to sample the Fermi-Dirac distribution on top of the spatial sampling.
- High velocity also restricts the size of time step at high redshifts.



**At every point,
let's say 16 momentum
bins, 6 directions,
~100 times more particles
required than CDM case**

**~140 bytes of storage per particle
(astro-ph/0505010)
10 billion CDM particles ~ 1.4 TB of ram
100 times that for neutrinos**

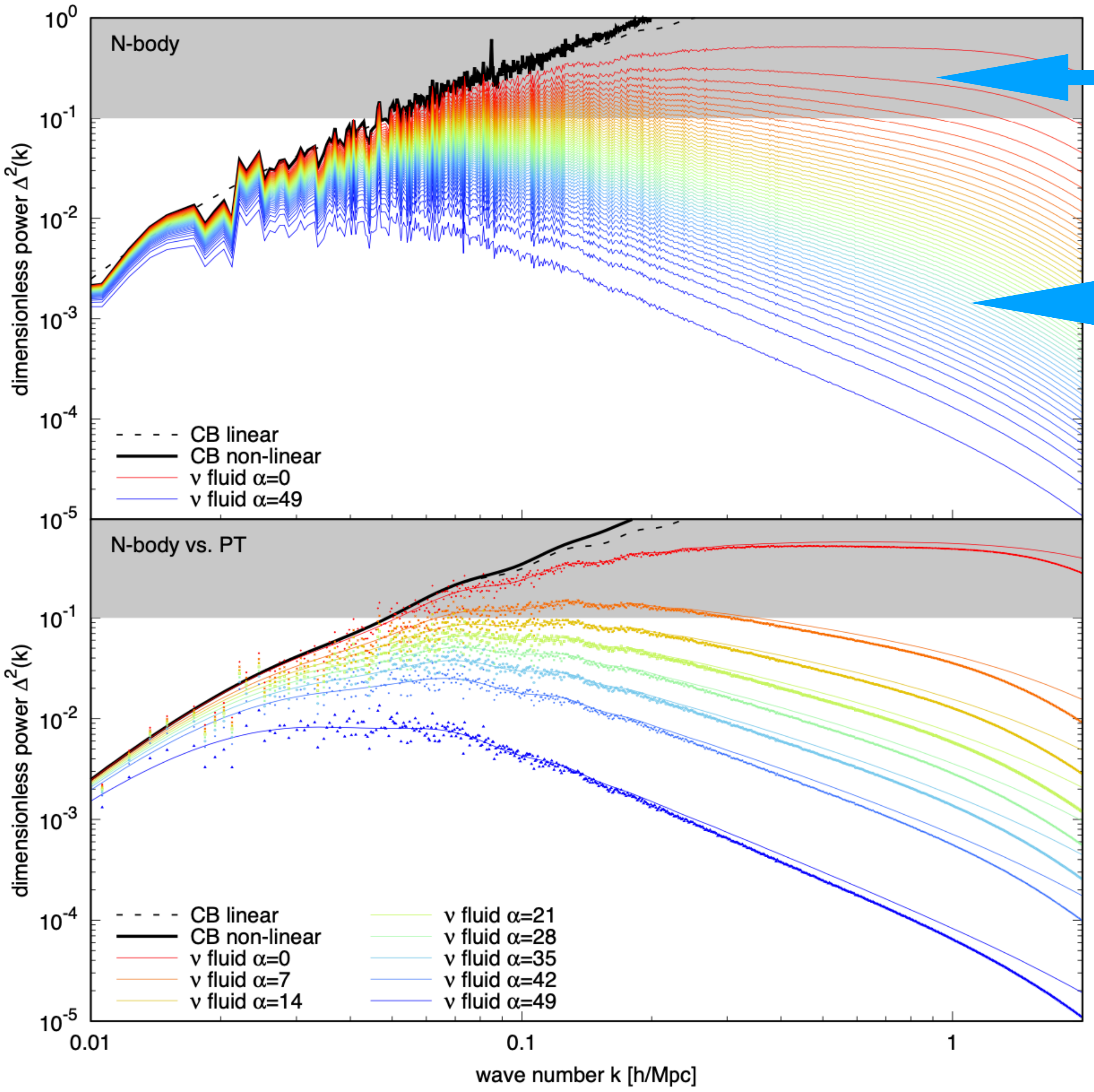
Alternatives

- Non-clustering neutrinos (Heitmann et al. 2015)
- Linear grid neutrinos (Brandbyge & Hannestad 2009)
- Fluid based neutrinos (Dakin et al. 2019)
- Gauge transform post-processing (Partmann et al. 2020)
- Linear response neutrinos (Haimoud & Bird 2012)

Linear response neutrinos

- The CDM component is represented as N-body particles.
- Solve the linearised neutrino Boltzmann equation in k-space, sourcing the nonlinear CDM gravitional potential.
- The neutrino contribution is added to the total gravitational potential when evolving the CDM motions.
- The Poisson equation for CDM particle dynamics is solved in k-space -> very easy to add in the neutrino part.

Validity of the linear approximation



A fraction of the (slower) population clusters non-linearly

Most neutrinos are linear throughout the evolution

Σm_ν **= 0.93 eV,** ω_m **= 0.1335**

ω_ν **= 0.01,** h **= 0.71**

Hybrid simulations

- Bird et al. (1803.09854), Brandbyge & Hannestad (0908.1969).
- At low redshift, when neutrinos become non-relativistic and cluster non-linearly, they are converted from perturbative representations to actual N-body particles.
- Question is:
  When to convert?
  Which neutrinos to convert?

Challenges

- Only a fraction of the neutrinos really needs to be tracked by N-body particles.

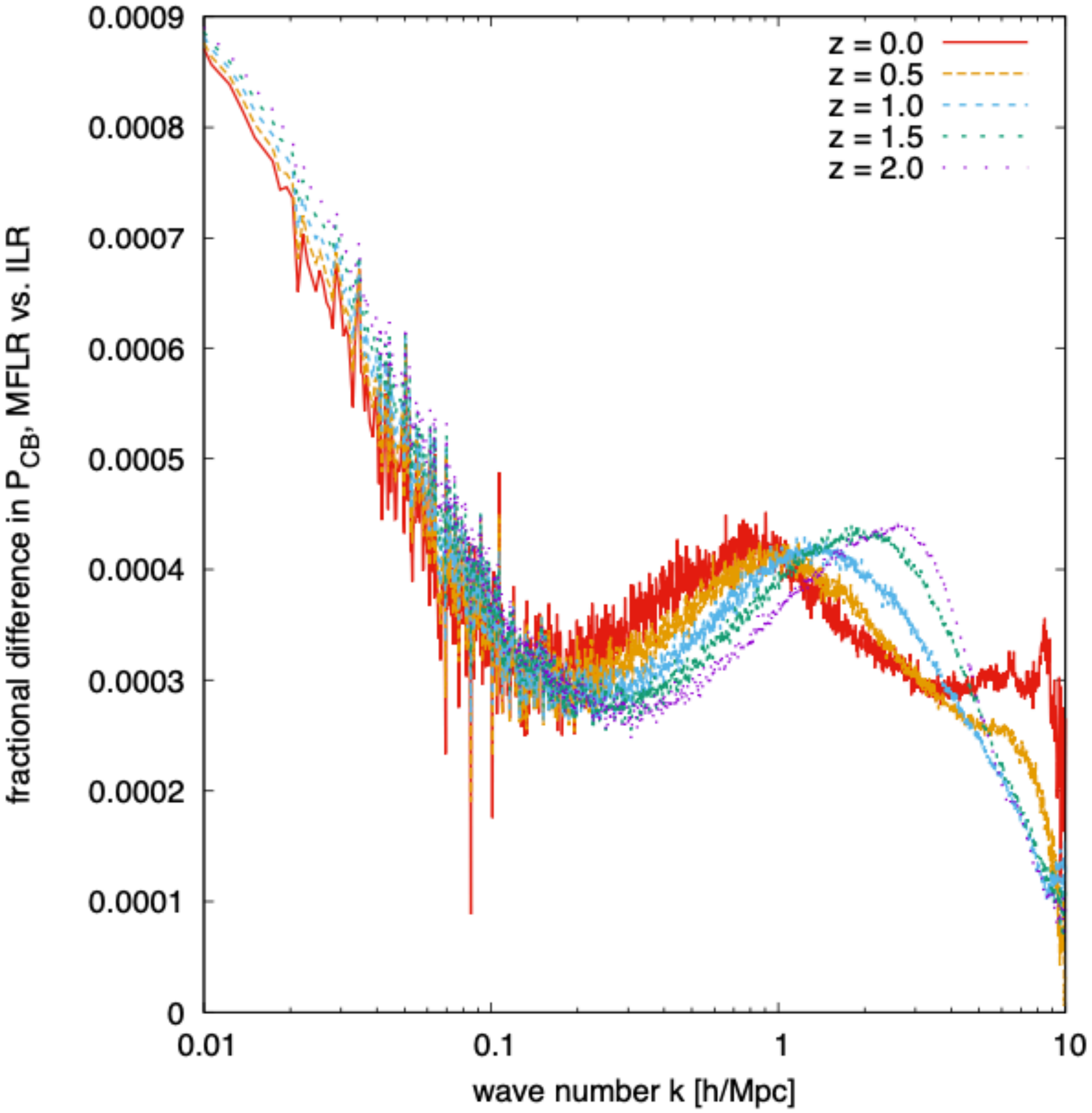
- The entire neutrino distribution is tracked together in 0908.1969, can’t simply ‘take out’ the lower momentum bins and make them into particles.

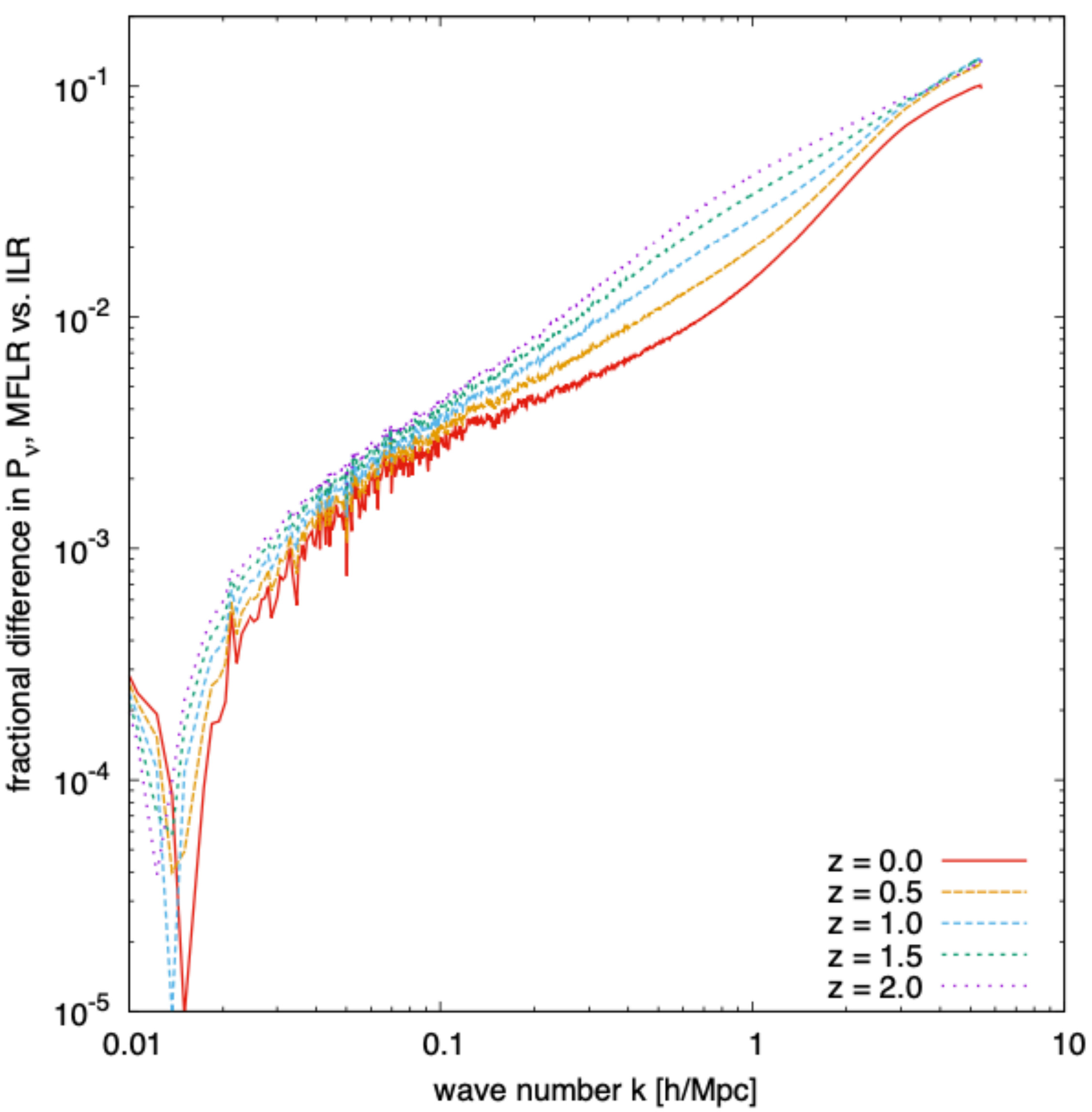
- The neutrino distribution is divided into fast and slow batches, each tracked individually (two Lagrangian labels) in 1803.09854. Convert only the slow batch.

The Multi-Fluid method

- First presented by Dupuy & Bernardeau (1311.5487, 1411.0428, 1503.05707).
- The entire Fermi-Dirac distribution is cut up into momentum bins, and each bin is tracked as an independent neutrino fluid with that particular initial Lagrangian velocity.
- Each stream is evolved indepedently and interacts with each other only via the gravitational potential.
- In the limit of large number of streams (>10), the method converges with the standard Boltzmann approach.

Results: comparison





Σm_ν = 0.93 eV, ω_m = 0.1335, ω_ν = 0.01, h = 0.71, 50 streams

The Multi-Fluid method

- Each stream is independent, can be converted to particles without disrupting other streams' evolution.

- The conversion timing can be staggered, each stream can be converted whenever they cross into non-linear regime.

- Each stream has it's own well defined thermal velocity + flow velocity from gravitational infalling, for initialising the N-body neutrino particles.

- The future work will be on actually coding this up as a hybrid simulation.

Thank you for your attention