

Axion strings and dark matter



UNSW
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QCD axion and dark matter

$$\mathcal{L}_{\text{CPV}} = \bar{\theta} \frac{\alpha_s}{8\pi} G_{a,\mu\nu} \tilde{G}_a^{\mu\nu}$$

$$\bar{\theta} = \theta - \arg \det M$$

$$d_n \simeq 1.52(71) \times 10^{-16} \bar{\theta} \, e \, \text{cm}$$

$$\bar{\theta} < 10^{-10} \quad [1902.03254]$$

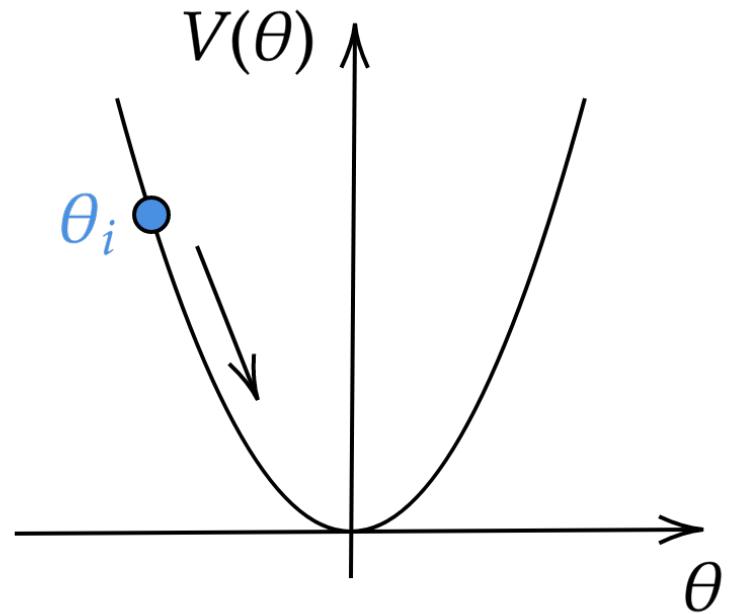
QCD axion and dark matter

$$\mathcal{L}_{\text{CPV}} = \bar{\theta} \frac{\alpha_s}{8\pi} G_{a,\mu\nu} \tilde{G}_a^{\mu\nu}$$

$$\bar{\theta}(x) \rightarrow a(x)/f_a$$

- Assume an abelian global symmetry, spontaneously broken at **high scale** f_a
- Axion appears as Goldstone boson, but mass term is generated at QCD era $m_a \propto f_a^{-1}$
- Oscillation around QCD potential behaves like NR condensate

$$\begin{aligned}\bar{\theta} &= \theta - \arg \det M \\ d_n &\simeq 1.52(71) \times 10^{-16} \bar{\theta} \text{ e cm} \\ \bar{\theta} &< 10^{-10} \quad [1902.03254]\end{aligned}$$



Clears strong CP problem and provides DM candidate!

Post-inflationary scenario

$$f_a \lesssim \frac{H_I}{2\pi} \quad \longrightarrow \quad \text{Formation of topological defects (strings, domain walls)}$$

- Evolution of string network in the ‘scaling’ regime
- *Domain wall problem*
- *Fate of miniclusters in late time Cosmology*
- *Uncertainty in the topological susceptibility*

Post-inflationary scenario

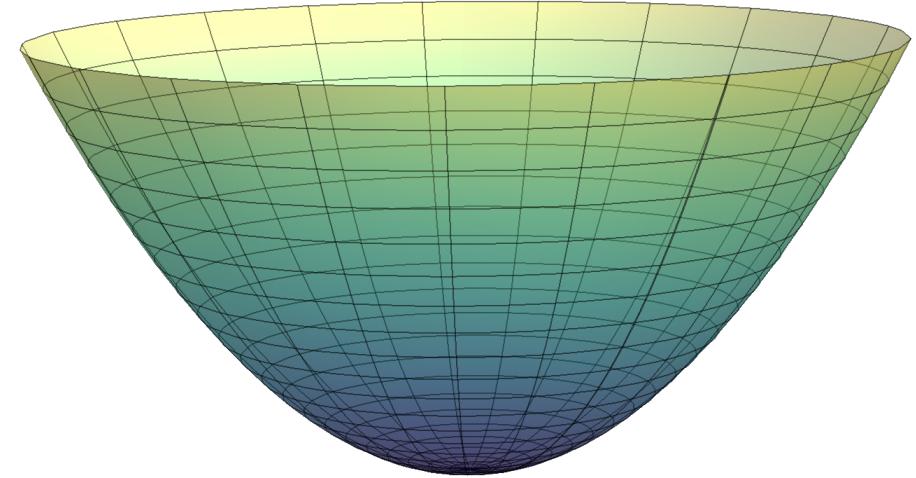
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- Evolution of string network in the ‘scaling’ regime This talk
- *Domain wall problem*
- *Fate of miniclusters in late time Cosmology*
- *Uncertainty in the topological susceptibility*

Global $U(1)$ field theory

$$\mathcal{L} = (\partial_\mu \Phi)^* (\partial^\mu \Phi) - V(\Phi)$$

$$V(\Phi) = \frac{\lambda}{4} (\Phi^* \Phi - f_a^2)^2 + \frac{\lambda}{6} T^2 \Phi^2$$



$$T \gg f_a$$

Global $U(1)$ field theory

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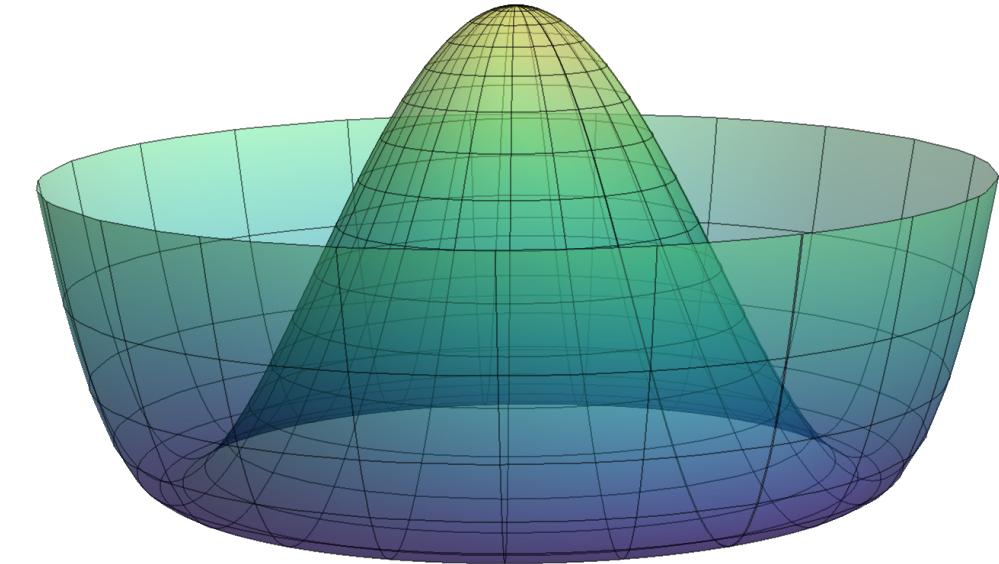
$$V(\Phi) = \frac{\lambda}{4} (\Phi^* \Phi - f_a^2)^2 + \frac{\lambda}{6} T^2 \Phi^2$$

$$\Phi(x) = \frac{1}{\sqrt{2}} [f_a + s(x)] e^{ia(x)/f_a}$$

Saxion

Heavy radial mode
with mass

$$m_s = \sqrt{2\lambda} f_a$$



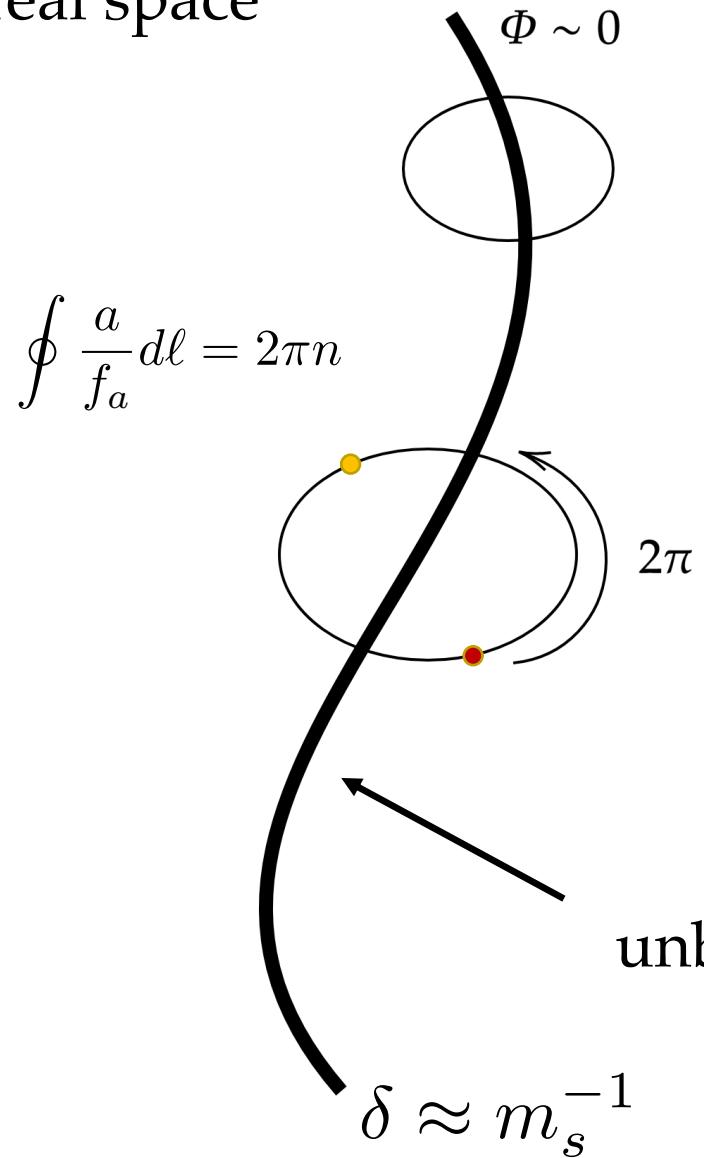
$$T \lesssim f_a$$

Axion

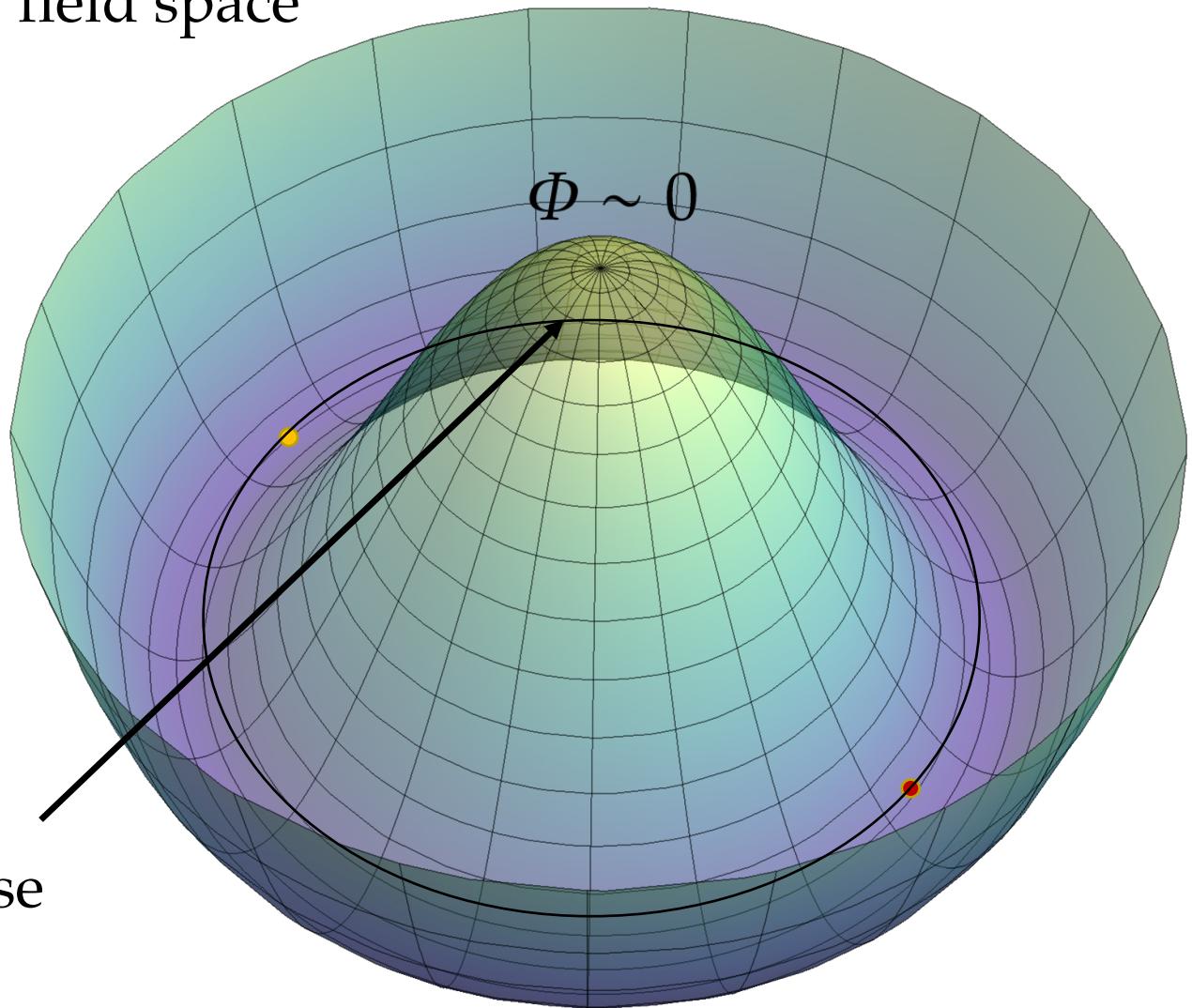
Massless goldstone
boson with shift
symmetry

$$a(x) \rightarrow a(x) + 2\pi f_a$$

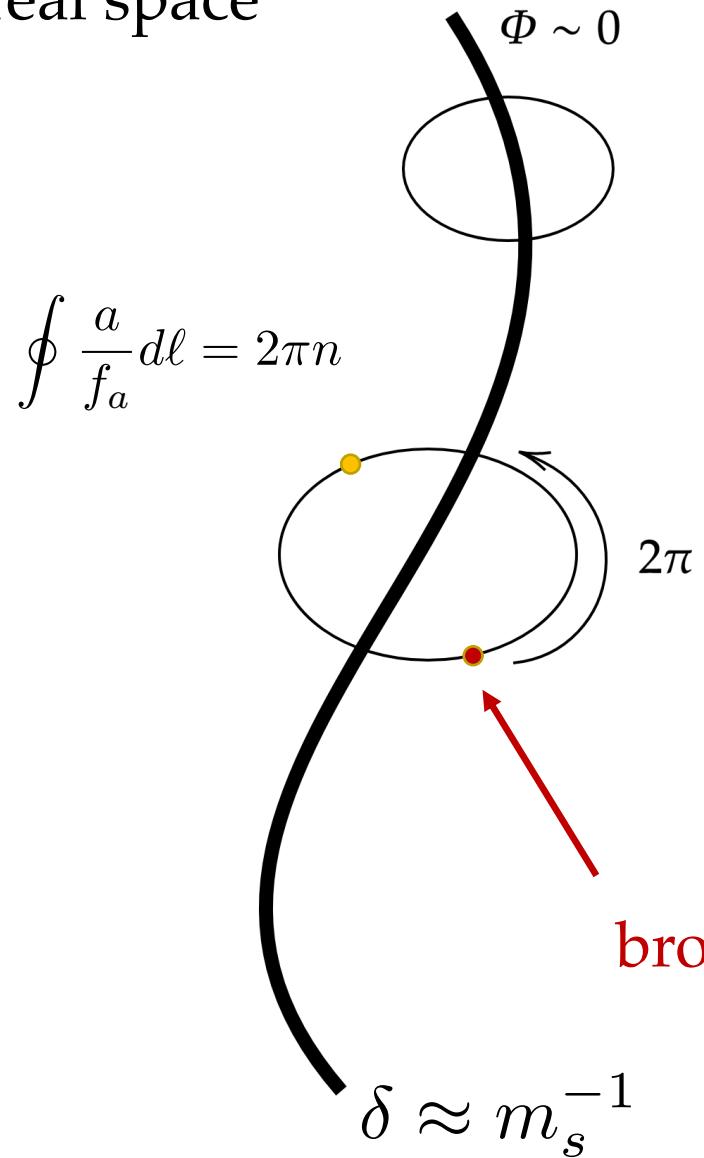
real space



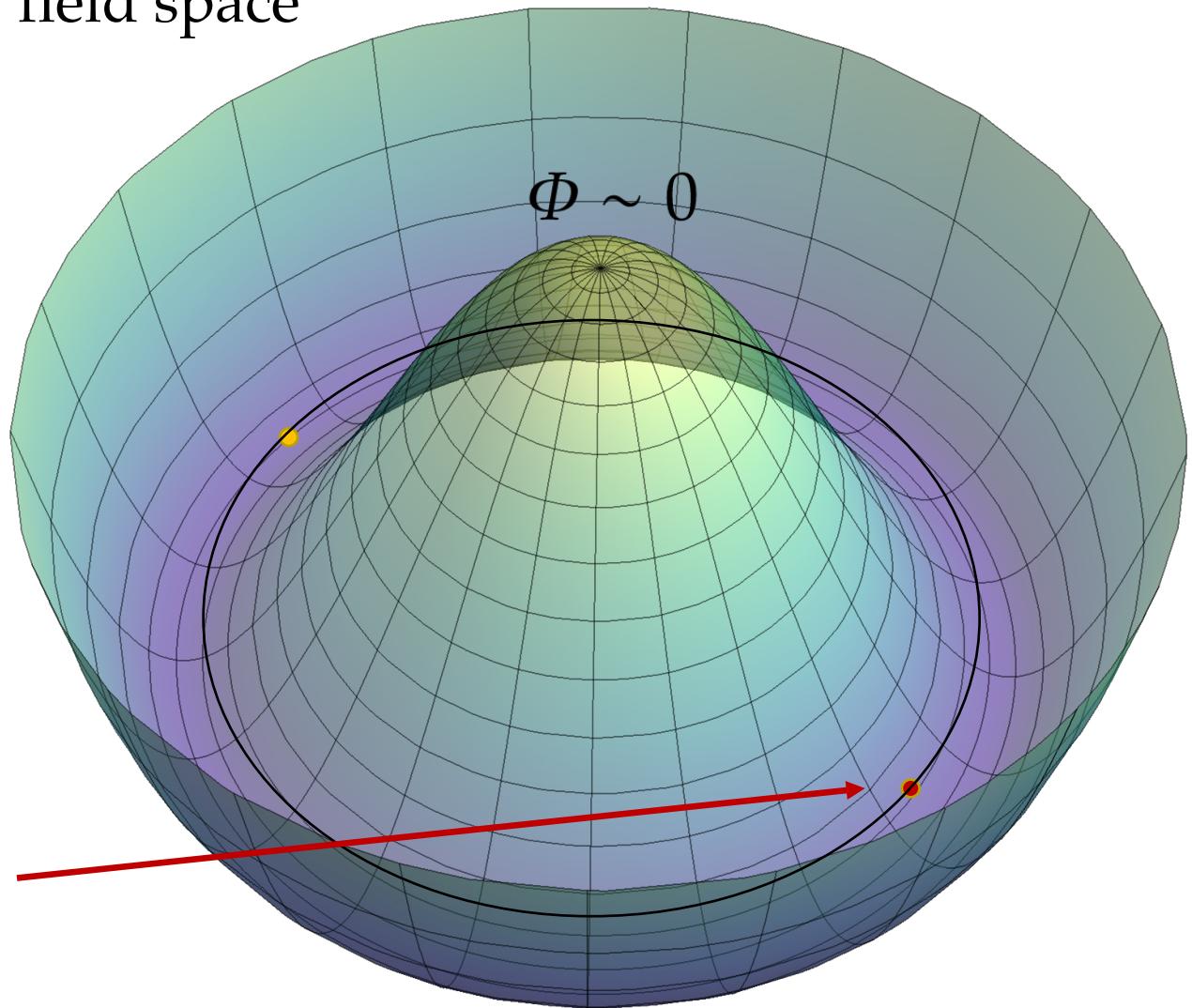
field space

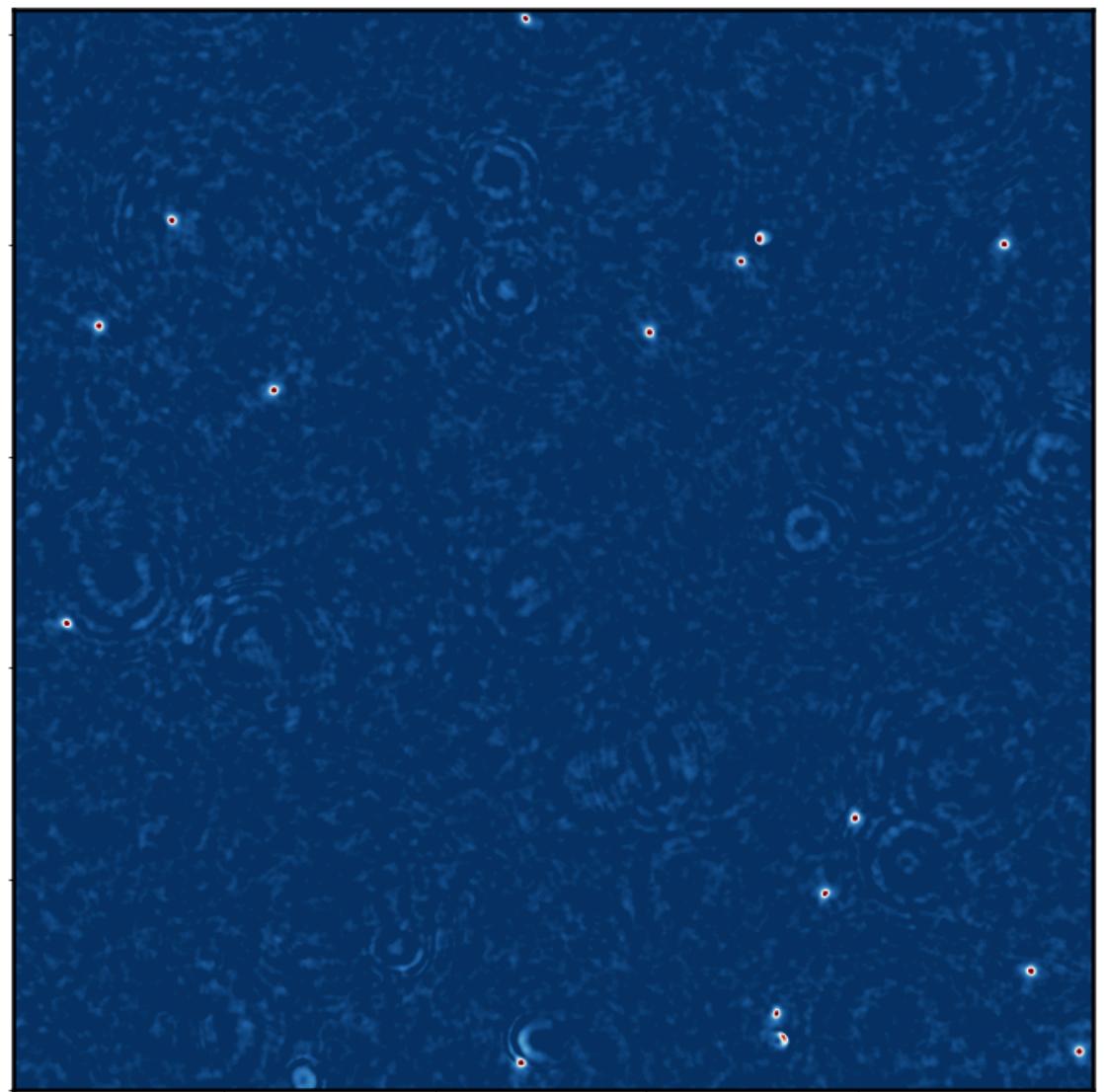
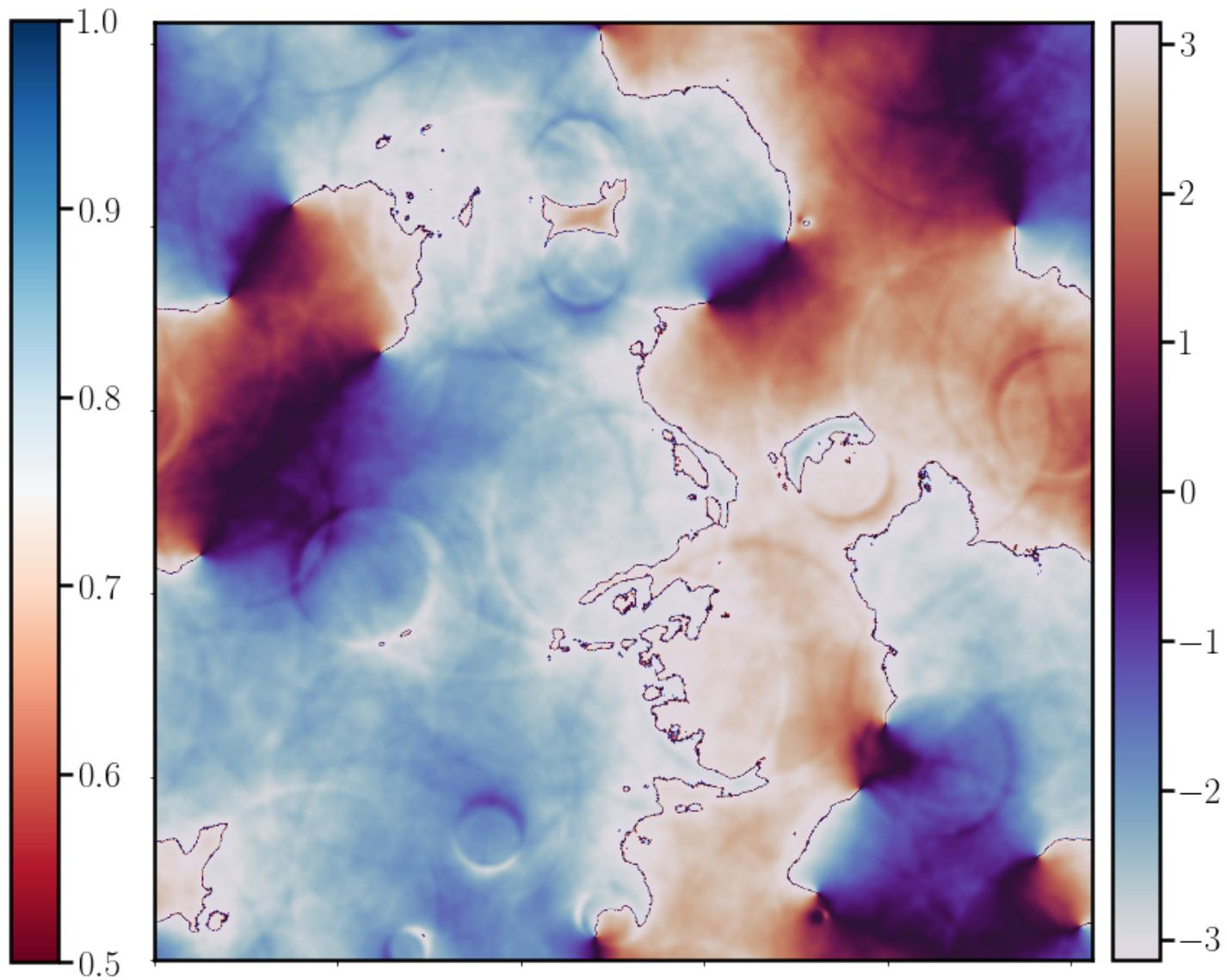


real space



field space



$|\Phi|/f_a$  a/f_a 

$$\ddot{\Phi} + 3H\dot{\Phi} - \nabla^2\Phi + \frac{m_s^2}{2f_a^2}(|\Phi|^2 - f_a^2) = 0$$

2D: vortex solution
3D: line-like solution

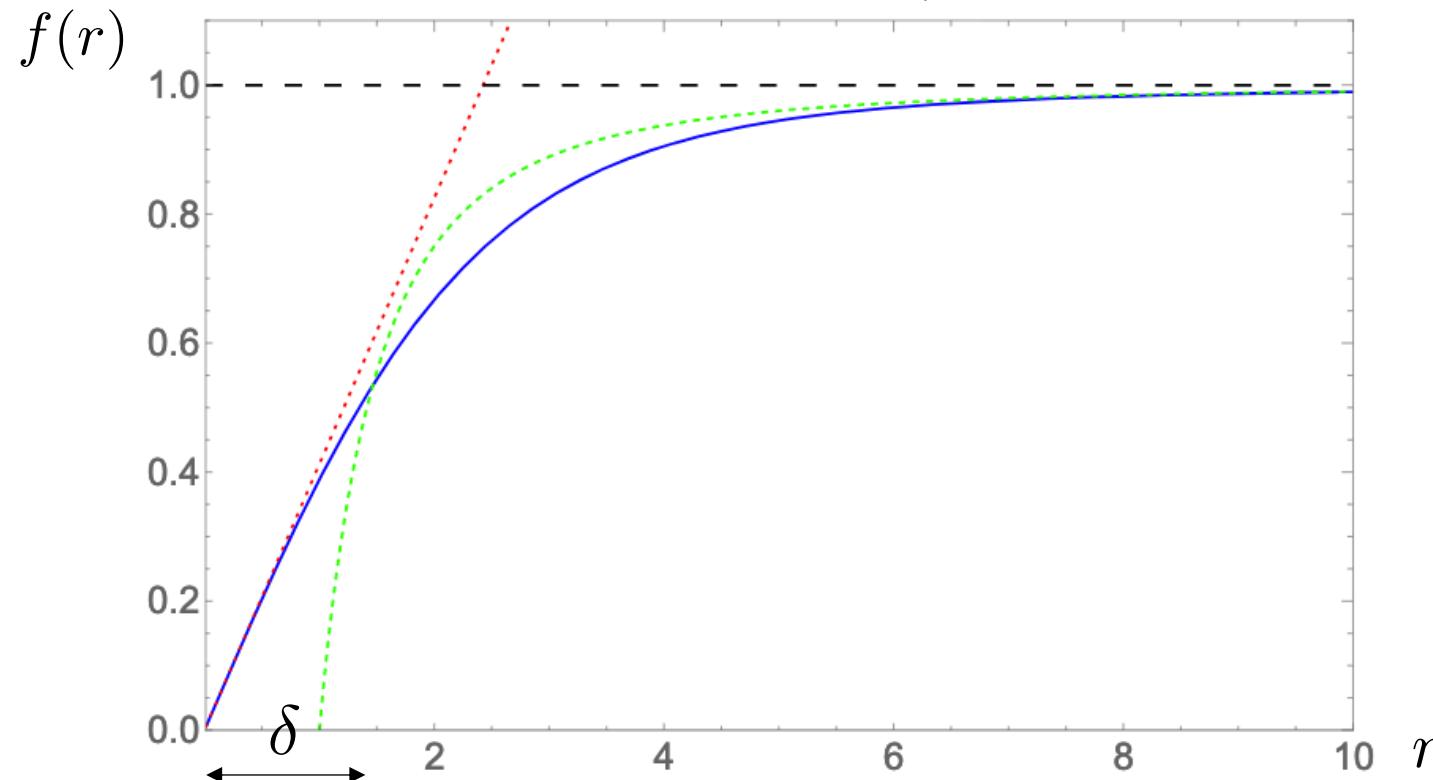
Ansatz: $\varphi_{\text{string}} = \underbrace{f(r)}_{\text{core profile}} e^{in\theta}$ winding number

Boundary conditions: $f(0) = 0$
 $f(r) \xrightarrow{r \rightarrow \infty} 1$

Drew, Shellard [1910.01718]

String core

$$\mathcal{O}(1)f_a^{-1} \simeq \mathcal{O}(1)m_s^{-1}$$



Away from string core

$$\varphi_{\text{string}} \simeq f_a e^{in\theta}$$

$$(\partial_\mu \partial^\mu + m_s^2)s(x) = 0$$

$$\partial_\mu \partial^\mu a(x) = 0$$

Two scale model

$$\varphi_{\text{string}} = f(r) e^{in\theta} \quad \text{winding around the core}$$

$$\rho_{\text{string}}(r) = \underbrace{(\partial_r f)^2 + \frac{\lambda}{4}(f^2 - 1)^2}_{\text{local core}} + \left(\frac{f}{r}\right)^2$$

Two scale model

$$\varphi_{\text{string}} = f(r) e^{in\theta}$$

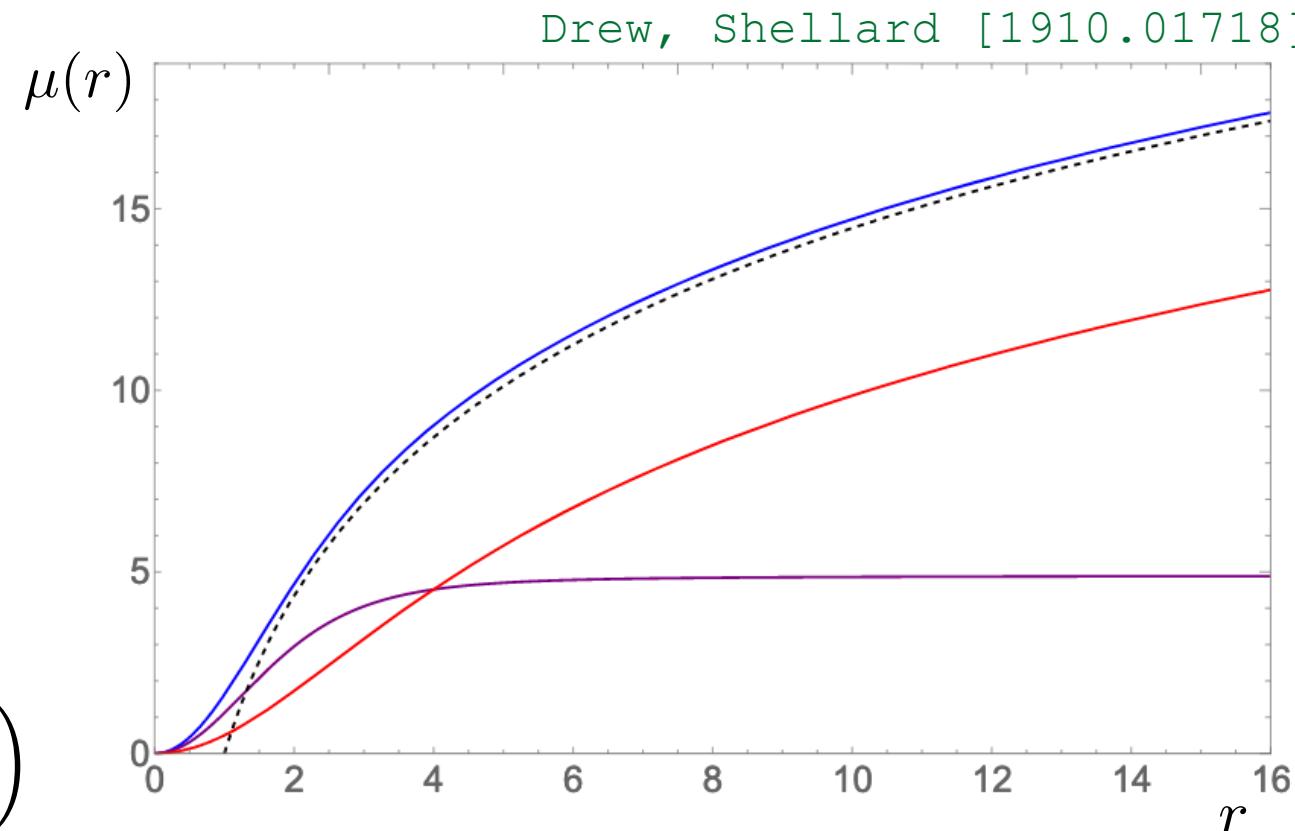
$$\rho_{\text{string}}(r) = \underbrace{(\partial_r f)^2 + \frac{\lambda}{4}(f^2 - 1)^2}_{\text{local core}} + \boxed{\left(\frac{f}{r}\right)^2}$$

String tension:

$$\begin{aligned} \mu(r) &= \mu_0 + \int_{\delta}^R 2\pi r \left(\frac{f(r)}{r}\right)^2 dr \\ &= \mu_0 + 2\pi f_a^2 \log\left(\frac{R}{\delta}\right) \text{IR} \quad \text{UV} \end{aligned}$$

For axion strings on cosmo scales:

$$\mu \approx 2\pi f_a^2 \log\left(\frac{m_s}{H}\right)$$



Axion string networks

- Difficult to model a network of strings
- Scaling solution is ‘natural’ for local strings (one scale)
- Axion string properties should show log corrections
- Network density:

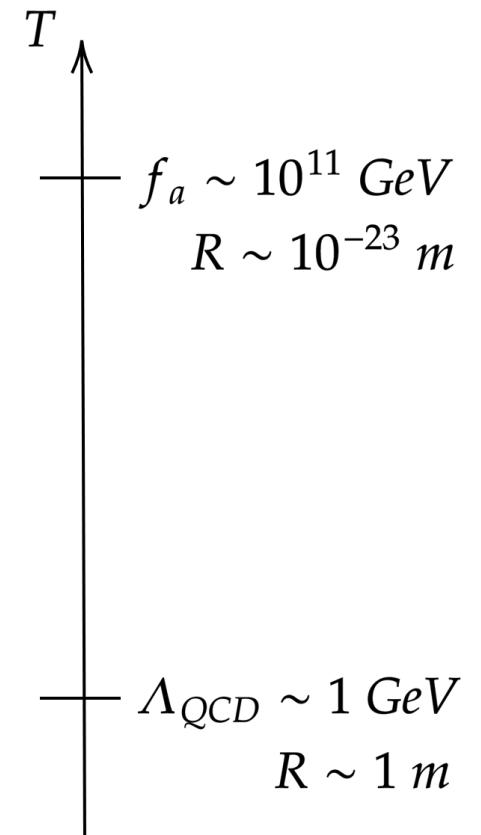
$$\rho_s \propto \xi(t) \quad \xi(t) = \lim_{L \rightarrow \infty} \frac{\ell(t)t^2}{L^3}$$

- String network collapses at the QCD crossover
- **Physical** values unrealistic to simulate:

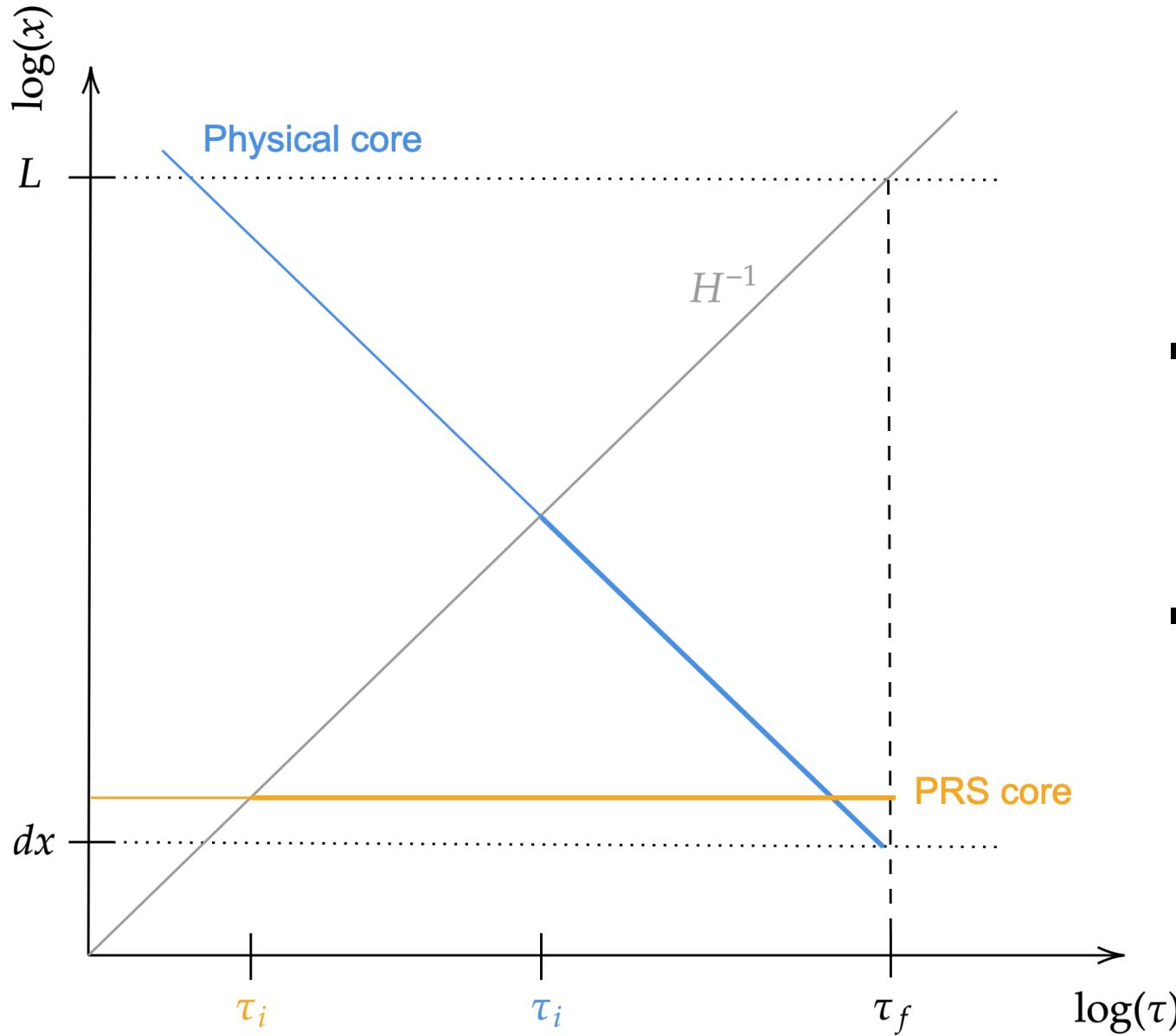
$$\log \left(\frac{m_s}{H_{QCD}} \right) \approx 70$$

vs.

$$\log \sim 6$$



Axion string simulations



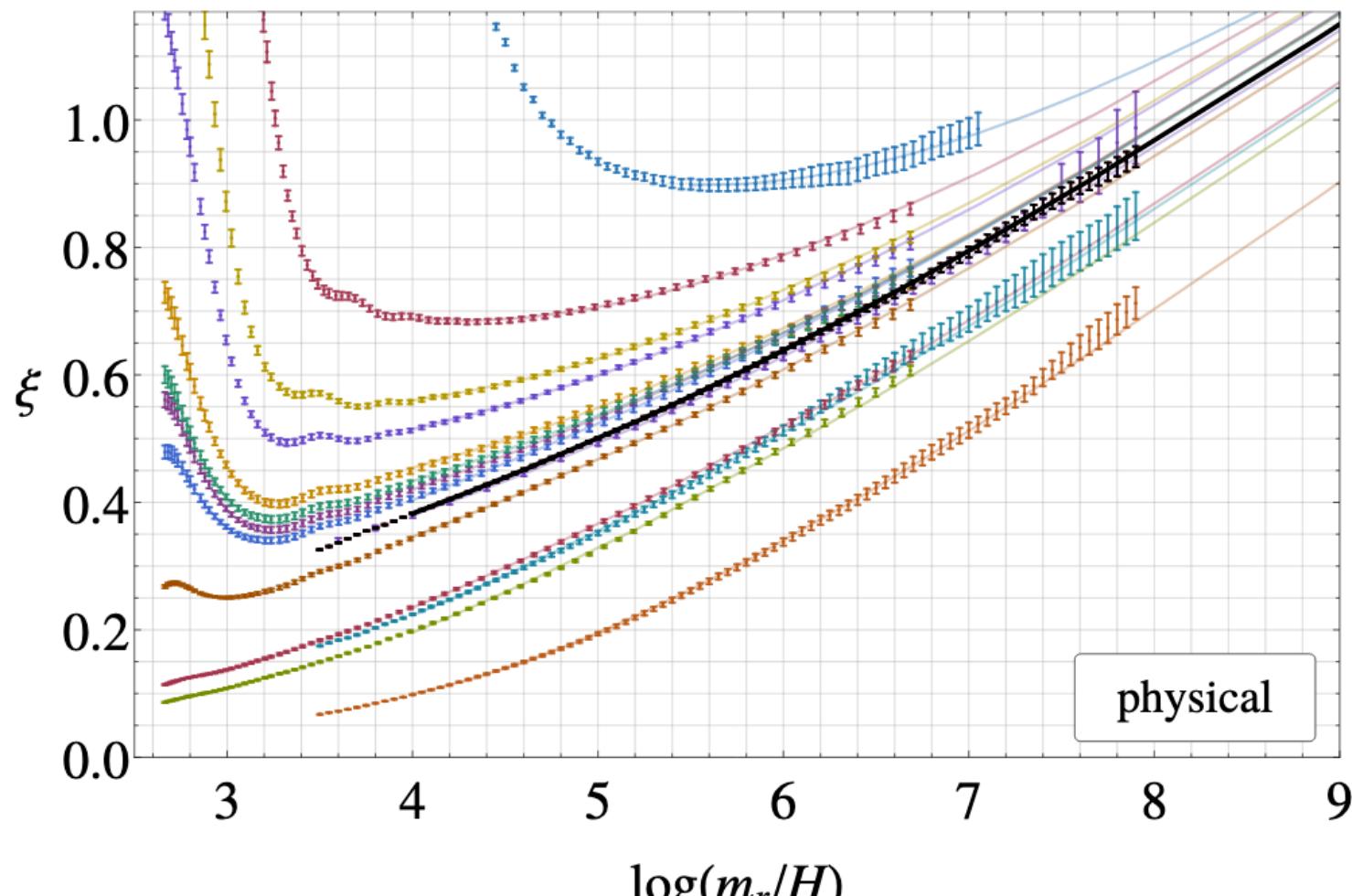
Physical vs 'fat' simulations

- Keep physical core constant
 $m_s = \text{const.}$
- Keep conformal core constant via PRS trick

$$m_s(t) = m_s \frac{\tau_0}{\tau}$$

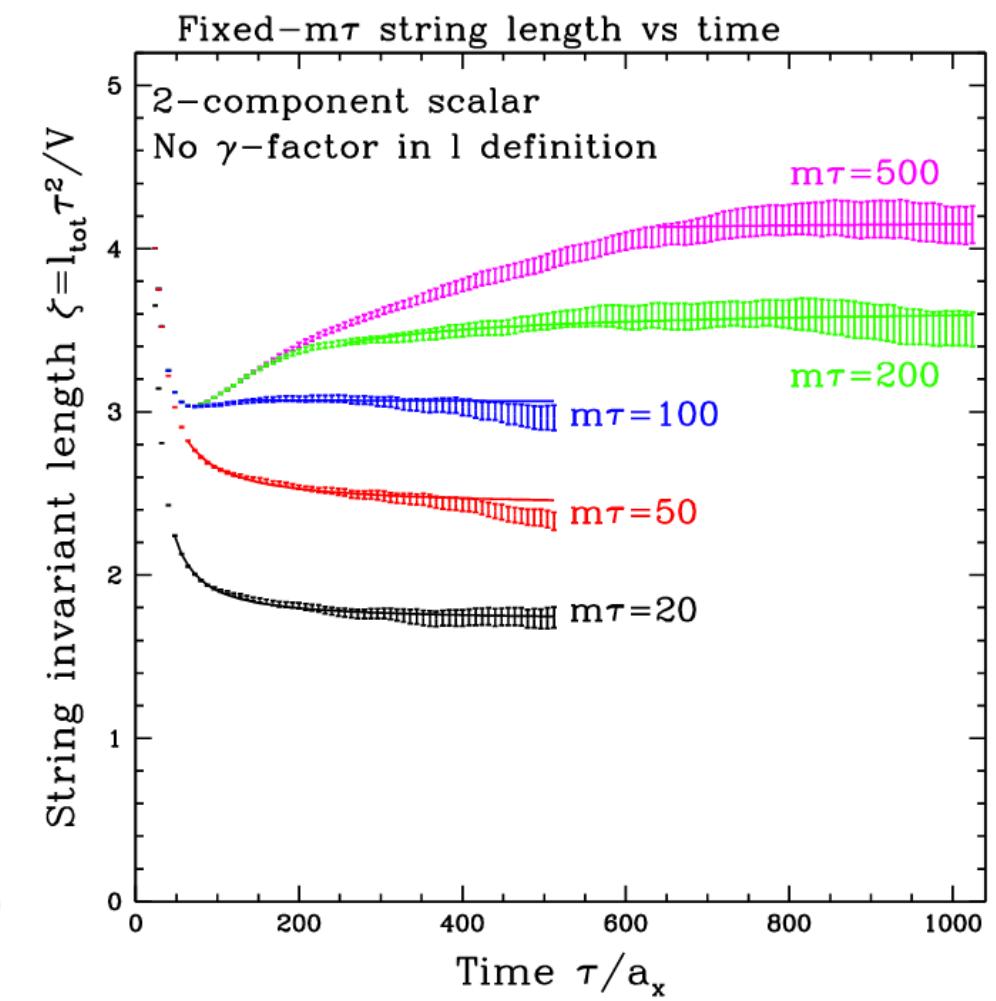
Attractor vs scaling

Gorghetto et al. [2007.04990]



Physical simulation

Klaer, Moore [1912.08058]



Fixed tension simulation

Axion radiation

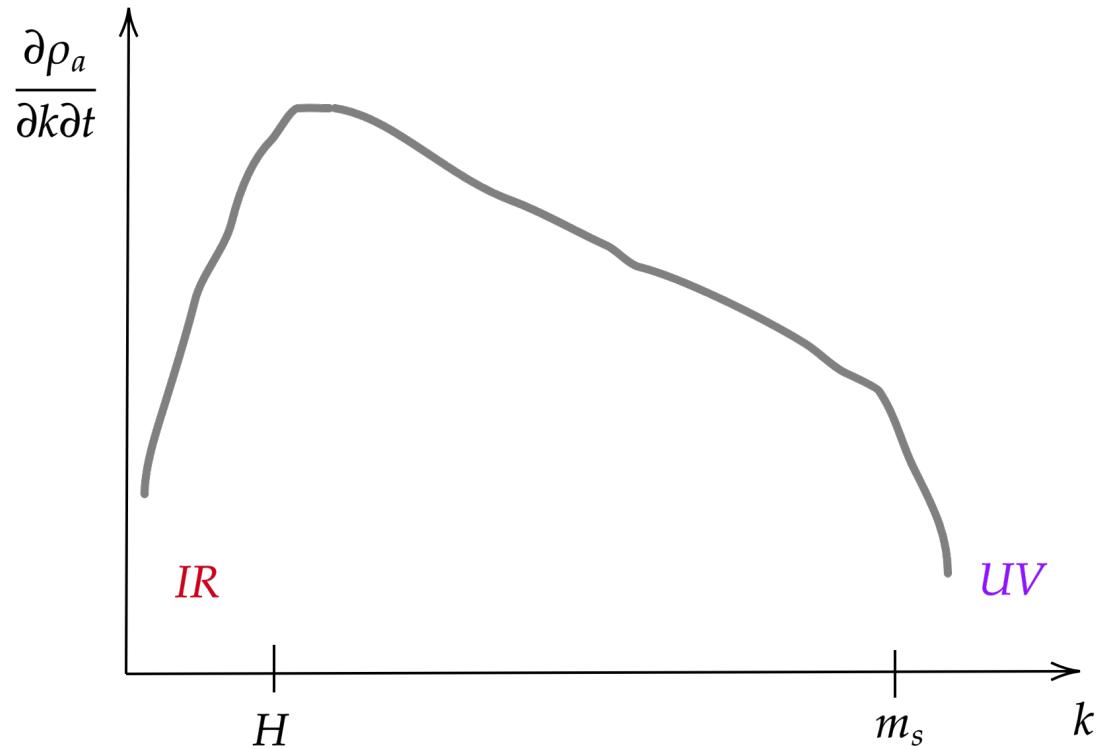
- ❖ Energy spectrum of axions emitted by strings
- ❖ Two scales imposing cutoffs in **IR** and **UV**

$$\frac{\partial \rho_a}{\partial k \partial t}$$

Axion radiation

- ❖ Energy spectrum of axions emitted by strings
- ❖ Two scales imposing cutoffs in **IR** and **UV**
- ❖ Parametrize with a spectral index

$$\frac{\partial \rho_a}{\partial k \partial t} \sim \frac{C}{k^q}$$



Axion radiation

- ❖ Energy spectrum of axions emitted by strings

$$\frac{\partial \rho_a}{\partial k \partial t} \sim \frac{C}{k^q}$$

- ❖ Two scales imposing cutoffs in **IR** and **UV**

- ❖ Parametrize with a spectral index

$$n_a = \int \frac{dk}{\omega_k} \frac{\partial \rho_a}{\partial k} = \int dk \frac{C}{k^{q+1}}$$

Case A: IR domination

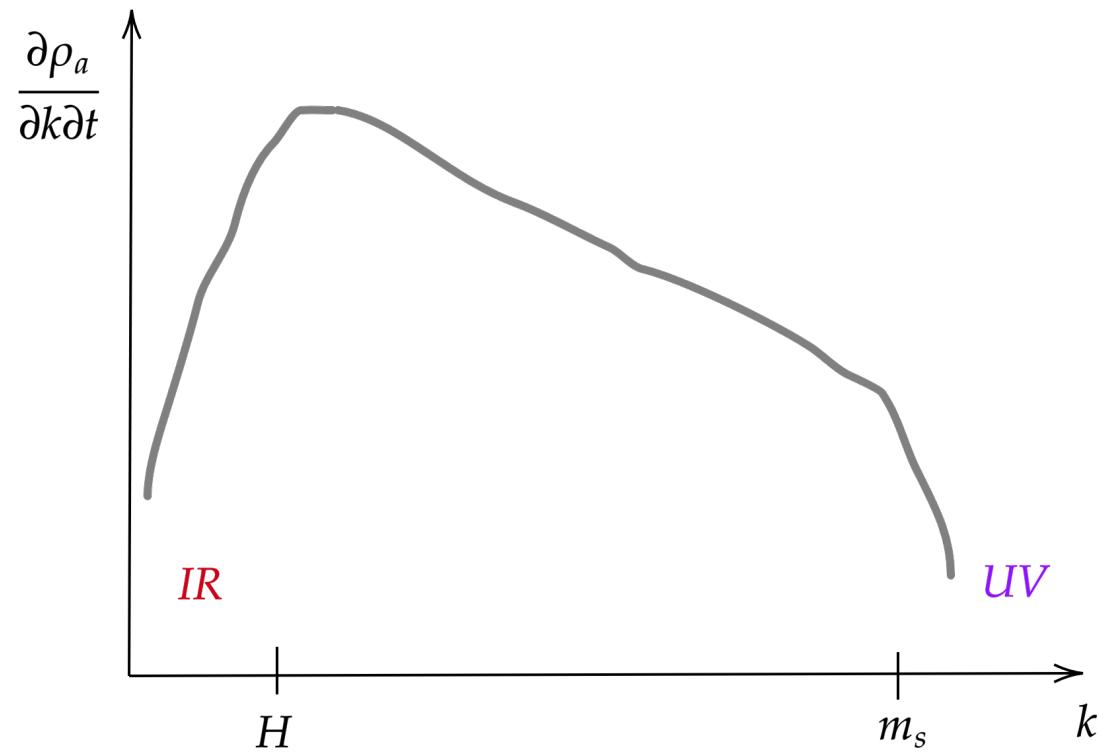
$$q > 1$$

Soft axions, large
number density

Case B: UV domination

$$q < 1$$

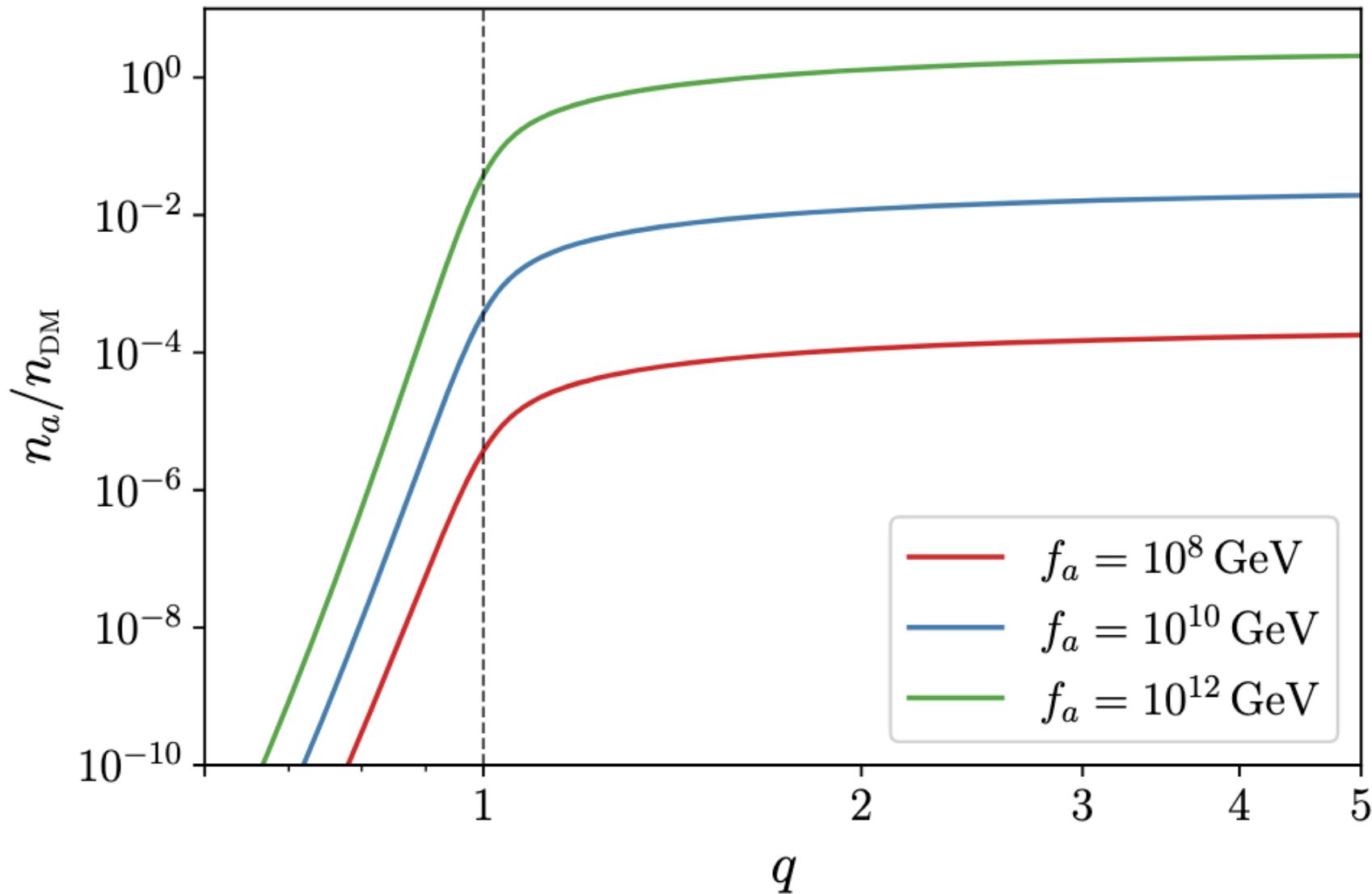
Hard axions, small
number density



- Expect a logarithmic growth on the spectral index

$$q = q(\log)$$

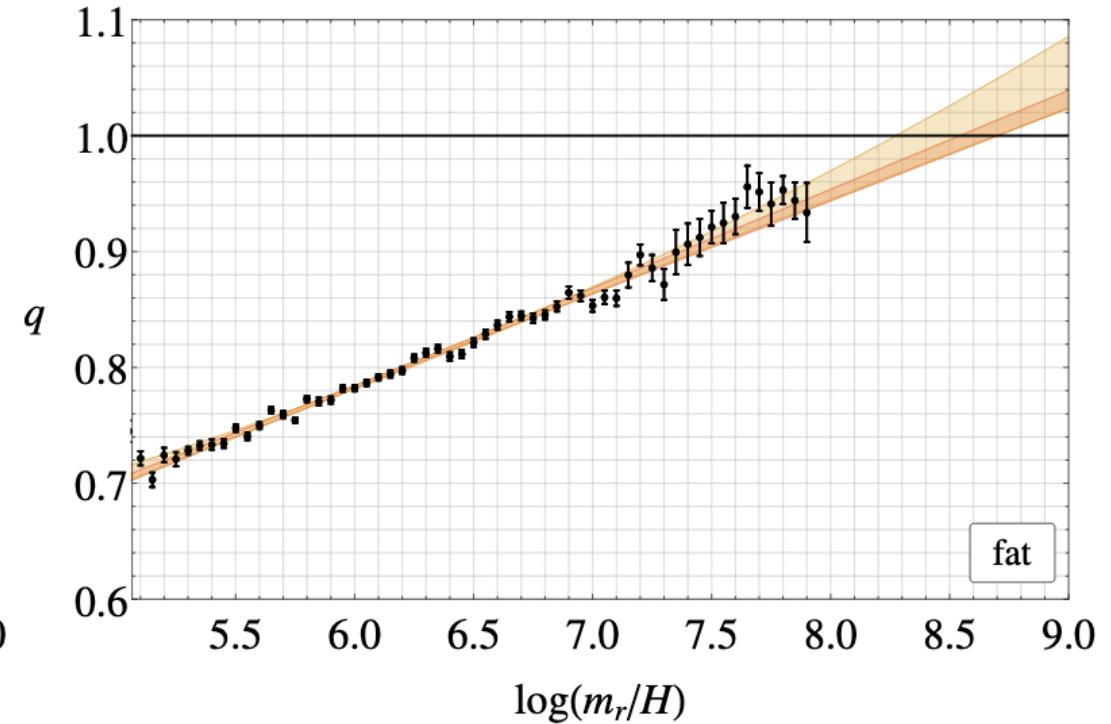
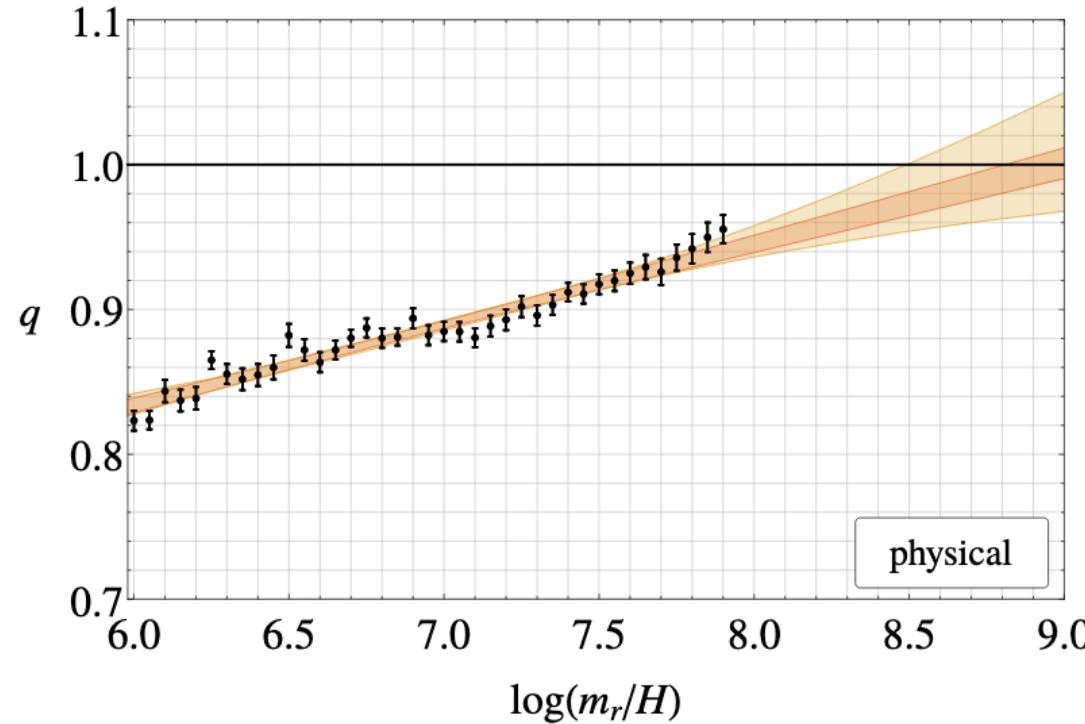
- Most of DM axions are produced at last Hubble time
- Spectral index at QCDPT determines number density of DM axions



Case $q > 1$

1. Evidence of running in the spectral index, linear extrapolation could give reasonable result

Gorghetto et al. [2007.04990]

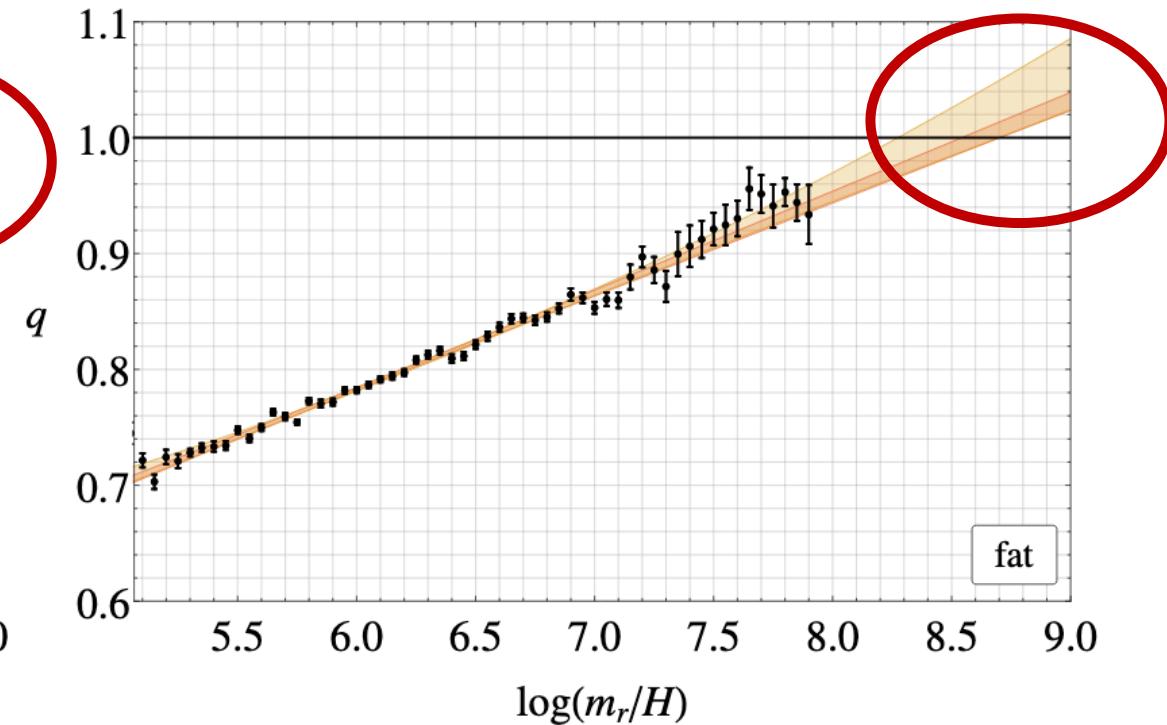
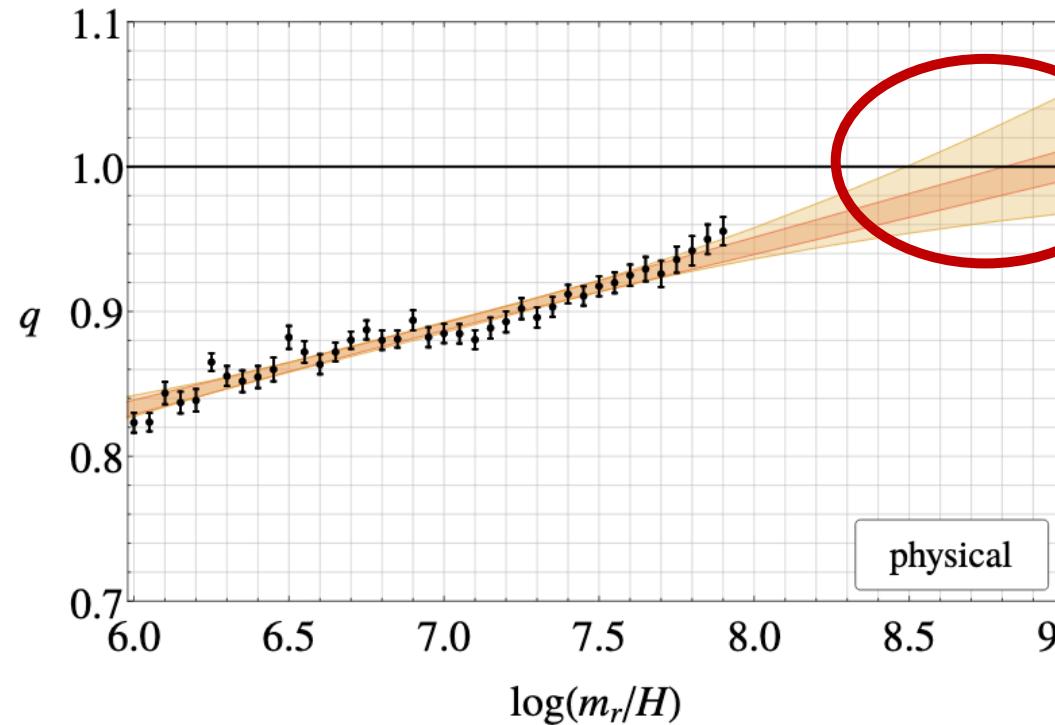


2. Axion strings approach the ‘thin string’ limit (Nambu-Goto strings)
Agreement with recent work by Drew and Shellard [1910.01718]

Case $q > 1$

1. Evidence of running in the spectral index, linear extrapolation could give reasonable result

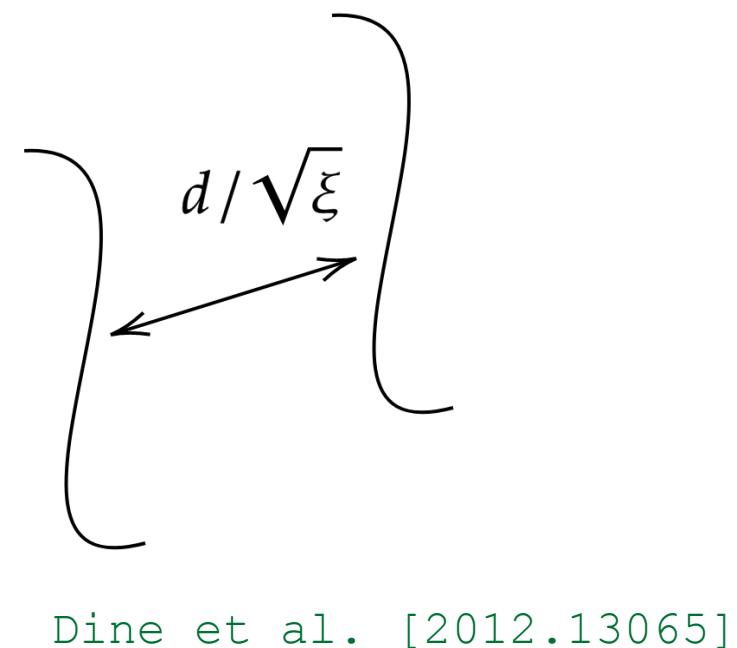
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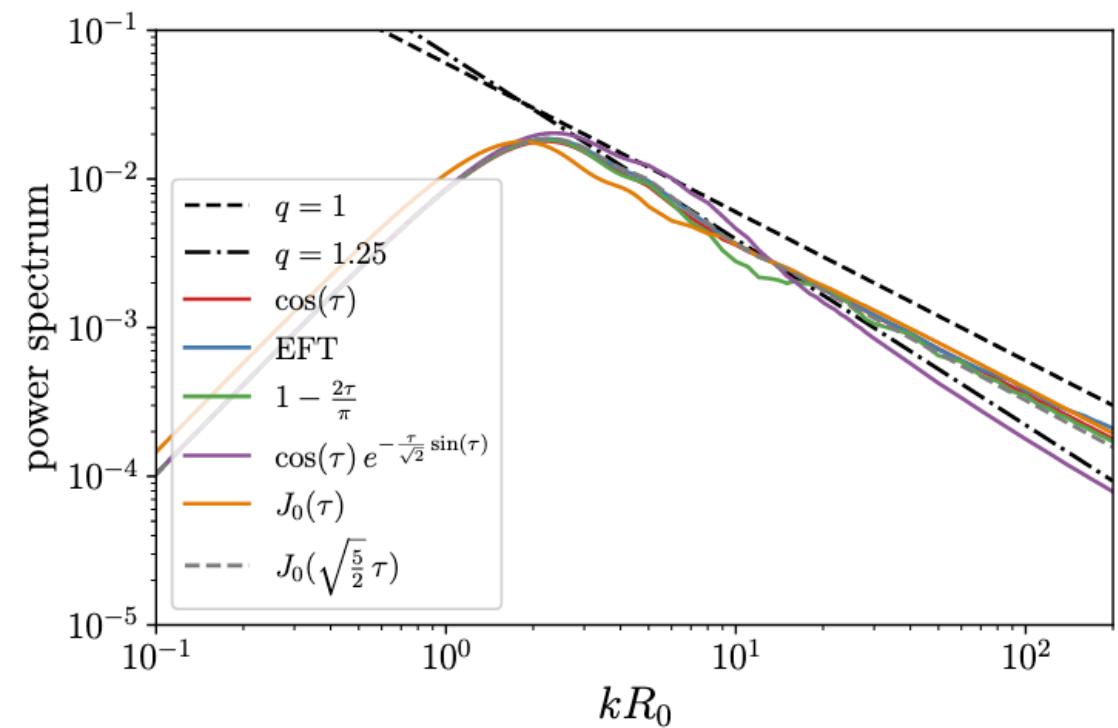
2. Axion strings approach the ‘thin string’ limit (Nambu-Goto strings)
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Case $q \lesssim 1$

- String tension is IR divergent and requires careful definition
- Enhancement in string density does not necessarily lead to enhanced axion number density
- IR cutoff suppresses low momentum axions
- Numerical evidence for circular strings with $q \approx 1$
- Only field simulations of string network is reliable



Dine et al. [2012.13065]



How to extend simulation range?

- ❖ Adaptive mesh refinement (AMR) might be the answer

- ❖ Adaptive space step

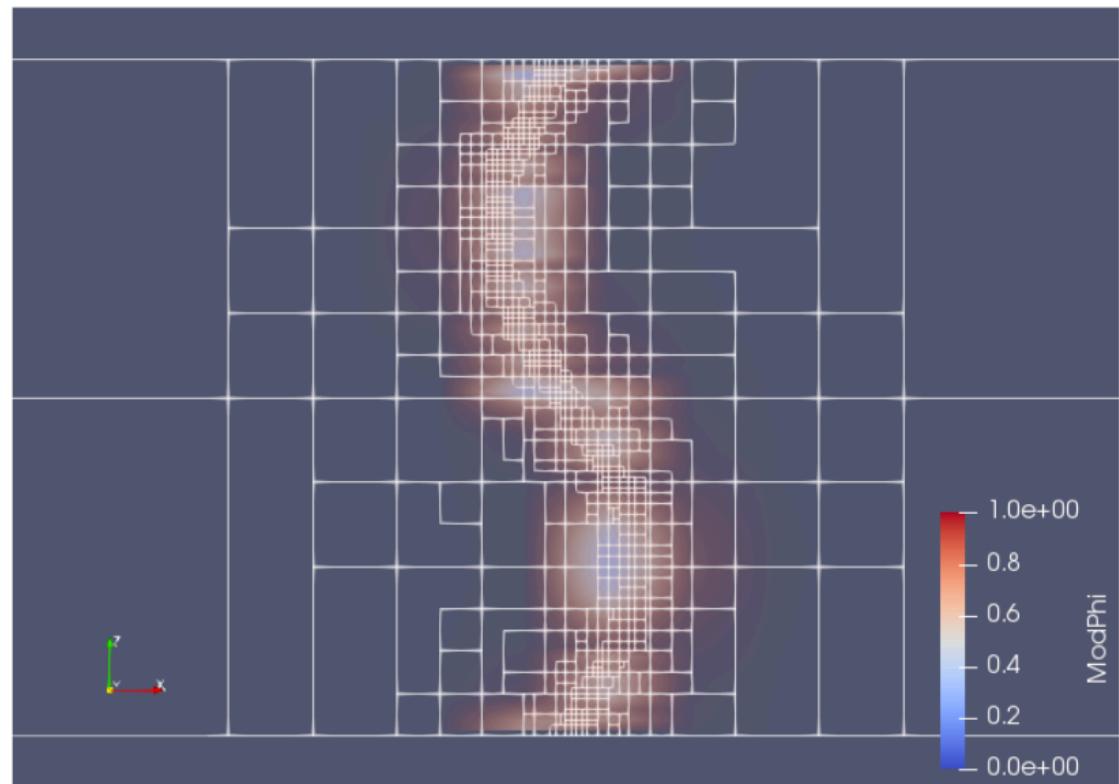
$$\Delta x \sqrt{(\nabla \phi_1)^2 + (\nabla \phi_2)^2} > |\phi_{\text{thr}}|$$

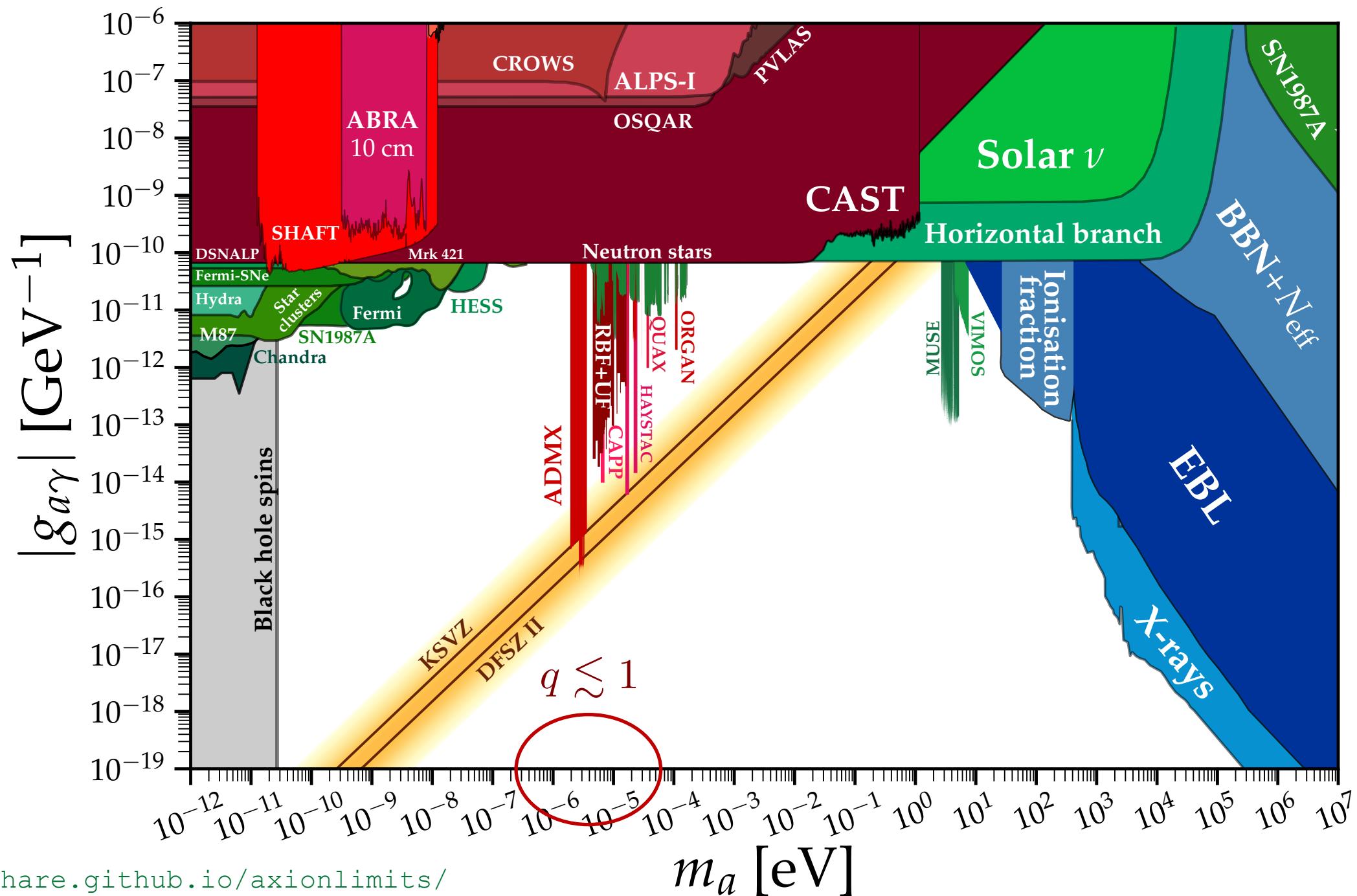
- ❖ Reach up to $\log \approx 14$

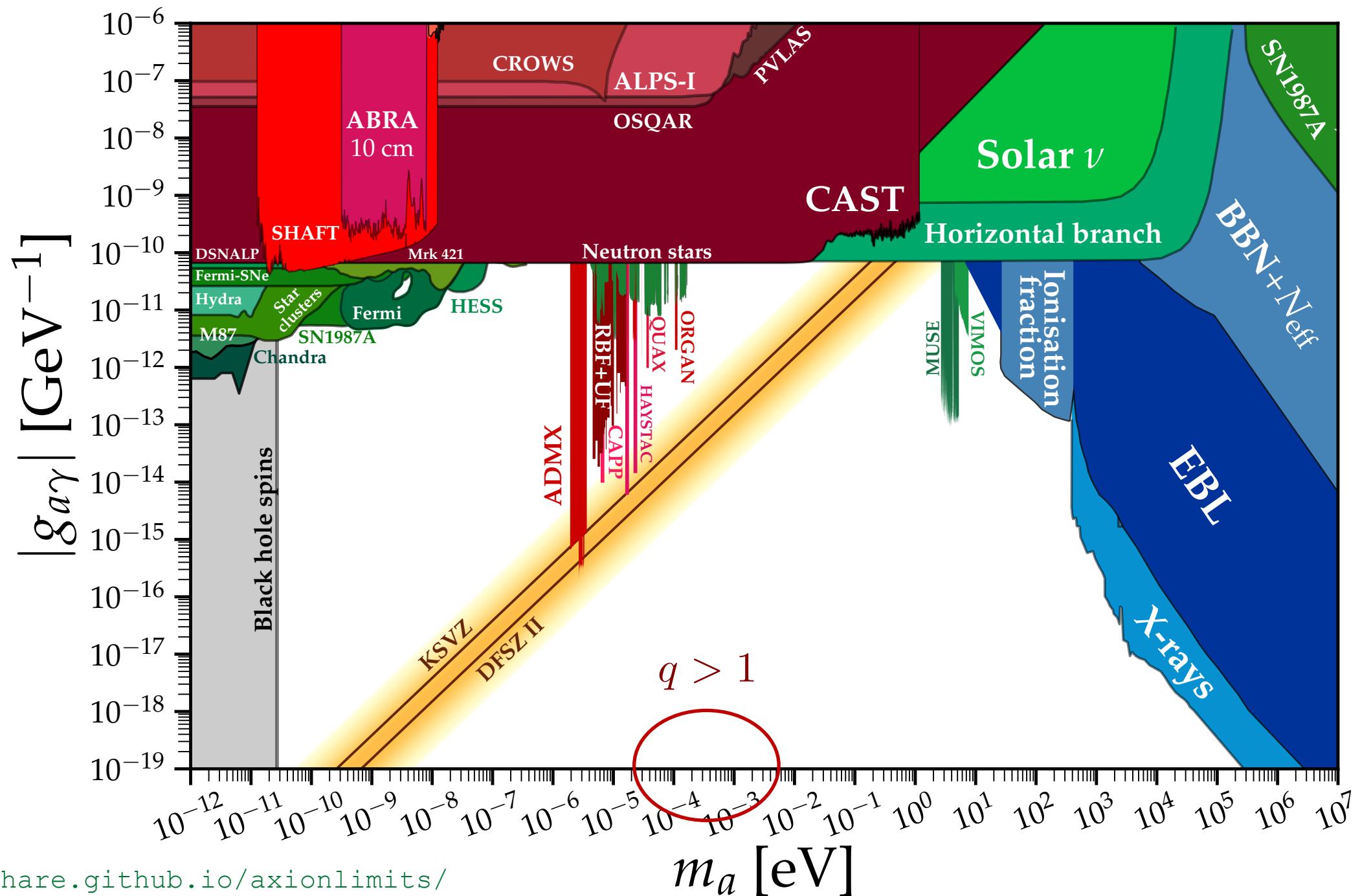
- ❖ Drew and Shellard implemented for a single string

- ❖ *Extend to axion string network*

Drew, Shellard [1910.01718]







Thank you!