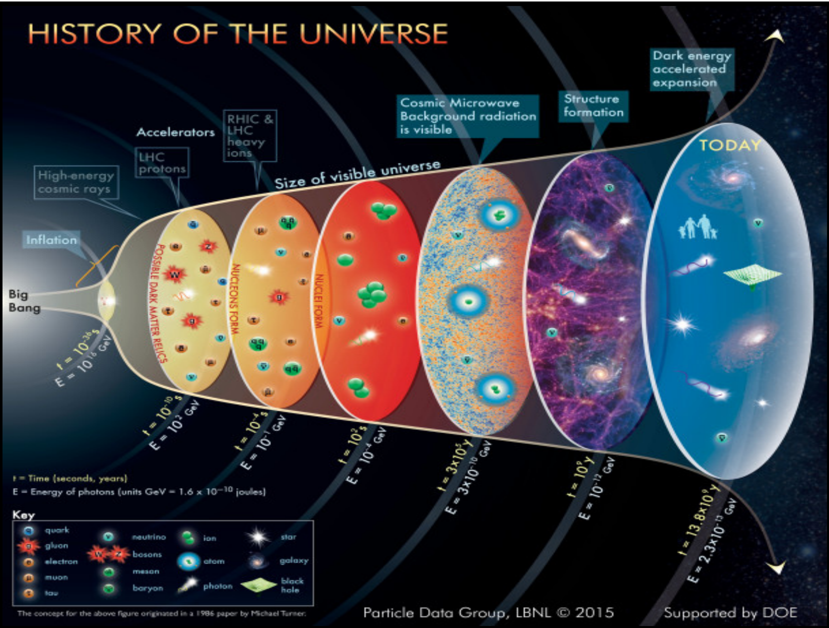
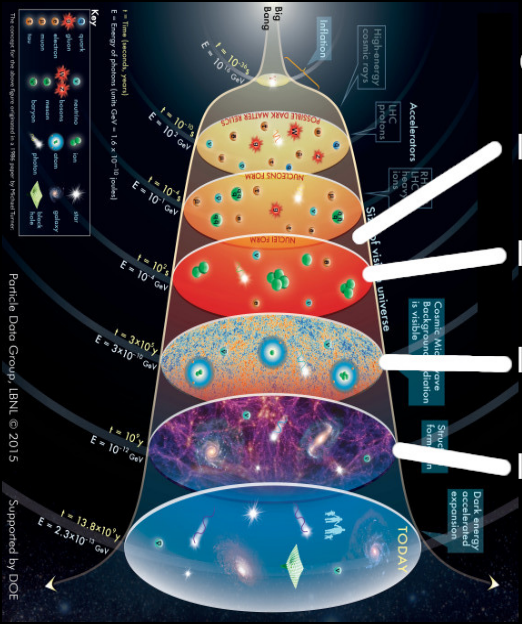
Amol Upadhye, UNSW-Sydney, February 19, 2021

History of the universe



History of the cosmic neutrinos



Outline

I. Neutrino decoupling and effective number

II. Neutrinos and big bang nucleosynthesis

III. Neutrinos and the cosmic microwave background

IV. Neutrinos and large-scale structure

Thermodynamics of the early universe

The number density, energy density, pressure, and entropy density of a particle of mass m, multiplicity g, chemical potential $\mu = 0$, and distribution function $f(\vec{x}, \vec{p}) = f(E)$ are:

$$n = \frac{g}{V} \int_V d^3x \int \frac{d^3p}{(2\pi)^3} f(\vec{x}, \vec{p})$$

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$$\rho = \frac{g}{2\pi^2} \int_m^\infty dE \sqrt{E^2 - m^2}\, E^2 f(E)$$

$$P = \frac{g}{2\pi^2} \int_m^\infty dE \sqrt{E^2 - m^2}\, E \frac{p^2}{3E} f(E) = \frac{g}{6\pi^2} \int_m^\infty dE (E^2 - m^2)^{3/2} f(E)$$

$$s = \frac{\rho + P}{T}$$

These assume *local thermal equilibrium:* rapid scattering maximizes entropy.

Mukhanov, *Physical foundations of cosmology* (2005)

Thermodynamics in the relativistic limit $T \gg m$

	Bosons	Fermions
n	$\frac{\zeta(3)}{\pi^2} g T^3$	$\frac{3\zeta(3)}{4\pi^2} g T^3 = \frac{3}{4} n_{\rm B}$
ρ	$\frac{\pi^2}{30} g T^4$	$\frac{7\pi^2}{240} g T^4 = \frac{7}{8} \rho_{\rm B}$
P	$\frac{1}{3} \rho_{\rm B}$	$\frac{1}{3} \rho_{\rm F} = \frac{7}{8} P_{\rm B}$
s	$\frac{4}{3} \frac{\rho_{\rm B}}{T}$	$\frac{4}{3} \frac{\rho_{\rm F}}{T} = \frac{7}{8} s_{\rm B}$

Decoupling temperature

- At temperatures $T \ll M_W$, we may use the Fermi theory as an effective theory for weak interactions.
- By dimensional analysis, the interaction rate of neutrinos with electrons is $\Gamma_\nu \sim G_{\mathrm{F}}^2 T^5$.
- When this drops below the Hubble rate $H \sim T^2/M_{\mathrm{Pl}}$, neutrinos decouple: $T \sim (G_{\mathrm{F}}^2 M_{\mathrm{Pl}})^{-1/3} \approx 1.4$ MeV. (Keeping track of numerical factors raises this a bit.)

e^+e^- annihilation

Electron-positron annihilation occurs at $T \approx 0.5$ MeV. Since neutrinos have already decoupled, the entropy of electrons, positrons, and photons must be conserved through this annihilation process.

$$
\begin{aligned}
2 \times \frac{2\pi^2}{45} T_{\text{after}}^3 &= 2 \times \frac{2\pi^2}{45} T_{\text{before}}^3 + 4 \times \frac{7}{8} \times \frac{2\pi^2}{45} T_{\text{before}}^3 \\
\Rightarrow T_{\text{after}}^3 &= T_{\text{before}}^3 + \frac{7}{4} T_{\text{before}}^3 \\
\Rightarrow \frac{T_{\text{before}}}{T_{\text{after}}} &= \left(\frac{4}{11}\right)^{1/3}
\end{aligned}
$$

To leading order, neutrinos remain at T_{before} while photons are heated to T_{after}, so $T_\nu / T_\gamma = (4/11)^{1/3}$.

Effective number of neutrinos

After electron-positron annihilation, the energy density of relativistic particles (photons and neutrinos) is described by

$$\rho_{\mathrm{r}} = \left[1 + \frac{7}{8}\left(\frac{4}{11}\right)^{\frac{4}{3}} N_{\mathrm{eff}}\right]\rho_\gamma \quad \text{where} \quad \rho_\gamma = \frac{\pi^2}{15} T_\gamma^4$$

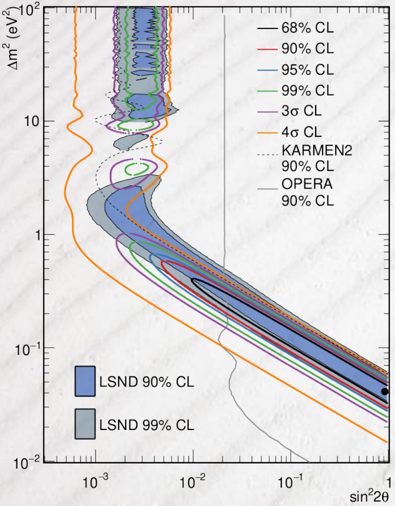
and N_{eff} is 3 to leading order.

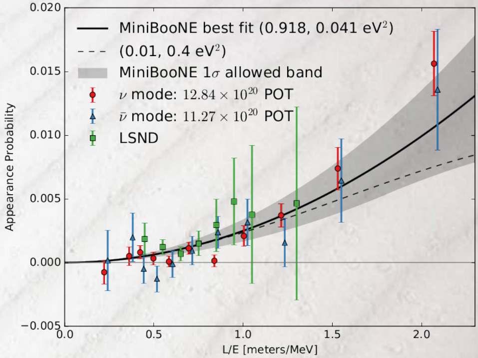
- 1.4 MeV not much greater than 0.5 MeV
  $\Rightarrow$ neutrinos not fully decoupled by e^+e^- annihilation
- Energy leakage from e^+e^- distorts ν distribution function
- Approximate this as a correction to N_{eff}. Accounting for oscillations, finite-temperature QED effects, etc, $N_{\mathrm{eff}} = 3.044$.
- Planck 2018 constraints: $N_{\mathrm{eff}} = 2.92^{+0.36}_{-0.37}$ (95% CL)

Bennett, Buldgen, Drewes, Wong, JCAP **03***:003(2020)[1911.04504]*

Bennett, et al. [2012.02726]

Could there be a fourth "sterile" neutrino?



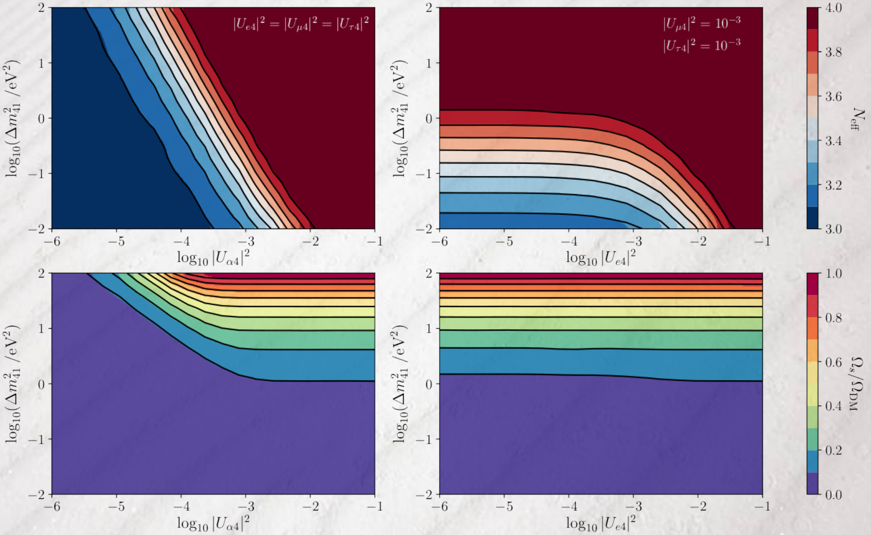


Standard Model neutrinos:

$$\Delta m^2_{21} = 7.55 \times 10^{-5}\ \text{eV}^2,$$
$$|\Delta m^2_{31}| = 2.5 \times 10^{-3}\ \text{eV}^2$$

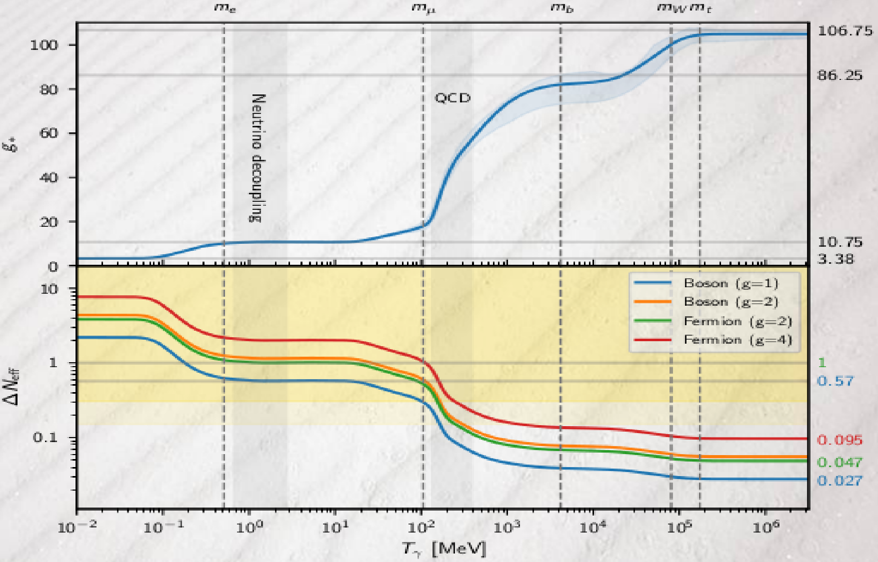
Aguilar-Arevalo, et al., PRL **121***:221801(2018)[1805.12028]*

Tensions with bound $N_{\rm eff} = 2.92^{+0.36}_{-0.37}$

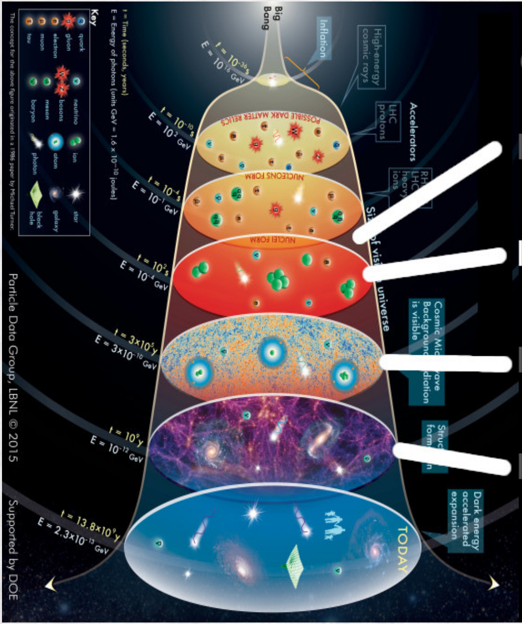


$sin^2(2\vartheta) = 4|U^2_{e4}||U^2_{\mu4}| \quad (\gtrsim 10^{-3})$ (Hagstotz, *et al.*, 2003.02289)

N_{eff} constraints bound all relativistic species



Aghanim, et al., A&A **641***:A6(2020)[1807.06209]* (Planck 2018)



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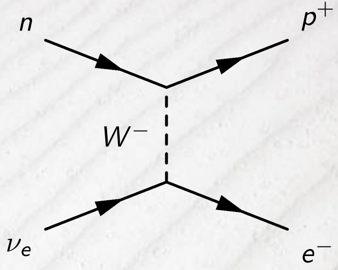
Big bang nucleosynthesis: overview

Key question: What is the mass fraction of ^{4}He?
$X_{^4\mathrm{He}} =$ mass of ^{4}He / total baryonic mass

- ~ 100 MeV: QCD phase transition, nucleons form
- ~ 1 MeV: neutron "freeze-out" ($n + \nu_e \rightleftharpoons p^+ + e^-$ and $n + e^+ \rightleftharpoons p^+ + \bar{\nu}$ out of equilibrium)
- $\gtrsim 0.1$ MeV: neutrons decay with lifetime $\tau_\mathrm{n} = 886$ sec
- ~ 0.1 MeV: photons cool enough for deuterium to form, most remaining neutrons end up in ^{4}He

Freeze-out occurs when $\Gamma_{n\to p}$ drops below H.
Neutrinos raise H, hence X_n at freeze-out.

Nucleon scattering



At $T \ll M_W$, use the Fermi EFT:

$$|\mathcal{M}|^2 = 16(1+3g_A^2)G_{\mathrm{F}}^2(p_n \cdot p_{\nu_e})(p_p \cdot p_{e^-})$$

$$\frac{d\sigma}{d\Omega} = \frac{|\mathcal{M}|^2}{(8\pi)^2(p_n + p_{\nu_e})^2}\left[\frac{(p_p \cdot p_{e^-})^2 - m_p^2 m_e^2}{(p_n \cdot p_{\nu_e})^2 - m_n^2 m_{\nu_e}^2}\right]^{1/2}$$

Neutron freeze-out

For $m_n,\, m_p \gg T \sim p_e \sim p_n$, we integrate over $\vec{p}_{\nu_e}$ to find the rate

$$\Gamma_{n\nu} \approx 1.07 \times 10^{-21}\ \mathrm{MeV} \times \left(\frac{T}{Q}\right)^3 \left(\frac{T}{Q} + 0.25\right)^2$$

where $Q = m_n - m_p = 1.293$ MeV. The rate for $n + e^+ \rightleftharpoons p^+ + \bar{\nu}_e$ is similar.

Freeze-out occurs when $\Gamma_{n\to p} \approx 2\Gamma_{n\nu}$ is just barely fast enough to keep X_n equal to its equilibrium value $X_n^{(\mathrm{eq})} = (1 + \exp(Q/T))^{-1}$:

$$-2\Gamma_{n\nu} = \frac{\dot{X}_{\mathrm{n}}^{(\mathrm{eq})}}{X_n^{(\mathrm{eq})}} = -\frac{e^{Q/T}}{1 + e^{Q/T}}\frac{Q}{T}H \quad \text{where} \quad H = \sqrt{\frac{\kappa}{3}}\frac{T^2}{M_{\mathrm{Pl}}}$$

$$\Rightarrow 0.184\kappa^{1/2} = \left(\frac{T}{Q}\right)^2 \left(\frac{T}{Q} + 0.25\right)^2 \left(1 + e^{-Q/T}\right)$$

Mukhanov, *Physical foundations of cosmology* (2005)

Neutrinos and neutron freeze-out

The Hubble expansion $H = \sqrt{\kappa/3}\, T^2/M_{\rm Pl}$ depends on

$$\kappa = \frac{\pi^2}{30}\left(g_{\rm B} + \frac{7}{8}g_{\rm F}\right) = \frac{\pi^2}{30}\left(2 + \frac{7}{8}(4 + 2N_{\rm eff})\right).$$

Given κ, we may solve $y^2(y+0.25)^2(1+e^{-1/y}) = 0.184\sqrt{\kappa}$ iteratively for $y = T/Q$.

- For $N_{\rm eff} = 3$ the temperature and mass fraction at freeze-out are $T_* = 0.800$ MeV and $X_{n*} = 0.1656$.
- For $N_{\rm eff} = 4$, $T_* = 0.816$ MeV and $X_{n*} = 0.1701$.

more ν $\Rightarrow$ higher H $\Rightarrow$ earlier freeze-out $\Rightarrow$ more neutrons

Helium-4 constraints

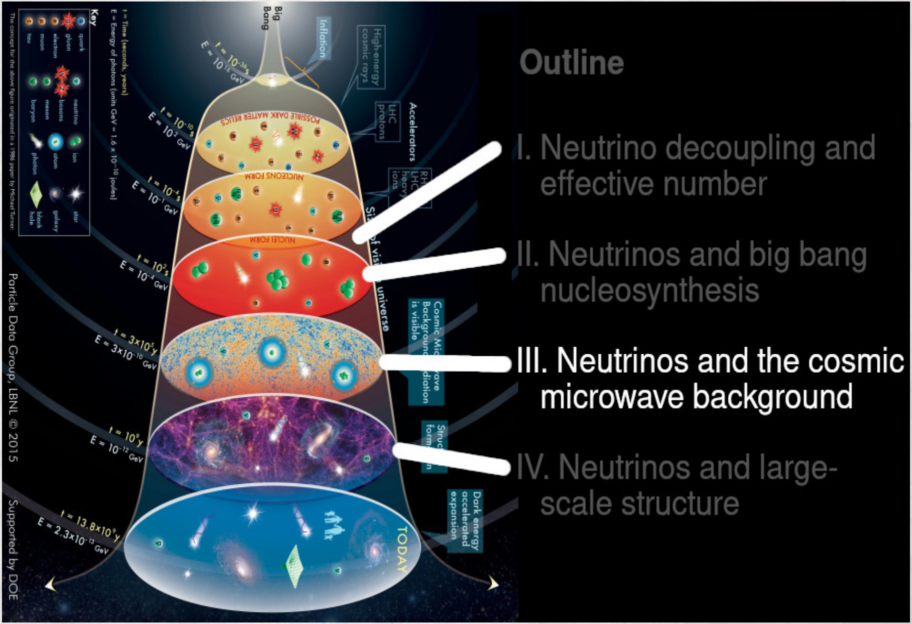
Deuterium, the first step in ^{4}He production, has a binding energy $B_{\mathrm{D}} = 2.23$ MeV. The fraction of photons above this energy, $e^{-B_{\mathrm{D}}/T}$, does not drop below the baryon-to-photon ratio η until $T_{\mathrm{D}} \sim 0.1$ MeV.

More accurate calculation: $T_{\mathrm{D}} = 0.07$ MeV, time $t_{\mathrm{D}} = 269$ sec, depends a bit on N_{eff} and η.

$$
\begin{aligned}
X_{^4\mathrm{He}} &= 2X_{n*}\exp(-t_{\mathrm{D}}/\tau_n) \\
&\approx 0.23 + 0.012(N_{\mathrm{eff}} - 3) + 0.005\log(10^{10}\eta) \\
\eta &= \Omega_{\mathrm{b0}}h^2 \times 2.72 \times 10^{-8} \approx 6 \times 10^{-10}
\end{aligned}
$$

Measurement $X_{^4\mathrm{He}} = 0.250 \pm 0.014$ implies $2.7 \leq N_{\mathrm{eff}} \leq 5.1$.

Cyburt, Fields, Olive, Yeh, Rev. Mod. Phys. **88***:015004(2016)[1505.01076]*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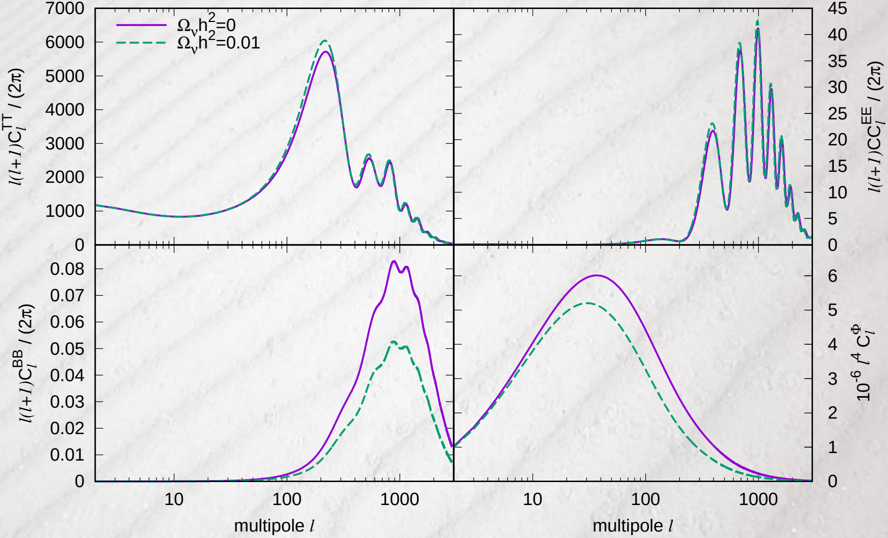
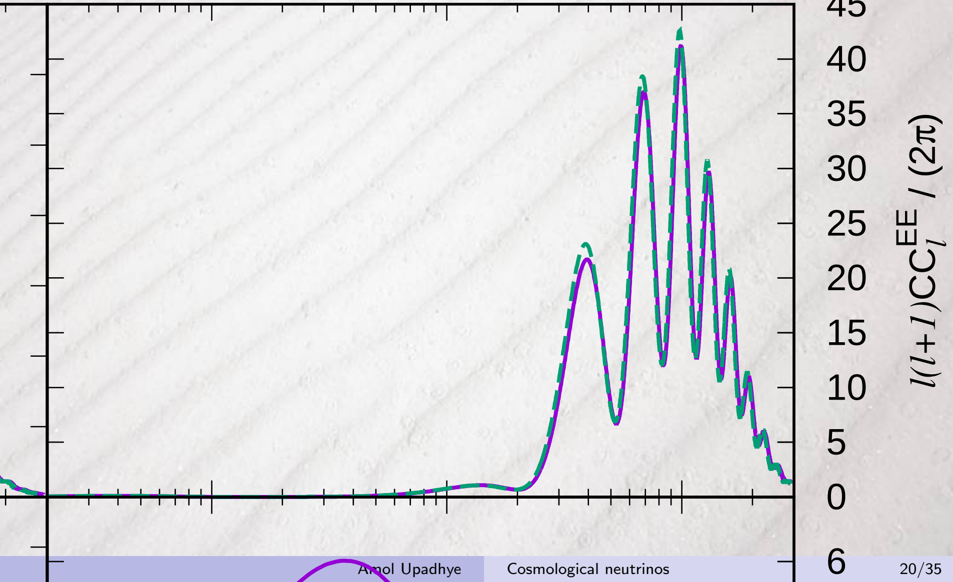
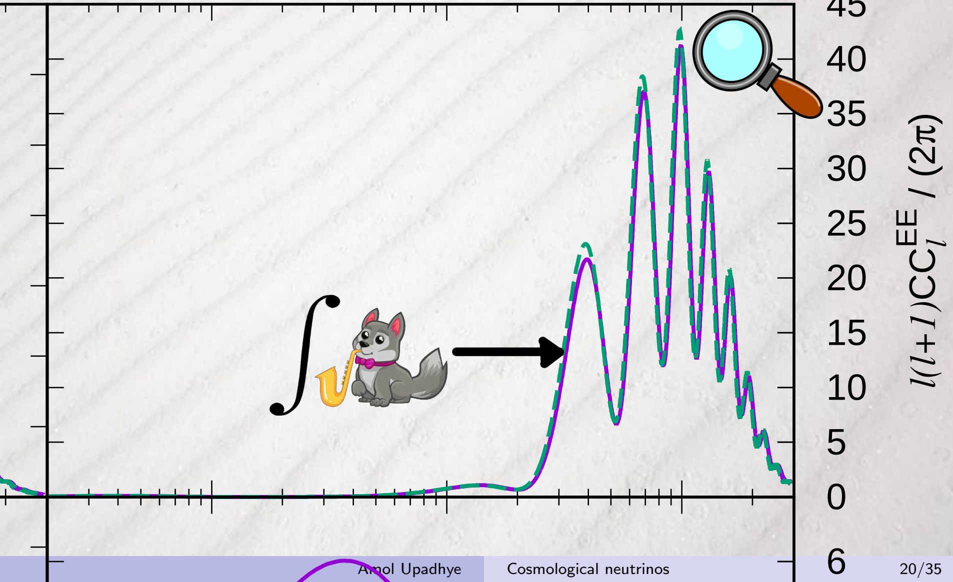
Big Bang
Inflation
High-energy cosmic rays
Accelerators
LHC protons
Cosmic Microwave Background Radiation is visible
Dark energy accelerated expansion
TODAY
POSSIBLE DARK MATTER RELICS
NUCLEONS FORM
NUCLEI FORM
Particle Data Group, LBNL © 2015
Supported by DOE

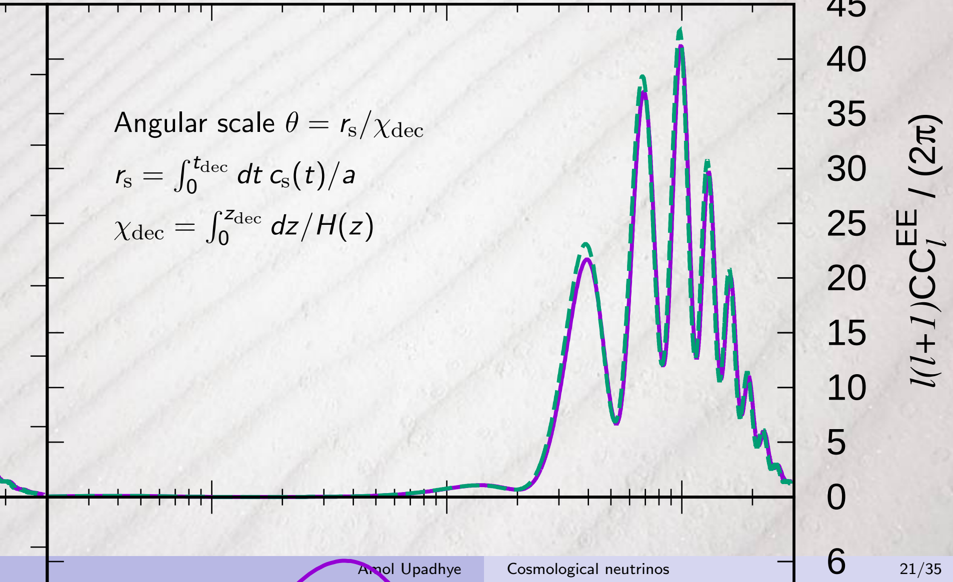
Overview of neutrino effects on CMB



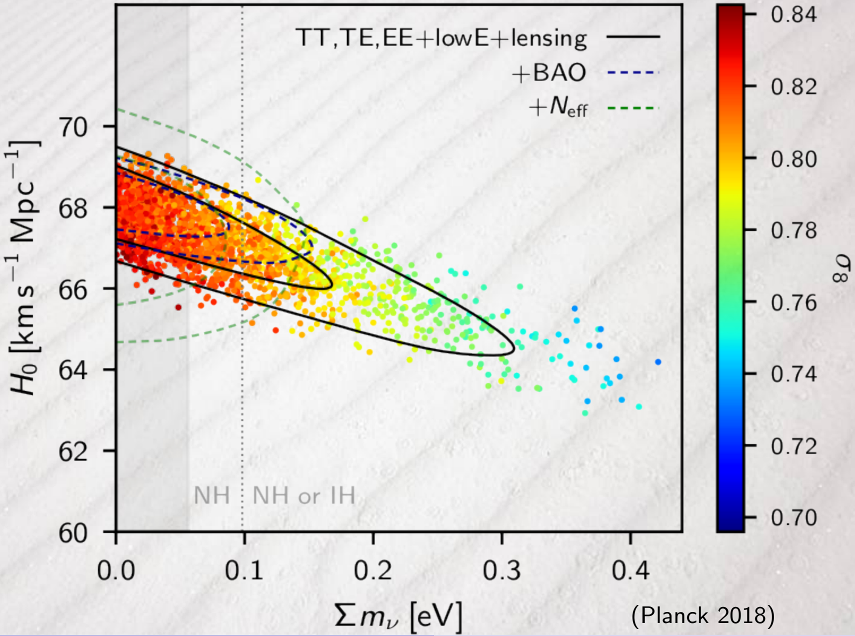
Neutrinos and the TT/EE power spectra




45
40
35
30
25
20
15
10
5
0
6
$l(l+1)CC_l^{EE} / (2\pi)$

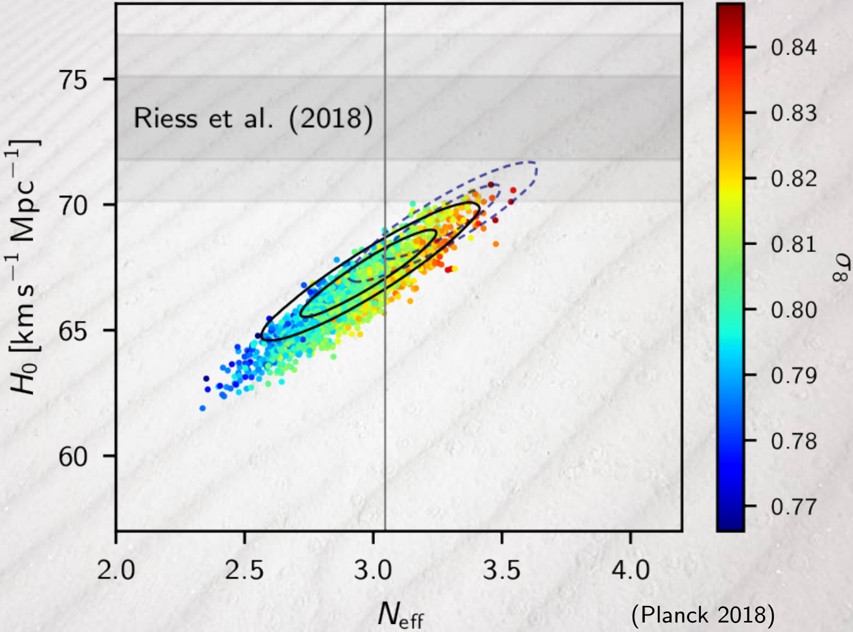

Angular scale $\theta = r_s/\chi_{dec}$
$r_s = \int_0^{t_{dec}} dt\, c_s(t)/a$
$\chi_{dec} = \int_0^{z_{dec}} dz/H(z)$
45
40
35
30
25
20
15
10
5
0
6
$l(l+1)CC_l^{EE} / (2\pi)$

Massive neutrinos shift the CMB peaks



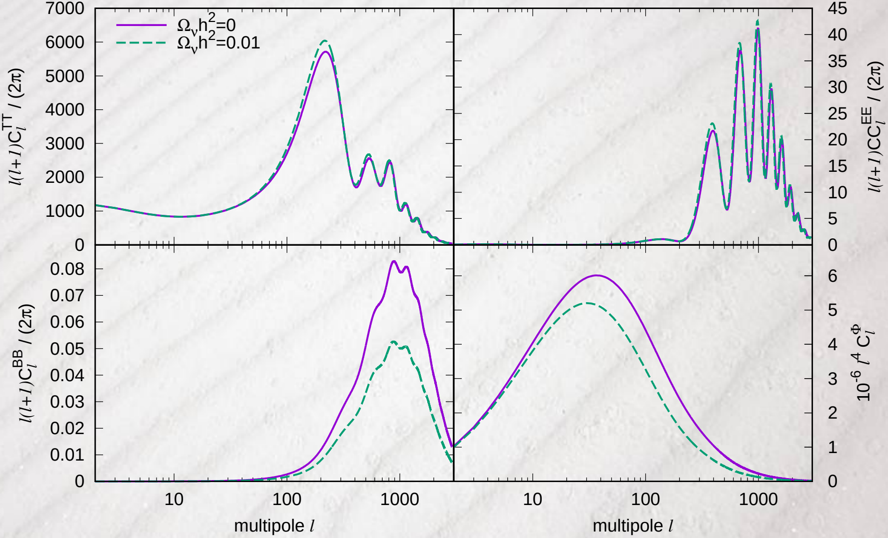
(Planck 2018)

More neutrinos suppress CMB power

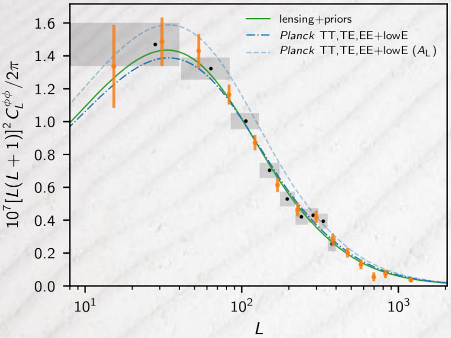


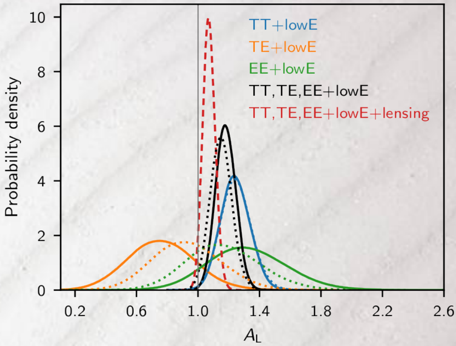
(Planck 2018)

Massive neutrinos reduce lensing



CMB lensing unexpectedly large

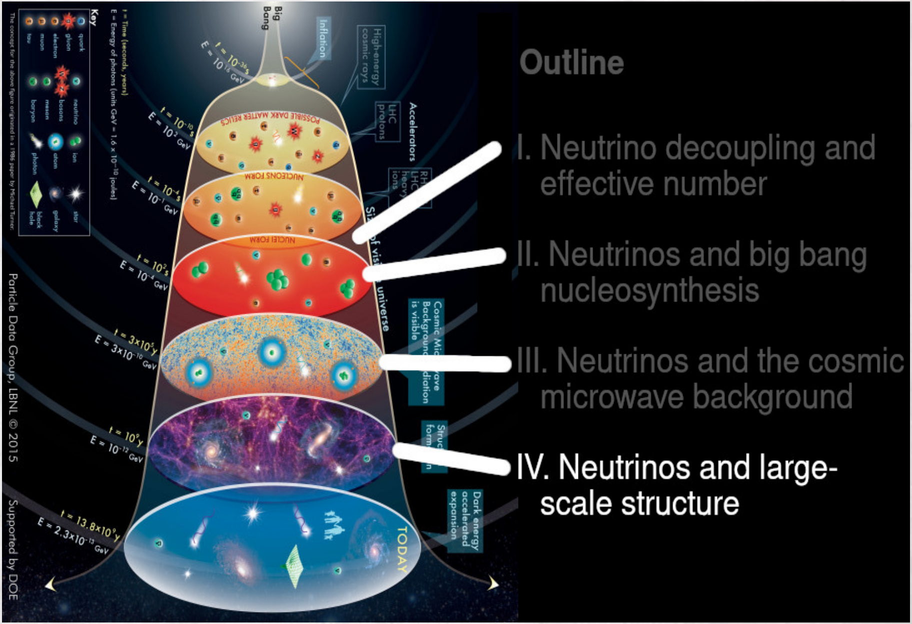




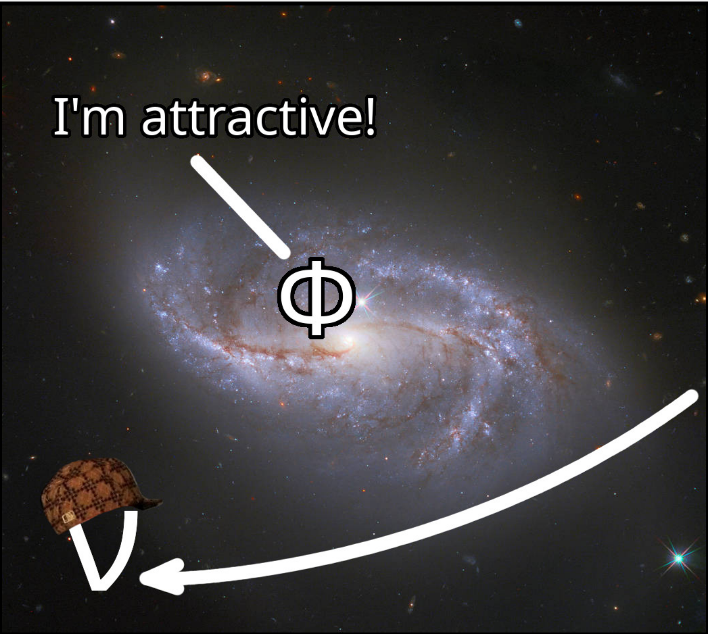
1. Planck sees more lensing than expected given constraints from other power spectra (blue dash-dots)
2. ΛCDM + $\sum m_\nu$ constraints: $\sum m_\nu < 0.24$ eV (95% CL) (without lensing: 0.26 eV or 0.38 eV depending on likelihood)
3. ΛCDM + $N_{\rm eff}$ constraints: $N_{\rm eff} = 2.92^{+0.36}_{-0.37}$ (95% CL)

(Planck 2018)

Part IV: Neutrinos and large-scale structure



Neutrinos cluster weakly on small scales



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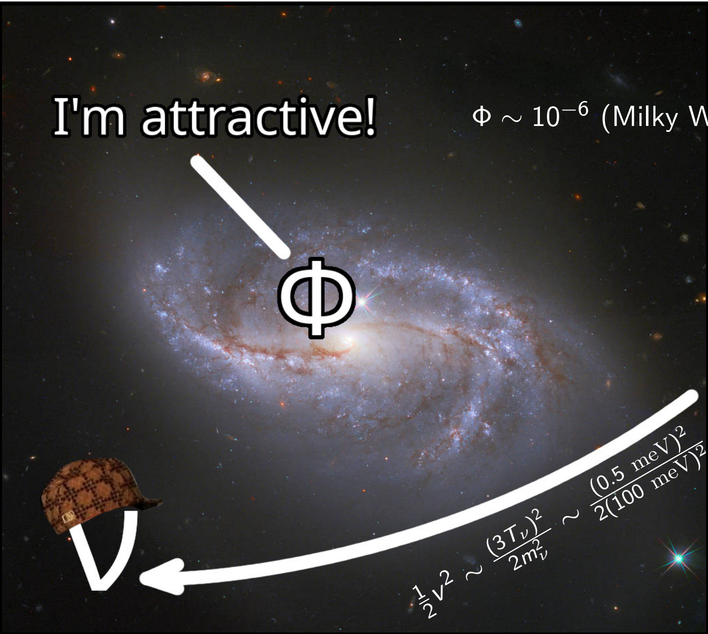

$\Phi \sim 10^{-6}$ (Milky Way)

$$\frac{1}{2}v^2 \sim \frac{(3T_\nu)^2}{2m_\nu^2} \sim \frac{(0.5\ \mathrm{meV})^2}{2(100\ \mathrm{meV})^2} = 10^{-5}$$

Neutrino clustering: limiting cases

Find density contrast $\delta_\nu = \rho_\nu/\bar{\rho}_\nu - 1$ from linearized Boltzmann-Poisson equation:

$$0 = \frac{\partial f}{\partial s} + \frac{i\vec{k}\cdot\vec{p}}{m_\nu} f - i m_\nu a^2 \Phi \vec{k}\cdot\nabla_{\vec{p}}\bar{f} \quad \text{where} \quad ds = dt/a^2$$

$$\Rightarrow \delta_\nu(\vec{k}, s) = -k^2 \int_{s_\mathrm{i}}^{s} ds' a(s')^2 \Phi(\vec{k}, s')(s - s') F\left[\frac{T_{\nu,0} k(s - s')}{m_\nu}\right]$$

$$F(q) = \frac{m_\nu}{\bar{\rho}_\nu} \int \frac{d^3p}{(2\pi)^3} \bar{f}(\vec{p}) e^{-i\vec{q}\cdot\vec{p}/T_{\nu,0}} = \frac{4}{3\zeta(3)} \sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{(n^2 + q^2)^2}$$

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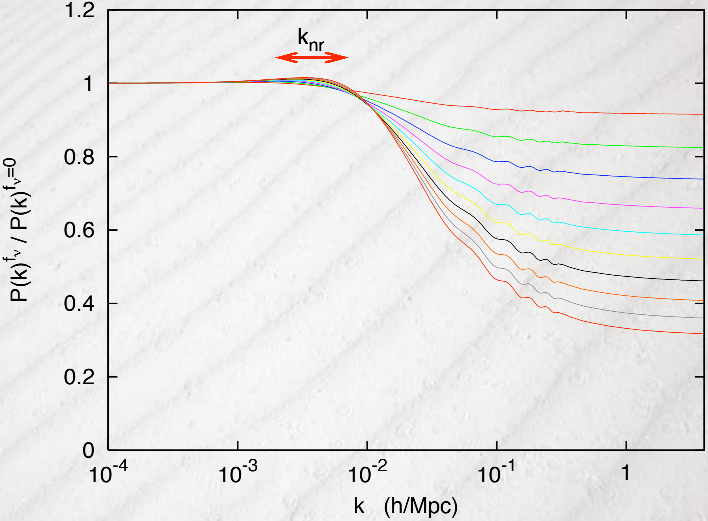
Small-scale (large k) "free-streaming" limit:

$$\delta_\nu = \frac{k_\mathrm{FS}^2}{k^2}\delta_\mathrm{m} \quad \text{where} \quad k_\mathrm{FS}^2 = \frac{3\mathcal{H}^2\Omega_\mathrm{m}(s)}{2c_\nu^2}, \quad c_\nu = \frac{T_\nu(s)}{m_\nu}\sqrt{\frac{3\zeta(3)}{2\ln(2)}}$$

Ringwald,Wong, JCAP **12**:*005(2004)[hep-ph/0408241], Chen,AU,Wong [2011.12504]*

Neutrinos suppress small-scale matter clustering



$\Omega_\nu/\Omega_\mathrm{m} = 0.01, 0.02, \ldots 0.10$

Lesgourgues and Pastor, Adv. High Energy Phys. **2012***:608515(2012)[1212.6154]*

Lesgourgues and Pastor, Phys. Rept. **429***:307(2006)[astro-ph/0603494]*

Neutrinos as fluids?

- Continuity equation: Mass is conserved. A change in density locally must be balanced by an inflow or outflow.

  $\frac{1}{a^3}\frac{\partial(a^3\rho)}{\partial t} + \vec{\nabla}\cdot(\rho\vec{v}) = 0$

- Euler equation: Changes in the velocity of a fluid element are driven by gradients in the gravitational potential.

  $\frac{1}{a}\frac{\partial(a\vec{v})}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v} + \vec{\nabla}\Phi = 0$

Problem: Neutrinos cannot be described as a fluid, since they have a distribution of velocities at each point in spacetime.

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  $$\frac{1}{a^3}\frac{\partial(a^3\rho)}{\partial t} + \vec{\nabla}\cdot(\rho\vec{v}) = 0$$

  $$\Rightarrow \frac{\partial\delta}{\partial t} + \mathcal{H}\theta = -\vec{\nabla}\cdot(\delta\vec{v}) \quad \text{where } \delta = \frac{\delta\rho}{\bar{\rho}},\ \theta = \frac{\vec{\nabla}\cdot\vec{v}}{\mathcal{H}}$$

- Euler equation: Changes in the velocity of a fluid element are driven by gradients in the gravitational potential.

  $$\frac{1}{a}\frac{\partial(a\vec{v})}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v} + \vec{\nabla}\Phi = 0$$

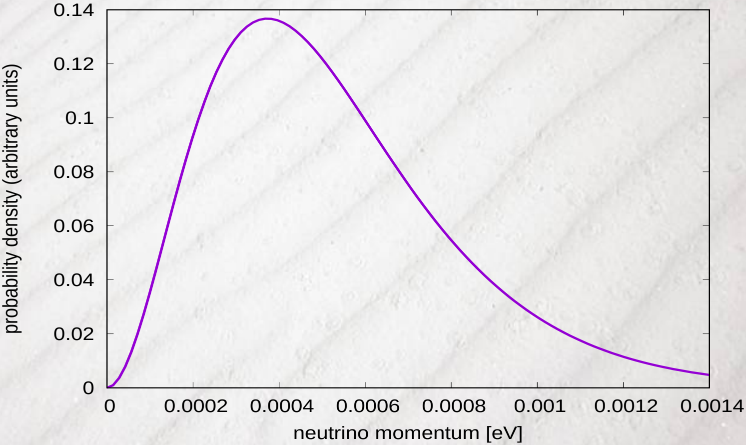
  $$\Rightarrow \frac{\partial}{\partial t}(\mathcal{H}\theta) + \mathcal{H}^2\theta + \frac{3}{2}\mathcal{H}^2\Omega_{\mathrm{m}}(t)\delta = -\vec{\nabla}\cdot[(\vec{v}\cdot\vec{\nabla})\vec{v}]$$

Problem: Neutrinos cannot be described as a fluid, since they have a distribution of velocities at each point in spacetime.

Neutrinos as multiple fluids

Idea: bin that distribution into streams, each of which is a fluid.

Dupuy and Bernardeau, JCAP **1401***:030(2014)*



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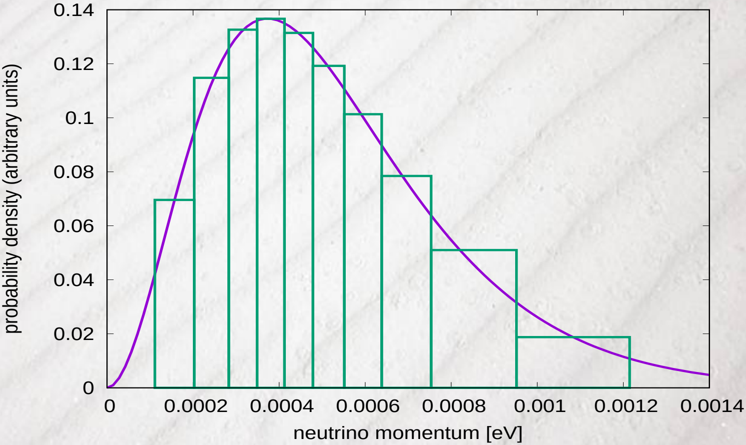
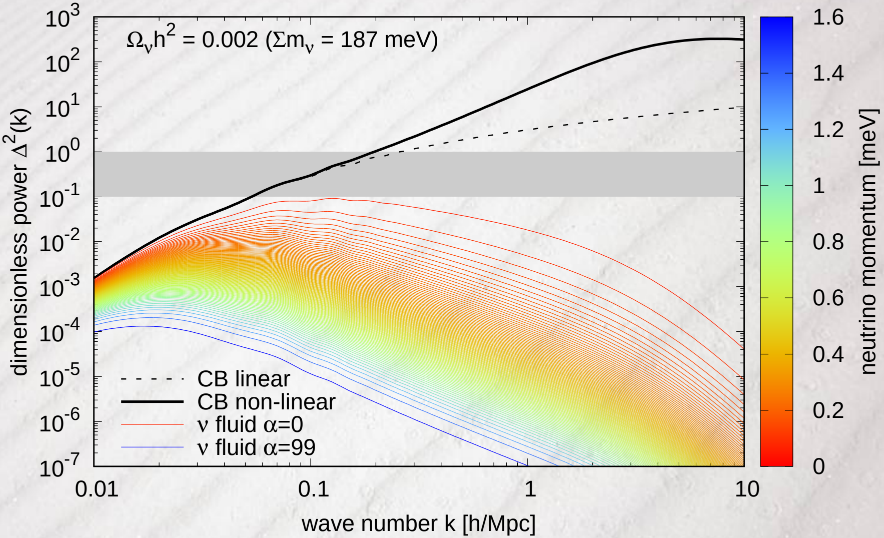
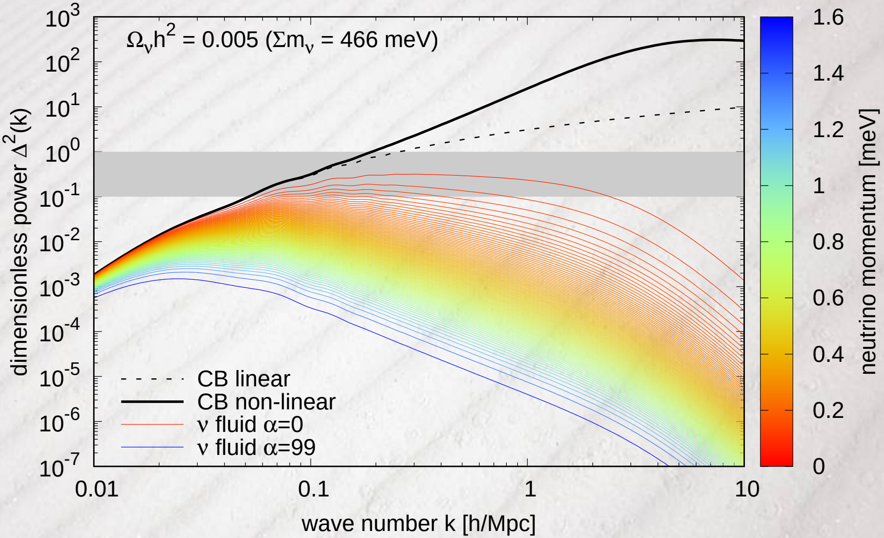


Clustering of neutrino fluids



Chen, AU, and Wong (to appear in JCAP, 2021) [2011.12503] See Joe's talk today!

Clustering of neutrino fluids



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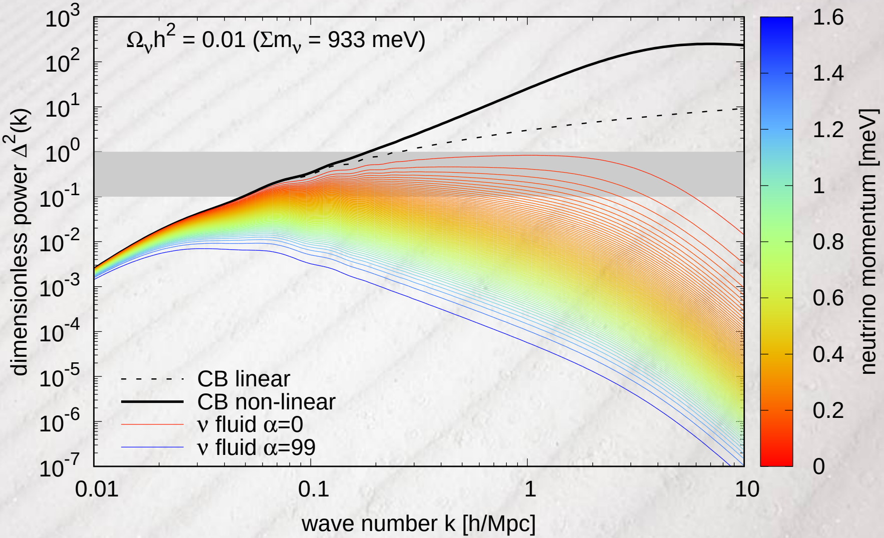
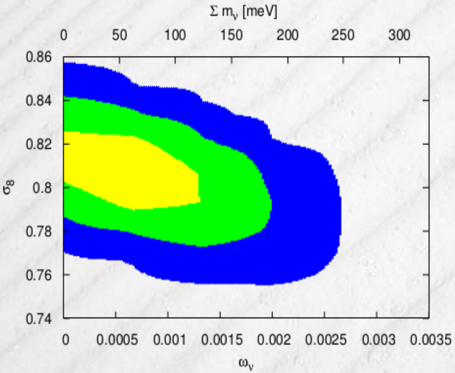
Clustering of neutrino fluids

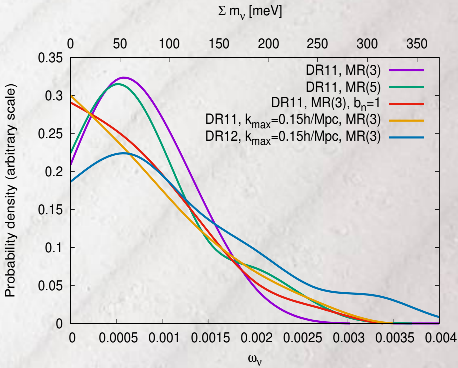


Chen, AU, and Wong (to appear in JCAP, 2021) [2011.12503] See Joe's talk today!

Massive neutrino constraints I

BOSS + Planck data for $\nu\Lambda CDM$ model:
$\sum m_\nu < 0.18$ eV ($\omega_\nu < 0.00197$) (95%CL)   (Planck 2018: 0.12 eV)
(5-parameter bias: $\sum m_\nu < 0.22$ eV, $\omega_\nu < 0.0024$)

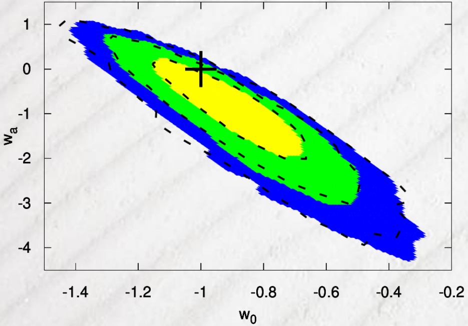


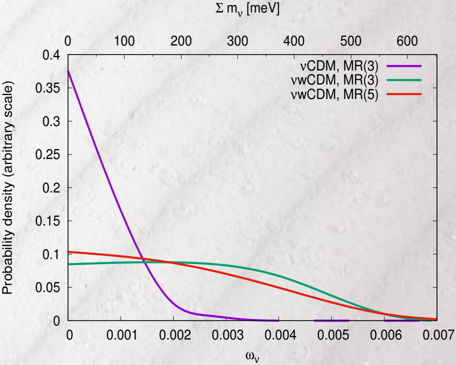


AU, JCAP 1905:041 (2019)[1707.09354]

Massive neutrino constraints II

BOSS + Planck + JLA for νwCDM [$w(z) = w_0 + w_a z/(1+z)$]:
$\sum m_\nu < 0.54$ eV ($\omega_\nu < 0.0058$) (95%CL)
(5-parameter bias: $\sum m_\nu < 0.57$ eV, $\omega_\nu < 0.0061$)





AU, JCAP 1905:041 (2019)[1707.09354]

Conclusions

1. The last two Standard Model parameters, $\sum m_\nu$ and $\delta_{\rm CP}$, are in the neutrino sector
2. Cosmology probes SM ($\sum m_\nu$) and new physics ($N_{\rm eff}$).
3. ^{4}He mass fraction provides a historically important $N_{\rm eff}$ bound.
4. Neutrinos alter the magnitudes (lensing, early ISW, early H) and angular scale (late H) of the CMB.
5. Neutrino free-streaming suppresses growth of matter density on small scales.
6. Combined constraints: $\sum m_\nu < 0.12$ eV. Cosmology will measure $\sum m_\nu$ soon!